THE DEVELOPMENT OF PEDAGOGICAL CONTENT KNOWLEDGE OF A MATHEMATICS TEACHING INTERN: THE ROLE OF COLLABORATION, CURRICULUM, AND CLASSROOM CONTEXT

A Dissertation

presented to

The Faculty of the Graduate School

University of Missouri-Columbia

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

---

by

BRIDGETTE BOND ALMOND STEVENS

Dr. James E. Tarr, Dissertation Supervisor

DECEMBER 2005

The undersigned, appointed by the Dean of the Graduate School, have examined the dissertation entitled

THE DEVELOPMENT OF
PEDAGOGICAL CONTENT KNOWLEDGE
OF A TEACHING INTERN: THE ROLE OF
COLLABORATION, CURRICULUM, AND THE CLASSROOM CONTEXT

presented by Bridgette Bond Almond Stevens

a candidate for the degree of Doctor of Philosophy

and hereby certify that in their opinion it is worth of acceptance.

James E. Tarr
Dr. James E. Tarr

E. Frances Arbaugh
Dr. E. Frances Arbaugh

John K. Lannin
Dr. John K. Lannin

Jay Scribner
Dr. Jay P. Scribner

Jessica J. Summers
Dr. Jessica J Summers

ACKNOWLEDGEMENTS

The teaching intern, Randy, began a new chapter in his learning to teach process when my study began. I, too, began a new journey in my professional development as a researcher and teacher educator. I am indebted to Randy and several colleagues and friends for their guidance and support. I feel blessed to have married my best friend, Jeff, who has supported the decisions I have made and whose patience has guided me while I pursued this degree. Rarely was it at the expense of our relationship and I appreciate each and every day I walk through life with him. I thank him for his willingness to uproot our lives so that this dream could be made possible and for understanding the type of work and dedication this degree entails.

I feel fortunate to have received tremendous support and encouragement from my committee, particularly my advisor and dissertation supervisor, James Tarr. I have admiration for James for his support during this dissertation process and graduate school. His guidance allowed me to understand my research interests and what I can bring to preservice mathematics teachers' education. Without James, this dissertation would not have been possible. I thank him for the time he has invested in me and in mathematics teacher educator preparation.

My committee members also provided support with their unique abilities and knowledge. Fran Arbaugh introduced me to the study of teacher education and was instrumental in helping me understand the, "So what?" Her encouragement and ability to listen led me to reach for more than I thought possible in my writing. John Lannin helped me understand and appreciate the intricacies of research in mathematics education and I thank him for un-muddying the new terrain. Jay Scribner offered much insight into the field of qualitative research and my beginning search for a better understanding of collaboration and context. Jessica Summers introduced me to the research of educational psychologists and learning theorists, which I am very thankful for in that I have been able to better define my research interests due to her guidance. Because of my committee members, this has been an educative experience.

I am grateful to Randy for his willingness to open his teaching internship experience to the mathematics education community. All who read this study will be more knowledgeable and better informed about the process of learning to teach. I thank Brad for his commitment to the teaching of middle grades mathematics students and preservice mathematics teachers. I thank him for his willingness to open his classroom to me. I believe I am a better researcher because of what I learned from Brad and Randy.

My support system extends to countless friends and colleagues. My Hy-Vee friends reminded me of life outside of graduate school and I am indebted to them for their friendship. They rarely asked about the big "D" and I appreciate it. My fellow graduate students were a great help during coursework and I thank them for their encouragement. The remaining faculty in the "Mathematics Education Suite" at MU have been helpful with their knowledge and expertise—Bob and Barb Reys, Doug Grouws, Kathryn Chval, and Oscar Chavez (and Joy Whitenack, formerly of MU). Last, but in no way least, my colleagues at Northern Iowa who have offered mentorship for so many years—Ed Rathmell, Larry Leutzinger, Diane Thiessen, Glenn Nelson, and Paul Trafton—I thank you for continuing to teach me long after the semester ended. Who would have thought I would follow through on my promise to Ed; well, he and I both knew.

TABLE OF CONTENTS

LIST OF TABLES

Table

LIST OF FIGURES

ABSTRACT

In this study I examined the role of collaboration, curriculum, and the classroom context in the development of pedagogical content knowledge of a mathematics teaching intern. Additionally, I investigated the nature of the collaborative process between the teaching intern and his mentor teacher as they collaborated *on action* (during structured planning time) and *in action* (while students were present). The teaching internship resided in a seventh-grade mathematics classroom during the teaching of a probability unit from a standards-based curriculum, *Connected Mathematics Project*.

Using existing research, a conceptual framework was developed and multiple data sources (audio taped collaborations, observations of the intern's teaching practices, semi-structured interviews, and a mathematics pedagogy assessment) were analyzed in order to understand the teaching intern's development of knowledge of instructional strategies, knowledge of student understandings, curricular knowledge, and conceptions of purpose for teaching probability.

Results identified numerous dilemmas related to planning and implementing instruction. Although the teaching intern developed pedagogical content knowledge, he often experienced difficulty accessing it while teaching. Through collaboration, curriculum, and the classroom context, the teaching intern learned to incorporate his pedagogical content knowledge in instruction. Analysis revealed that as he gained new knowledge he was able to shift his focus from content to the use of instructional strategies for teaching and learning. The curriculum was the primary focus of collaboration and initiated the intern's examination of the learning-to-teach process.

Collaboration *on action* and collaboration *in action* proved to be essential elements in the development of pedagogical content knowledge.

CHAPTER 1: THE PROBLEM AND ITS BACKGROUND

An expanding body of research has focused on the forms of knowledge required by teachers (e.g., Clandinin & Connelly, 1996; Darling-Hammond, 1996; Grossman, Wilson, & Shulman, 1989; McNamara, 1991; Shulman, 1986; Shulman & Sykes, 1986). Such research indicates teachers' instructional decision-making is directly influenced by personal knowledge (e.g., Borko & Putnam, 1996; Wallace & Louden, 1998). In particular, research indicates the knowledge base of expert mathematics teachers is comprised of rich teaching experiences (Hayes & Kelly, 2000; Leinhardt, 1989; Ropo, 1987), which influence the decision-making practices in their classrooms (Fennema & Franke, 1992; Thompson, 1992). It follows that defining a knowledge base for mathematics teaching serves several purposes. It can: (a) improve the quality of education for students, (b) enhance teacher education programs for preservice teachers, and (c) inform and guide professional development for inservice teachers. In mathematics education, reviews of research on teacher knowledge (e.g., Borko & Putnam, 1996; Hiebert, Gallimore & Stigler, 2002; Wilson, Floden & Ferrini-Mundy, 2001) suggest additional investigations are needed so we may understand whether teaching is a matter of knowing *what* to teach, knowing *how* to teach, or both.

Particularly relevant to research on learning to teach is Shulman's (1986) theoretical model of domains of teachers' professional knowledge. One such domain, pedagogical content knowledge, is a type of knowledge that is unique to teachers. Recent research has shown teachers' content and pedagogical content knowledge influence how they teach (Ball, 1990; Brown & Borko, 1992; Enderson, 1995; Hill, Rowan, & Ball, 2005; Loughran, Milroy, Berry, Gunstone, & Mulhall, 2001; Putnam & Borko, 2000; Shulman & Grossman, 1988). Research has also attempted to address the question of how preservice teachers learn to teach (e.g., Grossman, 1990; Grossman, Wilson, & Shulman, 1989; Gudmundsdottir, 1987; Marks, 1991). In addition, as researchers focus on constructivism, a learning theory of how people learn or gain knowledge, there are implications for teaching (Clements, 1997; Fogarty, 1999; Simon, 1995; Walker & Lambert, 1995; von Glasersfeld, 1995a) and teacher education. Novice teachers tend to have inadequate or underdeveloped pedagogical content knowledge of mathematics for use in practice (e.g., Borko et al., 1992; Borko & Putnam, 1996). Arguably, teacher training programs can never completely address all the components of pedagogical content knowledge a teacher needs (Magnusson, Krajcik, & Borko, 1999) because pedagogical content knowledge is constantly developing. Therefore, this raises the question of *how much* preservice teacher education programs can adequately develop pedagogical content knowledge through meaningful learning.

In the early 1990s the reform effort in mathematics education provided a new venue for teacher change that took the form of *standards-based* mathematics curricula as a means for addressing the kind of teaching and learning advocated by the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). The National Science Foundation (NSF) funded the development of several elementary, middle grades, and high school mathematics curriculum projects designed to reflect the *Standards*. At the middle grades level, the curricula projects funded were: *Connected Mathematics Project* [CMP] (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998), *Mathematics in Context* (National Center for

Research in Mathematical Sciences Education & Freudenthal Institute, 1997-1998), *MathScape* (Education Development Center, Inc., 1998), *MathThematics* (Billstein & Williamson, 1999), and *Pathways to Algebra and Geometry* (Institute for Research on Learning, 2000). These materials implemented in classrooms today are quite different from decades prior to the *Standards,* which necessitates changes in preservice teacher education courses (Tarr & Papick, 2004) because these curricula present a greater depth of mathematics than traditional texts have in the past. Furthermore, the teaching of probability and statistics is particularly challenging to middle grades teachers. In fact, the *Mathematical Preparation of Teachers* reports, "Of all the mathematical topics now appearing in middle grades curricula, teachers are least prepared to teach statistics and probability" (Conference Board on Mathematical Sciences, 2001; p. 114). Moreover, in the recent synthesis of the research on teaching and learning probability, Stohl (2005) argues that teachers are deficient in their pedagogical content knowledge for teaching probability. Therefore, preservice teacher education programs should seek to develop pedagogical content knowledge related to probability while preservice teachers are a part of their programs.

Willoughby (1990) argued that the single most important factor influencing teachers' thinking is the textbook. It is, therefore, important to ask how and when teachers develop the *depth* of pedagogical content knowledge needed to teach standards-based curricula, and probability, in particular. Since what teachers do in their classrooms depends largely on their knowledge, teachers need to learn a great deal to be able to enact reform-based curriculum (Borko & Putnam, 1996; Schneider, Krajcik, & Marks, 2000; Wallace & Louden, 1998). In committing to reform-based approaches, it is important for preservice teachers to experience, as learners, the type of instruction they are to create for their students (Frykholm, 1998) because classrooms are learning environments for teachers. Several researchers cite a need to prepare teachers in such a way that will allow them to implement these curricular materials successfully (Clark, 1997; Lappan, 1997; Lloyd, 1999; Papick, Beem, Reys, & Reys, 1999; Smith, 1999).

Improving teacher preparation is among the most prominent reforms suggested for education (Ginsberg & Rhodes, 2003). Pedagogical content knowledge of mathematics develops during the professional years of teaching, but it *begins* to develop during a preservice teachers' mathematics teacher education program. The mathematics content courses, methods courses, general education courses, and student teaching internship all influence the knowledge base of preservice teachers. Consequently, if preservice teacher education programs inadequately foster the development of pedagogical content knowledge, the development of pedagogical content knowledge appears to be postponed until a later stage in their professional development (Smith, 1999).

Resnick (1987) called for a need to bridge the gap between the theoretical learning in the formal instruction of the university classroom and the real-life application of the knowledge in the work environment. Preservice mathematics teachers need access to and opportunities for learning about standards-based mathematics curricula in the context of schooling. This brings forth the need to study how student teaching interns develop pedagogical content knowledge in mathematics classrooms where standards-based curricula have been implemented as embodied by the NCTM *Standards* (1989, 2000). With the growing trend of reform efforts in today's middle grades classrooms, preservice teachers need to develop the type of teacher knowledge consistent with the reform efforts.

Studying how teachers learn to teach mathematics is important because it has the potential to inform mathematics teacher education. Grouws and Schultz (1996) and Magnusson, Krajcik and Borko (1999) have noted the lack of studies that address the development of mathematics pedagogical content knowledge. These researchers and others call for an examination into the development of pedagogical content knowledge during teacher education programs (Lappan, 1997; Wilson, Floden, & Ferrini-Mundy, 2001), as preservice teachers progress through their university coursework, field experiences, student teaching internship, and enter the teaching profession. With the current dominance of constructivist learning theory taking hold (Bransford, Brown & Cocking, 2000; Holloway, 1999; Simon, 1995; von Glasersfeld, 1995b) and classrooms where instruction is broadly conducive to constructivism (Cobb, Wood, & Yackel, 1990), it is also appropriate to consider constructivism as a learning theory in the process for understanding how preservice teachers develop pedagogical content knowledge[1]. The purpose of this study was to examine the collaborative process found in a student teaching internship between a mentor teacher and his teaching intern. Additionally, this study investigated how the teaching intern's development of pedagogical content knowledge changed during the student teaching internship.

---

[1] Constructivism is widely accepted as a learning theory (Bransford, Brown, Cocking, 2000; Von Glasersfeld, 1995b), but is also considered an approach to teaching (Cobb, Wood & Yackel, 1990; Driver, 1995; Fosnot, 1992; Steffe, 1991; Von Glasersfeld, 1995a), teacher learning (Simon, 1995; Steffe & D'Ambrosio, 1995) and a curricula philosophy (Duit & Confrey, 1996; Romberg, 1992). For the purpose of this study, constructivism offers a theoretical perspective for understanding a teaching intern and his development of pedagogical content knowledge.

Statement of the Problem

Bullough's (2001) review of research on teacher knowledge concluded that research on the development of pedagogical content knowledge must take place within a teaching context and meaningful learning will only take place if it is embedded in the social and physical context within which it will be used (Brown, Collins, Duguid, 1989). The student teaching internship is one such teaching context that holds the potential for preservice teachers to foster the development of pedagogical content knowledge needed for teaching mathematics. Through a dynamic interaction with experiences in the environment (Abbott & Ryan, 1999; Walker & Lambert, 1995), including students and a mentor teacher, the learner constructs new knowledge (Cobb, 2000). In particular, Brindley (2000), Kelly (2000), and Lowry (2002) have found preservice teacher education courses that present active, learner-centered environments, like *Standards*-based classrooms (NCTM, 2000), are beneficial for preservice teachers because of the relevant problem situations embedded within this type of teaching context. Connecting such environments with standards-based mathematics classrooms proves essential in the initial development of pedagogical content knowledge for a teaching intern.

Bjuland (2004) and Carter and Gonzalez (1993) found the mentor teacher plays a prominent role in impacting the beliefs and teaching practices of the teaching intern. Collaboration between a mentor teacher and teaching intern can be highly effective (Oliver, 1997) because the social interaction plays a crucial role in learning (Vygotsky, 1962) and helps the teaching intern to progress through their zone of proximal development (Vygotsky, 1978) in the learning to teach process.

The classroom context is also influential in the development of the teaching interns' pedagogical content knowledge and their conceptions of teaching and learning to teach because knowledge is understood to be situated, highly impacted by the context, the setting, and the place where the learning happens (Putnam & Borko, 2000). Given the growing number of schools adopting standards-based mathematics curricula (for more information, see www.showmecenter.missouri.edu/sas), it has become more common for teaching interns to work with mentor teachers whom use, "forms of teaching compatible with constructivism" (Cobb, Wood, & Yackel, 1990) while implementing these curricula and innovative instructional practices. Few studies have examined how teaching interns learn to teach while using a standards-based curriculum. Furthermore, few studies have focused on the student teaching internship. After an exhaustive review of the educational literature, no research has addressed the teaching intern's development of pedagogical content knowledge with a standards-based curriculum at the middle grades level.

Purpose of the Study

The purpose of this study was to examine how the sources of collaboration, curriculum, and the classroom context influenced the teaching intern's development of pedagogical content knowledge. Examining these sources that influence the teaching intern enriches our understanding of the complex interaction found during the student teaching internship. Additionally, this study examined the nature of the collaborative process between a mentor teacher and teaching intern in order to clarify the nature of the collaboration with respect to collaboration *on action* and collaboration *in action* in the development of pedagogical content knowledge. Particular focus is placed on *what* collaboration took place rather than *how* the mentor teacher and teaching intern interacted. Therefore, the purpose of this study was two-fold:

The first purpose of this study was to investigate how the sources of collaboration, curriculum, and the classroom context influenced the development of pedagogical content knowledge of the teaching intern during the teaching internship. I also examined the process by which the teaching intern's development of conceptions of purpose for teaching mathematics, knowledge of student understandings, curricular knowledge, and knowledge of instructional strategies changed during the teaching internship.

The second purpose was to understand the nature of the collaborative process between the mentor teacher and teaching intern. In particular, what is the nature of the collaboration *on action* and collaboration *in action* with respect to the development of pedagogical content knowledge? Moreover, I looked for ways that the curriculum influenced the nature of the collaboration by examining specific instances when the collaboration was focused around the curriculum.

Collaboration *in action* was the instances when the teaching intern and mentor teacher collaborated during formal instruction, making adjustments to instruction as needed or when students transitioned between classes when collaboration could be brief. When students were not present, the collaboration took the form of collaboration *on action*, either during joint planning sessions or during instances throughout the school day when formal instruction was not enacted and time allowed for longer collaborations. Gathered from my perspective as an observer, I gained access into the collaborative process that existed during the teaching internship.

I gathered data during one unit of instruction, *What Do You Expect?* from the *Connected Mathematics Project* [CMP] (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998) curriculum. It was the only unit the teaching intern taught from start to finish and was of particular interest because it has been argued that teachers are least prepared to teach probability and statistics (Conference Board on Mathematical Sciences, 2001) and teachers need additional pedagogical content knowledge for teaching probability (Stohl, 2005). Two of the teaching intern's methods courses at the university emphasized learning to teach probability.

Conceptual Framework

To describe the knowledge base of teachers, several researchers have developed models of teacher knowledge (Carter & Doyle, 1987; Clandinin, 1986; Clandinin & Connelly, 1996; Darling-Hammond, 1996; Elbaz, 1983; Gess-Newsome, 1999; Leinhardt & Smith, 1985; Shulman, 1986, 1987; Wilson, Shulman, & Richert, 1987). The Knowledge Growth in a Profession project, conducted by Shulman and colleagues at Stanford University (Grossman, Wilson, & Shulman, 1989; Shulman & Grossman, 1987) focused on preservice teachers. While these researchers differed in their various components and definitions of teacher knowledge, Grossman (1990) identified four primary domains of professional knowledge for teaching: general pedagogical knowledge; subject matter knowledge; pedagogical content knowledge; and knowledge of context (see Figure 1.1).

General pedagogical knowledge is not subject-matter specific and includes "generic" teaching knowledge (e.g., management, instructional strategies) that might be applicable in a wide variety of educational settings. Subject matter knowledge is a teachers' knowledge of and about the content to be taught; in the case of this study, the teaching intern's knowledge of probability. Pedagogical content knowledge is a domain of teacher knowledge linking content with pedagogy. Specifically, it takes four forms: conceptions of purposes for teaching subject matter; knowledge of student understandings, conceptions, and misconceptions of particular topics in a subject matter; curricular knowledge; and knowledge of instructional strategies and representations for teaching particular topics. Lastly, knowledge of context included the knowledge of the school setting and the knowledge of learners in particular classrooms.

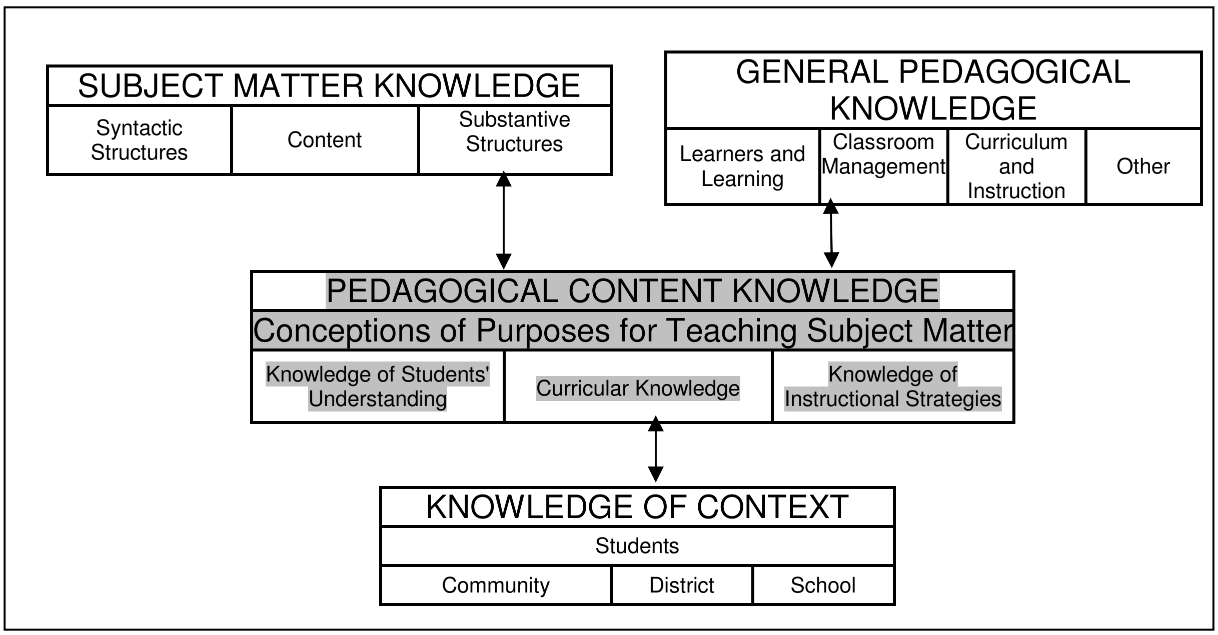


Figure 1.1: Grossman's model of teacher knowledge[2]

This study focused on the pedagogical content knowledge domain. Pedagogical content knowledge is an integration of general pedagogical and subject specific knowledge. Although Marks (1990) suggested the distinction among the types of

---

[2] From *The Making of a Teacher: Teacher Knowledge and Teacher Education* ©1990 by Pamela L. Grossman. Published by Teachers College Press, New York. Used by permission.

knowledge is somewhat arbitrary, the distinctions made here serve as a useful research agenda for the study of teacher knowledge.

Shulman (1987) stated that pedagogical content knowledge is a knowledge base specific to teaching. He advocated the need to explore the inherent relationship between content and pedagogy, emphasizing the need to examine the interaction between content and pedagogy as both come together in education. His research supported a new lens for conducting research about pedagogical content knowledge.

In addition to Shulman's work, Grossman (1990) outlined four central components within pedagogical content knowledge (see Figure 1.1): conceptions of purpose for teaching subject matter (i.e., forming goals); knowledge of student understandings, conceptions, and misconceptions of particular topics in a subject matter; curricular knowledge; and knowledge of instructional strategies for teaching particular topics. As the researcher, I defined pedagogical content knowledge as the type of knowledge teachers possess when they understand the mathematics they are teaching and are able to draw on that knowledge with flexibility by looking for multiple ways of representing mathematical ideas, including a variety of models, to foster students' conceptual understanding. With this in mind, I linked this definition of pedagogical content knowledge with Grossman's four central components within pedagogical content knowledge for this study.

One of Grossman's central components, conceptions of purpose, identify what the teacher knows and believes about the nature of the subject he/she teaches and what is important for students to learn. Vahey, Enyedy and Gifford (2000) contend,

> Every day, people are called upon to make decisions based on statistical and probabilistic information. Public opinion polls, advertising claims,

> medical risks, and weather reports are just a few of the everyday activities that draw on an understanding of probability. In addition, probability is applicable to many academic disciplines outside of mathematics, as it is routinely used in the professional activities of biologists, geneticists, and psychologists alike…..Students [should] be able to make predictions based on theoretical probabilities and empirical data, model probabilistic situations by representing all the possible outcomes for compound events, and understand and appreciate the pervasive use of probability in the real world. (p. 52)

In relation to the teaching of probability, conceptions of purpose are deepening the understanding of the basic concepts of probability. NCTM (2000) recommended:

> Teachers should give middle grades students numerous opportunities to engage in probabilistic thinking about simple situations from which students can develop notions of chance. They should use appropriate terminology in their discussions of chance and use probability to make predictions and test conjectures…students must grapple with many conceptual challenges in order to understand probability…. [in order] to correct misconceptions, it is useful for students to make predictions and then compare the predictions with actual outcomes. (p. 254)

Probability serves as a foundation to the collection, description and interpretation of data. This serves as a conceptual map for instructional decision-making and teachers' goals for teaching probability. In the case of this study, the curriculum goals for teaching probability were to develop a deeper understanding of experimental and theoretical probabilities, to analyze situations involving independent and dependent events, and to use probability and expected value to make decisions (CMP, 1998).

Knowledge of student understandings is knowledge of the understandings and misconceptions that students develop about a subject matter. It differs from general knowledge by its focus on content and the understanding of what students bring to class as prior knowledge. It also represents more than just rote learning. Shaughnessy (1992) contends that students of all ages experience difficulties in learning probability. As an example, the sense of variability is among the numerous misconceptions students hold regarding probability. To predict the likely variation range that will occur during repeated trials of a probability task, students must have some sense of the possible outcomes in that probability task (Shaughnessy & Ciancetta, 2002). If misconceptions exist about the connection between the data and the structure of the probability task and are not confronted or remediated, both, "the misconceptions and the scientific principles may coexist as separate islands of knowledge" (Carpenter & Hiebert, 1992, p. 89). Therefore, knowledge of student understandings and misconceptions help students relate probability to their work with data analysis.

Curricular knowledge is knowledge of curriculum and curricular materials. This component includes knowledge of the range of textbooks and instructional materials that are available for particular topics. It is also knowledge of how the subject matter is organized, and structured, not only at the particular grade level or course, but knowledge of vertical articulation. More specifically in this study, it was the knowledge of the CMP curriculum; its goals, key features, and resources provided. *Connected Mathematics* assists students and teachers in developing mathematical knowledge, understanding, and skill. Lappan, Fey, Fitzgerald, Friel, & Phillips (1998) state the overarching goal is:

> All students should be able to reason and communicate proficiently in mathematics. They should have knowledge of and skill in the use of the vocabulary forms of representation, materials, tools, techniques, and intellectual methods of the discipline of mathematics. This knowledge should include the ability to define and solve problems with reason, insight, inventiveness, and technical proficiency. (p. 1)

In the unit, *What Do You Expect?*, the unit organizer (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002) consists of pacing options, information about prerequisite units, alignment with NCTM's *Principles and Standards for School Mathematics* (2000), a teacher's guide and ancillary resources, a list of essential vocabulary, and a summary of the mathematics covered in the unit. Understanding the teaching intern's use and implementation of the CMP curriculum aided in understanding his curricular knowledge. The teaching intern learned about and worked with standards-based curricula in his two mathematics methods courses at the university, as well as his observations and teaching of lessons during his field experiences in the local school district were the CMP curriculum was implemented. Therefore, he began his teaching internship with some curricular knowledge as it pertained to CMP.

Finally, knowledge of instructional strategies included a repertoire of activities or explanations that are effective for teaching a particular topic for particular learners (Grossman, 1990), which has been researched to the greatest degree. For the purposes of this study, knowledge of instructional strategies included the ability to implement worthwhile mathematical tasks that elicit students' conceptions, and misconceptions, as they relate to probability, and how the teaching intern used students' thinking to build a shared understanding of probability. Particular focus was placed on the teaching intern's uses of representations to foster students' conceptual understandings, questioning skills, and classroom discourse.

In addition to the four components within pedagogical content knowledge, Grossman delineated four distinct *sources* in the development of pedagogical content knowledge: apprenticeship of observation, subject matter knowledge, teacher education, and classroom experience (see Figure 1.2). Because many teachers' ideas of how to teach come from memories of how their own teachers taught and how they learned the material, these experiences can shape their knowledge and beliefs about teaching particular subject matter.

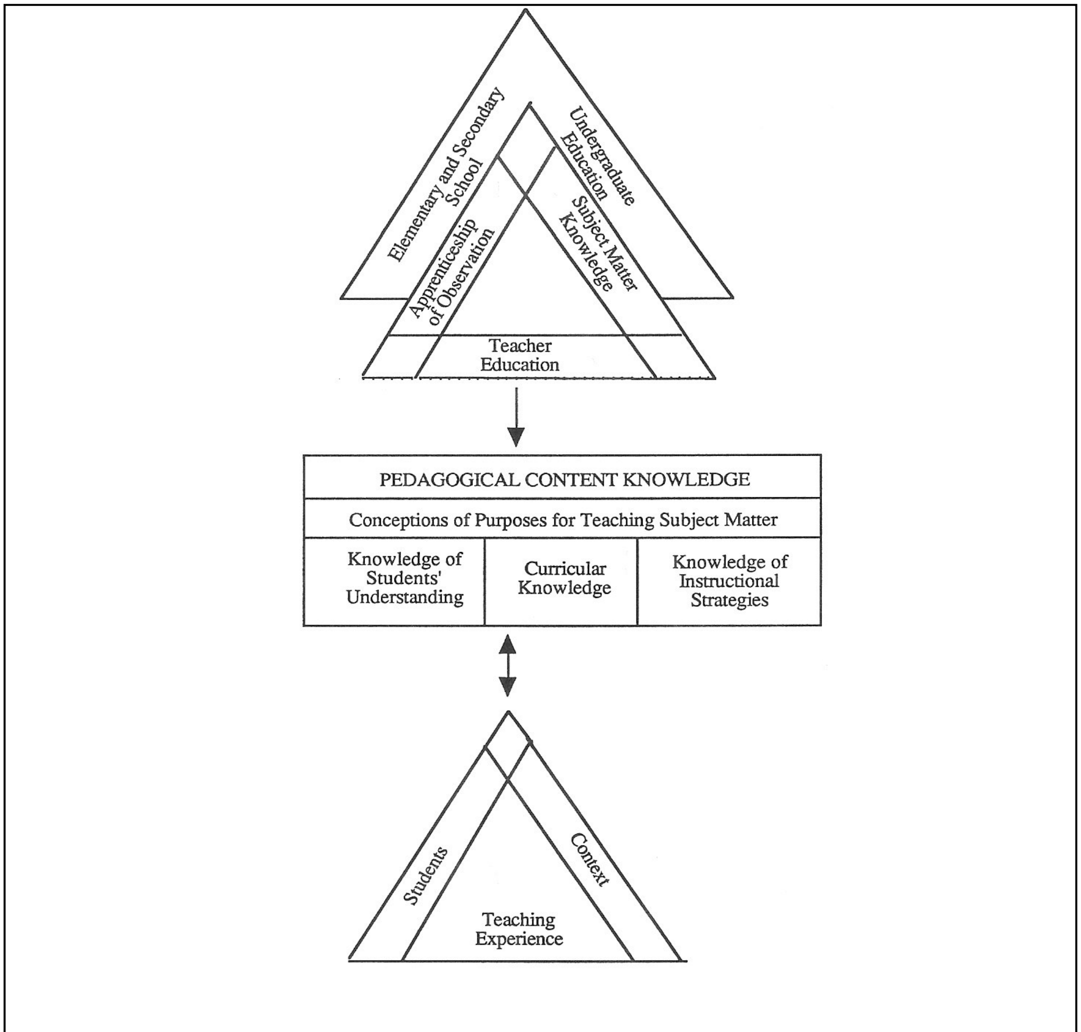


Figure 1.2: Grossman's conceptual framework[3]

This knowledge of the discipline—mathematics in this case—impacts the decisions teachers make with regard to particular content and the selection and sequencing of curricula. It can also affect the teachers' conception of what it means to teach a particular subject. Teacher education programs contribute to this source of knowledge as well, through general education courses, content courses, methods courses (including field experiences), and the student teaching internship required by the

---

[3] From *The Making of a Teacher: Teacher Knowledge and Teacher Education* ©1990 by Pamela L. Grossman. Published by Teachers College Press, New York. Used by permission.

university program. Whether experiences in these programs derive from behaviorist learning theory (e.g., a transmission model of learning) or constructivist learning theory (e.g., theory of learning through the process of "coming to know") (von Glasersfeld, 1995b), each of these sources offers a distinct opportunity for the development of teacher knowledge.

As the researcher, I have modified Grossman's conceptual framework to focus particularly on one source of pedagogical content knowledge (see Figure 1.3). Within the source of teacher education, I contend that teacher education takes the form of both inservice and preservice teacher education. Sources of inservice teacher education can include teaching, professional development, and advanced degrees. However, in preservice teacher education, teachers develop pedagogical content knowledge through their experiences in content courses, methods courses, general education courses, and the student teaching internship.

For the purpose of this study, I focused on the student teaching internship. It is here I pay particular attention to the sources of pedagogical content knowledge found in a student teaching internship—collaboration with a mentor teacher, the curriculum and the classroom context. I contend these sources impact the development of pedagogical content knowledge during the teaching internship. Therefore, I investigate how these sources of pedagogical content knowledge influence the teaching intern's pedagogical content knowledge. More specifically, how these sources influence the teaching intern's conception of purpose for teaching probability, knowledge of students' understandings, curricular knowledge, and knowledge of instructional practices. Moreover, I investigate the nature of the collaborative process between a mentor teacher and teaching intern.

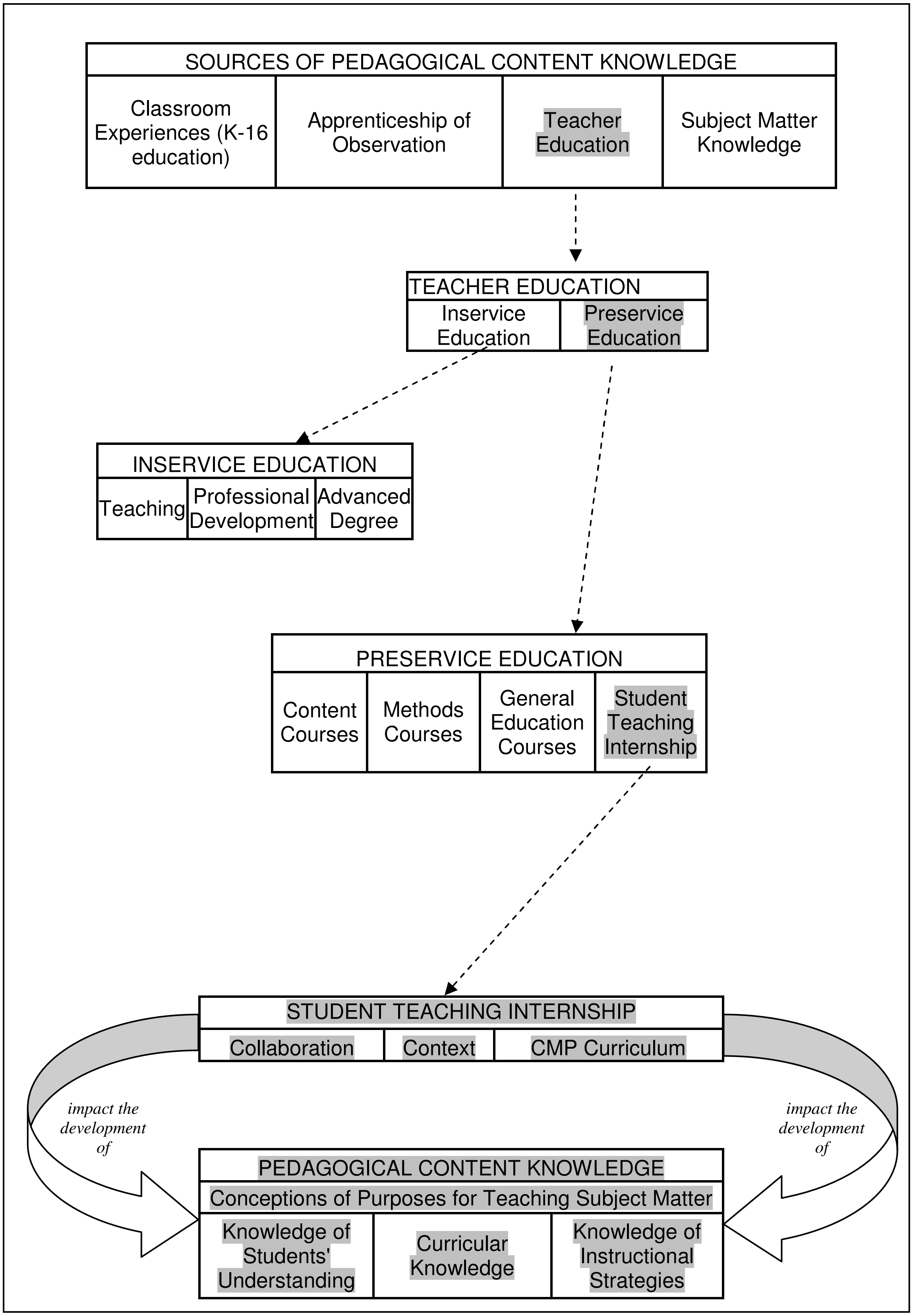


Figure 1.3: My conceptual framework

- *Collaboration*-I examined the nature of the collaborative process between the teaching intern and mentor teacher and looked for instances where the teaching intern and mentor teacher collaborated *in action* and *on action* in order to understand what collaboration took place. In particular, the triangulation of data focused on the development of conceptions of purpose for teaching mathematics, knowledge of student understandings, curricular knowledge, and knowledge of instructional strategies.
- *Classroom Context*-I described the classroom context in which the teaching intern was placed. A description of the mentor teacher assisted in framing the background of the mentor teacher's experiences with teaching the school district's adopted curriculum, working with teaching interns, and his instructional strategies. A discussion of the environment with which the students were accustomed to was addressed. I also considered the process by which the mentor teacher creates meaningful learning opportunities for the teaching intern to develop pedagogical content knowledge.
- The *CMP* Curriculum-I described the unit, *What Do You Expect?*, taught during the 10-week student teaching internship in order to understand the development of pedagogical content knowledge with regard to the curriculum and teaching probability. I analyzed the collaborations between the mentor teacher and teaching intern to further understand the role the curriculum played in collaboration.

Taken from the perspective as an observer, I considered the influence of these sources in the teaching intern's development of pedagogical content knowledge.

Research Questions

Through the research questions I investigated the teaching intern's development of pedagogical content knowledge. I also examined the collaborative process between the mentor teacher and teaching intern in the teaching intern's development of pedagogical content knowledge. Specifically, my study addressed the following research questions:

1. *How did the sources of collaboration, the CMP curriculum, and the classroom context influence the teaching intern's development of pedagogical content knowledge?* More specifically, how did these sources influence his development of conceptions of purpose for teaching probability, knowledge of student understandings, curricular knowledge, and knowledge of instructional strategies? What was the process by which the purpose for developing pedagogical content knowledge changed for the teaching intern?
2. *What is the nature of the collaboration between the teaching intern and mentor teacher?* In particular, what is the nature of the collaboration *on action* and the collaboration *in action*? In what ways did the CMP curriculum influence the nature of the collaboration?

Definition of Terms

The following definition of terms was used in this study (in alphabetical order):

Collaboration

Collaboration is the verbal interaction between the mentor teacher and teaching intern. Taken in two forms, collaboration *in action* are the instances when they collaborate within the act of teaching or between classes for brief moments; collaboration *on action* are the longer joint planning sessions when they collaborate on the act of teaching. Both are analogous to "reflection *in action*" and "reflection *on action*" as described by Schön (1983, 1987).

Conceptual Understanding

Conceptual understanding is the ability to comprehend concepts, operations, and relations that represent mathematical situations in different ways, knowing how different representations can be useful for different purposes, and the ability to make connections among these various forms of representations (National Research Council, 2001).

Content Knowledge

Content knowledge, also referred to as subject matter knowledge, is an understanding of the key facts, concepts, principles and frameworks in a discipline as well as the rules of evidence and proof within that discipline (Brown & Borko, 1992).

Constructivism

Constructivism is a theory of learning that states cognitive growth occurs through the transformation of mental structures. It is also the *theory of knowing* and the process of *coming to know* (von Glasersfeld, 1995b), which is influenced by, "reflection, mediation and social interaction" (Walker & Lambert, 1995; p. 2). Not through direct transmission, knowledge is constructed with the learner through cognitive conflict with the environment and others. The pedagogical implication of this theory is that both the mentor teacher and the teaching intern facilitate this construction of knowledge for the teaching intern.

Dilemma

When prior conceptions are inadequate and cause disequilibrium (Piaget, 1952), there is a need for cognitive reorganization (Lerman, 2001), thus initiating a search for a solution through assimilation and accommodation (Piaget, 1987). In response to Saxe (1995), Ackerman (1995) refers to "the learner's dilemma" as one in which a learner must build upon his own limited knowledge in order to make sense of another context which is not yet familiar. In this study, I refer to "dilemmas" as problematic situations in which the teaching intern requires additional knowledge in order to rectify matters.

Expert Mathematics Teacher

Shulman (1986, 1987) stated that expert mathematics teachers are those who have integrated knowledge of mathematics and pedagogy. In doing so, an expert mathematics teacher possesses pedagogical content knowledge and the knowledge of how to foster understanding of specific mathematical concepts.

Launch, Explore, Summarize

The CMP teacher materials and lesson design are organized around an instructional model that utilizes three phases. First, the teacher *launches* the investigation with the whole class in order to set the context for the problem. Then students *explore* the problem individually or in small groups by gathering data, looking for patterns, making conjectures, and developing other types of problem solving strategies. Last, the students in the class along with the teacher *summarize* by discussing the strategies used as the teacher helps to guide students' thinking in order to deepen their understanding of the mathematical ideas in the problem (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1996).

Mentor Teacher

This is the classroom teacher, along with his students, who was responsible for supervising the teaching intern during the student teaching internship. He was viewed by me as the teacher in the teaching intern's development of pedagogical content knowledge.

NSF-Funded Reform Mathematics Curricula

The National Science Foundation (NSF) funded the development of middle grades mathematics curricula designed to reflect the *Standards*. Five middle grades projects were created: *Mathematics in Context* (MiC) (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997-1998), *Connected Mathematics Project* (CMP) (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998), *MathThematics* (Billstein & Williamson, 1999), *MathScape* (Education Development Center, Inc., 1998), and *Pathways to Algebra and Geometry* (MMAP) (Institute for Research on Learning, 2000).

Pedagogical Content Knowledge

Pedagogical content knowledge is the type of knowledge teachers possess when they understand the mathematics they are teaching and are able to draw on that knowledge with flexibility by looking for multiple ways of representing mathematical ideas, including a variety of models, to foster students' conceptual understanding.

Pedagogical Knowledge

Pedagogical knowledge refers to more general knowledge of teaching, including classroom management techniques and instructional strategies (Shulman, 1986). For the purpose of this study, instructional strategies incorporate the development of questioning skills and the teacher's role in classroom discourse.

Standards-based Curricula

Standards-based curricula refers to one of the five NSF-funded curricula projects designed to embody the recommendations outlined in the NCTM documents (1989, 1991, 1995, 2000). Trafton, Reys, and Wasman (2001) contend standards-based mathematics curricula contain five strongly developed characteristics: comprehensive, coherent, develop ideas in-depth, promote sense-making, engage students, and motivate learning.

Student Teaching Internship (Teaching Internship)

The culmination of a preservice teacher education program during which the preservice teacher in this study participated in a sixteen-week supervised student teaching experience.

Teaching Intern

The teaching intern was the preservice teacher who participated in the student teaching internship in the mentor teacher's classroom. He was viewed by me as the learner in his own development of pedagogical content knowledge.

Methodology

Because the proposed study described and interpreted the collaborative process as a way to enrich our understanding of the complex interaction between a mentor teacher and teaching intern, I selected a *qualitative case study* as the most promising mode of inquiry (Creswell, 1998; LeCompte, Milroy & Preissle, 1992). A case study methodology (Stake, 1995; Yin, 2003) illuminated the complexity of the pairing of these two experts in their respective fields, teaching and preservice teaching. This study represents a case of understanding the nature of the collaborative process in the initial and emerging development of the teaching intern's pedagogical content knowledge and the influence of the CMP curriculum while a teaching intern participated with a mentor teacher in his student teaching internship.

I anticipated the nature of the collaboration between the teaching intern and mentor teacher as a successful pairing that yielded substantial development in the teaching intern's pedagogical content knowledge due to the dynamic interaction with the teaching intern's experiences with the environment as he constructed new knowledge. Both individuals played a role in initiating and guiding the teaching intern's learning efforts. Therefore, the practical, real-life examples and dilemmas the teaching intern encountered assisted in bridging the gap between theories and practice that existed in the research literature on the student teaching internship. I sought to make interpretations in order to provide a greater depth of understanding about the sources that interact in the development of pedagogical content knowledge of a teaching intern during the student teaching internship.

The process of learning to teach is complex and not readily captured by a single frame of reference (Lortie, 1975). Hiebert, Gallimore, and Stigler (2002) contend that teacher knowledge is, "intertwined, organized not according to type of knowledge but according to the problem the knowledge is intended to address" (p. 6). Therefore, it is my contention that in order to study the process by which the teaching intern's development of pedagogical content knowledge changed, I viewed teacher knowledge as one based on a constructivist view of learning (von Glasersfeld, 1995a) where a teaching intern constructs new knowledge about teaching probability.

Theoretical Perspective

Constructivist learning theory (von Glasersfeld, 1995b) can be used to explain teachers' learning (Simon, 1995; Steffe & D'Ambrosio, 1995) and the development of teacher *knowing*. First, constructivists believe that learning occurs when the learner encounters new experiences and concepts, and seeks to assimilate these into their existing cognitive structures or adjust these schemas to accommodate the new information. "To solve a problem intelligently, one must first see it as one's own problem. That is, one must see it as an obstacle that obstructs one's progress toward a goal" (von Glasersfeld, 1995a, p. 14). Therefore, learning is not the result of development; learning *is* development. This learning experience is personal, one that, in the case of this study, the teaching intern must resolve through dissonance and develop new understandings and constructs about what it means to teach probability.

Second, when challenges arise for which prior conceptions is inadequate, "disequilibrium" (Piaget, 1952) occurs and the learner is faced with new dilemmas. Learning becomes an adaptive response to the problematic situation. Therefore, disequilibrium facilitates learning. Learning is then seen as cognitive reorganization by the individual (Lerman, 2001), a self-regulated process of resolving inner cognitive conflicts that often become apparent through concrete experience, collaborative discourse and reflection (Brooks & Brooks, 1999) that the learner experiences. This outcome initiates a search for a solution through assimilation and accommodation (Piaget, 1987), and thus, thinking changes and becomes more complex. Moreover, when learning is shared among two individuals, as in the case of the mentor teacher and teaching intern, "one cannot assume an exact match among the understandings of the learners involved" (von Glasersfeld, 1987, 1995a). The mentor teacher cannot simply give his understandings to the teaching intern. The teaching intern must adapt the direction of his development towards the cognitive structure of better teaching. Therefore, learning is not viewed as a passive absorption of ideas, but rather as active, sense-making and construction of ideas by the teaching intern as he learns to teach probability.

Von Glasersfeld (1988) summarizes Piaget's theory of cognition as consisting of two basic concepts, assimilation and accommodation. He states, "The learning theory that emerges from Piaget's work can be summarized by saying that cognitive change and learning take place when a scheme, instead of producing the expected result, leads to perturbation, and perturbation, in turn, leads to accommodation that establishes a new equilibrium" (p. 128). Thoughts evolve from both the experiences and maturation process of the individual and become internalized into higher forms of cognitive functions when the individual's consciousness evolves from mediated activities (Vygotsky, 1978). Von Glasersfeld (1996) describes his form of constructivism as a theory of rational knowing in that,

> We come to realize that 'understanding' is a matter of fit rather than match. Put in the simplest way, to understand what someone has said or written means no less but also no more than to have build up a conceptual structure that, in the given context, appears to be compatible with the structure the speaker had in mind—and this compatibility, as a rule, manifests itself in no other way than that the receiver says and does nothing that contravenes the speaker's expectations. (p. 134).

Research from a constructivist perspective generally focuses on what students learn, the processes by which they come to know, and their performance insofar as it reflects their understanding (Cobb, 1994; von Glasersfeld, 1995b). Through the lens of constructivism I examined the teaching intern's development of pedagogical content knowledge, comparing his vision of mathematics teaching with his actions in the classroom, and identified dilemmas he faced in teaching mathematics, and how he tried to resolve these dilemmas. I sought to identify how the teaching intern's dilemmas arise, and to identify factors by which these dilemmas are resolved. Consequently, I assumed the teaching intern entered the study with prior experiences that would influence his development of pedagogical content knowledge. Moreover, I assumed his engagement in the study would facilitate his reflection on practice and, in turn, his development of pedagogical content knowledge. Placed with an expert mathematics teacher (Shulman, 1987) in a standards-based mathematics classroom, the teaching intern learns firsthand the importance of learning from teaching.

Using a case study approach to examine one or more individuals (Creswell, 2003; Yin, 2003) I employed several data collection procedures in order to provide a wide array of information pertaining to the teaching internship. The teaching intern and mentor teacher were carefully selected to permit a close examination of their collaboration and the teaching intern's development of pedagogical content knowledge. I used the following data sources to understand how the sources of pedagogical content knowledge found in a student teaching internship--collaboration, classroom context, and the curriculum--influenced how the teaching intern's development of pedagogical content knowledge changed.

Subjects

The subjects were a teaching intern, Randy, and his mentor teacher, Brad, in a school district in a large metropolitan area in the Midwest. The teaching internship resided in a seventh-grade mathematics classroom within a middle school building. In order to examine the teaching intern's development of pedagogical content knowledge, this study included a classroom that used the *Connected Mathematics Project* (CMP) curriculum, and a mentor teacher who had taught CMP for several years, attended CMP workshops for advanced users, and is a district leader in professional development of middle grades teachers.

Data Sources

I used several data sources to capture the teaching intern's development of pedagogical content knowledge and the nature of the collaboration between the teaching intern and mentor teacher.

Audio taped collaborations. Fourteen collaboration *on action* sessions were audio taped during which the teaching intern and mentor teacher prepared for upcoming lessons and discussed previous lessons. Ten of these collaborations coincided with observations and interviews related to lessons from Investigation 2, "Analyzing Number-Cube Games", and Investigation 4, "Analyzing Two-Stage Events". The audiotapes were transcribed for analysis and served as documentation for the second research question, understanding the nature of the collaboration episodes. Fourteen collaboration *in action* sessions, documented via audiotapes, were also gathered in order to understand this type of collaboration that occurred outside the joint planning sessions.

Observations. Eight classroom observations were conducted at pre-arranged dates and times. Observations were made of the teaching intern's teaching with two core groups of students. The cores of students are identified as Core B and Core C. I observed two lessons from both cores for Investigation 2, "Analyzing Number-Cube Games", and Investigation 4, "Analyzing Two-Stage Events". The teaching intern was observed in order to address both research questions. For research question 1, the observations were designed to document the teaching intern's pedagogical content knowledge for use in practice and compare his planning of a lesson with the implementation of the lesson in order to understand the similarities and differences in planning when students became an influence in instruction. For my second research question, the observations allowed me to see firsthand the collaboration *in action* employed. I observed the role played by the mentor teacher, the teaching intern, and the students when a classroom event initiated the *in action* episode.

Field notes of observations. Field notes were recorded for the observations to document the teaching intern's use of pedagogical content knowledge when students were present in instruction. The field notes provided evidence of the teaching intern's teaching for three specific domains related to teaching for student learning as defined by PRAXIS III [ETS](Educational Testing Service, 2001): organizing content knowledge for student learning, creating an environment for student learning, and teaching for student learning. The *Assessment Criteria and Scoring Rules* (ETS, 2001) were used at the conclusion of the observations and recorded field notes so to summarize and evaluate the events of the observation and to gain an understanding of the teaching intern's interaction with the CMP curriculum and the classroom context. More specifically, his pedagogical content knowledge while in the act of teaching when students were present and his interaction with the curriculum.

