Spatially Explicit and Stochastic Forest Landscape Model of Fire Disturbance and Succession

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Presented by Jian Yang
A candidate for the degree of Doctor of Philosophy

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Spatially Explicit and Stochastic Forest Landscape

Model of Fire Disturbance and Succession

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Under the supervision of Dr. Hong He
At the University of Missouri – Columbia

Abstract

Fire disturbance plays an important role in shaping ecosystem dynamics and vegetation patterns in many forested landscapes. Simulation modeling is an effective tool to study such interactive dynamics over large areas and long time periods. This dissertation is dedicated to the modeling of fire disturbance in spatially explicit and stochastic forest landscape models. I chose LANDIS, a spatially explicit model of forest landscape disturbance, management, and succession as my research model. My research includes both theoretical and technical aspects of modeling fire occurrence patterns and fire spread behavior. For modeling fire occurrence I proposed a hierarchical fire frequency model in which the joint distribution of fire frequency is factorized into a series of conditional distributions. The model is consistent with the framework of statistically based approaches in that a fire occurrence is divided into two stages – fire ignition and fire initiation. The model possesses great flexibility for simulating temporal variation in fire frequency for various forest ecosystems. I implemented an improved fire module in LANDIS and conducted experiments within forest landscapes of northern Wisconsin and southern Missouri. The results demonstrate this new fire module can
simulate a wide range of fire regimes across heterogeneous landscapes with a few model parameters and a moderate amount of input data. For modeling fire spread, I implemented four representative fire spread simulation methods (complete uniform, dynamic percolation, fire-size-based elliptical wave propagation, and duration-based-elliptical wave propagation) in LANDIS. I compared temporal and spatial fire patterns simulated using these four fire spread simulation methods under two fire occurrence process scenarios that are fuel-independent and fuel-dependent. The results showed that although primary characteristics of simulated fire regimes (e.g., fire cycle, distribution of fire frequency, fire size) were similar, spatial pattern of fire occurrence, temporal pattern of fire frequency, and the shape of burned patches were different. Furthermore, I found that the incorporation of fuel into fire occurrence modeling greatly changes fire patterns, suggesting that a mechanistic representation of fire occurrence with fuel and possible other drivers is important in the model building process. Lastly, I used point process modeling approach to study the effects of proximity to road, land cover, topography (slope, aspect, and elevation) on the probability of fire occurrence in the Missouri Ozark Highlands, where more than 90% of reported fires are human-caused. The spatial distribution of fire occurrence density, which is one of the results from point pattern modeling, can be further used in LANDIS as an input map for simulating (human-caused, lightning-caused, or both) fire occurrence.
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Introduction

Research problems

Forest landscape modeling has a deep root in ecological theory and concepts. Ecological theories of succession (Odum, 1969; Pickett et al., 1987), disturbance (Pickett et al., 1989; Clark, 1991; Turner et al., 2003), and equilibrium or non-equilibrium ecological systems (Levin 1992) lay the foundations for landscape modeling (Mladenoff and Baker, 1999). Early applications of Markov transition models in studying vegetation dynamics (Waggoner and Stephens, 1970), vital attribute models (Noble and Slatyer, 1980; Roberts, 1996), distributed and aggregated models (Wu and Levin, 1994), forest growth and yield models (Dale et al., 1985), forest stand and gap models (Botkin et al., 1972; Shugart, 1984), forest planning and decision support models (Potter et al., 1979) provided fertile soil for the development of spatially explicit forest landscape models.

Forest landscape change has been traditionally modeled by simulating changes in sample plots of up to a few hundred square meters, selected within various forest types or along environmental gradients (e.g., Pastor and Post, 1988; Bugmann, 1996). These models are referred to as gap models and they typically do not account for spatial interaction among plots or across a large landscape. In gap models, large-scale contagious processes such as seed dispersal usually have been assumed to be either constant or random. Recent models, such as FIRESUM (Keane et al., 1989), SORTIE (Pacala et al., 1993), and FACET (Urban et al., 1999), have incorporated more spatial interaction than the earlier JABOWA-FORET types of gap models. FACET considers interaction of directly neighboring plots when simulating seed dispersal (Urban et al., 1999); SORTIE tracks individual tree locations and
simulates seed dispersal distances and seedling density defined by each species (Ribbens et al., 1994). But even with state-of-the-art computers, these models are still limited to simulating relatively small sections of landscapes (less than 10 ha) because the computational loads increase exponentially with simulation area (He et al., 1999). To simulate large areas using these models, spatial inexplicit scaling-up (e.g., Acevedo et al., 1995; Keane et al., 1996; Urban et al., 1999) or simplifying the representations of some ecosystem processes, are needed.

One family of computer simulation models that simulate forest change across large heterogeneous landscapes ($10^3$ – $10^7$ ha) over a long temporal scale ($10^1$ – $10^3$ yr) is the spatial explicit and stochastic forest landscape model (SESFLM), in which spatial information is explicit and the randomness in the ecological process is modeled. SESFLMs address how forest change across heterogeneous landscapes driven by tree species succession, competition for environmental constraints, natural disturbances such as fire, windthrow, or biological disturbance; and human management and activities. They offer great potential by providing synergy with experiments and observational research, understanding the dynamics of ecological processes, and evaluating the long-term effects of management activities.

This dissertation is dedicated to the modeling of fire disturbance in SESFLM. Fire disturbance is one of the most important processes driving landscape patterns and forest succession (Turner and Romme, 1994). A large variety of models have been developed for simulating landscape-scale patterns of fire effects, here I chose LANDIS, a spatially explicit model of forest landscape disturbance, management, and succession (Mladenoff et al., 1996; Mladenoff and He, 1999, Gustafson et al. 2000) as my research model. My research included both theoretical and technical aspects on how to improve LANDIS for a better understanding
fire disturbance in driving forest compositional change and feedbacks in ecosystem processes. The purpose of my research is to advance modeling approaches for simulating fire occurrence and fire spread, to implement the proposed approaches in LANDIS, and to explore the potential of the improved LANDIS fire module in (1) simulating various fire regimes on heterogeneous landscapes, (2) examining the influence of the fuel configuration, topography and wind on the spatial pattern of fires, and (3) calibrating the parameters against the observed patterns of fires.

**Chapter outline**

There are three chapters in my dissertation. Chapter one develops a hierarchical fire frequency model to substitute for current widely used Exponential and Weibull models in simulating fire occurrence. Chapter two systematically compares a full spectrum of fire spread simulation (i.e., purely statistical, statistical-probabilistic, and statistical-physical) methods on simulated spatial and temporal forest fire patterns. Chapter three applies statistical modeling (specifically, spatial point process modeling) to estimate the intensity of fire occurrence (intensity here is a term used in spatial statistics, it can be loosely interpreted as density), from which we can further derive a fire occurrence probability map serving as an input for many SESFLMs (including LANDIS) in simulating fire occurrence.

**Chapter 1** uses the theory of hierarchical modeling and mixture distributions to account for a commonly used strategy in fire occurrence modeling: to divide a fire occurrence into two consecutive events – fire ignition and fire initiation. A fire occurrence begins with an ignition from an external heat source, which heats the forest fuel complex up
to its ignition temperature. Fire ignition agents are either natural (e.g., lightning) or anthropogenic (e.g., arson or accidental). A fire initiation event starts with the ignition until a certain area whose size is equal to the grain of the model is burned (Li, 2000). However, the Exponential and Weibull fire interval distributions, which model a fire occurrence as a single event, are often inappropriately applied to these two-stage models for calculating fire probability. The hierarchical fire frequency model that I proposed is consistent with this two-stage modeling. The Exponential and Weibull models are actually special cases of my hierarchical model. I implemented this approach as an improved fire module in LANDIS and conducted experiments within forest landscapes of northern Wisconsin and southern Missouri. The results demonstrate this new fire module can simulate a wide range of fire regimes across heterogeneous landscapes with a few model parameters and a moderate amount of input data. The hierarchical fire frequency model possesses great flexibility for simulating temporal variation in fire frequency for various forest ecosystems and can serve as a theoretical framework for future statistical modeling of fire regimes.

**Chapter 2** compares temporal and spatial fire patterns simulated using the four representative fire spread simulation methods (complete uniform, dynamic percolation, fire size based elliptical wave propagation, and duration based elliptical wave propagation) under two fire occurrence process scenarios that are fuel-independent and fuel-dependent. Specifically, I examined how statistical characteristics of fire frequency and fire size distribution, temporal structure (trend, autocorrelation) of fire frequency series, point pattern of fire occurrences, and shapes and spatial configuration of burned patches varied by those forest fire simulation methods. The results demonstrated that although primary characteristics of fire regimes (e.g., fire cycle, distribution of fire frequency, fire size) were similar, spatial
pattern of fire occurrence, temporal pattern of fire frequency, and the shape of burned patches were different. Furthermore, we found that the incorporation of fuel into fire occurrence modeling greatly changed fire patterns, suggesting that a mechanistic representation of fire occurrence with fuel and possible other drivers is important in the model building process.

Chapter 3 uses the point process modeling approach to study the effects of proximity to road, land cover, and topography (slope, aspect, and elevation) on the probability of fire occurrence in the Missouri Ozark Highlands, where more than 90% of fires are human-caused. Our results show that fire locations are spatially clustered, and high fire occurrence probability is found in areas that are close to roads (<500 meters away from the nearest road) and in lower (<25 degree) slopes, where forest are more accessible to human. In addition, fire occurrence probability is higher in pine-oak forests and woodlands and is lower at low (<900 ft) elevations, which reflects the effects of natural factors on fire occurrence process. The spatial distribution of fire occurrence density, which is one of the results from this study, can be further used in LANDIS and other SESFLMs as an input map for simulating (human-caused, lightning-caused, or both) fire occurrences.
References


Chapter 1. A Hierarchical Fire Frequency Model to Simulate Temporal Patterns of Fire Regimes in LANDIS

Abstract

Fire disturbance has important ecological effects in many forest landscapes. Existing statistically based approaches can be used to examine the effects of a fire regime on forest landscape dynamics. Most examples of statistically based fire models divide a fire occurrence into two stages -- fire ignition and fire initiation. However, the exponential and Weibull fire interval distributions, which model a fire occurrence as a single event, are often inappropriately applied to these two-stage models. We propose a hierarchical fire frequency model in which the joint distribution of fire frequency is factorized into a series of conditional distributions. The model is consistent with the framework of statistically based approaches because it accounts for the separation of fire ignition from fire occurrence. The exponential and Weibull models are actually special cases of our hierarchical model. In addition, more complicated non-stationary temporal patterns of fire occurrence also can be simulated with the same approach. We implemented this approach as an improved fire module in LANDIS and conducted experiments within forest landscapes of northern Wisconsin and southern Missouri. The results of our experiments demonstrate this new fire module can simulate a wide range of fire regimes across heterogeneous landscapes with a few parameters and a moderate amount of input data. The model possesses great flexibility for simulating temporal variations in fire frequency for various forest ecosystems and can serve as a theoretical framework for future statistical modeling of fire regimes.
1. Introduction

Disturbances such as fire are often key factors driving the dynamics of forested landscapes in many ecosystems such as boreal forests (Heinselman, 1970), coastal western chaparral shrublands (Keeley et al., 1999; Moritz, 2003), and the northern and central hardwood forests (Bormann and Likens, 1979; Guyette and Larsen, 2000). Small and large fires of varying intensity strongly affect species composition and age distribution (Hough and Forbes, 1943). Fire creates a mosaic of burned and unburned forest patches, leaving complex heterogeneous patterns across the landscape. The resulting landscape heterogeneity can further influence successional processes, which in turn may affect the spatial spread of subsequent fires (Turner and Romme, 1994). In this light, modeling the temporal and spatial pattern of fire disturbance, as well as the interaction between fire disturbance and landscape heterogeneity, is very important for understanding forest landscape dynamics.

A large variety of fire models have been developed to examine the effects of fire regimes (e.g., fire frequency, severity, and extent of disturbances) on the recovery of disturbed landscapes. Because different models have varying purposes, applicable spatial and temporal extents, and levels of ecological detail, they use different approaches to simulate fire occurrence, behavior and effects. Detailed reviews of the approaches can be found in Albright and Meisner (1999), and Gardner et al. (1999). Here we discuss only one family of these approaches, called statistically based approaches, that are often applied to forest landscape simulations over large spatial and temporal domains. Statistically based approaches use the distribution of fire frequency and fire size, and estimates of the renewal rate of disturbed forests from fire history studies (e.g., Heinselman, 1973) to simulate a given fire regime. The approaches have evolved from the theory of Weibull and exponential fire
history models (Van Wagner, 1978; Johnson and Van Wagner, 1985) and are used in many models such as DISPATCH (Baker et al., 1991), the model of Antonovski et al. (1992), REFires (Davis and Burrows, 1994), FLAP-X (Boychuk and Perera, 1997), ON-FIRE (Li et al., 1997), LANDIS (He and Mladenoff, 1999), and LADS (Wimberly et al., 2000). All these spatial models simulate fire ignition from certain probability distributions, use fire probability to determine whether an ignition can become an active fire, and randomly generate a pre-defined fire size to simulate the extent of a given fire occurrence. The distributions of fire frequency and fire size are used to estimate fire probability and the predefined fire size for the simulation of fire occurrence and spread, respectively. The main differences among these models are (1) in the distributions used to model fire frequency and fire size, and (2) in the way fire probability is calculated. For example, the fire frequency distribution in DISPATCH is uniform while the distribution in LADS is Poisson, and the fire size distribution in LANDIS is lognormal while the distribution in LADS is exponential.

The term fire occurrence here refers to a detected active fire that happens when the fire begins to spread through the forest fuel complex as a surface fire or a crown fire and emits significant amounts of smoke and energy (Anderson et al., 2000). Some modelers use the term fire ignition to refer to fire occurrence, and use other terms such as potential fire ignition (Davis and Burrows, 1994) or fire source (Antonovski et al., 1992) to refer to fire ignition as used here. No matter what terms are employed in the models, the essence of implementing a statistically based approach is to divide a fire occurrence into two consecutive events -- fire ignition and fire initiation (Li, 2000). Separating fire ignition from fire occurrence helps to separate the factors influencing fire ignition such as climate, topography, and human activities from the influences on fire spread of fuel accumulation and vegetation structure.
Although current statistically based fire regime models have been successfully applied in various studies, they typically are not flexible enough to simulate the full range of fire regimes observed in forested ecosystems. This is mainly because the modeling of fire ignition and calculation of fire probability in these models cannot fully account for the fire frequency distribution of various fire regimes. Fire frequency distributions in these models are deduced from the exponential or Weibull model in fire history studies, in which a fire occurrence is treated as one single event rather than two events in fire regime models. It is, therefore, conceptually inaccurate to apply the exponential or Weibull model directly into fire regime modeling. Moreover, in statistically based fire regime models, including the current version of LANDIS (version 3.7), there is a common mistaken practice of using the fire hazard function provided by the exponential or Weibull model as the fire probability function. Fire hazard is defined as the instantaneous rate of burning (Johnson and Van Wagner, 1985). In discrete time, fire hazard, denoted as $h(t)$, is the probability of fire in year $t$ given that a fire has not yet occurred (Clark, 1989). It represents a combination of the rate of ignition and the probability of the fire spreading given the presence of ignition sources (McCarthy et al., 2001). On the other hand, fire probability in fire regime models is the probability of a fire occurrence given the presence of an ignition. It is determined primarily by the process of fuel build-up, which is often assumed to be a function of time since last fire. Thus, fire probability is different from fire hazard. However, many fire regime models assume that fire hazard equals fire probability. This often causes problems in model parameterization because the discrepancy between fire hazard and fire probability is not properly accounted for in the calculation of fire probability in these models.
In this study, we use the theory of hierarchical modeling and mixture distributions to model fire frequency. Hierarchical modeling in statistics refers to modeling a complicated process by a sequence of relatively simple models placed in a hierarchy (Casella and Berger, 2001). It is based on the fact that the joint distribution of a collection of random variables can be decomposed into a series of conditional models. That is, if $A$, $B$, and $C$ are random variables, we can write a factorization such as $[A, B, C] = [A | B, C][B | C][C]$. The notation $[A]$ denotes the probability distribution of $A$; $[A | B]$ represents the conditional distribution of $A$ given the random variable $B$. Random variable $A$ has a mixture distribution, because the distribution of $A$ depends on a quantity $B$ that also has a distribution. Because statistically based fire regime models simulate fire occurrence as two consecutive stages, it is natural to use the theory of hierarchical modeling to model fire frequency distribution as a mixture distribution.

