Frames and Projections

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Abstract

In this dissertation we explore several ways in which the concept of projections arise in finite frame theory. In the first chapter we show that the Paulsen problem from frame theory is equivalent to a long standing open problem about orthogonal projections with constant diagonal. In the second chapter we introduce the idea of nonorthogonal fusion frames and derive some conditions for when tight nonorthogonal fusion frames exist. In particular, we give a classification of how to factor a self-adjoint matrix into a product of projections. The third chapter explores the idea that the cross gramian of a dual pair of frames forms a projection. We use this to give a classification of when two tight frames form a dual pair. We also introduce a notion of Naimark complement of dual pairs and derive some of its basic properties. The fourth chapter is devoted to questions that relate to applying an invertible operator to a given frame to get a new frame with some desired properties. The last chapter looks at frames as sets of rank one projections rather than as sets of vectors. In this chapter we discuss two problems: the first is the question of rescaling a given frame in order to get a tight frame, the second is known as phase retrieval.