Interviews with teaching intern. Following two observations of the same lesson, an interview was conducted which focused on the events of the lesson and the evolving internship experience. This resulted in four interviews from eight observations. An additional interview occurred after the teaching intern taught Investigation 5, "Expected Value." The interview questions served to address both research questions, in that they explored the nature of the collaboration related to teaching the lesson, the process by which the teaching intern's use of pedagogical content knowledge changed and its importance to teaching, and the teaching intern's use of the curriculum for the specific lessons taught. The interviews were designed to unpack the decisions made by the teaching intern related to his internship experience.

Mathematics pedagogy assessment. A pedagogical content knowledge assessment was adapted for the teaching of probability from constructed-response items of the Mathematics: Pedagogy Test (#30065) of *The Praxis Series: Professional Assessments for Beginning Teachers* (additional information can be found at: www.ets.org/praxis/prxtest.html) developed by Educational Testing Service [ETS] (1993, 2000). The assessment was administered at the beginning of the internship to both the teaching intern and mentor teacher separately, focusing on their pedagogical content knowledge and reform notions of teaching, specific to the teaching of probability. The assessment provided a measure of the type of knowledge the mentor teacher and teaching intern possessed when they understood the teaching of probability and were able to draw on their knowledge with flexibility by looking for multiple ways of representing probabilistic reasoning through a variety of models to foster students' conceptual understanding. In addition, a post assessment was given to the teaching intern at the end of the teaching internship in order to assess pre/post gains in his pedagogical content knowledge for use in practice.

The test was administered with a 1-hour time limit and no resources were allowed. This data source allowed me to address the first and second research questions. For the first research question, the test documented the teaching intern's initial pedagogical content knowledge for teaching probability at the beginning and end of the teaching internship in order to assess pre/post-test gains. For the second research question, results of the test had the potential to provide reasons for particular collaboration episodes related to learning to teach probability.

Significance of the Study

This study was significant because it provided insight into the process of learning to teach probability. Because it was situated in a student teaching internship context that is common in preservice teacher education programs, this study has implications for program planning in preservice teacher education programs with a similar structure. The role the teaching internship played in a teaching intern's development of pedagogical content knowledge was of importance because the most difficult factor to manage for beginning teachers is pedagogical content knowledge (Grouws & Schultz, 1996). During the student teaching internship, the collaboration with the mentor teacher, the classroom context, and the role of the curriculum influenced the learning to teach process and the development of pedagogical content knowledge. Previous studies considered these elements to some degree (Borko & Putnam, 1996), but this study brought together a middle grades mathematics classroom where CMP was used with a mentor teacher who embraced *reform* notions of teaching and taught the curriculum as embodied by the NCTM *Standards* (1989, 2000). As well, consideration was made with regard to the process by which the teaching intern's development of pedagogical content knowledge changed.

Opportunities for teachers to discuss mathematical ideas and instructional practices as well as to discuss student learning are important for teachers to learn how to teach mathematics effectively (Feiman-Nemser & Parker, 1990; Wilson & Berne, 1999). Examining the collaborative process from my perspective as an observer enriched my understanding of the complex interaction among the sources that influence the development of pedagogical content knowledge during the student teaching internship. As more is learned about teaching, research will assist in refining the teacher knowledge base Shulman (1986, 1987) described. Understanding the characteristics of expert teachers, their knowledge base for teaching, their sources of that knowledge, and their role in aiding teaching intern's in the learning to teach process, will make preparation of quality teachers more likely. Researchers agree that understanding teacher knowledge is critical to understanding teaching and learning how to teach (Clark & Dunn, 1991; Grossman, 1995; Leinhardt, Putnam, Stein & Baxter, 1991).

Limitations of the Study

There are various limitations involved with this study. First, as it was difficult to collect data within a context where the researcher was not embedded, audio taped collaborations between the teaching intern and mentor teacher were gathered and represented an insider's view of the collaboration. It was impossible to document and audiotape every instance of collaboration between the mentor teacher and teaching intern, however, every attempt was made to collect data and the data gathered documented a snapshot of the teaching internship.

Second, due to the focus of this study on the teaching intern, the role of the mentor teacher came second to learning about the teaching intern's understandings in learning to teach. The data I collected about the role the mentor teacher played in this teaching internship consisted only from his role in the nature of the collaborations, both *in action* and *on action*, and what the teaching intern shared about the mentor teacher as a result of his interaction with him. Focusing on the mentor teacher's decisions and actions was beyond the scope of this study.

Third, the duration of the study spanned one semester. More specifically, the teaching intern completed a ten-week practice-teaching experience in an expert teacher's classroom at the end of his university teacher preparation coursework. The teaching intern was seeking dual-certification in middle grades mathematics teaching as well as Art teaching so accommodations were made to the length of his internship in order to gain certification in both areas. Because I examined one teaching intern during his teaching internship, I did not follow the teaching intern into his first year of teaching when pedagogical content knowledge would continue to develop.

Last, the teaching intern entered the study with prior experiences that would influence his development of pedagogical content knowledge. Moreover, I assumed his engagement in this study would facilitate his reflection on practice and, in turn, his development of pedagogical content knowledge. The involvement in the study would facilitate a focus on the curriculum and in turn, the teaching intern's instructional practices.

Summary

Defining a knowledge base for mathematics teaching serves several important purposes. It can improve the quality of mathematics education for students, enhance teacher education programs for preservice teachers, and add to the professional development of inservice teachers. More specifically, pedagogical content knowledge is a key component of this knowledge base. Because it represents the intersection of how teachers relate their pedagogical knowledge (what they know about teaching) to their subject matter knowledge (what they know about what they teach), it is crucial in understanding the forms of knowledge required of expert teachers. Some research that has stemmed from the introduction of pedagogical content knowledge has attempted to address the question of how preservice teachers learn to teach. What remains to be seen is *how much* a teacher-training program can foster the development of pedagogical content knowledge a teacher needs.

First, this study investigated how the development of pedagogical content knowledge of the teaching intern changed during the teaching internship. I looked for ways the teaching intern developed his conceptions of purpose for teaching mathematics, knowledge of student understandings, curricular knowledge, and knowledge of instructional strategies. I paid particular attention to the process through which the teaching intern developed pedagogical content knowledge. I considered the dilemmas he faced and how he overcame these dilemmas. Second, this study was intended to examine the nature of the collaborative process between the mentor teacher and teaching intern. In particular, I looked at the nature of the collaboration *on action* and *in action*. Moreover, I looked for ways the curriculum influenced the nature of the collaboration by examining specific instances when the collaboration was focused around the curriculum. Pedagogical content knowledge is constantly developing and it is an impossible goal for undergraduate teacher education programs to fully develop a preservice teacher's pedagogical content knowledge; however, preservice teacher education programs can influence preservice teachers' initial development. The student teaching internship offers the potential for preservice teachers to learn to teach from an expert mathematics teacher in the context of schooling.

CHAPTER 2: RELATED LITERATURE

Researchers have called for an examination into the development of teacher knowledge (e.g., Hiebert, Gallimore & Stigler, 2002; Wilson, Floden & Ferrini-Mundy, 2001). One domain, pedagogical content knowledge, is a type of knowledge unique to teachers. With this said researchers report that teacher-training programs can never completely address all the components of pedagogical content knowledge a teacher needs (Magnusson, Krajcik, & Borko, 1999) because pedagogical content knowledge is always developing (Borko & Putnam, 1996). However, the student teaching internship affords an opportunity for the mentor teacher and teaching intern to collaborate about pedagogical content knowledge. More specifically, conceptions of purpose for teaching subject matter, knowledge of student understandings, curricular knowledge, and knowledge of instructional strategies are components of pedagogical content knowledge. As teaching interns are influenced by the collaboration with their mentor teachers, the type of teacher knowledge they develop will be carried into the beginning years of teaching.

Given the interrelatedness of pedagogical content knowledge and teacher development, it might be considered important to review research on teacher knowledge and teacher education separately, however, a thorough review of research on these two areas is well beyond the scope of this study. Accordingly, this chapter provides a focused review of the literature on pedagogical content knowledge, preservice teacher education, and more specifically, the student teaching internship, as they relate to my study.

This chapter is divided into four major sections. In the first section on pedagogical content knowledge, I situate my study within the related research on the development of pedagogical content knowledge. I provide an overview of Grossman's (1990) key components of pedagogical content knowledge, including studies related to her work that inform the design of this study. Examples of such components are provided. In section two I discuss research on preservice teacher education programs. A review of this literature provides a frame for the significance of studying the student teaching internship. In section three, I provide an overview of studies related to pedagogical content knowledge in preservice teacher education. It is here I discuss three sources of preservice teacher education programs that influence the development of pedagogical content knowledge. Finally, in the last section I summarize the final source of pedagogical content knowledge in preservice teacher education, the student teaching internship, as it relates to collaboration, the classroom context, and curriculum, which informed the conceptualization of this study. It is here I describe constructivism as a theory for understanding the personal, complex interaction of these elements in a teaching internship. It was my intention to set the stage for the reader to understand the need for additional research on the pedagogical content knowledge preparation of preservice mathematics teachers.

Pedagogical Content Knowledge

Pedagogical content knowledge represents knowledge that is unique to the teaching profession. In particular, pedagogical content knowledge is a blending of content knowledge and pedagogical knowledge that underlie the understanding teachers need to promote student comprehension. Shulman (1987) stated pedagogical content knowledge represents the best knowledge base for teaching:

> The key of distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students. (p. 15)

Scholars have elaborated on Shulman's work by proposing different conceptualizations of the features included or integrated within pedagogical content knowledge (Grossman, 1990; Loughran, Milroy, Berry, Gunstone, & Mulhall, 2001; Magnusson Krajcik & Borko, 1999; Marks, 1990). For example, Segall (2004) contended pedagogy cannot be considered simply a method, but rather pedagogy and content become one. She stated,

> Conceiving of [content and pedagogy] as one opens the possibility for educators and teacher educators to examine not only how people and issues are represented in subject-area texts but also how audiences are constructed and constituted as they are invited, pedagogically, to interact with those texts. (p. 495)

This research is relevant because Segall suggested additional research is needed in the examination of the influences on a teachers' development of pedagogical content knowledge. My study considered the influence of collaboration between a mentor teacher and teaching intern on the teaching intern's development of pedagogical content knowledge, which is discussed at great length later in this chapter. Furthermore, I define pedagogical content knowledge in mathematics teaching as the type of knowledge teachers possess when they understand the mathematics they are teaching and are able to draw on that knowledge with flexibility by looking for multiple ways of representing mathematical ideas, including a variety of models to foster students' conceptual understanding.

After working with Shulman at Stanford University, Grossman's (1990) work with English teachers brought forth four central components of pedagogical content knowledge. Specifically, these components are: (a) conceptions of purposes for teaching subject matter (i.e., forming goals); (b) knowledge of student understandings, conceptions, and misconceptions of particular topics in a subject matter; (c) curricular knowledge; and (d) knowledge of instructional strategies and representations for teaching particular topics. Grossman acknowledged that, "these components are less distinct in practice than in theory" (p. 9), but her general framework is useful for considering the ways in which the teachers develop their pedagogical content knowledge. Grossman's framework for pedagogical content knowledge is relevant for review in this chapter because her four components of pedagogical content knowledge were used to guide my research on a teaching intern's development of pedagogical content knowledge. It is important to note that in review of studies related to these components (Borko & Putnam, 1996), pedagogical content knowledge is relatively underdeveloped in preservice teachers, which suggests preservice teachers leave their undergraduate programs with limited knowledge of these four components and how these four components are interconnected. A review of the related research on Grossman's components and preservice teachers follows.

Conceptions of Purpose

In pivotal writings, Grossman (1989, 1990) described a teacher's overarching conception of a subject for teaching as closely related to the teacher's beliefs about the nature of the subject itself. It is, however, more specifically linked to how the teacher thinks about the subject matter domain for students; in relation to this study, it is what students should learn about mathematics. As described in a number of studies (Borko et al., 1992; Clift, 1987; Grossman; National Center for Research on Teacher Education, 1991) that examined preservice teachers' overarching conceptions, how preservice teachers change during teacher education experiences, and how these changes influence their teaching practices, it is understood inconsistent evidence exists for whether these conceptions can be changed as a result of teacher education experiences (Clift & Brady, 2005). However, Borko and Putnam (1996) contend that such overarching conceptions do influence the instructional decisions and approaches of novice teachers in the classroom. Therefore, understanding preservice teachers' development of this component of pedagogical content knowledge is needed so that we may understand how their conceptions change during preservice teacher education experiences.

As an illustration of conceptions of purpose for teaching probability, I draw upon the *Principles and Standards for School Mathematics* (NCTM, 2000), in which the authors state:

> The amount of data available to help make decisions in business, politics, research, and everyday life is staggering: Consumer surveys guide the development and marketing of products. Polls help determine political-campaign strategies, and experiments are used to evaluate the safety and efficacy of new medical treatments. Statistics are often misused to sway public opinion on issues or to misrepresent the quality and effectiveness of commercial products. Students need to know about data analysis and related aspects of probability in order to reason statistically—skills necessary to becoming informed citizens and intelligent consumers...
>
> In addition, the processes used in reasoning about data and statistics will serve students well in work and in life. Some things children learn in school seem to them predetermined and rule bound. In studying data and statistics, they can also learn that solutions to some problems depend on assumptions and have some degree of uncertainty. The kind of reasoning used in probability and statistics is not always intuitive, and so students will not necessarily develop it if it is not included in the curriculum. (p. 48)

Teachers should build on students' base experiences and move toward new representations that are added to students' repertoire about concepts related to probability (Stohl, 2005). In this study, I looked at the development of pedagogical content knowledge of a teaching intern as he collaborated with his mentor teacher and was influenced by the mentor teacher's conceptions of purpose for teaching probability and their shared knowledge of the CMP curriculum. Conceptions of purposes for teaching probability are one factor that contribute to the development of pedagogical content knowledge.

Knowledge of Student Understandings

Several studies (e.g., Borko & Putnam, 1996; Grouws & Schultz, 1996) report novice teachers often do not recognize the informal knowledge students bring to the classroom and have low expectations for what students can learn. Consequently, as reported by Borko, Livingston, McCaleb, and Mauro (1988), a limited understanding of student understandings is another factor that can inhibit the development of pedagogical content knowledge, which can lead to difficulties in planning and implementing appropriate instructional activities. In addition, Civil (1992) reported limitations in preservice teachers' knowledge of student understandings due to a lack of understanding the unique ways students look at problems, the experience of teaching children, and possibly even how to listen to students.

To illustrate this point about knowledge of student understandings, Moritz and Watson (2000) used the context of tossing coins to start a game of cricket where middle grades mathematics students were to consider the probability of obtaining four tails in a row. They found that although 32% to 55% of students in the middle grades (6^{th} – 8^{th})

knew the numerical value of obtaining one tail in a single toss was ½, virtually no middle grades students and only 3% of ninth grade students could determine the chance of four tails in a row as 1/16. Understanding compound events is difficult for middle grades students. But, more importantly, understanding *why* compound events are difficult is more of a challenging endeavor for a preservice mathematics teacher all together. Understanding the utility of mathematical concepts and techniques, as in the probability example, in the sense that middle grades students learn how and why that concept is useful, can best be applied by illustrating its utility in a purposeful context (Pratt, 2005).

Grossman (1989, 1990) reported that student teachers have difficulty learning from their teaching experience or incorporating student understandings into their instructional planning. This can stem from a disjoint between enacting a desirable practice learned in methods courses in the field experience (Clift & Brady, 2005), causing preservice teacher education programs to have little impact on preservice teachers' thinking (Frykholm, 1996). Findings from these studies suggest novice teachers enter the classroom with little information about what students know about the subject matter being taught, consequently, inhibiting their development of pedagogical content knowledge. This lack of information affects teachers' knowledge of specific understandings and misunderstandings that children have within particular content areas. Additional research is needed to explore how novice teachers can be helped to acquire such knowledge (Borko & Putnam, 1996) and develop the disposition that pedagogical content knowledge is constantly developing. It is an impossible goal for undergraduate teacher education programs to fully develop a preservice teacher's pedagogical content knowledge.

Curricular Knowledge

Another component of pedagogical content knowledge, curricular knowledge, pertains to an understanding of the scope and sequence of content (mathematics, in this case), and the instructional materials used in teaching (Grossman, 1990). Textbooks are not the only source of curricular knowledge, however. Indeed, multiple perspectives on teaching arise from district and state curricula frameworks, videos and case studies, instructional materials, technology, preservice teacher education programs, and research literature, to name a few. Studies have shown that little is known about what novice teachers know and learn about curriculum and curricular materials (De Jong and Van Driel, 2004; Grossman, 1989; McDuffie, 2004; Van Zoest & Bohl, 2002) and what is known varies across preservice teacher preparation programs (Ball & Feiman-Nemser, 1988; Borko & Putnam, 1996). Only recently has research on how teachers interact and use mathematics curriculum materials taken hold (Lloyd, 1999; Remillard & Geist, 2002). Given the recent infusion of reform-oriented curricular materials in mathematics classrooms, research on the use of standards-based mathematics curricula at the middle grades is warranted.

Teacher beliefs and knowledge about teaching, learning, and content influence their decisions about what and how to teach more so than what is simply presented in curricular materials (Putnam, 1992; Remillard, 1992). Curricular materials afford a unique learning opportunity for teachers (Ball & Cohen, 1996) because, “a dynamic interaction between teachers and curriculum has the potential to be highly educative for teachers” (Lloyd, 1999, p. 247). New teachers need opportunities to have conversations *around* curricula with experienced teachers as they analyze and critique curricula material (Grossman & Thompson, 2004). Haller (1997) advocated for professional development on the use of new curricula so that teachers may capitalize on contexts already presented with the curricula, as well as the potential to understand how students think about such contexts. Haller's work suggests an understanding of curricular knowledge appears to contribute to the development of pedagogical content knowledge. "Teachers' abilities to capitalize on tasks posed in curricula are dependent on the robust nature of their knowledge of both probability and the teaching of probability" (Stohl, 2005; p. 358). Therefore, additional research is needed to fully understand preservice teachers' development of curricular knowledge and its relation to teaching probability. The student teaching internship is one such environment where this learning opportunity exists. Therefore, my study considered the use of CMP and the probability unit, *What Do You Expect?*, and its influence on the teaching intern's development of pedagogical content knowledge.

Knowledge of Instructional Strategies

At the heart of teaching is the notion of instructional strategies and forms of representation, key components of pedagogical content knowledge. Teaching entails knowing about and understanding ways of representing and formulating subject matter so that students can understand it. This requires teachers to have a sophisticated understanding of content as well as methods of teaching. Shulman (1986) stated,

> Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstrations-in a word, the ways of representing and formulating the subject that makes it comprehensible to others. (p. 9)

Knowledge of strategies for conducting lessons and creating learning environments is influenced by knowledge of how to structure classroom activities, as well as repertoires of strategies and routines for interacting with students, ensuring student participation and engagement, and for keeping lessons running smoothly (Leinhardt & Greeno, 1986). Consequently, without such knowledge, a teacher may be likely to implement rote-learning experiences over lessons that support multiple strategies and student participation, inhibiting the teachers' development of pedagogical content knowledge. Therefore, having the knowledge of ways to represent concepts of probability influence the instructional strategies used in teaching probability.

Recent focus on the use of questioning as an instructional strategy in teaching mathematics (Carpenter, Fennema, Franke, Levi & Empson, 2000; Moyer & Milewicz, 2002) supports the idea that teachers' questioning strategies are crucial to the instructional process because questioning is the most commonly used instructional strategy (Wassermann, 1991). However, the relationship between questioning skills of the teacher and students' thinking (Baroody & Ginsburg, 1990) is more than just understanding students' knowledge. It takes worthwhile mathematical tasks, and an experienced expert teacher who can analyze their use of questioning and forms of representation to know how to use student understandings to build a shared understanding of probability.

It is difficult to distinguish the pedagogical content knowledge of instructional strategies and representations from subject matter knowledge (Shulman, 1987). To illustrate this point, in a study of preservice teachers, Ball (1990) contends preservice teachers typically believe they already know how to teach and that their undergraduate programs offer little value. She describes a novice teachers' inability to represent division of fractions, which was evident of inadequate subject matter knowledge, but also demonstrates teachers' limited knowledge of ways to represent division of fractions. Regardless of how it is labeled, McDiarmid, Ball, and Anderson (1989) report novice teachers lack adequate repertoires of powerful representations for teaching in their subject matter areas, which limits their development of pedagogical content knowledge. Therefore, developing instructional strategies and representations should be a major component of preservice teacher education (Borko et al., 1992; Wilson, Shulman, & Richert, 1987) because it is vital in the development of pedagogical content knowledge.

Standards-based mathematics curricula have recently aided teachers in their knowledge of instructional strategies and development of pedagogical content knowledge. The *Connected Mathematics Project* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998) provides a lesson design that is intended to assist teachers in understanding how students may think during particular tasks about the investigation. As a part of the unit, *What Do You Expect?*, Investigation 4, "Analyzing-Two Stage Events", Lesson 4.1, Choosing Paths, introduces students to the use of an area model to compute probabilities in situations involving a sequence of events. Before introducing this model, students are encouraged to create a method of their own for analyzing the game. The teacher pages that appear along with the lesson suggest in detail how students might think about the problem. Several methods are shown and lend themselves as examples of how the lesson might play out with students. Additionally, in the teacher pages several questions are posed to aid in assessing student understandings so to build a shared understanding of the mathematics in the lesson.

Given the questioning strategies and forms of representation in standards-based curricula such as this, teachers are more apt to develop knowledge of instructional strategies and pedagogical content knowledge for use in practice. My study sought to understand the teaching intern's pedagogical content knowledge in teaching probability. Specifically, I sought to understand the knowledge the teaching intern possessed when he understood the mathematics he was teaching and was to draw on that knowledge with flexibility by looking for multiple ways of representing mathematical ideas, including a variety of models, intended to foster students' conceptual understanding.

As teachers become more experienced in teaching, they expand their repertoires of teaching strategies and experience an array of classroom situations in a more informed manner. Thus, as teachers become more experienced, they have a greater likelihood of becoming expert teachers because pedagogical content knowledge is constantly developing. Through their development of pedagogical content knowledge, it is important to point out, however, experienced teachers are not always experts; thus, experience alone does not yield expertise in teaching.

To some extent, Grossman's (1990) framework reflects aspects of teacher beliefs that are more specifically related to how they think about content they teach. Teacher beliefs influence how they teach (Brown & Borko, 1992; Kennedy, 1996). Obviating from a discussion about beliefs, however, I contend this framework will allow access to the type of pedagogical content knowledge teaching interns develop while learning to teach. Grossman's work provides a lens for which the study of research on preservice mathematics teacher education can exist. Moreover, her work also considers four sources that contribute to the development of pedagogical content knowledge: classroom experiences (K-16 education), apprenticeship of observation (Lortie, 1975), subject matter knowledge, and teacher education. For the purposes of my study, I focus my literature review on one source, the area of preservice teacher education.

Preservice Teacher Education

Many recent studies (Artzt, 1999; Chappell, 2003; De Jong & Van Driel, 2004; Ebby, 2000; Hatton & Smith, 1995; Kahan, Cooper, & Bethea, 2003; McDairmid, Ball, & Anderson, 1996; Moyer and Milewicz, 2002; Ward & McCotter, 2004; Wubbels, Korthagen, & Broekman, 1997) were designed to examine what preservice teachers learned from their undergraduate teacher education program. Little research has addressed instruction within the teacher education program and its impact on that instruction (Clift & Brady, 2005). In the past, traditional preservice teacher education programs concentrated on classroom management (e.g., pedagogical knowledge, organizing activities, and discipline) and tended not to address the content that teachers would be teaching. Recently a number of studies have examined the actual content of preservice teacher education programs in more depth (Darling-Hammond, 1996; Feiman-Nemser, 1987; Goodlad, 1988; Kennedy, 1999) and have found that teachers with a greater knowledge of teaching and learning are more effective in fostering student learning. To further support this claim, a review of recent research on preservice teacher education programs follows.

In research on preservice mathematics teacher education programs, Lampert and Ball (1999) discussed preparing teachers *to know*. They identify the work of Feiman-Nemser and Buchmann (1986) who said:

> Educators learn theories and methods of teaching, and in classroom settings, they practice using what they have been taught. The assumption,

> held by instructors and learners at the university as well as by teachers, field supervisors, and learners in classrooms, is that knowledge is acquired in course work and applied in practice. (p. 38)

The researchers concede the effect of preservice teacher education is small and gaps exist in the reform visions of teaching and the traditional pedagogy of preservice teacher education because fundamental tensions within coursework and programs exists (Munby, Russell, & Martin, 2001). In addition, the connection between preservice teacher education program goals and student teaching experiences is rare (Guyton & McIntyre, 1990) because beginning teachers resist practices advocated by teacher education programs. Such programs are inconsistent with preservice teachers' beliefs (Clift & Brady, 2005). Not only do preservice teachers need to learn about the ideas discussed in the studies above, they must also make use of these ideas. Additional research is needed in preparing preservice teachers for the middle grades and high school because most studies have focused on prospective elementary education teachers (Clift & Brady, 2005). Trying to view teaching and classroom events from the perspectives of the students who experience them is one way to engage preservice teachers in an analysis of the effects of their actions and the needs of their students (Darling-Hammond & Snyder, 2000). Additionally, interacting with the curriculum as students can aid in understanding how situations are interpreted and responded to differently (Chappell, 2003; Graeber, 1999; Haller, 1997). The teaching intern in my study had prior experience with the CMP curriculum used in his teaching internship, so prior experience aided his understanding how situations are interpreted and responded to differently.

Kennedy (1999) and Artzt (1999) noted that teacher educators should understand teachers' interpretations of classroom situations and their responses to these situations because preservice teachers cannot draw from experiences they have not had. Therefore, preservice teacher education programs should require preservice teachers to think beyond their past experiences. Crespo (2003) examined the changes in problem-posing strategies of preservice teachers through letter writing with students. This course provided access to problem solving experiences the preservice teachers had not previously experienced. Her study provided a view of the interaction between preservice teachers and students, as well as preservice teachers and problem solving. Her study provides rich practical implications for preservice teacher education due to the nature of the experiences the preservice teachers acquired through their interaction with students.

In a related study, Ross, Johnson, and Smith (1991) examined preservice teachers' development of reflective judgment and mastery of the knowledge base about teaching. The researchers concluded that learning to teach is influenced by a complex array of factors, one of the most significant of which is the entering perspective of the preservice teacher. This perspective acts as a filter that determines how experiences within the preservice teacher education program are interpreted. While research on preservice teachers' prior school experiences exists (Ball, 1989; Brown, Cooney, & Jones, 1990), it is beyond the focus of this literature review. However, images of teaching can be useful in teacher preparation programs for helping novice teachers understand themselves as teachers and their knowledge of teaching (Calderhead & Robson, 1991; Johnston, 1992). In particular, these images can provide an understanding of the nature of novice teachers' professional knowledge base because these images can be linked to classroom practices and can provide opportunities for teachers to refine and clarify their images of teaching (Borko & Putnam, 1996).

In studies related to pedagogical content knowledge and teacher education, An, Kulm, and Wu (2004) studied middle school mathematics teachers in China and the U.S. They concluded that deep and broad pedagogical content knowledge is not only important, but also, *necessary* for effective teaching. They argue that effective teaching requires the use of focused questions and activities that promote students' thinking. Moreover, they argue that expert teachers not only have knowledge of content and curriculum, but also knowledge of students' thinking to inform their instructional decision-making practices. Although their study offered detailed descriptions of the types of pedagogical content knowledge mathematics teachers possess, few studies have considered the *development* of pedagogical content knowledge. This study is referenced because of its implications for preservice teacher education; not only do preservice teachers need a deep and broad understanding of pedagogical content knowledge, they also need an understanding of content, curriculum, and students' thinking in order to inform their practices. These components are vital in the development of pedagogical content knowledge during preservice teacher education; therefore, my study attends to these aspects of pedagogical content knowledge during the student teaching internship.

Pedagogical Content Knowledge in Preservice Teacher Education

Ginsberg and Rhodes (2003) contend improving teacher preparation is among the most prominent reforms suggested for education because preservice teacher education programs contribute to the development of pedagogical content knowledge through several sources. Therefore, an understanding of how to develop teachers with adequate pedagogical content knowledge has the potential to inform the design of preservice teacher preparation programs and, in so doing, yield better-prepared teachers. Research needs to focus on the process of acquiring expertise so that we may understand the sort of knowledge preservice teachers must bring to their professional years of teaching (Lowry, 2002; Smith, 1996). Lederman, Gess-Newsome, and Latz (1994) point out that making the learning more meaningful by learning from teaching will influence the development of preservice teachers' practical knowledge base. However, even through preservice teacher education programs incorporate the NCTM *Standards* (1989, 2000), little evidence of that is seen in teaching intern's instruction (Frykholm, 1996, 1999). The sources with which these experiences can take place during preservice teacher education are: (a) general education courses, (b) content courses, (c) methods courses (including field experiences), and (d) the teaching internship. Consequently, it is from these sources that teacher educators influence the initial and emerging development of pedagogical content knowledge.

General Education Courses

Because of their prior beliefs about teaching, prospective teachers may not see the relevance of general education courses in the process of learning to teach (Borko & Putnam, 1996) and, consequently, they may not attend closely to the information or experiences offered by these courses. Data collected by Ball (1988), McDiarmid (1990), and Comeaux (1992) suggest that when courses in pedagogy explicitly attempt to challenge preservice teachers' beliefs, changes in those beliefs can and do take place. McDiarmid cautions, however, that students who are willing to reexamine their understandings and beliefs about teaching may not be prepared to transfer the lessons learned to their own teaching. This has considerable implication for the development of pedagogical content knowledge in preservice teacher education because these programs may be a weak intervention (Borko et al., 1992; Clift, 1987; Grossman, 1989, 1990; National Center for Research on Teacher Education, 1991). Although this study does not specifically take general education courses into consideration, the teaching intern in this study was influenced by his previous experiences in his preservice teacher education program. His general education courses are part of the building blocks that shape the teaching intern's beliefs, preconceptions, and conceptions about what it means to teach.

Content Courses

Exploring teachers' subject matter knowledge might help us determine what and how teachers teach. Many studies have contributed important insights into how deeper knowledge of content helps teachers select the best metaphors, examples, and explanations (McDairmid, Ball, & Anderson, 1996), which are key aspects of pedagogical content knowledge. However, content knowledge alone does not ensure effective teaching performance and may not be the best investment of teacher development (Kahan, Cooper, and Bethea, 2003).

People who have never studied teaching or learning often have a difficult time understanding how to convey material that they may have learned effortlessly (Darling-Hammond, 2000). A number of studies (Ball, 1988; Grossman, Wilson, & Shulman, 1989) have suggested that, in general, teachers with greater subject matter knowledge tend to emphasize the conceptual aspects of their subject matter. Likewise, less knowledgeable teachers tend to emphasize procedural knowledge and stick closely to detailed lesson plans, sometimes neglecting to foster important connections among ideas. There is little doubt that teachers' subject matter knowledge shapes their instructional practices (Borko & Putnam, 1996), which has great potential to influence the initial development of pedagogical content knowledge with preservice teachers.

Content-specific teaching orientations influence how teachers perceive the teaching and learning of content (Gess-Newsome, 1999). Many studies have documented and analyzed the influence of subject matter on pedagogical content knowledge (Ball, 1996; Grossman, 1990; Hillocks, 1999; Rovegno, 1994; Wilson & Wineberg, 1988). Wilcox, Lanier, Schram, and Lappan (1992) point out that modeling new practices and conceptions of mathematical pedagogy in the study of content may be insufficient to develop that knowledge of teaching is more than telling or a matter of technical competence. As well, subject matter knowledge has priority over teaching for student learning. McDiarmid, Ball, & Anderson (1996) state:

> Recent research highlights the critical influence of teachers' subject matter understanding on their pedagogical orientations and decisions…Teachers' capacity to pose questions, select tasks, evaluate their pupils; understanding, and make curricular choices all depend on how they themselves understand the subject matter. (p. 198)

However, Monk (1994) concluded, "Courses in undergraduate mathematics pedagogy contribute more to pupil performance gains than do courses in undergraduate mathematics" (p. 130). Without formal pedagogical preparation, teachers rely on their disciplinary knowledge and personal experiences. Therefore, when the learning of mathematics is embedded in a context of learning to teach, developing subject matter knowledge can be linked to developing pedagogical content knowledge (Ball & Bass, 2000).

Methods Courses

Methods courses also influence the initial and emerging development of pedagogical content knowledge in preservice teacher education, but it is different from other kinds of teacher education courses. Methods courses are often about acquiring new ways of thinking about teaching and learning and incorporate a field-based experience for preservice teachers to learn from observing teachers and students. Moreover, these courses are intended to integrate knowledge about subject matter with knowledge about children and how students learn, and about the teacher's role in student learning. Therefore, methods courses have some potential to influence preservice teachers' conceptions about what it means to teach mathematics and the disposition to learn *from* teaching. While in past research preservice teacher education programs have had little impact on preservice teachers' thinking (Frykholm, 1996), more recent research on methods courses and field experiences can impact the thoughts about practice and actual teaching practices of preservice teachers (Clift & Brady, 2005).

Foss & Kleinsasser (1996) studied 22 elementary preservice teachers' practical knowledge and beliefs during a mathematics methods course. They found their beliefs about mathematics as computational applications remained unchanged and rejected the methods instructor's emphasis on knowledge construction. In analyzing the preservice teachers' teaching (including lesson plans, interviews, and a surveys), the preservice teachers continued to believe repetition and drill are appropriate teaching techniques. Little influence was made with regard to understanding mathematics learning as construction of knowledge.

Researchers suggest the role of the methods course should engage preservice teachers in learning *from* the classroom (Kagan, 1992; Thompson, 1984), much like the student teaching internship. As such, field experiences are often a component of methods courses, where preservice teachers observe and experience the role of teaching. Through these experiences, Chappell (2003) points out that preservice teachers should experience firsthand the mathematics and instructional pedagogy modeled in classrooms, explore the depth of mathematics, as well as develop an understanding of how traditional mathematics curriculum compare with standards-based curriculum. This is true not only for the experiences in preservice methods courses, but also the student teaching internship. My study considered Chappell's research in that the teaching intern gained firsthand experience during his teaching internship in developing pedagogical content knowledge through the use of a standards-based curriculum and through his collaboration with a mentor teacher who implemented the CMP curriculum as embodied by the NCTM *Standards* (1989, 2000).

Teachers need experiences in *teaching* particular topics in order to develop pedagogical content knowledge (De Jong & Van Driel, 2004) because having an understanding of students' learning difficulties concerning these topics is important. Because pedagogical content knowledge is always developing, gaining experiences such as these before the student teaching internship has the potential for the teaching intern to enter the student teaching internship with already well-conceived notions about what it means to teach mathematics. Consequently, the internship can then focus more on the teaching intern's development of pedagogical content knowledge related to teaching particular topics *in practice* and gaining an understanding of students' learning difficulties concerning these topics.

Ebby (2000) explored the connections between fieldwork and coursework that aimed to integrate these two contexts. His experiences suggested that learning to teach mathematics differently (reform methods of teaching) is not a linear process (i.e., knowledge learned in the methods course is simply applied to the fieldwork classroom):

> Coursework helped each of the student teachers in this study to think about the children in their fieldwork classroom in new ways, and observing children's learning in the fieldwork classroom helped them clarify their thinking about what they were learning in coursework. (p. 92)

Ebby's study illustrated the knowledge preservice teachers possess and the different ways fieldwork and coursework interact. Methods courses need to be facilitated towards learning *from* fieldwork, much like the student teaching internship. As in Ebby's research, other research on the student teaching internship has provided insight into how methods courses can better prepare preservice teachers for the student teaching internship (Bjuland, 2004; Hatton & Smith, 1995; Bullough, Knowles, & Crow, 1989; Calderhead, 1988). These studies suggest that a deeper understanding of reform practices may result if we required fieldwork and student teaching internships in only reform-minded classrooms.

Several studies have demonstrated preservice teacher beliefs about learners and learning can change, and that their existing knowledge and beliefs can influence what they learn from their preservice teacher education program (Bird, 1991; Hollingsworth, 1989; Holt-Reynolds, 1992). Other research has focused on the kinds of knowledge about learners that preservice teachers do acquire during their preservice teacher education programs, especially through their teaching internship experiences (Calderhead, 1988; Kagan and Tippins, 1991; Levin & Ammon, 1992; Bullough, Knowles, & Crow, 1989, Wubbels, Korthagen, & Broekman, 1997). This research highlighted the important role that novice teachers' initial teaching experience can play in learning about students. Notwithstanding these contributions to the literature base, additional research is needed on student teaching internship experiences in classrooms where a standards-based curriculum is implemented as characterized by the NCTM (2000) *Standards*. This study has the potential to fill this gap in research on teaching interns' development of pedagogical content knowledge in a middle grades mathematics classroom where standards-based mathematics curriculum was implemented in the spirit of the *Standards*.

The Student Teaching Internship

The student teaching internship experience is a major component of traditional undergraduate teacher preparation programs, yet little is known about the impact of specific experiences on teaching interns' emerging pedagogical knowledge (Johnston, 1992). What is known is that without guidance, preservice teachers find it difficult to make the transition to pedagogical thinking (Brown & Borko, 1992; Grouws & Schultz, 1996). Given the time constraints and other demands on teaching interns, researchers have been skeptical of preservice teacher education programs ability to produce teachers who actively possess conceptions of good mathematics instruction (Clift & Brady, 2005). The role of the mentor teacher continues to be the primary influence on the initial development of preservice teachers' knowledge during the student teaching internship (Frykholm, 1996) because the interaction between the teaching intern and mentor teacher in the student teaching internship plays a critical role (Blanton et al., 2000; Lanier & Little, 1986; Wideen et al., 1998). If teaching interns are placed with mentor teachers that teach using traditional methods, the teaching internship experience often offers little in the way developing pedagogical content knowledge in such a manner consistent with the reform-based content and methodology taught in university courses (Frykholm, 1996; Van Zoest & Bohl, 2002).

Because the teaching internship experience is generally regarded as the most formative and significant element of the preservice preparation process (Lortie, 1975; Zahorik, 1988), Zeichner and Gore (1990) suggest the mathematics education community take a more proactive role in increasing the likelihood that a beginning teacher will support the visions of reform. The teaching internship is the last intervention for preservice teacher education programs. Therefore, my study sought to understand this last intervention where a teaching intern was placed in an internship experience that supported reformed notions of teaching compatible with constructivism and a standards-based curriculum.

Much of the research on constructivism in education has been focused on students' construction of knowledge and classrooms where teachers are basing instruction on constructivist principles (Cobb, Wood, & Yackel, 1990). For the purpose of this study I considered the teaching intern as the student in a learning environment where he encountered new experiences and sought to assimilate these experiences through cognitive conflict. A discussion on constructivist learning theory is warranted.

Constructivism is a theory of learning that states cognitive growth occurs through the transformation of mental structures (von Glasersfeld, 1995a) which support the development of teacher knowledge. It is also the *theory of knowing* and the process of *coming to know* (von Glasersfeld, 1995b), which is influenced by, "reflection, mediation and social interaction" (Walker & Lambert, 1995; p. 2). As well, knowledge is constructed with the learner through cognitive conflict with the environment and others. Therefore, the mentor teacher influences this knowledge construction. It is a personal learning experience in that the learner is on a quest for understanding and seeks to resolve any dissonance in understanding. It is also social in that learning is a process of resolving "inner cognitive conflicts" as a result of experiences, collaborative discourse and reflection (Brooks & Brooks, 1999; Forman, 1993). Consequently, the teaching internship experience influences the manner in which the teaching intern constructs knowledge about teaching and his emerging pedagogical content knowledge development.

While it has been said, the constructivist view of learning has implications for what role teachers can play in facilitating new experiences as students construct meaning (Dolk, Uittenbogaard, & Fosnot, 1996; Good & Brophy, 1994), I contend the same has implications for the role of the mentor teacher and the teaching intern in the student teaching internship. Teachers today are faced with the challenge of producing students who know *how* to learn (McLaughlin & Overman, 1996). Expert teachers more likely facilitate experiences that result in this, and as a result, offer a classroom environment with the potential for teaching interns to learn to teach in much the same manner. I contend constructivism is a viable perspective for understanding the learning to teach process and the dilemmas teachers face when the interact with curriculum and students in instruction.

Embedded within a type of teaching context that is an active, learner-centered constructivist environment, the teaching internship utilizes an apprenticeship model where the mentor teacher plays a considerable role in impacting the beliefs and initial teaching practices of the teaching intern (Zeichner, 1996). Placing teaching interns in schools that have adopted standards-based mathematics curricula, as well with mentor teachers who embrace current reform efforts and innovative instructional practices, affords a teaching opportunity where teaching interns can develop pedagogical content knowledge (Frykholm, 1998). Few studies have considered such an environment for the student teaching internship. Therefore, my study considered an apprenticeship model where standards-based mathematics curriculum was implemented. The examination of the nature of the collaborative process between the mentor teacher and teaching intern in order to identify the sources associated with the emerging development of pedagogical content knowledge of a teaching intern framed the purpose of this study. Therefore, a review of recent research on these aspects of a teaching internship is warranted. I contend that as the teaching intern constructs teacher knowledge, the nature of the collaboration, the classroom context, and the curriculum are three sources that influence the teaching intern's emerging development of pedagogical content knowledge.

Context

Contextual factors, such as school community (traditional and/or reformed notions of teaching), curricula, and students as learners, influence teacher knowledge. Whether a teaching situation is challenging, offers support and guidance from colleagues, or considers different expectations from teachers, it is difficult to interpret how these contextual differences might influence teacher knowledge (Kennedy, 1996). However, Lloyd (1999) considered the elements of a teacher's context that enabled and supported the teacher's instructional and conceptual change in teaching while implementing a standards-based mathematics curriculum. The role the curriculum played in her study, along with Remillard (2000), suggest features of the teaching context can affect teachers' efforts to change. While a discussion about the role of the curriculum appears later in this chapter, it is important to recognize a standards-based mathematics classroom context is influential in affecting teachers' efforts to change.

Wilcox, Lanier, Schram, and Lappan (1992) examined three beginning teachers to understand the choices teachers make and how those choices were influenced by the interaction of their views about knowledge and pedagogy within the perceived context. The researchers contended that teachers placed in a context (school community) that is supportive, collaborative, and where teachers share the same vision positively influence teacher knowledge. Consequently, when teachers are able to understand the classroom context, they will learn more from the classroom context. Placing interns with mentor teachers in such a context ensures the types of experiences where the teaching intern can learn from his experiences.

To better understand the teaching intern in the context of teaching, one avenue researched greatly is the use of reflection. Bjuland (2004) studied teaching interns as they collaboratively reflected in small groups on their learning process in a problem-solving context. The reflective dialogue among the teaching interns illustrated how preservice teachers reflect, both on experiences as learners of mathematics and as teachers of mathematics. As teachers focus on the mathematical understanding of children, reflecting more often on their teaching will ensure better planning for instruction. I contend this type of context of the teaching intern's collaboration with the mentor teacher additionally influences the teaching intern's emerging development of pedagogical content knowledge. It follows that research on teaching interns' development of pedagogical content knowledge could include a reflection component, or at a minimum, data explaining the instructional decisions made by the teaching intern. Moreover, Artzt (1999) used a framework for structured reflection to study a teaching intern and found the structured reflection assisted the university supervisor in understanding the teaching intern in the context of teaching.

There is growing value placed on reflection in teacher development. Its essential quality is known as "thinking about practice in order to improve" (Hatton & Smith, 1995, p. 34). Moreover, several researchers agree reflection is an important process for teachers (Darling-Hammond & Snyder, 2000; Thompson, 1992; Zeichner, 1996). Although, reflection is not a specific component in this study, it is important to make the point that the teaching intern in this study was characterized as highly reflective by both of his mathematics methods instructors. As the teaching intern interacts with the context of his student teaching internship, I contend his reflective nature will play a considerable role in his initial development of pedagogical content knowledge. As well, it was my goal to offer a contextual view of an expert teacher's classroom as an arena for where this learning took place. Making his classroom context public enabled it to be shared with the mathematics education community.

Curriculum

Ball and Cohen (1996) report curricular materials historically played an uneven role in teachers' practice. Research suggests standards-based curriculum materials can provide rich opportunities for not only student learning, but for the teachers who teach them (Grossman & Thompson, 2004; Haller, 1997; Lloyd, 1999; Remillard & Geist, 2002). Remillard (2000), and Grossman and Thompson point out that there is a need to increase the use of reform-oriented curriculum in professional development programs because of the potential for teacher learning. Those same recommendations are suggested for improving teacher preparation (Ginsberg & Rhodes, 2003; Tarr & Papick, 2004).

Because it has been argued the textbook is the single most important factor in influencing teachers' thinking (Willoughby, 1990) and with the growing number of schools adopting standards-based mathematics curricula (for more information, see www.showmecenter.missouri.edu/sas), teachers need to learn a great deal to be able to enact standards-based curriculum (Wallace & Louden, 1998; Borko & Putnam, 1996; Schneider & Krajcik, & Marks, 2000). Few studies have described teacher's use of reform-based curriculum materials. As stated by Chappell (2003), the teacher, "should have a willing and committed posture as well as a mathematics disposition that is open to change" (p. 294) in order to teach from a standards-based curriculum. The teacher plays a critical role in ensuring that the curriculum is implemented as intended. Therefore, additional research is needed to understand the role of curriculum in the development of pedagogical content knowledge.

The NCTM *Standards* (1989, 2000) are also considered curriculum. Frykholm (1999) studied sixty-three teaching interns and their implementation of the NCTM *Standards* (1989), and the influence their mentor teachers had on their teaching philosophy. He found the most significant influence on the teaching interns' philosophy and instructional practices was the mentor teacher. Moreover, because their mentor teachers often did not model or discuss the *Standards*, they needed additional information regarding how to implement the *Standards*. Observations of the teaching interns' teaching indicated more traditional, teacher-centered approach to teaching; however, a few interns reported concern for the mismatch between how they taught and how they wanted to teach. Frykholm's research suggests a dichotomy remains between what is taught in preservice teacher education programs and what is observed in teaching intern's instructional practices once they leave said programs.

Enacting reform-based curricula is not easy because it requires learning new instructional practices where teachers will need access to the knowledge of content and pedagogy required for teaching such materials as they think about their students in a particular context. In recent studies related to standards-based curricula, Collopy (1999) reported that *Investigations in Numbers, Data and Space* (TERC, 1995) presented a promising source for teacher learning. On the other hand, Gess-Newsome (1999) and Schneider, Krajcik, and Blumenfeld (2005) found teachers' subject matter knowledge might hinder the adoption of innovative teaching materials because *enacting* the materials is not sufficient. Professional development opportunities are needed for teachers to learn new instructional practices that align with reform notions of teaching.

Remillard (2000) studied the relationship between two elementary teachers' use of a reform-oriented mathematics textbook and their learning about and the teaching of mathematics. She concluded the curriculum is not enough. The pedagogical change required to support reform of mathematics instruction needs to be more than just a set of activities for students to do. The curriculum should provide explicit reasons and purpose for certain activities, as well as content, that provide opportunities for teachers to engage in decision-making instructional practices. Consequently, the initial development of pedagogical content knowledge needed to teach standards-based curriculum can begin in preservice teacher education programs. De Jong and Van Driel (2004) found that nearly all of the teaching interns in their study did not expect teaching and student-learning difficulties from the textbook they were using prior to teaching, but soon became aware of the specific shortcomings in their textbooks (i.e., understanding something that is very clear to themselves, but not to their students). The opportunity to learn from teaching during the teaching internship appeared to be an effective way for the teaching interns to experience teaching a textbook firsthand and become aware of specific teaching and student-learning difficulties.

Wang and Paine (2003) explored how a Chinese middle school beginning teacher developed her professional knowledge of mathematics instruction. They found the teacher's instructional practices were both directly and indirectly influenced by the nature of the mandated curriculum and teaching organization in the context of her work because systematic observations by and discussions with other teachers helped the beginning teacher develop and refine her teaching strategies when pedagogical content knowledge was the focus. The study indicated that the ways in which the mandated curriculum is structured and teachers are organized might help teachers develop the necessary professional knowledge for teaching. Further, research is still needed to understand the role of standards-based curriculum in the development of pedagogical content knowledge of a middle grade teacher.

Role of the Collaboration

Preservice teacher education programs have historically focused more on the development of individual knowledge and competencies thought to be important for teaching (Kennedy, 1999). However, the view of knowledge as socially-constructed makes it clear that an important part of *learning to teach* is becoming enculturated into the teaching community (Putnam & Borko, 2000) because the classroom is a powerful environment for shaping (and constraining) how practicing teachers think and act. Therefore, the nature of collaboration in a professional community can be vital in the learning to teach process. Consequently, the student teaching internship is one area where collaboration plays a substantial role in the initial and emerging development of pedagogical content knowledge because the mentor teacher impacts the thinking of the teaching intern (Frykholm, 1996).

Blanton, Berenson, and Norwood (2001) studied a teaching intern's change from authority-based teaching to one that offered encouragement for students to reflect and analyze their thinking. The teaching intern began to listen to students' thinking as illustrative of their beliefs and problem solving practices. Therefore, the students' language and discourse influenced the teaching intern's instructional practices. The researchers acknowledge the mentor teacher, who was open-minded and inquiry-oriented, also assisted the teaching intern's development.

An apprenticeship model includes features of authentic activity, social interaction, collaborative learning, and a teacher/coach experience that supports the teaching interns' learning to teach. An application of this model would include the teaching internship where the teaching intern's learning is "situated" in the context of practice. This enables learning to be embedded in "authentic" activity (Brown, Collins, & Duguid, 1989). Consequently, the teaching intern has the opportunity to think and act like a teacher by observing teaching, engaging in activities alongside a teacher to a gradual withdraw of support so that the candidate can transition from teaching intern. Accordingly, the teaching intern would be more likely to develop pedagogical content knowledge if their learning is situated in practice. In such a context, teaching interns are able to explore the richness and complexity of genuine pedagogical problems before they begin their professional years of teaching.

The role of the nature of collaboration during the student teaching internship is of importance because one factor associated with successful learning during the teaching internship is having a mentor teacher or university supervisor who serves as a role model and facilitator of change (Calderhead, 1988; Hollingsworth, 1989). Novice teachers often acquire the management strategies of the experienced teachers with whom they are working (Borko & Putnam, 1996) because mentor teachers' beliefs and teaching strategies impact the thinking of teaching interns (Frykholm, 1999). Von Glasersfeld contends learning is not a simple stimulus-response occurrence, but the building of concepts through reflection and abstraction, with meaning based upon the learner's experiences (1995a, 1995b). Constructivists believe individuals are active participants in the learning process, and that they do not learn just by doing, but by doing and thinking. Action combined with reflection helps individuals construct new understandings (Ammon & Levin, 1993).

Feiman-Nemser and Beasley (1997) explored mentoring in their case study of a mentor teacher (Beasley) working with a teacher intern. While the teachers together learned about the new content to be taught, Beasley's scaffolding played a critical role in guiding the intern's initial pedagogical thinking because mentor teachers are a significant influence on the development of teaching interns' philosophy and instructional practices (Zeichner & Gore, 1999). In relation to research on constructivism, because knowledge is constructed from existing beliefs and understandings, teachers must be aware of students' incomplete understandings that they bring to the classroom and strive to build upon their knowledge in order to help them reach a more mature understanding of these concepts (Bransford, Brown, & Cocking, 2000). The same could also be said for the mentor teacher's role in a teaching intern's learning to teach where mentor teachers need to be aware of teaching interns' existing beliefs and understandings so to assist them in building upon their knowledge in order to help them reach a more mature understanding of what it means to learn from teaching.

