The existence and uniqueness of invariant measure, a way of measuring probability that does not change in time, is an important problem both mathematically and in practice. For example, the atmospheric sciences use invariant measures to calculate correlation coefficients and other physical quantities. The support of the measure, i.e. what is given measure/probability, is also an important property of a measure. While previous works considered this problem on flat domains, we consider it in the more physically relevant case of the 2-dimensional sphere, which is a simplified model for the atmosphere.

We show the existence and uniqueness of an invariant measure for the kick-forced Navier-Stokes system on the 2-dimensional sphere, first without deterministic force and then with a time-independent deterministic force. The existence and uniqueness of an invariant measure for the white noise forced Navier-Stokes system on the 2-dimensional sphere without a deterministic forcing is also shown.

We examine the support of the invariant measure and give a description of the support of the measure in general, and in several special cases, for the kick-forced flow. The support of the invariant measure for the white noise forced equations is shown to be the entire space of admissible vector fields of the sphere.