The objectives of this study are: (1) to design a hierarchical fire frequency model that accounts for the separation of fire ignition from fire occurrence; (2) to implement the hierarchical model as an improved fire module in LANDIS; and (3) to explore the potential of the improved module to simulate a wide range of temporal patterns of various fire regimes on heterogeneous landscapes.

2. Fire frequency models

2.1 Terms describing temporal patterns of fire regimes

The combination of certain aspects of wildland fire disturbance, especially fire
frequency, size, severity, and seasonality, may be used to characterize a fire regime. Here we clarify the terms that will be used in fire frequency models, that include fire frequency, fire interval, fire cycle, and mean fire size. Fire frequency is the number of fires per unit time in a specific area (Agee, 1993). The size of the specific area will affect fire frequency: larger areas will have a higher fire frequency (Johnson and Van Wagner, 1985). The reciprocal of fire frequency is fire interval, which is the elapsed time between two successive fires in a specific place (McPherson et al., 1990). Fire interval often is modeled using a Weibull distribution or an exponential distribution, a special case of the Weibull distribution where the fire hazard is held constant (Johnson and Van Wagner, 1985). Fire cycle is the number of years necessary for an area equal to the entire area of interest to burn (Johnson and Van Wagner, 1985; Turner and Romme, 1994). This definition does not imply that the entire area would burn during a cycle; some sites may burn several times, while others do not burn at all. Fire cycle also is referred to as fire rotation (Agee, 1993). The distribution of fire size is usually difficult to estimate. However, mean fire size also is a common descriptor in the study of fire regimes, and the relationship among the size of study area (AREA), mean fire size (MFS), mean fire frequency (MFF), and fire cycle (FC) is depicted in the following equation (Boychuk et al., 1997).

\[ \text{AREA} = \text{MFS} \times \text{MFF} \times \text{FC} \quad (1) \]

### 2.2 Exponential model

If fire hazard is constant, then fire interval has an exponential distribution, and fire frequency is distributed as a Poisson process (Van Wagner 1978). Following statistical conventions, random variables are denoted by uppercase letters and their observed numerical values are denoted by lowercase letters. The probability density function (PDF) of a random
variable, such as $X$, is denoted as $f_X(x)$, and its cumulative density function (CDF) is denoted as $F_X(x)$. The relational symbol ~ means “is distributed as”. We use $U$ to denote the number of fire occurrences per unit time in the study area, and $T$ to denote the time since last fire. $U$ follows Poisson distribution with parameter $\alpha$. $T$ then has an exponential distribution with parameter $\beta$, which is the inverse of $\alpha$ (Appendix 1). The probability density function of $U$ and $T$ are equation 2 and equation 3, respectively.

$$f_U(u) = \frac{e^{-\alpha u}}{u!} \quad (2)$$

$$f_T(t) = \frac{1}{\beta} e^{-t/\beta} \quad (3)$$

MFF is the expected value of $U$ that equals $\alpha$, and mean fire interval (MFI) is the expected value of $T$ that equals $\beta$. Fire hazard is independent of time since last fire and it equals $1/\beta$, or $\alpha$.

2.3 Weibull model

Fire interval is widely modeled as a Weibull distribution because it permits fire hazard to increase or decrease with time since last fire (Johnson and Van Wagner, 1985; Clark, 1989; Johnson and Gutsell, 1994; McCarthy et al., 2001). The probability density function of time since last fire with parameters $\beta$ and $\gamma$ is

$$f_T(t) = \frac{\gamma}{\beta} t^{\gamma-1} e^{-t/\beta} \quad (4)$$

The fire hazard function for the Weibull distribution is
\[ h(t) = (\gamma / \beta)t^{\gamma-1} \quad (5) \]

When \( \gamma \) equals 1, fire hazard is then a constant \( 1/\beta \), and equation 4 becomes the same as equation 3. Hence the exponential model is a special form of Weibull model.

There is no explicit distributional form for fire frequency when fire interval is modeled as a Weibull distribution. Instead, a fairly complicated renewal function, denoted as \( R(t) \), is used to give the expected number of fire occurrences during time \((0,t)\) (Clark 1989).

\[ R(t) = F(t) + \int_{0}^{t} R(t-x)dF(x) \quad (6) \]

2.4 Hierarchical fire frequency model

Unlike the previous two fire interval models, our hierarchical fire frequency model divides a fire occurrence into two consecutive events -- fire ignition and fire initiation. A fire occurrence begins with an ignition from an external heat source that heats the forest fuel complex up to its ignition temperature. Fire ignition agents are either natural (lightning) or anthropogenic (e.g., arson or accidental). A fire initiation event starts with the ignition until a certain area whose size is equal to the grain of the model is burned (Li, 2000). Whether a fire ignition can result in fire initiation is dependent on the fuel loading, fuel arrangement, and fuel moisture content.

Let \( X \) denote the number of fire ignitions per unit time in a specific area. \( X \) is a discrete random variable and follows a Poisson distribution with the parameter intensity \( \lambda \), that is the expected number of ignitions per unit time (Cunningham and Martell, 1973; Van Wagner, 1978; Anderson et al., 2000; Pennanen and Kuuluvainen, 2002). The fire initiation
process can be modeled as a Bernoulli trial. Given \( x \) ignitions during the unit time, we assume there exist \( x \) random variables \( Y_i \) (\( i = 1, 2, ..., x \)) taking a value 1 if a fire ignition results in fire initiation and 0 otherwise. Each \( Y_i \) has a Bernoulli distribution with the parameter fire probability \( (P_i) \). The sum of these Bernoulli trials (i.e., fire occurrence per unit time) is fire frequency \( (U) \). The conditional distribution of \( U \mid X \) (conditional distribution of fire occurrence given the fire ignitions) primarily is determined by how we define the fire probability function. If we assume fire probability \( (P) \) is independent of time since last fire, and constant across the fire regime, then \( U \mid X \) follows a binomial distribution with the parameters \( X \) and \( P \), and \( U \) follows a Poisson distribution with the parameter \( \lambda P \) (Appendix 2). In this case, the fire frequency distribution is identical to the one in the exponential model, where fire hazard \( (\alpha) \) equals the product of fire ignition intensity \( (\lambda) \) and fire probability \( (P) \). Hence our hierarchical fire frequency model is consistent with previous fire frequency models, except that fire hazard is more accurately modeled as the combination of ignition rate and fire probability.

Fire probability also can be modeled as a function of time since last fire. Here the form of the fire probability function is determined primarily by the fuel accumulation within the ecosystem that can be estimated from data on the rate of fuel accumulation and occurrences of fire. As long as fire probability is not a constant, fire occurrence is a complex inhomogeneous Poisson process, whose distribution is often difficult to explicitly formulate in a single equation. However, this fairly complex process can be factorized into much simpler probabilistic distributions as shown in equations 7-10.

\[
[U] = [U \mid X][X \mid \lambda] \quad (7)
\]
\[ [X | \lambda] \sim \text{Poisson}(\lambda) \quad (8) \]
\[ U | X = \sum_{i=1}^{X} Y_i \quad (9) \]
\[ [Y_i | P_i] \sim \text{Bernoulli}(P_i) \quad (10) \]

3. **Fire module in LANDIS**

3.1 **LANDIS overview**

LANDIS is a spatially explicit and stochastic raster-based model that simulates forest landscape change over long time domains (10^1 - 10^3 yr) and large heterogeneous landscapes (10^3 – 10^7 ha). The model currently operates on 10-year time step. It is designed to model ecological dynamics and interactions of temporal processes such as succession, and spatial processes such as seed dispersal, disturbances, and forest management (Mladenoff et al., 1996; Mladenoff and He, 1999, Gustafson et al. 2000). In LANDIS, a large landscape is stratified into several small relatively homogeneous fire regime units such as ecoregions or land types where the meteorological, physical, and biological properties as well as ecological factors are more uniform. LANDIS simulates fire regime units based on their fire cycles and statistics of fire sizes that are specified by the users. Further details about the simulation of fire regime and its interactions with succession can be found in He and Mladenoff (1999).

3.2 **Fire module design**

We incorporated the hierarchical fire frequency model into LANDIS by dividing the fire process into three stages: fire ignition, fire initiation, and fire spread. For a given time step
(i.e., 10 years), LANDIS first generates the number of ignitions \( X \) in the given fire regime unit from the Poisson distribution with the parameter \( \lambda \) (i.e., average fire ignitions per decade). For each ignition, LANDIS performs a Bernoulli trial, whose result is denoted by \( Y \), with the parameter fire probability \( P \), whose value is determined by the time since last fire of the ignited cell. If the ignition becomes an initiation, LANDIS will select a pre-defined fire size, denoted by \( Z \), from a lognormal distribution with parameters \( \mu \) and \( \sigma^2 \) to simulate fire spread (Figure 1).

LANDIS uses a percolation algorithm similar to the algorithms of Gardner et al. (1987), Clarke et al. (1994), Hargrove et al. (2000), and Wimberly et al. (2000) to simulate fire spread. Fires simulated by the percolation algorithms spread from a burning cell to forested cells in the cardinal directions (up, down, left and right). These cells are entered into a priority queue in a random order. The first cell in the queue has a higher priority of fire spread. The fire will continue to spread until it reaches its pre-determined size. LANDIS does not allow a forested site to be burned more than once within one time step, and non-active land types or ecoregions (e.g., roads, lakes) may serve as fuel breaks in the landscape. Therefore, it is possible for a fire to be extinguished prior to burning its pre-determined size if the fire reaches fuel breaks or newly burned sites. In a real landscape, fires may spread across the boundaries of fire regime units where the fire size distribution changes. When a fire spreads into a different fire regime unit, the module will simulate a new ignition. If the new ignition results in an active fire, a new pre-determined fire size will be selected based on the fire size distribution for the new fire regime unit.

For each fire regime unit, LANDIS needs to know its size of area, fire cycle, mean fire size (MFS), and the standard deviation of fire size (DFS). Mean fire frequency can be
calculated using equation 1. Fire size follows a lognormal distribution that is negatively skewed, consisting of many small fires and some rare large fires (He and Mladenoff, 1999; Wimberly, 2002). The parameters $\mu$ and $\sigma^2$ can be derived from the MFS and DFS:

$$
\mu = 2 \log MFS - \frac{1}{2} \log (DFS^2 + MFS^2) \quad (11)
$$

$$
\sigma^2 = \log (DFS^2 + MFS^2) - 2 \log MFS \quad (12)
$$

The fire probability function is essential to the distribution of fire frequency and hence very important in determining the realism of fires simulated by LANDIS. Different forest ecosystems may have different fire probability functions because of fuel accumulation. Our model uses Olson’s approach (Olson, 1963; McCarthy et al., 2001), in that fuel accumulation is assumed as a constant rate of litter input and constant decomposition of a proportion of the litter. Assuming fire probability is proportional to fuel load, the change in fire probability is given in the following equation, where $t$ is the time since last fire, and FC is the fire cycle.

$$
P(t) = 1 - e^{-t/FC} \quad (13)
$$

4. Experimental design and analysis

4.1 Case study landscapes

To demonstrate the capability of the LANDIS fire module to simulate multiple fire regimes on heterogeneous landscapes at various spatial resolutions, we applied the module to two landscapes with distinct fire regimes. The first landscape is characterized by 2 forested ecoregions and 23 species of trees, and is located in northern Wisconsin, USA (Figure 2).
The area comprises more than 700,000 ha, of which the pine barren ecoregion is about 90,000 ha and the lakeshore ecoregion is about 150,000 ha. Non-forested ecoregions (e.g., agriculture lands, lakes), shown as background in the map, are treated as fuel breaks in the simulation. Ecoregion boundaries are derived from an existing quantitative ecosystem classification (Host et al., 1996). This area is a largely forested, glacial region, with little topographic relief. Dominant tree species include sugar maple (*Acer saccharum*), northern red oak (*Quercus rubra*), eastern hemlock (*Tsuga canadensis*), yellow birch (*Betula alleghaniensis*), paper birch (*B. papyrifera*), quaking aspen (*Populus tremuloides*), white pine (*Pinus strobus*), red pine (*P. resinosa*), and jack pine (*P. banksiana*) (Curtis, 1959).

The second landscape for our case study has 8 land types and 4 dominant species, and is located in southern Missouri, USA. The study area is in the Ozark Highlands Section (Kabrick et al., 2000), approximately 130,000 ha. The area is largely forested. White oak (*Quercus alba*), post oak (*Quercus stellata*), black oak (*Quercus velutina*) and shortleaf pine (*Pinus echinata*) are the dominant tree species. Forest age structure is relatively simple due to historical harvesting practices. Topographic variation is high, with elevations ranging from 140-410 m, and many slopes are greater than 30 degrees. There are eight land types: southwest slopes, northeast slopes, ridges or flat uplands, upland drainages, mesic coves or bottoms, sites with limestone, savannas, and glades (Figure 3). Private land is the collection of the sites where there is no national forest; we include it in our study to allow fires to spread across the entire forested landscape.

Historical fire regime statistics (FC, MFS, DFS) for the fire regime units are interpreted from empirical studies in the Wisconsin and the Missouri regions (Heinselman, 1973, 1981; Cleland et al., 1997; He and Mladenoff, 1999; Gustafson et al., 2000; Shifley et al., 2000;
Mean fire frequency (MFF) is calculated from equation 2. Fires in the lakeshore ecoregion are infrequent and fire sizes tend to be very large, whereas fires in the barrens ecoregion are relatively frequent and fires tend to be smaller. Fires in the land types of the Ozark region are frequent and fire sizes are small.