In an additional study on the role of the nature of collaboration, Van Zoest and Bohl (2002) found the nature of the collaboration between teaching intern and mentor teacher, *along with the role of the curriculum*, impacted the teaching intern's initial development of pedagogical content knowledge. The teaching intern's collaborations with the mentor teacher centered *around* a standards-based curriculum. The mentor teacher had limited experience in teaching a high school standards-based mathematics curriculum so together they gained new knowledge through this learning to teach process.

The majority of the collaborations focused on the content presented in the curriculum and how to teach from the curriculum. McDuffie (2004) found this collaboration between the teaching intern and mentor teacher should occur in a more in-depth way because teaching interns with knowledge of reform mathematics curricula still have an inability to teach with it in an innovative way. Both studies were situated in an elementary and high school student teaching internship respectively; however, no study has investigated the emergence of pedagogical content knowledge in a middle grades mathematics classroom in which a standards-based curriculum is used. Therefore, research is needed to understand how standards-based mathematics curricula impact the teaching intern's development of pedagogical content knowledge with a middle grades standards-based mathematic curriculum.

Summary

With reform efforts in today's classrooms taking hold, preservice teachers need to acquire the type of teacher knowledge consistent with the reform efforts (Cochran-Smith, 1991). Schön (1983, 1987) discussed the importance of community in the learning-to-teach process because, in particular, he described the collaboration that must take place in order for teaching interns to cultivate reformed, reflective practices. One purpose of my study was to investigate the development of pedagogical content knowledge of the teaching intern and to look for ways the curriculum helped shape the teaching intern's pedagogical content knowledge. From a constructivist learning perspective, I paid particular attention to the process through which the teaching intern developed pedagogical content knowledge. Additionally, another purpose was to closely examine the student teaching internship in order to better understand the nature of the collaborative process between the mentor teacher and teaching intern and to look for ways the curriculum influence the nature of the collaboration. Learning from teaching implies that teaching interns learn in an active way, involving real practice situations, to make their learning more meaningful for themselves (Lampert & Loewenberg, 1998).

McDuffie (2004) found collaboration beneficial for the teaching intern. She found a teacher intern's amount of learning could be expanded by conducting in-depth collaborations with the mentor teacher in advance of teaching; however, it is not clear whether it is reasonable for mentor teachers to conference in an in-depth, lesson-specific pedagogical content knowledge way all of the time. Van Zoest and Bohl (2002) found collaboration is influenced by standards-based mathematics curriculum (*Core-Plus Mathematics Project*, Coxford et al., 1997) because the joint planning sessions between the mentor teacher and teaching intern assisted the teaching intern in dealing with issues of content and pedagogical content knowledge. Therefore, the standards-based curricula materials has the potential for a strong and positive influence on the character of the teaching intern's learning and experience. Using such materials has potential to offer teaching interns opportunities to increase their always developing knowledge.

Both Van Zoest and Bohl and McDuffie's (2004) studies validated the work by Brown and Borko (1992) of the need for, "teacher educators to place interns in school contexts that support and extend the work begun in teacher education programs" (p. 284). Additionally, both found teaching interns heavily relied upon the mentor teacher to assist with pedagogical content knowledge issues. Grouws and Schultz (1996) reported that by providing teachers with certain types of pedagogical content knowledge, teachers' classroom practices changed and, consequently, student learning increases. The use of standards-based mathematics curricula during the student teaching internship afforded the opportunity to examine teaching interns' development of pedagogical content knowledge. It follows that additional research is needed to better understand how much a preservice teacher education program can aid in a preservice teacher's development of pedagogical content knowledge. In addition, research is needed to better understand the role of the collaborative process between the mentor teacher and teaching intern, within the classroom context, and the role of standards-based mathematics curriculum in a middle grades mathematics classroom. Accordingly, my study sought to address these existing gaps in the research literature.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

Research has focused on the forms of knowledge required by teachers (Clandinin, 1986; Clandinin & Connelly, 1996; Darling-Hammond, 1996; Grossman, Wilson, & Shulman, 1989; McNamara, 1991; Shulman, 1986; Shulman & Sykes, 1986). Teachers' content and pedagogical content knowledge influences how they teach (Ball, 1990; Brown & Borko, 1992; Enderson, 1995; Loughran, Milroy, Berry, Gunston, & Mulhall, 2001; Putnam & Borko, 2000; Shulman & Grossman, 1988). This study documented the story of one teaching intern, describing the nature of the collaborative process between the teaching intern and his mentor teacher and how the sources of collaboration, the *Connected Mathematics Project* [CMP] (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998). The purpose of this study was to investigate how the teaching intern's emerging development of pedagogical content knowledge changed during the student teaching internship. In addition, this study sought to clarify the nature of collaborative process between a mentor teacher and teaching intern; in particular, the nature of the collaboration *on action* and *in action*. In considering both the nature of the collaborative process and the teaching intern's development of pedagogical content knowledge, I looked for ways the CMP curriculum shaped the teaching internship experience. Examining these sources will enrich our understanding of the complex interaction found during the student teaching internship. Next, I discuss the methods used to achieve this purpose.

Case studies are appropriate for examining one or more individuals (Creswell, 2003; Yin, 2003). A case study methodology (Stake, 1995) was used because I wanted to illuminate the complexity of the pairing of two individuals, an expert mentor teacher and a teaching intern, in their respective fields, teaching and preservice teaching. This study represented a case of understanding the initial and emerging development of pedagogical content knowledge, the nature of the collaborative process, and the influence of the CMP curriculum while a teaching intern participated with a mentor teacher in his student teaching internship. I expected a successful pairing of these two individuals in that I anticipated the nature of their collaborations would yield substantial development in the teaching intern's pedagogical content knowledge. Both individuals were characterized by colleagues, instructors, and peers as reflective learners and highly motivated in understanding the kinds of questions to pose to students and how to use student understandings to build a shared understanding of mathematics. Therefore, the practical, real-life examples the teaching intern encountered assisted in bridging the gap between theory and practice that currently exists in the research literature on the student teaching internship.

I examined the process by which the teaching intern's development of conceptions of purpose for teaching probability, knowledge of student understandings, curricular knowledge, and knowledge of instructional strategies changed during the teaching internship. In doing so, I employed several data collection procedures. The case of these two experts permitted a close examination of the role of the collaboration, the classroom context, and the use of the CMP curriculum influenced the development of pedagogical content knowledge for the teaching intern. In order to understand how these factors influenced the teacher intern's emerging development of pedagogical content knowledge, I used several data sources. Interviews, observations, a mathematics pedagogy assessment, and audio taped collaborations *on action* and of collaborations *in action* with both participants, informed the study.

In order to build the story of the teaching internship, a mentor teacher and teaching intern participated. The data I collected during the unit, *What Do You Expect?* (CMP, 1998), was analyzed. I generated inferences regarding the initial and emerging development of the teaching intern's pedagogical content knowledge and the nature of the collaborative process between the teaching intern and mentor teacher related to the teaching internship. More specifically, I collected data that addressed the following research questions:

1. *How did the sources of the collaboration, the CMP curriculum, and the classroom context influence the teaching intern's development of pedagogical content knowledge?* More specifically, how did these sources influence his conceptions of purpose for teaching probability, knowledge of student understandings, curricular knowledge, and knowledge of instructional strategies? What was the process by which the purpose for developing pedagogical content knowledge changed for the teaching intern?
2. *What is the nature of the collaboration between the teaching intern and mentor teacher?* In particular, what is the nature of the collaboration *on action* and the collaboration *in action*? In what ways did the curriculum influence the nature of the collaboration?

The research design of this study was driven by my conceptual framework (Figure 1.3) and reflected the complexities involved in answering the research questions. In the following pages I describe the design of the study, a description of the cases selected, the data collection methods, and data analysis techniques employed.

Research Design

LeCompte and Preissle (1993) stated that a qualitative study allows for an examination of a particular culture where a holistic focus on the phenomena can be studied in its natural setting. Because the study described and interpreted the nature of the collaborative process between a mentor teacher and teaching intern as a way to enhance our understanding of the interaction found in the teaching internship, I selected a qualitative case study as the mode of inquiry (LeCompte, Milroy & Preissle, 1992; Creswell, 1998). By using a case study methodology, my study illuminated the pairing of these two individuals. This study represented a case of the two experts in understanding the development of pedagogical content knowledge, the nature of the collaborative process with respect to collaboration *on action* and collaboration *in action*, and the influence of the CMP curriculum while a teaching intern participated with a mentor teacher in his student teaching internship.

The nature of the collaboration between these two individuals proved successful because their pairing yielded substantial development in the teaching intern's pedagogical content knowledge due to the dynamic interaction with the teaching intern's experiences with the environment as he constructed new knowledge about teaching probability. Both the collaboration, the classroom context, and the curriculum played considerable roles in initiating and guiding the teaching intern's learning efforts. Therefore, the practical, real-life examples the teaching intern encountered assisted in bridging the gap between theory and practice that existed in the research literature on the student teaching internship.

I sought to make interpretations in order to provide a greater depth of understanding about the process by which the purpose for developing pedagogical content knowledge changed for the teaching intern. A mentor teacher plays a considerable role in impacting the beliefs and teaching practices of the teaching intern (Carter & Gonzalez (1993). As well, a classroom utilizing a standards-based mathematics curriculum yields a unique opportunity in learning to teach (Remillard, 2000; Van Zoest & Bohl, 2002). Placed with an expert mathematics teacher, as defined by Shulman (1987), in a standards-based mathematics classroom, the teaching intern learned firsthand the importance of how to use student understandings to build a shared understanding of probability. Therefore, a description of the research context in which the mentor teacher and teaching intern resided follows.

The Research Context

The teaching internship resided in a school district located in a city of approximately 90,000 people in the Midwest. The district, with a total of approximately 17,000 students, has three middle grades schools for students in grades 6 and 7. Given the growth of standards-based mathematics curricula, it has become more common for teaching interns to work with mentor teachers implementing standards-based curricula so this district was chosen because of its adoption of the CMP curriculum. Moreover, the district supported implementation of CMP through on-going professional development. This professional development included: inservice days, workshops, and Saturday planning and implementation sessions.

The internship was in a seventh-grade mathematics classroom, which resided in a middle school with approximately 450 students per grade. In a school supportive of a "middle school philosophy" (Hopkins, 1997), the students remained with their regular education teachers (including the mentor teacher who teaches mathematics) for two years and were "looped" from sixth grade into seventh grade. The mentor teacher had taught most of his students for 1 2/3 years by the beginning of this study. The approximately 125 students on his team were divided among five groups, called "cores," that transitioned from one core content class to the next. The cores were identified by Core A, B, C, D, and E respectively. Due to time constraints and scheduling, only 4 of the 5 cores met for their core classes (language arts, science, mathematics, and social studies) on any given day. Therefore, the cores were "snaked" in an alternating schedule (see Table 3.1). Thus, the students had mathematics class four out of the five days of the scheduled school week. For this study, observations of lessons taught by the teaching intern occurred for Cores B and C, which resulted in varying times for the observations. The teaching intern chose to have observations of these two cores due to the make-up of the students in each core. Core B was primarily low-ability mathematics students. Core C was high-achieving mathematics students.

Monday	Tuesday	Wednesday	Thursday	Friday
Core A	Core E	Core D	Core C	Core B
Core B	Core A	Core E	Core D	Core C
Core C	Core B	Core A	Core E	Core D
Core D	Core C	Core B	Core A	Core E

Table 3.1: Team schedule for mathematics class ('snaking')

The length of each class period was 45 minutes. The students had experienced six CMP units in sixth grade (none of which dealt specifically with probability), and were working with their fifth unit in seventh grade. All students were enrolled in an enrichment skills class at the beginning of each school day where students reviewed skills related to mathematics, reading, and spelling. Four students took an advanced mathematics class before the school day began and were also enrolled in the regular mathematics class with fellow classmates. All students on the team chose two exploratory courses as electives, as well. Options for the electives included art, band, choir, computers, drama, family consumer science, foreign language, Industrial Technology, and orchestra.

With the teachers on a team teaching four of seven cores a day, the remainder of their time was devoted to an academic lab period for students to explore literature, a team planning time to collaborate about students, scheduling, and meet with parents. The last core period was a personal planning period to prepare for teaching. Furthermore, the mathematics teachers from each team in the school frequently collaborated as well. Some of their collaboration occurred during meetings held before and after school. Other times the mathematics teachers met during district professional development days, such as early dismissal sessions and Saturday sessions.

The mentor teacher in this study, Brad, is characterized as an expert mathematics teacher who possesses pedagogical content knowledge and knowledge of how to foster the understanding of specific mathematics concepts. A description of Brad, his teaching practices, investment in CMP, commitment to preservice teacher education students, and his prior years of teaching follows to provide evidence that supports the claim that he is indeed an expert mathematics teacher.

The Mentor Teacher

In relation to Brad's teaching, his instructional practices reflected an investigative approach to teaching and learning mathematics. From my observations of Brad facilitating inservice teacher workshops for his school district, the formal conversations we shared about instruction, along with my observations of him interacting with students in his classroom, it was clear Brad has an excellent reputation as a middle grades mathematics teacher. My observations of his students' small group collaborative discourse, the availability and use of manipulatives that lined the classroom walls made use by his students, it was evident Brad's style in conducting his classroom was one of discovery where mathematicians were busy at work.

In relation to Brad's use of CMP in his school district, he exhibited considerable "buy-in" in terms of teaching CMP in the spirit of the NCTM *Standards* (2000). In particular, he participated in two users' conferences sponsored by CMP, one of which was an advanced users' workshop, where he learned about best teaching practices so to implement the curriculum in the spirit of the NCTM *Standards*. Not only had Brad participated in his school district's professional development opportunities, but he also served as a teacher leader for professional development related to middle school mathematics within his district and across the state. He was instrumental in leading professional development sessions about his experiences in teaching CMP at his school, with teachers in his district, and across the state. In an informal conversation with Brad I learned he had been open about his CMP instructional practices from his beginning use of CMP and actively looked for avenues where he could learn about CMP from other classroom teachers.

Brad's commitment in working with preservice mathematics education students and beginning teachers in his district was substantial. In fact, methods students and teaching interns are often placed with him for field experiences. From my experience as a methods instructor whose preservice teachers participated in field experiences with Brad, I was aware of how he encouraged his preservice students to actively participate with his middle grades students so that they learned more about student understandings of mathematics. He engaged the preservice teachers and teaching interns in reflective thought, posing questions about mathematics teaching, which these conversations were often shared in preservice teachers' reflective journals for their methods courses as a component of learning from the field experiences.

Brad had been a middle grades mathematics teacher for 12 years residing at the same middle school for most of those years. He holds a Bachelor's in Elementary Education with a minor in Psychology and a Master's in Educational Technology. He was certified to teach K-8 mathematics and Social Studies. At the time of this study he had taught CMP for seven years. Since his school district adopted full implementation of the curriculum, he taught CMP exclusively for the past four years. Considered an expert mathematics teacher as evidenced by the factors I have described, a teaching intern was assigned to work within this expert teacher's mathematics classroom. Therefore, Brad was invited to participate in this study. He willingly opened his classroom, his instructional practices, and his students to a semester of data collection so that I may study the pairing of two experts in their respective fields, mathematics teaching and preservice mathematics teaching.

The Teaching Intern

The teaching intern, Randy, was in his final semester of undergraduate work at the local university where he majored in middle school mathematics education. Upon completion of his degree, his certification included a middle grade teaching certification, with endorsements in the areas of mathematics (grades 5-9) and Art (K-12). During his teaching internship, he devoted six weeks to an internship in Art within the same building as his ten-week internship in a middle grades mathematics classroom. In a personal conversation with me, Randy mentioned his primary area of interest was teaching the lower middle grades mathematics, either fifth- or sixth-grade. This was partly due to feeling uncomfortable with teaching older students at a level of mathematics he felt he was not adequately prepared to teach. He had particular concerns with regard to his mathematics content knowledge in the secondary grades.

Prior to his participation in this study, Randy's middle grades mathematics methods instructors—my dissertation advisor and I—characterized him as one of the best methods students in their preservice mathematics teacher education program. He was resourceful in that he continuously sought after the best approaches to teaching mathematics. In a conversation with one of his methods instructors, he described Randy as a preservice teacher who actively looked for ways to understand how students learn mathematics. From interviewing classroom teachers to interviewing students, he honed his questioning skills and made each experience a learning experience. He sought opportunities to plan and teach lessons with students and often stayed after his scheduled field time to talk about teaching, reflect on his instructional practices, and learn from his supervising teachers' observations and knowledge about teaching. I learned of his eagerness from his journal writings and through a conversation with a field placement teacher.

Through journal writing and personal conversations with Randy, I learned of his reflective nature. This enabled him to grow in his experiences as a future classroom teacher and prepared him for a teaching internship where he could learn even more about how to teach CMP; more specifically, he could learn how to pose questions to students and how to use student understandings to build a shared understanding about mathematics. Randy seemed to be an ideal fit for a teaching internship with Brad.

It is worth noting that prior to his teaching internship, his methods courses at the university, along with his field experiences, gave Randy the opportunity to teach lessons from CMP. His methods courses showcased several of the standards-based mathematics curricula. Randy took the opportunity to learn about each and compare the curricula with more traditional textbooks in order to make informed judgments about the differences in instructional design. For one assignment in his methods courses, Randy conducted an in-depth analysis of one of the CMP units and observed the teaching of that unit in his field placement. This experience led him to discover the strength of the mathematics presented in such curricula and the dilemmas the field teacher and students faced. He was eager to observe teachers teaching CMP, and ultimately, eager to teach the curriculum. Randy was given opportunities to teach students with CMP and asked for additional opportunities to teach even more. His preservice mathematics methods instructors both commented that it was rare to teach a preservice teacher with the drive and determination that he had so early in his career.

The Classroom Context

In describing the classroom context, focus is placed on the grouping of students in each core and the style of teaching the students had become accustom to in Brad's classroom. Early in the school year and prior to this study, the team of students was divided among five cores. I gathered data from two of those five; Core B and C. The students in Core B were characterized by Brad as below-average mathematics students. Some struggled to comprehend mathematics concepts and skills, while others felt less confident about mathematics and struggled to feel comfortable around those students who did excel. Therefore, students in Core B experienced mathematics class, at times, differently from the other cores by how Brad tailored his lessons. Brad described this difference as more traditional, teacher-directed lessons than problem-centered or with handouts, which he prepared in advance to assist students in organizing their thinking. Rarely was assigned homework different from the other cores, however. Students in Core C were characterized by Brad as above-average students. The majority of students in this core excelled at mathematics. They were competitive and I observed how much they enjoyed sharing how they thought about mathematics with their peers.

In addition, both cores of students in Brad's classroom were accustomed to how the *he* taught mathematics. As mentioned in an earlier section of this chapter, these students transitioned with Brad from 6th grade to 7th grade. Therefore, the students were accustomed to learning mathematics from him. They experienced mathematics through an investigative, problem-centered approach where Brad's style in conducting his classroom was one of discovery. It became evident to me through my observations of Brad interacting with his students at the start of the teaching internship that his students explored mathematics, were encouraged to describe their thinking and were prepared to share their thinking because of the value Brad placed on understanding how they think about mathematics.

The Connected Mathematics Project Curriculum

The Connected Mathematics Project (CMP) curriculum was developed for the middle grades through funding by the National Science Foundation. The curriculum's stated intent was to help students, "develop understanding of important concepts, skills, procedures, and ways of thinking and reasoning in number, geometry, measurement, algebra, probability, and statistics" (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998; p. 1). Key features of this curriculum are that it is problem-centered, and organized around a select number of important mathematical content and process goals. It also emphasizes significant connections, making the mathematics meaningful within the discipline, as well in other disciplines. The instructional model in CMP emphasizes inquiry and discovery of mathematical ideas based in rich problem situations where students are encouraged to develop their own thinking in a collaborative environment.

The unit of focus for this study was *What Do You Expect?*(CMP, 1998). This probability unit was intended to deepen student understandings of the basic concepts of probability.

> [The students] develop several powerful strategies for finding and interpreting experimental and theoretical probabilities, such as simulations to gather experimental data; using counting trees and other listing techniques to determine all the possible outcomes in a situation; and creating area models, in which the probability of each possible outcome is represented as portion of the whole. In addition, student will use areas models to determine probabilities in two-stage situations and to informally develop an understanding of expected value, or long-term average. (Lappan, Fey, Fitzgerald, Friel, and Phillips, 1997; p. 1a)

Brad relied on the CMP curriculum as the only source used for instruction. Therefore, Randy utilized this probability unit as his main curriculum source for determining what and how he taught students in Brad's classroom.

Sources and Data Collection

In order to understand the nature of the collaborative process between the mentor teacher and teaching intern and to investigate how the sources of pedagogical content knowledge found in a student teaching internship influence the teaching intern's development of pedagogical content knowledge, several data were required. Specifically, to understand how the teaching intern was influenced by collaboration, the classroom context, and the curriculum, I used the following data sources: (a) audio taped collaborations, (b) observations of the teaching intern, (c) interviews with the teaching intern, (d) and a mathematics pedagogy assessment administered to the teaching intern at the beginning and end of the internship, and to the mentor teacher at the beginning of the internship. These data sources allowed investigation into the initial and emerging development of pedagogical content knowledge for the teaching intern and functioned as a method of interpreting the mentor teacher and teaching intern's understandings of teaching and teaching probability.

I collected data throughout the school day. A sample overview of the data collection schedule for Investigation 2, "Analyzing Number-Cube Games", is provided in Table 3.2; data collected on planning and implementing Investigation 4, "Analyzing Two-Stage Events", was essentially identical. This sequence of events reflected how Brad and Randy collaborated, when observations took place, the coding and summary of the lessons taught, and when Randy was interviewed.

Audio Taped Collaborations

In order for me to understand the nature of the collaborative process between Brad and Randy, the nature of the collaboration *in action* and *on action*, and to look for

Monday	Tuesday	Wednesday	Thursday	Friday
	Possible Collaboration *On Action* audio taped (focusing on Investigation 2 or Lesson 2.1)	Collaboration *On Action* about Lesson 2.1 audio taped; Listened to *On Action* Audiotape	Collaboration *On Action* about Lesson 2.1 audio taped	Collaboration *On Action* about Lesson 2.2 audio taped;
		Observe the teaching of Lesson 2.1 to Core B; Record field notes and *In Action* that took place	Observe the teaching of Lesson 2.1 to Core C; Record field notes and *In Action* that took place	Observe the teaching of Lesson 2.2 to Core B; Record field notes and *In Action* that took place
				Observe the teaching of Lesson 2.2 to Core C; Record field notes and *In Action* that took place
		Code and Summarize the teaching of Core B	Code and Summarize the teaching of Core C	Code and Summarize the teaching of Core B and C
	Possible Collaboration *In Action* audio taped during Lesson 2.1 to Core A		Interview Teaching Intern about the teaching of Lesson 2.1 to Core B and C	Interview Teaching Intern about the teaching of Lesson 2.2 to Core B and C
	Collaboration *On Action* audio taped (focusing on Lesson 2.1)	Collaboration *On Action* audio taped regarding Lesson 2.1	Collaboration *On Action* audio taped regarding Lesson 2.1 and Lesson 2.2	Collaboration *On Action* audio taped regarding next investigation

Table 3.2: Sequence of events during data collection[4]

ways the curriculum influenced the nature of the collaboration, Brad and Randy documented their collaboration episodes so that I could address the research questions. To capture these collaborations, they audio taped their joint planning sessions (*on action*).

[4] The Monday column is blank due to the timing of when the lessons for Investigation 2 and Investigation 4 occurred. No lessons for either of these investigations occurred on a Monday.

As well, they audio taped the instances when they collaborated *in action*, during transitions in the lesson when Randy was teaching, or between classes. They found carrying the audiotape recorder around with them and recording when necessary was more accommodating than writing about their collaborations.

Collaboration on action. Fourteen joint planning sessions were audio taped and transcribed during which Brad and Randy prepared for lessons so that examination could be made about the *on action* employed. Six of these collaboration episodes occurred before the observations of the Randy's teaching of Investigation 2, "Analyzing Number Cube Games." A sample excerpt of collaboration *on action* appears in Figure 3.1.

Randy: ...What do you think about printing worksheets for that lower group? As opposed to having them work through the book? Now, do they have that?

Brad: Do you mean, do they have transparencies to go along with it?

Randy: Yeah, here we go (turns in the teacher pages). A Labsheet for them to record their numbers. I know one of the students said, 'Where is our piece of paper?' And I said, 'Well, you have to figure out how to keep track of it.' I think for that class, if I had a Labsheet, like page 126,

Brad: (looking at Labsheet) Instead of just odd or even. It has a place for them to mark.

Randy: So they mark their sum. And so it's a place for them to keep organized. They don't have to, but I think that when there *IS* a Labsheet available, they should use it…

Figure 3.1: Sample of collaboration *on action* episode

Collaboration in action. Fourteen collaboration *in action* episodes occurred during the teaching internship. Both Brad and Randy documented via audiotape the instances where collaboration occurred outside of the joint planning sessions. Some of these instances occurred in the middle of Randy's teaching and others occurred between classes. All instances were brief in nature. Furthermore, because I was not embedded in the teaching internship, it was important to gather information regarding the collaboration that existed outside of joint planning sessions. A sample excerpt of collaboration *in action* appears in Figure 3.2.

Brad: Okay, we're still on *Making Purple* in the second class (students working at desks during this collaboration). What did you want to get to in the first class that you didn't?

Randy: Well, it's basically laying the ground work for Expected Value.

Brad: Did they get to list the possibilities?

Randy: No, we didn't get to the Follow-Up.

Brad: How many in the class do you think actually finished the problem A through F? (pause) No Follow-Up. Just A through F? 3 of the groups? All 6 groups?

Randy: They just weren't doing it. I guess they didn't understand the directions? I walked by and they were just sitting there….

Figure 3.2: Sample of collaboration *in action* episode

Classroom Observations

Classroom observations included my taking field notes during Randy's lessons and audio of Brad and Randy's collaborations *in action* that occurred during my observations. The data collected during observations related to both research questions. The first research question was to identify how Randy's initial and emerging development of pedagogical content knowledge changed during the teaching internship. Therefore, observations of Randy's teaching allowed me to gain access into his use of pedagogical content knowledge in practice. I observed Randy's use of integration of knowledge and instructional strategies while in the act of teaching students. Also, by comparing the lessons provided in the CMP curriculum to the lessons taught by Randy, the observations served to document his ability to draw upon his knowledge with flexibility by looking for multiple ways of representing mathematical ideas, including a variety of models to foster students' conceptual understanding while in the act of teaching. Part of the second research question was to understand the nature of the collaborative process between Brad and Randy, and the nature of collaboration *in action* that occurred, which allowed me to observe when and why these collaboration episodes took place. Collaboration *in action* occurred when Brad and Randy collaborated during formal instruction, making adjustments in Randy's instruction as needed, or while students transitioned between classes. On the day of my observations I was better able to understand these collaborations by observing Brad and Randy's interactions in the classroom firsthand.

Randy spent most of his internship teaching one CMP unit, *What Do You Expect?* When he arrived, Brad finished the CMP unit, *Filling and Wrapping*, where Randy observed his students taking the school district's common unit assessment for that unit. When Randy assumed responsibilities of teaching, students learned about probability exclusively from him through his teaching of *What Do You Expect?*

Eight classroom observations of Randy occurred over the course of four lessons. I observed Randy teach each lesson twice, typically back-to-back within the same school day. Due to the daily change in his schedule (described earlier as "snaking"), there were a few instances in which Core B occurred at the end of one day and Core C started the beginning of the next day. Observations occurred during Investigation 2, "Analyzing Number-Cube Games", and Investigation 4, "Analyzing Two-Stage Games". More specifically, I observed Randy when he taught Lesson 2.1, Playing the Addition Game, Lesson 2.2, Playing the Multiplication Game, Lesson 4.1, Choosing Paths, and Lesson

4.2, Finding the Best Arrangement, to Core B and Core C. This observation schedule allowed the opportunity to delve further into Randy's conceptions of purpose in teaching probability and his use (or nonuse) of student understandings to plan subsequent lessons.

Investigations 2 and 4 were chosen due to the nature of the probability content presented in these two investigations. It also allowed time for Randy to become comfortable in his new surroundings as a teaching intern. As well, audio taped collaborations *on action* served as documentation early in the internship of his concerns, misconceptions, and questions regarding teaching probability before my observations of his teaching. Four audio taped collaborations *on action* between Brad and Randy were recorded prior to the observations. These collaborations also served as documentation of the mentor teacher/teaching intern relationship and how Brad aided Randy in how his development of pedagogical content knowledge changed during his teaching of probability. Furthermore, during the first investigation of the unit, the students' schedules were altered due to several days of state-mandated testing. Randy was juggling the newness of his internship, working with students and Brad, and becoming accustomed to the structure of his daily teaching schedule.

Once observations of Investigation 2 began, every attempt was made to listen to the audio taped collaborations prior to the observations or at least before the post-instructional interviews at the end of the sequence of two lessons. Some of the collaborations *on action* occurred during the thirty minutes immediately before Randy's teaching and my observations, so listening to and analyzing those collaborations proved difficult. However, having information regarding the collaboration *on action* and *in action* that occurred, along with two observations of the same lesson taught and coded, the questions in the interviews (discussed later) were in direct relation to these earlier events.

I chose Investigation 4 as it occurred towards the end of the teaching internship. This allowed for a break in observations as Randy discussed and taught Investigation 3 and transcription of data from Investigation 2 occurred. Additionally, the students in Brad's school piloted a state test during this time and schedules were again altered. Randy was able to teach two lessons from Investigation 3 uninterrupted by my research or evaluations from the university supervisor or principal at his school. During this time, Brad and Randy continued to collaborate *on action* and *in action* as Randy were still responsible for full-time teaching. The observations ended after Lesson 4.2 due to the end of Randy's full-time teaching requirements during his teaching internship.

Field notes of classroom observations. The purpose of the field notes was to address the first research question in that the field notes documented Randy's pedagogical content knowledge while interacting with the classroom context, the use of the CMP curriculum, his instructional strategies, specifically questioning skills. The second research question was also addressed in the field notes by documenting the collaboration *in action* employed. During each lesson, a narrative was kept which described the lesson and its timeline, the role of Randy, the role of the students, the types of instructional strategies used, and how the curriculum and manipulatives were used. Field notes were recorded using a classroom observation narrative (see Figure 3.3). This allowed for comparisons to be made regarding what Brad and Randy collaborated about with regard to how Randy planned to teach and what actually occurred once students became participants in these lessons.

Observation Narrative

Date of Observation: ________________ Core ___________

Lesson Observed: ____

Start Time: __________ End Time: ___________

Time Stamp	Observations of Teaching Intern, students, and instructional strategies used	Code
	(1)	
	(2)	
	(3)	
	(4)	
	(5)	
	(6)	
	(7)	
	(8)	
	(9)	
	(10)	
	(11)	
	(12)	

Figure 3.3: Classroom observation narrative

I gathered additional data on Randy's use of pedagogical content knowledge in practice by listening to and documenting his involvement with the mathematics and student understandings while in the act of teaching. I wanted to connect Randy's actions with students in the classroom to prior collaborations about his intentions regarding the teaching of the lessons. The collaboration *in action* between Brad and Randy was documented firsthand, which afforded an on-site observation of what caused the *in action* to occur and how it was remedied. Therefore, eight sets of field notes were gathered. Two sets for each lesson were then coded and scored using an observation protocol before each of the interviews took place.

Interviews

Following the pair of observations of each lesson, I interviewed Randy for approximately 45-90 minutes to gain information regarding his understandings of the lesson. The purpose of the interviews was to address both research questions, in that the interviews serve to explore Randy's initial and emerging development of pedagogical content knowledge and its importance to teaching, the nature of the collaborative process with Brad, and Randy's use of the CMP curriculum (see Figure 3.4). The interviews included core questions that were designed to understand the decisions Randy made related to the use of the curriculum, his instructional practices, and his knowledge of student understandings. I wanted to capture Randy's thinking in order to clarify what was observed and to determine why specific decisions were made regarding the lesson. The interview questions also served to probe broader issues related to Randy's teaching internship. All interviews were semi-structured in nature in order to create a friendly conversation between Randy and myself, as well as to incorporate questions that arose in response to the statements made by Randy.

Mathematics Pedagogy Assessment

The assessment of pedagogical content knowledge in mathematics was designed to assess pedagogical content knowledge needed for teaching probability in middle grades mathematics. Pedagogical content knowledge is the type of knowledge teachers possess when they understand the mathematics they are teaching and are able to draw on

Sample Interview Questions

1. How well prepared did you feel to teach Lesson 4.1 and why?
    a. What helped you prepare for teaching this lesson?
2. Today's lesson was about designing a simulation and dependent events.
    a. Why should students learn about simulations and dependent events?
    b. How did you decide what you were going to teach?
    c. Why did you decide to teach the lesson in the manner that you did?
    d. How did you decide how to teach the lesson?
    e. Why did you decide to teach the lesson in the manner that you did?
3. What mathematics content was the main focus for today's lesson?
4. How does the area model help students understand the mathematics?
5. What modifications, if any, did you make after teaching once?
    a. Why were those changes made?
    b. What was the influence behind making that decision?
6. Area models were used to represent probability of dependent events.
    a. What role did the model play in students' understanding of experimental and theoretical probability?
    b. What is it about (dependent events; area models) you want your students to understand?
    c. How do you assist your students in making a connection between dependent events and area models?
    d. Tell me whether your students are comfortable with independent and dependent events?
        i. Do they understand the differences between the two?
        ii. Do they recognize when a problem context is one or the other?
7. When you and your mentor teacher collaborated before this lesson,
    a. What did you mainly talk about?
    b. How did you decide to focus on that?
    c. How did this collaboration assist you in teaching Lesson 4.1?
    d. Having enacted the lesson, what do you wish you would spend more time talking about?
8. When you collaborate *on action*,
    a. What do you primarily focus on?
    b. How does this influence your teaching?
    c. Having collaborated *on action*, what do you wish you would spend more time talking about?
9. When you collaborate *in action*,
    a. what do you primarily focus on?
    b. How does this influence your teaching?
    c. Having collaborated *in action*, what do you wish you would spend more time talking about?
10. How will you use what you learned today from teaching Lesson 4.1, collaborating with your mentor teacher, and listening to your students in Lesson 4.2?

Figure 3.4: Sample interview questions

that knowledge with flexibility by looking for multiple ways of representing mathematical ideas, including a variety of models, to foster students' conceptual understanding. This data source allowed me to address both research questions in that it documents the reason for the collaboration episodes that addressed the dilemmas the teaching intern faced with regard to learning to teach probability and it also provides evidence of the initial pedagogical content knowledge of the teaching intern for teaching probability at the beginning of the internship and at the end in order to measure pre/post-test gains. It was adapted for the teaching of probability from constructed-response items of the Mathematics: Pedagogy Test (#30065) of *The Praxis Series: Professional Assessments for Beginning Teachers* (additional information can be found at: www.ets.org/praxis/prxtest.html) developed by Educational Testing Service [ETS] (1993, 2000). The three-question assessment (see Appendix A) was administered with a one-hour time limit and no other available resources. The assessment focused on the Process Standards in relation to pedagogy, as embodied by the *Principles and Standards for School Mathematics* (NCTM, 2000). It addressed the preparation for teaching probability: planning (i.e., the preparation for teaching); implementation (i.e., the presentation of material); and assessment (i.e., the evaluation of student understanding and teacher performance) as the teaching intern organized his content knowledge related to probability into lesson plans for teaching probability. The mathematics content of the assessment focused on experimental and theoretical probabilities, two-stage events, and independent and dependent events, which are concepts taught in lessons related to probabilistic reasoning in the middle grades.

Randy was assessed at the beginning and immediately following his teaching internship, in order to consider pre/post gains. This assessment was used to capture his initial pedagogical content knowledge of probability in order to understand his growth in such knowledge. It was also administered to Brad at the beginning of the internship in order to understand his pedagogical content knowledge in teaching probability.

Data Analysis

These multi-data sources were analyzed using a constant-comparison method (Strauss & Corbin, 1990). In the form of a case study, the purpose for using a constant-comparison method was to triangulate the data in order to develop a rich description of the participants' collaborations, influences on how the teaching intern developed pedagogical content knowledge, and the influence of the CMP curriculum as both the mentor teacher and teaching intern's thoughts played out in their actions in the classroom. I looked for key issues and activities in the data that become categories of focus. With new data informing existing data, patterns emerged. I began to reduce the data by coding these core categories and integrating the new material back in order to refine the codes, which allowed new themes and dilemmas to emerge. These themes assisted in developing theory to funnel through while continually searching for new incidents.

Because the data analysis informed data collection, analysis involved several phases (see Table 3.3). In the first phase, I analyzed the nature of the initial collaborations from the transcripts of the *on action* audiotapes that occurred before Randy was observed teaching Investigation 2. The analysis consisted of understanding what collaboration took place, who initiated the collaboration, whether the focus of the collaboration was about the CMP curriculum, the mathematics, or the teaching of the

Data Collection Phases	Steps in Data Collection:
Phase I: Pre Observations	1. Listen to and begin analysis of collaboration *in action* and *on action* audiotapes before the teaching of Lesson 2.1; 2. Record notes concerning themes that emerged
Phase II: Investigation 2 Data	1. Listen to collaboration *on action* audiotapes; 2. Observe and take field notes of Lessons 2.1 and 2.2; 3. Code field notes; score coding; 4. Interview teaching intern about teaching Lesson 2.1 and 2.2; 5. Transcribe all collaborations and the two interviews; 6. Triangulate data sources(collaborations, interviews, observations) and analyze new data by reading and coding
Phase III: Investigation 4 Data	1. Listen to collaboration *on action* audiotapes; 2. Observe and take field notes of Lessons 4.1 and 4.2; 3. Code field notes; score coding; 4. Interview teaching intern about teaching Lesson 4.1 and 4.2; 5. Transcribe all new collaborations, the two interviews; 6. Triangulate data sources (collaborations, interviews, observations) and analyze new data by re-reading and re-coding
Phase IV: Data Collection Complete	1. Triangulate all data sources; 2. Analyze so to refine themes and dilemmas; 3. Begin written analysis

Table 3.3: Data collection and analysis phases

mathematics.

In the second phase, I collected data regarding Randy's teaching of Investigation 2, "Analyzing Number-Cube Games". This consisted of Brad and Randy's collaboration *on action*, the observations, and field notes of both lessons taught twice (to Core B and Core C) by Randy. Next, the field notes were coded and scored. Details about the coding process are provided in a subsequent section. Then, an interview with Randy was conducted where the interview questions were adapted to consider themes that emerged from the audio taped collaborations and the observations. In the time between Investigation 2 and Investigation 4, audiotapes were transcribed (the collaborations and the two interviews), and triangulation of these data sources began by reading and coding the new data.

The third phase consisted of the same routine as the second phase, but with observations and I collected data for Investigation 4, “Analyzing Two-Stage Events.” Some of the collaboration *on action* audiotapes in this phase was collaborations regarding Investigation 3, “Probability and Area.” While data on the teaching of Investigation 3 were not collected, the collaborations *on action* and *in action* were collected and analyzed for the purpose of understanding the nature of their collaboration.

Again, after the last interview of Investigation 4, audiotapes were transcribed (the collaborations and the two interviews) and I re-read and re-coded the data in order to capture the themes and dilemmas that existed. As themes derived from already developed categories, new categories were defined. No incidence of data was discarded; rather, a miscellaneous category was used in order to consider these data in the end. Therefore, through this learning to teach process, the categories were continuously revised until fewer modifications were needed and the themes became well defined and saturated with data.

Audio Taped Collaborations

All audio taped collaborations were analyzed in order to allow for investigation into how the teaching intern’s initial and emerging development of pedagogical content knowledge changed during the teaching internship and to clarify the nature of the collaboration. Analysis began by first identifying what collaboration took place and who initiated the collaboration. Delving further into data, it was apparent collaboration focused on Randy’s understanding of pedagogical content knowledge, the influence of the CMP curriculum, the classroom context, and the influence these collaboration sessions had on his pedagogical content knowledge. These recurrent events produced additional key issues that revealed how Randy's development of pedagogical content knowledge changed was limited due in part by his content knowledge related to probability. As well, the nature of the collaboration regarding teaching CMP followed only after Randy understood the content first. Therefore, recoding these core categories to include the influence of Randy's content knowledge allowed new themes to emerge. To ensure the accuracy of codings, check-coding was performed by my dissertation advisor. In particular, he applied the coding scheme described above to the transcription of one audio taped collaboration session. Agreement was achieved on the assignment of 18 out of 20 codes; that is, a reliability of 90%. Disagreements in the codings were resolved through discussions and the negotiated codings were used in subsequent qualitative analyses.

As additional audio taped collaborations were analyzed, my analysis of the data continued to inform existing data as the analysis kept doubling back. Other data sources, the interviews and observations, continued to saturate and refine the codes. Constant comparisons enabled variations to be found in the data. Ultimately, the analysis of the data assisted me in developing a theory of the process in the development of pedagogical content knowledge for Randy. Consideration was given with regard to the process by which Randy's pedagogical content knowledge changed and the dilemmas Randy faced in his thinking about mathematics teaching and how he tried to resolve these dilemmas.

Mathematics Pedagogy Assessment

The assessment of pedagogical content knowledge in mathematics was scored using a mathematics pedagogy scoring rubric that was adapted from a rubric used for scoring the Mathematics: Pedagogy Test (#30065) of *The Praxis Series: Professional Assessments for Beginning Teachers* (additional information can be found at: www.ets.org/praxis/prxtest.html) developed by ETS (1993, 2000)) (see Table 3.4).

Score	A score was assigned based on the following scoring rubric:
5	• Clearly demonstrates an understanding of the mathematics to be presented • Clearly explains how to present the mathematics to students in a way that is likely to achieve the desired goals(s) • Gives a clear and complete response • Develops the mathematics in a way that is well-motivated (that is, students can clearly see why the mathematics being presented is worth studying and/or see the mathematics as the logical consequence of previously studied mathematics)
4	• Clearly demonstrates an understanding of the mathematics to be presented but may have a notational error or minor mathematical misstatement • Explains how to present the mathematics to students in a way that can reasonably be expected to achieve the desired goal(s) • *Either* gives an almost complete response and a well-motivated development of the material OR gives a complete response and a fairly well-motivated development of the material
3	• Demonstrates an understanding of the mathematics to be presented • Indicates how to present the mathematics to students in a way that can reasonable be expected to achieve the desired goal(s) • *Either* gives an almost complete response and a well-motivated development of the material OR gives a complete response and a fairly well-motivated development of the material
2	• *Either* demonstrates a limited understanding of the mathematics to be presented (and may or may not indicate how to teach the mathematics to students in a way that is likely to achieve the desired goal(s)) OR demonstrates an understanding of the mathematics but gives little or no indication of how to present the mathematics to students in a way that is likely to achieve the desired goal(s) • Gives an unclear and incomplete response
1	• *Either* demonstrates a very limited understanding of the mathematics to be presented OR fails to discuss the mathematics at all
0	• Blank, almost blank, or off topic

Table 3.4: Mathematics pedagogy scoring rubric

The assessment focused on the Process Standards in relation to pedagogy, as embodied by the *Principles and Standards for School Mathematics* (NCTM, 2000) and was used to capture Randy's pedagogical content knowledge of probability and his growth in such knowledge. Each of the three questions were scored and analyzed in relation to Randy's pre/post-test gains and sought to capture his use of pedagogical content knowledge.

Coded Field Notes of Observations

Once field notes of the observations were recorded for two corresponding lessons, the field notes were coded for the purpose of evaluating Randy's teaching (see Table 3.5) in order to understand Randy's instructional practices as they related to student learning. Coding of the field notes consisted of identifying the events and actions of Randy's lesson with regard to the *Assessment Criteria for the Praxis Series: Classroom Assessments for Beginning Teachers*™ (ETS, 1993, 2000). Three of the four domains were used in order to understand Randy's instructional practices during instruction when students became an additional source in his development of pedagogical content knowledge. The domains included: organizing content knowledge for student learning, creating an environment for student learning, and teaching for student learning. The fourth domain, teacher professionalism, involving professional relationships with colleagues and communicating with parents, was beyond the scope of this study. Within each domain were four to five criteria, "which represent important, interrelated, aspects of the complex phenomenon of teaching as defined in the Praxis III guiding conception of teaching" (Dwyer, 1998; p. 169). These criteria were used to code the field notes and it further supported documentation and evaluation for the observations.

Coding for Field Notes

Domain A: Organizing Content Knowledge for Student Learning	Domain B: Creating an Environment for Student Learning
A1: Becoming familiar with relevant aspects of students' background knowledge and experiences. A2: Articulating clear learning goals for the lesson that are appropriate to the students. A3: Demonstrating an understanding of the connection between the content that was learned previously, the current content, and the content that remains to be learned in the future. A4: Creating or selecting teaching methods, learning activities, and instructional materials or other resources that are appropriate to the students and that are aligned with the goals of the lesson. A5: Creating or selecting evaluation strategies that are appropriate for the students and that are aligned with the goals of the lesson.	B1: Creating a climate that promotes fairness. B2: Establishing and maintaining rapport with students. B3: Communicating challenging learning expectations to each student. B4: Establishing and maintaining consistent standards of classroom behavior. B5: Making the physical environment as safe and conducive to learning as possible.
Domain C: Teaching for Student Learning	**Domain D: Teacher Professionalism**
C1: Making learning goals and instructional procedures clear to students. C2: Making content comprehensible to students. C3: Encouraging students to extend their thinking. C4: Monitoring students' understanding of content through a variety of means, providing feedback to students to assist learning, and adjusting learning activities as the situation demands. C5: Using instructional time effectively.	D1: Reflecting on the extent to which the learning goals were met. D2: Demonstrating a sense of efficacy. D3: Building professional relationships with colleagues to share teaching insights and to coordinate learning activities for students. D4: Communicating with parents or guardians about student learning.

Table 3.5: *Pathwise/Praxis III* domains (ETS, 1993, 2000)
Additional information can be found at:
www.bgsu.edu/colleges/edhd/programs/MentorNet/pathprax.html

Observation Protocol

Following the coding of the observation field notes, I scored the coding using the *Scoring Rules for Praxis III: Classroom Performance Assessments*™ (ETS, 1993) (see sample in Table 3.6) as a means for evaluating Randy's teaching in relation to student learning. A scale of 1.0 to 3.5 in 0.5 intervals with 3.5 as the highest score, allowed for varying degree of evaluation of Randy's lessons. I trained for use of the *Assessment Criteria and Scoring Rules* by participating in *The Praxis III: Recalibration System* (ETS, 2002). This was an instrument used by assessors to collect data, analyze, and score

Score	Domain B, Code 3: Communicating challenging learning expectations to each student
1.0	The teacher communicates explicitly or implicitly to individuals, to groups within the class, or to the class as a whole that they are incapable of learning or that they teacher's expectations for their learning are very low.
1.5	Above level 1.0, but below level 2.0
2.0	The teacher does noting to communicate to any student that he or she is incapable of meeting learning expectations.
2.5	Above level 2.0, but below level 3.0
3.0	The teacher actively encourages students to meet challenging learning expectations.
3.5	Above level 3.0

Table 3.6: Sample of the *Assessment Criteria and Scoring Rules* (ETS, 1993) as provided in Dwyer (1998).

observations of beginning teachers. The training aided in my ability to facilitate consistent, accurate, and fair assessments of Randy's teaching. The components used allowed me to focus on evaluating Randy's instructional practices for the purpose of understanding how Randy's emerging development of pedagogical content knowledge changed. As stated in previous chapters, pedagogical content knowledge is the type of knowledge teachers possess when they understand the mathematics they are teaching and are able to draw on that knowledge with flexibility by looking for multiple ways of representing mathematical ideas, including a variety of models, to foster students' conceptual understanding. I summarized the coding of the field notes for each lesson by breaking each domain into its four or five criteria. Each criterion was considered separately, allowing summary statements to be written.

Eight separate scores summarized the observations, allowing for comparisons to be made regarding the lessons taught during Core B and Core C, along with lessons taught over two days for Investigation 2 and Investigation 4. As well, comparisons were made regarding Randy's scores over the duration of his teaching internship.

Audio Taped Interviews

The interviews were transcribed and analyzed to understand the teaching intern's interpretations of the teaching internship experience. More specifically, questions presented in the semi-structured interviews explored Randy's pedagogical content knowledge and its importance to teaching, the nature of the collaboration with Brad and its influence on how his pedagogical content knowledge changed, and Randy's use of the CMP curriculum. Understanding the decisions he made related to these components provided further insight into Randy's thinking, the dilemmas in teaching he still faced, and the specific decisions he made regarding teaching and offered clarification with what was observed. My analysis of this data was folded into the analysis of the collaboration data where recoding of core categories again allowed new themes to emerge. The interviews revealed the influence the curriculum, Brad, and the students made on Randy's emerging pedagogical content knowledge, the dilemmas he faced and his interpretation of that knowledge. Ensuring the accuracy of my coding, check-coding was applied by my dissertation advisor for one interview. Agreement was achieved on the assignment of 27 out of 29 codes; that is, a reliability of 93%. Disagreements in the codings was resolved through discussions, and the negotiated codings were used in subsequent qualitative analyses.

The lessons revealed differences in Randy's teaching from what was initially collaborated about and planned with the Brad prior to the lesson. The interviews served to document Randy's response to the decisions he made regarding his teaching and why differences were found. The additional dimension from the act of teaching further clarified Randy's *true* development and use of pedagogical content knowledge and the interview data further aided my analysis of Randy's decisions regarding his teaching internship experiences. Information gleaned from the interviews were constantly compared to the other data sources in order to develop a theory of the development of pedagogical content knowledge of the teaching intern.

Summary

As the researcher, it was my role in this study to investigate how the sources of collaboration, curriculum, and the classroom context influenced the development of pedagogical content knowledge for the teaching intern and I examined the nature of the collaborative process between the teaching intern and mentor teacher. As it was important for the findings in this study to be grounded in the data, I collected multiple data sources and triangulated the data by conducting several iterations of data analysis. Prior to this study, I developed a positive working relationship with both participants, which was important for the type of data collection described due to the fact that the interviews in particular were semi-structured in nature, allowing a conversation-type atmosphere to exist. The participants were assured that my role, as researcher, was strictly to gain an understanding of the student teaching internship and not to judge or evaluate any aspect of the internship experience.

Through extensive observations and interviews, combined with written and audio taped collaborations, and a mathematics pedagogy assessment, I generated inferences about the nature of the collaborative process in developing pedagogical content knowledge of a teaching intern as he constructed new knowledge about teaching probability. In the next chapter, I offered results of these data sources in order to gain insight into the student teaching internship.

CHAPTER 4: ANALYSIS OF THE DATA AND RESULTS

The purpose of this study was to examine how the sources of collaboration, curriculum, and the classroom context influenced the teaching intern's development of pedagogical content knowledge. I examined the process by which the teaching intern's development of pedagogical content knowledge changed during the teaching internship as it pertains to teaching probability. Additionally, this study examined the nature of the collaborative process between the mentor teacher and teaching intern in order to clarify the nature of the collaboration with respect to collaboration *on action* and collaboration *in action* in the development of pedagogical content knowledge. I looked for ways the CMP curriculum influenced the nature of the collaboration by examining specific instances when the collaboration was focused around the curriculum. Examining the sources of pedagogical content knowledge that influence the teaching intern will enrich our understanding of the complex interaction found during the student teaching internship.

