

THE POISSON PROBLEM ON LIPSCHITZ DOMAINS

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ABSTRACT

The aim of this work is to describe the sharp ranges of indices, for which the Poisson problem for Laplacian with Dirichlet or Neumann boundary conditions is well-posed on the scales of Besov and Triebel-Lizorkin spaces on arbitrary Lipschitz domains.

The main theorems we prove extend the work of D. Jerison and C. Kenig [JFA, 95], whose methods and results are largely restricted to the case $p \geq 1$, and answer the open problem # 3.2.21 on p. 121 in C. Kenig's book in the most complete fashion. When specialized to Hardy spaces, our results provide a solution of a (strengthened form of a) conjecture made by D.-C. Chang, S. Krantz and E. Stein regarding the regularity of the Green potentials on Hardy spaces in Lipschitz domains.

The corollaries of our main results include new proofs and various extensions of: Hardy space estimates for Green potentials in convex domains due to V. Adolfsson, B. Dahlberg, S. Fromm, D. Jerison, G. Verchota and T. Wolff and the $L^p - L^q$ estimates for the gradients of Green potentials in Lipschitz domains, due to B. Dahlberg.