### 4.2 Parameterization and simulation of fire disturbance

According to the dendrochronological study and other historic records of fires of the two tested landscapes (He and Mladenoff, 1999; Guyette and Larsen, 2000), initial time-since-last-fire for the barrens and lakeshore ecoregions in northern Wisconsin landscape was set to 50 and 200 years, respectively; initial time-since-last-fire for the land types in the Missouri landscape was set to 50 years, except that it was set to 10 years for savanna. The ignition rate for each fire regime unit was held constant through simulation time with the assumption that there were no changes of climate and human dimensions. The input of mean fire size was larger than the expected mean fire size due to landscape fragmentation and configuration. Simulation runs of northern Wisconsin were carried out on a 328 x 535 grid of 200 x 200 m cells for 400 years. Simulation runs of Missouri Ozark highlands were carried out on an 1185 x 1207 grid of 30 x 30 m cells, for 300 years using the calibrated parameters (Table 2). The reasons for choosing different resolutions of analysis of the two study areas were that: (1) the ecoregions in northern Wisconsin are fairly contiguous, which reduces the need for analyzing fire spread at a finer resolution; and (2) an important purpose of the excise was to demonstrate that the new fire module in LANDIS can simulate forest landscape dynamics at various spatial resolutions.
5. Results

5.1 Frequency and extent of fire disturbance

Simulation results show distinct differences between the Wisconsin and the Missouri fire regime units in terms of mean fire frequency and mean fire size, as expected (Table 3). The module also captures subtle differences of fire pattern at the finer land type scale. For instance, in Missouri, simulated mean fire size on southwest slopes is slightly larger than on northeast slopes, whereas simulated mean fire frequency is less than on northeast slopes. After calibration, the percentage absolute errors of simulated MFS and MFF on the fire regime units are very small (less than 10%), and simulated fire cycles on the fire regime units are close to the expected fire cycles (Table 3). Meanwhile, the temporal dynamics of fire frequency exhibit high variability, reflecting the non-stationary behavior of fire occurrences (Figures 4 and 5). Although fire ignition is simulated as a stationary process in our model (i.e., ignition rate is held constant during the simulation time), the simulated fire occurrences on the tested landscapes are still non-stationary because fire probability in our model increases exponentially with time-since-last-fire rather than being held constant.

5.2 Fire spread behavior over heterogeneous landscape

Simulated mean fire size for the lakeshore ecoregion seems to have a negative temporal autocorrelation (Figure 6), which suggests that if few fires occur for a relatively long period, then there will be more suitable, contiguous fuel left in the lakeshore ecoregion,
resulting in large, severe fires during the subsequent period. A similar pattern also is found in
the simulation results for the relatively contiguous land types (e.g., Flat) in southern Missouri
(Figure 7). This demonstrates that our module can simulate the correlations between fire
frequency and fire size to some extent by having fires spread across the patches with
different time since last fire, thus different fire probabilities within a fire regime unit.

Another scale of landscape heterogeneity also affects fire spread -- the spatial
configuration of patches of different fire regime units. In our simulations of northern
Wisconsin fires, fires initiated in the barrens ecoregion often stop in the lakeshore ecoregion
or at the boundary of non-forest ecoregions, which serve as fuel breaks. Fire occurrences in
the lakeshore ecoregion are usually caused by fire spreading from the barrens ecoregion.
Similar fire spreading patterns occur in the southern Missouri highlands, where land types are
highly dispersed. For example, only 35% of fires reach their pre-determined size completely
within the southwest slope land type in the simulation; the other 65% of the fires spread into
other land types, especially the northeast slope land type.

6. Discussion

6.1 Implications of the hierarchical fire frequency model

The hierarchical fire frequency model presented here depicts fire frequency as a mixture
distribution where parameters also follow relatively simple distributions. Although our model
does not have an explicit fire frequency distribution or fire interval distribution as do the
exponential and Weibull models, it is more flexible in that it can represent a wider range of
fire regimes than these other models. The exponential and Weibull models are stationary in
the sense that the parameters are fixed and the sampling occurs from a single fixed
probability distribution assumed static (Polakow and Dunne, 1999). On the other hand, our hierarchical model incorporates variability about the parameter estimates; hence it can be used to replicate more complicated, non-stationary temporal patterns of fire occurrence. Moreover, the hierarchical model conceptually is consistent with statistically based fire regime models, because it separates fire ignition from fire occurrence as most of these models do. Therefore, ours is a better model for application in statistically based modeling of fire regimes than widely used exponential or Weibull fire interval models.

6.2 Improvements in LANDIS fire simulation using the hierarchical fire frequency model

The fire module implemented in LANDIS (version 4.0) is an application of our hierarchical fire frequency model. The results of our experiments demonstrate this new fire module can simulate multiple fire regimes across heterogeneous landscapes with a few parameters and a moderate amount of input data. The simulated fire cycle, fire frequency distribution, and fire size distribution are consistent with historical data on fire occurrences. Compared to previous fire simulations using LANDIS 3.x, which applies the exponential model to simulate fire occurrences, four major advances have been achieved: 1) In earlier versions of LANDIS, the fire algorithm fails to simulate fire regimes characterized by many small fires, and some applications of LANDIS have had to artificially modify the fire algorithm to circumvent such limitations (e.g., Sturtevant et al., 2004). As our results demonstrate, the new fire module can simulate a much wider range of fire regimes. 2) Earlier versions of LANDIS assume fire hazard is constant across the fire regime throughout the entire simulation period. Results attained from these versions simulated stationary temporal
patterns of fire disturbance (He and Mladenoff, 1999). However, even fire ecologists who utilize the exponential model assess fire frequency in terms of temporally distinct epochs and recognize the variability in the parameters for fire frequency distribution (Johnson and Gutsell, 1994; Reed et al, 1998). The new LANDIS fire module simulates fire probability as increasing with fire interval, and hence can simulate non-stationarity of temporal patterns of fire disturbance. 3) Our new fire module is able to simulate subtle differences among multiple fire regime units within a large landscape, which are often hard to simulate in earlier versions of LANDIS due to the difficulty in parameterization. For instance, simulated mean fire size on southwest slopes is slightly larger than on northeast slopes; this is consistent with the fact that prevailing winds in the Ozark highlands are southerly and strongest in the spring season (Kabrick et al., 2000). 4) The new fire module captures more realistic fire-spread patterns than do previous approaches that use only one distribution of fire size for the entire landscape. From the simulation results, we observe that if few fires occurred in some decades, then the subsequent fires tend to be larger and more intense (unpublished data), and that fires often extinguish near the boundaries of less flammable fire regime units. This is consistent with empirical observations by Bergeron (1991) on the influence of island and lakeshore landscapes on boreal fire regimes.

6.3 Further research needs

The new LANDIS fire module assumes ignition density is uniform within a fire regime unit without explicitly considering the effects of human population, site, topography, and vegetation. A generalized linear mixed model (GLMM) can be used to describe temporal and
spatial distributions of ignition density (Diaz-Avalos et al. 2001). The module also assumes
fire probability increases exponentially with the time-since-last-fire. This assumption may
not be valid for forest ecosystems different from those selected (McCarthy et al., 2001). A
model of fire probability with respect to fuel loading and weather conditions can be
incorporated into the module. Such future work will allow us to model fire regimes more
dynamically with fewer predetermined fire regime statistics required as inputs to the module;
simulated fire cycle and fire size will be emergent properties of the simulations rather than
predetermined inputs.
7. Appendix

7.1 Relation between Poisson process and exponential process

We use $U$ to denote the number of fire occurrence per year, and $T$ to denote the time since last fire. If $U$ follows Poisson distribution with parameter $\alpha$, then $T$ has an exponential distribution with $\beta = 1/\alpha$.

Proof:

The cumulative distribution function for $T$ is given by

$$F_T(t) = \Pr[T \leq t] = 1 - \Pr[T > t]$$

$\Pr[T > t]$ is the probability of having the first fire occurring after time $t$, which means no fire occurrences in the time interval $[0, t]$. Let $W$ denote the number of fire occurrences in this time interval. $W$ is a Poisson process with parameter $\alpha t$. Thus

$$\Pr[T > t] = \Pr[W = 0] = \frac{e^{-\alpha t} (\alpha t)^0}{0!} = e^{-\alpha t}$$

By substitution we obtain

$$F_T(t) = 1 - e^{-\alpha t}$$

Because the PDF is the derivative of CDF for a continuous random variable

$$f_T(t) = F_T'(t) = \alpha e^{-\alpha t} = \frac{1}{\beta} e^{-t/\beta}$$

This is the PDF for an exponential random variable with $\beta = 1/\alpha$. 

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7.2 A special case of hierarchical fire frequency model

When both ignition rate and fire probability are constant, fire ignition is a Poisson process with \( \lambda \), conditional distribution of fire occurrence given fire ignition is binomial with \( X \) and \( P \), i.e.,

\[
[X \mid \lambda] \sim \text{Poisson}(\lambda)
\]

\[
U \mid X \sim \text{Binomial}(X, P)
\]

Then the mixture distribution of fire occurrence becomes a Poisson distribution with \( \lambda P \).

Proof:

The PDF of discrete random variable \( U \) (fire frequency) is

\[
f_U(u) = \Pr[U = u] = \sum_{x=0}^{\infty} \Pr[U = u, X = x] \quad \text{(definition of marginal distribution)}
\]

\[
= \sum_{x=0}^{\infty} \Pr[U = u \mid X = x] \Pr[X = x] \quad \text{(definition of conditional distribution)}
\]

\[
= \sum_{x=u}^{\infty} \left[ \binom{x}{u} p^u (1-p)^{x-u} \frac{e^{-\lambda \lambda x}}{x!} \right], \quad \text{(conditional probability is 0 if } x < u)\]

where we substitute PDFs of binomial and Poisson distribution into the expression. If we now simplify this expression, we get

\[
f_U(u) = \Pr[U = u] = \frac{(\lambda p)^u e^{-\lambda \lambda}}{u!} \sum_{x=u}^{\infty} \frac{((1-p)\lambda)^{x-u}}{(x-u)!}
\]

\[
= \frac{(\lambda p)^u e^{-\lambda \lambda}}{u!} \sum_{t=0}^{\infty} \frac{((1-p)\lambda)^t}{t!} \quad \text{(change of variable)}
\]
\[
\frac{(\lambda p)^u}{u!} e^{-\lambda} e^{(1-p)i} \quad (\text{Maclaurin series for exponential function})
\]

\[
= \frac{(\lambda p)^u}{u!} e^{-\lambda p} \quad (\text{a kernel for a Poisson distribution}), \text{ therefore}
\]

\[
[U] \sim \text{Poisson}(\lambda p)
\]
References


### Tables

Table 1. Characteristics of fire regimes of test landscapes for LANDIS simulations

<table>
<thead>
<tr>
<th>Fire regime unit</th>
<th>Area (ha)</th>
<th>FC (years)</th>
<th>MFS (ha)</th>
<th>DFS (ha)</th>
<th>MFF (# per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barren, WI</td>
<td>87,752</td>
<td>100</td>
<td>400</td>
<td>300</td>
<td>2.19</td>
</tr>
<tr>
<td>Lakeshore, WI</td>
<td>151,200</td>
<td>800</td>
<td>2000</td>
<td>2000</td>
<td>0.09</td>
</tr>
<tr>
<td>Private land, MO</td>
<td>57,585</td>
<td>300</td>
<td>5.4</td>
<td>1.8</td>
<td>35.55</td>
</tr>
<tr>
<td>Southwest slope, MO</td>
<td>21,054</td>
<td>415</td>
<td>2.7</td>
<td>1.8</td>
<td>18.79</td>
</tr>
<tr>
<td>Northeast slope, MO</td>
<td>18,177</td>
<td>415</td>
<td>2.3</td>
<td>1.8</td>
<td>19.04</td>
</tr>
<tr>
<td>Flat, MO</td>
<td>26,141</td>
<td>415</td>
<td>2.3</td>
<td>1.8</td>
<td>27.39</td>
</tr>
<tr>
<td>Savanna, MO</td>
<td>255</td>
<td>10</td>
<td>1.5</td>
<td>0.9</td>
<td>17</td>
</tr>
<tr>
<td>Updrain, MO</td>
<td>3,484</td>
<td>415</td>
<td>2.3</td>
<td>0.9</td>
<td>3.65</td>
</tr>
<tr>
<td>Lime, MO</td>
<td>842</td>
<td>415</td>
<td>1.8</td>
<td>0.9</td>
<td>1.13</td>
</tr>
<tr>
<td>Mesic, MO</td>
<td>1,189</td>
<td>415</td>
<td>1.8</td>
<td>0.9</td>
<td>1.59</td>
</tr>
</tbody>
</table>
Table 2. Calibrated parameters of fire regimes for LANDIS simulations.

<table>
<thead>
<tr>
<th>Fire regime unit</th>
<th>Initial time-since-last-fire (years)</th>
<th>Ignition density (# per decade)</th>
<th>Input MFS (ha)</th>
<th>Input DFS (ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barren, WI</td>
<td>50</td>
<td>48</td>
<td>500</td>
<td>200</td>
</tr>
<tr>
<td>Lakeshore, WI</td>
<td>200</td>
<td>3.15</td>
<td>3000</td>
<td>2000</td>
</tr>
<tr>
<td>Private land, MO</td>
<td>50</td>
<td>1520</td>
<td>5.4</td>
<td>1.8</td>
</tr>
<tr>
<td>Southwest slope, MO</td>
<td>50</td>
<td>1020</td>
<td>4.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Northeast slope, MO</td>
<td>50</td>
<td>920</td>
<td>3.33</td>
<td>1.8</td>
</tr>
<tr>
<td>Flat, MO</td>
<td>50</td>
<td>1300</td>
<td>2.7</td>
<td>1.8</td>
</tr>
<tr>
<td>Savanna, MO</td>
<td>10</td>
<td>350</td>
<td>25</td>
<td>7</td>
</tr>
<tr>
<td>Updrain, MO</td>
<td>50</td>
<td>210</td>
<td>3</td>
<td>0.9</td>
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<tr>
<td>Lime, MO</td>
<td>50</td>
<td>46</td>
<td>2.5</td>
<td>0.9</td>
</tr>
<tr>
<td>Mesic, MO</td>
<td>50</td>
<td>61</td>
<td>2.5</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Table 3. Simulated results of fire regimes from LANDIS simulations.

<table>
<thead>
<tr>
<th>Fire regime unit</th>
<th>MFF (# per year)</th>
<th>MFS (ha)</th>
<th>FC (years)</th>
<th>Error of MFF (%)</th>
<th>Error of MFS (%)</th>
<th>Error of FC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barren, WI</td>
<td>2.24</td>
<td>390</td>
<td>100</td>
<td>5.0</td>
<td>-2.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Lakeshore, WI</td>
<td>0.095</td>
<td>1997</td>
<td>797</td>
<td>5.6</td>
<td>-0.2</td>
<td>-0.4</td>
</tr>
<tr>
<td>Private land, MO</td>
<td>38.34</td>
<td>5.17</td>
<td>290</td>
<td>7.8</td>
<td>-4.3</td>
<td>-3.3</td>
</tr>
<tr>
<td>Southwest slope, MO</td>
<td>19.39</td>
<td>2.54</td>
<td>428</td>
<td>3.2</td>
<td>-5.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Northeast slope, MO</td>
<td>19.22</td>
<td>2.16</td>
<td>438</td>
<td>0.9</td>
<td>-6.1</td>
<td>5.1</td>
</tr>
<tr>
<td>Flat, MO</td>
<td>26.59</td>
<td>2.14</td>
<td>459</td>
<td>-3.0</td>
<td>-7.0</td>
<td>9.6</td>
</tr>
<tr>
<td>Savanna, MO</td>
<td>18.23</td>
<td>1.36</td>
<td>425</td>
<td>7.2</td>
<td>-9.3</td>
<td>2.4</td>
</tr>
<tr>
<td>Updrain, MO</td>
<td>3.67</td>
<td>2.24</td>
<td>424</td>
<td>0.5</td>
<td>-2.6</td>
<td>2.1</td>
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<tr>
<td>Lime, MO</td>
<td>1.15</td>
<td>1.78</td>
<td>411</td>
<td>1.8</td>
<td>-1.1</td>
<td>-0.9</td>
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<tr>
<td>Mesic, MO</td>
<td>1.47</td>
<td>1.96</td>
<td>412</td>
<td>-7.5</td>
<td>8.9</td>
<td>-0.6</td>
</tr>
</tbody>
</table>
**Figures**

\[ X \sim \text{Poisson}(\lambda) \]

\[ Y_i \sim \text{Bernoulli}(p_i) \]

\[ Y_1 = 0 \quad Y_2 = 0 \quad \cdots \quad Y_s = 1 \]

\[ U = \sum_{i=1}^{s} Y_i \]

X : The number of ignitions per decade.