I selected a mentor teacher who was considered to be an expert mathematics teacher in his school district and a teaching intern who was characterized by his undergraduate mathematics methods instructors as a reflective learner. The teaching intern had some prior experience with CMP through mathematics methods courses at the university and field experiences within the same district the mentor teacher was employed. Data sources included audio taped collaborations, a pre/post mathematics pedagogy assessment, interviews, and observations of the intern's teaching practices.

In this study I sought to address the following research questions:

1. *How did the sources of the collaboration, the CMP curriculum, and the classroom context influence the teaching intern's development of pedagogical content knowledge?* More specifically, how did these sources influence his development of conceptions of purpose for teaching probability, knowledge of student understandings, curricular knowledge, and knowledge of instructional strategies? What was the process by which the purpose for developing pedagogical content knowledge changed for the teaching intern?
2. *What is the nature of the collaboration between the teaching intern and mentor teacher?* In particular, what is the nature of the collaboration *on action* and collaboration *in action*? In what ways did the curriculum influence the nature of the collaboration?

This chapter is organized into two sections, addressing the research questions listed above. Within each section I summarize the results of the data analyses in order to understand how the teaching intern's use and knowledge of pedagogical content knowledge changed and the process by which the teaching intern's development of pedagogical content knowledge changed. I also discuss the nature of the collaboration and how the CMP curriculum influenced the collaboration. In the first section I report results related to how the sources of pedagogical content knowledge—collaboration, the CMP curriculum, and the classroom context—influenced the teaching intern's development of pedagogical content knowledge, and the process by which his pedagogical content knowledge changed. The data sources were the audio taped collaborations, pre/post mathematics pedagogy assessment, observations, and interviews. In the second section, I report results related to the nature of the collaboration *on action* and collaboration *in action*, and how the CMP curriculum influenced the nature of the collaboration utilizing data from audio taped collaborations, interviews, and observations.

Teaching Intern's Pedagogical Content Knowledge

For the purpose of this study, I defined pedagogical content knowledge as the type of knowledge teachers possess when they understand the mathematics they are teaching and are able to draw on that knowledge with flexibility by looking for multiple ways of representing mathematical ideas to foster students' conceptual understanding. In the following results, I describe how the sources of the CMP curriculum, the collaboration with the mentor teacher, Brad, and the classroom context influenced Randy's development of pedagogical content knowledge and the process by which the purpose for developing this specialized knowledge changed for Randy during the teaching internship. These sources proved vital in Randy's development of pedagogical content knowledge. Similarly, the process by which Randy's purpose for developing his pedagogical content knowledge changed involved a three phase transformation.

How the Sources Influenced Pedagogical Content Knowledge

The extent of how the sources of the CMP curriculum, the collaboration with Brad, and the classroom context influenced Randy's development of pedagogical content knowledge was analyzed by triangulating several data sources in order to investigate the initial and emerging development of pedagogical content knowledge in practice. The findings (see Figure 4.1) indicate the CMP curriculum initiated Randy's examination into his learning to teach process because the curriculum was where Randy began to build upon his own limited knowledge as a way to make sense of what he was not yet familiar with, i.e., teaching probability. After this initial view of learning to teach probability from the curriculum, Randy called upon his collaborations with Brad to address the dilemmas he still faced after learning what he could from the curriculum. However, the CMP curriculum and the collaborations with Brad could not fully prepare Randy's initial and emerging development of pedagogical content knowledge. The final source, the classroom context, became important in how Randy developed pedagogical content knowledge because he learned about how to teach probability from listening to and incorporating students' thinking in instruction. A discussion of how these three sources influenced Randy's conceptions for teaching probability, knowledge of student understandings, knowledge of instructional strategies, and curricular knowledge during the teaching internship follows.

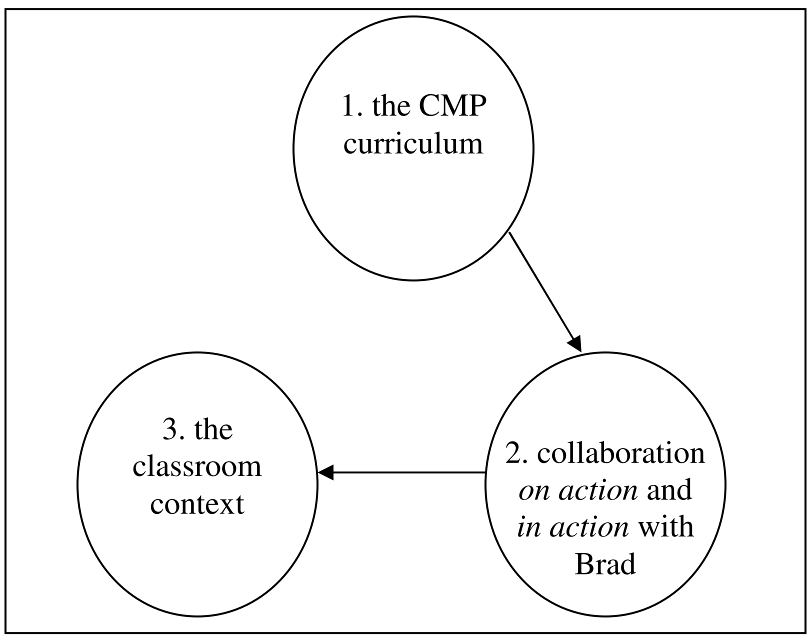


Figure 4.1: Order of sources in Randy's development of pedagogical content knowledge

Randy's pre-assessment. To offer a baseline measurement of Randy's initial pedagogical content knowledge, the mathematics pedagogy assessment (Appendix A) was administered at the beginning of the teaching internship. The findings from Randy's assessment (Appendix B) indicate that from the first task in the pre-assessment, Randy scored 4 out of 5 for adequately designing a lesson related to basic probability concepts. His lesson demonstrated an understanding of one-stage events, explained how to present one-stage events to students in a way that would achieve the specified goal, and gave an almost complete response and a fairly well-motivated development of the material. His lesson conveyed the purpose for knowing when games of chance are fair (or unfair), which indicates prior knowledge related to conceptions of purpose for teaching probability. His use of curricular knowledge and pedagogy was illustrated by a lesson design much like that of the *Launch*, *Explore*, *Summarize* used in CMP where Randy's knowledge of instructional strategies enabled him to pose questions that would launch the lesson and made use of a game of chance to explore probability. His attempt to summarize was lacking, however, which suggested minimal knowledge of student understandings and how to incorporate assessment into a lesson involving a game of chance. The only form of assessment consisted of informal questions posed in the *Launch*.

As tasks in this pre-assessment became more difficult, Randy attempted to use his conceptions of purpose, pedagogical knowledge, and curricular knowledge to plan the lesson, however his content knowledge limited his ability to do so. For the second task related to independent and dependent events, Randy scored 2 out of 5. The goal of the lesson was for students to identify and model the probabilities in situations that involve drawing an item from a container with and without replacement, analyzing situations to find all possible outcomes, and confront the issue of how the dependence of the outcomes of one action on another action affects the probabilities in a situation. However, Randy's lesson plan demonstrated a limited understanding of the mathematics to be presented and he was unable to indicate how to teach the mathematics to students in a way that was likely to achieve the goals of the lesson. Therefore, his limited experience related to conditional probability content and the use of instructional strategies related to teaching conditional probability proved evident in this pre-assessment. Randy focused on demonstrating the mathematics rather than on how to assist students in understanding the mathematics; thus resulting in a low score.

In his response, Randy provided an example of a dependent event without replacement where the students were asked to find the number of ways one can arrange the letters, ABC. He then provided a listing of the possible outcomes and an algorithm that demonstrated these outcomes (e.g., 3 choices x 2 choices x 1 choice = 6 outcomes). Although this is correct, further explanation or examples were not provided. Therefore, Randy lacked the knowledge of how to model the probabilities in situations that involve drawing an item from a container with or without replacement and confront the issue of how the dependence of the outcomes of one action on another action affects the probabilities in a situation.

In designing the lesson associated with the third task in the pre-assessment, Randy once again attempted to demonstrate his curricular knowledge, knowledge of instructional strategies and conceptions of purpose with a lesson that began with a *Launch*, including probing questions to initiate and support classroom discourse. He also suggested the use of applets for students to explore, but a limited attempt to confront the issue of how dependent events affect the probabilities of all events and resulted in an incomplete response and a score of 2 out of 5. Therefore, the pre-assessment revealed

Randy's limited pedagogical content knowledge and his inability to coordinate components of pedagogical content knowledge for more difficult probability concepts.

As Randy began to interact with the CMP curriculum and collaborate with his mentor teacher, his pedagogical content knowledge began to further develop. Data to support this claim regarding these two major sources follows.

The CMP curriculum. The CMP curriculum played a substantial role in how Randy developed pedagogical content knowledge and was a driving force in both *what* Randy taught and *how* he taught it. More specifically, the CMP curriculum was the first source Randy utilized when he planned his lessons. Randy began the planning process by working through the lesson and subsequent homework problems, which influenced his development of curricular knowledge. Recall curricular knowledge is knowledge of curriculum and curricular materials. The following excerpt offers a view of Randy's emerging development of knowledge of student understandings and knowledge of instructional strategies from the CMP curriculum:

Bridgette: Let's talk more about lesson 2.1… How did you decide *what* you were going to teach when you planned Lesson 2.1?

Randy: I used the teacher pages a lot. I actually went through and I worked the entire problem (the lesson along with advised homework problems. I figured out the ways by simulating 36 trials. I rolled the dice.

Bridgette: So you did the problem yourself?

Randy: Yeah, [because I wanted to know] what might I possibly get. How many odds and how many evens. To get a feel for what [the students] were going to be doing in class. I listed different ways. A counting tree to find out the outcomes and a matrix to find out all the possibilities. Then I answered the Follow-Up questions. I've tried to have these questions typed up on a sheet of paper to refer back to, but I don't [always] get back to them [when presenting the lesson to students.]

(Randy was pointing out a need to incorporate the Follow-Up problems when teaching the lesson, but found he often either forgot about them or ran out of class time. Thus, the Follow-Up questions were ultimately not a part of the lesson with students. This was evident in my observations of his teaching, as well.)

Bridgette: Tell me, specifically; when you were getting ready [to teach] and you were deciding what to teach, you said you looked through the teacher pages. Be more specific…

Randy: For instance, one of the first questions it asks is, 'How can you and your partner keep track of the results?' And, that just made me start wondering, 'Well, is this lesson about keeping track of your results?' At first, I thought if you just look at the objectives before you teach the lesson, like on page 21n? The mathematical and problem solving goals are listed and it's (the lesson) doing the same thing again, 'A deeper understanding of experimental and theoretical…' It's the same stuff we've been doing, um, finding the probability, determining whether a game is fair. Those are three things that we did in the last [investigation] so I wondered, 'What's different about this [investigation]? What are we doing that's new?' When I saw these [Follow-Up] questions on how do you keep track of your results? There's a big focus on using all the different methods...for organizing our data. So that helped me to understand what was important in this lesson, by doing the problems first. Before teaching it.

Bridgette: Okay.

Randy: And that was the difference between that and [Investigation 1] because they've already done the relationships between experimental and theoretical. [The students] should know what each one of those is, how they relate to each other. We've done fair and unfair [games]. So for me it was really, 'How can we represent the numbers so that we can retrieve the information in the most efficient way.'

[Randy felt Investigation 2 was a repeat of Investigation 1 and that he had already 'covered' this material. He assumed that through Investigation 1, the students had formalized an understanding of experimental versus theoretical probability and fair versus unfair games. However, in later interviews he contemplated this assumption. It was also evident in future observations a limited connection made between experimental probability and theoretical probability, which caused confusion for the students.]

(Interview #2)

Next in the same interview, Randy's responses to how he planned his teaching shifted from the curriculum as a source in his development of pedagogical content knowledge to his mentor teacher, Brad, before he first taught the lesson:

Bridgette: Okay. So you had recognized that they needed to organize their information and they gave a couple different models that they were looking for (offered as possible strategies in the teacher pages). The teacher pages suggested what you should emphasize, look for… If we move beyond just the model of how to represent experimental probability, then how did you decide how you were going to teach?

Randy: Brad and I were talking and [the curriculum] has ways to organize [the students'] experimental data. And then throughout our discussions we decided the most important thing in here (the teacher pages on how to teach the lesson) is, 'How do you find all the different outcomes…' And then tying it to the theoretical. [In Investigation 2] there was a lot more emphasis on the theoretical [than in Investigation 1].

Bridgette: Did your collaborations focus on the CMP curriculum?

Randy: A lot of the times, it did. We talked through the lessons, the teacher pages and the student pages [before I taught the lesson].

(Interview #2)

In planning to teach Lesson 4.1, Choosing Paths, the following excerpt proves the CMP curriculum was the first source Randy considered in how he planned his lessons. However, he struggled to make sense of the CMP curriculum at times. Still faced with the dilemma of envisioning how the lesson unfolds with students and in an attempt to develop his knowledge of instructional strategies and student understandings from this newly-formed curricular knowledge, Randy collaborated with Brad regarding his interpretation of the teacher pages and how he planned to teach the lesson regarding dependent events. This collaboration *on action* suggested Randy prepared to teach the lesson first by, again, working through the lesson before collaborating with Brad. When faced with remaining dilemmas, he sought guidance from Brad:

Randy: There's a couple initial questions. I'm looking at the way [the teacher pages] suggest teaching the investigation. And it is basically introducing the game and then playing the game with the class. And it says have ½ the class hide the treasure and the other half proceed through the maze, randomly choosing paths. Is that necessary? I mean, it sounds fun. Let ½ the class do it against the other half, but all I see is a potential for problems like, 'Okay, whose choosing the paths?' And, 'How are they choosing the paths? Who is going to choose where to put the treasure?' I know to simulate the game-

Brad: So you're saying it's ½ of the class versus the other ½?

Randy: Yes, but, at that point we haven't talked about how you can randomly even select a path. Or maybe that is where you *would* talk about it?

Brad: Um-

Randy: I'm just curious. It's just to demonstrate how to play the game, right?

Brad: Right. I wouldn't spend very much time doing that. You could split them up into the left and the right and then *you* decide who is going to hide. And you could just say, 'We're going to play this side against this side. But not all of you are going to get to play. We're just going to choose a couple of people.' Then pick a person. Have them choose a room to hide the treasure. Write it down on paper and have them show someone at their table. Then go to the other side of the room. I mean, I think this is what it is saying. Go to the other side of the room. '[Student], which path are you going to take? 1? 2 or 3? When you get to the first part…' And then, based on what he says, you can go through where the treasure would end up. Am I in the right spot?

Randy: Is that the? Oh, that is this one (flipping through book). On page 42. It just, it looked different.

Brad: Well, on page 42 they use numbers to help them walk through the paths.

Randy: Now, I think that is a necessary part of it.

Brad: You could draw this up on the board. Or a big piece of paper and have a [student] come up to the board. At the first intersection, chose 1, 2, or 3. And then the next one. And then you can say, 'Okay, who is going to end up in Room A? Was the treasure in Room A? Is it a 50/50 chance?' Because there is only 2 rooms. 'Is it a 50/50 chance you are going to get it right?' Are these the type of questions that were popping up in the book?

Randy: Yes.

Brad: We (other teachers in the school district) talked on a CMP Saturday about the

Follow-Up, which is quite an extensive Follow-Up. Which goes onto page 43. 'Draw a square on your paper…' (reads problem from book). So, basically it is walking through how to make an area model.

Randy: I did this myself. That's how I learned [to use the area model]. You've shown me a couple of times, too, but this is the first time I have really sat down and looked at it. I just followed those directions (in the Follow-Up) and I was able to make one.

(Collaboration on action #13)

While Randy used the CMP curriculum first to plan lessons, he struggled to understand some of the fundamental ideas the CMP lessons envisioned, and consequently, this led him to focus on the wrong key concepts while teaching. Therefore, his misinterpretations of the curriculum allowed only minimal development of some aspects of pedagogical content knowledge because he could not envision how the lessons would play out in instruction. Without enacting the lesson with students to better understand how it would play out in instruction, and without a full grasp of the purposes for teaching probability as described by CMP (Randy planned one lesson at a time and could not foresee a lesson's purpose in subsequent lessons), Randy continued to face the dilemma of developing the type of knowledge teachers possess when they understand the mathematics they are teaching and are able to draw on that knowledge with flexibility so to foster students' conceptual understanding of probability.

Through further examination in how the CMP teacher pages suggest to *Summarize* and incorporate Follow-Up questions in the summary, the class's analysis should naturally lead to a discussion of whether the game was fair and how to award points to make games of chance fair. Experimental data from the students should have been pooled and serve as the basis of student conjectures regarding the fairness of games. Subsequently, finding the theoretical probability through the use of listing the possible outcomes allows students to make a connection between what they found *experimentally* to what should happen in *theory*. Because Randy could not fully comprehend the purpose for connecting experimental probability with theoretical probability, Randy enacted the lesson by focusing more on procedures for listing the possible outcomes, rather than connecting experimental with theory.

In a later interview Randy expressed his belief that students should know and be able to use all the strategies suggested in the teacher pages, including solving probability problems analytically, which ultimately took precedence over an analysis of experimental data. In addition, the Follow-Up questions at the end of the lesson were left out, eliminating the possible *Summarize* component, in which the ideas from the *Launch* and *Explore* are brought together to build a shared understanding of probability. Therefore, Randy's development of curricular knowledge from CMP only led to a partial understanding of curricular knowledge when enacting the lessons. Consequently, the *Launch* and *Explore* portion of the lesson received greater emphasis than the *Summarize*, even though Randy's planning appeared to consider all three as equally important. In order to summarize the lesson as the curriculum intended, it is essential the concepts are *Summarize*d so that the goals of the lesson are achieved. Randy's interpretation of how to enact the lesson illustrates the dichotomy of *intended* versus *enacted* curricula in school mathematics.

One catalyst for Randy's development of pedagogical content knowledge was his own personal desire to improve his knowledge of instructional strategies as related to developing classroom discourse. Recall knowledge of instructional strategies includes the ability to implement worthwhile mathematical tasks that elicit students' conceptions, and misconceptions, as they relate to probability, and how to use students' thinking to build a shared understanding of probability. In several interviews and through collaborations *on action* and *in action* with his mentor teacher, Randy expressed his desire to monitor the questions he posed to students. This desire stemmed from experiences he gained from his two middle grades mathematics methods courses during his undergraduate coursework at the university. Both courses stressed the importance of asking open-ended questions to elicit students' thinking rather than merely supplying facts or performing a learned procedure.

At the beginning of his internship, Randy videotaped his teaching so he could review his questioning skills and improve them. Randy shared this desire to improve questioning techniques with his mentor teacher, who then paid particular attention to this component of knowledge of instructional strategies. The CMP curriculum afforded the opportunity for this focus via the lesson design format (*Launch*, *Explore*, *Summarize*) and the questions to pose provided in the teacher pages.

The CMP lesson design format emphasized a problem-centered approach to learning probability and envisioned classroom discourse where students explore concepts, experiment and make conjectures related to these concepts, which lead to fostering a shared understanding of probability. With this type of lesson design, the questions teachers pose are vital in the learning process. To that end, Randy utilized the teacher pages in order to understand the type of questions to pose when planning and implementing lessons. Utilizing this problem-centered approach to teaching, his questions mirrored those posed in the curriculum. It was evident at times his newfound knowledge of instructional strategies did not always transfer into his lessons, but this may have been due to his inability to incorporate his knowledge of student understandings along with his questioning skills; that is, how to understand and utilize students' responses to his questions while in the act of teaching. Once this became apparent, Randy began to utilize what he learned from students to gain new knowledge regarding an additional component of pedagogical content knowledge, namely knowledge of student understandings.

The CMP curriculum initiated Randy's examination into this learning to teach process, and how he came to understand the mathematics he was teaching, how he first learned about students' thinking regarding probability, and the blending of these two components. However, after learning what he could on his own with the CMP curriculum, he brought any dilemmas he still faced to his collaborations with Brad.

Brad's pre-assessment. Brad's pedagogical content knowledge related to teaching probability was examined in order to understand the type of knowledge he possessed when he understood the mathematics he was teaching and whether he was able to draw upon that knowledge with flexibility in order to foster students' conceptual understanding because the type of knowledge Brad possessed could influence Randy's knowledge development. Therefore, at the beginning of the student teaching internship, Brad took the mathematics pedagogy assessment (Appendix C).

The findings from this assessment indicate that from planning, implementing, and assessing student understandings and his instructional practices, the assessment served to gauge how Brad thought about teaching probability. Results indicate Brad's understanding of teaching probability was aligned with a problem-centered approach to teaching, similar to CMP, however, his responses were original in nature. He scored 2-4s and a 5 out of 5 on each of the three assessment tasks.

To illustrate one of Brad's responses to the pedagogy assessment, the third assessment task related to conditional probability where Brad composed a lesson that would motivate students' interest, achieve the goals of the lesson, as well as, how he would summarize these goals and reinforce student understanding of the material presented in the lesson. He exhibited an understanding for conceptions of purpose for teaching probability through a simulation involving a variety of prizes, varying in monetary value and importance, where names were drawn from a hat with and without replacement in order to demonstrate the likelihood of winning a prize. Also, he posed questions at specific well-chosen points in the lesson, indicating his ability to understand the mathematics deeply in order to assist students in learning about dependent events.

Collaboration with Brad. In relation to the collaborations with Brad as a second source in Randy's development of pedagogical content knowledge, Randy learned what he could individually from the CMP curriculum and took any remaining dilemmas he faced to his collaboration episodes with Brad the following morning. Randy utilized both collaboration *on action* and *in action* to make sense of the mathematics he was teaching and how the lessons would look in instruction with students. Ultimately, the majority of their collaborations focused on Randy's construction of new pedagogical content knowledge through the enactment of the CMP lessons and Randy's questioning skills. The following interview transcript highlighted Randy's perception of Brad's role in his development of pedagogical content knowledge:

Bridgette: Why do you need reassurance from [Brad]?

Randy: Cause, he's been doing it (laughs) well for so long! He knows what he's doing. He's taught this before. He's strong in math. He's a strong teacher. The [students] respect him and it seems like they really pay attention when he teaches.

Bridgette: Are you getting a vibe from the kids like, 'I better check with Brad and see if this is going well'? Or,

Randy: Well, I just think it would be stupid not to get his opinion, not to ask, 'What do you think about this?' I mean, I want his feedback because every time he's got a suggestion, it's something that makes a lot of sense. And he always validates why he makes a suggestion. He doesn't ever just say, 'Well, you should just do it this way.' He's never once said that to me. He says, 'In the past I've done this and this seems to work pretty well.' And then we'll talk about why that might work. He helps me in understanding when and why something is important. And then I might try it in the class. [Brad] always justifies [the advise he shares with me].

Bridgette: That's the type of collaboration *in action* you have?

Randy: Yeah. He'll say, 'Yeah, it's going good.' Or, if he notices that time is [running short] he'll give me a signal, 'You need to... No [students] have introduced the area model and you need to go ahead and introduce it.' He's able to [*summarize*] and watch the time at the same time. Because that's what happened the first time I taught it. I kept waiting for [the students] to introduce it and nobody did, and then time was out. So in the last couple of minutes it got shoved in there.

Bridgette: Which makes it difficult to *Summarize*?

Randy: Yes. In the teacher pages it says, 'Let the students discover the area model.' But, then it also does say in the teacher pages that when you're getting towards the end of class, if nobody has presented this as a method, then you need to introduce it. And so [Brad] gives clues like, 'Okay, it's time.' Or, 'You haven't hit that yet. You need to do that.' And he actually said that to me. Was it with Core B? No, it might have been Core A. The very first time I taught it, he said that… so he had to, 'Okay, you gotta stop.' (laughs)

(Interview #3)

From planning before teaching to reflecting on and making changes to lessons after teaching, the collaborations focused on instructional practices. Brad offered guidance, validating Randy's concerns, and providing insight from his years of teaching CMP and his expert teacher's pedagogical content knowledge. Therefore, the collaborations with Brad were influential in Randy developing pedagogical content knowledge related to knowledge of instructional practices.

Brad and Randy collaborated *on action* about how to teach CMP. Randy had difficulty in understanding how to implement a CMP lesson where the notes in the teacher pages say, "Some students *may* suggest…," or "If students are struggling with…", and "You might suggest… if students do not come up with it on their own." In the following excerpt, Randy attempted to understand how he would *Launch* Lesson 4.1, Choosing Paths (see Appendix D), in which students would use an area model to compute probabilities in situations involving a sequence of actions (additional discussion about *on action* and *in action* is described in response to the second research question that follows). In this lesson, students analyze a maze that starts at a common point and finishes in one of two rooms. Students first simulate the game. Then they analyze it by using an area model to represent the probabilities of selecting various paths that led to either Room A or Room B. Before using the area model with students in the lesson, the notes in the teacher pages stressed the importance of giving students the opportunity to create their own method for analyzing the game. Trusting students could create their own method was difficult for Randy. This may have resulted from inadequate knowledge of student problem solving strategies or his propensity to simply *show* students how to simulate the game, model the mathematics, and eliminate confusion:

Randy: Should I go ahead? I mean, [the teacher pages] focus a lot on, "How do we make random choices?" In the teacher pages. Like, "What can we use to do a random choice? What does a random choice mean? Can we use dice?" And let [the students] choose their own way? Like with dice, a spinner, or flipping a coin?

That seems like that will eat up a lot of the time where they could be just playing. And, would be it? Is it not a good idea just say, "Use a number cube. If you roll an odd, then pick the odd path." You know what I'm saying?

[Randy is suggesting he tell the students which manipulative to use and how to use it, rather than take class time for the students to develop their own method for simulating the game.]

Brad: Well, there are several ways you can set this up. You can give [the students] spinners, dice, coins. We've got two colored flippers. Red and yellow. And their groups can decide how they are going to [simulate the game]. The decision they are going to make at the first intersection (three paths so a manipulative with three outcomes). What they are going to make at the top intersection (at stage two)… If they are going to flip a coin for that one (a different manipulative because there are just two choices). I mean, you could have each group do that, but that would take away from the mathematics. *They're* figuring out how they are going to set up their own rules. And I think it is important for *them* to do that.

[Brad treated his role as a mentor as one who aided his teaching intern in thinking about how to guide students rather than lead. Brad modeled an approach to teaching where one considers the actions of the teacher as a facilitator for learning rather than the 'keeper' of learning in much the same way he facilitated Randy's learning. Through collaboration, Brad offered support for Randy's concerns while at the same time offering suggestions from his personal experiences in teaching middle grades students, and preservice teachers, as well.]

Brad: At the same time, showing some guidance in the beginning so that [the students] could create their area model for the Follow-Up is probably going to be more an effective use of time. Just because I think the main idea is not to, 'How do you choose randomly?' but, 'How to analyze this game with an area model.' Do you know what I mean?

[Brad validated Randy's concerns for teaching this lesson and understood the importance of learning to use an area model to represent the mathematics.]

Randy: That is what I'm saying. How are they going to [know which manipulative to use]? They could pull out colored blocks, but then they have to subscribe colors,

Brad: Well, yeah. You could have 3 colored blocks (to make a decision at a stage where there are three choices) and blindly reach in and pull one. And that's the path that they go.

Randy: Well, I guess it is about having *them* figure out what to do.

(Collaboration on action #13)

When the notes in the teacher pages suggested, “Students will create their own method for analyzing the game,” Randy expressed in the interviews his reservations for this type of teaching strategy because he had limited experience in teaching from a problem-centered approach. However, Brad’s experiences with CMP influenced the way Randy thought about his instructional practices, and challenged Randy to step outside of his comfort zone. As Randy constructed new knowledge about instructional practices, Brad questioned and supported Randy’s thinking and development of this new knowledge. In an interview that centered on Randy’s teaching of Lesson 4.1, I asked how collaborating with Brad influenced his instructional decisions in planning lessons:

Bridgette: When you and Brad were planning for Lesson 4.1, what did you collaborate about?

Randy: Okay, I asked him about simulating how the students choose the paths. Should we let them decide, as suggested, or should I prompt them. Should I tell them to use number cubes to pick random paths? That was a part of our discussion. He thought it was important that they try and figure out how.

Bridgette: To simulate the game.

Randy: That they can simulate a random choice. And he thought that was an important part of that.

Bridgette: So why did you ask Brad about that?

Randy: I asked him about that because for me it was like a time issue. I was wondering the importance of it… This book thinks that it’s important.

Bridgette: Okay.

Randy: And I didn’t really know why, though. So I came in and asked Brad and he said, “This is why I think it’s important.” I knew that it was important, I just hadn’t really figured out why. So we decided to do that. I also asked him if we should label the paths in the beginning.

(In the initial *Launch* of the lesson, the paths are not labeled. Randy contemplated whether he should lead with the paths numbered when he taught the lesson.)

Randy: When I was doing the demonstration. I wanted to know if it was important because on the next page, the paths are labeled. We decided not to label them and waited to see if the students looked at the following page for help.

(Interview #3)

Randy was developing pedagogical content knowledge through his collaborations with Brad. His experiences and initiative to develop knowledge of instructional strategies and curricular knowledge were catalyst in his learning to teach process. He was learning to think about the mathematics he was teaching by looking for multiple ways of representing mathematical ideas to foster student learning. As I present below, observational data support the notion that collaboration about pedagogical content knowledge did not consistently influence Randy's teaching. Consequently, transferring Brad's pedagogical content knowledge into his own proved difficult.

In teaching Core B, Randy began Lesson 4.1 with a discussion of making random choices and asked the students to predict the best room to hide the treasure. Using prediction as a strategy can afford teachers the opportunity to assess students' initial conceptions (including misconceptions) before the simulation. Brad suggested this instructional strategy in Collaboration *On Action* #4. It was a strategy used often by Brad and offered as a suggestion for Randy to use. When the *Launch* did not go as planned, Randy referred the students to the illustration where the paths were numbered, and directed them with a method to simulate the game. In doing so, however, Randy eliminated the main goal of the lesson: to allow students to create their own method to simulate the experiment and analyze the game. These resulting changes to the lesson were not consistent with their prior collaboration *on action*. Therefore, not all of the collaboration experiences influenced Randy's teaching. When lessons were not implemented the way they planned in collaborations, it was typically Randy who decreased the time allocated for the *Explore* portion of the lesson and became more teacher-directed, eliminating a problem-centered approach envisioned by CMP and modeled by Brad. Consequently, Randy had difficulty learning to develop his knowledge of instructional strategies from a problem-centered approach and then make use of it in practice, which may result from his inexperience in incorporating student learning in instruction; however, my analysis of the data suggested early signs of the classroom context influencing Randy's knowledge of student understandings, as I will discuss in the next section.

The classroom context. After learning from the CMP curriculum and all he could from his collaborations with Brad, Randy still faced the dilemma of enacting the CMP lessons with students because his initial conceptions in planning could not prepare Randy for how his lessons would actually unfold with students. Therefore, Randy's involvement with the classroom context aided him in his development of pedagogical content knowledge. The curriculum and collaboration with Brad about his questioning skills allowed Randy to change his focus from the *types* of questions to ask to *listening to* how his students responded to questions. Ultimately, Randy began to further develop knowledge of student understandings by learning from students' thinking in instruction. While the CMP curriculum and Brad assisted in his understandings of how to teach probability, the classroom context was a requirement for Randy resolving his teaching dilemmas. The findings from The *Praxis III Domains* (Dwyer, 1994) (see Table 3.4), suggest Randy's teaching experiences aided in his development of pedagogical content knowledge.

I interpreted Randy's ability to teach in accordance with the *Praxis III Domains* (Table 3.3). The following table (See Table 4.1) reports results of Randy's teaching as measured by the *Assessment Criteria and Scoring Rules* (ETS, 1993). Domain A, organizing content knowledge for student learning, assessed how teachers use their understanding of subject matter and knowledge of students to think about the content to be taught. It is evident in how a teacher organizes instruction and then how that instruction plays out in the classroom with students. In comparing the five criteria within this domain over the course of observations of Randy teaching Core B, Randy scored an

Domains	Criteria	Lessons									
		2.1 B	2.1 C	2.2 B	2.2 C	4.1 B	4.1 C	4.2 B	4.2 C	Avg. B	Avg. C
A: Organizing Content Knowledge for Student Learning	A1	2.0	2.0	2.0	2.0	2.0	2.5	2.5	2.5	2.1	2.3
	A2	1.0	1.5	2.0	2.0	1.5	2.5	1.5	2.5	1.5	2.1
	A3	2.0	2.0	2.0	2.0	1.5	2.0	1.5	2.0	1.8	2.0
	A4	3.0	3.0	1.5	2.0	2.0	2.5	2.0	2.0	2.1	2.4
	A5	2.0	2.0	1.5	2.0	2.0	2.0	2.0	2.0	1.9	2.0
	Avg.	2.0	2.1	1.8	2.0	1.8	2.3	1.9	2.2	1.9	2.2
B: Creating an Environment for Student Learning	B1	2.5	2.5	1.5	3.0	2.5	3.0	2.5	3.0	2.3	2.9
	B2	2.0	2.5	2.0	3.0	2.5	3.0	2.5	2.5	2.3	2.8
	B3	2.0	2.0	1.5	2.5	2.5	2.5	2.5	2.5	2.1	2.4
	B4	2.0	2.5	2.0	2.5	3.0	3.0	3.0	3.0	2.5	2.8
	B5	3.0	3.0	2.0	3.0	3.0	3.0	3.0	3.0	2.8	3.0
	Avg.	2.3	2.5	1.8	2.8	2.7	2.9	2.7	2.8	2.4	2.8
C: Teaching for Student Learning	C1	1.5	2.0	2.0	2.0	1.5	2.5	1.5	2.5	1.6	2.3
	C2	2.0	2.5	2.0	2.5	1.5	2.5	2.5	2.5	2.0	2.5
	C3	1.5	1.5	1.5	1.5	1.5	1.5	2.0	2.0	1.6	1.6
	C4	2.0	2.0	1.5	2.0	1.5	2.5	2.0	2.5	1.8	2.3
	C5	2.5	2.5	2.0	2.0	2.0	2.5	2.0	2.5	2.1	2.4
	Avg.	1.9	2.1	1.8	2.0	1.6	2.3	2.0	2.4	1.8	2.2

Table 4.1: Summary of Praxis III scores

average of 1.9 (on a scale of 1.0 to 3.5). This score is consistent with that of a beginning teacher of average ability (a score of 2.0) to: (a) demonstrate some understanding of why students' background knowledge is important, (b) articulate clear learning goals, (c) demonstrate an understanding of making connections to previous and future lessons, (d) create and select appropriate teaching methods aligned with the goals of the lesson, and (e) create or select evaluation strategies that are aligned with the goals of the lesson. His averaged score for Core C was 2.2, an improvement. This may have resulted from several factors. First, the additional collaboration with Brad after Core B but before Core C could have influenced Randy's decision making practices. Secondly, learning from enacting the lesson to Core A and Core B before Core C may have allowed changes in Randy's teaching to occur, thus resulting in an improvement of these criteria. Also, students in Core B were considered average ability, where Core C was a class of above-average ability so planning and implementing lessons may have differed as a result. In any event, Randy's average score from Core B to Core C showed improvement in his ability to organize content knowledge for student learning.

Domain B, creating an environment for student learning, related to the social and emotional components of learning and the emotional and physical safety and well-being of students. Teachers must be able to use their knowledge of students to interpret student behavior and establish a sense of a community with clear standards. A grounded sense of respect for all members of the community is important. With respect to creating an environment for student learning, Randy scored a 2.4 in Core B and 2.8 in Core C. These scores indicate Randy was fair in the treatment of students, established a basic level of rapport, encouraged students to meet challenging learning expectations, attempted to respond to disruptive behavior, and created a physical environment that did not interfere with learning. Differences in scores may have resulted from the manner in which students were ability-grouped or perhaps can be attributed to learning from collaboration with Brad or the classroom context.

Domain C, teaching for student learning, focused on the act of teaching. It is the teacher's ability to connect students with content. Teachers should monitor learning, aid students in their ability to move beyond the limits of their current knowledge and apply what they have learned to new knowledge. Randy's ability to do so resulted in an average score of 1.8 for Core B and 2.2 for Core C. The mathematics content was accurate and appeared to be comprehensible. He encouraged students to think independently and critically in the context of the content to be studied and monitored students' understanding of the content. While above-average scores in this domain are what we hope effective teachers strive for, it is not uncommon for a teaching intern to demonstrate an average ability at such an early stage in their learning to teach process. Comparing these results with collaboration and interview data, it is evident Randy's reflective nature and interest in improving his instructional practices would aid him in continuing to improve his instructional practices.

Randy's post-assessment. As a final assessment of the extent of Randy's development of pedagogical content knowledge, the mathematics pedagogy post-assessment revealed gains in Randy's pedagogical content knowledge. For simple mathematical tasks related to probability, such as one-stage events, Randy continued to demonstrate improvements in his ability to design a lesson for students to develop conceptual understanding of one-stage events (see Appendix E). With a rubric score of 4, his post-test revealed a more detailed lesson design that showed connections among conceptions of purpose for teaching probability, knowledge of instructional strategies, curricular knowledge, content knowledge and pedagogical knowledge. Again, he explained how to present one-stage events to students in a way that would achieve the specified goal, and gave an almost complete response and a fairly well-motivated development of the material. His effective use of pedagogy and knowledge of instructional strategies was illustrated by a lesson design much like that of the *Launch*, *Explore*, *Summarize* used in CMP where he posed questions to launch the lesson and made use of a game of chance to explore probability. His attempt to summarize the lesson showed improvement from the pre-assessment where a better connection between experimental and theoretical probability was demonstrated.

Randy's second assessment task showed gains in planning and implementing a lesson on independent and dependent events. His score of a 2 on the pre-assessment improved to 4 on the post-assessment. The focus of this task was to discuss how you could get students to develop an effective strategy for solving the problem. He described the type of questions he would pose to pique interest, made connections to prior learning experiences, considered possible misconceptions students may face, and described how he would encourage his students to participate in classroom discourse through his questioning techniques and prior experiences with buckets and colored blocks. Therefore, his plans aided students in identifying and modeling probabilities in situations that involve drawing an item from a container with and without replacement, analyzing situations to find all possible outcomes, and confront the issue of how dependent events affect the probabilities of all events. While his pre-assessment showed his concern for developing procedural techniques for thinking about independent and dependent events, his post-assessment focused on teaching problem-solving techniques for interpreting tasks related to independent and dependent events. Therefore, Randy achieved the goal of this assessment task.

The third assessment task taken by Randy also showed improvement in his ability to motivate students' interest. He outlined what he would do to achieve the goals of the lesson, as well as how he would summarize these goals and reinforce student understanding of the material presented in the lesson. The post-assessment score improved from 2 to 4, which showed Randy exhibited an understanding of more advanced content knowledge related to probability, a better understanding for conceptions of purpose, a more informed knowledge of instructional strategies and knowledge of student understandings related to teaching. Moreover, he was able to effectively blend knowledge of instructional strategies and knowledge of student understandings related to the teaching of probability. Consequently, Randy's ability to blend content knowledge with pedagogical knowledge at the end of this teaching internship demonstrated his growth in pedagogical content knowledge for use in practice. Therefore, the major sources of collaboration, the CMP curriculum, and the classroom context influenced how Randy developed pedagogical content knowledge.

Process of Change in Pedagogical Content Knowledge

Analysis of Randy's development of pedagogical content knowledge revealed that as he gained new knowledge he was able to shift his focus from content to the use of instructional strategies for teaching and learning (See Table 4.2). This resulted in Randy's transition from understanding the content *for him* to focusing on *how to teach it*, and finally understanding the content in a way for *students to learn*. A description of this transformation follows.

Process by which Randy Developed Pedagogical Content Knowledge
For Self: • Focused on understanding probability for Self
For Teaching: • Focused on instructional strategies for teaching probability
For Student Learning: • Focused on instructional strategies that developed through knowledge of student understandings and classroom discourse

Table 4.2: Process of change in Randy's development of pedagogical content knowledge

For self. Randy planned lessons by working through each lesson as though he were a student. In doing so, he learned he had trouble understanding the mathematics presented in the CMP curriculum and was faced with the dilemma of understanding probability in a way to teach it. He described in his first interview that he sometimes resorted to textbooks from his university mathematics courses to check the accuracy of his mathematical thinking. In the following excerpt, Randy describes this state of disequilibrium:

Bridgette: You had background knowledge of probability. You've taken classes in high school about probability. You've possibly talked about probability in some of your statistics classes in college. Did you have an understanding of what theoretical probability was [before teaching Lesson 2.1]?

Randy: Yes. I did have an understanding. It's funny that you mention it. I was like, "Oh, this is why I had to take discrete mathematics." Not so much with this investigation, but before I even taught the first [lesson]. I had to get myself back into the groove of finding probabilities. I was like, "Okay, what's going on here?" With the product rule. I went and got my actual discrete math book out and was looking at it and was like, "Why does it work to multiply?" To get your outcomes, to multiply the possibilities. So there were things that I had to go back and revisit. If I thought it might come up in class. So I had to make sure… I wanted to make sure that I knew how to answer those questions. That I could do all the math while I was up there [teaching in front of the class].

(Interview #2)

This suggests a state of disequilibrium because Randy understands concepts related to probability, however, the CMP curriculum presents this information in an unfamiliar way. Consequently, through assimilation, he attempts to take the new information presented in CMP and fit it with what he understands about probability from his university coursework.

Randy admittedly recognized his university textbooks aided him in knowing one way to solve a problem related to probability. However, the CMP curriculum would require him to understand basic probability concepts in more than one way. Therefore, he not only needed to understand the content of basic probability, but he must also understand multiple ways of representing these concepts for students.

Bridgette: So you find the teacher pages have been helpful in your understandings of how [students] might think about the problem? But, also how you could approach the teaching of this problem?

Randy: Yeah, because I never would have really thought about [showing multiple ways to solve a problem] as being an important part. Just to save time, I would have just looked at one of these methods [provided in the teacher pages] and been like, "Here's a method. Now, everybody use it."

(Interview #4)

Through making accommodations in his disequilibrium of understanding the probability presented in CMP, he had to adjust his new experiences by revising his old plan of only understanding one way to solve a problem to fit the new plan of understanding multiple ways to represent the mathematical ideas to foster students' conceptual understanding. In his search to understand the mathematics he would teach to his middle grades students, Ryan found utilizing only his university textbooks and the CMP curriculum in understanding the mathematics he would be teaching was insufficient. Consequently, the dilemmas that remained regarding understanding the mathematics in more than one way would require assistance from another source.

Randy sought to clarify how he thought about problems through collaboration *on action* with Brad. In the C*ollaboration On Action #2* discussed earlier in this chapter, Ryan brought his remaining concerns about the mathematics to Brad to assist him in clarifying what he was learning regarding how to use multiple ways to represent mathematical ideas. Therefore, he used a self-regulated process of resolving any remaining dissonance through collaborative discourse with Brad.

Randy discussed in an interview that working through a lesson as though he was a student helped him in recognizing how to plan and pace the lesson for a 45-minute class period. Also, he considered all the problems as suggested in the curriculum, making note of where the students might struggle. His primary concern was in understanding the mathematics first, before planning the lesson. Once he overcame this dilemma, he faced a new one. Randy found understanding the mathematics was now compounded by understanding the instructional strategies needed for teaching probability.

For teaching. While Randy was developing background understandings of the mathematics in the CMP curriculum, he faced the new challenge of planning to teach the mathematics in instruction. In *Interview #1* he discussed his attempt to think about the mathematics while simultaneously planning his instruction; however, learning the mathematics is not synonymous with planning to teach the mathematics. In continuing *Interview #1* discussed earlier, I asked Randy how he planned for teaching Lesson 2.1, Playing the Addition Game. His response was:

Randy: As far as the thoughts I had before [teaching this lesson]. I was. (pause) I came to Brad and I was like, "How am I suppose to do this?" Or basically, that's what I asked. There were so many questions that the [teacher pages] expect you. Or not expect you. They offer so many questions!

Bridgette: And you're looking at the teacher pages, page 31A?

Randy: Yeah. I mean, there's 1, 2, 3, 4. There's about 5 pages for [planning this lesson] and most of the lessons have maybe two [pages]. It's like they want you to hit on so many things and I couldn't see how it would all fit. I was very apprehensive about teaching this [lesson] beforehand.

Randy lacked prior experiences in teaching that would allow him to assimilate this new information by fitting it with prior conceptions regarding teaching probability. In the past, he had thought about the teaching of probability based on what he had observed from his teachers. The textbooks used in his university coursework were quite different from the CMP curriculum so he faced the dilemma of adjusting his existing cognitive structure to accommodate this new information. This learning experience led Randy to search for a better understanding of how to teach probability. Therefore, he called upon his mentor teacher, the expert mathematics teacher, to assist him in learning to teach the lessons as described by the CMP curriculum.

Randy generally planned his lessons the night before and utilized the collaboration *on action* the following morning to discuss the steps he would take in teaching. Brad offered support as they collaborated about specific aspects of the lesson, focusing on the teacher pages and student pages of the curriculum. They discussed timing and the use of manipulatives.

To illustrate Randy's concern for teaching, consider the collaboration *on action* for teaching Lesson 1.4, Making Counting Trees. The lesson introduced the use of counting trees as a way to analyze some probability situations (see Appendix F). In situations in which each of the outcomes is equally likely, counting trees are a useful tool for listing all possible outcomes. Randy collaborated with Brad how he planned to teach this lesson:

Randy: Okay, 1.4. [The students] figure out how to make a counting tree. That's basically what this one is about. The book shows you how to do it. It walks you through it; through the steps. I really am not expecting that much of a problem. The first example wants you to play the Match/No Match Game where you [use a] 3-choice spinner.

Brad: Uh huh.

Randy: It wants you to find the probability. And, I really think [the students] are going to understand. The Follow-Up is asking you to go back and do the other game. The outcomes for Making Purple. Then there is a different one, which I thought was interesting…where the students use both the spinner and the coin.

Brad: That's good.

Randy: So. I had thought of some extension [questions] because there is a possibility that we are going to breeze through this lesson. If they understand how to [make] the tree and how to read the branches, then, it really shouldn't take them long at all.

Brad: Right, I would have extra [questions] ready [to pose]. Like, look in the Follow-Up. They might have some extras. There was a question on one of the homework problems. Was it number 5? 'If you toss a coin three times… (reading problem)'. That might be one [to incorporate into the lesson] since they've all done it as a homework assignment.

Randy: And that's one that there was confusion on yesterday.

Brad: Whether it was equally likely?

Randy: We started doing a tree diagram.

Brad: And maybe now, afterward (working through these problems)-

Randy: It was a bad place for me to [assign and discuss #5 at that point]. I shouldn't have done it [yesterday].

Brad: Okay. Well, now, after they do counting trees, do you think they would be able to do that one without much help? With counting trees?

Randy: Yes. So that would be a good way to assess. Is have them [go back and think about #5 from the homework]. I had thought of one on my own, too, to make sure they understand a 3-stage outcome where, you could have one spinner as red or blue, then another that's 1, 2, and 3, and then flip a coin.

Brad: Uh huh.

Randy: I mean, you're not really going to play a game like that, but it can really test to see if you can do the branches and see how many outcomes there are.

(Collaboration on action #7)

Randy began to focus on his instructional practices once he felt comfortable with the content of the lesson and could devote his attention to how the lesson would play out with students. Talking through his anticipated instructional practices with Brad enabled Randy to clarify his thinking and gain the insight of his mentor teacher's expertise and wisdom of experience.

Lesson 4.2, Finding the Best Arrangement, exemplified Randy's use of the curriculum and collaboration in understanding how to model dependent events, and how to assist student understandings of dependent events in this lesson (see Appendix G). The following audio taped collaboration depicts Randy's initial conceptions in planning Lesson 4.2. The problem requires students to analyze two-stage games involving dependent probabilities using an area model. The area model representation allowed students to focus on one stage at a time:

(Randy began to see at this point in the unit that the lessons are more than playing games. In Lesson 4.2, the students were asked to find the number of arrangements when given four marbles and two cups. The possible arrangements are the focus, not playing the game.)

Randy: For the game in 4.2. When are [the students] actually playing the game? Or, *are* they actually playing the game? Or are they just analyzing the game?

Brad: When I've [taught] 4.2. I had two partners come up and let them know what they were going to do. Sent one out into the hall and told the student that was still in here to put the marbles in the cups so that when your partner comes back you'll want your partner to pick. You know, they're going to choose randomly a cup and they are going to reach in that cup. If they pick orange, they win.

(Randy needed further explanation regarding how to introduce the lesson with students because he wasn't sure if the students were supposed to act out the game and keep track of points, or only find the possible arrangements.)

Randy: Now what if I did that. Should I do that 5 times? To get the five different arrangements? Or just one in front of the class to illustrate? Then have the students find all the arrangements?

(As Randy taught from CMP, he struggled with understanding how much he should tell the students up front. His intent was to be more directive, while Brad understood the intention of CMP was to allow students to explore and to teach the lesson in relation to how students responded to it. This collaboration is of particular interest in Randy's phase of focusing on learning in that Randy sought from Brad his knowledge of student understandings and possible classroom discourse so to better understand how to assist students' learning during the lesson.)

Brad: That's a good question. Finding the different arrangements?

Randy: Cause we could talk about the arrangements while the person is out in the hall? 'Okay, we did it this way. What is another way we could do it?' Switch them up. Do all the arrangements as a class?

Brad: I think what I would do is introduce the game. Take a couple of volunteers first and explain the game…and have them act out like I mentioned to [launch the lesson]. Get their interest. They will be excited. And then while everyone is in the classroom say, 'List all the different ways we can [arrange four marbles in two cups].'

Randy: But then anybody who didn't get to [come to the front], who wasn't one of the two volunteers is not actually playing the game at this point?

(Randy recognized this lesson has the students analyze the theoretical probability so that they can choose the arrangement that will lead to them winning the game when they play against their partner. The goal of the lesson is to find the possible arrangements and use the arrangement where the probability of drawing an orange is the greatest.)

Brad: Right, and I think that is what was intended by this lesson. In the teacher pages? The *Summarize* has the students find the probability (of drawing an orange) for each arrangement, where the students use an area model…that is going to be the most important part of this lesson.

(Collaboration on action #14)

Once Randy began teaching, he continued to work through the lessons as a student and planned his lessons on his own until he could collaborate with Brad; however, a new dilemma surfaced. Randy not only wanted to understand the mathematics and how to plan his lessons. He focused on these two important aspects of teaching, but began to incorporate new learning he gained from listening to his students.

For student learning. In the process by which Randy developed pedagogical content knowledge, he focused his learning on how to teach probability by developing instructional strategies that incorporated students' probabilistic reasoning. He collaborated with Brad regarding what he was learning from students. What he could not decipher on his own or with further explanation from the students, he sought guidance from Brad to aid in this learning experience. The following excerpt demonstrates this type of learning experience:

Randy: ... Brad and I talked a lot. I've been asking him about probabilities.

Bridgette: What specifically have you talked about? In terms of the mathematics.
Randy: ...In the lesson, Making Purple, you have 3 choices on the first [spinner]. 3 different choices on the second and there's only one outcome that's purple. So you got a thirty...uh. Sorry, an 11 % change. 1 out of 9... A student knew that there was a 33% chance on the first spin and then he divided it by 3 to get the 11%.

Bridgette: The 1 out of 9?

