

Public Abstract

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The Poisson problem on Lipschitz domains

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The shift theorem for the Laplacian identifies the functional-analytic settings in which the solution  $u$  of the equation  $\Delta u = f$  in a domain  $\Omega$  (with homogeneous Dirichlet or Neumann boundary conditions) is two units smoother than the datum  $f$ . When the domain in question is smooth, such a result holds on all the classical smoothness spaces, of Hardy-Besov-Sobolev-Triebel-Lizorkin type, with no restrictions on the indices involved (integrability and smoothness). On the other hand, counterexamples due to D. Jerison, C. Kenig and B. Dahlberg paint a fundamentally different picture when the boundary of the domain is allowed to have irregularities.

In this work we present a complete, sharp answer to the question of the regularity of Green potentials in domains whose boundaries are locally described by graphs of Lipschitz functions. One remarkable corollary of this work is a solution of the conjecture made by D.-C. Chang, S. Krantz and E. Stein in the 90s to the effect that two derivatives on the Green potential yield a bounded mapping in the context of local Hardy spaces  $H^p$  for  $p < 1$  sufficiently close to 1.