Yi : The result of the ith ignition. 0 is failed, 1 means an active fire (initiation)

U : The sum of all ignition trials, the number of fire occurrences per decade

Z : Fire size

Figure 1. The overview of the fire module design
Figure 2. Study region, location, and land types within the study area in northern Wisconsin.
Figure 3. Study region, location, and land types within the study area in southern Missouri
Figure 4. Simulated fire frequency for barrens land type in northern Wisconsin and lakeshore land type in northern Wisconsin
Figure 5. Simulated fire frequencies for the eight land types in southern Missouri
Figure 6. Simulated dynamics of mean fire size (MFS) for barren and lakeshore land types in northern Wisconsin
Figure 7. Simulated dynamics of mean fire size for the eight land types in southern Missouri
Chapter 2. Comparing the effects of different modeling methods on simulated spatial and temporal forest fire patterns

Abstract

We compared temporal and spatial fire patterns simulated using four representative fire spread simulation methods (complete uniform, dynamic percolation, fire size based elliptical wave propagation, and duration based elliptical wave propagation) under two fire occurrence process scenarios that are fuel-independent and fuel-dependent. Specifically, we examined how statistical characteristics of fire frequency and fire size distribution, temporal structure (trend, autocorrelation) of fire frequency series, point pattern of fire occurrences, and shapes and spatial configuration of burned patches varied by forest fire simulation methods. The comparison was carried out using a spatially explicit and stochastic forest landscape model (LANDIS) that has the entire spectrum of fire spread simulation methods implemented and is capable of simulating the interaction of fire, fuel, and vegetation succession. The results demonstrate that primary characteristics of fire regimes (e.g., fire cycle, distribution of fire frequency, fire size) are similar among fire spread and fire occurrence simulation methods, but spatial pattern of fire occurrence, temporal pattern of fire frequency, and the shape of burned patches are somewhat different. Incorporation of fuel into fire occurrence modeling greatly changes fire patterns, suggesting that a mechanistic representation of fire occurrence with fuel and possible other drivers is important in the model building process.
1. Introduction

Fire disturbance plays an important role in shaping ecosystem dynamics and vegetation patterns in many forested landscapes (e.g., Heinselman, 1970; Romme, 1982; Johnson, 1992; Miller and Urban, 1999; Ehle and Baker, 2003). Small and large fires of varying intensities create a mosaic of burned and unburned patches, affect tree species composition and age classes, and generate spatially heterogeneous fuel beds. The resulting fuel heterogeneity in turn influences the spatial pattern of subsequent fires (Turner and Romme, 1994). The dynamic interaction of fire and vegetation at landscape scales is further complicated by other important ecological drivers such as tree species dispersal, successional recovery of disturbed landscapes, weather and climate shifts, and land management (Bessie and Johnson, 1995; Schoennagel et al., 2004).

Although empirical studies using dendrochronology (e.g., Agee, 1996) and charcoal stratigraphic analysis (e.g., Clark 1990) that provide insights into the complex interaction of fire, vegetation, climate and human activities, simulation modeling is also regarded as an effective tool to study such interactive dynamics over large areas and long time periods (Keane et al., 2004). Forest landscape fire simulation models can be used to reconstruct the historical range and variability of landscape patterns (Boychuk and Perera, 1997; Li, 2000; Keane et al., 2002; Wimberly, 2002), to study the effects of climate change on the structure of landscapes subject to fire disturbance (Baker et al., 1991), to examine human influence on fuel heterogeneity and fire patterns (Sturtevant et al., 2004), and to evaluate forest management alternatives and fire suppression plans (Gustafson et al., 2004).
A large variety of forest landscape fire simulation models have been developed. These models use different methods to simulate fire occurrence, fire spread, and fire effects at landscape scales and are implemented with varying levels of ecological detail.

Because of the prevalence of forest landscape fire simulation models and the diversity of simulation methods, it is important to understand the premises and behavior of various modeling methods.

Efforts have been made to classify landscape fire simulation models. For example, Gardner et al. (1999) provided a 6-category (Theoretical, Exploratory, Physical, Probabilistic, Shape, and Statistical) classification scheme to characterize a broad range of models that simulate fire effects at landscape scales. Keane et al. (2004) described each of the four simulation components (succession, fire ignition, fire spread, fire effects) of a landscape fire succession model in three dimensions by the gradients of stochasticity, complexity, and mechanism inherent in the simulation component. They proclaimed that the resulted 12 elements (4 components by 3 gradients) could represent a formal description of the model that can be objectively compared to other models. Although these classification schemes have organized the diversity of model approaches in a systematic way, the lack of a comparison of simulated fire patterns across different fire simulation methods limits their use for fire managers/modelers in selecting an appropriate model/method to best meet their needs. There are only a few published papers that compared simulated fire patterns across different methods. Li et al. (1997) compared mean interval between successive fires simulated using four fire probability functions in investigating modeling effects on temporal fire disturbance patterns on a boreal forest landscape. Cary et al. (in press) assessed the sensitivity of four existing landscape-fire-succession models (EMBRY, FIRESCAPE, LANDSUM and SEM-
LAND), in terms of area burned, to variation in environmental factors and complexity of model formulation. Their studies laid groundwork for examining the effects of different methods on fire patterns, but such examinations are far from comprehensive because there are many important aspects of fire patterns need to be compared beside the primary characteristics of fire interval and fire size.

Our study expands the scope of measures of fire patterns into the following five aspects: (1) statistical characteristics (central tendency and variability) of fire frequency distribution and (2) fire size distribution, (3) temporal structure (trend, autocorrelation) of fire frequency series, (4) spatial pattern of fire occurrences, and (5) fire shapes. The aspects (1), (3) and (4) are primarily affected by the way that a model simulates fire occurrence, the aspect (2) and (5) by fire spread simulation methods. We are interested in comparing spatial and temporal differences in fire pattern simulated using various fire spread and occurrence algorithms, to understand the implications of selecting a specific algorithm to answer research or management questions. Moreover, we want to find out if there are any interactive effects of fire spread simulation methods and fire occurrence simulation methods.

Our approach was to examine a spectrum of fire spread simulation methods from the simplest statistical methods, which essentially do not simulate fire growth, to the probabilistic-based percolation methods, and to the most complicated physical methods. These fire spread methods were further compared under two fire occurrence process scenarios. The first scenario assumes fuel loads have no effects on fire occurrence, whereas the second scenario assumes that the probabilities of fire occurrence increase with fuel loads. This modeling factorial design allows us to study the interactions between fuel and fire simulated across different simulation methods. We tested the null hypothesis that there is no
significant difference among spatial and temporal fire patterns simulated using different fire spread methods under the two fire occurrence process scenarios. The test was carried out using a single spatially explicit and stochastic forest landscape model that has the entire spectrum of fire spread simulation methods implemented and is capable of simulating the interaction of fire, fuel, and vegetation succession for our study requirements. We held all other ecological processes such as succession and dispersal constant. Because of the stochasticity inherent in these fire simulation methods, comparison of cell-by-cell patterns carries very little meaning for our purposes. Instead, we examined aggregate measures of simulated landscape patterns of burned patches and statistical characteristics (e.g., frequency, size, timing, intensity) of the simulated fire regime.

2. Methods

2.1 Fire simulation methods

Fire simulation consists of three components (fire occurrence, fire spread, and fire effects). The fire occurrence component simulates the initiation of a fire event defined as a fire start that consumes at least one cell in the grid which represents the simulation landscape (Keane et al., 2004). The most commonly used method to simulate fire occurrence uses fire frequency distributions (e.g., Poisson, Weibull) to derive a fire probability function (Johnson and Van Wagner, 1985) in which time-since-last-fire is the independent variable. A fire occurrence is usually modeled as two consecutive stages – fire ignition and fire initiation (Antonovski et al., 1992; Davis and Burrows, 1994; Li, 2000). We developed a hierarchical fire frequency algorithm to simulate this two-stage process (Yang et al., 2004). In a given
time step, the number of ignitions in the given fire regime unit (a spatial area where the characterization of historic, natural fires are homogeneous, e.g., an ecological land type or an ecoregion) is generated from a Poisson distribution whose parameter ignition intensity is assumed to be homogeneous within the fire regime unit. For each ignition attempt, the simulator performs a Bernoulli trial with parameter fire probability, whose value is determined by the amount of fine fuels in the cell. If the result is successful, a fire is initiated. The amount of fine fuels is derived primarily from vegetation types and stand age, and modified by land type and disturbance history (He et al., 2004).

In this study, we developed two types of fire probability functions. In the constant fire probability function, the fire probability is fuel-independent, and in the step fire probability function the fire probability is a function of the level of fuel, which is typical of many fire occurrence simulators (e.g., Keane et al., 2002). This comparison enables us to explore the effects of vegetation dynamics (through fuel) on fire occurrence.

The fire spread component simulates the growth of individual fire events. There are many simulation techniques requiring different levels of computation effort, parameterization, and input data preparation. We implemented four representative methods for this study: (1) complete random, (2) probability-based percolation, (3) fire size distribution based elliptical wave propagation, and (4) duration based elliptical wave propagation.

The complete random method is the simplest but it is used when only coarse scale characteristics of a fire regime need to be simulated (Baker, 1991; Lenihan et al., 2003). It selects a maximum fire size from a user specified size distribution and lets the fire to spread from the burning cell to its (four directional or eight directional) neighboring forested cells uniformly until it reaches the maximum fire size or all available forested cells have been
disturbed. The effects of vegetation characteristics on fire spread are not simulated in this method.

The percolation method represents the landscape as a lattice of square, triangular, or hexagonal sites. A fire in one cell spreads to neighboring sites that contain forest, just as the complete random method does. However, the direction of fire spread can be adjusted by a spread probability that is affected by factors such as wind direction and speed, topography, and fuel types (Gardner et al. 1999; Hargrove et al., 2000; Wimberly et al., 2000). We employed a version of dynamic percolation in which spread probability interacts with fuel types, topography and wind, and fire size distribution. The simulator randomly selects a maximum size from a user-defined lognormal distribution. The spread probability increases in the beginning, but the rate of change decreases as the fire grows toward its maximum size and fluctuates with respect to the effects of fuel types, topography, and wind. When the rate of change becomes a negative value, the probability will begin to decrease hence the spread reaches an extinguishment phase. The algorithm can produce burned patches with a compact core and a fractal boundary, similar to the geometric features of real wildfires as shown by Caldarelli et al. (2001).

The elliptical wave propagation methods are based on Huygens’ Principle, which asserts that wave fronts can be propagated independently from discrete points. The method we chose is the minimum travel time method developed by Finney (2002). It uses algorithms developed from graph theory (e.g., Dijkstra, 1959) to search for minimum cumulative travel times of waves along straight-line paths among cells of a lattice. Travel times along the line segments were calculated from rate of fire spread in the underlying cells of the lattice using Rothermel model in which the shapes of fires are assumed to be elliptical under uniform fuel
conditions (Rothermel, 1972; Andrews, 1986). The paths producing minimum travel time between cells were then interpolated to reveal the fire perimeter positions. The method can produce spatial fire growth and behavior data identical to perimeter expansion techniques used in models such as FARSITE (Finney, 1998). Furthermore, it is simpler to implement and computes much faster than perimeter expansion techniques. We designed two versions of minimum travel time methods. The fire size based version uses a fire size randomly selected from a user-defined fire size distribution to truncate the elliptical wave propagation simulation of fire spread when the fire size reaches the pre-selected fire size. The duration based version randomly selects a burning time from a user-specified burning duration distribution and uses it to determine when the fire is extinguished.

Fire effects were simulated using a rule-based strategy. Fire intensity is calculated by the amount of fine fuel and coarse fuel consumed. Because fire is a bottom-up disturbance, fires affect younger age classes first and older cohorts are killed by fires of greater intensity. However, fire tolerance varies among species and age classes so that not all cohorts are affected similarly by a fire of a given intensity. To implement these characteristics, species fire tolerance classes, containing five categories from 1 to 5, are designed to reflect the differences of fire tolerance among species, and species age susceptibility classes are designed to reflect differences related to age within a species. Fire tolerance class combined with age susceptibility class determines whether an age cohort of a certain species can survive a fire event of a given intensity class (He and Mladenoff, 1999).

The combination of the two fire occurrence process scenarios (with and without) and the four fire growth modeling methods results in eight fire modeling experiments, which were carried out using LANDIS 4.0 (Table 1).
2.2 LANDIS model

The LANDIS model simulates spatial forest dynamics including forest succession, seed dispersal, species establishment, various disturbance including wind, fire and forest management, and their interactions (Mladenoff and He, 1999; Gustafson et al., 2000) across large ($10^3 – 10^7$ ha) landscapes and over long time domains ($10^1 - 10^3$ years). The model currently operates on 10-year time step. In LANDIS, a landscape is represented by a grid of cells, where each cell contains information on the presence and absence of tree species and their 10-year age-cohorts. The model does not track individual trees, which differs from most forest stand simulation models. It has been demonstrated that tracking age cohorts rather than individuals does not significantly reduce realism for landscape-scale applications (Bugmann, 1996). Additionally, computational loads are greatly reduced (He et al., 1999).

LANDIS stratifies a heterogeneous landscape into land types, which are generated from GIS layers of climate, soil, or terrain attributes. The model requires parameters for species establishment, fire disturbance characteristics, and fuel accumulation regime for each land type. LANDIS 4.0 (He et al., 2005) allows users to stratify the landscape into fire regime units, which can be different from the land type delineation (Yang et al., 2004). The fuel module tracks fine fuel and coarse fuel for each cell. Fine fuel load is derived from vegetation types (species composition) and species age, and coarse fuel load is derived from stand age in combination with disturbance history (He et al., 2004).
2.3 Case study area

Our case study area, 128,725 ha in size, is located in the Mark Twain National Forest and within the Missouri Ozark Highlands (Figure 1) in southern Missouri, USA. The area is largely forested, with white oak (*Quercus alba*), post oak (*Quercus stellata*), black oak (*Quercus velutina*) and shortleaf pine (*Pinus echinata*) as the dominant tree species. Forest age structure is relatively simple due to historical harvesting practices. Historical records of wildfire and dendrochronology studies in the study area show that the fire cycle (average fire return interval) varied from 17.7 years in the depopulated period (1580 – 1700) and 12.4 years in the Native American repopulation period (1701 – 1820), to 3.7 years in the Euro-American settlement period (1821 – 1940) (Guyette et al., 2002). Recent effective fire suppression policy has resulted in a dramatically extension of fire cycle, which is about 450 years. The average of fire size of current fire regime in the study area is about 8 hectares (Guyette and Larsen, 2000).

2.4 Comparing the effects of different modeling methods

We conducted 10 replicates of a 450-year simulation for each of the eight fire simulation experiments. The parameters of each experiment were calibrated interactively to produce acceptable simulated fire cycle and mean fire size (within 5% of actual values of empirical estimates for the case study fire regime). The simulated results were then analyzed using box plots, time series analysis, point pattern data statistics, and analysis of variance of landscape metrics to explore the following four aspects of fire pattern.
2.4.1 Fire frequency and fire size distributions

We used box plots to explore simulated fire frequency and fire size distributions. A box plot provides an excellent visual summary of many important aspects of a distribution. The box stretches from the lower hinge (defined as the 25th percentile) to the upper hinge (the 75th percentile) and therefore contains the middle half of the scores in the distribution. The median is shown as a line across the box. A group of box plots is particularly useful for detecting and illustrating central tendency and variation changes between different experimental treatments.