Randy: Yes, and I had to talk to Brad to realize that I was doing the same thing. When I take 1 and multiply by 1/3, because I think of it as 1/3 times 1/3, which gives you 1/9. [I didn't see them as the same so Brad] asked me questions, just like a teacher would. Brad asked me the questions and then I realized that, "Oh, that's just the same. Dividing by 3 is the same as multiplying by 1/3 and 1/3." The two of us talked about how students think about these situations.

Bridgette: Well, the student talked about the mathematics in a different way than you understood the mathematics.

Randy: Yes, yes. When it came up in class I was like, "Well, yeah. 33 divided by 3 is 11." But, does that always work? Because that is not the way I thought about it.

In an interview with Randy after he taught Lesson 4.2 several times, he identified changes he made in teaching this lesson to Core B after noticing problems in teaching the lesson to Core A. In particular, he learned to make adjustments to lessons from what he was learning from his students regarding his instructional practices:

Bridgette: What happened during [Core] A that made you change? Did something happen in A that helped you make the decision to not use cups and marbles [to illustrate the possible arrangements]?

Randy: Well, we noticed [the students] were playing. There was a lot of rattling of the cups… But, they weren't really using [the materials] to see the different arrangements. They're just using it to rattle the marbles around and just shake them back and forth from one cup to the other. They played with them. They didn't really use them to solve the math.

Bridgette: So, describe how you made changes for Core B.

Randy: Now with B. We demonstrated two different arrangements in the beginning. And the questions, well, I let them ask questions. I know a question came up, 'Well, do we have to use them all?' 'Yes, you have to use all four marbles.' 'Okay.' Somebody asked, 'Can one of the cups be empty?' …Just to get them thinking about all the different arrangements was basically by just asking questions. I had listed two on the board. There were two different arrangements that were tried, um, when I demonstrated how to play the game. So I went ahead and listed those two. Then, I walked around. I saw what arrangements they were making. For a couple groups, I actually went and grabbed the cups. If I saw [some students not working] or [struggling with it] I was actually physically taking the marbles and saying, 'Now is that arrangement?' And then I would put one in the other one. So they started moving them around. I think they got it. Just from the two examples they saw and asking the questions. Knowing they had to use all the marbles. Knowing they could have an empty cup and just setting them out and giving them time to do it. I don't think they really needed anything, anything else. And, when I brought it together, I asked, 'Who has more than three ways of doing this?'

(Interview #4)

I observed Randy teach this same lesson to Core B and Core C. In launching both lessons, Randy made changes after Core A and utilized Brad's suggestion of placing a student in the hall while another came to the front of the class to arrange the marbles. When the student returned to the classroom, he drew a marble from a cup. My field notes revealed Randy then proceeded in asking the students about the choices they would need to make in finding an orange marble. He posed the question, "What is the first choice you are going to have to make? Which cup, right?" Then, he posed the question, "What is the probability of choosing a cup?" When the class responded with, "1/2," Randy remarked, "Once you choose that cup, you have additional choices to make." He called upon two more students to demonstrate the game. Once they demonstrated in front of the class, Randy prepared the class to *Explore* the lesson by saying, "Individually, write down all the ways you can arrange the marbles."

The manner in which Randy launched this lesson was of particular interest because he not only piqued students' sense of wonderment in modeling the game, but he focused their exploration around finding the arrangements, which is more problem-centered and less teacher-directed. Through collaborations and experience in teaching the lesson to students, Randy found ways to make changes to the lesson while remaining true to the pedagogical orientation of the curriculum. Additionally, Randy became more accustomed to listening to students and adapting instruction accordingly. As his pedagogical content knowledge became more developed, it also became more useful in practice.

In summarizing both lessons with students, Randy deviated from the *Summarize* notes provided in the teacher pages for Lesson 4.2. The lesson gave students another opportunity to analyze dependent probabilities using an area model. An area model allows students to focus on one stage at a time, which is the equivalent to adjusting the sample space for the second stage to account for the result in the first stage. Before working with this curriculum, Randy had not heard of an area model for representing dependent events. He described in an interview how he thought about the problem before teaching the area model:

Randy: Well, I mean, I've never used an area model before. And there are different ways that I can think about it now. Now that I know this method, I'm trying to think of how I would have solved a two-stage event before. I guess I would have definitely just multiplied the fractions. I would probably have done something similar to a counting tree. And just on each branch, I probably would have just wrote 1/3 for the first branch and then ½ for the second one.

Bridgette: You didn't do that with [your students]. I was waiting to see that.

Randy: And, they didn't like the counting tree. I got the feeling that most of them didn't like the counting trees. So I just kind of stayed away from it.

Bridgette: Well, …I think [the CMP authors] are wanting [students] to understand the area model. Another way to represent two-stage events so that makes sense, too. Now that you've taught it several times you have a new way of thinking about how to solve two-stage events?

Randy: I do. I don't think I've ever thought of it as, 'This is one-third of one-half.' And if you would have said that to me, 'This is a third of a half.' I would have been like, 'What are you talking about?' Um, now I've also seen [it used] in two different classes. Somebody was doing it on straight percentages. And getting 133 out of 200 percent.

Bridgette: Oh, that's right. What did that mean?

Randy: …Each cup was one hundred percent. And they would look at it and, 'Okay, if this cup was divided into 3. Okay so that's 33%. And look at the other one and it was...,' I think it was two blue and an orange in one and then one orange in the other cup.

Bridgette: So, 100%.

Randy: So they had 100% and then 33%. So 133 out of 200%. That's an easy way to solve the problem. It really is.

(Interview #4)

Randy's newfound appreciation for learning from his students demonstrates the shift made in focusing on understanding mathematics through multiple representations. Randy began to listen to his students while teaching and orchestrated classroom discourse based on *their* contributions, and this often represented a departure from Randy's initial intentions. He might have taught in a more procedural manner, but he now recognized new ways to think about probability. Ultimately, his concerns for student learning became the driving force in Randy's process of learning to teach.

Summary of Teaching Intern's Pedagogical Content Knowledge

Within this section, I reported results with regard to how the sources of the CMP curriculum, the collaboration with Brad, and the classroom context influenced Randy's development of pedagogical content knowledge and the *process* by which the purpose for developing this specialized knowledge changed for Randy during the teaching internship.

With regard to how the sources influenced Randy's development of pedagogical content knowledge, he began the internship having difficulty accessing and utilizing his pedagogical content knowledge while in the act of teaching. Although the curriculum and collaboration influenced his pedagogical content knowledge, he continued to struggle to make use of this new knowledge. Students' thinking served as a catalyst in Randy's development of pedagogical content knowledge, enabling him to become more comfortable soliciting students' thinking in instruction. The three sources of curriculum, collaboration, and classroom context continued to prove vital in how Randy developed pedagogical content knowledge.

With regard to the process by which the purpose for developing pedagogical content knowledge changed for Randy during the teaching internship, Randy was able to shift his focus from a *concern for content* to his use of instructional strategies *for teaching* and ultimately, toward his use of instructional strategies *for learning*. Often times this cyclic process was evident in each lesson he taught. First, he faced the dilemma of knowing how to solve the problems himself, working the lesson as a student would. When he learned all he could from the CMP curriculum and prior college experiences, he sought assistance from Brad, his mentor teacher. Through collaboration, Randy was able to talk through the mathematics and how he planned to teach the lesson, making adjustments based on Brad's guidance. As he taught lessons, he listened to students and adjusted his instructional strategies based on what he was learning from his students. This new knowledge afforded the opportunity to focus his concern on whether students were learning and by looking for multiple ways of representing mathematical ideas in order to foster students' conceptual understanding. This process made the purpose for developing pedagogical content knowledge a worthwhile learning experience for Randy.

With regard to his concern for learning how students think about probability, knowledge of his students' thinking made it possible for Randy to focus on how he could utilize their thinking in instruction. It was at this point that Randy possessed the greatest learning potential for developing and making use of pedagogical content knowledge in practice. Therefore, all three factors—collaboration, the CMP curriculum, and the classroom context—were essential in Randy's learning to teach process.

The Nature of the Collaborative Process

Brad and Randy collaborated often during the student teaching internship. In the findings that follow, I recount two forms of collaboration, the nature of the collaboration *on action* and the nature of the collaboration *in action*. As the data will indicate, these collaborations proved fruitful for Randy as he learned to teach probability to middle grades mathematics students. Moreover, Brad's involvement as the mentor teacher was critical in Randy's learning to teach because Brad served as an expert teacher who continuously analyzed teaching and considered ways to optimize classroom discourse. Several themes emerged from the triangulation of my data sources. The majority of the collaborations focused on how students think about probability and how classroom discourse should incorporate such reasoning to foster student learning.

The Nature of Collaboration On Action

Brad and Randy collaborated *on action* in two specific time frames during the school day, in addition to before and after school. Specifically, most of their collaboration *on action* occurred in the morning, during their scheduled planning period. The length of these collaborations was approximately 25 minutes. In addition, during the final period of the school day, they collaborated *on action* when students participated in a daily structured reading period. While the latter time yielded the fewest number of collaborations, the two found it possible to collaborate *on action* while students were present engaging in a reading activity. In addition, they devoted time to collaboration *on action* after school. It was during these times Brad and Randy collaborated at great lengths, beyond Brad's scheduled contract time with the district.

Brad and Randy used collaborations to discuss three main topics: the CMP curriculum, the mathematics, and the teaching of the mathematics. Within each of these topics Randy grappled with specific dilemmas (see Table 4.3), but as the data indicate, through the nature of their collaboration *on action* he became more capable of addressing these problems.

Domain	Dilemmas
The CMP Curriculum	• Allocating time to *Launch* and *Explore* with adequate time to sufficiently *Summarize*
The Mathematics	• Understanding the mathematics content (probability) of each lesson in order to focus the lessons
The Teaching of the Mathematics	• Posing questions that would aid students in the discussion of mathematics • How to listen to, correctly interpret and incorporate students' thinking in instruction

Table 4.3: Central topics in collaboration on action

The CMP curriculum. In the unit *What Do You Expect?,* there are many opportunities for students to collect data through experimentation and assign experimental probabilities. An important learning outcome of this unit is for students to realize that larger samples can offer good estimates of theoretical probabilities that are typically unknown to students, as they consider long-run trends in experimental data. Therefore, conducting experiments and analyzing data should promote an understanding of the relationship between experimental and theoretical probability; however, conducting experiments and analyzing data takes time. Randy struggled to manage all the components of a CMP lesson in the time provided. The *Launch, Explore, Summarize* lesson design in CMP is intended to encourage student exploration in making connections between data and chance. The nature of the collaboration *on action* between Brad and Randy consisted of how to enact the CMP lessons during the scheduled 45-minute class periods.

Much of Brad and Randy's collaboration *on action* focused on teaching CMP. In one collaboration they discussed Randy's concern regarding teaching Lesson 1.2, Matching Colors, where students analyze a game in which a spinner is spun twice and a match/no-match in color is recorded in order to review the idea of a fair game of chance (see Appendix H). The information in the teacher pages suggests students work in pairs to collect data with each partner taking 12 turns. Once data is collected, the entire class would determine the experimental probability of each event—match and no-match—using the pooled set of class data. Randy would then review how to use experimental probabilities to make predictions of whether the game was fair. One dilemma Randy faced regarding the *Explore* portion of the lesson was the allocation of time devoted to the experiment so that there would be sufficient time to *Summarize*:

(Randy began the collaboration with how he would organize the student data collected for use later in the lesson.)

Randy: I was wondering if I should just have [the students] count up the total of all of the matches they have. And I guess that would only be eight [sets of group data] because they are going to count 24 times.

Brad: Right.

Randy: Okay.

Brad: Have [the students] work in pairs?

Randy: Yeah, there's enough spinners for every two [students].

Brad: You could have them call out their number. Or you can have each pod (2 tables; a pair per table), each table get a total. You'll end up with ninety-six numbers.

[Randy became concerned about how much time it would take to collect the data from the students, while also allotting time to use those data to summarize the key concepts of the lesson.]

Randy: Okay. [Pause; sounding frustrated]. It's just trying to figure out how to fit everything [suggested in the teacher pages] in. I mean, when they give you stuff like this (referring to teacher pages). And it's like, this is all the, what do you call this book?

Brad: The *Launch, Explore, Summarize*? The teacher's notes?

[Randy equated the quantity of teacher pages with longer lessons and was concerned for time needed to summarize the lesson.]

Randy: The teacher's notes. Well, they give you one, two, 2 ½ pages of information that's suppose to fit in the class [period], along with the actual activity. And all the recording and everything. Plus, anything else that comes up that [the teacher pages] don't cover. That you think, 'Oh well, I should talk about this because it is important.' And it seems, some of it is only one page [in length] (the teacher pages). Like the cone activity (referring to *Variables and Patterns*, the previous unit). [The teacher pages were only] front/back. There wasn't too much to it. [For today's lesson] they have a lot of questions in here, good questions, but, how can this possibly all fit into one class [period]? Especially with the experiments?

Brad: Spinning with spinners shouldn't take very long.

Randy: I guess,

[Brad attempted to reassure Randy of his ability to conduct the experiment and have adequate time to *Summarize*.]

Brad: That part goes much quicker (referring to teacher pages). The book, here, is focusing on how you analyze that information you just got. 'How can you theoretically, if you didn't have spinners, find the probability? Is there a way to find it?' The teacher stuff (referring to teacher pages) is important…in order to understand the 'discover' part. If you follow these pages, these instructions, and ask these questions during class, you'll be able to move on. There is a lot of merit in this textbook, the wording in the teacher section. I do not follow 100% what they, especially [now after teaching it for] so many years. And, [these books] are good. The authors knew where they were coming from. They knew where they were going…and the questions for them (the students) to think about are [crucial] in each investigation. Along with the experimental data.

Randy: I agree. It's a really good question to use. Yet, frustrating to see [how it will all come together]…

(Collaboration on action #2)

Before this collaboration, Randy taught Lesson 1.2, Matching Colors, to Cores A and B. The next morning, Randy taught the same lesson to Cores C, D, and E. The following excerpt is a portion of the collaboration *on action* that occurred after Randy finished teaching all five classes. The nature of this discussion focused on how the lessons went and whether adequate analysis of experimental probability led to an understanding of theoretical probability. Emphasis was placed upon how well the *Launch, Explore, Summarize* components were implemented:

Brad: Okay, your comments first on how the lessons progressed today.

Randy: Oh, better than yesterday. I think I got my objectives. Yesterday I didn't meet the objectives. There were things that were changed after reflecting yesterday. More time was spent showing them how to play the game, which paid off in the end. I structured it more. This last class I just said, 'Okay, player A take twelve spins. Then player B.' I designated it right from the beginning. Because they were still taking a little bit longer than I thought [they would need] when they spinned it. One of the reasons why [was because] some [of the students] were seeing how hard and how fast they could spin [the spinner] and it would take awhile before [the spinner] would stop.

Brad: Right.

Randy: Some people were getting three spins in the amount of time it took other groups to get one…

(briefly talked about commercially made spinners and homemade pencil/paper clip spinners)

Randy: ...Which, they are enjoying playing the game. There's something that-

Brad: They're exploring mathematics! They are making comments in other classrooms. I noticed you started to keep track of the last three cores up there (on the board). Are you going to try to figure out how close it is to 50/50 (for the entire team's data)?

(The teacher pages suggested the students analyze the data and discuss whether the game was fair; Brad focused his discussion with Randy on the analysis of the data.)

Randy: Yeah, I was going to do the totals. I didn't know if I wanted to, if it was stronger to leave it up as the individual numbers (by classes) like it is now or go ahead and total up?

Brad: Right.

[Randy was unsure how to make use of the student data from each class as well as combining the data from all classes. The data collected, both individual class data and all classes were not analyzed as suggested in the teacher pages.]

Randy: Just work with the numbers (total for all classes), like 144 matches, 101 no-matches. And then,

Brad: I like what you did when, let's see, you had one with 15 matches and 9 no-matches. And one with 9 matches and 15 no-matches. I like how you pointed that out. One of the things I was going to comment on. Seeing all those data, if you asked them if they had any more examples that are likely? Like that example, 15 and 9, 9 and 15. There are some more examples. In that same one there are 10 and 14, and 14 and 10. And, just getting them, to look up there and analyze the data. I like, I especially like the question, but I don't know if I would have asked it. Or at least noticed that [about the data]. You noticed it. You pointed it out with the kids. I wasn't sure what you were looking for, but then close to the end I noticed your line of thinking. And I do that. As soon as you have a good idea, I am like, 'Oh. Here there is data like…,' 'Are there any other examples that show 50/50?' Just so they analyze the data, by looking for answers, other answers that equal 24.

Randy: I didn't even think that about adding two totals. I was looking for just points to plot, but I hadn't really…I really want kids to understand that you approach 50%.

Brad: I think that you really, you're really hitting that hard now.

(Collaboration on action #4)

In the lessons discussed in the planning sessions, Randy launched all lessons through the use of experimentation as the CMP curriculum intended. He attempted to use the questions posed in the curriculum to *Launch* the lesson and walk among groups during *Explore* to observe and discuss their data collection. However, an additional dilemma he faced was how to *Summarize* the key concepts as suggested by CMP once students were arranged for whole class discourse. He often expressed how he felt unprepared to work with students' data because of his uncertainty in what the students would report. Whether it was a need to know the data beforehand (better preparation) or a concern for students biasing the data (having messy data to work with), Randy voiced his frustration with how to use students' data and still adequately *Summarize* the lesson. Although their collaborations fostered growth in Randy's ability to enact the *Launch* and *Explore* components, the *Summarize* component continued to be problematic.

With 45-minute class periods, Randy repeatedly expressed the dilemma that time was the culprit for not meeting the goals of the CMP lesson. He found it was difficult to grade homework, *Launch* the lesson, manage materials needed for experiments, allow time for students to *Explore*, and adequately *Summarize* the lesson by discussing students' findings and pulling together the key concepts as suggested by the CMP teacher pages. Randy perceived that CMP lessons require more time than his scheduled 45-minute class periods. Therefore, if he was going to teach CMP as Brad had modeled in the previous weeks, then Randy would have to learn how to effectively manage a 45-minute class period, incorporating all the components of a CMP lesson in the time allotted.

In the interviews, Randy discussed how he grappled with the teacher pages provided by CMP, both, the *quantity of* information provided and specifically *how to utilize* this information in a 45-minute lesson. There was an element of uncertainty that continued to plague Randy's process of learning to teach because he felt his instructional strategies should align with the information in the teacher pages. However, he had a difficult time visualizing the lesson unfolding with students as evidenced in the following interview excerpt in which he described how Brad assisted him in understanding the CMP teacher pages:

> I thought that there was a lot of information [in this lesson]. I wasn't sure that I knew what was the really important part of it? And after talking to [Brad], I decided [the big idea was] the outcome, or how to organize and access the information. And I think [Brad] was kind of in agreement with

me. That was one of the big ideas. Where as before I hadn't really known what the big idea was. There's so many aspects to this, there's so many places where you could stop and go more in depth (with regard to the information provided in the teacher pages)... And, usually with the questions about what to ask? I don't think you could ask every question that they suggest or you wouldn't be able to go into any depth. Because they suggest a lot of different ideas. And it was hard to try and pick out which one do I focus on? I would rather find one of these big ideas and go deeper with it than to just shallowly ask all of these questions and not really go anywhere with it.

(Interview #1)

Lesson 2.1, Playing the Addition Game, asked students to find a way to make an organized list of the possible sums of two numbers rolled on a pair of number cubes. The sample space is larger and more complex than previously encountered in Investigation 1, "Evaluating Games of Chance." In collaborating about this lesson, Randy discussed suggestions given in the teacher pages (Appendix I). Randy was interested in understanding how to adequately discuss the assigned homework and still have time to teach the lesson. He grappled with whether the lesson would take as much time as previous lessons when the goal of the lesson is really an extension of earlier lessons:

Randy: …I like having warm-up problems (problems posed to students on the board when they enter class) but at the same time [the students] need some feedback on the homework.

Brad: Right. Well, I wouldn't do both, homework and the warm-up. That would [take] too much time. Especially since [Lesson] 2.1 has all these questions (referring to teacher pages).

Randy: There are a ton of suggestions. And a lot of really big ideas, even though the objectives are basically the same. [The students are] performing an experiment and are looking at experimental and theoretical probability. Looking at the number of outcomes and deciding whether the game is fair.

[Randy's first impression was that the *Explore* and *Summarize* phases of the lesson would not take long because it was an extension of earlier lessons.]

Brad: Uh huh.

Randy: So it's all the same things (as in Investigation 1, "Evaluating Games of Chance"), but to actually look at the teacher [pages]. It says, 'To develop a deeper understanding of,' but then it's the same objectives.

(Randy explains to Brad the suggestions in the teacher pages were misleading because he was concerned about looking at all the different ways to represent the data and allowing adequate time to *Explore* and then *Summarize* the lesson. [Randy appeared to think he would teach all of the content in the teacher pages.])

Randy: But, in the teacher [pages], they are listing all the different ways [the students] could categorize their data and it just doesn't seem like it could all fit into that lesson. All the suggestions that they have. Which I think are really good suggestions, um,

Brad: Uh huh.

Randy: On page 31b [of the teacher pages], they're talking about doing a line plot. Which is a really good way [to organize their data], but [do I suggest this strategy] before [the students] even start the problem?

(Randy explained to Brad how he solved the problem the night before and the strategies he found useful.)

Randy: And, nowhere does it say anything about looking at different [sums]… it's just asking about evens and odds. It's not asking anything about probability of rolling a 6, a 7, an 8, which is what a line plot would be good for.

[Randy may be focusing on the fact that 7 occurs most often, then 6 and 8, etc. when actually students mainly explore odds and evens in this lesson. It is in the *Summarize* phase when they discuss how often each sum occurs.]

[Brad listens to Randy describe his thoughts and actions and waits to interject when needed. Randy sought to understand how to best *Launch* the lesson by discussing with Brad the questions he should pose to the students.]

Randy: It's a good way to have their data organized so you can see, 'Oh, seven is rolling more often.' But there is no prompting here for that. So, does that mean that when I launch this activity, do I need to talk about that? Talk about the different kinds of things we might want to look at, with our data? You know, 'Hey, which number comes up most often?' 'Which one comes up the least?' These are things that we might want to do with this data. So we need to find out a way to organize this. That would make it efficient for us to pull these numbers out [when we summarize or discuss the Follow-Up questions]. And, is making a big long list [of the possible sums] the best way to do that?

Brad: Ah.

Randy: I mean, if I could still go through here and say, 'Okay, how many times did I have an 8? How many times did I have a 9?' Or I could take this and transfer it to a line plot.

Brad: Uh huh, good questions. I think you just turn them loose with the problem. Make sure they know what they're doing. I'd just make sure they know the rules (of the game). 'We're rolling 36 times.' They're just going to record whether it's even or odd, right? They're not even going to write it down what the sum was.

[Brad recognized Randy was moving away from the main objective in conducting the experiment; a focus on odd and even sums.]

Randy: Well, that's-

[Randy's collaborations suggest he has concern for 'turning [the students] loose with the problem.' While it was an effective strategy Brad was accustomed to using, Randy grappled with letting go and trusting the lesson to develop as described in the teacher pages.]

Brad: They'll do tally marks. One will have an even. One will have an odd and they'll do tally marks to keep track of how many times they added. And then they'll just do that 36 times.

Randy: Some people might do that.

Brad: They won't even write down what the sum was. And that is not what this problem asks, right? But the Follow-Up, then.

Randy: No, that's the thing. All of the suggestions back here in the teacher part. 'How could you keep track of your results? What are possible outcomes?' It's asking you stuff that you really wouldn't be getting at until the Follow-Up. It's asking you to discuss all this before you actually do the problem.

(Here, Randy is talking about the examples provided in the teacher pages that show strategies students might use in finding the theoretical probability; however, Randy was confusing these *possible* solution paths with what he was suppose to teach.)

Brad: Well, I don't think that's a bad idea. 'How could you and your partner keep track of your results?' So that they got some idea of how they're going to record it. 'What are the possible outcomes of adding the numbers shown?'

[Brad suggested posing these questions, but not necessarily answering them—a topic discussed in later collaborations; when these questions are posed in the *Launch,* they are used as a way to assist students in what they should be thinking about while exploring the lesson.]

Randy: Yeah, here it says, '*Some* students may suggest keeping a list of the sums. Others might suggest keeping track of the outcomes for each number cube as well as the sums.' You're saying we might not even have the sums.

(Collaboration on action #2)

Although this excerpt documents Randy's concern for time, it also demonstrates his attention to how to *Launch* the lesson by focusing on even/odd outcomes, while at the same time gathering enough data to *Summarize* in the Follow-Up. Randy was looking for places where he could tighten the lessons. Leaving the students to discover on their own would require time that he was not sure they would have if they did not keep track of the right type of data (not only even/odd outcomes, but also the sum). Therefore, he was altering the design of the lesson, which resulted in *telling* the students what to look for when they conducted the experiment.

As he progressed through his teaching internship, Randy became more capable of addressing concerns regarding teaching CMP. Understanding what questions to pose and when to pose them aided his understanding of how to effectively make use of the 45-minute class period. His in-depth analysis of the teacher pages and listening to and understanding how the students interacted with the curriculum proved fruitful. In one interview, he made the following comment regarding how the teacher pages helped him understand how students think about probability:

Bridgette: So you find the teacher pages have been helpful in your understanding of how [students] might think about the problem? But, also how you could approach the teaching of this problem?

Randy: Yeah, because I never would have really thought about [showing multiple ways to solve a problem] as being an important part. Just to save time, I would have just looked at one of these methods [provided in the teacher pages] and been like, 'Here's a method. Now, everybody use it.'

(Interview #4)

Randy struggled to manage all the components of a CMP lesson in the time provided. Therefore, the nature of Brad and Randy's collaboration *on action* focused on teaching CMP. The *Launch, Explore, Summarize* lesson design in CMP is intended to encourage student exploration in making connections between data and chance. Randy sought guidance from Brad's knowledge and experience in teaching CMP during a 45-minute class period.

The mathematics. Randy faced the dilemma of inadequate content knowledge related to probability. He discovered this when he began his lesson preparation by working through the CMP lessons and student pages as though he were a student in the classroom. He found it important to read, solve, and understand the mathematics from a student perspective both prior to and during his teaching experience. Consequently, through this approach, he came to realize that he thought about many of the problems in only one way. Moreover, Randy often commented that he knew how to solve problems related to probability, but was not sure why his methods worked. Consequently, Randy utilized the CMP teacher pages to validate his solutions and learn new ways students solve problems, which he considered to be important.

In an excerpt from a collaboration *on action*, Randy asked for Brad's assistance in understanding Lesson 1.2, Matching Colors (see Appendix H). Discussions centered around Problem 1.2, Parts A and B. In the following excerpt, Randy questioned whether he was thinking about the mathematics in the lesson correctly and whether his students could correctly determine the theoretical probability:

Randy: I'm sort of having trouble with, I don't know why, but, how the kids were seeing that it's 2 out of 4? Without listing all of the outcomes? Was that the only way that you could see it? A match/no match? Do you recall that one?

Brad: I do recall that one. (Brad turns to the page in the book regarding Randy's question.)

Randy: I was having a hard time, that intuitively seeing it was 2 out of 4. That it was ½. And I had to list all the possibilities. Is that what [the CMP curriculum developers are] expecting?

Brad: Listing all the outcomes helps [the students in finding the possible outcomes].

Randy: Because it doesn't actually say to give [the students] that strategy. Am I suppose to let them discover that that's the only way to [solve it]?

Brad: Here, let me look. (Brad reads from page 7 in the teacher pages.) '… play with 24 turns. For each turn, record the colored pair on Labsheet 1.2, and award points… Use the results you collected to find the experimental probabilities of a match and a no-match…' As a summary, you might go back through (walk among the groups of students working) and check to see how [the students are] doing that. (Continues to read from text)

Randy: Here, it tells you to [list all the possible outcomes of a turn (two spins)]. I'm just confused myself because for some reason I didn't understand. I wasn't seeing why it was 2 out of 4. Like, I was trying to do it just by the numbers. By listing the outcomes. Like, discrete mathematics stuff. Like the product rule.

Brad: Uh huh.

Randy: Or something where you're multiplying.

Brad: Right.

Randy: And that's the way I'm trying to see it beforehand (when he solves the problems before teaching the lesson). So, 2 out of 4 is just not making sense to me.

Brad: Right. And similar to our discussion yesterday about rolling the dice. You still have 2 events because you have 2 dice. You get 6 outcomes so 1/6 so those would be two different. And that's the conversation that we have with the kids. Like the blue/yellow and yellow/blue are two different things.

Randy: Because you got the first spin *and*-

Both: the second spin.

(Brad changes the conversation slightly to talk about a blue/yellow spin is different from a yellow/blue spin.)

Brad: So those are two different ones. And that might come up today. The kids might say you really only have 3 options. So you have 1 in 3. That would be something to talk about as far as what [the students] will be looking at. They can, hopefully when they list it on the Labsheet 1.2 and then they list it by what happens, they look to see if they got two different ones. They've got B/Y and Y/B.

Randy: Well then, in the Follow-Up.

(Pause)

Brad: (reads from book, page 7, Problem 1.2 Follow-Up, #1.) 'A match/no-match are equally likely?'

Randy: Where it asks (reads from book, page 7, Problem 1.2 Follow-Up, #2), 'How many times do you expect two yellows, two blues, one yellow, one blue.' But then it doesn't. I think that's kind of confusing because one yellow and one blue. It doesn't say anything about order.

Brad: Right, it doesn't say anything [about order].

Randy: The way I was trying to do it. Like, the first spin you got ½ of getting, you got 50% chance of.

Brad: And then you were talking about the next one?

Randy: Yeah, so I was getting ¼. So that's, I guess, that *is* right because 1/4, one of the combinations. Right?

Brad: Right.

Randy: It's 1/4 for (recognizes the same answer for a different problem).

Brad: Right.

Randy: For a blue and then a yellow. It's ¼ for a blue then a blue or yellow/yellow.

Brad: That will tell you the possible outcomes.

Randy: So that does work, to do that. It's 1/4.

Brad: Because it's a match of possibilities.

Randy: Because you're just adding. It's 1/4 plus 1/4, which is giving your 2/4.

Brad: Sure, sure.

(pause)

Randy: [sounding frustrated] It just *really* was confusing me. I would follow this (referring to text) and I would just think the kids were going to look at it and be like, except for the higher level ones (meaning the more able students), 'What is this?'

Brad: (Referring back to book). 'The P of a match, the P of a no-match.' (pause) The P of the number of turns, the total, the total number of turns that are matches. You might have to demonstrate when you do it. You can, you can play with the spinner, you know, from one class to the next, but you might have to explain.

(Collaboration on action #2)

Brad's experience in teaching probability to middle grades students helps Randy understand how students think about probability. The CMP teacher pages provide some insight into how students might think, but it is through the collaboration with Brad that Randy develops a clearer understanding of how the lesson and the mathematics develop and how it will play out with students.

In the following excerpt Randy described his experience with working through a lesson when he only had a student book with him to plan. This exchange with Brad pertained to Lesson 2.2, The Multiplication Game. In this game, points are assigned based on whether the product of the two numbers rolled was odd or even (see Appendix J). Students need to investigate the possible products, counting all the ways a product can be rolled. From the excerpt of *on action* collaboration, Randy grappled with the mathematics and understanding what the book was looking for in a solution:

Brad: You want to talk at all about 2.2 today?... Just, the last core of today, we'll have 2.2. Which is the Multiplication [Game]. Anything different (different from Lesson 2.1, The Addition Game)?

Randy: (pause) Basically, it's kind of the same. I mean we're switching to multiplication, but the only thing is [whether or not] we want to predict [beforehand]. 'What is the probability of getting an odd product?' Do we want to say that beforehand? Or, 'How many ways can you get an odd? How many ways can you get an even?' Cause if they do that right beforehand, then they should see that, 'No, it's not.' It's not going to be the same. It can't be the same because you've got more ways of getting an even. You've got three ways of getting an even and only one way of getting an odd.

(Randy begins to discuss the mathematics behind the multiplication game.)

Brad: Uh huh. I'm not following. 3 and 1?

Randy: …The only way you can get an odd [product] is to have an odd *times* an odd. The only way you can get an even [product] is even *times* an even.

Brad: Oh, okay. With the rules.

Randy: Odd *times* an even. Even *times* an odd. [Both will result in an even product.] So it is 3 to 1. And when you work it out, you get 9 out of 36 ways to get an odd [product]. And,

Both: 27 ways out of 36 ways to get an even [product].

Brad: You wonder. That's pretty cool. I'm not sure if you want [the students] to share that with you out loud, before they do the experiment.

Randy: That is kind of what I was thinking.

Brad: You might get them thinking about it and have them write down a prediction. And you could ask them a question. 'When multiplying, think of evens and odds. Think of multiplying them together. When is the answer going to be even? What kind of numbers give you an odd?' Without them calling out. And then, write down their prediction. And you know, 'Is this going to be the same as the Addition Game (Lesson 2.1) with the evens and the odds?' If you wanted to get them thinking about it before they started playing the game, but didn't want to give them an answer, just [ask] a couple of questions… And then, as far as how [the students] play the game. I would do it exactly the same as you did it yesterday. With [Lesson] 2.1? Just set it up the same. It's just the questions you might ask at the end will be a little different…

(Brad and Randy continued to discuss where the lesson takes the students mathematically. In the Follow-Up for this lesson, Randy expressed his concern regarding a question that he found difficult to solve. Brad brought up this problem in their collaboration.)

Brad: ...Follow-Up #2 on [Lesson] 2.2, page 23.

Randy: Okay, I had a problem with that [question]. Because I'm like, 'I can't change to points (create another game where two players have an equally likely chance of winning.' I was trying to read what [the book] said because I didn't have the teacher's book last night (used a student book to work through the lesson before teaching). I sat there and looked at it. I made my little matrix. I looked at all the different numbers. I looked at multiples of 3. Then I looked at multiples of 4, multiples of 5 and 6. And none of them were [the same number of products]. I was looking for ones that would even out. I was looking for, well, there are 18 multiples of 3 and 18 multiples of 5. And, nothing worked out. And I really had a hard time finding out how to play *this* game (laughs). I don't know if I ever [figured it out].

Brad: If you even figured that out?

Randy: It was blank. I didn't know what to do with it. Their suggestion (in the teacher pages) is multiples of 10 and prime numbers.

Brad: Interesting. Not even-

Randy: I wasn't thinking that at all.

Brad: Yeah, not even every (flipping through book). What the kids will do is probably come up with numbers that are equal. You know, when they list all the possible outcomes, like on page 31. 31f. If they list all the different possible products, they might just pick 6s and 10s because they are equally likely to get a 6 and a 10. Or you can put different ones together. Like, '6s and 8s will be mine and 10s and 20s will be yours.' And the kids-

(both talked at once)

Randy: If I'd done it on a line plot, I probably could have come up with some rules for the problem.

Brad: That's what I'm saying. The kids might have it easier if they made a line plot. I think it is easier with a line plot than it is-

Randy: Yeah, it is difficult with this because I had to go through and just count. Where as, if you just do this the right way (organize your data), it is really easy to see the equal outcomes as opposed to counting the individual ones (comparing the matrix with the line plot).

Brad: It might depend upon how your classes are going and what you have for time, but you might. After they have listed all the possible outcomes, the products? When do they do that? It doesn't ask them to do it, does it? (pause). If you have time, you might suggest to them, like you did with the addition game. You might have them list all the possible outcomes. And then from that, make a line plot. I think it's just good practice for them to organize their data. But, it's not required for this particular problem.

Randy: Because you could even do a 16 and a 24, and a 30. And someone else gets an 8 and a 20.

Brad: Yeah, yeah.

Randy: So there are all kinds of runs. (laughs) And I couldn't come up with a single one!

Brad: If you had that with you (the teacher pages) that would have made that easier.

(Collaboration on action #11)

Randy found the student pages difficult to follow at times, which helped him to anticipate how students might react when working on homework. After working through the lesson as a student and then planning the lessons he would teach, Randy incorporated suggestions from the teacher pages. He also made notes of where he thought students might struggle based on where he struggled with the mathematics as well. He also paid particular attention to the lesson objectives to understand which problems in the lesson needed more emphasis. Randy's commitment to understanding the mathematics (including different solution paths) might not have influenced his lesson planning if he had not taken the time to first experience the lesson as a student himself.

During Investigation 4, "Analyzing Two-Stage Events," Randy found the mathematics puzzling due to the manner in which it was represented in CMP. In this investigation, students encounter probability situations in which one event depends on another. Moreover, they use a new representation to understand conditional probability, an area model, to analyze the probabilities. In these problems, outcomes are not equally likely, so simply listing the possible outcomes will not suffice. While Randy attempted to make sense of the mathematics by solving the problems on his own and then using the teacher pages, he regularly called upon Brad to assist with this dilemma. The following excerpt from an interview describes Brad's involvement in assisting Randy with the mathematics:

> I read things in the book. I read about the area model and I was still like, 'I don't know if I really get this.' But Brad actually sat down and explained it to me. As soon as he did that I was like, 'Oh, okay. This, this is not that hard (possibly referring more to how to use it, than how to teach using it).'
>
> *(Interview #3)*

As Randy began to understand how students think about probability, he was more capable of deciphering the mathematics shared by his students while in the act of teaching. In another interview, it became clearer how Randy tackled his limited understanding of the mathematics and how his collaborations *on action* with Brad enabled him to think about probability differently:

Randy: Brad and I talked a lot about probabilities.

Bridgette: What specifically have you and Brad talked about? In terms of the mathematics?

Randy: There's a student who. Now, I've got to think of what it was (pause). We had three choices on one spinner, three choices on the second spinner. (Lesson 1.3) Making Purple. You have three choices on the first one, three different choices on the second one and there's only one outcome that's purple. So you got eleven percent chance. 1 out of 9. And we talked about things like this student knew that there was a 33 percent chance on the first spin and then he divided it by 3 to get the 11 %.

Bridgette: The 1 out of 9.

Randy: And, I had to talk to Brad to realize… that I was doing the same thing. When I take one and multiply by one-third, because the way I think of it is one-third times one-third, which gives you one-ninth…And just those instances where [Brad]. He asked me the questions, *just like a teacher would*. Brad would ask *me* the questions and then I realized that, 'Oh, that's just the same, dividing by three is the same as multiplying by one-third.' So, we talked about how we (as teachers) have to look at how the students are…

Bridgette: Well, the student you mentioned talked about the mathematics in a different way than you understood the mathematics?

Randy: Yes, yes.

Bridgette: And so you had to wrap your mind around his line of thinking to see if he was right.

Randy: And we did…I didn't actually [understand it] when that came up in class. I knew 33 divided by 3 is 11. But, does that always work? Because that's not the way I thought about it. I didn't actually realize that it worked until I talked to Brad later and we sat down and looked at it on paper…I can't recall a specific thing that he said to me to make me understand, but just working through it with him and the questions he asked me. I was like, 'Oh, okay, I understand this more.'

(Interview #1)

Randy's content knowledge –that is, understanding mathematics via only one method – typically limited his knowledge of students' reasoning which then limited his ability to utilize pedagogical content knowledge in practice. In their collaborations *on action*, Brad focused discussion on the mathematics and aided Randy's understanding of the type of content knowledge needed in order to foster student learning. As a result, the focus of the collaborations moved from mathematics content towards the teaching of the mathematics, which is the focus of the next section.

The teaching of the mathematics. Through his experiences in teaching, Brad understood students' reasoning related to probability and therefore had a large repertoire of questions he could pose to build upon student understandings. Most importantly, he shared with Randy the kinds of questions he would pose to improve Randy's pedagogical content knowledge. Pertaining to the teaching of the mathematics, Brad and Randy's collaborations *on action* focused on the kinds of questions that promote classroom discourse. Brad also aided Randy in how to use students' thinking to build a shared understanding of probability. Specific collaborations *on action* highlight how Randy learned to teach the mathematics.

Developing classroom discourse through effective questioning skills was a primary focus for Brad and Randy's collaboration *on action*. More specifically, their collaborations *on action* episodes addressed posing questions that aid students in the discussion of mathematics. In an informal conversation with me, Randy shared his concern for inadequate questioning abilities largely due to his limited experience in teaching probability to middle grades students. It was his own desire to improve his instructional strategies; specifically related to posing questions that aid students in understanding the mathematics. Therefore, Randy capitalized on Brad's ability to focus classroom discourse around the mathematics through appropriate questioning in order to develop a problem-centered approach to teaching. The following excerpt is indicative of collaborations that focused on questions that would aid students in understanding the mathematics:

Brad: Um, whatever it is we want them to find during that lesson, that's where the questions need to be focused… Simple questions that will put everybody on the same page. Every question does not have to be a discovery type of question. Questions that surround the objectives should be more of, 'What do you think, what would happen if, or do you see any patterns, or are you sure you have all the choices there?' You know, those are the questions that you want to be asking.

Randy: Okay, am I doing that?

Brad: I think you did-

Randy: Because, in my mind I think I'm doing that. I'm thinking this is what I want to do, but, what I'm thinking about and what I actually say? When I'm interacting with [the students], it's two different things.

Brad: Yeah, well, you had the kids up there (brought students to the board). I think you were asking them then. I do. I think you could ask more of those questions, though.

Randy: I'm just trying to do a better job of doing that.

(The collaboration shifts to a specific event.)

Brad: To address that question again, about percents. You didn't have them 'discover' how to find the percent. You were telling them, 'Take this one on top, then divide it by…,' and that's okay. Because this lesson wasn't about [changing a fraction to a percent]. The lesson was not about, 'How do I find the percent?' If you want to find out the percent, 'It's 144 out of 312. What percent is that?' Certainly you could have asked them questions about [calculating percent], but it's not about that. You can get them all on the same page by saying, 'Okay, let's divide this by this. Now that we've [divided], what would you say? Would you say it's 52%?' Or, 'What are your observations? Are you seeing patterns up there? Do you see two numbers that show this?' So, you kind of lead them through with those questions to get at what you need to get at. 'Now, you know, theoretical and we have experimental probability. Which one is this? Compare. How do these results compare?' So you're leading them to try get as close. 'So, what would happen if we added these up?' You listen to their ideas. I think that is where you need work, your questioning is going to really pay off. Try to summarize it. That you're talking about it at the end of class. Your questioning is what leads them through. Like with the volume questions (from a previous unit), 'What's the volume?' Well, then you need to define volume so they can compare. Then, let's get them all on the same page and then you can say, 'Now let's compare these. Here's the volume,'

Randy: 'What's the relationship?'

Brad: Right! Otherwise, we're going to be, knocking heads together (figuratively). And then the graphing! Well, you could have spent forever on how to make a graph and how you plot those numbers. OR, you could put them up there and you can ask the more important questions, like, 'What's happening to these data points? Oh, they're moving closer to 50%. What does that mean?' And that's where the questions, the conversation focuses.

Randy: Yeah.

Brad: I think you're doing great. And I really hope my advice is helping.

Randy: I'll take all the advice I can get.

Brad: You've got a good control of the classroom. Trust yourself and your answers. Your ability to *do the math*. And, if you don't know the answer, don't let on. Throw it back to the students. Take time to understand what the students are

> thinking. Let them share their ideas. See where it takes the class. Don't worry, it comes with time.
>
> *(Collaboration on action #4)*

But a focus on questions to pose also led to a concern for too much questioning. Randy discussed in an interview Brad's concern for the rhetorical questions he posed to students:

> …it's something I've been concerned about; the questioning. [Brad] has told me, 'You ask too many questions. You string too many questions together.' So I'm thinking about that now. He's actually told me you can just tell [the students] those little clues like, 'Don't answer right now. I just want everybody to think about this.' And so that preemptively stops all the hand raising and someone blurting out the answer when you don't want them to…Or, he's even suggested. I haven't done it yet but like, 'Okay I want everybody to write on a little piece of their paper. Write down what you think the answer is.' That way I don't have the same students always answering my questions, blurting it out. I haven't tried that one yet, but it's definitely something I've thought about.
>
> ...I pose too many questions at times, not giving [the students] time to think about them. Or, I see [some of the questions] as rhetorical, where I don't want [the students] to answer, but they don't know that. They give me these blank stares. I've asked Brad why that is and he said I string too many questions together and [the students] become confused… So I'm trying to pair-down the questions I pose.
>
> *(Interview #2)*

Not only was Randy concerned with the questions he should pose, but also with *how to listen* to the answers given by students. Randy sought to ask open-ended questions, but unfamiliarity with how to incorporate students' responses while, at the same time, teaching the lesson as he had planned proved to be difficult. He understood his questioning skills would elicit what his students were thinking, but that entailed better understanding on his part of not only how students thought about probability, but how to listen to students' thinking and then respond to and incorporate students' thinking in classroom discourse.

A second dilemma regarding the teaching of the mathematics was demonstrating an understanding of students' thinking while in the act of teaching, which was a source of anxiety for Randy. In particular, Brad and Randy's collaboration *on action* planning sessions focused on how to listen to, correctly interpret and incorporate students' thinking in instruction, an important component of pedagogical content knowledge. Randy was unsure if he could always understand the ideas presented by the students in instruction. The following excerpt described the pressure he felt to understand and incorporate student understandings in classroom discourse. Brad was aware of this problem because he, too, had grappled with how to use student understandings in instruction:

Brad: Sometimes my thinking is not as quick in my head as it is with the kids. So don't be so hard on yourself.

[Brad attempts to reassure Randy that he, too, experiences the same concerns Randy has regarding understanding students' thinking while in the act of teaching.]

Randy: It's things like that. I have a slow processor, a slow word processor (laughs). Sometimes that's not very helpful when I'm teaching.

Brad: But I think that *allows* you to teach. Not like someone that is very quick.

Randy: Oh, I know. I know what it is like. There are people that know all the math in the world and can't teach it.

Brad: Right. Even if you work slow and methodical, sometimes [you] are better. It doesn't mean you're not as smart. It means you have a way of breaking something down into parts. Where as other people may not.

Randy: I think it does hurt me, though. And I'm trying to understand some of the ideas from these kids.

Brad: That's the real challenge with this math (teaching CMP). Because it use to be that you were in charge. 'This is how you do it. Then work.' Give an example and have [the students] do the problems. Now we're asking [the students] what they think. They throw something back at you, and you're like, 'Oh, why?' And that's the hard part. I've seen kids have extreme ways of doing it and I don't [understand how they are thinking about it in that way]. At first, when they explain it to me, how they did it. And that it actually works every time. So, when

I'm up there teaching, I should know whether this works, but then I possibly have never seen this way of solving it before... (talks about problems from earlier CMP units)... Now [as teachers] we're forced to look at all the ways the kids are able to solve it. And make a decision, 'Is this something to go with? Or no?'

Randy: There is somebody who did a process to find the 16 blocks that worked (for a particular problem), but I couldn't understand why they were doing it that way… Because I knew it was 16 blocks, but now I've seen so many different ways. I don't remember how I attacked that problem. In fact, I think I actually saw the answer (in the teacher pages) before I ever let myself work through it. Because it tells you to get 16 blocks ready for the lesson.

Brad: Oh, yeah.

Randy: So, I don't know how I would have figured it. I mean, I would have figured it out, but I don't know what method I would have used to do it. And somebody did one that was different and it worked. But, I couldn't figure out why they were doing it [that way].

Brad: I did a tree diagram. Then looked at the totals.

Randy: Well, it's just the *explorations* in general…

(Both discuss a particular exploration where the mathematics seemed obvious to them, but were unsure if it was as obvious to the students and whether additional questioning would help.)

Brad: You're not sure if it is obvious to the kids?

Randy: Yeah.

Brad: Then, it's another place where you can question. You can then say, 'What does it have to be?'

Randy: Have them tell me why.

Brad: 'Why does that have to be? Why?'

Randy: Because I am somewhat hesitant. 'Is that the best way to think about it?' So maybe, maybe in questioning? Maybe they will say something that's like, 'Oh, that's a good way for me to think about it, too. Because it makes more sense.' Because, if I'm struggling with the explanation? Then, how well do I actually understand it? Am I thinking about it the best way I know?

Brad: Sure.

(Collaboration on action #6)

Brad assisted Randy with questions to pose, but Randy's implementation in practice was uneven. He struggled to make use of such questions in practice. When comparing my field notes of the questions Randy asked with the questions in the CMP teacher pages and the questions Brad posed in their audio taped collaborations, I found Randy did not always ask the questions he originally planned to ask or had collaborated about with Brad. Therefore, some collaborations about the teaching of the mathematics did not make it into Randy's instructional practices.

With regard to the teaching of the mathematics in the CMP lessons, my analysis of the collaborations *on action* also yielded what Randy perceived to be lacking from his collaborations with his mentor teacher. In particular, in one interview, Randy disclosed that he and Brad collaborated just before lessons or immediately after the lessons were taught; never were there collaborations about future lessons or investigations. Randy expressed his concern of foreseeing the direction of where a series of lessons or investigations would take the students mathematically:

> A lot of the collaboration is last minute. I might have some questions right at the end [of our planning time], right before I teach it. I look at the book and I see how I want to attack [the lesson]… And then I come to [Brad], if we have time. We don't always. But, I will say, 'I was thinking about doing this' or, 'I had a question about this.'…A lot [of the collaboration] is [last minute].

When asked if he would like more time spent collaborating on how to teach the lesson, Randy responded:

> Yes, beforehand. Like, more than the day before. A lot of times, it's [during our] planning period and I've got all these questions. Here it is, the day of and I've still got questions about how I'm going to teach this. I don't like that. I feel like I should be prepared already…So I would like to have more…pre-pre-planning.
>
> *(Interview #2)*

Because Randy typically planned his lessons the night before teaching them, he never felt prepared enough to go beyond the next day's lesson and consider future content. Essentially, he was precluded from planning ahead because of how planning time was utilized during the school day; namely, *one lesson at a time*.

Both the nature of the audio taped collaborations and the interviews allowed investigation into the initial and emerging development of pedagogical content knowledge for the teaching intern. The nature of the collaborations *on action* provided an opportunity to examine Randy's experience in learning to teach. More specifically, as a result of his collaboration *on action* with his mentor teacher, Randy learned more about the CMP curriculum, probability, and the teaching of probability.

The Nature of Collaboration In Action

Randy utilized another form of collaboration during his teaching internship: collaboration *in action*. Collaboration *in action* refers to discussions "in the act of teaching," while students are present or transitioning between classes when they could only collaborate briefly. Brad and Randy collaborated *in action* at critical points that Brad or Randy deemed necessary. Some of the collaboration *in action* existed while Randy was in the act of teaching during the *Explore* portion of the day's lesson. Others occurred as students transitioned between classes. Both instances were typically short in duration, lasting less than three minutes, and in some cases, lasting less than one minute.

Evidence gleaned from the audio taped collaborations, interviews and observations of collaboration *in action* produced two common topics. First, Randy looked to Brad for reassurance. These collaboration *in action* episodes often occurred after Randy taught a lesson once. Second, the *in action* episodes focused on Randy's role in orchestrating classroom discourse. Within orchestrating classroom discourse it was evident Randy grappled with three specific dilemmas (see Table 4.4).

