ON THE SPECTRA OF SCHRÖDINGER AND JACOBI OPERATORS WITH COMPLEX-VALUED QUASI-PERIODIC ALGEBRO-GEOMETRIC COEFFICIENTS

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ABSTRACT

In this thesis we characterize the spectrum of one-dimensional Schrödinger operators $H = -d^2/dx^2 + V$ in $L^2(\mathbb{R}; dx)$ with quasi-periodic complex-valued algebro-geometric potentials V (i.e., potentials V which satisfy one (and hence infinitely many) equation(s) of the stationary Korteweg–de Vries (KdV) hierarchy) associated with nonsingular hyperelliptic curves. The spectrum of H coincides with the conditional stability set of H and can explicitly be described in terms of the mean value of the inverse of the diagonal Green's function of H.

As a result, the spectrum of H consists of finitely many simple analytic arcs and one semi-infinite simple analytic arc in the complex plane. Crossings as well as confluences of spectral arcs are possible and discussed as well. These results extend to the $L^p(\mathbb{R}; dx)$-setting for $p \in [1, \infty)$.

In addition, we apply these techniques to the discrete case and characterize the spectrum of one-dimensional Jacobi operators $H = aS^+ + a^-S^- - b$ in $\ell^2(\mathbb{Z})$ assuming a, b are complex-valued quasi-periodic algebro-geometric coefficients. In analogy to the case of Schrödinger operators, we prove that the spectrum of H coincides with the conditional stability set of H and can also explicitly be described in terms of the mean value of the Green's function of H. The qualitative behavior of the spectrum of H in the complex plane is similar to the Schrödinger case: the spectrum consists of finitely many bounded simple analytic arcs in the complex plane which may exhibit crossings as well as confluences.