2.4.2 Temporal structure of fire frequency series

We used the techniques in time series analysis to identify the internal structure (such as autocorrelation, trend or seasonal variations, and stationarity) for the simulated fire frequency series. We used the autocorrelation function (ACF) and partial ACF to test for randomness (i.e., that there is no time dependence in the data) and to identify an appropriate time series model (e.g., white noise, autoregressive or moving average process) for our simulated fire frequency data. Structureless white noise is an independent and identically-distributed random process with a mean of zero through time and zero autocorrelation for all time lags except lag zero. With the exception of lag 0, this is always 1 by definition, almost all of the autocorrelations fall within the 95% confidence limits (Figure 2a). Non-stationary processes (e.g., random walk) exhibit a characteristic where the correlogram does not decay to zero (Figure 2b). The ACF of autoregressive (AR) processes usually decays exponentially to zero, and there is a sharp cut-off in partial ACF (Figure 2c). The ACF of moving
averaging (MA) processes has one or more spikes, and the rest are essentially zero (Figure 2d). We also used the Box-Jenkins (Ljung and Box, 1978) approach to fit autoregressive integrated moving average (ARIMA) processes to the data. A non-stationary time series \( \{Y_t\} \) is an ARIMA process of order \( p, d, q \), written \( Y_t \sim \text{ARIMA}(p,d,q) \), if the \( d \)th difference of \( Y_t \) is a stationary autoregressive moving averaging (ARMA) process of order \( p, q \).

\[
X_t = \nabla^d Y_t
\]

\[
X_t = \mu + a_1 X_{t-1} + \cdots + a_p X_{t-p} + Z_t + b_1 Z_{t-1} + \cdots + b_q Z_{t-q}
\]

where \( X_t \) is the \( d \)th difference of \( Y_t \), \( \mu \) is the intercept (mean) of differenced series \( \{X_t\} \), \( \{Z_t\} \) is the white noise with zero mean, \( a_1 \cdots a_p \) are coefficients of AR(\( P \)), and \( b_1 \cdots b_q \) are coefficients of MA(\( q \)) (Brockwell and Davis, 1996). If our fitted ARIMA has zero order \( d \), we can then conclude that our simulated fire frequency is a stationary process.

2.4.3 Spatial pattern of fire occurrence

We chose the \( K \) function (also called Ripley’s \( K \)) and kernel estimation for spatial point pattern analysis of simulated fire occurrence maps to ascertain whether there is a tendency for fire occurrences to exhibit a systematic pattern (regularity or alternatively clustering), as opposed to being distributed randomly, and to determine whether or not fires are more likely to occur in some regions than in others. The \( K \) function is defined as

\[
\lambda K(h) = E(\text{number of events within distance } h \text{ of an arbitrary event})
\]
where $E()$ is the expectation operator, and $\lambda$ is the intensity or mean number of events per unit area. The estimate of $K$ is usually compared to the true value of $K$ for a homogeneous Poisson process, often called complete spatial randomness (CSR), which is $K(h) = \pi h^2$. Deviations between the empirical and theoretical $K$ curves may suggest spatial clustering or spatial regularity: under regularity $K(h) < \pi h^2$, whereas under clustering $K(h) > \pi h^2$ (Bailey and Catrell, 1995). The $K$ function measures the second order effects (local-scale variations resulting from spatial correlation structure) of point pattern data, while the kernel estimation is concerned with exploring the first order properties (large-scale variations in the mean value) of a spatial point pattern, i.e., estimating the way in which the intensity varies in the study region. The $K$ function and kernel estimation of simulated fire occurrence data were computed using a spatial point pattern analysis package (spatstat) of R (Baddeley and Turner, 2005).

### 2.4.4 Simulated fire shapes

We created a map of the simulated fires for each replicate and each time step, in which a pixel value is either 0 when it never burned or 1 if it burned at least once in that time step (10 years). These maps were imported into the FRAGSTATS (McGarigal et al., 2002) to quantify the shape and spatial configuration of burned patches. We selected only the shape index (SHAPE) and clumpiness index (CLUMPY) to avoid analysis of correlated indices (Hargis et al., 1998). SHAPE equals patch perimeter divided by the minimum perimeter possible for a maximally compact patch of the corresponding patch area. SHAPE equals 1 when the patch is maximally compact (i.e., square or almost square) and increases without
limit as patch shape becomes more irregular. CLUMPY equals the proportional deviation of the proportion of like adjacencies involving the corresponding class from that expected under a spatially random distribution. CLUMPY equals -1 when the focal patch type is maximally disaggregated, equals 0 when the focal patch type is distributed randomly, and approaches 1 when the patch type is maximally aggregated.

3. Results

3.1 Primary characteristics of simulated fire frequency and size distributions

The location (i.e., central tendency) and variability of simulated fire frequency and fire size distributions resulting from the eight fire simulation experiments are shown in the box plots in figure 3. The statistical measures of location, both the mean and median, for the simulated fire frequency and fire size are not distinctively different. The mean fire frequency simulated from A2 and D2 experiment (Table 1) exhibits observable departure from the empirical mean fire frequency, but such departure is less than 5% and is an artifact of the calibration process in which a 5% departure from empirical values was tolerated. Nevertheless, the variability of fire frequency data simulated using the step fire probability function (A2, B2, C2, and D2) is much higher than those simulated from the constant probability function (A1, B1, C1, and D1) (Figure 3a). The larger inter-quartile range (IQR) in D1, D2 (Figure 3b) illustrates that the duration based elliptical wave propagation fire growth simulation method produces higher variability of fire size. The distributions of fire size resulting from all eight experiments are skewed to the right, manifested by the median being less than the mean and the many outliers above the upper whisker in the box plots.
(Figure 3b). Such strongly skewed fire size distributions are consistent with the observation of the fire regime in our study area, which consists of many small-size fires and very few large fires (Guyette et al., 2002).

3.2 Temporal structure of chronological fire frequency

By examining the ACF and partial ACF of each one of the total 80 simulated fire frequency series, we found that fire frequency data simulated using a constant fire probability function exhibit no temporal structure (i.e., white noise, Figure 4e, 4g). When using a step function there is a discernible trend (Figure 4b, 4d), and a strong serial dependence: significant strong positive autocorrelation at the first few lags (10-year to 60-year lag), and weak negative autocorrelation for longer lags (around 300-year lag), as shown in the correlograms (Figure 4f and 4h).

All the simulated fire frequency series can be identified as realizations of a stationary process. The ones simulated from constant fire probability function (experiment A1, B1, C1, and D1) can be fitted to a white noise process $X_t = \mu + Z_t$ (table 2), and there is no significant difference in the fitted intercept among all the four fire spread methods (ANOVA, $p$ value $= 0.396$, table 2). The fire frequency series simulated from the step function (experiment A2, B2, C2, and D2) can be fitted to a first order autoregressive processes $X_t = \mu + a_t X_{t-1} + Z_t$, and fire spread approach D (duration based elliptical wave propagation) produces significantly (alpha $= 0.05$) lower autocorrelation (coefficient $a_t$) than the other fire spread methods (Table 2). This can be seen when the realizations take relatively short excursions above or below the intercept level (Figure 4d vs. 4b).
3.3 Point pattern of fire occurrences

The point pattern exhibited from our simulated fire occurrence data is greatly affected by fire probability functions; whereas fire spread simulation does not play a role. For fire occurrence data simulated using the step function, regardless of fire spread simulation approaches, the kernel estimation maps show the same pattern that the spatial intensity of fire occurrence is high in the center of the landscape and it progressively decreases towards the boundary of the landscape (Experiments A2 and D2 are shown in the right part of figure 5). B2 and C2 produce the same pattern, although not shown in the figure. The computed $K$ function begins to show spatial clustering at approximately a scale of 100 cells (i.e., 3 kilometers). Clustering becomes more evident as the scale gets larger (example A2 is shown in figure 6, experiment B2, C2, and D2 produce a similar pattern). As a test of significance, the computed $K$ function is compared with not only the theoretical $K$ function under complete spatial randomness (CSR), but also the upper and lower “simulation envelopes”: -- the 99 $K$ functions calculated from point data simulated independently under CSR but with the same intensity as the test data (Figure 6 and figure 7). The $K$ function graphs and spatial intensity maps suggest that the point pattern of fire occurrence data simulated using constant fire-fuel probability function exhibit CSR: the spatial intensity of fire occurrence is homogeneous across the entire landscape (as shown in the left part of figure 5) and there is no spatial dependency among fire occurrences (as shown in figure 7).
3.4 Shape and configuration of burned patches

The shapes simulated using the fire spread method of elliptical wave propagation (i.e., method C and D) are much simpler than those simulated using completely uniform fire spread and percolation methods (Figure 8a). This reflects the algorithms of the fire spread methods since elliptical wave propagation is deterministic (in particular, physical-statistical) and is fire shape driven, while the other two are probabilistic and are burn extent driven. The Percolation method produces the most irregular burned patch shapes, and fire size based elliptical wave propagation produces the most regular shapes. ANOVA tests showed that fire probability functions have no significant (p> 0.05) effects on the simulated fire shapes. Similarly, the burned patches simulated using elliptical wave propagation methods are most aggregated and the two fire probability functions have no significant effects on the simulated spatial configuration of burned patches given the same fire spread simulation method is used (Figure 8b).

4. Discussion

Our study suggests that – although all our designed experiments are able to simulate primary characteristics of fire regimes (e.g., fire cycle, distribution of fire frequency, fire size) – spatial pattern of fire occurrence, temporal pattern of fire frequency, and the shape of burned patches are generally different by fire spread and fire occurrence simulation methods. Thus, the null hypothesis is rejected. The fire probability step function, which is used to simulate a fuel-dependent fire occurrence process, greatly increases simulated variability of fire frequency distribution (Figure 3a), and imposes conspicuous temporal autocorrelation in
fire frequency series (Figure 4b, 4d) and spatial point pattern in fire occurrences (e.g., Figure 6). This result suggests that it is important for landscape fire succession models to incorporate fuel in simulating fire occurrence. This may be especially true for many eastern U.S. forest ecosystems in which heterogeneous fuel beds are common (Barbour and Billings, 1988) in contrast with relatively homogenous fuel beds (largely due to live fuels) found in boreal forest and many western U.S. forest ecosystems (Bessie and Johnson, 1995). Fire spread simulation methods (statistical, probabilistic, and physical ones) primarily affect the shape of burned patches (Figure 8), but exert very little influences in fire frequency, fire size and fire occurrence. This result implies that choosing a suitable fire spread method is important only when the shape of burned patches is of model’s major concern. For example, the complete random fire spread method appears to be suitable for coarse scale dynamic global vegetation models in which the explicit simulation of fire growth is not needed or too computationally expensive (Keane et al. 2004). The probabilistic fire growth simulation methods (e.g., percolation), by their stochastic nature, produce more irregular and fractal shape of burned patches than the physical methods such as those fire perimeters based elliptical wave propagation methods. If we are to model a forest landscape that is topographically rugged and experiences surface fire regimes where fuel fragmentation becomes a major factor limiting the propagation of surface fires, the probabilistic fire growth simulation methods are then preferable.

Furthermore, we found very little interactive effects of fire occurrence and fire spread simulation methods in our results. This finding first appeared to contradict intuition that different fire spread simulation methods produce different fuel beds, which in turn should produce different fire patterns, especially when the step fire probability function is used to
simulate fire occurrence. Our explanation lies in the characteristics of the fire regime of our study area and the scale that we chose for this study. The case study area (Missouri Central Hardwood region) has a predominantly anthropogenic fire regime, where ignitions are abundant and mostly caused by humans (Guyette et al., 2002). Hence the effect of fuel on fire occurrence process has been mitigated. In addition, the fuel types are fairly simplex (largely oak-pine forests) in this area, and because low-intensity and small-size fires are much more common than the catastrophic crown fires, fuel loads are not greatly reduced in the burned patches. Moreover, fine fuel of the disturbed sites needs only a few years to accumulate to undisturbed levels (Kolaks et al., 2004) while we used a 10-year time step. All these factors contributed to make the presumed interactive effects imperceptible. We intend to apply our comparison to some other forest ecosystems with different fire regimes in the future to find out what fire regimes may bring such interactive effects discernible.

Evaluating inter-model behavior is an important issue and a systematic framework is lacking. This study fills in the gap by providing a framework of comparing model behavior in a statistically rigorous manner. Previous modeling studies have rarely examined the temporal pattern of fire frequencies. Currently used primary measure of temporal fire patterns is the average interval between two successive fires (e.g., Li et al., 1997), which not only conveys little information on temporal structure such as trend and autocorrelation but also impractical when fires are frequent enough to make the average interval less than the research temporal resolution. In this study we used time series analysis to compare different modeling experiments against temporal structure (trend, autocorrelation, stochasticity) of fire frequency. Our results demonstrate that this method is effective in examining simulated
temporal fire patterns. This method is not limited to fire frequency series and it can also be used for chronological series of burned area, landscape metrics, and so on.

It is not our intention in this study to examine the validity of the fire simulation approaches that were compared in this study. All these approaches have been implemented in the respective forest landscape fire models and have demonstrated that they can achieve satisfactory results for the intended use. As Rykiel (1996) pointed out, validation demonstrates that a model meets some specified performance standard under specified conditions, which include all implicit and explicit assumptions about the real system the model represents as well as the environmental context. A model is declared validated within a specific context which is an integral part of the certification. In that sense, all of the chosen approaches have been validated elsewhere before. Yet, each of the approaches embodies its own modeling purpose, the degree of realism it intends to simulate, and assumptions and context it upholds. Therefore, the difference in simulated spatial and temporal landscape forest fire patterns discovered here should not lead us to discuss which simulation approach is superior to others without providing an appropriate context. Rather, it sheds lights on the exploration of modeling behavior of each approach and lay grounds for modelers/decision makers to select any particular type of forest fire simulation method to meet their modeling purpose and context. In our specific context, fire probability function is proved to be the crucial component in determining simulated temporal fire patterns and interactions of forest fires and fuel dynamics, hence particular attention to the functional form and corresponding parameters is needed during the model building process.
References


McGarigal, K., S. A. Cushman, M. C. Neel, and E. Ene. 2002. FRAGSTATS: Spatial Pattern Analysis Program for Categorical Maps. Computer software program produced by the authors at the University of Massachusetts, Amherst. Available at the following web site: www.umass.edu/landeco/research/fragstats/fragstats.html


Table 1. Eight fire modeling experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Fire occurrence</th>
<th>Fire spread</th>
<th>Fire effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Complete uniform</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>Percolation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1 Constant function</td>
<td>Fire size based elliptical wave propagation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>Duration based elliptical wave propagation</td>
<td>Rule-based</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>Complete uniform</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>Percolation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C2 Step function</td>
<td>Fire size based elliptical wave propagation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>Duration based elliptical wave propagation</td>
<td></td>
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</tbody>
</table>

Note: We use the alphabet A-D to represent four different fire spread simulation methods and number 1 and 2 to represent two fire probability functions used in fire occurrence simulation.
Table 2. The coefficients of ARIMA (Autoregressive integrated moving average) processes that the simulated 80 (10 replicates x 8 experiments) fire frequency time series were fitted into

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Intercept $\mu$</th>
<th>The coefficient $a_1$</th>
<th>average $p$ value for Ljune-Box statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
<td>Mean</td>
</tr>
<tr>
<td>Fitted to white noise: $X_t = \mu + Z_t$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>357</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>B1</td>
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<td>3</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>356</td>
<td>4</td>
<td></td>
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<tr>
<td>D1</td>
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<td>2</td>
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<tr>
<td>Fitted to the 1st order autoregressive process: $X_t = \mu + a_1X_{t-1} + Z_t$</td>
<td></td>
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<tr>
<td>A2</td>
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<td>10</td>
<td>0.870</td>
</tr>
<tr>
<td>B2</td>
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<td>4</td>
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<tr>
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</tr>
<tr>
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<td>3</td>
<td>0.798*</td>
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</table>

* The mean is significant different at the 0.05 level.
Figures

Figure 1. Study area
Figure 2 A realization of some typical time series processes and its corresponding autocorrelation function (ACF) and partial ACF: (a) white noise, (b) random walk, (c) autoregressive (AR) process, and (d) moving average (MA) process.
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Figure 8 landscape metrics calculated from the burned patch maps simulated from the eight fire simulation experiments.