Domain	Dilemmas
Reassurance	• Randy looked to Brad for reassurance after teaching a lesson for the first time
Classroom Discourse	• Randy's questioning skills and questions to pose • Randy struggled with how to listen to and incorporate students' thinking in instruction • Allowing time to *Summarize* the key concepts

Table 4.4: Central topics in collaboration *in action*

Reassurance. The collaboration *in action* episodes occurred throughout the school day, but usually occurred after Randy taught a lesson once. Most times, it was initiated by Randy. It was after observing him teach the lesson for the first time that Brad collaborated on what he observed. A sample of Randy's perspective of this type of collaboration *in action* follows:

> A lot of the time [the teaching of the lesson] is on me. What am I bringing to it first and [Brad] lets me teach it. I'll teach it and then that's when we will actually get together after the first time I teach it. It's almost like he feels like that's the way it should be. I should discover for myself. If I ask him, he will tell me, "This came up and you should have really done this." But I think part of this he has confidence in me. He knows I'll get what's suppose to be done.
>
> *(Interview #3)*

In another excerpt, Randy was asked why he looked to Brad for reassurance:

> Because he's been doing it (laughs). He knows what he's doing. He's taught this before. He's strong in mathematics. He's a strong teacher. The kids respect him and it seems like they really pay attention when he teaches… It would be stupid of me not to ask, 'What do you think about this?' I want his feedback because his suggestions make a lot of sense. And he always validates why he makes a suggestion. He doesn't just say, 'Well you should just do it this way.' He's never once said that to me.
>
> …In teaching the lessons in the past, he has said, 'In the past, I've done this and this seems to work pretty good.' And so, and then we'll talk about why that might work. And then I might try it in the class. He always justifies.

> …(when teaching) He'll just give me a signal that, 'You need to [move along]. Nobody has introduced the area model and you need to go ahead and introduce it.'
>
> *(Interview #4)*

Randy faced several dilemmas during his teaching internship, but through the collaborative process, he learned to mediate these dilemmas. Analysis of the collaborations that occurred throughout the internship show Randy sought reassurance less and less during his student teaching internship experience.

Classroom discourse. Brad assisted Randy with his thinking about learning to teach by examining the questions he posed to students. The *in action* episodes revealed that Brad used questions with Randy in order for Randy to think about the questions he used with students. Therefore, questioning was an instructional practice used by both Brad and Randy.

With regard to Randy's questioning skills, Randy and Brad's collaborations *in action* typically focused on Randy's questioning skills and the questions to pose, which is representative of Randy's emerging development of pedagogical content knowledge. More specifically, the collaborations addressed the questions Randy posed during the *Launch* of the lessons. When Randy went to Brad for an *in action* episode, he typically asked for validation of his instructional practices because in an interview with Randy he told me he specifically asked Brad to monitor the questions he posed in instruction. The *in action* episodes often occurred directly after Randy *launch*ed a lesson. However, when Brad initiated an *in action* episode, analysis of the data revealed his concern was looking beyond the *Launch* for key points during the lesson where Randy could have influenced the direction of discourse through the questions he posed.

In the following *in action* exchange regarding Randy's teaching of Lesson 1.1, What's in the Bucket, Brad assisted Randy with the types of questions he could pose to build upon students' understanding of probability. The lesson consisted of reading a story about placing colored blocks in a bucket (see Appendix K). The students were to discuss the probability of a block drawn from the container, without knowing the number of blocks in the bucket or their colors. The following exchange between Brad and Randy occurred between his first and second teachings while students transitioned to class:

Randy: Okay, what do I need to?

Brad: Ah, I liked a lot of things. I like how you brought up the percents and how the percents equaled 100%. That was good. Probably at the end, what I might have done. Instead of saying 'This is experimental. This is theoretical,' talk about the difference between the two.

Randy: Okay.

Brad: And then, say, 'These are two words that I have. There is experimental and there is theoretical. Which one of these (examples given on the board) is experimental? Which one of these would you think is theoretical?' And then, maybe at the end, I thought there was one question you could have asked at the end, with the flipping of the coin. 'If you get eight out of ten heads, was that a theoretical or an experimental probability?' Or something similar to that. I thought your timing was fine. Did you think so?

Randy: Ah, I felt rushed on this. We were almost five minutes late getting back (from lunch).

(Collaboration in action #1)

During the teaching of his second lesson, Randy asked the students, "Is possible to have more than 100% in the context of probability?" Although Randy knew the answer, he was unsure of how to explain it. Struggling while in the act of teaching, Randy sought Brad's help:

Brad: (to Bridgette) Randy just asked the class if you can have more than 100% pulled out of the bucket. Or 100% of something happening. Or a better chance of 1, that something would happen, and some of the kids were saying yes. He said this is hard to explain. [Randy sought Brad's assistance during instruction.] So my suggestion was to ask, 'If you have three blocks, can you pull six blocks out of the bucket?' And then that was a way for the kids to understand it. The probability of more than 100%.

(Collaboration in action #2)

Not only did Randy grapple with knowing what questions to pose, he also had difficulty knowing *when* to pose key questions that would stimulate discussions about probability.

As an additional example, during Lesson 2.2, Playing the Multiplication Game, Brad and Randy collaborated between classes. Previously, Lesson 2.1, Playing the Addition Game, specifically asked students to list all possible outcomes (Appendix I, Part B, page 22); however, the same question was omitted in Lesson 2.2, leading Brad and Randy to discuss whether this key question should be posed anyway (see Appendix J). Brad suggested Randy observe how the students respond to the lesson and include the question only if they did not consider it on their own. My observation and field notes of Randy teaching this lesson to Core B revealed he taught the lesson without asking students to list all possible outcomes, as presented in the student pages. The following is an excerpt of how this played out in Randy's initial teaching of Lesson 2.2, Playing the Multiplication Game, in a collaboration *in action* episode between classes:

Brad: Why didn't that go the way you wanted it to?

Randy: I don't know. I had all of this outlined. They just kind of dictated where it went. I covered how they calculate the experimental and theoretical. That was one of my objectives. I talked about the relationship between the two. We listed all the outcomes. No, we didn't, actually. We did the matrix, but I don't know if they were really realizing that the matrix lists the possible outcomes. We didn't discuss it. I wanted to fit the line plot in, too, but we didn't have time.

Brad: So, those two. List the possible outcomes and using an effective method to organize? Are those the two that maybe you were more teacher-driven, instead of student-driven?

Randy: Yes, but maybe that's okay because I've already done that?

(Randy had touched on listing the possible outcomes in the last lesson so he contemplated what he needed to reinforce in this lesson.)

Brad: That group (class) has its own personality and letting them dictate, there is nothing wrong with letting them dictate. I would still have them list all the outcomes. I still think that is an important thing. How to organize it.

Randy: Well, it doesn't actually ask that.

Brad: You're right, it doesn't (refers to questions in the lesson that are posed to the students).

Randy: But it *is* important. You have to list the outcomes if you want to understand where theoretical probability comes from, right? I better go and erase the board before the next class.

(pause in collaboration; Randy erases board, talked more with Brad and then instructed the next class)

Brad: (to Bridgette) Randy and I were just talking about how he was pleased with the class. I think you heard that. I think one of the main objectives they didn't get to is having them list the individual outcomes. The problem doesn't ask for that. It just asks them to find the theoretical probability. But my comment to him was that (after observing how the lesson went) I think they need to still list the outcomes. That is going to be very important for them, to determine the theoretical probability in future problems in this book. They are going to have to become comfortable with an organized method of listing their outcomes. Hopefully with this next class he'll get to that particular part. We had to stop talking so he could erase the board and get ready for this class coming in.

(Collaboration in action #12)

In this excerpt, Randy illustrated his ability to adjust the lesson as needed, as evidenced by his outline of questions to pose and his willingness to let students influence the direction of the lesson; however, Randy continued to struggle with *getting to the mathematics* in classroom discourse. An objective of Lesson 2.2, Playing the Multiplication Game, was to develop the relationship between experimental and theoretical probabilities, to review familiar methods of computing experimental and theoretical probabilities, including making an organized list of possible outcomes. While the latter appeared to be his main concern and his collaboration with Brad addressed this, the mathematics was less of a concern for Randy and more of a concern for Brad. Randy struggled with simultaneously addressing both dilemmas while teaching.

It is important to note what Brad focused on during collaboration *in action* with regard to Randy's process of learning to teach. Brad encouraged Randy to observe how the students respond to his questions, making note of when students struggle, when they need his support and when to let go. Randy's ability to know what questions to pose and when to pose them influenced classroom discourse so Brad focused *in action* collaboration on this aspect of teaching. This proved vital in this teaching internship because Randy struggled to understand how to learn from student understandings. Consequently, from my interpretation of the collaboration *in action* episodes, it was apparent that Brad emphasized the need to learn from students' responses in order to use them in instruction.

An additional data source that documented collaboration *in action* was the interviews. Randy's reflective nature brought forth new information not captured in the collaboration audiotapes. Some of the instances he discussed in the interviews further supported an analysis of his own teaching, with regard to his development of questioning skills and knowledge of student understandings, a component of pedagogical content knowledge, and the collaboration with Brad in his learning to teach process. The following excerpt from an interview with Randy described a time when he initiated collaboration with Brad while he was in the act of teaching:

> Sometimes it's just as simple as I'll walk over and I'll be like, "Boy, nobody got that question that we thought everybody was really going to

> get and I threw it out there and I just got all these blank stares." And then I'm like, "I wonder if I should ask that again?" And sometimes Brad will be like, "No, I still think it's a good question. I think you should ask it," or "Yeah, maybe they're not ready for that. Maybe we just need to push it off to the side because…" We'll make a decision about whether or not it (the question) is important enough to keep.
>
> …Now I know today in a lesson I was really hesitant. I asked the students to predict beforehand; to vote on whether they thought a game was going to be fair…the game of 2.1 (Lesson 2.1, The Addition Game*)* was going to be fair or unfair. And they made their votes. And then I was struggling because one of them wanted to tell me why they thought it was unfair. I was struggling for a second because I'm like, "Do I want to ask them?" Because, depending on whom I asked, they could have, you know, ruined the whole experiment.
>
> …But then I walked over to Brad and asked him. I was like, "Do you think I should have asked that question?" Just to see if he agreed. And he was in agreement that, "I think it's perfectly fine to ask that question and, you know even if somebody does get it right," or says something, it's kind of you just, 'Okay, that's interesting.' And just move on to the experiment. You don't really tell them whether they're right or wrong anyway, so it's okay.
>
> *(Interview #1)*

Brad suggested a response that Randy hadn't considered prior to teaching this lesson. While Randy expressed concern for students "ruining the experiment," Brad was familiar with how this unpredictability of student data commonly occurs when teaching from CMP. It was also common for the *Launch* to spark student conjectures that might "spoil" the exploration at times. Randy was apprehensive about using students' thinking at times, unsure of whether he should allow classroom discourse down an unintended path because his knowledge of instructional strategies was limited. In collaborating with Brad, he learned a technique for considering students' conjectures without "spoiling" the exploration. He learned to validate students' thinking with a sense of wonderment by saying, "I am not sure. How about we think about that as we collect data," and continued on with the exploration.

In unpacking the topic of classroom discourse further, the data from the collaborations *in action* revealed Randy struggled with incorporating students' thinking in instruction. In Lesson 4.1, Choosing Paths, students are introduced to an area model to compute probabilities in situations involving a sequence of actions (see Appendix D) where students are asked to place a treasure in either room A or room B. In the following excerpt Randy's ability to listen to and incorporate students' thinking in instruction was addressed. Brad discussed with Randy the questions he posed and how students responded:

Brad: What I've seen in here so far today, you've been building on to the questions and their responses are building on it. Taking it to the next level. Or if they didn't answer the question the way you wanted it, you rephrased it. Like, I wrote it down. You said, 'How can we pick a path at random?' You didn't get very many responses to that. They were still just sitting there. And I know they know what the word random means. You even asked them, 'What does random mean?' Which was good. Let's define random. A good way to get them to understand the question. Then you said, 'Can I just pick the middle path or is that just because it's my preference?' Do you remember saying something like that? And the kids said, 'Oh, that would just be picking based on a preference.' So then you rephrased. And you asked the question again, 'How do we choose something that is random then? So it is not based on a preference?' Then someone said, 'Flip a coin.' Was that [student] in the back? I think it was [student].

(Collaboration in action #12)

Randy was able to adjust his instructional strategies through information he gained from listening to his students respond to questions he posed while teaching. Brad mentioned to me in an informal conversation that he recognized growth in Randy's ability to take student understandings and build upon them. Therefore, Randy began to overcome his dilemma of incorporating students' thinking in instruction.

While teaching Lesson 2.1, Playing the Addition Game, collaboration *in action* was initiated by Randy when a student asked if rolling a 2 and a 3 is the same as rolling a 3 and a 2. During the *Launch* portion of the lesson Randy addressed this key concept with the class and I observed the students agree the two rolls were different. Seemingly puzzled by the question, Randy suggested they think of the dice as different colors or by recording their rolls in order, and also referred them to a problem they encountered in an earlier lesson. Despite these efforts, students still struggled with these suggestions. Therefore, Randy moved to the side of the classroom and conversed with Brad in an *in action* episode. Brad assured him he had adequately addressed this issue. Rather than involve the entire class once again in this discussion, Brad suggested Randy talk with the individual group of students.

When teaching the same lesson to a different class the following morning, I observed again Randy *telling* students these were different rolls. He then walked over to Brad with a look of dismay. Brad reassured Randy the *Launch* was adequate and that most students understood the difference. Furthermore, Brad continued to point out this lesson was in the early stages of the unit. Students were just beginning to explore key concepts of probability and future lessons would assist the students in understanding when order matters. Brad's knowledge of the curriculum and student understandings aided Randy while in the act of teaching. As Randy gained new knowledge about the curriculum and his students, he sought Brad's assistance less frequently. This was evident by the fact that four of the 14 collaboration *in action* episodes occurred after Investigation 2.

With regard to time to summarize key concepts, the collaborations *in action* also revealed time management was of great concern to Randy. Although these concerns were discussed minimally in the audio taped collaborations, they were discussed at great length in Randy's interviews when he talked about his collaboration *in action* experiences. In one interview, Randy recounted how Brad insisted he have ample time to develop the fundamental mathematics of the lesson. Randy described an *in action* episode he had when students transitioned to their next class:

> A lot of times, it is timing things. "Okay, you spent too long on the *Summary*." Sometimes we'll have a warm-up problem, but that's the hard thing, trying to decide between a warm-up or homework. Because there's not enough time to have a warm-up problem and have them check answers to homework. So it'll be things like, "You spent too long on the *Summary*." …I focus too much on summarizing the previous day's material in a warm-up. (Randy often summarized during the next lesson's *Launch*.) And Brad's always like, "We're not teaching it to mastery. These are things that are going to come up again."
>
> *(Interview #2)*

Additional insight into time management concerns occurred in another interview in which Randy commented:

> A lot of times, he's just given those clues like, 'Okay, it's time.' Or, 'You haven't hit that yet. You need to do that.' And he actually said that to me. Was it with Core B? No, it might have been Core A. The very first time I taught it, he said that. But he had to. We had an assembly today. The last two classes were cut short. And so he had to, 'Okay, you gotta stop. You gotta do this because we got to get [going]' (laughs).
>
> *(Interview #3)*

Randy sought Brad's assistance in his collaboration *in action* episodes for finding ways to "tighten up" his lessons, spending less time summarizing the previous day's lesson and allowing more time for launching and exploring the new lesson. Here, Randy reported that Brad knew the direction of the lessons, when it was important that students have mastered the concept, and when it is necessary to go back and review concepts and skills that connect with the current unit. Randy adopted Brad's position that not every topic is mastered the first time it is presented in an investigation:

> It's usually the *Summary* [where I spend too much time] (Again, Randy seems to summarize the following day). Yes, I'm too in-depth. [The students] should have already known this so I don't have to go in-depth

> now. Brad will say, "This is supposed to be about the big ideas. You're not trying to re-teach it again and we're not teaching it to mastery." …Because I'll spend too much time on it and then realize that I don't have enough time left for the main part of the lesson.
>
> *(Interview #2)*

In earlier collaborations when Brad shared his knowledge of the curriculum and student understandings related to teaching probability, Brad taught Randy how to pace his lessons. With this assistance and as Randy taught additional CMP lessons, Randy became more aware of how the CMP curriculum unfolded, paying attention to when lessons explored particular concepts versus when lessons emphasized mastery.

Influence of CMP on the Nature of the Collaborative Process

As described earlier in this chapter, Brad and Randy extensively collaborated throughout the teaching internship. Additional analysis of the data revealed that CMP influenced the nature of the collaboration through its role as the adopted curriculum and *the* source for planning. However, collaborations occurred *around* the curriculum and were not necessarily word-by-word-specific, suggesting collaboration focused on the specifics of *teaching* and not the specifics of CMP per se (see Table 4.5).

Domain	Influence
Connected Mathematics Project	• Served as *the* source for planning and implementing instruction as the adopted curriculum

Table 4.5: CMP influence on the nature of the collaboration

Adherence to CMP in mentor teacher's school district. One year prior to Randy's student teaching internship, the school district implemented district-wide requirements for the content and pacing of the seventh grade mathematics curriculum. With three middle schools and approximately ten full-time mathematics teachers for seventh grade, the district's Secondary Mathematics Specialist, in consultation with Brad and other mathematics teachers, developed a pacing chart for each of the six CMP units taught in seventh grade. A pacing chart existed for the sixth-grade curriculum as well, which Brad abided by in teaching sixth grade mathematics in the previous year.

In formulating a schedule for Randy's teaching responsibilities at the outset of the internship, Brad emphasized his desire to adhere to the district-mandated pacing schedule, which allotted a one-lesson instructional day for each lesson in the unit that Randy would teach. Randy used this information to assist him in planning and implementing the unit, *What Do You Expect?* Given the aforementioned struggles with time-management and Brad's strict adherence to the CMP curriculum pacing, Randy said in an interview with me he contemplated whether the schedule hindered the development of concepts across lessons. He wanted time to review concepts at the end of each investigation.

It is clear that much of Brad and Randy's collaborations focused on the *Launch, Explore, Summarize* lesson format utilized by CMP. Lesson 1.2, Matching Colors, had students analyze a game played with a spinner and review the idea of a fair game of chance. Their collaboration focused on the specifics of how to organize students into pairs where the students conduct an experiment using spinners and data. Randy sought Brad's guidance with regard to allocation of time so that students could gather data and analyze the class's results during the summarize portion of the lesson. Collaborations specific to the CMP curriculum included understanding how Brad enacted the lesson in prior years, what problems may arise, and whether one 45-minute class period was enough time to teach the lesson.

My synthesis of the interview data also indicated Randy utilized the CMP curriculum for planning and implementing instruction. Randy was concerned his narrow view of the mathematics limited what his students could learn from him, but the CMP curriculum enabled him to think about the mathematics in ways that middle grades students do. Specifically, the curriculum provided samples of students' thinking and possible strategies to utilize in assisting students with the mathematics. Without the CMP curriculum, Randy may have only emphasized strategies he understood and felt comfortable teaching, thus limiting the problem-centered approach. The CMP curriculum provided Randy with a way of understanding how middle grades students think about the probability, before he began his initial teaching. His need to work through the lesson and solve the problems for himself first also proved his desire to understand the mathematics in CMP and its lesson format structure.

Specific instances when Brad and Randy talked about the CMP curriculum were collaborations when they posed questions to one another through reading from the teacher pages. In the following collaboration *in action*, Brad suggested to Randy a need to change a portion of his instruction so that additional time could be allotted to discuss the key concepts in Lesson 1.3, Making Purple:

Brad: …I saw you made an adjustment. You made an adjustment on the homework assignment, too, so you would have more time. What would happen if you skipped part B? If you skipped part B and C, to go over later? And, at least had [the students] get to part D. Part B is, 'Graph the information.' Part C is, 'What would the graph look like if you did it 100 times, 200, or a thousand?' D is, 'List your outcomes and what is the theoretical probability?' If that is what you think is the *meat* of the lesson, perhaps, maybe go right to D and E, and then go back and look at part B to see if it actually heads towards that theoretical probability that they discovered in part E.

(Randy initially prepared and taught Lesson 1.3 as suggested in the CMP teacher pages, however, through observing the lesson, Brad recognized a need to slightly change the order of questions so to allot time for discussing the key concepts of the lesson. [Randy was concerned this change might eliminate Part B and C where students graph their data; an important strategy for analyzing data that Randy did not want to skip.])

Randy: I thought part B was really powerful, but,

Brad: B is a good one, but …part B is tripping them up, preventing them from getting to D and E. While B is still important, what happens if we put the hurdle at the end? Instead of putting it in the middle? Do you see what I'm saying?

Randy: Yeah.

Brad: I'm not saying that what you are doing is wrong, but, myself, if I was looking at this problem and struggling with B, I might just change it for the next time and say, 'Okay, we're going to do part B later.' It's just so that I can make sure they get the bulk of the lesson and not get so hung up on one problem. Part B has a lot of value in it. It's just tripping them up. They need a lot of extra help with it the graphing in B and C.

(Collaboration in action #10)

This collaboration illustrated specific instances when Brad and Randy discussed the CMP curriculum and Randy's teaching of it. Much collaboration such as this stemmed from the teacher pages, suggesting that CMP played a central role in the collaborative process.

Summary of the Nature of the Collaborative Process

Within this first section of Chapter 4, I reported results related to the nature of the collaborative process between the mentor teacher, Brad, and the teaching intern, Randy. More specifically, I reported results related to the nature of the collaboration *on action* and the nature of the collaboration *in action*. I collected data from audio taped collaborations, interviews, and observations. In my analysis of these sources of data, it was evident collaboration *on action* played a significant role in Randy learning about the CMP curriculum, the mathematics, and the teaching of the mathematics. While Randy addressed specific dilemmas among these three topics, Brad proved influential in what Randy learned in his teaching internship experience.

Brad and Randy collaborated *on action* extensively about the CMP curriculum. In particular, they collaborated about allocating time to *Launch* and *Explore* the lessons, allowing for adequate time to sufficiently *Summarize*. Randy learned ways to structure his lessons so to make effective use of time, and Brad assisted by providing a view of future lessons so that Randy could see the big picture.

Also, the collaboration *on action* proved crucial in Randy's understanding of the mathematics. While he demonstrated adequate content knowledge with regard to probability, he often understood just one way to solve particular problems. It was through his collaboration with Brad, their discussions about the CMP curriculum and how students think about probability, that Randy demonstrated growth in understanding probability in a way that would help others to understand it. Moreover, the use of multiple representations of mathematical ideas to foster student learning became more important to Randy.

The *on action* collaborations assisted Randy's teaching of the mathematics. Their collaborations addressed Randy's question-posing abilities that would aid students in the discussion of mathematics. As well, their collaborations addressed how to listen to, correctly interpret and incorporate students' thinking in instruction.

In relation to the collaboration *in action* episodes, Randy sought reassurance from Brad after the first time he taught a lesson. It was during this time Brad provided insight into how Randy's lesson unfolded and his impressions of Randy's instructional practices. Their collaborations *in action* also dealt with Randy's orchestration of classroom discourse. It is in these collaborations where Brad and Randy evaluated Randy's questioning skills and the questions he posed to students in instruction. Randy also sought guidance about how to use students' thinking in instruction and how to allow time for *Summarize* where Randy needed to pull together all the components of his lessons.

The CMP curriculum also influenced the nature of the collaboration through its role as the adopted curriculum and *the* source for planning and implementing instruction. The nature of the collaborative process between Brad and Randy proved fruitful in Randy's process of learning to teach during his student teaching internship.

CHAPTER 5: DISCUSSION, SUMMARY, AND RECOMMENDATIONS

This study examined how the sources of collaboration, curriculum, and the classroom context influenced the teaching intern's development of pedagogical content knowledge and the process by which the purpose for developing such knowledge changed. In addition, this study investigated the collaborative process between a mentor teacher and teaching intern in order to clarify the nature of the collaboration with respect to collaboration *on action* and *in action* in the development of pedagogical content knowledge. I also looked for ways the CMP curriculum influenced the nature of the collaboration. A framework was developed to examine these converging forces so that we may understand the complex interaction found during the student teaching internship.

In this chapter, I summarize the study and discuss its findings. Specifically, this chapter is organized into seven sections: (a) Summary of the Study and its Findings, (b) Results of the Study, (c) Discussion of Findings, (d) Study Implications and Recommendations, (e) Recommendations for Future Research, (f) Limitations of the Study, and (g) Final Thoughts.

Summary of the Study and its Findings

Research has sought to address the question of how preservice teachers learn to teach (e.g., Grossman, 1990; Grossman, Wilson, & Schulman, 1989; Gudmundsdottir, 1987; Marks, 1991). There is a general consensus that novice teachers have inadequate or underdeveloped pedagogical content knowledge of mathematics for use in practice (e.g., Borko et. al, 1992; Borko & Putnam, 1996). Although novice teachers are typically deficient in pedagogical content knowledge for a wide range of mathematics content, the teaching and learning of probability concepts represents particular challenges for teachers (Jones, 2005). In particular, research supports the notion that mathematics teachers need additional pedagogical content knowledge for teaching probability (Stohl, 2005). An additional challenge faced by today's preservice teachers is the fact that materials implemented in classrooms today are quite different from the decade prior to the *Principles and Standards for School Mathematics* (NCTM, 2000) in that, such curricula are problem-centered and likely represent a departure from preservice teachers' experiences learning mathematics. Consequently, if preservice teacher education programs inadequately foster pedagogical content knowledge, then the development of pedagogical content knowledge appears to be postponed until a later stage in their professional development (Smith, 1999). Therefore, given the interrelatedness of pedagogical content knowledge and teacher development, it is important to ask *how much* preservice teacher education programs can adequately develop pedagogical content knowledge.

Purpose of the Study

This study examined the development of pedagogical content knowledge of a student teaching intern as he was influenced by three sources found in a student teaching internship: collaboration, the curriculum, and the classroom context. I defined pedagogical content knowledge as the type of knowledge teachers possess when they understand the mathematics they are teaching and are able to draw on that knowledge with flexibility by looking for multiple ways of representing mathematical ideas, including a variety of models, to foster students' conceptual understanding. This study represented the case of a strong preservice teacher paired with a district leader in mathematics education. This case was studied in order to understand the nature of the collaborative process in the development of pedagogical content knowledge. Specifically, my study sought to address the following research questions:

1. *How did the sources of collaboration, the CMP curriculum, and the classroom context influence the teaching intern's development of pedagogical content knowledge?* More specifically, how did these sources influence his development of conceptions of purpose for teaching mathematics, knowledge of student understandings, curricular knowledge and knowledge of instructional strategies? What was the process by which the purpose for developing pedagogical content knowledge changed for the teaching intern?
2. *What is the nature of the collaboration between the teaching intern and mentor teacher?* In particular, what is the nature of collaboration *on action* and collaboration *in action*? In what ways did the CMP curriculum influence the nature of the collaboration?

Collaboration *in action* was the instance when the teaching intern and mentor teacher briefly collaborated during formal instruction, making adjustments to instruction as needed. When students were not present, the collaboration took the form of collaboration *on action*, either during joint planning sessions or during instances throughout the school day when formal instruction was not enacted and time allowed for longer collaborations.

Methodology

Because this study described and interpreted the collaborative process as a way to enrich our understanding of the complex interaction between a mentor teacher and teaching intern, I selected a *qualitative case study* as the most promising mode of inquiry

(Creswell, 1998; LeCompte, Milroy, & Preissle, 1992; Stake, 1995). A case study methodology (Stake; Yin, 2003) illuminated the complexity of the pairing of these two experts in their respective fields, teaching and preservice teaching. Through the lens of understanding the teaching internship, I sought to make interpretations in order to provide a greater depth of understanding about how sources influenced the teaching intern's development of pedagogical content knowledge, and the process by which the teaching intern's development of pedagogical content knowledge changed. I also sought to understand the collaborative process found between the mentor teacher and teaching intern.

Using a case study approach (Creswell, 2003; Yin, 2003), I employed numerous data collection and analysis procedures. The teaching intern and mentor teacher were carefully selected to permit a close examination of several sources with the potential to influence the development of pedagogical content knowledge during a student teaching internship: the role of the collaboration, the classroom context, and the use of the curriculum. How Randy's development of pedagogical content knowledge changed was analyzed by triangulating several data sources in order to investigate the initial and emerging development of pedagogical content knowledge. These data sources functioned as a method of interpreting Randy's development in learning to teach probability. The following subjects and data sources were used to understand how these sources influenced the teaching intern's development of pedagogical content knowledge.

Subjects

The subjects were a teaching intern, Randy, and his mentor teacher, Brad, in a school district in a large metropolitan area in the Midwest. Randy was characterized as one of the best methods students in his preservice mathematics teacher education program. He was resourceful in that he sought after the best approaches to teaching mathematics. He was the type of preservice teacher who looked for opportunities to plan and teach lessons with students, reflected on his instructional practices, and learned from his field experiences. The teaching internship resided in a seventh-grade mathematics classroom within a middle school. This study included a classroom that used *Connected Mathematics Project* [CMP] (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998), a standards-based mathematics curriculum, and a mentor teacher who had taught CMP for several years, attended CMP workshops for advanced users, and is a district leader in professional development of middle grades teachers.

Data Sources

Several data sources were used to capture the nature of the collaboration between the teaching intern and mentor teacher and the teaching intern's development of pedagogical content knowledge.

Collaborations. The collaborations were analyzed in order to allow for investigation into both research questions: the nature of the collaborations and how the teaching intern's initial and emerging development of pedagogical content knowledge changed during the teaching internship. Fourteen collaboration *on action* sessions were analyzed during which the teaching intern and mentor teacher prepared for both upcoming lessons and discussions of previous lessons. Ten of these collaborations coincided with observations and interviews related to lessons from Investigation 2, "Analyzing Number-Cube Games", and Investigation 4, "Analyzing Two-Stage Events." In addition, fourteen collaboration *in action* sessions were documented to understand this type of collaboration that occurred outside of the joint planning sessions. Delving into the data, it was apparent collaboration focused on Randy's understandings of pedagogical content knowledge and how Brad, the curriculum, and the students aided in this development. In addition, the data revealed the process by which Randy's development of pedagogical content knowledge changed and the dilemmas he faced in his thinking about teaching probability and how he tried to resolve these dilemmas.

Observations. Eight classroom observations were conducted at pre-arranged dates and times. I observed the teaching intern in order to document and evaluate his teaching in relation to student learning, along with collaboration *in action*. In particular, I observed the teaching intern's teaching with two core groups of students, identified as Core B and Core C. I observed two lessons from both cores for Investigation 2, "Analyzing Number-Cube Games", and Investigation 4, "Analyzing Two-Stage Events." I kept field notes in order to analyze the role of the teaching intern, the mentor teacher, the students, and the types of instructional strategies used during the lessons. I coded the field notes in order to understand the complex phenomenon of teaching as defined by *Pathwise/Praxis III Domains* [ETS] (Educational Testing Service, 2000). These domains include: organizing content knowledge for student learning, creating an environment for student learning, and teaching for student learning. Once coded, I evaluated the teaching intern's teaching using the *Assessment Criteria and Scoring Rules* (ETS, 2001) to further support the documentation and evaluation of the observations.

Interviews. Following two observations of the same lesson, I interviewed the teaching intern. The interviews served to document Randy's understandings of his teaching and the teaching internship experience. The core questions were designed to address the decisions Randy made related to the use of the curriculum, his instructional practices, and his knowledge of student understandings. Understanding the decisions he made related to these components provided further insight into Randy's learning, the dilemmas he faced in teaching, and the decisions he made regarding teaching, which offered clarification with what I observed. The information gleaned from the interviews was constantly compared to the other data sources in order to develop an understanding of how the teaching intern developed pedagogical content knowledge.

Mathematics pedagogy assessment. An assessment was administered at the beginning and end of the internship to Randy, and in the beginning to Brad, focusing on their pedagogical content knowledge and reform notions of teaching, specific to the teaching of probability. The assessment provided a measure of the type of knowledge a teacher possesses when they understand the teaching of probability and are able to draw on their knowledge with flexibility by looking for multiple ways of representing probabilistic reasoning to foster students' conceptual understanding. Randy's assessment was analyzed in order to understand his pedagogical content knowledge related to probability at the beginning and end of the internship experience and to assess pre/post gains in his pedagogical content knowledge. In addition, the test provided reason for the collaboration episodes that addressed the dilemmas Randy faced with learning to teach probability. Brad's assessment was analyzed in order to understand his pedagogical content knowledge related to probability.

Results of the Study

Teaching Intern's Pedagogical Content Knowledge

Analysis of multiple data sources—audio taped collaborations, observations and evaluations of Randy's teaching, interviews, and pre/post mathematics pedagogy assessments—revealed that although Randy began his teaching internship with some pedagogical content knowledge, he experienced difficulty accessing it while teaching. Through the sources of pedagogical content knowledge found in a teaching internship—collaboration, curriculum and the classroom context—Randy was able to learn to and incorporate his pedagogical content knowledge in instruction.

How the sources influenced pedagogical content knowledge. Analysis of the data indicates the CMP curriculum initiated Randy's examination into his learning to teach process. The curriculum aided him as he developed new curricular knowledge, knowledge of instructional strategies, and knowledge of student understandings. Once he learned what he could from the curriculum, Randy collaborated with Brad regarding the dilemmas he still faced.

Randy had difficulty understanding how to use students' thinking in instruction. The CMP curriculum provided Randy with examples of students' thinking; however, his collaborations with Brad assisted him with this new knowledge because Brad helped him decipher the CMP teacher pages. In addition, Randy struggled to understand the key concepts of the CMP lessons and, consequently, focused more on procedural skills rather than in-depth analyses of experimental data. Brad offered guidance by validating Randy's instructional practices, and providing insight drawn from his years of teaching CMP and his own pedagogical content knowledge by using questions during the collaboration that helped Randy to make sound decisions regarding his instructional practices. Therefore, Brad aided Randy in his knowledge of conceptions for teaching probability, instructional practices, and curricular knowledge.

The classroom context aided Randy in his knowledge of student understandings by Randy learning to listen to and incorporate students' thinking in instruction. Randy's initial conceptions in planning did not prepare him for how his lessons would actually unfold with students. Therefore, Randy began to develop additional pedagogical content knowledge by *listening to* how students responded to his questions. Students' thinking served as a catalyst in Randy's development of pedagogical content knowledge because he learned to solicit and interpret students' thinking by responding to and participating in classroom discourse. From his study of the CMP curriculum and through collaboration with Brad who provided some insight, observations of Randy's teaching indicate his use of building upon students' thinking in instruction. My analysis suggests that both the curriculum and collaborations focusing on Randy's questioning skills allowed him to change his focus from the types of questions to pose to *listening to* how his students responded to questions.

Process of change in pedagogical content knowledge. Randy's development of pedagogical content knowledge evolved by a shift from focusing on content to the use of instructional strategies for teaching and learning. First, data reveal Randy faced the dilemma of knowing how to solve the problems in the CMP curriculum for him. He worked through the lessons as though he were a student and found he had trouble understanding the mathematics the way it was presented in the CMP curriculum. He resorted to his university textbooks to check the accuracy of his mathematical thinking.

But his university textbooks did not provide multiple ways of thinking about probabilistic reasoning. Consequently, the CMP curriculum's teacher pages present various representations for modeling basic probability concepts so Randy used the teacher pages to build his understanding of how to solve the problems in more than one way. After he had learned all he could from the CMP curriculum and his college textbooks, he sought assistance from Brad. It was through his collaborations with Brad that Randy was able to move beyond his concern for understanding the content toward the focus of learning how to plan for instruction.

Data revealed Randy attempted to think about the mathematics he was learning simultaneously with planning for instruction; however, learning probability was not synonymous with planning to teach probability. This dilemma led Randy to search for a better understanding of how to teach probability. Brad offered support by collaborating about specific aspects of the CMP lessons, focusing on the teacher pages and student pages of the curriculum. Randy devoted his attention to how the lesson would play out with students and the collaboration data revealed less time spent collaborating about probability content. More time was spent talking through his anticipated instructional practices with Brad so that he may clarify his thinking and gain insight from Brad's expertise and wisdom of experience. After focusing on his questioning techniques, Randy faced a new dilemma: listening to and incorporating students' thinking in instruction.

Randy began to focus more on student learning after he realized he posed many questions to his students without considering how to incorporate students' responses in instruction. He collaborated with Brad regarding what he was learning from his students because he could not decipher on his own how to use this information. Brad focused their collaborations around how to listen to students and incorporate their thinking by adapting instructional strategies to fit the needs of the students so to foster students' conceptual understandings. Learning from his students demonstrated the shift Randy made in focusing on understanding mathematics through multiple representations. Randy listened to students while teaching and orchestrated classroom discourse based on students' contributions.

Nature of the Collaboration

Analysis of the audio taped collaborations and interviews revealed the collaborations permitted Randy to learn to teach probability to middle grades students. Brad served as an expert teacher who analyzed teaching and suggested ways for Randy to optimize classroom discourse. Through collaboration, Brad shared his knowledge and expertise for teaching with Randy.

Collaboration on action. My analysis of data revealed Randy faced several dilemmas during his teaching internship. He utilized collaboration *on action* with Brad to overcome these dilemmas. Therefore, Brad and Randy focused their collaboration *on action* on three main topics: the CMP curriculum, the mathematics, and the teaching of the mathematics. With regard to CMP, Randy grappled with understanding how to present the lessons as suggested in the teacher pages of the curriculum. More specifically, their collaborations primarily centered on the *Launch* and *Explore* components of the lessons, where Randy focused on the questions he would pose with students. The *Summarize* component received less emphasis both in Randy's lessons and in their collaborations of the lessons. Consequently, this made it difficult for Randy to learn how to use students' experimental data to develop an understanding of experimental and theoretical probability. He struggled to learn how to develop the *Summarize* portion of lessons in ways that would build a shared understanding of probability concepts among students.

In focusing on the mathematics, Randy faced the dilemma of inadequate content knowledge related to probability. He discovered this when he began his lesson preparation by working through the CMP lessons as though he were a student. He began most of his collaborations with Brad by verifying his understanding of the mathematics; although outside of the scope of this study, Randy's actions could indicate that his content knowledge limited his ability to understand how students think about probability as described in and modeled by the CMP curriculum. Once more comfortable with the content, however, their collaborations subsequently addressed how to teach probability.

With regard to the teaching of the mathematics, Randy and Brad's collaborations revealed Randy had concerns about his ability to understand students' reasoning while in the act of teaching as well as apprehension in that the lessons would unfold as described by CMP. Much of their collaboration *on action* sessions focused on developing key questions to pose to students during instruction.

Collaboration in action. Analysis of data revealed two common topics. First, Randy used collaboration *in action* to seek reassurance from Brad about his instructional practices. These *in action* episodes often occurred after Randy taught a lesson once or during *Explore*. Brad provided comments regarding what he observed and suggestions of how he has taught the lesson in the past. His comments offered reassurance in Randy's ability to implement his lessons. Second, the *in action* episodes focused on Randy's role in orchestrating classroom discourse. These instances related to Randy's questioning skills and questions to pose, how to listen to and incorporate students' thinking in instruction, and allowing time to *Summarize* key concepts. The collaborations addressed the questions Randy posed during the *Launch* because Randy asked Brad to monitor his questioning abilities. Brad emphasized the need to learn from students' reasoning and to use their thinking in instruction; however, at times, Randy's concern for time management took precedence over teaching probability for understanding. Brad offered assistance in finding ways to "tighten up" his lessons in order to allow time to *Summarize* the day's lesson. More importantly, Brad and Randy's collaborations offered assurance in knowing when a concept was introduced as an exploration and when it was important that students master the concept because Randy did not know the curriculum well enough to make that distinction.

Influence of CMP on the nature of the collaboration. The CMP curriculum served as *the* source for planning and implementing mathematics instruction. Due to the school district and Brad's adherence to the CMP curriculum, Randy used only the unit, *What Do You Expect?* as his curriculum source; stated alternatively, he did not supplement the adopted curriculum. The collaborations focused on the *Launch, Explore, Summarize* lesson format and both Brad and Randy posed questions to one another through reading from the teacher pages. Their collaborations illustrated specific instances of Randy learning from the CMP curriculum. In addition, the CMP curriculum enabled Randy to think about mathematics in ways that middle grades students do.

Discussion of Findings

Focus on Content Knowledge before Pedagogical Content Knowledge

For this study, the teaching intern had to overcome his underdeveloped content knowledge related to probability in order to focus on the development of pedagogical content knowledge for *teaching* probability. Moreover, because of the limited time to learn *how* to use his newly developed pedagogical content knowledge, Randy placed less emphasis on the *Summarize* component of CMP lessons. In the following sections, I offer a discussion of these two key elements as well as connections to pertinent research literature.

Underdeveloped content knowledge. Research has shown the need for strong preparation in content prior to the teaching internship (Brown & Borko, 1992); however, subject matter knowledge alone does not ensure effective teaching performance (Kahan, Cooper, and Bethea, 2003). In this study, Randy became miscognizant of his limited knowledge related to probability when he began to solve the problems in the CMP curriculum. Consistent with interacting with the curriculum as a student so to understand how situations are interpreted and responded to differently (Chappell, 2003; Graeber, 1999; Haller, 1997), Randy was able to make sense of the mathematics he was going to teach. He took the initiative of focusing his learning of probability from the CMP lessons prior to his preparation in *teaching* the lessons. Consequently and consistent with McDiarmid, Ball, & Anderson (1996), Randy used planning time *outside* of the collaborations which afforded the opportunity to bring to the collaboration episodes his final concerns regarding content that he had not resolved on his own or with the CMP curriculum. Thereafter, the collaborations focused on *teaching* probability.

The teaching of probability depends greatly on the teachers' pedagogical understandings of probability (Stohl, 2005). Randy first experienced the *teaching* of probability during the teaching internship. Consistent with Ball and Bass (2000) by embedding Randy in a context of learning to teach, Randy's development of subject matter knowledge was also linked to his development of pedagogical content knowledge.

Limited time to use newly developed pedagogical content knowledge. Because concerns for Randy's content knowledge were addressed during the teaching internship, little time remained for Randy to use his newly developed pedagogical content knowledge. By the time Randy was able to question his pedagogical content knowledge in collaboration episodes, he was only able to focus on two of the three lesson components, *Launch* and *Explore*, resulting in little time to learn how the *Summarize* component should be implemented. Rarely did Brad and Randy have time to discuss *Summarize* in great depth. Consistent with Blanton, Berenson, and Norwood's (2001) research on classroom discourse, this study found the lack of time and attention to develop and orchestrate classroom discourse in *Summarize* was evident in Randy's learning to teach process. This final component holds the greatest potential for aiding students in building a shared understanding of probability, which is consistent with recommendations for improving teacher preparation (Ginsberg & Rhodes, 2003). Research has yet to document whether a teaching intern can effectively implement the *Summarize* component of CMP lessons.

Valuable Collaborations

Giebelhaus and Bowman (2002) report collaboration with inservice teachers improves the instructional practices of beginning teachers. In my analysis of this teaching internship, Brad's collaboration with Randy proved highly effective for Randy for several reasons. Brad brought to the collaborative process years of experience in teaching, and this experience was important because Randy faced several dilemmas when it came time to teaching probability.

Collaboration *on action.* The nature of the collaboration *on action* assisted Randy in making sense of the mathematics presented in the CMP curriculum. Working with the curriculum alone proved difficult in understanding the mathematics within the probability lessons. As a result, Randy gained Brad's perspective, which made teaching the curriculum from a problem-centered approach more likely.

Before the teaching internship, Ryan had little experience in teaching from a problem-centered approach. Collaborations *on action* aided Randy's development of this type of instructional practice because it was an approach used by Brad. The *on action* planning episodes consisted of Randy developing an understanding of how to *Launch* and *Explore* the CMP lessons with students. Consistent with Alleman, Cochran, Doverspike, and Newman (1984), Schmidt and Wolfe (1980), and Schein (1978), Brad, in his role as mentor teacher, assisted in the development of Randy's teaching practices. Without Brad's expertise and the nature of their collaboration, Randy may not have developed pedagogical content knowledge.

Collaboration in action. My study found that when it came time for Randy to teach, collaboration *in action* was at least, if not more important than collaboration *on action*. During and immediately after Randy taught a lesson for the first time, he sought reassurance from Brad regarding his instructional practices. He looked for immediate feedback regarding how the events in the lesson unfolded, the questions he posed to students, and effective use of class time. The nature of the collaboration with Brad was constructive, in that he assured Randy of his concerns; however, he pushed Randy to think beyond concerns for time management and instead focus on effectively orchestrating classroom discourse. Brad helped Randy to consider the big picture of what students were learning regarding probability. Without collaboration *in action*, Randy may have evaluated his teaching abilities solely on effective use of class time, and perhaps had not considered his orchestration of classroom discourse. Although many teaching interns fixate on issues related to management (e.g., Van Zoest & Bohl, 2002), Brad pushed Randy to think beyond these issues and instead focus on the disposition to learn from teaching.

In learning more about the collaborations that occurred in this teaching internship, it became evident they focused on substantive issues related to teaching and learning probability. The disposition of Brad and Randy during the teaching internship exemplified a distinctive pairing that allowed collaborations to focus on the learning to teach process.

Richness of the Collaborations

The mentor/teaching intern relationship in my study proved to be highly effective because of the nature of Brad and Randy's collaboration. Brad viewed Randy's learning to teach as a process of Randy learning to analyze his own instructional practices. Brad took extreme efforts to avoid giving direct advice. Instead, his suggestions and guidance resulted in advances in Randy's pedagogical development. Ball (1988) contends, "We need to examine the influence of different kinds of teacher education experiences on teacher candidates' knowledge about and orientations towards mathematics and mathematics teaching and learning, as well as on what they actually do in their classrooms" (p. 46). My study found a pairing of Randy, who learned about teaching through his experiences with Brad who was capable of analyzing Randy's process of learning to teach.

As expert teachers learn from their experiences with students, questioning the teaching intern's experiences can also prompt sense-making (Blanton, Berenson, & Norwood, 2001). Brad used open-ended questions with Randy in order for him to learn from his teaching because it formulates the need to justify thinking about teaching mathematics and consequent actions in the classroom. Unlike what Van Zoest and Bohl (2002) found in their study of a mentor teacher and teaching intern, my study found Brad taught Randy in much the same manner he taught his students. In particular, through the collaborative process, Brad modeled good questioning techniques to prompt Randy's sense-making of teaching. Consistent with Philippe (2000), the questions teachers pose often reflect the underlying goals they hold for instruction, which result in preservice teachers that will more likely teach in ways they were taught (Borko & Mayfield, 1995; Feiman-Nemser, 1983).

Questioning as an effective instructional practice. Studies in professional development of mathematics teachers (Cobb, Yackel, & Wood, 1991; Wood, 1995) have discussed the importance of classroom discourse as it relates to teacher learning. Consistent with this research, collaboration served as a method for Randy to examine the questions he posed to students because Brad could monitor those questions while Randy was teaching. He then provided immediate feedback regarding Randy's effectiveness. What Brad found was that Randy posed too many questions, many of which appeared to be rhetorical in nature. The collaborative process was consistent with the research of Blanton, Berenson, and Norwood (2001) in that both Brad and Randy were interested in listening to students' thinking for the purpose of understanding and continuing the dialogue in order to foster a shared understanding of mathematics. Eventually, the nature of Brad and Randy's collaborations *on action* and *in action* enabled Randy to learn to monitor his own questioning skills by listening to and interpreting students' thinking in instruction. Therefore, as evident in the results in this study, classroom discourse informs both our understanding of students' thinking about mathematics and also teachers' thinking about teaching mathematics.

The influence of the CMP curriculum as a source in learning from teaching. The collaboration episodes were strongly influenced by the CMP curriculum because it was *the* source for planning and implementing instruction. This result is consistent with Remillard (2000) who concluded that the type of pedagogical change required to support reform-oriented curriculum should invoke opportunities for teachers to engage in decision-making instructional practices. My study found the CMP curriculum was a source in the learning to teach process because its use during this internship invoked the opportunity for collaboration between Brad and Randy.

Although few studies have reported teachers' use of reform-based curriculum materials (e.g., Remillard, 2000), there are even fewer studies of teaching interns' use of curricular materials. Results of this study indicate CMP played an essential role in the collaborative process because nearly all collaborations focused on the student and teacher pages of CMP. Whether the discussions were addressing Randy's content knowledge or development of his instructional practices, the collaborations centered *around* CMP.

Therefore, consistent with research that has found reform-based curriculum to be highly educative for teachers (Ball & Cohen, 1996; Lloyd, 1999; Schneider, Krajcik, & Marks, 2000), CMP influenced what Randy learned about probability, what Randy and Brad discussed, as well as what and how Randy planned and implemented lessons. Without Brad's experience in using CMP, Randy might have interpreted the CMP curriculum differently, which could have resulted in a mediocre internship experience for Randy.

Conceptual Framework Revisited

This study focused on the pedagogical content knowledge domain as defined by Grossman (1990). In particular, Grossman outlined four central components within pedagogical content knowledge (see Figure 1.1). For the purpose of this study, I considered Grossman's framework and its components in formulating my conceptual framework (see Figure 1.3) to investigate how three sources found in a student teaching internship—curriculum, collaboration, and the classroom context—impact the development of pedagogical content knowledge. In light of the results of this study, a refinement of my conceptual framework is warranted.

As depicted in Figure 5.1, my revised framework reflects the relationship between content knowledge and the four components of pedagogical content knowledge: curricular knowledge, knowledge of student understandings, knowledge of instructional strategies, and conceptions of purpose. Moreover, this new framework incorporates the roles of collaboration, curriculum, and the classroom context in the development of pedagogical content knowledge. Results of this study clearly suggest that not only did collaboration play an important role in Randy's development of pedagogical content knowledge, it was, in fact, truly *essential*. As previously noted , working in isolation

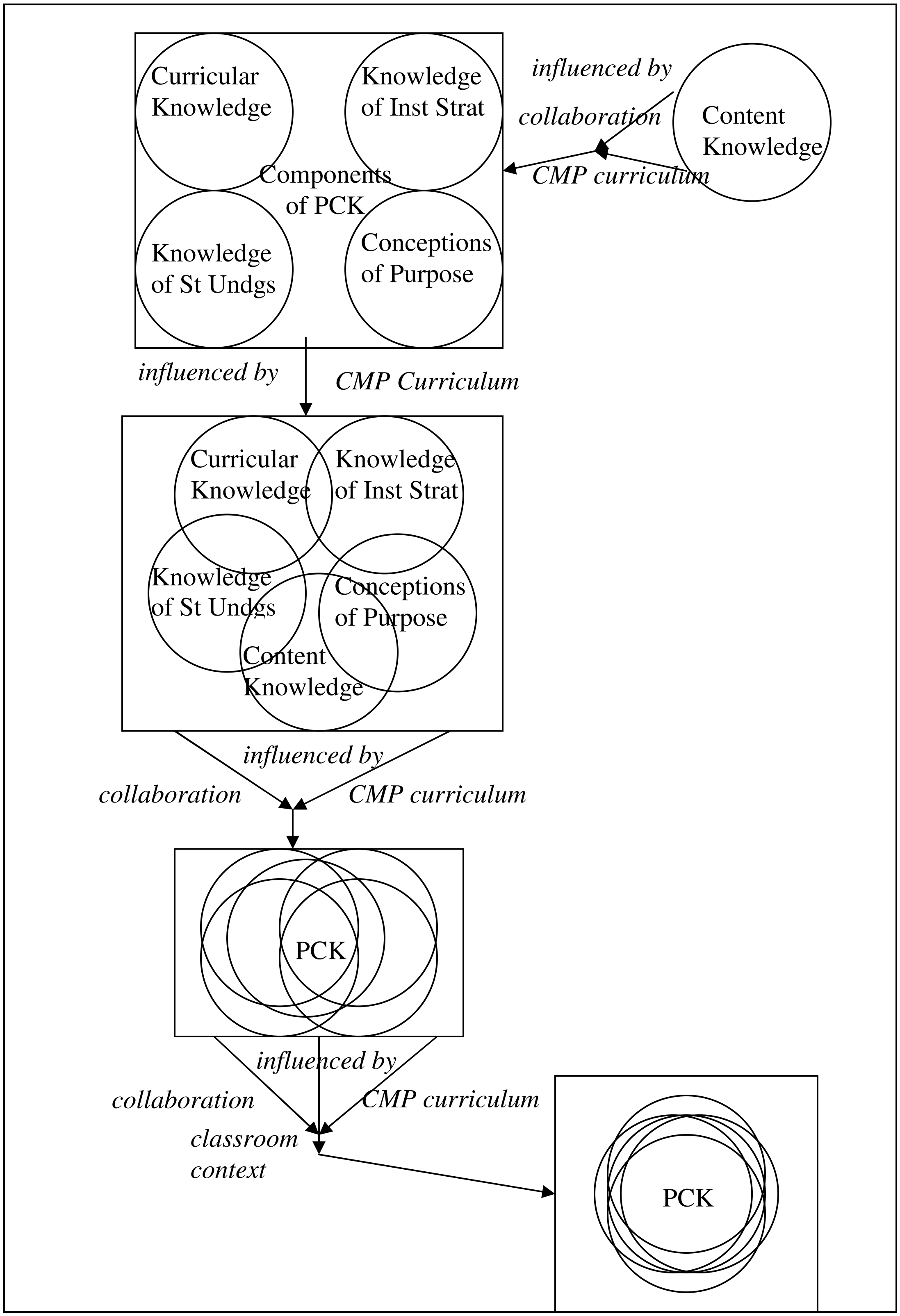


Figure 5.1: The integration of sources enacting change in the development of pedagogical content knowledge of a mathematics teaching intern

Randy faced numerous dilemmas when utilizing curricular resources. Moreover, his inability to access existing pedagogical content knowledge and understand students' thinking during instruction represented dilemmas in the learning-to-teach process. Analyses determined that it was primarily through the collaborative process that Randy solved these dilemmas and enhanced his pedagogical content knowledge. Therefore, it is my strong belief that without such rich collaboration with his mentor teacher, the teaching intern's development of pedagogical content knowledge would have been severely limited or delayed.

