Geometry, Technology, and the Reasoning and Proof Standard with the Geometer's Sketchpad®

Óscar Chávez
University of Missouri
chavezo@missouri.edu

December 5, 2003

> "When I started as a graduate student at Berkeley [...]
> I didn't really understand what a 'proof' was."
>
> William Thurston

Geometry Standard

Instructional programs from prekindergarten through grade 12 should enable all students to—

- — Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- — Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- — Apply transformations and use symmetry to analyze mathematical situations
- — Use visualization, spatial reasoning, and geometric modeling to solve problems.

Grades 9-12 Expectations (some of them)

In grades 9-12 all students should—

- — analyze properties and determine attributes of two- and three-dimensional objects;
- — explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;

— establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;

Reasoning and Proof Standard

Instructional programs from prekindergarten through grade 12 should enable all students to—

— Recognize reasoning and proof as fundamental aspects of mathematics

— Make and investigate mathematical conjectures

— Develop and evaluate mathematical arguments and proofs

— Select and use various types of reasoning and methods of proof

Students in grades 3–5 should frequently make conjectures about mathematical relationships, investigate those conjectures, and make mathematical arguments that are based on their work. They need to know that posing conjectures and trying to justify them is an expected part of students' mathematical activity.
NCTM (2000)

The particular format of a mathematical justification or proof, be it narrative argument, "two-column proof," or a visual argument, is less important than a clear and correct communication of mathematical ideas appropriate to the students' grade level.
NCTM (2000)

Dynamic Geometry Principles:

— Student Use: Construction, Model Making, Creative Exploration

— Instructor Use: Demonstration/Presentation, Productivity, Research

— Generalization and Induction through Dynamic Manipulation

— Focus on Mathematical "Behavior" as well as "Properties"

Sketchpad and Geometry

Elementary Level:

— Basic Shapes (Definitions & Properties)

— Symmetries and Transformations

— Triangle Geometry & Theorems

Secondary Level:

— Full Compass-and-Straightedge Toolbox

— Locus Constructions

— Analytic & Metric Geometry

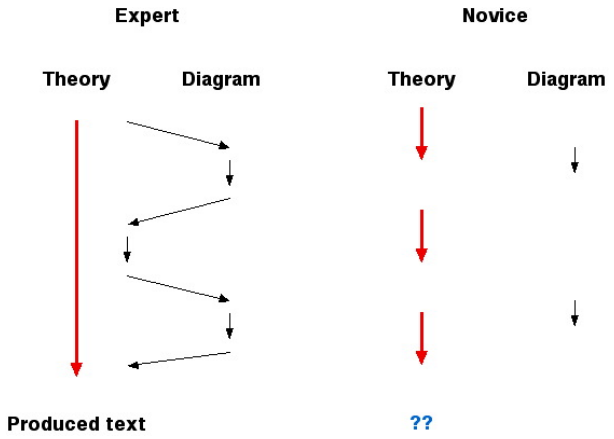
— Projective, Hyperbolic & Other Non-Euclidean Geometries

How and for what purpose do mathematicians prove?

> . . . having verified the theorem in several particular cases, we gathered strong inductive evidence for it. The inductive phase overcame our initial suspicion and gave us a strong confidence in the theorem. Without such confidence we would have scarcely found the courage to undertake the proof which did not look at all a routine job. When you have satisified yourself the theorem is true, you start proving it.
>
> George Pólya

> Actually the mathematician does not rely upon rigorous proof to the extent that is normally supposed. His creations have a meaning for him that precedes any formalization, and this meaning gives the creation an existence or reality ipso facto. . . . Great mathematicians know before a logical proof is ever constructed that a theorem must be true. . .
>
> Morris Kline

Colette Laborde has explained how an expert produces the final text of a proof, while a novice can't see the connections that the expert fluently uses. These connections can be stimulated with Sketchpad.



Michael de Villiers' model for the functions of proof in mathematics classes:

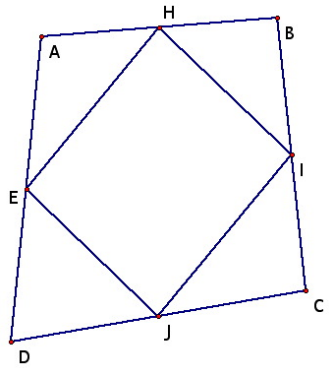
- Verification (concerned with the truth of a statement)
- Explanation (providing insight into why it is true)
- Discovery (the discovery or invention of new results)
- Systematization (the organization of various results into a deductive system of axioms, major concepts and theorems)
- Intellectual Challenge (the self-realization/fulfillment derived from constructing a proof)
- Communication (the transmission of mathematical knowledge)

De Villiers' suggested progression for teaching proof in geometry classes:

In general, start at top.

Explanation
↓
Discovery
↓
Intellectual Challenge
↓
Verification
↓
Systematization

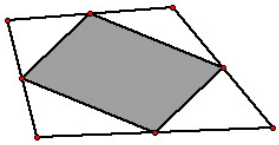
What figure has its vertices at the midpoints of a quadrilateral's sides?



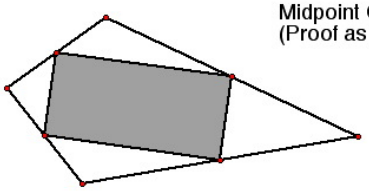
In this case, proof serves an explanatory function.

A more complex conjecture

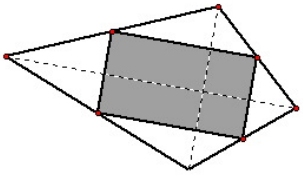
While the figure with vertices on the midpoints of a kite is a rectangle, it is true for figures other than kites. This is a case of proof as discovery.



Midpoint Quadrilateral in a Quadrilateral
(Proof as Explanation)



Midpoint Quadrilateral in a Kite
(Proof as Explanation)



Midpoint Quadrilateral in a Quadrilateral
with Perpendicular Diagonals
(Proof as Discovery)

Why do students need to prove?

It is not reasonable to expect from our high-school geometry students a sudden appreciation of sophisticated proofs if they have not been exposed to both the necessity of proving to understand and proving as an essential feature of doing mathematics. Students in the middle grades should experience proof and argumentation as an integral part of their daily experiences with mathematics.

The moral of the story

— Proof is not a topic, is an essential part of doing mathematics.

— A statement is true not because I, the teacher, say so, but because you can give a convincing argument.

— Sketchpad is not a tool for proving. It is a tool that can generate examples that can help students to reason inductively and make conjectures.

— Proofs serve different purposes. Its different roles should be experienced by students. High-school students should be expected to be able to generate proofs on their own, with increasing mathematical sophistication.

References

National Council of Teachers of Mathematics (2000). *Principles and standards of school mathematics*. Reston, VA: NCTM.

de Villiers, Michael (1999). *Rethinking proof with the Geometer Sketchpad*®. Emeryville, CA: Key Curriculum Press.

Dr. de Villiers has many resources about proof and about Sketchpad (go to page 4):

`http://mzone.mweb.co.za/residents/profmd/homepage.html`

His original article on *The role and function of proof with Sketchpad* is at:

`http://mzone.mweb.co.za/residents/profmd/proof.pdf`

Manouchehri, A., Enderson, M., & Pugnucco, L. (1998). Exploring geometry with technology. *Mathematics Teaching in the Middle School (3)*, 6. 436–442.

Sketchpad presentations, talks, and workshops offered by members of the Sketchpad group at KCP Technologies and Key Curriculum Press (some were used to prepare this presentation):

`http://www.keypress.com/sketchpad/sketchtalks.html`