Abstract

The spatial pattern of forest fire locations is important in the study of dynamics of fire disturbance. Many existing fire occurrence studies focus on only natural (e.g., lighting) caused fires, hence are not adequate in studying anthropogenic fire regimes where human-caused fires are predominant. In this paper, we used spatial point process modeling approach to quantitatively study the effects of proximity to road, land cover, and topography (slope, aspect, and elevation) on the probability of fire occurrence in the Missouri Ozark Highlands, where more than 90% of fires are human-caused. Data on the starting location of fires in our study area between 1970 and 2002 and covariate information were mapped using GIS. Kernel intensity estimation and Ripley’s $K$ function were derived to explore spatial patterns in the data. We used the AIC (Akaike Information Criterion) method to select an appropriate spatial point process model to account for both spatial heterogeneity (‘trend’ or covariate effects) and dependence among fire occurrences (inter-point interaction, such as clustering or regularity). The fitted model was then diagnosed using residual analysis. Our results show that fire locations were spatially clustered, and high fire occurrence density was found in areas that are close to road (<500 meters away from nearest road) and on lower (<25 degree) slopes, where forests are accessible to humans. In addition, fire occurrence density was higher in pine-oak forests and woodlands and lower in low (<900 ft) elevation, which reflects the effects of natural factors on fire occurrence. The spatial point process modeling approach
presented here can be utilized in other forest ecosystems where spatial distribution of fire occurrence is inhomogeneous because of complex biotic and abiotic factors.
1. Introduction

The spatial pattern of fire occurrence is one of major aspects in the study of dynamics of forest fire and its role in determining landscape structure and vegetation community composition (Turner and Romme, 1994; Guyette et al., 2002; Ryan, 2002; Larjavaara et al., 2005; Mermoz et al., 2005), and has drawn of attention from forest landscape ecologists, fire historians, and spatial statisticians.

There are many biotic and abiotic factors that, besides the inherent stochasticity, may determine occurrence of forest fires. Among them fuel characteristics (fuel type, fuel loading, fuel moisture, and fuel inflammability) are considered to be prominent (Latham and Williams, 2001; Wotton and Martell, 2005). However, the effect of fuel characteristics on fire occurrences in different forest ecosystems at different spatial and temporal scales can vary. For example, Tanskanen et al. (2005) showed that differences in the moisture regime of surface fuels between stands of different age classes dominated by *Picea abies* or *Pinus sylvestris* can result in significantly different ignition conditions in boreal forests of southern Finland. On the other hand, Moritz et al. (2004) analyzed several hundred wildfires over a broad expanse of California shrublands, and their results revealed that there was generally not a strong relationship between fuel age and fire probabilities in their study area. These appeared contradicting results suggest that fire occurrence is a complex process in which fuel characteristics alone cannot provide the full explanation. Recent studies have incorporated other important abiotic factors such as elevation, slope, and aspects in describing spatial patterns of lightning-caused fires (e.g., Kushla and Ripple, 1997; Diaz-Avalos et al., 2001).
Human effects, however, are not yet considered in modeling spatial distribution of fire occurrences.

Fire ecologists and historians have found that fire regimes of many forest ecosystems are anthropogenic and shaped largely by human settlement and management (e.g., Veblen et al., 1999; Guyette et al., 2002; Bergeron et al., 2004; Hessburg et al., 2005). They have successfully identified, mainly using dendrochronology, temporal stages (e.g., pre-European-settlement and modern eras) of fire regimes related to human population density levels and shifting cultural behaviors. There are usually more human-caused fires than lightning-caused fires in modern-era fire regimes. For example, lightning-caused fires only contribute 35% of the fires reported in the boreal forest of Canada (Weber and Stocks, 1998) and less than 1% of the fires reported in Missouri in 1970 – 1989 (Westin, 1992). Many existing fire occurrence studies focus on only natural caused fires. Hence they are not adequate in describing, modeling, or predicting the locations of fires in modern-era anthropogenic fire regimes.

In this study, we used spatial point pattern (SPP) analysis to examine spatial patterns of both human-caused and lightning-caused fires reported in 1970 – 2002 in the Missouri Ozark Highlands. Analyzing fire occurrence data is challenging because conventional statistical approaches are not adequate to address spatial correlation exhibited in the data. Spatial statistics are therefore necessary (Wagner and Fortin, 2005). Cressie (1993) identifies three major branches of spatial statistics, namely geostatistics (the variable of interest is spatially continuous), lattice data statistics (the variable of interest has values only within a fixed set of areas or zones covering the study area) and spatial point pattern analysis (the important variable to be analyzed is the location of “events”). Whilst these three topics are to
som extent interlinked, they nevertheless give rise to distinct stochastic models and
associated statistical methods, and can therefore be applied into different research problems.
Point pattern and marked point pattern analysis methods have been used in forestry,
particularly for describing the variability of forest stands in which the “points” are tree
locations and the “marks” are tree characteristics such as tree species and diameter at breast
height (e.g., Stoyan and Penttinen, 2000; Kokkila et al., 2002). To our knowledge Podur et al.
(2003) are the first group who applied SPP in analyzing forest fire occurrences. They
computed the \( K \) function (also called Ripley’s \( K \)) and kernel intensity estimation of the
lightning-caused fires reported from 1976 to 1998 in Ontario, Canada. They found that fire
locations were clustered at a scale of approximately 150 – 200 km and these clusters were
located in the northwest and in the southeastern regions of northern Ontario. The statistics
that they used are non-parametric and exploratory, and offered limited inferential power
because different point pattern processes may result in the same spatial pattern (Wiegand and
Moloney, 2004). On the other hand, recent theoretical development within the realm of SPP,
the provision of formal likelihood-based methods of inference for a wide range of models,
provides tools for statistically rigorous modeling of spatial patterns of fire occurrence. This
study uses both non-parametric and parametric statistical analysis to describe and model
spatial point patterns of fire occurrences in a modern-era anthropogenic fire regime. The
primary objectives of our study are (1) to characterize spatial point patterns exhibited in the
fire occurrence data, (2) to model the human-caused fire occurrence process with the
consideration of both environmental heterogeneity (including human effects) and interactions
of neighbored fire occurrences (if there are any), (3) to estimate spatial distribution of fire
occurrence density and fire occurrence probability, and (4) to determine how specific spatial covariates (biotic or abiotic factors) contribute to the probability of fire occurrence.

2. Study area

The study area, located in the Missouri Ozark Highlands, is measured about 1287 km² in size (Figure 1). The geographical bounding coordinates are: 91.5°W, 91.1°W, 37.0°N, 36.7°N. It includes portions of four Missouri counties: Shannon, Carter, Ripley, and Oregon. A large proportion (86%) of the study area is forested, and 71% of the area is in the Mark Twain National Forest. White oak (*Quercus alba*), post oak (*Quercus stellata*), black oak (*Quercus velutina*) and shortleaf pine (*Pinus echinata*) are the dominant tree species.

Topographic variation is high, with elevations averaging 260 m and ranging from 140 m to 350 m. There is a river (Eleven Point) across the study area, and the landscape is dissected by steep ridges and streams. There were 1,299 fires reported to either Mark Twain National Forest or Missouri Department of Conservation (MDC) in this study area during 1970 – 2002, among which only 16 (1.2%) fires were caused by lightning, 608 (46.8%) fires were caused by arson, and others were caused by campfire, smoking, debris burning, or not-specified (unknown) reasons.

3. Approach and Methods

The overall approach is shown in the Figure 2. Reported fire locations, which are geographically referenced using the UTM (Universal Transverse Mercator) coordination
system, and spatial covariates such as roadway coverage, land cover, and topography data were mapped and processed using the GIS (detailed description in section 3.1). Non-parametric spatial statistics such as Kernel intensity estimation and Ripley’s $K$ function (described in section 3.2.1) were calculated to characterize spatial patterns (e.g., clustering or regularity pattern of points) of the fire occurrence data. Parametric modeling was then performed in order to study the effects of humans and other biotic or abiotic factors on spatial patterns of fire occurrence. The fire occurrence process was modeled as an inhomogeneous Poisson process. That is, we assumed there were no effects of interaction between fire occurrence points, and spatial patterns of fire occurrence solely resulted from the effects of environmental heterogeneity in the landscape. Maximum likelihood estimation method (described in section 3.2.2 and 3.2.3) was used to estimate model parameters. We used residual analysis (section 3.2.4) and the AIC method to select a best inhomogeneous Poisson process model to fit the data (section 3.3). Finally, the assumption of inhomogeneous Poisson process was validated by calculating generalized $K$ function of the best fit model we found.

3.1 Data acquisition and processing

The most important factor affecting spatial distribution of fire occurrence that we suspected is the roadway coverage, because most fires in this area are human-caused and spatial distribution of roadway determines the human access to forests. Places that are close to roads should have higher probability of human-caused fires. We obtained the digital roadway coverage published by the Missouri Department of Transportation (MoDOT) in 2004. The data layer contains all US highways, state highways (Missouri numbered routes and Missouri lettered routes), county roads, and city streets in this area. The spatial accuracy is generally $\pm 3$ meters. We calculated the proximity to roads as a measure of human
accessibility. We used ArcGIS Spatial Analyst tool to calculate the Euclidean distance from each cell (30 meter resolution) to its nearest roads. The resultant raster grid image (Figure 3) was then used as one spatial covariate for our point process modeling.

A 30-meter resolution grid of the land cover of the state of Missouri published by Missouri Resources Assessment Partnership (MoRAP) in 1999 was used as another spatial covariate in the modeling. This dataset is an amalgam of classified Landsat Thematic Mapper (Landsat TM) satellite image data ranging from 1991 to 1993. There are 12 classes of land cover in our study area, and we aggregated them into 4 classes (Table 1) because otherwise some of land cover classes (e.g., open water, Urban Impervious) contain such a small portion of cells in the study area that regression estimation would fail. We assumed that land cover for any cell within our study area was constant over the time 1970 – 2002. The grid image of aggregated land cover (Figure 4) was also used as one spatial categorical covariate.

We also obtained a 60-meter resolution grid of digital elevation model (DEM) data (Figure 5) published by Missouri Spatial Data Information Service (MSDIS) in 1999. We calculated slope (Figure 6), aspect (Figure 7) from the DEM data using the surface analysis provided in the ArcGIS Spatial Analyst tool.

3.2 Spatial point process modeling

3.2.1 The basics of SPP

A spatial point pattern data is a dataset \( x = \{x_1, \ldots, x_n\} \) with \( n \) points observed in a bounded region \( D \) of the plane \( \mathbb{R}^2 \). A spatial pattern is often the result of a mixture of both first order and second order effects. First order effects are related to variation in the mean
value of the spatial process, a global or large scale spatial trend. Second order (local or small scale) effects result from the spatial correlation structure or the spatial dependence in the process (Bailey and Gatrell, 1995). The first-order properties in spatial point pattern data are described by an intensity function. For any location \( u \in D \), the intensity (the mean number of events per unit area) is defined as

\[
\lambda(u) = \lim_{|\delta u| \to 0} \left\{ \frac{E[N(du)]}{du} \right\},
\]

Where \( E[\cdot] \) denotes the expectation of a random variable, and \( N(du) \) denotes number of events in the infinitesimal region of \( u \).

For a stationary process, \( \lambda(u) \) assumes a value \( \lambda \) constant over the entire space. To avoid confusion of statistical intensity with fire intensity used in fire ecology, we shall use fire occurrence density to denote the intensity used in spatial statistics in the results section.

The second-order intensity function is similarly defined as

\[
\lambda_2(u, v) = \lim_{|\delta u|, |\delta v| \to 0} \left\{ \frac{E[N(du)N(dv)]}{du \, dv} \right\}
\]

An alternative statistics that characterizes the second-order properties of a stationary, isotropic process is the \( K \) function. It is defined as

\[
K(h) = \frac{E(\text{number of events within distance } h \text{ of an arbitrary events})}{\lambda}
\]

The \( K \) function is widely used to describe the small-scale spatial correlation structure of the point pattern. The estimate of \( K \) is usually compared to the true value of \( K \) for the null model of complete spatial randomness (CSR), which is \( K(h) = \pi h^2 \). Deviations between the
empirical and theoretical $K$ curves may suggest spatial clustering or spatial regularity: under regularity $K(h) < \pi h^2$, whereas under clustering $K(h) > \pi h^2$ (Bailey and Gatrell, 1995). Edge effects (i.e., the circles centered at the point that is close enough to the boundary cannot be evaluated without a bias) can be either mitigated by edge-correction methods or safely ignored for some kind of exploratory analysis (Diggle, 2003). Any spatial patterns suggested by the estimated $K$ function could be due more to spatial inhomogeneity rather than to inter-point interactions if the intensity is not approximately constant in the study region. A generalisation of the $K$ function $K_i$ for inhomogeneous point processes, proposed by Baddeley et al. (2000), can be used in evaluating the second-order patterns after taking into account of the first-order effects.

### 3.2.2 Conditional intensity

For any finite spatial point process whose probability density function $f_\theta(x)$ satisfies the positivity condition

$$f_\theta(x) > 0 \text{ implies } f_\theta(y) > 0 \text{ for all } y \subset x,$$

the Papangelou conditional intensity for any location $u \in D$ is defined as

$$\lambda_\theta(u; x) = \begin{cases} 
\frac{f_\theta(x \cup \{u\})}{f_\theta(x)}, & u \notin x \\
\frac{f_\theta(x)}{f_\theta(x \setminus \{x_i\})}, & u = x_i \in x 
\end{cases} \quad (1)$$

It may be loosely interpreted as the conditional probability that there is a point at $u$ given that the rest of the point process coincides with $x$. For the homogeneous Poisson
process in which intensity function is a constant $\lambda$, the conditional intensity is the same constant $\lambda$. The inhomogeneous Poisson process with intensity function $\lambda_\theta(\cdot)$ has conditional intensity $\lambda(u; x) = \lambda(u)$. The fact that the conditional intensity at any points $u$ does not depend on the configuration of $x$ is a consequence of the independence properties of the Poisson process.