Generalizibility

The construction of knowledge. All forms of knowledge are interactive, in that one form of knowledge can directly or indirectly influence other knowledge (Voss et. al, 1986; Alexander et. al, 1989; Walker, 1987). As Randy developed a specific type of knowledge (namely, pedagogical content knowledge), the dilemmas he faced and his subsequent learning demonstrate an interactive perspective of other forms of knowledge on the development of pedagogical content knowledge. Consistent with Alexander, Schallert, and Hare (1991), it is important that consideration be given to other forms of knowledge—tacit and explicit, sociocultural, conceptual, content, discourse, and metacognitive knowledge—so that we may understand the knowledge teachers generate as a result of their experiences.

It is worth noting that Randy's learning was self-regulated to the degree that he was metacognitively, motivationally, and behaviorally active in his own learning process as he acquired new knowledge. Consistent with Regehr and Norman (1996), Randy used the process of *first* considering the CMP curriculum, followed by collaboration with

Brad, and finally listening to his students in instruction. Such results suggest that specified self-regulated learning strategies (see Zimmerman and Martinez-Pons, 1986) represented one way that Randy used to make sense in learning to teach probability, thus achieving academic goals on the basis of self-efficacy perceptions as described by Bandura (1986).

Supervision

Although the student teaching internship is a widely studied component of formal teacher preparation, the influence of supervision on teacher learning is still unclear (Borko & Mayfield, 1995). As well, Borko and Mayfield found that supervisors focused on superficial aspects of teaching, such as paperwork, lesson plans, and behavioral objectives and avoided in-depth discussions about content and pedagogy, which offered no specific directives on student teaching interns' process of learning to teach. Given Randy's transformation in this study, improving the role university supervisors play has the potential to significantly influence the development of student teaching interns.

Blanton, Berenson, and Norwood (2001) suggest *educative supervision* that conceptualizes the ideology that the supervisor, as a more knowing other, can guide student teaching interns' development of pedagogical content knowledge to a greater extent than the teaching intern can alone, which provides an instructional, rather than evaluative, purpose for supervision. In this study, Brad's collaboration with Randy was highly effective in Randy's learning from teaching. Therefore, one way to formulate an approach, such as Brad's, is through the use of open-ended questions to aid the teaching intern in learning to justify their thinking about teaching and their consequent actions in the classroom. This approach would require supervision that shifts the focus to listening to the teaching intern, which is more challenging and time-consuming than the summative evaluation. Consistent with Zeichner's (1996) view of learning from teaching, this study reinforces the need for an educative approach to the supervision of the student teaching internship.

Brad's disposition toward Randy learning from his teaching suggests feedback from supervisors and mentor teachers could be less evaluative and more of a study for the teaching intern to learn from his/her experiences. As in the case of Randy, the disposition for learning from teaching should be instilled in teaching interns' learning to teach process early in their preservice teacher education program.

Research Methodology

Limited view of Randy's content knowledge. My study utilized a conceptual framework for understanding the development of pedagogical content knowledge for a teaching intern during his student teaching internship. It was here that I paid particular attention to the sources of pedagogical content knowledge found in a teaching internship. While my framework allowed me to focus my research, Eisenhart (1991) warns the use of frameworks can also limit your research because your framework influences the research questions, decisions about data collection and analysis, and how the results of the study are interpreted and reported. Through analyzing the collaboration and interview data, I found Randy shared with both Brad and me his concern for limited content knowledge related to probability; it should be noted, however, my theoretical framework did not adequately make provisions for assessing Randy's content knowledge and its role in learning to teach. Nevertheless, and consistent with McDiarmid, Ball, and Anderson (1996), it appears that Randy's subject matter knowledge influenced his pedagogical orientation and decisions, as well as his ability to make curricular choices. Collecting data regarding Randy's content knowledge related to probability could have provided additional insight into how his content knowledge influenced his instructional practices.

Capturing collaborations. In order to understand the nature of the collaborative process between Brad and Randy, their collaborations were audio taped. Because I, as researcher, was not fully embedded in the internship, collecting data required reliance on Brad and Randy to audiotape their collaborations. Heretofore, research studies on collaboration (e.g., Van Zoest & Bohl, 2002) have asked participants to write and reflect upon the teaching internship experience, requiring participants to make time available to write or respond to the events of the collaboration after the fact. For the purpose of my study, I wanted an account of *all* of the actual collaborations – vis-à-vis *conversations*, captured word-for-word, which is where other studies have fallen short in documenting collaborations. I approached Brad and Randy regarding possible methods for documenting their collaborations that were relatively unobtrusive. They considered scheduling a writing period each day but felt their time was better suited for additional planning. Ultimately, it was determined the least obtrusive data collection method was the use of an audiotape recorder.

Brad and Randy were provided with two battery-operated tape recorders, tapes, and batteries. When a collaboration began, they started the recording with the date and a description of the lesson the collaboration addressed. The two quickly developed the habit of turning on the recorder whenever they began to speak with one another. They changed tapes when a tape was full or at the start of a new collaboration related to the next lesson. Brad found it helpful to carry the recorder during Randy's teaching to prepare for documenting collaboration *in action*. Despite their concerted efforts to document all collaborations, there were a few instances when Brad or Randy failed to capture data on collaboration *in action.* At these times, either Brad or Randy took a moment to reiterate on audiotape the missed collaboration. Additionally, the participants quickly learned they had to hold the recorder very close to their mouths in order to be heard over the background noise. Although my study represented improved methods of data collection (and yielded robust collaborations); nevertheless, future research could consider the use of less obtrusive methods (i.e., microphones and continuous audiotape recording) for collecting collaboration data.

Study Implications and Recommendations

Implications for Teacher Education

This study identifies two distinct types of collaboration that were important to Randy's growth in pedagogical content knowledge: collaboration *on action* and collaboration *in action*. Randy utilized collaboration *on action*, the joint planning sessions when he collaborated with Brad about the act of teaching, to clarify his concerns for understanding the mathematics content and to capitalize on Brad's perspective regarding his plans for instruction. Collaboration *in action* so occurred either when Randy was in the act of teaching or between classes. These collaborations were due in part by concerns from either participant regarding the questions Randy posed during instruction or the timing of events during the lesson. This finding suggests both types of collaboration experiences were essential in Randy's development of pedagogical content knowledge. *Therefore, teaching interns and mentor teachers should focus not only on* *collaborating during planning (on action), but also on collaborating while the teaching intern is in the act of teaching (in action).*

Analysis of the nature of the collaborations suggest Brad's experience in learning about and teaching the CMP curriculum resulted in advances in Randy's pedagogical content knowledge development. Brad demonstrated a well-developed conceptualization of teaching CMP as embodied by the NCTM *Standards*. Results of this study indicate that Randy's experiences in teaching the CMP curriculum was influenced by Brad's knowledge and expertise in teaching the CMP curriculum. *Therefore, teacher educators should seek to identify mentor teachers who have several years of experience in teaching standards-based mathematics curricula.*

This study reports that in Randy's desire to monitor his questioning skills, he began to listen to and interpret students' thinking in instruction. In doing so, he developed an understanding of how middle grades students think about probability, as well as how the posing of questions can help (or hinder) student learning. Therefore, classroom discourse has the potential to foster *student* and *teacher* learning: students' understanding of mathematics and teachers' learning to teach mathematics. *Hence, teacher educators should seek to support the continued education of mathematics teachers so that they can develop the disposition to learn from students and from instruction.*

Evidence from this study suggests that Brad's use of open-ended questions enabled Randy to overcome numerous dilemmas in learning to teach. In particular, Brad's supportive nature guided Randy in his development of pedagogical content knowledge to a greater extent than Randy could have alone. In some sense, Brad taught

Randy in much the same manner that he taught his own students, through questioning techniques focused on sense-making. *Therefore, teacher education programs should change the culture of the teaching intern/mentor teacher relationship to one that is co-supportive and less hierarchical.*

Analysis of the nature of the collaborations and interviews revealed Randy's awareness of his underdeveloped content knowledge related to probability. He became aware of his limited knowledge related to probability when he began to solve the problems in the CMP curriculum and tried to understand how to represent solutions in more than one way. The CMP curriculum, Brad, and the students contributed not only to Randy's content knowledge development, but similarly to his pedagogical content knowledge of probability. *Consequently, preservice teachers need to have coursework that enables them to learn probability in an in-depth way.*

Implications for Researching Teacher Education

This study makes the distinction that the depth of Randy's content knowledge related to probability was limited; however, my framework offered a limited view of content knowledge as a source in the development of pedagogical content knowledge. Collecting data on Randy's content knowledge related to probability could have provided additional insight into how his content knowledge influenced his instructional practices. *Therefore, when using a pedagogical content knowledge framework, be aware that content knowledge may play a role in teacher learning.*

Documentation of collaboration *in action* revealed Randy sought support and guidance regarding his instructional strategies *while in the act of teaching*. He looked for immediate feedback about his lessons as they unfolded, the questions he posed, and use of class time. He made adjustments to his lessons immediately following the *in action* collaborative episodes with Brad. Nevertheless, capturing the *in action* episodes was obtrusive at times due to the spontaneity of the episode and the participants' proximity to the audiotape recorder. *For this reason, tools for data collection need to be designed so to allow us to capture collaboration in action in a less obtrusive manner.*

Recommendations for Future Research

The study of the development of pedagogical content knowledge is a complex endeavor. Several sources contribute to preservice teachers' initial and emerging development of pedagogical content knowledge. This study reports collaboration with a mentor teacher in a student teaching internship impacts the teaching intern's conceptions of purpose for teaching probability, knowledge of student understandings, curricular knowledge, and knowledge of instructional practices. Additional research is needed so that we may continue to examine the complex area of learning to teach.

The framework used in this study may easily be modified to examine the content knowledge needed for teaching probability in a standards-based mathematics classroom. It is apparent in my study Randy's content knowledge related to probability was not adequate for understanding how to solve probability problems in more than one way. It is imperative teacher education programs study the mathematical knowledge needed for teaching probability with standards-based curricula so that we may understand the type of preparation preservice teachers need prior to the student teaching internship.

This study focused on the nature of the collaborative process between a mentor teacher and a teaching intern. The richness of the collaborations proved highly effective for Randy because Brad approached his role as the mentor teacher to Randy as he approached his role as a middle grades mathematics teacher. Brad taught Randy in much the same manner he taught his students; through good questioning techniques. The questions Brad posed to Randy reflected the underlying goals he holds for instruction and for learning from teaching. Future research should examine the learning of a mentor teacher like Brad.

Consideration regarding the nature of the collaboration between the mentor teacher and teaching intern should also be extended to include the university supervisor. Taking into consideration the perspective that all teaching interns develop pedagogical content knowledge through the transformation Randy experienced could improve the role supervisors play in the teaching intern's learning to teach process. Future research could formulate an educative approach to the supervision of teaching interns by examining the collaborative relationship among a university supervisor, a mentor teacher and a teaching intern.

Limitations of the Study

There are various limitations involved with this study. Through the data collection process, I found it difficult to collect data in a school where I was not embedded in the classroom context. Ideally, I would have liked to observe the teaching intern more often in this classroom setting, interacting with his mentor teacher and his students; however, Brad and Randy made every attempt to capture their collaborations on audiotape in order for me to gain a better understanding of this teaching internship. If collaboration was missed, either Brad or Randy provided after the fact a recorded message detailing the collaboration for the record. Despite this limitation, both participants nevertheless felt confident they had captured almost, if not all, their collaborations during the data collection period.

Second, because this study primarily focused on the teaching intern, a relatively modest set of data was collected on the mentor teacher, namely only audio taped collaborations and a mathematics pedagogy assessment. Because I did not capture Brad's perspectives of the teaching internship experience through interviews, my understanding of some of the decisions he made remain somewhat unclear. Although I was able to interpret his involvement in the teaching internship, it was only from the perspectives of Randy and me.

Third, the duration of this study spanned one semester. More specifically, because data collection was initiated after Randy's teaching internship in his second endorsement area, this study resided in the last ten weeks of the semester. In fact, in order to capture data regarding Randy's teaching of *What Do You Expect?* I focused the majority of intense data collection during five weeks. By the time I initiated data collection, Randy had likely already become acclimated with his students, the school experience, and established a working relationship with Brad.

Final Thoughts

This study represented the first documented attempt to analyze the nature of the collaborative process in the development of pedagogical content knowledge of a teaching intern in a standards-based mathematics classroom at the middle grades level. Collaboration *in action* and *on action* both proved highly effective for Randy in learning to teach mathematics. Brad's involvement in the collaborative process suggests there is so much more to understand about the use of collaboration with experienced teachers in standards-based mathematics classrooms. My research into the nature of collaboration has stimulated my ambition to conduct future research on the role of the university supervisor in this collaborative process, a piece not addressed in this study.

My study documented how numerous sources in a student teaching internship influenced the development of pedagogical content knowledge. Through this dissertation process, I have come to appreciate that preservice teachers have the capability to learn from their earliest teaching experiences because they have a disposition to learn from teaching. Accordingly, my future research interests include understanding more about preservice teacher education programs and their potential influence in the emerging and ongoing development of pedagogical content knowledge.

Additionally, as a result of my dissertation research, I am interested in developing meaningful learning experiences for preservice teachers that incorporate students' thinking. In order for preservice teachers to understand the relevance of multiple representations to foster authentic student learning, mathematics content courses need to address the learning of mathematics in an in-depth way to promote sense-making. Despite my contributions to the research literature, there is still so much more to learn about the preparation of preservice teachers in teaching from a standards-based mathematics curriculum and in the spirit of the NCTM *Standards*.

Last, I appreciate Brad and Randy in their willingness to participate in my study. They opened their classroom and their souls for examination into the collaborative process for the mathematics education community. I learned so much and grew as a researcher and teacher educator. Recently I learned the phrase, "to teach is to twice learn." What I have learned from my experience in this study with Brad and Randy is that the potential is there for preservice teachers to learn from their experiences in teaching, which suggests there is so much more to understand about the complex area of learning to teach.

REFERENCES

Abbott, J. & Ryan, T. (1999). Constructing knowledge, reconstructing schooling. *Educational Leadership, 57*(3), 66-69.

Ackerman, E. (1995). Construction and transference of meaning through form. In L.P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 341-354). Hillsdale, NJ: Erlbaum.

Alexander, P.A., Schallert, D.L., Hare, V.C. (1991). Coming to terms: How researchers in learning and literacy talk about knowledge. *Review of Educational Research, 61,* 315-343.

Alleman, E., Cochran, J., Doverspike, J., & Newman, I. (1984). Enriching mentoring relationships. *The Personnel and Guidance Journal, 62,* 329-332.

Ammon, P. & Levin, B.B. (1993). Expertise in teaching from a developmental perspective: The developmental teacher education program at Berkeley. *Journal of Learning and Individual Differences, 5*(4), 319-326.

An, S., Klum, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education, 7,* 145-172.

Artzt, A.F. (1999). A structure to enable preservice teachers of mathematics to reflect on their teaching. *Journal of Mathematics Teacher Education, 2*, 143-166.

Ball, D.L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics, 8,* 40-48.

Ball, D.L. (1989, March). *Breaking with experience in learning to teach mathematics: The role of a preservice methods course*. Paper presented at the annual meeting of the American Education Research Association, San Francisco, March, 1989.

Ball, D.L. (1990). Prospective elementary ad secondary teachers' understanding of division. *Journal for Research in Mathematics Education, 21,* 132-144.

Ball, D.L. (1996). Teaching learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappan, 77*(7), 500-508.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Ball, D.L, & Cohen, D.K. (1996). Reform by the book: What is-or might be-the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher, 25*(9), 6-8.

Ball, D.L., & Feiman-Nemser, S. (1988). Using textbooks and teachers' guides: A dilemma for beginning teachers and teacher educators. *Curriculum Inquiry, 18*(4), 401-423.

Baroody, A.J. & Ginsburg, H.P. (1990). Children's mathematical learning: A cognitive view. In R.B. Davis, C.A, Maher & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 51-64). Reston, VA: NCTM.

Billstein. R., & Williamson, J. (1999). *Middle grades Math Thematics* (Books 1-3). Evanston, IL: McDougal Littell.

Bird, T. (1991, April). *Making conversations about teaching and learning in an introductory teacher education course.* Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Bjuland, R. (2004). Student teachers' reflections on their learning process through collaborative problem solving in geometry. *Educational Studies in Mathematics, 55,* 199-225.

Blanton, M.L., Berenson, S.B., & Norwood, K.S. (2001). Using classroom discourse to understand a prospective mathematics teacher's developing practice. *Teaching and Teacher Education, 17,* 227-242.

Borko, H., Eisenhart, M., Brown, C., Underhill, R., Jones, D. & Agard, P. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23*(3), 194-222.

Borko, H., Livingston, C., McCaleb, J., & Mauro, L. (1988). Student teachers' planning and post-lesson reflections: Patterns and implications for teacher preparation. In J. Calderhead (Ed.), *Teachers' professional learning* (pp. 65-83). New York: Falmer.

Borko, H. & Mayfield, V. (1995). The roles of the cooperating teacher and university supervisor in learning to teach. *Teaching and Teacher Education, 11*(5), 501-518.

Borko, H. & Putnam, R.T. (1996). Learning to teach. In R.C. Calfee & D.C. Berliner (Eds.), *Handbook of educational psychology* (pp. 673-708). New York: Macmillan.

Bransford, L.K., Brown, A.L., Cocking, R.R. (Eds.). (2000). *How people learn: Brain, mind, experience and school*. (Exp. Ed.). Washington, DC: National Academy Press.

Brindley, R. (2000). Learning to walk the walk: Teacher educators' use of constructivist epistemology in their own practice. *Professional Educator, 22*(2), 1-14.

Brooks, M.G. & Brooks, J.G. (1999). The courage to be constructivist. *Educational Leadership, 57*(3), 18-24.

Brown C.A., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). New York: Macmillan.

Brown, C., Cooney, T.A., & Jones, D. (1990). Mathematics teacher education. In W.R. Houston (Ed.), *Handbook of research on teacher education* (pp. 639-656). New York: Macmillan.

Brown, J., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32-42.

Bruner, J. (1986). *Actual minds, possible worlds*. New York: Plenum Press.

Bullough, Jr., R.V. (2001). Pedagogical content knowledge circa 1907 and 1987: A study in the history of an idea. *Teaching and Teacher Education, 17,* 655-666.

Bullough, Jr., R.V., Knowles, J.G., & Crow, N.A. (1989). Teacher self-concept and student culture in the first year of teaching. *Teachers College Record, 91,* 209-233.

Calderhead, J. (1988). The development of knowledge structures in learning to teach. In J. Calderhead (Ed.), *Teachers' professional learning* (pp. 51-64). London: Falmer.

Calderhead, J., & Robson, M. (1991). Images of teaching: Student teachers' early conceptions of classroom practice. *Teaching and Teacher Education, 7,* 1-8.

Carpenter, T.P., & Hiebert, J. (1992). Learning and teaching with understanding. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). Reston, VA: NCTM.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (2000, September). *Cognitively guided instruction: A research-based teacher professional development program for elementary school mathematics.* National Center for Improving Student Learning and Achievement in Mathematics and Science, Report No. 003. Madison, WI: Wisconsin Centre for Education Research, The University of Wisconsin-Madison.

Carter, K., & Doyle, W. (1987). Teachers' knowledge structures and comprehension processes. In J. Calderhead (Ed.), *Exploring teachers' thinking* (pp. 147-160). London: Cassell.

Carter, K. & Gonzalez, L.E. (1993). Beginning teachers' knowledge of classroom events. *Journal of Teacher Education, 44,* 223-232.

Chappell, M. (2003). Keeping mathematics front and center: Reaction to middle grades curriculum projects' research. In S. Senk & D. Thompson, (Eds.), *Standards-oriented school mathematics curricula: What does the research say about student outcomes?* (pp. 285-296). Mahwah, NJ: Erlbaum.

Civil, M. (1992, April). *Prospective elementary teachers' thinking about mathematics.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Clandinin, D.J. (1986). *Classroom practice: Teacher images in action*. Philadelphia: Falmer Press.

Clandinin, D.J., & Connelly, M.F. (1996).Teachers' professional knowledge landscapes: Teacher stories. *Educational Researcher, 25*(3), 24-31.

Clark, D.M. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education, 28*, 278-308.

Clark, C., & Dunn, S. (1991). Second-generation research on teachers' planning, intentions, and routines. In H. C. Waxman & H. J. Walberg (Eds.), *Effective teaching: Current research* (pp. 183-201). Berkeley, CA: McCutchan.

Clements, D. (1997). (Mis?)Constructing constructivism. *Teaching Children Mathematics, 4*(4), 198-200.

Clift, R.T. (1987). English teacher or English major: Epistemological differences in the teaching of English. *English Education, 19,* 229-236.

Clift, R.T., & Brady, P. (2005). Research on methods courses and field experiences. In M. Cochran-Smith and K.M. Zeichner (Eds.), *Studying teacher education: The report of the AERA panel on research and teacher education* (pp. 309-424). Mahwah, NJ: Lawrence Erlbaum Associates.

Cobb, P. (1990). A constructivist perspective on information-processing theories of mathematical activity. *International Journal of Educational Research, 14*, 67-92.

Cobb, P. (1994). *Learning mathematics: Constructivist and interactionist theories of mathematical development.* Dordrecht: Kluwer.

Cobb, P. (2000). The importance of a situated view of learning to the design of research and instruction. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 45-82). Stamford, CT:Ablex.

Cobb, P. & Steffe, L.P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14*(2), 83-94.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Journal for Research in Mathematics Education. Monograph: Vol. 4. Constructivist Views on the Teaching and Learning of Mathematics*. Reston, VA: NCTM.

Cobb, P., Yackel, E., & Wood, T. (1991). Curriculum and teaching development of psychological and anthropological perspectives. In E. Fennema, T.P. Carpenter, & S.J. Lamon (Eds.), *Integrating research on teaching and learning mathematics.* Albany, NY: State University of New York Press.

Cochran-Smith, M. (1991). Learning to teach against the grain. *Harvard Educational Review, 61*(3), 279-310.

Collopy, R. (1999). *Teachers' use of and learning from curriculum materials*. Unpublished Doctoral Dissertation, University of Michigan, Ann Arbor.

Comeaux, M. (1992, April). *Challenging students' views about teaching and learning: Constructivism in the social foundations classroom.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Conference Board on Mathematical Sciences. (2001). *The Mathematical Education of Teachers*. Providence, RI: American Mathematical Society.

Connected Mathematics Project [CMP] (1998). *What Do You Expect?* Michigan State University: Dale Seymour.

Coxford, A.F, Fey, J.T., Hirsch, C.R., Schoen, H.L., Burrill, G., Hart, E.W., Watkins, A.E., with Messenger, M.J. & Ritsema, B. (1997). *Contemporary Mathematics in Context: A Unified Approach (Course 1).* Chicago, IL: Janson.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics, 52,* 243-270.

Creswell, J.W. (1998). *Qualitative Inquiry and Research Design: Choosing Among Five Traditions.* Thousand Oaks, CA: Sage.

Creswell, J.W. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: Sage.

Darling-Hammond, J. (1996). The right to learn and the advancement of teaching: Research, policy, and practice for democratic education. *Educational Researcher, 25*(6), 5-19.

Darling-Hammond, J. (2000). How teacher education matters. *Journal of Teacher Education, 51*(3), 166-173.

Darling-Hammond, L, & Snyder, J. (2000). Authentic assessment of teaching in context. *Teaching and Teacher Education, 16*(5-6), 523-545.

De Jong, O., & Van Driel, J. (2004). Exploring the development of student teachers' pedagogical content knowledge of the multiple meanings of chemistry topics. *International Journal of Science and Mathematics Education 00,* 1-15.

Dolk, M., Uittenbogaard, W., & Fosnot, C.T. (1996). Teaching to facilitate progressive schematization. *The Constructivist, 11*(1), 10-14.

Driver, R. (1995). Constructivist approaches to science teaching. In P. Steffe and J. Gale (Eds.), *Constructivism in education* (pp. 385-400). Hillsdale, NJ: Lawrence Erlbaum Associates.

Duit, R., & Confrey, J. (1996). Reorganizing the curriculum and teaching to improve learning in science and mathematics. In D.F. Treagust, R. Duit, and B.J. Fraser (Eds.), *Improving teaching and learning in science and mathematics*. (pp. 79-93). New York: Teachers College Press.

Dwyer, C.A. (1994). *Development of the knowledge base for the Praxis III: Classroom Performance Assessment criteria.* Princeton, NJ: Educational Testing Service.

Dwyer, C.A. (1998). Psychometrics of Praxis III: Classroom performance assessments. *Journal of Personnel Evaluation in Education 12*(2), 163-187.

Ebby, C.B. (2000). Learning to teach mathematics differently: The interaction between coursework and fieldwork for preservice teachers. *Journal of Mathematics Teacher Education, 3*, 69-97.

Education Development Center, Inc. (1998). *Mathscape: Seeing and thinking mathematically* (6-8). New York: Glencoe/McGraw Hill.

Educational Testing Service. (1993). *Using Praxis III: Performance Assessments for teaching licensing.* Princeton, NJ: Author.

Educational Testing Service. (2000). *Pathwise/Praxis III Domains.* Princeton, NJ: Author.

Educational Testing Service. (2001). *Teacher Performance Assessments: Assessor*

Manual. Preston, NJ: Author.

Educational Testing Service. (2002). *The Praxis Series: Praxis III Recalibration System.* Princeton, NJ: Author.

Eisenhart, M.A. (1991). *Conceptual frameworks for research circa 1991: Ideas from a cultural anthropologist: Implications for mathematics education researchers.* Paper presented at the Thirteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Blacksburg, VA.

Elbaz, F. (1983). *Teaching thinking: A study of practical knowledge.* New York: Nichols.

Enderson, M.C. (1995). *Assessment practices of three prospective secondary mathematics teachers*. An unpublished doctoral dissertation, The University of Georgia, Athens.

Feiman-Nemser, S. (1983). Learning to teach. In L. Shulman & G. Sykes (Eds.), *Handbook of teaching and policy* (pp. 150-170). New York: Longman.

Feiman-Nemser, S. (Ed.). (1987). *Teacher education and learning to teach: Proceedings of the First Annual NCRTE Retreat.* (Conference Series 87-1). East Lansing, MI: National Center for Research on Teacher Education.

Feiman-Nemser, S., & Beasley, K. (1997, April). *Discovering and sharing knowledge: Inventing a new role for cooperating teacher.* Paper presented at the Annual Conference of the American Educational Research Association, Chicago, IL.

Feiman-Nemser, S., & Buchmann, M. (1986). The first year of teacher preparation: Transition to pedagogical thinking. *Journal of Curriculum Studies, 18,* 239-256.

Feiman-Nemser, S., & Parker, M.B. (1990). *Making subject matter part of the conversation or helping beginning teachers learn to teach* (Research Report (90-3). East Lansing: National Center for Research on Teacher Learning.

Fennema, E., & Franke, M.L. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.

Fogarty, R. (1999). Architects of the intellect. *Educational Leadership, 57*(3), 76-78.

Forman, C. (1993). The constructivist perspective to early education. In J. Roopnarine & J. Johnson (Eds.), *Approaches to Early Childhood Education*. New York: MacMillan.

Fosnot, C.T. (1992). *Center for constructivist teaching/teacher preparation project*.

Final Grant Report, FIE #R 215A93232.

Foss, D.H. & Kleinsasser, R.C. (1996). Preservice elementary teachers' views of pedagogical and mathematical content knowledge. *Teaching and Teacher Education, 12*(4), 429-442.

Frykholm, J.A. (1996). Preservice teachers in mathematics: Struggling with the Standards. *Teaching and Teacher Education, 12*(1), 665-682.

Frykholm, J.A. (1998). Beyond supervision: Learning to teach mathematics in community. *Teaching and Teacher Education, 14*(3), 305-322.

Frykholm, J.A. (1999). The impact of reform: Challenges for mathematics teacher preparation. *Journal of Mathematics Teacher Education, 2*, 79-105.

Gess-Newsome, J. (1999). Secondary teachers' knowledge and beliefs about subject matter and their impact on instruction. In J. Gess-Newsome and N. B. Lederman (Eds.), *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 51-94). Kluwer.

Giebelhaus, C.R., & Bowman, C.L. (2002). Teaching mentors: Is it worth the effort? *The Journal of Educational Research, 95*(4), 246-254.

Ginsberg, R., & Rhodes, L.K. (2003). University faculty in partner schools. *Journal of Teacher Education, 54*(2), 150-162.

Good, T.L. & Brophy, J. (1994). *Looking in classrooms*. New York: HarperCollins.

Goodlad, J. (1988). Studying the education of educators: A value-driven inquiry. *Phi Delta Kappan, 70* (2), 104-111.

Graeber, A.O. (1999). Forms of knowing mathematics: What preservice teachers should learn. *Educational Studies in Mathematics 38,* 189-208.

Grossman, P.L. (1989). A study in contrast: Sources of pedagogical content knowledge for secondary English. *Journal of Teacher Education, 40*(5), 24-31.

Grossman, P.L. (1990). *The making of a teacher: Teacher knowledge and teacher education.* New York: Teachers College Press.

Grossman, P.L. (1995). Teachers' knowledge. In L. W. Anderson (Ed.), *International encyclopedia of teaching and teacher education* (2nd ed., pp. 20-24). New York: Pergamon.

Grossman, P.L. & Thompson, C. (2004). *Curriculum materials: Scaffolds for new teacher learning?* (Document R-04-1). Seattle, WA: Center for the Study of

Teaching and Policy.

Grossman, P.L., Wilson, S., & Shulman, L. (1989). Teachers of substance: Subject matter knowledge for teaching. In M. Reynolds (Ed.), *Knowledge base for the beginning teacher* (pp. 23-36). Oxford, England: Pergamon.

Grouws, D.A, & Schultz, K.A. (1996). Mathematics teacher education. In J. Sikula (Ed.), *Handbook of research on teacher education* (2nd ed., pp. 442-458). New York: Macmillan.

Gudmundsdottir, S. (1987). *Pedagogical Content Knowledge: Teachers' Ways of Knowing.* East Lansing, MI: National Center for Research on Teacher Learning. (ERIC Document Reproduction Service No. ED290701).

Guyton, E., & McIntyre, D.J. (1990). Student teaching and school experiences. In W.R. Houston, M. Habeman, and J. Sikula (Eds.), *Handbook of research on teacher education* (1st ed., pp. 535-548). New York: Macmillan.

Haller, S.K. (1997). *Adopting probability curricula: The content and pedagogical content knowledge of middle grades teachers.* Unpublished doctoral dissertation, University of Minnesota.

Hatton, N., & Smith, D. (1995). Reflection in teacher education: Towards definition and implementation. *Teaching and Teacher Education, 11*, 33-49.

Hayes, T.H. & Kelly, M. (2000). Transgressed boundaries: Reflections on the problematics of culture and power in developing collaborative relationship with teachers at an elementary school. *Curriculum Inquiry, 30*(4), 451-472.

Hiebert, J., Gallimore, R., & Stigler, W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational Researcher*, 31(5), 3-15.

Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal,*

Hillocks, G. (1999). *Ways of thinking, was of teaching.* New York: Teachers College Press.

Hollingsworth, S. (1989). Prior beliefs and cognitive change in learning to teach. *American Educational Research Journal, 26,* 160-189.

Holloway, J.H. (1999). Caution: Constructivism ahead. *Educational Leadership, 57*(3), 85-86.

Holt-Reynolds, D. (1992). Personal history-based beliefs as relevant prior knowledge in coursework: Can we practice what we teacher? *American Educational Research Journal, 29,* 325-349.

Hopkins, G. (1997). Grade configuration: Who goes where? *Education World* [Online]. Available: http://www.education-world.com/a_admin/admin017.shtml.

Institute for Research on Learning. (2000). *Pathways to algebra and geometry*. Dallas, TX: Voyager Expanded Learning.

Johnston, S. (1992). Images: A way of understanding the practical knowledge of student teachers. *Teaching and Teacher Education, 8,*123-136.

Jones, G. (2005). Reflections. In G.A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning.* (pp. 367-372). Springer.

Kagan, D.M. (1992). Professional growth among pre-service and beginning teachers. *Review of Educational Research*, *62*(2), 129-169.

Kagan, D.M., & Tippins, D.J. (1991). How student teachers describe their pupils. *Teaching and Teacher Education, 7,* 455-466.

Kahan, J.A., Cooper, D.A., & Bethea, K.A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education, 6,* 223-252.

Kelly, J. (2000). Rethinking the elementary science methods course: A case for content, pedagogy and informal science education. *International Journal of Science Education, 22*(7), 55-77.

Kennedy, M.M. (1996). Research genres in teacher education. In F.B. Murray (Ed.), *The teacher educator's handbook: Building a knowledge base for the preparation of teachers* (pp. 120-152). San Francisco: Jossey-Bass.

Kennedy, M.M. (1999). The role of preservice teacher education. In L. Darling-Hammond & G. Sykes, (Eds.), *Teaching as the learning profession* (pp. 54-85). San Francisco: Jossey-Bass.

Lampert, M. & Ball, D.L. (1999). Aligning teacher education with contemporary k-12 reform visions. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 33-53). San Francisco: Jossey-Bass.

Lampert, M. & Loewenberg, D.B. (1998). *Teaching, multimedia, and mathematics: Investigations of real practice.* New York: Teachers College, Columbia University.

Lanier, J. & Little, J. (1986). Research on teacher education. In M. Wittrock (Ed.), *Handbook of research on teaching* (3rd. ed., pp. 527-569). New York: Macmillan.

Lappan, G. (1997). The challenges of implementation: Supporting teachers. *American Journal of Education, 106,* 207-239.

Lappan, G., Fey, J.T., Fitzgerald, W., Friel, S.N., & Phillips, E.D. (1996). *Getting to Know Connected Mathematics: A Guide to the Connected Mathematics Curriculum.* Palo Alto, CA: Dale Seymour.

Lappan, G., Fey, J.T., Fitzgerald, W., Friel, S.N., & Phillips, E.D. (1998). *Connected mathematics series*. Palo Alto, CA: Dale Seymour.

Lappan, G., Fey, J.T., Fitzgerald, W., Friel, S.N., & Phillips, E.D. (2002). *Connected Mathematics: Lesson Planner*. Glenview, IL: Prentice Hall.

LeCompte, M.D., Milroy, W., & Preissle, J. (1992). *The handbook of qualitative research in education.* New York: Academic Press.

LeCompte, M.D., & Preissle, J. (1993). *Ethnography and qualitative design in educational research* (2nd Ed.). San Diego: Academic Press.

Lederman, N.G., Gess-Newsome, J., & Latz, M.S. (1994). The nature and development of preservice science teachers' conceptions of subject matter and pedagogy. *Journal of Research in Science Teaching*, *31*, 129-146.

Leinhardt, G. (1989). Math lessons: A contrast of novice and expert competence. *Journal of Research in Mathematics Education, 20*(1), 52-75.

Leinhardt, G., & Greeno, J.G. (1986). The cognitive skill of teaching. *Journal of Educational Psychology, 78,* 75-95.

Leinhardt, G., Putnam, R.T., Stein, M.K., & Baxter, J. (1991). Where subject knowledge matters. In J. E. Brophy (Ed.), *Advances in research on teaching* (pp. 87-113). Greenwich, CT: JAI Press.

Leinhardt, G., & Smith, D. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology, 77*, 241-271.

Lerman, S. (2001). A review of research perspectives on mathematics teacher education. In L.F. Lin and T.J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 33-51). Kluwer.

Levin, B.B., & Ammon, P. (1992). The development of beginning teachers' pedagogical thinking: A longitudinal analysis of four case studies. *Teacher Education Quarterly, 19*(4), 19-37.

Lloyd, G. (1999). Two teachers' conceptions of a reform-oriented curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education, 2*, 227-252.

Lortie, D. (1975). *School teacher: A sociological study.* Chicago: University Press.

Loughran, J., Milroy, P., Berry, A., Gunstone, R., & Mulhall, P. (2001). Documenting science teachers' pedagogical content knowledge through pap-ers. *Research in Science Education, 31,* 289-307.

Lowery, N.V. (2002). Construction of teacher knowledge in context: Preparing elementary teachers to teach mathematics and science. *School Science and Mathematics, 102*(2), 68-83.

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources, and development of pedagogical content knowledge. In J. Gess-Newsome and N. B. Lederman (Eds.), *Examining pedagogical content knowledge: The construct and its implications for science education* (pp. 95-132). Kluwer.

Marks, R. (1990). Pedagogical content knowledge from a mathematical case to a modified conception. *Journal of Teacher Education, 41,* 3-11.

Marks, R. (1991). *When Should Teachers Learn Pedagogical Content Knowledge?* East Lansing, MI: National Center for Research on Teacher Learning. (ERIC Document Reproduction Service No. ED335338).

McDiarmid, G.W. (1990). Challenging prospective teachers' beliefs during early field experience: A quixotic undertaking? *Journal of Teacher Education, 41*(3), 12-20.

McDiarmid, G.W., Ball, D.W., & Anderson, C.W. (1989). Why staying one chapter ahead doesn't really work: Subject-specific pedagogy. In M. C. Reynolds (Ed.), *Knowledge base for the beginning teacher* (pp. 193-205). Pergamon.

McDuffie, A.R. (2004). Mathematics teaching as a deliberate practice: An investigation of elementary pre-service teachers' reflective thinking during student teaching. *Journal of Mathematics Teacher Education, 7,* 33-61.

McLaughlin, M.W. & Overman, I. (Eds.). (1996). *Teacher learning: New policies, new practices*. New York: Teachers College Press.

McNamara, D. (1991). Subject knowledge and its application: Problems and possibilities for teacher educators. *Journal of Education for Teaching*, *17*(2), 113-128.

Monk, D., (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 12*(2), 125-145.

Moritz, J.B. & Watson, J.M. (2000). Reasoning and expressing probability in students' judgments in coin tossing. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australia, Vol. 2, pp. 448-455). Perth, WA: MERGA.

Moyer, P.S. & Milewicz, E. (2002). Learning to question: Categories of questioning used by preservice teachers during diagnostic mathematics interviews. *Journal of Mathematics Teacher Education, 5*, 293-315.

Munby, M., Russell, T., & Martin, A.K. (2001). Teachers' knowledge and how it develops. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 877-904). Washington, D.C.: AERA.

National Center for Research in Mathematical Sciences Education & Freudenthal Institute. (Eds.). (1997-1998). *Mathematics in context*. Chicago: Encyclopedia Britannica.

National Center for Research on Teacher Education (1991). *Final report: The teacher education and learning to teach study.* East Lansing: College of Education, Michigan State University.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (1991). *Professional standards for the teaching of school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (1995). *Assessment standards for the teaching of school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Oliver, R. (1997). *Using computer-based learning environments to connect learners: Learning through collaboration, communication and talk*. Retrieved July 18, 2005 from http://elrond.scam.ecu.edu.au/oliver/docs/97/Taipeido.pdf.

Papick, I., Beem, J., Reys, B., & Reys, R. (1999). Impact of the Missouri Middle Mathematics Project on the preparation of prospective middle school teachers. *Journal of Mathematics Teacher Education, 2*, 301-310.

Philipp, R.A. (2000, Winter). Questions to ask and issues to consider while supervising elementary mathematics student teachers. *Teacher Education Quarterly*, 69-86.

Piaget, J. (1952). *The origins of intelligence in children.* New York: International Universities Press.

Piaget, J. (1987). *Possibility and necessity, vol. 1: The role of possibility in cognitive development.* Minneapolis: University of Minnesota Press.

Pratt, D. (2005). How do teachers foster students' understanding of probability? In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning.* Springer.

Putnam, R.T. (1992). Teaching the "hows" of mathematics for everyday life: A case of a fifth-grade teacher. *Elementary School Journal, 93*(2), 163-177.

Putnam, R.T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4-15.

Remillard, J.T. (1992). Teaching mathematics for understanding: A fifth-grade teacher's interpretation of policy. *Elementary School Journal, 93*(2), 179-193.

Remillard, J.T. (2000). Can curriculum materials support teachers; learning? Two fourth-grade teacher's use of a new mathematics text. *Elementary School Journal, 100*(4), 331-350.

Remillard, J.T., & Geist, P.K. (2002). Supporting teachers' professional learning by navigating openings in the curriculum. *Journal of Mathematics Teacher Education 5,* 7-34.

Resnick, L. (1987). Learning in school and out. *Educational Researcher, 16*(9), 13-20.

Romberg, T.A. (1992). Problematic features of the school mathematics curriculum. In P.W. Jackson (Ed.), *Handbook for Research on Curriculum* (pp. 749-788). New York: Macmillan.

Ropo, E. (1987, April). *Teachers' Conceptions of Teaching and Teaching Behaviour: some differences between expert and novice teachers.* Paper presented at the annual meeting of the American Educational Research Association, Washington, 20-24 April.

Ross, D.D., Johnson, M., & Smith, W. (1991). *Developing a professional teacher at the University of Florida.* Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Rovegno, I. (1994). Teaching within a curricular zone of safety: School culture and the situated nature of student teachers' pedagogical content knowledge. *Research Quarterly for Exercise and Sport*, 65(3), 269-279.

Saxe, G.B. (1995). From the field to the classroom: Studies in mathematical understanding. In L.P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 287-312). Hillsdale, NJ: Erlbaum.

Schneider, R.M., Krajcik, J., & Blumenfeld, P. (2005). Enacting reform-based science materials: The range of teacher enactments in reform classrooms. *Journal of Research in Science Teaching, 43*(3), 283-312.

Schneider, R.M, Krajcik, J., & Marks, R. (2000). The role of educative curriculum materials in reforming science education. In B. Fishman, & S. O'Connor-Divelbiss, S. (Eds.), *Proceedings of the international conference of learning science.* New Jersey: Erlbaum.

Schein, E.H. (1978) *Career dynamics: Matching individual and organizational needs.* Reading, MA: Addison-Wesley.

Schmidt, R.A., & Wolf, J.S. (1980). The mentor partnership: Discovery of professionalism. *NASPA Journal, 17*(3), 45-51.

Schön, D. (1983). *The reflective practitioner: How professionals think in action.* New York: Basic Books.

Schön, D. (1987). *Educating the reflective practitioner. Towards a new design for teaching and learning in the professions.* San Francisco: Jossey Bass.

Segall, A. (2004). Revisiting pedagogical content knowledge: the pedagogy of content/the content of pedagogy. *Teaching and Teacher Education 20*, 489-504.

Shaughnessy, J.M. (1992). Research in probability and statistics: Reflections and directions. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465-494). New York: Macmillan.

Shaughnessy, M., & Ciancetta, M. (2002). Students' understanding of variability in a probability environment. In B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching Statistics: Developing a Statistically Literate Society*, South Africa. Available: http://www.stat.auckland.ac.nz/~iase/publications/1/6a6_shau.pdf.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15,* 4-14.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57,* 1-22.

Shulman, L.S., & Grossman, P.L. (1987). *Final report to the Spencer Foundation* (Knowledge Growth in a Profession Technical Report). Stanford, CA: School of Education, Stanford University.

Shulman, L.S., & Grossman, P.L. (1988). *Knowledge growth in teaching: A final report to the Spencer Foundation*. Stanford, CA: Stanford University.

Shulman, L.S., & Sykes, G. (1986). *A National Board for Teaching? In search of a bold Standard*. A Report for the Task Force for Teaching as a Profession. New York: Carnegie Corporation.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145.

Smith, D.C. (1999). Changing our teaching: The role of pedagogical content knowledge in elementary science. In J. Gess-Newsome and N. B. Lederman (Eds.), *Examining pedagogical content knowledge: The construct and its implications for science education,* (pp. 163-197). Kluwer.

Smith, J.P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*, 387-402.

Stake, R. (1995). *The art of case study research.* Thousand Oaks, CA: Sage.

Steffe, L. (1991). The constructivist teaching experiment: Illustrations and implications. In E. v. Glasersfeld (Ed.), *Radical constructivism in mathematics education*. (pp. 177-194). The Netherlands: Kluwer.

Steffe, L., & D'Ambrosio, B.S. (1995). Toward a working model of constructivist teaching: A reaction to Simon. *Journal for Research in Mathematics Education, 26*(2), 146-159.

Stohl, H. (2005). Probability in teacher education and development. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning.* Springer.

Strauss, A. & Corbin, J. (1990). *The basics of qualitative research: Grounded theory procedures and techniques.* Newbury Park, CA: Sage.

Tarr, J.E. & Papick, I.J. (2004). Collaborative efforts to improve the mathematical preparation of middle grades mathematics teachers: Connecting middle school and college mathematics. In T. Watanabe & D. R. Thompson (Eds.), *The work of mathematics teacher educators*, Association of Mathematics Teacher Educators (AMTE) Monograph 1 (pp. 19-34). San Diego, CA: Author.

TERC. (1995). *Investigations in Numbers, Data and Space.* Palo Alto, CA: Dale

Seymour Publications.

Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 115-126). New York: Macmillan.

Trafton, P.R., Reys, B.J., & Wasman, D.G. (2001). Standards-based mathematics curriculum materials: A phrase in search of a definition. *Phi Delta Kappan*, *83,* 259-264.

Vahey, P., Enyedy, N., & Gifford, B. (2000). Learning probability through the use of a collaborative, inquiry-based simulation environment. *Journal of Interactive Learning Research, 11*(1), 51-84.

Van Zoest, L., & Bohl, J. (2002). The role of reform curricular materials in an internship: The case of alice and gregory. *Journal of Mathematics Teacher Education 5,* 296-288.

von Glasersfeld, E. (1988). *The Construction of Knowledge: Contributions to Conceptual Semantics*. Seaside, CA: Intersystems.

von Glasersfeld, E. (1995a). A constructivist approach to teaching. In L. Steffe & J. Gale (Eds.), *Constructivism in education*. (pp. 3-15). Hillsdale, NJ: Lawrence Erlbaum.

von Glasersfeld, E. (1995b). *Radical constructivism: A way of knowing and learning*. Washington, DC: The Falmer Press.

von Glasersfeld, E. (1996). Introduction: Aspects of constructivism. In C.T. Fosnot, (Ed.), *Constructivism: Theory, Perspectives, and Practice.* (pp. 3-7). New York: Teachers College Press.

Vygotsky, L.S. (1962). *Thought and Language*. Cambridge, MA: MIT Press.

Vygotsky, L.S. (1978). *Mind in Society*. Cambridge, MA: Harvard University Press.

Walker, D. & Lambert, L. (1995). Learning and leading theory: A century in the making. In L. Lambert, D. Walker, D.P. Zimmerman, J.E. Cooper, M.D. Lambert, M. Gardner, & P.J. Slack (Eds.), *The constructivist leader*. (pp. 1-27). New York: Teachers College Press.

Wallace, J., & Louden, W. (1998). Curriculum change in science: Riding the waves of reform. In B. J. Fraser & K. G. Tobin (Eds.), *International handbook of science* *education* (pp. 471-485). Kluwer.

Wang, J. & Paine, L.W. (2003). Learning to teach with mandated curriculum and public examination of teaching as contexts. *Teaching and Teacher Education, 19,* 75-94.

Ward, J.R., & McCotter, S.S. (2004). Reflection as a visible outcome for preservice teachers. *Teaching and Teacher Education, 20*, 243-257.

Wassermann, S. (1991). The are of the question. *Childhood Education, 67*(4), 257-259.

Wideen, M.J., Mayer-Smith, J., & Moon, B. (1998). A critical analysis of the research on learning to teach: Making the case for an ecological perspective on inquiry. *Review of Educational Research, 86*(2), 130-178.

Wilcox, S.K., Lanier, P., Schram, P. & Lappan, G. (1992). *Influencing Beginning Teachers' Practice in Mathematics Education: Confronting Constraints of Knowledge, Beliefs, and Context* (Research Report No. 92-1). National Center for Research on Teacher Education, Michigan State University.

Willoughby, S.S. (1990). *Mathematics education for a changing world*. Alexandria: Association for Supervision and Curriculum Development.

Wilson, S.M., & Berne, J. (1999). Teacher learning and the acquisition of professional Knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & P.D. Pearson (Eds.), *Review of research in education* (pp. 173-209). Washington, DC: AERA.

Wilson, S.M., Floden, R.E., & Ferrini-Mundy, J. (2001). *Teacher preparation research: Current knowledge, gaps, and recommendations.* (Document R-01-3). Seattle, WA: Center for the Study of Teaching and Policy.

Wilson, S.M., Shulman, L.S., & Richert, A.E. (1987). "150 different ways" of knowing: Representations of knowledge I teaching. In J. Calderhead (Ed.), *Exploring teachers' thinking* (pp. 104-124). London: Cassell.

Wilson, S.M., & Wineburg, S.S. (1988). Peering at history through different lenses. The role of disciplinary perspective in teaching history. *Teachers College Record, 84,* 525-539.

Wood, T. (1995). An emerging practice of teaching. In P. Cobb, & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 203-227). Hillsdale, NJ: Lawrence Erlbaum.

Wubbels, T., Korthagen, F. & Broekman, H. (1997). Preparing teachers for realistic mathematics education. *Educational Studies in Mathematics, 32*, 1-28.

Yin, R.K. (2003). *Case study research: Design and methods* (3rd Ed.). Thousand Oaks, CA: Sage.

Zahorik, J. (1988). The observing-conferencing role of university supervisors. *Journal of Teacher Education, 39*(2), 9-16.