3.2.3 Parametric model-fitting using likelihood-based methods

The Papangelou conditional intensity is used in the definition of the pseudo-likelihood of a point process (Besag, 1982). For any sub-region $A \subset D$, the pseudo-likelihood is defined to be

$$PL_A(\theta; x) = \prod_{x_i \in A} \lambda_\theta(x_i; x) \exp \left( - \int_A \lambda_\theta(u; x) du \right)$$

(2)

Hence the log-pseudo-likelihood when $A = D$ is

$$\log PL(\theta; x) = \sum_{i=1}^{N(x)} \log \lambda_\theta(x_i; x) - \int_D \lambda_\theta(u; x) du$$

(3)

The integral term on the right-hand side of (3) can be further approximated by a finite sum using different quadrature schemes (Berman and Turner, 1992).

$$\int_D \lambda_\theta(u; x) du \approx \sum_{j=1}^{m} \lambda_\theta(u_j; x) w_j$$

(4)

Where $\{u_j, j = 1, \ldots, m\}$ are points in $D$ and $w_j$ are quadrature weights summing into $|D|$. Baddeley and Turner (2000) showed that with a proper quadrature scheme ensuring all data
points are in quadrature points list, \{x_i, i = 1, \ldots, N(x)\} \subset \{u_j, j = 1, \ldots, m\}, equation (3) and (4) can yield

\[
logPL(\theta; x) \approx \sum_{j=1}^{m} \left( \frac{z_j}{w_j} \log \lambda_0(u_j; x) - \lambda_0(u_j; x) \right) w_j
\]

(5)

Where \(z_j\) are indicators, defined as

\[
z_j = \begin{cases} 
1 & \text{if } u_j \in x \\
0 & \text{if } u_j \notin x 
\end{cases}
\]

Expression (5) can be maximized using standard statistical software for generalized linear or additive models, provided that the conditional intensity takes an exponential family form (Baddeley and Turner, 2000).

In practice, the conditional intensity is often specified through a log-linear regression model with two components:

\[
\lambda_0(u; x) = \exp(\theta_1 B(u) + \theta_2 C(u, x))
\]

(6)

or equivalently,

\[
\log \lambda_0(u; x) = \theta_1 B(u) + \theta_2 C(u, x)
\]

(7)

where \(\theta = (\theta_1, \theta_2)\) are parameters to be estimated.

The trend term \(B(u)\) depends only on the spatial location \(u\), so it represents spatial trend or spatial covariate effects. The interaction term \(C(u, x)\) depends on not only the point \(u\), but also the configuration of \(x\), hence it represents stochastic interactions between the points. The term \(C(u, x)\) is reduced to zero for the Poisson process.
3.2.4 Residual analysis

Residual analysis for spatial point processes, recently formulated by Baddeley et al. (2005), is considered a milestone of the development of point pattern statistics in that it provides us an excellent technique for determining the need of model refinement or for diagnosing the quality of the model-fitting for spatial point process in a statistically rigorous way. The method is analogous to the usual residual analysis for non-spatial generalized linear models. The basis of various forms of residuals is the Georgii-Nguyen-Zessin (GNZ) formula:

\[
E \left[ \sum_{x_i \in X} h(x_i; X \setminus \{x_i\}) \right] = E \left[ \int_D h(u; X) \lambda_\theta(u; X) du \right] 
\]

where \( \lambda_\theta(u; x) \) is the conditional intensity defined in (1), and \( h(u; x) \) is any non-negative function. The \( h \)-weighted innovations for spatial point process (analogous to errors in a linear model) are defined as

\[
I(B, h, \lambda) = \sum_{x_i \in X \cap B} h(x_i; X \setminus \{x_i\}) - \int_B h(u; X) \lambda_\theta(u; X) du 
\]

for any set \( B \subseteq D \). The innovations have mean zero, from (8).

The \( h \)-weighted residuals after fitting a parametric model to data \( x \) are defined by

\[
R(B, \hat{h}, \hat{\theta}) = I(B, \hat{h}, \hat{\lambda}) = \sum_{x_i \in X \cap B} \hat{h}(x_i; x \setminus \{x_i\}) - \int_B \hat{h}(u; x) \hat{\lambda}(u; x) du 
\]

where \( \hat{\theta} \) is the estimated parameters of the model (often using maximum-likelihood based methods), \( \hat{\lambda} \) is the fitted conditional intensity \( \hat{\lambda}(u; x) = \lambda_\theta(u; x) \), and \( \hat{h} \) is the fitted weight function when the weight function depends on the parameter \( \theta \), \( \hat{h}(u; x) = h_\theta(u; x) \).
Different weight functions can yield different types of innovations (errors) and residuals. Two types are used in this study. One choice is \( h(u; x) = 1 \), which yields the raw innovation measure

\[
I(B, 1, \lambda) = N(X \cap B) - \int_B h(u; X) \lambda_\theta(u; X) du
\]

and raw residual measure

\[
R(B, 1, \hat{\theta}) = N(x \cap B) - \int_B \hat{\lambda}(u; x) du .
\]

The other choice is \( h(u; x) = \frac{1}{\sqrt{\lambda(u; x)}} \). It yields the Pearson innovation measure

\[
I(B, \frac{1}{ \sqrt{\lambda} }, \lambda^\theta) = \sum_{x_i \in X \cap B} \frac{1}{\sqrt{\lambda}(x_i; X)} - \int_B \sqrt{\lambda(u; X)} du,
\]

which has mean zero and variance \( |B| \) (it is analogous to the usual Person residuals for generalized linear models in that the variance is standardized, ignoring effects of parameter estimation), and the corresponding Pearson residual measure

\[
R(B, \frac{1}{ \sqrt{\lambda} }, \hat{\theta}) = \sum_{x_i \in X \cap B} \frac{1}{\sqrt{\lambda}(x_i; x)} - \int_B \sqrt{\lambda(u; x)} du .
\]

One important application of residual analysis is to plot the residual against a spatial covariate (or one of Cartesian coordinates) to investigate the presence of spatial trend for point process models, called lurking variable plot. For a spatial covariate \( Z(u) \), which must be spatially continuous in the study region \( D \), we may evaluate the residual measure on each sub-region defined by

\[
B(z) = \{ u \in D : Z(u) \leq z \},
\]

(11)
yielding a cumulative residual function

\[ A(z) = R\left( B(z), \hat{h}, \hat{\theta} \right). \] (12)

The function \( A(z) \) should be approximately zero if the fitted model is correct, and any systematic pattern in the lurking variable plot suggests an appropriate modification of the model to better account for that spatial covariate.

### 3.3 Model selection

The compiled fire occurrence data and GIS-processed spatial covariates were analyzed and modeled using a spatial point pattern analysis package (\textit{spatstat}) of R (Baddeley and Turner, 2005). The major task at this step was to identify a best choice of model that would balance fit to the data, model size (parameters within the model), and model complexity. The goal was: 1) to determine the interaction term \( C(u, x) \) specified in the log-linear regression model of equation 7, and 2) to select predictor variables (Cartesian coordinates, spatial covariates, and their transformations) for the trend term \( B(u) \). Three major tools were used in this stage: residual analysis (described in section 3.2.4), the generalized \( K \) function \( K_i \) (described in section 3.2.1), and the Akaike information criterion (AIC). The AIC (Akaike, 1974) is widely used as a measure for selecting the best among competing models for a fixed data set. It is defined as

\[ AIC = -2 \max(\text{log-likelihood}) + 2(\text{number of parameters}). \]

The model with the smaller AIC is considered to be a better fit to the data. It is sometimes useful to note that the log-likelihood ratio statistic takes the form
\[-2\log(L_0 / L_1) = AIC(H_0) - AIC(H_1) + 2d,\]

where \(L_0, L_1\) denote the maximized likelihood under model \(H_0, H_1\) respectively, and \(d\) denotes the difference between the numbers of parameters in \(H_0\) and \(H_1\). The statistic \(-2\log(L_0 / L_1)\) is expected to have a chi-squared distribution with \(d\) degrees of freedom when the null model \(H_0\) is true (Ogata, 1988).

We started our quest in computing non-parametric statistics \(K\) function, and the kernel smoothed intensity function from our fire occurrence point data for exploratory purposes. The data were first fitted to the null model (a homogeneous Poisson model), and we used residual analysis (especially, lurking variable plots) to find out whether the point pattern depends on tested spatial covariates. We then assumed the fire occurrence process is an inhomogeneous process (hence \(C(u, x)\) is zero) and included those spatial covariates that were revealed to affect the fire occurrence patterns into the trend term \(B(u)\) of the log-linear regression model one by one. Those included spatial covariates could be transformed, and we used polynomial function (up to power of four) to transform spatial covariates or Cartesian coordinates. We calculated AIC as well as performed residual analysis for every alternative model. We kept “diagnostics \(\Rightarrow\) variable selection (with possible transformation) \(\Rightarrow\) diagnostics” strategy until we were satisfied with the model-fitting results. Finally, we computed the generalized \(K\) function \(K_i\) for the fitted model for checking of our assumption that fire occurrence is a Poisson process. If there were still any second-order patterns after we weighted the trend fitted from the model, then the Poisson assumption should not be considered to be sound, and other point process (e.g., pair-wise interaction point process, Strauss process) might be assumed.
4. Results

4.1 Characteristics of fire occurrence patterns

The kernel-smoothed intensity (Figure 8) estimated from the fires reported in 1970 – 2002 showed a “hot bed” (where fire occurrence density is high) on the north center of the study area (near the town Winona) and a “cold bed” (where fire occurrence density is low) on the southwest (near the town Alton), suggesting fire occurrence process is far from homogeneous. The estimated $K$ function (Figure 9) is larger than that of theoretical CSR from the scale of about 500 meters. Spatial clustering pattern is observable for almost any scale that is larger than 500 meters. However, it doesn’t necessarily suggest there is a strong dependence among fire occurrences, since the environmental heterogeneity (e.g., clustering of camping sites or vegetation distribution) may also be the reasons for explaining this clustering pattern. Our model-fitting exercises showed that the spatial clustering pattern could be in deed explained by the environmental heterogeneity (section 4.2).

4.2 The fitted model and its diagnostics

The null model (homogeneous Poisson) fitted to the data has a constant intensity, valued 1.009116e-06 per m² over the entire region. This constant intensity can be interpreted as an average of 1.009116 fire occurrences every 1 km² in our study area over 33 years (1970 – 2002). We plotted the cumulative Pearson residuals against the four spatial covariates (distance to nearest road, slope, aspect, and elevation) and the two (x and y) Cartesian
coordinates for the null model (Figure 10). The cumulative residual function defined by equation (12) should approximate zero if the null model adequately addresses effects of the spatial covariate. In Figure 10a, the cumulative Pearson residuals are larger than $+2\sigma$ limit for the scale of distance to nearest road < 1500 m. This suggests the null model underestimates fire occurrence at this scale. In other words, there are more fires occurring at the spatial locations that are close (< 1500 m) to roads than the null model predicts. The peak of the curve rises in the 500 m scale and the cumulative Pearson residuals decrease with the increase of the distance to nearest road after the 500 m scale, which indicates that there are actually fewer fires occurring at 500 m – 1500 m scale than the null model predicts. The steepest increases in the curve are observed at less than 150 meters away from roadways, implying that many human-caused fires were set in the places that are very close to roads (refer to Figure 3). The peak in Figure 10b rises in slopes of about 25 degree and the steepest increase falls within the range of slopes of 10-20 degrees, suggesting that more fires occur on lower slopes than higher slopes. Effects of aspect on fire occurrence patterns are less conspicuous based on Figure 10c. Nevertheless, we can infer that there are fewer fires in the northeast/east extreme than the south/southwest extreme because there is a relatively large decrease of cumulative Pearson residuals within the range of aspects of 25-100 degrees (northeast to east) and an increase within the range of 150-210 degrees (south to southwest). There are obvious systematic patterns observed in Figure 10d: the nadir of the curve is much less than the $-2\sigma$ limit of error bounds, and there is a steep increase of cumulative residuals after the nadir point (900-1050 ft). The patterns suggest that the null model overestimates intensity of fire occurrence for spatial locations with less than 900 ft (i.e., 274 m) elevation, and more fires are ignited at higher elevations (refer to Figure 5). The patterns revealed from
Figure 10e and Figure 10f are easier to explain using the kernel smoothed intensity map (Figure 8). The intensity of fire occurrence estimated from kernel smoothed estimation is in general smaller than the constant intensity 1.009116e-06, estimated from the null model, within the area that is x-coordinated less than 64,550 m (x-coordinate of the first inflection point in Figure 10e), and larger than the average within the area that is x-coordinated between 64,550 m and 65,550 m (x-coordinate of the second inflection point in Figure 10e). Similarly, kernel-smoothed intensity is in general lower than the intensity estimated from the null model within the area that is y-coordinated between 4,065,000 m and 4,085,000 m (y-coordinates of the first and second inflection point in Figure 10f respectively), and is higher than the average within the area that is y-coordinated larger than 4,085,000 m. These patterns manifested in lurking variable plots against Cartesian coordinates are possibly resulted from diverse drivers, which may include road proximity, topography, and other factors that were not included in our study.

Table 2 gives the maximum log-likelihoods and AIC values for the models considered in our model selection stage. The models were fitted to the data using the maximum pseudo-likelihood method that was described in section 3.2.3. All models considered are for Poisson point processes, in which case maximum pseudo-likelihood is equivalent to maximum likelihood (Baddeley and Turner, 2005). Although AIC is useful for the comparison of competing models, we didn’t rely solely on AIC in selecting the best model. Residual analysis was also exercised for every fitted model. When we didn’t include Cartesian coordinates as predictor variables in the log-linear regression models, the best one, denoted as $H_1$, has identified the predictor variables as the distance to the road, land cover, elevation and the square of elevation, and slope. Aspect, however, was eliminated in the
model selection procedure. The lurking variable plots (Figure 11) for the fitted model, showed that the model has captured the effects of road proximity, slope, and elevation on fire occurrence patterns fairly well comparing to the lurking variable plots for the null model (Figure 10). Nevertheless, the glaring violation of error bounds in the lurking variable plots against Cartesian coordinates as shown in Figure 11e and Figure 11f implied that there was still presence of spatial trend not captured in this model. Hence proposed spatial covariates (road proximity, land cover, and topography) were not sufficient for characterizing spatial variation of fire occurrence density. We then added Cartesian coordinates into the models. The selected model $H_2$ having 22 parameters (Table 2) was much more complicated than $H_1$, but also showed greater improvement in terms of model diagnostics (Figure 12).

We calculated the generalized $K$ function for inhomogeneous Poisson point process (Baddeley et al., 2000), $K_i$ for our final full model $H_2$. The calculated $K_i$ function (Figure 13) was within the envelopes of 99 simulations of inhomogeneous Poisson point process with the same distribution of conditional intensity as the one of model $H_2$, indicating our assumption of inhomogeneous Poisson process was valid.

4.3 Distribution of fire occurrence density and fire occurrence probability in space

The spatial intensity calculated from our point process model can be loosely interpreted as fire occurrence density. Figure 14 provided two spatial distributions of fire occurrence density calculated from the two models $H_1$ and $H_2$. Both parametric models showed spatial trend affected by the road proximity, land cover, and topography, which were
not characterized in the non-parametric estimated intensity of fires (Figure 8) using the
kernel smooth technique. Based on these spatial covariates alone (in $H_1$ model), fire
occurrence density is estimated to be high for all spatial locations that are close to roads,
even for those within the buffer areas of the town of Birch Tree and Alton (northwestern and
southwestern areas in Figure 14a) – where very few fires were actually reported in 1970 –
2002 (Figure 3). We suspected that the effects of spatial covariates that we included were
overwhelmed by some factors we did not have knowledge of. The factors could be the
efficient execution of fire suppression for these two towns, the lack of fire reporting, or
others. Nevertheless, the more complicated model $H_2$ did not show this contradiction to the
fire occurrence data in the estimated spatial distribution of fire occurrence density (Figure
14b) by incorporating Cartesian coordinates in the regression. The estimated fire occurrence
density in the buffer area of Alton is smoothed low (left bottom area in Figure 14d), and high
fire occurrence density is mostly found in the places that are not only along roadways but
also within the specific “hot bed” areas (e.g., the upper middle area right below Winona,
Figure 14b).

We therefore used the model $H_2$ as our final full model to derive fire occurrence
probability in space. The study area was represented as a grid in which each cell has the size
of 1 ha (100 by 100 meters). The normalized fire occurrence density ($\lambda$, number of fires per
m² per decade) was then estimated using the model $H_2$. The fire occurrence probability of
one cell is defined as the probability of having at least one fire occurrence within the 1-ha
cell over a decade. It is calculated from the Poisson probability density function as

$$P(x \geq 1) = 1 - P(x = 0) = 1 - e^{-\lambda \cdot 1000},$$
Where \( x \) denotes number of fire occurrence within the cell. The calculated fire occurrence probability for each cell is shown in Figure 15. It is obvious that spatial pattern of fire occurrence probability is identical to the pattern of fire occurrence density shown in Figure 14b.

### 4.4 Effects of proximity to roads, topography, and land cover on fire occurrence probability

The lurking variable plots against spatial covariates for the null model have already provided some rough ideas on how these factors affect fire occurrence. The coefficients of the chosen models estimated from the fire occurrence data further provided quantitative information on how each spatial covariate affects fire occurrence density, and consequently fire occurrence probability. Table 3 gives the coefficients of predictor variables related to the spatial covariates for the model \( H_2 \). The model is a log-linear regression of intensity as showed in equation (7), where the conditional intensity is equivalent to the intensity for Poisson point processes. The intensity is a product of several multiplicative components, and each component represents the contribution of a predictor variable. Because the variables in the model take non-negative values, variables with positive coefficients have positive contributions to fire occurrence density and hence fire occurrence probability while the ones with negative coefficients have negative contributions. Fire occurrence probability in the locations that are away from roads is in general lower, and a one-meter increase in distance to nearest road makes log-intensity decrease 9.144732e-04 when all other factors are fixed. Fire occurrence probability is also lower in steep places, and one-degree increase in slope makes log-intensity decrease -3.336604e-03 provided other factors are fixed. Fire occurrence
probability first decreases then increases with regard to elevation, since the coefficient for elevation is negative and relatively high (-6.094904e-03) but the coefficient for the square of elevation is positive and relatively low (5.418072e-06). The inflection value is about 1125 ft (i.e., 343 m) when other factors are not considered. Land cover is modeled as a factor in the regression, and the ranking of land cover class in terms of fire occurrence probability is urban (corresponding coefficient is -1.189284) < grassland (corresponding coefficient is 0) < deciduous forest and woodland (corresponding coefficient is 0.5558712) < mixed forest and woodland (corresponding coefficient is 0.6762037).

5. Discussion

5.1 Why topography and fuel are still important in anthropogenic fire regimes

Studies of dendrochronological fire histories from the Missouri Ozarks have identified cultural effects such as roads and agricultural development are replacing fuel and topography to affect the spread, frequency, and size of fires during the modern-era fire regime stage (Guyette et al. 2002). Our results further showed that fire occurrences are spatially clustered, and the accessibility to humans is one of the most important determinants of spatial locations of human-caused fires. Proximity to roads is one aspect of human-accessibility, and the places close to roads (< 500 m) are in general associated with high fire occurrence probability. The roads are often located either on the ridge tops or in the valley bottoms in this region, but most fires are set along the roads in ridge tops (elevation > 900 ft a.s.l.) as shown in the results (e.g., Figure 10d). This is may be because that (1) many roads
in the valleys are eroded more or less by floods, and (2) the moisture of fuels along the valleys is higher than ridge tops, making these places unfavorable for people to start a fire. Steep slope is also a limiting factor because high (> 25 degree) slopes (e.g., bluffs) are difficult for humans to access. The differential effects of various land cover classes (substitute for fuel types) on human-caused fire occurrence probability can also be explained by the characteristics of human behaviors. If an arsonist intends to set a fire, he or she is more likely to choose flammable fuel types such as shortleaf pine and oak forests and woodlands in this region. Therefore, fuel type and topography, which are often modeled in lightning-caused fire occurrence process (e.g., Wotton and Martell, 2005), may still be prominent ingredients in human-caused fire occurrence process, although the explanations of such effects are different. This finding suggests that for an anthropogenic fire regime, vegetation dynamics still interacts with dynamics of fire occurrence process.

5.2 Applications of modeling results

The map of fire occurrence probability estimated from the model can be directly used in fire risk analysis for forest managers. The map helps us not only to pinpoint “hot spot” (high fire occurrence probability) areas, but also to identify areas that should be paid more attention in terms of fire risk. For example, The Irish Wilderness, a 66 km² wilderness area established in 1984 and a part of the Mark Twain National Forest, is located in the southeast of our study area. The area is primarily covered by shortleaf pine and oak forests and woodlands, and deciduous forests. There are no drivable roads in the Wilderness area and motorized or mechanical equipment including bicycles is prohibited. Hence the fire
occurrence probability in this area is very low (Figure 15). However, the fire occurrence probability surrounding this area (especially to the east and west) is high (Figure 15). This makes the chance for the Wilderness area to be burned by the propagation of fire ignited from surroundings be relatively high.

Another important application of our results is in forest landscape modeling of succession and fire disturbance. Fire occurrence simulation is one of the three components in fire simulation. The other two are fire spread simulation and fire effects simulation. A fire occurrence is usually modeled as two consecutive stages – fire ignition and fire initiation (e.g., Antonovski et al., 1992; Davis and Burrows, 1994; Li, 2000; Yang et al., 2004). For these forest landscape models, the number of ignitions is often simulated from a Poisson distribution whose parameter intensity (i.e., fire ignition density) is provided by users as a model input. The most common parameterization of the fire ignition density is purely based on fire frequency distributions in which fire cycle is the only descriptor variable. It hence fails to incorporate the effects of roads and topography on the fire occurrence process. We can derive fire ignition density map from our modeling results and use it as the input data for these models. It will then greatly improve simulation realism in terms of spatial patterns of fire occurrence.

5.3 Implications of the approach and methods

We used a spatial point process modeling approach in analyzing spatial patterns of human-caused fires in Mark Twain National Forest in the southern Missouri Ozark Highlands, reported in 1970 – 2002. Inhomogeneous Poisson process was assumed and a log-
linear regression was used for modeling intensity of fire occurrence with regard to road proximity, elevation, slope, aspect, and land cover. Spatial distribution of fire occurrence density was also affected by some unobserved factors, which were evaluated using Cartesian coordinates as substitutes in the model. The residual analysis was proved to be a very powerful tool in selecting predictor variables and charactering the effects of spatial covariates. The approach used here is statistically rigorous and can be applied elsewhere in the modeling of human-caused or natural-caused fire occurrence process and other point processes in forestry. The approach allows input spatial grid data to have different resolutions (e.g., the resolution of DEM data is 60 m, whilst the resolution of land cover GIS data is 30 m), but it requires all the spatial covariates to be either spatially continuous (e.g., distance to nearest road) or lattice data (e.g., land cover). It is possible in the future studies that a newly added explanatory variable is observed only in the locations of fire occurrence. In this case, one could use geostatistical methods to interpolate from the observed values to the entire region (Rathbun, 1996).
References


Tables

Table 1. Land cover classes from the MoRAP data and the aggregated classes used in the analysis.

<table>
<thead>
<tr>
<th>Land cover class</th>
<th>Count of cells</th>
<th>Aggregated class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban Impervious</td>
<td>1260</td>
<td>Urban</td>
</tr>
<tr>
<td>Urban Vegetated</td>
<td>1392</td>
<td></td>
</tr>
<tr>
<td>Barren or Sparsely Vegetated</td>
<td>456</td>
<td></td>
</tr>
<tr>
<td>Grown Crops</td>
<td>158</td>
<td></td>
</tr>
<tr>
<td>Glade Complex</td>
<td>1407</td>
<td></td>
</tr>
<tr>
<td>Open Water</td>
<td>595</td>
<td></td>
</tr>
<tr>
<td>Cool-season Grassland</td>
<td>220606</td>
<td>Grassland</td>
</tr>
<tr>
<td>Eastern Redcedar-Deciduous Forest and Woodland</td>
<td>28887</td>
<td>Deciduous</td>
</tr>
<tr>
<td>Deciduous Woodland</td>
<td>47433</td>
<td></td>
</tr>
<tr>
<td>Upland Deciduous Forest</td>
<td>742449</td>
<td></td>
</tr>
<tr>
<td>Shortleaf Pine-Oak Forest and Woodland</td>
<td>239844</td>
<td>Shortleaf Pine-Oak</td>
</tr>
<tr>
<td>Shortleaf Pine Forest and Woodland</td>
<td>145808</td>
<td></td>
</tr>
<tr>
<td>SUM</td>
<td>1430295</td>
<td></td>
</tr>
</tbody>
</table>
$$\text{Table 2. Comparison of Poisson point process models fitted to the fire occurrence data}$$

<table>
<thead>
<tr>
<th>Model</th>
<th>Max log-likelihood</th>
<th>Number of parameters</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null model (homogeneous Poisson)</td>
<td>-19233.56</td>
<td>1</td>
<td>38469.12</td>
</tr>
</tbody>
</table>

**Alternative models that only includes spatial covariates**

<table>
<thead>
<tr>
<th>Model</th>
<th>Max log-likelihood</th>
<th>Number of parameters</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>poly (D,1)</td>
<td>-19084.67</td>
<td>2</td>
<td>38173.34</td>
</tr>
<tr>
<td>poly (D,3)</td>
<td>-19081.01</td>
<td>4</td>
<td>38170.02</td>
</tr>
<tr>
<td>poly (D,2) + factor(L)</td>
<td>-19002.73</td>
<td>6</td>
<td>38017.46</td>
</tr>
<tr>
<td>poly (D,1) + factor(L) + poly (E,2)</td>
<td>-18974.99</td>
<td>7</td>
<td>37963.98</td>
</tr>
<tr>
<td>poly (D,1) + factor(L) + poly (E,1) + poly(S,1)</td>
<td>-18971.76</td>
<td>7</td>
<td>37957.52</td>
</tr>
<tr>
<td>*poly (D,1) + factor(L) + poly (E,2) + poly(S,1)</td>
<td>-18969.36</td>
<td>8</td>
<td>37954.72</td>
</tr>
<tr>
<td>poly (D,1) + factor(L) + poly (E,2) + poly(S,1) + poly(A,1)</td>
<td>-18968.43</td>
<td>9</td>
<td>37954.86</td>
</tr>
</tbody>
</table>

**Cartesian coordinates are also considered in the models**

<table>
<thead>
<tr>
<th>Model</th>
<th>Max log-likelihood</th>
<th>Number of parameters</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>poly(x,y,4)</td>
<td>-18896.74</td>
<td>15</td>
<td>37823.48</td>
</tr>
<tr>
<td>poly(x,y,4) + poly (D,1) + factor(L) + poly (E,1) + poly(S,1) + poly(A,1)</td>
<td>-18754.29</td>
<td>22</td>
<td>37552.58</td>
</tr>
<tr>
<td><strong>poly(x,y,4) + poly (D,1) + factor(L) + poly (E,2) + poly(S,1)</strong></td>
<td>-18752.33</td>
<td>22</td>
<td>37548.66</td>
</tr>
</tbody>
</table>

Where poly() denotes polynomial function, D denotes distance to nearest road, L denotes land cover class, E denotes elevation, S denotes slope, and A denotes aspect.

* denote this model is the best one based on AIC in the group of models that only use spatial covariates as predictor variables. The model is denoted as $H_1$ in this paper.

** denote this model is the best one based on AIC in the group of models that use both spatial covariates and Cartesian coordinates as predictor variables. The model is denoted as $H_2$ in this paper.
Table 3. Coefficients of spatial covariates for the model $H_2$

<table>
<thead>
<tr>
<th>Predictor variable</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-6.308161e+09</td>
</tr>
<tr>
<td>Distance to nearest road</td>
<td>-9.144732e-04</td>
</tr>
<tr>
<td>Grassland</td>
<td>0 (assumed)</td>
</tr>
<tr>
<td>Deciduous Forest and Woodland</td>
<td>5.558712e-01</td>
</tr>
<tr>
<td>Shortleaf Pine (mixed with Oak) Forest and Woodland</td>
<td>6.762037e-01</td>
</tr>
<tr>
<td>Urban/Barren</td>
<td>-1.189284e+00</td>
</tr>
<tr>
<td>Slope degree</td>
<td>-3.336604e-03</td>
</tr>
<tr>
<td>Elevation height</td>
<td>-6.094904e-03</td>
</tr>
<tr>
<td>Square of elevation height</td>
<td>5.418072e-06</td>
</tr>
</tbody>
</table>
Figure 1. Study area
Figure 2. The flowchart of the overall approach. Each rectangle stands for a specific procedure, and each rounded rectangle stands for the method used in the corresponding procedure.
Figure 3. The grid image of proximity to primary roads overlaid with primary roadway coverage and reported fire occurrences. The major roads includes US highway 60 and 160, numbered state highway 19 and 99, and lettered state highways (e.g., route K, route AA). The three towns (Alton, Birch Tree, and Winona) are located in the boundary of the study area.
Figure 4. The grid image of aggregated land cover overlaid reported fire occurrences
Figure 5. 60-meter resolution DEM data, coupled with reported fire occurrences
Figure 6. The grid image of slope Calculated from the DEM data, and fire occurrences
Figure 7. The grid image of aspect Calculated from the DEM data, and fire occurrences
Figure 8. Kernel smoothed intensity (expected number of fires per meter) estimated from the fire occurrence data reported in 1970 – 2002
Figure 9. Estimated $K$ function for the observed fires in 1970 – 2002
Figure 10. Lurking variable plots against (a) distance to nearest road, (b) slope, (c) aspect, (d) elevation, (e) x-coordinate, and (f) y-coordinate for the null model fitted to the data. The solid lines are empirical curve of cumulative Pearson residuals. The dotted lines denote two-standard-deviation error bounds.
Figure 11. Lurking variable plots against (a) distance to nearest road, (b) slope, (c) aspect, (d) elevation, (e) x-coordinate, and (f) y-coordinate for the $H_1$ model fitted to the data. The solid lines are empirical curve of cumulative Pearson residuals. The dotted lines denote two-standard-deviation error bounds.
Figure 12. Lurking variable plots against (a) distance to nearest road, (b) slope, (c) aspect, (d) elevation, (e) x-coordinate, and (f) y-coordinate for the $H_2$ model fitted to the data. The solid lines are empirical curve of cumulative Pearson residuals. The dotted lines denote two-standard-deviation error bounds.
Figure 13. Generalized $K$ function, $K_i$ for the final full inhomogeneous Poisson point process model $H_2$. The solid line represents the observed $K_i$, the dotted line is the theoretical $K_i$ function, and the two dashed lines are the upper and lower bounds calculated from 99 simulations.
Figure 14. Fire occurrence density (fires per m$^2$) for all the years 1970 – 2002 calculated from the model (a) $H_1$ and (b) $H_2$, and their corresponding 3D perspective graphs: (c) $H_1$ and (d) $H_2$. 
Figure 15. The grid image of estimated fire occurrence probability, defined as the probability of having at least one fire occurrence for the given 1-ha size cell over a decade.
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