Zeichner, K.M. (1996). Teachers as reflective practitioners and the democratization of school reform. In K.M. Zeichner, S. Melnick, & M.L. Gomez (Eds.), *Currents of reform in preservice teacher education* (pp. 199-214). New York: Teachers College Press.

Zeichner, K.M. (1999). The new scholarship in teacher education. *Educational Researcher*, *28*(9), 4-15.

Zeichner, K.M., & Gore, J. (1990). Teacher socialization. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 329-348). New York: Macmillan.

APPENDIX A: MATHEMATICS PEDAGOGY ASSESSMENT

Assessment of Pedagogical Content Knowledge in Mathematics
Question 1

Describe how you would present an introductory lesson on ratio and proportion to a class of average ability middle school students.

The goals of the lesson are to have the students identify and write rational numbers, recognize equivalent forms of rational numbers, and solve simple proportions.

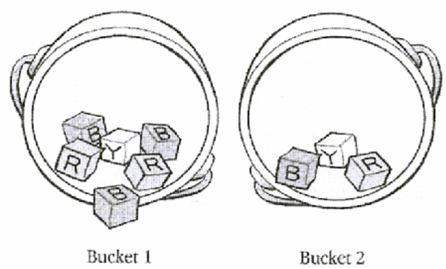
Your description should include:

A. How you would motivate the students' interest in ratio and proportion
B. An outline of what you would do to achieve the goals of the lesson
C. Sample exercises that could be used to reinforce student understanding of the material presented in the lesson

Assessment of Pedagogical Content Knowledge in Mathematics
Question 2

Suppose that you are teaching problem-solving techniques to a middle school class of average ability and that you have posed the following problem:

> All the winners from the Gee Whiz Everyone Wins! Game show get an opportunity to compete fro a bonus prize. Each contestant draws one block at random from each of the buckets shown below. If the blocks are the same color, the contestant wins a prize. What is the probability that a contestant will draw two blocks of the same color?
>
> (CMP, *What Do You Expect,* 1998)



Discuss how you could get the students to develop an effective strategy for solving this problem. Include in your discussion questions relating to the problem that you might ask the class that would facilitate the development of a strategy.

Assessment of Pedagogical Content Knowledge in Mathematics
Question 3

Describe how you would present a lesson introducing the concept of experimental and theoretical probabilities to a middle grades class with below-average ability. The goal of the lesson is for students to develop a conceptual understanding of what experiemental and theoretical probability is, and how it is represented notationally.

Your description should include:

A. How you would motivate the students' interest in the concept of experimental and theoretical probability
B. An outline of what you would do to achieve the goals of the lesson
C. Sample exercises that could be used to reinforce student understanding of the material presented in the lesson

APPENDIX B: RANDY'S PRE-ASSESSMENT

Question 1

To motivate student interest in probability I would probably try to use different kinds of games to engage them (we could discuss things like how some people claim they're good at rolling doubles) other "chance" games are also possibilities (cards, flipping coin). I think that students a genuinely interested in playing games and having fun and I would let them know that a better understanding of probability would allow them to be more successful at games (or at least avoid games that are unfair)

Achieving the Goals

- Background information on theory vs. experime
  - → check w/ science teacher, have them try to work this concept into a science investigati to reinforce in the math class.
- Ask students to predict what ~~theory is~~ & theoretical prob & experimental prob. is based on what they already know about theory & experiments
- When we have a working definition of what the students think this is we can actually start experimenting
  - → flipping a coin is best place to start (only 2 outcomes)
  - → ask students questions like, if I flip 4 heads in a row, is a tails more likely to be ne

(critical thinking questions, prediction questions)
- Asked before exp. conducted
  - chart/organize outcomes w/ small # of trials

- Really focus on fact that it can only be heads or tails. Hopefully students can see that there is a 1 out of 2 chance for getting either H or T
  - I think flipping coin for themselves will help students understand.

- After small # of trials have students predict what will happen if we flip the coin more often
  - → collect data from entire class to increase sample size (each group flips a coin like 20 times & pools data w/ other groups)

→ look at sample size and ~~try~~ to generalize what is happening to the # of H & # of tails as we conduct more experiments.

→ Move onto an computer applet where we can control the # of trials & keep steadily increasing the # until we see that 50% of time its H, 50% → T

Discuss that increasing the # of trials will move us from exp. prob to theor. prob. & discuss the law of large numbers.
~~Notationally students~~ should be able to see that the prob. of a H is 1 out of 2 or 1/2 = 50% chance of flipping a head

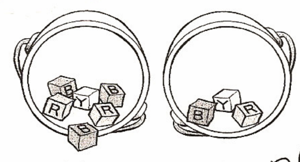
Assessment of Pedagogical Content Knowledge in Mathematics
Question 2

Suppose that you are teaching problem-solving techniques to a middle school class of average ability and that you have posed the following problem:

> All the winners from the Gee Whiz Everyone Wins! Game show get an opportunity to compete for a bonus prize. Each contestant draws one block at random from each of the buckets shown below. If the blocks are the same color, the contestant wins a prize. What is the probability that a contestant will draw two blocks of the same color?
>
> (CMP, *What Do You Expect,* 1998)

$y = \frac{1}{6}$
$B = \frac{3}{6}$
$r = \frac{2}{6}$ →

$\frac{1}{3}$ for all



Bucket 1 Bucket 2

$p(2\ \text{yellow}) = \frac{1}{3}\left(\frac{1}{6}\right) = \frac{1}{18}$

$p(2\ \text{blue}) = \frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{6}$

$p(2\ \text{red}) = \frac{1}{3}\left(\frac{1}{3}\right) = \frac{1}{9}$

Discuss how you could get the students to develop an effective strategy for solving this problem. Include in your discussion questions relating to the problem that you might ask the class that would facilitate the development of a strategy.

- 1st we would have to establish the prob. for drawing a certain color from bucket 1 & from bucket 2.

Possible ?'s

Q: what is the chance that I will draw a blue from Bucket 1?

Q: Is prob. of drawing a blue (buck. 1) 1/3, 3/6, or 1/6? Why?

(could modify to ask about diff. colors or bucket # 2)

~~Pull 1 cube from # 1 & 1 from # 2 record outcome~~

#2

Try to list possible outcome w/ tree diagram

Bucket 1	Bucket 2
B	B
B	
B	Y
R	
R	r
Y	

Assessment of Pedagogical Content Knowledge in Mathematics
Question 3

Describe how you would present an introductory lesson on independent and dependent events to a class of average ability middle school students. The goals of the lesson are to have the students identify and model the probabilities in situations that involve drawing with and without replacement, analyzing situations to find all possible outcomes, and confront the issue of how the dependence of the outcomes of one action on another action affects the probabilities in a situation.

Your description should include:

A. How you would motivate the students' interest in independent and dependent events
B. An outline of what you would do to achieve the goals of the lesson
C. Sample exercises that could be used to reinforce student understanding of the material presented in the lesson

- This could build on the lesson of flipping the coin.
  - ie: flipping 4 heads, does that mean a T is more likely on 5th toss?
  - use ~~manipulatives~~ to simulate exp. 2 drawing from bucket & replacing or
  - use applets to reinforce/explore 3 ie: "monty hall game"
  - record results from experiments
  - make predictions based on outcomes

To find all possible out comes
(ie: how many ways can you arrange the letters A B C?)

→ listing strategies 1st

ABC	BAC	CAB
ACB	BCA	CBA

lead to $\underbrace{3}_{\text{3 choices for 1st}} \cdot \underbrace{2}_{\text{2 choices 2nd}} \cdot \underbrace{1}_{\text{1 choice last}} = \underbrace{6}_{\text{outcomes}}$

APPENDIX C: BRAD'S PRE-ASSESSMENT

Question #1

Below-average ability

I have a bucket with some red, some blue, and some yellow blocks in it. I will ask you to guess before you draw out a color. If you guess it correctly, you get a piece of candy. After guessing, draw a block, announce it to the class and place it back in the bucket. I will need a volunteer to keep our results on the board.

As I go through the room and we record our data kids should start using the results from the experiment to decide which color gives them the best chances of winning the candy.

Which color do you think I have more of?
Which color do I have the least?
After showing the blocks we compare the REAL blocks to the board (experimental data).
Does our EXPERIMENTAL data match would you think we should have gotten?
How does it differ and why?

Next I have a bucket with 8 red, 4 blue and 1 yellow. We will play again but we won't record our results or tell out loud our guess. You need to write down your guess before I get to you and then reach in, but DO NOT SHOW the results.

This should lead us into a discussion of what you expected to get at first (THEORY), versus basing your decision on the EXPERIMENTAL outcomes.

Exercise.

I have 3 red, 4 yellow and 5 blue marbles in a cup. What is the probability of drawing the following…Explain why.
a red?
A yellow
A blue
A green

Question #2

Pose the problem, then ask…
What are the different possibilities for drawings?
Give time to create a list. Then ask to share at tables and find an easy way to list.
I'll circulate while they are doing this to find efficient methods.
During sharing I'll call on the efficient methods to share.

How can I be sure I have all options listed?
Can I organize the list in any way to help me?
I see 3 blues, in the first bucket. Do I count them 3 times?

I hope for the following explanation from a group of kids…
One efficient method is to list all of the first possibilities, then branch a 3 way tree off each one showing the second choice. Now it can be seen visually and we can answer the question. They should have a total from the list and can count how many are matching.

Eventually, you can use fractions from the 1st bucket and 2nd bucket combined. For example, the chances of drawing blue out of bucket 1 is 3/5 and blue out of bucket 2 as 1/3. So the chances are 3/15 of drawing matching blues. You can do the same with the other colors. This is where we are headed with the idea, and we can go there with some of the bright kids.

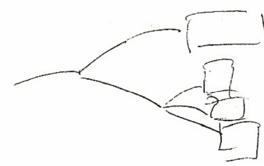
3 Independent events | dependent events

4 Experimental | Theoretical

2 Buckets

3 drawing cards
marbles
blocks

Game:



Question 3

"I have a 5 prizes I would like to give away to five random students. In this hat I have placed 5 names. The five prices are a piece of corn, a used dry-erase marker, an autographed picture of your favorite math teacher, a free homework assignment pass, and a pass to the movies."

I'll reach in and pull out the names and award the prizes accordingly, without placing the names back in the hat.
Next, I'll collect the prizes back and start over this time placing the names back in the hat after each drawing. If lucky, a kid could win a piece of corn AND the autographed picture, or maybe all 5. Or maybe one of the 5 names never gets called.

I would then ask the kids to do the following individually;
How did it feel the first time?
How did it feel to have your name in the hat the second time?
Compare the chances of winning both times?

Now as a small group of 4 discuss your findings and be ready to share out.

Sharing questions:
Drawing without replacement. Were you assured of getting a prize if your name was in the hat the first time? If you won the old marker, did you have any shot at winning the movie pass? What is the probability of winning something else? What's the probability of winning the movie pass when the game started? Did that change as the names were drawn?
Drawing with replacement. Were you assured a prize if your name was in the hat? If you won the piece of corn, could you have won something else? What is the probability of winning something else? What's the probability of winning the movie pass when the game started? Did that change as the names were drawn?

The task for the groups will be to show a way of finding the THEORETICAL probability of winning the movie pass under each scenario. Using a tree diagram or other method, I need to see their thinking.

Sample exercises:
John has a bucket containing 3 red and 3 blue marbles. He is going to draw three marbles out separately without replacement. Theoretically what are the chances that he will draw all three reds? Show how you solved this.

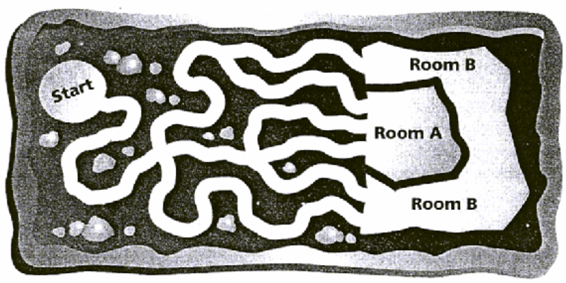
INVESTIGATION 4

Analyzing Two-Stage Games

In the Treasure Hunt game and in games involving spinners and dartboards, you used area to find probabilities. In this investigation, you will learn how to use area in a slightly different way to analyze more situations involving probability.

4.1 Choosing Paths

Kenisha designed a computer game called Deep in the Dungeon. The game pits a player against a computer character named Zark. The game screen is shown below.



The player puts the treasure in one of the two rooms in the dungeon. Zark begins at "start" and makes his way toward the dungeon, *randomly* selecting a path at each fork. If Zark ends in the room with the treasure, he wins. If he ends in the room without the treasure, the player wins.

4.1 Choosing Paths

At a Glance

Launch

- Introduce the Deep in the Dungeon game, and ask the class how they might simulate Zark *randomly* choosing paths.
- Play a game with the class, letting half the class hide the treasure and the other half proceed through the maze, randomly choosing paths.
- Have groups of two or four work on the problem.

Explore

- Circulate as groups work, asking questions that highlight the differences between these multistage outcomes and single-stage outcomes.

Summarize

- Discuss strategies for choosing paths and assigning experimental probabilities.
- As a class, pool the data and find the overall experimental probabilities.
- Work through the follow-up questions as a class.

Assignment Choices

ACE questions 1, 4, 7, and unassigned choices from earlier problems

4.1

Problem 4.1

A. If you were playing Deep in the Dungeon, in which room would you put the treasure in order to have the best chance of beating Zark? Explain your choice.

B. Work with a partner to find a way to simulate Deep in the Dungeon so it can be played without a computer. Your simulation should be a two-person game. One person should hide the treasure, and the other should play the role of Zark. You will need to figure out a way for Zark to make a random selection at each fork.

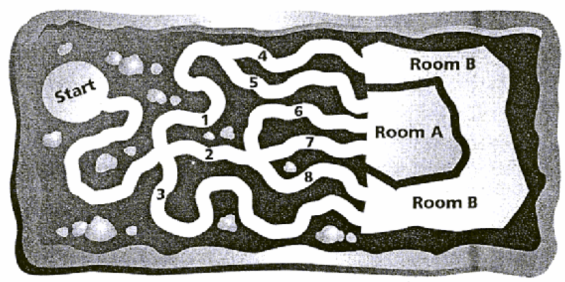
C. Play your simulation of Deep in the Dungeon 20 times with your partner. Take turns hiding the treasure and playing Zark. For each game, record the room that Zark ends in.

D. Based on your results from part C, what is the experimental probability that Zark will end in room A? What is the experimental probability that Zark will end in room B?

Problem 4.1 Follow-Up

You and your classmates may have found several ways to simulate Deep in the Dungeon in order to find experimental probabilities. How could you determine the theoretical probabilities of Zark ending in each room?

One way to find the theoretical probabilities is by using an *area model*. To make it easier to talk about the game, we'll number the paths as shown below.



Answers to Problem 4.1

A. Possible answers: Room A because it is smaller. Either room because they each have three paths leading to them. Room B because the lowest path goes right to it.

B. Simulations will vary. See the Explore section for several ways to simulate the game.

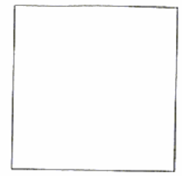
C. Results will vary.

D. Answers will vary. Combining the entire class's data should bring the experimental probabilities close to $\frac{7}{18}$ for room A and $\frac{11}{18}$ for room B.

Answers to Problem 4.1 Follow-Up

See the Summarize section for an example of how to use an area model to represent the game. The theoretical probability that Zark will end in room A is $\frac{7}{18}$; the theoretical probability that he will end in room B is $\frac{11}{18}$.

1. Draw a square on your paper. Suppose that the square has an area of 1 square unit, representing a probability of 1. At the first fork, there are three equally likely choices: path 1, path 2, and path 3. Divide and label the square so the areas of the sections represent the probabilities of these three choices.



2. If Zark selects path 1 at the first stage of his journey, he will reach a fork where he must randomly select path 4 or path 5. Subdivide your diagram to represent the probabilities that Zark will choose path 1 and then choose path 4 or path 5.
3. If Zark selects path 2 at the first stage of his journey, he will reach a fork where he must randomly select path 6, path 7, or path 8. Subdivide your diagram to represent the probabilities that Zark will choose path 2 and then choose path 6, path 7, or path 8.
4. On your diagram, color the sections that represent paths leading to room A with one color and the sections that represent paths leading to room B with a second color.
5. What is the theoretical probability that Zark will end in room A? What is the theoretical probability that he will end in room B?

APPENDIX E: RANDY'S POST-ASSESSMENT

Assessment of Pedagogical Content Knowledge in Mathematics
Question 1

Describe how you would present a lesson introducing the concept of experimental and theoretical probabilities to a middle grades class with below-average ability. The goal of the lesson is for students to develop a conceptual understanding of what experiemental and theoretical probability is, and how it is represented notationally.

Your description should include:

A. How you would motivate the students' interest in the concept of experimental and theoretical probability
B. An outline of what you would do to achieve the goals of the lesson
C. Sample exercises that could be used to reinforce student understanding of the material presented in the lesson

- Discuss what prob. is and where we might see it in everyday life.
- Talk about what it means to perform an experiment. (We will collect data and analyze it)
- We can collect data by playing games of chance! (motivator)
- Get their initial response to the question "What chance do I have of getting a tail when I flip a fair coin?
- Hopefully everyone at this point understands it's a 50-50 chance - Now I would have everyone flip a coin/counter about 20 times & record their results,
- Did anybody get heads 50% of the time?

We could then look @ the data from 5 people (to make 100 trials) on the board & discuss what the results were. (We could later combine all the class data)

- Start w/ a hundred trials b/c ~~this class will~~ it will be easier w/ this class to understand percentages.

- Discuss how even though we expect ~~to~~ get 50% H & 50%T, that are experiments might show something diff

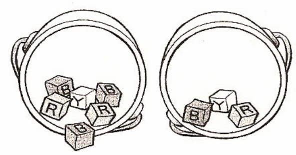
- Ask students ~~that~~ if they should trust their data or the fact that it's a 50-50 chance.

- Bring in some "pre-found" data to demonstrate that as we increase the # of trials we will approach the theoretical prob. of 50% H/50%

Assessment of Pedagogical Content Knowledge in Mathematics
Question 2

Suppose that you are teaching problem-solving techniques to a middle school class of average ability and that you have posed the following problem:

> All the winners from the Gee Whiz Everyone Wins! Game show get an opportunity to compete for a bonus prize. Each contestant draws one block at random from each of the buckets shown below. If the blocks are the same color, the contestant wins a prize. What is the probability that a contestant will draw two blocks of the same color?
>
> (CMP, *What Do You Expect,* 1998)



Bucket 1 Bucket 2

Discuss how you could get the students to develop an effective strategy for solving this problem. Include in your discussion questions relating to the problem that you might ask the class that would facilitate the development of a strategy.

We could start out by talking about the outcomes A Questions such as "do I have an equal chance of choosing each color in bucket 1? How about bucket 2?

Other questions: what is similar about buckets 1 & 2? what are the differences?

What chance do I have of pulling a blue block from bucket 1? Bucket 2?

Or maybe even — If I told you there was a 50% chance of pulling a blue out of bucket 1 would you agree or disag

Hopefully this last question could lead us to discuss the fact that we have to consider ~~all of t~~ each blue block individually.

If we are at a place where the students can solve this type of problem I am assuming that they have already used ways to list outcomes in previous activities.

→ ~~what are~~ How do we find the total # of outcomes? (listing, count. tree, ~~area model~~, etc.)

→ if no one suggests a counting tree after discussing possibilities I would mention it myself and then ask a student to draw one on the board or have individual students work on one of their own.

Once you have the counting tree, the problem becomes more simple. They should be abl to see where they can actually draw two of the same color.

We could also check comprehension by adjusting the # of blocks. Possibly start Ⓐ with a bucket that doesn't have mult. blocks of same color (Actually, maybe just focus on 2 bucke that ~~just~~ to contain only 3 blocks - one of each color Having them do this 1st will give them the confidence to move on to the actual problem (would be eas

Assessment of Pedagogical Content Knowledge in Mathematics
Question 3

Describe how you would present an introductory lesson on independent and dependent events to a class of average ability middle school students. The goals of the lesson are to have the students identify and model the probabilities in situations that involve drawing with and without replacement, analyzing situations to find all possible outcomes, and confront the issue of how the dependence of the outcomes of one action on another action affects the probabilities in a situation.

Your description should include:

A. How you would motivate the students' interest in independent and dependent events
B. An outline of what you would do to achieve the goals of the lesson
C. Sample exercises that could be used to reinforce student understanding of the material presented in the lesson

A) I think one of the best ways to do this is with the situation of shooting "free-throws". Students are familiar with this concept & can really see how the math can apply to a real-life situation. I could also ask students ~~how the~~ what they think independent/dependent means in a probabilistic sense. We could maybe try to list some events that students think are ind/dep?

B) Start w/ questions
- ~~Do you think flipping a coin (more than once) is ind/dep? Why?~~
- If I flip a coin 4 times and get 4 tails in a row, what is the chance my next flip will be a head? (could use # cubes as well)
maybe discuss how this is an independ. situation (allow students to discover)
- Introduce the situation of a b-ball player shooting a one-and-one foul shout.

- Discuss whether/not this is an ind or dep. situation.

→ Questions to ask

— What are the possible outcomes of a 1 and 1 free throw situati
(0 pts, 1 pt, 2 pts)

- How can you get 0 pts?
" " 1 pt?
" " 2 pts?

- Is it possible to get 1 pt. after missing the 1st shot? Why or why not?

- Discuss that taking the 2nd shot depends on making the 1st shot.

However: Maybe should focus discussion on how the opportunity to shoot a 2nd time is dependent, but the chance of making or missing that 2nd shot is independent. The basketball does not know if you made or missed your last shot.

*From here move on to the situation of drawing marbles from a cup w/wo replacement (The CMP activity worked well)

- Compare the prob. of drawing a certain color when drawing 2 times in a row w & wo replacement.)

Kids should physically do this (manip.) b/c they can actually see how the odds change when they do not replace the marble.

— Again, outcomes could be listed by any of the methods that we've been studying

Q. 3

~~Exercises:~~ Activities Similar to CMP "Deep in the Puz proved to be popular w/ students and is a good way to get them familiar w/ area models. Many different variations of paths could make this exercise more simple or complicated depending on where you need to go w/ it. Students could also make their own & analyze

- one & one free throws
- 2 shot free throws
- Choosing clothes from drawer
- etc., etc.

1.4

Making Counting Trees

At a Glance

Launch

- Use the example in the student edition to illustrate how to make a counting tree.
- Talk about how to read a counting tree.
- Have students work individually to make their counting trees, then gather in groups to share results and to answer the questions.

Explore

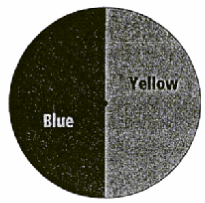
- Assist students who need help getting started.
- Have groups move on to the follow-up when they finish the problem.

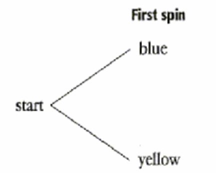
Summarize

- Ask several students to share their counting trees.
- Verify that the class can read a counting tree.
- Review the follow-up questions.

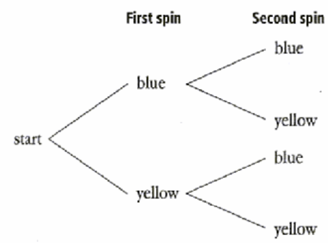
Making Counting Trees

You can find all the possible outcomes of a situation by making an organized list. Creating a **counting tree** can help you make sure you find all the possibilities. April used a counting tree to show all the possible outcomes for the Match/No-Match game (from Problem 1.2). First, she listed the equally likely outcomes of the first spin as shown in the tree at right below.





A turn consists of two spins, so from each of the possible results of the first spin, April drew two branches and labeled them to show the possible results of the second spin.



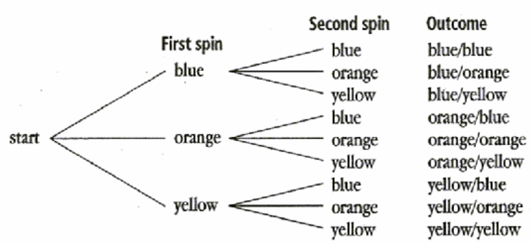
By following the paths from left to right, April can read all the possible outcomes of a turn. For example, she can follow the upper branch from start to blue, and then from there follow the upper branch to blue. This path represents the outcome blue/blue.

Assignment Choices

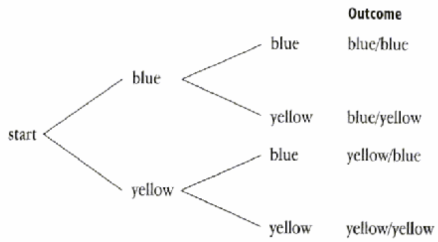
ACE questions 9, 12, 14, 16, 17, 19, 20, and unassigned choices from earlier problems

Answers to Problem 1.4

A.

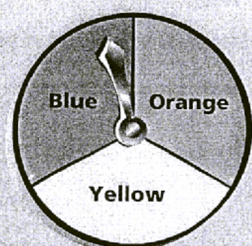


The column to the right of the tree below lists the possible outcomes.



Problem 1.4

April and Tioko decide to play the Match/No-Match game on the spinner below. As in the original game, a turn consists of two spins. Player A scores 1 point if the spins match, and Player B scores 1 point if they do not match.



A. Use a counting tree to find all the possible outcomes for this game.

B. What is the theoretical probability of getting a match on a turn?

C. What is the theoretical probability of getting a no-match on a turn?

D. Do you think this is a fair game? If you think the game is fair, explain why. If you think it is not fair, explain how the rules could be changed to make it fair.

B. P(match) = $\frac{3}{9} = \frac{1}{3}$

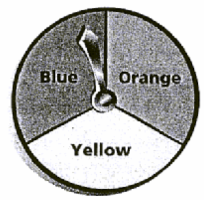
C. P(no-match) = $\frac{6}{9} = \frac{2}{3}$

D. The game is not fair because the chances of scoring are not equal. If a match is awarded 2 points rather than 1, the expected number of points scored will be the same for both players and the game will be fair.

1.4

■ **Problem 1.4 Follow-Up**

1. a. Find all the possible outcomes for the Making Purple game in Problem 1.3 by creating a counting tree.
   b. Use your counting tree to find the theoretical probability of making purple on a turn.
   c. How does the theoretical probability you found by using a counting tree compare with the theoretical probability you found in Problem 1.3?
2. Shondra played a game with a spinner and a coin. For each turn, she spun the spinner once and tossed the coin once. For example, one possible outcome would be blue/head.



   a. Create a counting tree to find all the possible outcomes of a turn in Shondra's game.
   b. Are all the outcomes equally likely? Explain why or why not.
   c. What is the probability that Shondra will spin blue and toss a head on a turn?

Answers to Problem 1.4 Follow-Up

1. a. See page 21k.
   b. P(purple) = $\frac{1}{9}$
   c. The probabilities are the same.
2. a. See page 21k.
   b. The outcomes are all equally likely because each has the same probability of occurring, $\frac{1}{6}$.
   c. P(blue/head) = $\frac{1}{6}$

4.2 • Finding the Best Arrangement

Launch

This problem gives students another chance to analyze two-stage games, involving dependent probabilities, using an area model. An area model allows students to focus on one stage, or decision, at a time; the model accounts for the dependent relationship. The second stage, or decision, is restricted as a portion of the total area. This is the equivalent of adjusting the sample space for the second stage to account for the result in the first stage.

Begin by telling the story of the game Brianna and Emmanuel are to play. Ask students for one way that two orange and two blue marbles could be arranged in two containers.

When students understand the game, have groups of three or four work on the problem. Each group will need two blocks, marbles, or other manipulatives in each of two colors and two identical opaque containers.

Explore

This is a difficult problem for students; they often struggle to find all the ways the four marbles can be arranged in two containers. Also, they may not see a relationship between this problem and Problem 4.1 and thus not realize that an area model is a reasonable way to find the theoretical probability of drawing an orange marble from each of the possible arrangements.

It is likely that many students will not initially realize that reversals of the contents of the two containers are not separate arrangements in a probabilistic sense. That is, the probability of drawing an orange marble when two blue marbles are in container 1 and two orange marbles are in container 2 is equal to the probability of drawing an orange marble when two blue marbles are in container 2 and two orange marbles are in container 1; there is no need to analyze both arrangements. Rather than pointing out that the containers are interchangeable and that these are equivalent situations, you may want to let students struggle with this idea, reach decisions about it within their groups, then clear up any misunderstanding during the summary.

As groups explore the problem, ask questions to help them make sense of their work.

> How do you know that you have considered all the possible ways to arrange the marbles in the containers?

If a group is really struggling with finding all the arrangements, model another arrangement with them.

> Once Brianna has put the marbles in the containers, what is the first choice that Emmanuel must make? *(He must select a container.)*
>
> What does Emmanuel do after he selects a container? *(He reaches in and, unless the container is empty, draws out a marble.)*

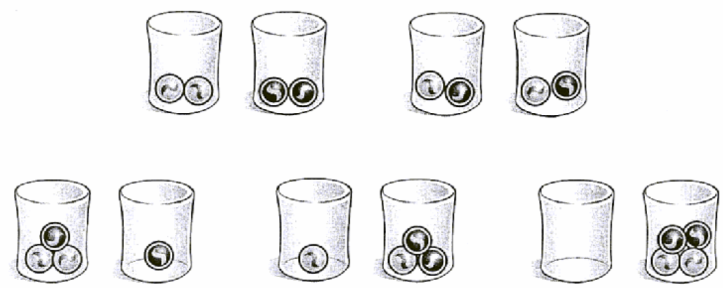
Think about the two choices Emmanuel has to make. What do you think is the probability of him drawing an orange marble in each of the arrangements? How might you show these two choices in your analysis?

Can you use what you learned in the last problem about area analysis to help you analyze this problem?

Summarize

Ask for all the ways the four marbles can be placed in the two containers. If students do not give all five arrangements (not counting reversals), say that you have another, and ask whether they can discover it. Putting all the marbles into one container is an arrangement students often miss.

Here are the five basic arrangements.



The associated theoretical probabilities of drawing an orange marble from each arrangement are as follows:

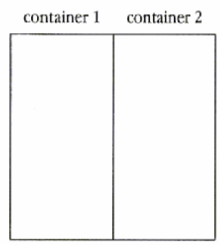
Container 1	2 orange	1 blue, 1 orange	1 blue, 2 orange	1 orange	—
Container 2	2 blue	1 blue, 1 orange	1 blue	2 blue, 1 orange	2 blue, 2 orange
P(orange)	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$

Ask which arrangement will give Brianna and Emmanuel the best chance of winning. For each arrangement the groups suggest, have someone from that group illustrate on the board how they determined that it would give the friends the best chance of winning. If a particular group does not use an area analysis, ask another group to show how an area model could have been used to analyze that arrangement. If groups have other strategies, give them a chance to offer these for class consideration.

If no group presents an area model as a means of determining the theoretical probabilities for the various arrangements, demonstrate how to apply an area model to one of the arrangements. Some students have trouble understanding the probability of drawing an orange marble when all the marbles are placed in one container. You may want to work through this situation with the class.

Let's use an area model to represent putting all four marbles in one container. What is the first decision Emmanuel must make? *(He must select a container.)* How do we represent this with an area model?

A square could be drawn to represent the total probability, and it could be divided into two equal parts to represent the choice between the two containers. For example:

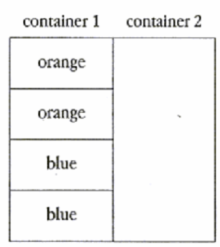


After selecting a container, what must Emmanuel do to complete the game?

Emmanuel must reach in and select a marble. If he chooses the empty container, when he reaches in he will get nothing.

How can we represent this step with our area model?

The region representing the container of four marbles—for example, container 1—must be divided into four equal parts, two for orange and two for blue. The region for the second container needs no further division.



Based on our area model, what is the probability of drawing an orange marble with this arrangement? ($\frac{1}{4}$) What is the probability of drawing a blue marble? ($\frac{1}{4}$) What is the probability of drawing nothing? ($\frac{1}{2}$)

Have groups work on follow-up question 1, which asks which arrangement gives the friends the *worst* chance of winning. Collect all ideas, again having students illustrate their analysis of each arrangement they suggest. If no one uses an area analysis, ask someone to show how an area model could be used.

Now, turn the class's attention to follow-up question 2. Let groups present their ideas about how to analyze the various options in this new game.

1.2 Matching Colors

At a Glance

Launch

- Explain that students will be analyzing April and Tioko's spinner game.
- Demonstrate how to take one turn (two spins).
- Distribute Labsheet 1.2 and a paper clip to each pair of students.

Explore

- Circulate as pairs collect their data.
- Have pairs answer the problem and follow-up questions.

Summarize

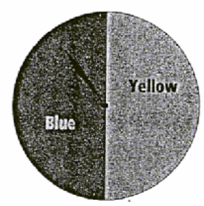
- Discuss the students' findings.
- As a class, combine all the data collected and use the experimental probabilities to make predictions.
- Talk about how to find the theoretical probabilities and how to determine whether the game is fair.

Assignment Choices

ACE questions 1–7 and unassigned choices from earlier problems

1.2 Matching Colors

April and Tioko invented a two-player spinner game called Match/No-Match. A player spins this spinner twice on his or her turn. If both spins land on the same color (a match), Player A scores. If the two spins land on different colors (a no-match), Player B scores. Since there are two matching combinations—blue/blue and yellow/yellow—they decided that Player A should score only 1 point for a match and Player B should score 2 points for a no-match.



1.2

Problem 1.2

Play the Match/No-Match game with a partner. Take a total of 24 turns (12 turns for each player). For each turn, record the color pair on Labsheet 1.2, and award points to the appropriate player.

A. Use the results you collected to find the *experimental probabilities* of a match and a no-match. The experimental probability of a match is

$$P(\text{match}) = \frac{\text{number of turns that are matches}}{\text{total number of turns}}$$

The experimental probability of a no-match is

$$P(\text{no-match}) = \frac{\text{number of turns that are no-matches}}{\text{total number of turns}}$$

B. List all the possible outcomes of a turn (two spins). Write the outcomes as pairs of the form *color on first spin / color on second spin,* such as blue/blue. Use your list to determine the *theoretical probabilities* of a match and a no-match. Since all the outcomes are equally likely, the theoretical probability of a match is

$$P(\text{match}) = \frac{\text{number of outcomes that are matches}}{\text{number of possible outcomes}}$$

The theoretical probability of a no-match is

$$P(\text{no-match}) = \frac{\text{number of outcomes that are no-matches}}{\text{number of possible outcomes}}$$

C. How do your results for parts A and B compare?

D. Is Match/No-Match a fair game? If you think the game is fair, explain why. If you think it is not fair, explain how the rules could be changed to make it fair.

Problem 1.2 Follow-Up

1. Are a match and a no-match equally likely? Explain your reasoning.

2. In 100 turns of the Match/No-Match game, how many times would you expect each of the following to occur?

a. two yellows

b. two blues

c. one yellow and one blue

d. at least one yellow

Answers to Problem 1.2

A. Answers will vary. In one game played by two students, there were 13 matches and 11 no-matches, making P(match) = $\frac{13}{24}$ and P(no-match) = $\frac{11}{24}$.

B. Of the four possible outcomes—blue/blue, blue/yellow, yellow/blue, and yellow/yellow—two are matches and two are no-matches. The theoretical probability of a match is $\frac{2}{4}$ or $\frac{1}{2}$, and the theoretical probability of a no-match is $\frac{2}{4}$ or $\frac{1}{2}$.

C. The results should be close.

D. The game is not fair, as the two events—spinning a match and spinning a no-match—are equally likely but the players do not receive the same number of points for each event. Scoring 1 point for a match and 1 point for a no-match would make it fair.

Answers to Problem 1.2 Follow-Up

1, 2. See page 21i.

APPENDIX I: LESSON 2.1, PLAYING THE ADDITIONA GAME, LESSON PAGES AND TEACHER PAGES

2.1

Playing the Addition Game

At a Glance

Launch

- Introduce the Addition Game, and demonstrate a few rolls of a pair of number cubes.
- Ask the class how they might keep track of their results.
- Ask for conjectures on whether the game is fair.

Explore

- Have pairs play the Addition Game and answer the questions.
- Help pairs who are having trouble finding a systematic way of recording their results.

Summarize

- Ask the class whether the Addition Game is a fair game.
- Have students share how they kept track of their results.
- As a class, review all the possible outcomes of a roll of two number cubes.
- Discuss the follow-up questions.

INVESTIGATION 2

Analyzing Number-Cube Games

In Investigation 1, you used various strategies to find probabilities associated with games of chance. You found *experimental probabilities* by playing a game several times and evaluating the results, and you found *theoretical probabilities* by analyzing the possible outcomes of a game. In this investigation, you will explore experimental and theoretical probabilities involved in some number-cube games.

2.1 Playing the Addition Game

In this problem, you will play the Addition Game with a partner and try to determine whether it is fair.

Addition Game Rules

- Player A and Player B take turns rolling two number cubes.
- If the sum of the numbers rolled is odd, Player A scores 1 point.
- If the sum of the numbers rolled is even, Player B scores 1 point.
- The player with the most points after 36 rolls wins.

Problem 2.1

Play the Addition Game with a partner. Keep track of your results.

A. Based on your data, what is the experimental probability of rolling an odd sum? An even sum?

B. List all the possible pairs of numbers you can roll with two number cubes.

C. What is the theoretical probability of rolling an odd sum? An even sum?

D. Do you think the Addition Game is a fair game? Explain why or why not.

Assignment Choices

ACE questions 1–6, 11–15, 18, 20–23, and unassigned choices from earlier problems

Answers to Problem 2.1

A. Answers will vary. See the Summarize section for one pair's results.

B. All the possible pairs of numbers can be found by making an organized list, a counting tree, or a chart; see the examples in the Summarize section.

C. Both theoretical probabilities are $\frac{18}{36}$, or 50%.

D. The Addition Game is a fair game because each player has the same chances of scoring and receives the same number of points for a score.

■ **Problem 2.1 Follow-Up**

1. Min-wei invented a game based on the sum of two number cubes. In her game, Player A scores 1 point for sums of 6 or 7, and Player B scores 1 point for any other sum. Min-wei thought this would be a fair game because sums of 6 and 7 occur so often. Is this a fair game? Explain why or why not.
2. Royce invented a game based on the sum of two number cubes. In his game, Player A scores 3 points if the sum is a multiple of 3, and Player B scores 1 point if the sum is *not* a multiple of 3. Is Royce's game a fair game? Explain why or why not.

Answers to Problem 2.1 Follow-Up

1. Min-wei's game is not a fair game of chance. The probability of a score by Player A is $\frac{11}{36}$, the probability of a score by Player B is $\frac{25}{36}$, and the players both receive 1 point each time they score.
2. Royce's game is not a fair game of chance. The probability of a score by Player A is $\frac{12}{36}$, and the probability of a score by Player B is $\frac{24}{36}$. This means that of 36 rolls, Player A could expect 12 scoring opportunities, and Player B could expect 24 scoring opportunities—thus Player A would score 36 points and Player B would score only 24 points.

TEACHING THE INVESTIGATION

2.1 • Playing the Addition Game

Launch

In Investigation 1, students made lists of possible outcomes to determine theoretical probabilities. To find the theoretical probabilities of the outcomes in the Addition Game in Problem 2.1 and the Product Game in Problem 2.2, students must find a way to make an organized list of a larger sample space than they have previously encountered. There are 36 possible pairs of numbers that can be rolled with two number cubes.

Introduce the Addition Game, and demonstrate a couple of rolls of two number cubes. If possible, use two different colors of number cubes to help students to see, for example, that 2 and 3 is not the same as 3 and 2. After each roll, ask the class who scores on that roll. If the sum of the two numbers shown is odd, Player A scores 1 point. If the sum is even, Player B scores 1 point.

> How could you and your partner keep track of your results?

Some students may suggest keeping a list of the sums. Others might suggest keeping track of the outcomes of each number cube as well as the sums.

> What are the possible outcomes of adding the numbers shown on a roll of two number cubes? *(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12)*

> Based on this information, the rolls we have taken, and your experience with rolling number cubes, do you think the Addition Game is fair?

Let students offer their conjectures. If some claim that the game is unfair, ask whom the game favors and why they think so. Don't confirm any of their conjectures at this time; you will return to this idea in the summary of the problem. Some students think the game favors the evens because there are six even sums but only five odd sums. Again, this can be explored mathematically in the summary.

Take a few minutes to talk about how to conduct this experiment.

> We want the results of the number-cube rolls to be *random*—that is, we do not want anything about the way we roll the number cubes to bias the results. What sorts of things might bias our results? *(always holding the cubes a certain way, using cubes that are flawed, and so on)*

Distribute two number cubes to each pair of students. Hand out Labsheet 2.1 for pairs to record their data, or let them find their own methods of keeping track of their results.

Explore

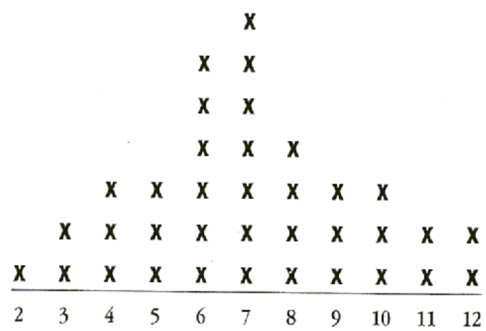
As you watch students play, ask them whether they think the game is fair. Look for interesting ways they have of reasoning about the game and recording their experimental results.

If students are having trouble finding a way to systematically list the outcomes in part B so they can find the theoretical probabilities of rolling even and odd sums, you might suggest that they use a counting tree.

Summarize

Once students have finished answering the questions, bring the class back together. Ask students to share how they kept track of their results.

A line plot is one useful method for collecting such data. You may want to demonstrate this by making a line plot on the board of one pair's data; for example:



From the line plot, we can quickly see that rolling a sum of 6, 7, or 8 is more likely than rolling a sum of 2, 3, 11, or 12. Based on the data shown in this line plot, the probability of rolling an even sum is $\frac{19}{36}$, and the probability of rolling an odd sum is $\frac{17}{36}$. If one were to continue to add data to the line plot, not only would the probability of rolling an odd or an even sum get very close to $\frac{1}{2}$, but the data would yield probabilities for specific sums that were very close to their theoretical probabilities.

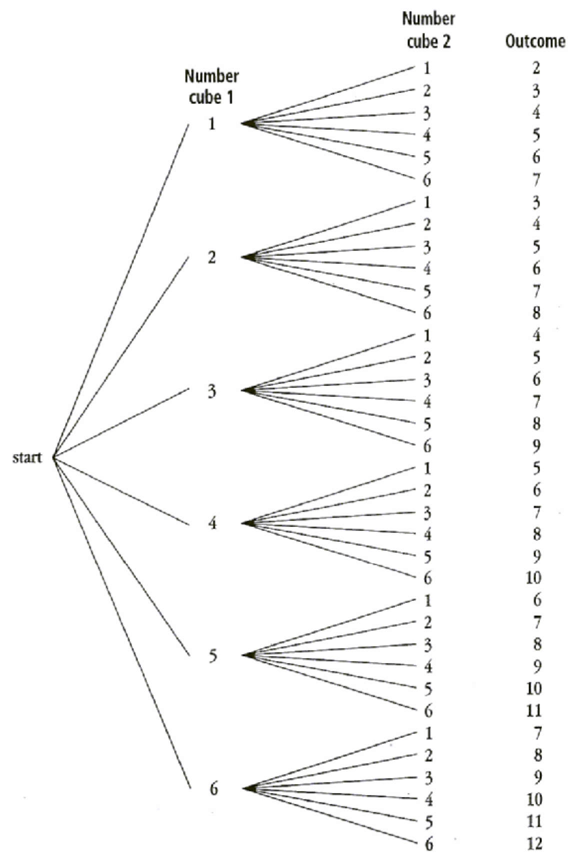
> When you played the game, did you get an equal number of odd and even sums?
>
> Do you think the Addition Game is a fair game? Why or why not?

Most, or all, of the students will say that the Addition Game is a fair game of chance.

Discuss with the class how you might pool, in a uniform way, all the data that the class has generated. Then, collect all the data.

> For our combined data, what is the experimental probability of rolling an even sum? Of rolling an odd sum?
>
> How did you find these probabilities?
>
> Do you think we have enough rolls of the number cubes to feel confident about the accuracy of the experimental probabilities we have found?

Part B of the problem asks students to list all the possible pairs of numbers that can be rolled with two number cubes. To find the theoretical probabilities in part C, students will have to have found a way to list all the possible sums. There are several ways students might have approached this. For example, students might have made counting trees.



Some students might have used an organizing scheme such as the following:

(1, 1) → 2	(2, 1) → 3	(3, 1) → 4	(4, 1) → 5	(5, 1) → 6	(6, 1) → 7
(1, 2) → 3	(2, 2) → 4	(3, 2) → 5	(4, 2) → 6	(5, 2) → 7	(6, 2) → 8
(1, 3) → 4	(2, 3) → 5	(3, 3) → 6	(4, 3) → 7	(5, 3) → 8	(6, 3) → 9
(1, 4) → 5	(2, 4) → 6	(3, 4) → 7	(4, 4) → 8	(5, 4) → 9	(6, 4) → 10
(1, 5) → 6	(2, 5) → 7	(3, 5) → 8	(4, 5) → 9	(5, 5) → 10	(6, 5) → 11
(1, 6) → 7	(2, 6) → 8	(3, 6) → 9	(4, 6) → 10	(5, 6) → 11	(6, 6) → 12

Students who have completed the grade 6 probability unit *How Likely Is It?* might have used a chart like the one below. Each cell in the chart represents the sum of the numbers rolled. For example, in the first row, the roll (1, 1) gives a sum of 2, the roll (1, 2) gives a sum of 3, and the roll (1, 3) gives a sum of 4.

Number cube 1

	+	1	2	3	4	5	6
Number cube 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

If none of your students suggests such a chart as a means of finding all the possible sums, present the idea yourself. Finding the theoretical probabilities from this chart is easy.

Next, help the class use this chart to compare the theoretical probabilities to the class's experimental probabilities. Ask questions to help students understand this analysis.

How many ways are there to get an even sum? *(18)*

How many ways are there to get an odd sum? *(18)*

Does each player have an equal chance of winning this game? Why or why not? *(Yes; each player has an $\frac{18}{36}$, or 50%, chance of winning the game.)*

With this chart displayed, you can ask other questions to help students review concepts about factors and multiples.

What is the probability of getting a sum that is

- a prime number? *($\frac{15}{36}$ or $\frac{5}{12}$)*
- a multiple of 5? *($\frac{7}{36}$)*
- a multiple of 2 and 3? *($\frac{6}{36}$ or $\frac{1}{6}$)*
- a factor of 24? *($\frac{17}{36}$)*
- a multiple of 15? *(0)*

If you roll the number cubes 100 times, how many times could you expect to get a sum that is a factor of 15? *(about 17 times)*

How did you find this amount? *(Of the possible sums, $\frac{6}{36}$, or $\frac{1}{6}$, are factors of 15, and $\frac{1}{6}$ of 100 is $16\frac{2}{3}$, or about 17.)*

This analysis leads naturally into a discussion of the follow-up questions. When students have agreed that the game in follow-up question 2 is unfair, ask for new scoring rules that would change it to a fair game. For example, if the rules gave Player A 2 points each time a multiple of 3 was rolled, the game could be fair because each player could expect 24 points.

APPENDIX J: LESSON 2.2, PLAYING THE MULTIPLICATION GAME, LESSON PAGE

Playing the Multiplication Game

In the Addition Game, players score points based on the sum of the numbers rolled on two number cubes. In the Multiplication Game, scoring depends on the *product* of the numbers rolled.

Multiplication Game Rules

- Player A and Player B take turns rolling two number cubes.
- If the product of the numbers rolled is odd, Player A scores 1 point.
- If the product of the numbers rolled is even, Player B scores 1 point.
- The player with the most points after 36 rolls wins.

Problem 2.2

Play the Multiplication Game with a partner. Keep track of your results.

A. Based on your data, what is the experimental probability of rolling an odd product? An even product?

B. What is the theoretical probability of rolling an odd product? An even product?

C. Do you think the Multiplication Game is fair? Explain why or why not.

D. If the game consisted of 100 rolls instead of 36, how many points would you expect each player to have at the end of the game?

Problem 2.2 Follow-Up

1. How could you make the Multiplication Game a fair game?
2. Invent a fair two-person game based on the product of two number cubes. A player should score 1 point each time he or she scores. You will need to decide which player scores on which kinds of products. Explain why your game is fair.

Answers to Problem 2.2

See page 31g.

Answers to Problem 2.2 Follow-Up

See page 31g

2.2

Playing the Multiplicatio Game

At a Glance

Launch

- Introduce the Multiplication Game, and demonstrate a few rolls of two number cubes.
- Ask the class how they might keep track of their results.
- Ask for conjectures on whether the game is fair and how close the theoretical and experimental probabilities will be.

Explore

- Have pairs play the game and answer the questions.
- Look for students with interesting ways of forming the theoretical probabilities.
- Have pairs move on to the follow-up.

Summarize

- Review the answers to the questions.
- Help the class pool all their data and compare the theoretical and experimental probabilities.
- Discuss the follow-up.

Assignment Choices

ACE questions 7–10, 16, 17, 19, 24, and unassigned choices from earlier problems

Assessment

It is appropriate to use Check-Up 1 after this problem.

INVESTIGATION 1

Evaluating Games of Chance

In this investigation, you will explore several games involving chance. In each situation, you are asked to determine the chance, or *probability*, that certain outcomes will occur. In some situations, you will also be asked to determine whether a particular game is fair. What do you think it means for a game to be fair?

1.1 What's in the Bucket?

One day, Ms. MacAfee brought a mysterious bucket to class. She did not show her students what was in the bucket, but she told them that it contained blue, yellow, and red blocks. She asked if they could predict, without emptying the bucket, the fraction of the blocks that were blue, the fraction that were yellow, and the fraction that were red.

The class conducted an experiment to help them make their predictions. Each student randomly selected a block from the bucket, and the result was recorded on the board. After each draw, the block was returned to the bucket before the next student selected a block. In this problem, your class will conduct a similar experiment.

1.1 What's in the Bucket?

At a Glance

Launch

- Show the class the container, and tell them that it contains different colors of blocks.
- Ask students how they might predict how many blocks of each color are in the container.
- Talk about conducting an experiment to determine the number of blocks of each color.
- As a class, collect data by having each student draw a block, record its color, and return it.

Explore

- Have pairs or small groups work on the problem and follow-up.
- Have the class count the blocks of each color and then answer parts D and E.

Summarize

- Have students share their answers to the problem, and review basic ideas about experimental and theoretical probabilities.
- Talk about the follow-up.

Assignment Choices

ACE question 18

VITA

Bridgette Bond (Almond) Stevens is the daughter of Larry and Sandra Almond and was born on October 21, 1971, in Sioux City, Iowa. She graduated from Independence High School, Independence, Iowa, in 1990. She received a Bachelor of Arts in Elementary Education/Mathematics degree in 1995 and a Master of Arts in Middle Grades Mathematics Education degree from the University of Northern Iowa, Cedar Falls, Iowa. She taught middle grades mathematics for nine years. She received a Doctor of Philosophy degree in Curriculum and Instruction-Mathematics Education from the University of Missouri-Columbia in 2005 and is presently an Assistant Professor in Mathematics Education at the University of Northern Iowa, Cedar Falls, Iowa, where she teachers elementary education mathematics courses and elementary education mathematics methods courses. She resides in Waterloo, Iowa, with her husband, Jeff.