DESIGN OF DISTRIBUTION CHANNEL:

DIRECT SALE vs. MIXED SALE

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by

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ABSTRACT

The design of the distribution channel is one of main decisions for manufacturers and should be determined in the early stage of the production. In this paper, two types of channel distributions are suggested for single product manufacturers. One is a direct sale, which is considered as a general tendency of the times, and the other is a mixed sale, which combines wholesale and retail sale strategies. The expected profit functions for these strategies are formulated and evaluated by different assumptions about customer demands. In particular, the customer demand function is assumed to be uniformly distributed, depending on the sales price. As a special case a linearly bounded demand distribution is further investigated. Based on these conditions, the optimal wholesale quantity and retail price are determined. In conclusion, the results indicate a channel strategy that guarantees the maximum profits based on business conditions.

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CHAPTER 1

INTRODUCTION

1.1 Research Motivation

Business environments have become more complicated and various. Companies have been meeting with cutthroat competition in their market. Consequently, it has led to the rapid growth of distribution strategy to increase profit. As Tsay and Agrawal [32] point out, a number of major developments in business environments also accelerate the diversification of sales channels between manufacturers and consumers. The expanding role of internet and the economics of materials delivery are sufficient reasons to reconsider the distribution strategy. It is a good example that Avon products Inc., a representative company of direct sales, extends their channel to on-line sales [1]. It seems to have an effect on increasing sales. Besides, the successes of competitors, who adjust well to the dramatically changing circumstance, also incite them to change rapidly. For example, HP has revamped its channel strategy to allow its customers to request products shipped directly from its factory [21]. It is without saying that Dell computer's success served as the stimulus for HP. Dell computer is a great success since it cut out intermediaries and built computers to customers' orders and shipped them within days [16]. Dramatically changed business environments and well-adjusted competitors menace most companies to develop their marketing strategies for survival. They realize that they cannot boost their sales with their traditional channel any longer. However, this trend is not only for the survival, but also for the maximum profit. We notice that if manufacturers produce popular products then they have a power to control distribution channels. For example, Logitech Inc., which is a leading company concerning cordless peripherals, such as mice, keyboards, gaming controllers and headsets, makes OEM contracts with big computer companies and retails directly to customers simultaneously [36]. Therefore, the manufacturer sells the products with both OEM brand and its own brand. Another example is Prius, a gasoline-electronic hybrid sedan from Toyota. Though Toyota is producing Prius, a used one is sold for higher price than a new one because customers cannot stand to be on a waiting list [34]. In this case, the manufacturer controls the price and the distribution channel.

In this research, we assume three kinds of basic channels of distribution: a direct sale, an indirect sale, and a mixed sale. Direct sale strategy is to retail to customers directly, so the expected profit is high if demand is high. Indirect sale strategy is to wholesale to intermediaries, so the expected profit is carefree from customer demand. Hence, if customer demand is high then direct sale strategy is more profitable, but if customer demand is low then indirect sale strategy is more profitable. Mixed sale strategy is using both of them, so the manufacturer controls the amount of wholesale and decides channel according to business environment. Since we consider customer demand function, we only compare the direct sale strategy and the mixed sale strategy, excluding the indirect sale strategy, which is not affected by customer demand. We suggest the best distribution channel, the optimal wholesale quantity, and the optimal retail price through considering business environments to maximize the manufacturer's profit.

1.2 Research Scope and Objectives

The primary concern of this research is to find the best distribution channel for a single product produced manufacturer. This research proposes a proper distribution channel between two channels. One is the direct sale and the other is the mixed sale where a manufacturer sells a part of the products to an intermediary and sells the rest of them to customers directly. The expected profit models establish parameters and generate optimal parameters to maximize the profit. Next, the decision making for the distribution channel is considered under the different practical assumptions.

1.3 Research Construction

This research is divided into 6 chapters. First, the research background is described in chapter 1, and the literature review is presented in chapter 2. Next, chapter 3 formulates and analyzes the expected profit functions regarding two cases of distribution channels. Chapter 4 and 5 support the previous chapter through adapting several scenarios. In the subsequent section, the numerical experiments are presented.

Finally, the paper concludes with a summary of results of chapters 3 and 4. All detail computations are given in the appendix.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

Design of distribution channel is no less necessary than production of goods for manufacturers. Telser [29] expresses that manufacturers need to intervene among the retailers in a market. If manufacturers dispose of all products to retailers and leave them as they are, retailers may confer on the maintaining of retail price. It reduces competition among retailers and consequently leads manufacturers to decrease profits. Therefore, manufacturers need to take an active hand in the design of distribution channel. Manufacturers' ultimate purpose is to maximize their profits, so they intend that all players in the distribution channel harmonize well in a flow. Many economists and industrial engineers have studied channel distributions for decades, so there is abundant research regarding distribution channels. Almost all research is roughly divided into two streams. One is the channel structure and the other is the channel coordination. Therefore, we need to review the literature separately, and we find the achievements of them in chronological order.

2.2 Channel Structure

"Channel structure refers to how the players are arranged in the system."

『Bell, Wang and Padmanabhan [3](p.5)』

Distribution channel consists of many parts according to its type. For example, the channel may include a manufacturer, a wholesaler, a retailer and a consumer, etc. No matter what the distribution channel consists of, the members affect one another. Therefore, the channel design and arrangement of players in a flow of a supply chain have been good sources of study in this area.

McGuire and Staelin [23] 1983

Their research is based on channel structure study. They assume that there are two single-item manufacturers, who hold most power in producer-retailer dyads. If they distribute products through either manufacturer owned stores or privately owned retailers then three cases of structure are formed as follows: 1) decentralized case (Both manufacturers distribute products through private retailers.); 2) centralized case (Both manufacturers distribute products through company stores.); and 3) mixed case (One manufacturer distributes products through private retailer and the other manufacturer distributes products through own store.) Since each player in structure affects the other, they view channel selection as a game. An issue of great interest in their research is the price sensitivity between two manufacturers. What they found using this concept is that a channel decision is determined by the uniqueness of a manufacturer's product. Highly substitutable products tend to be sold through the decentralized distribution and lowly substitutable products tend to be sold through centralized distribution. In other words, if a manufacturer produces less competitive products, then a centralized distribution is more profitable than a decentralized distribution.

Coughlan [9] 1985

She extends and generalizes the research of McGuire and Staelin [23]. She examines several scenarios to show the importance of middlemen. She claims that middlemen reduce price competition and increase profits for the manufacturers. That is, if a manufacturer produces less competitive products then using middlemen is profitable though they need to pay costs for it.

Rangan [27] 1987

His research shows how manufacturers should decide on their policy for distribution channel. He proposes distribution functions and determines the kind of distribution channel, the number of intermediaries and the level of service. Though his research is not well received, it is important because it differs from others in that it considers levels of service for customers.

Choi [5] 1991

Almost all researchers who study channel structure or channel coordination reference this work. His research is an epoch-making attempt in terms of channel interactions. He considers three kinds of channel interactions: manufacturer stackelberg, vertical nash, and retailer stackelberg. His game rule is given as follows, where w_i and p_i are wholesale price and retail price, respectively (274).

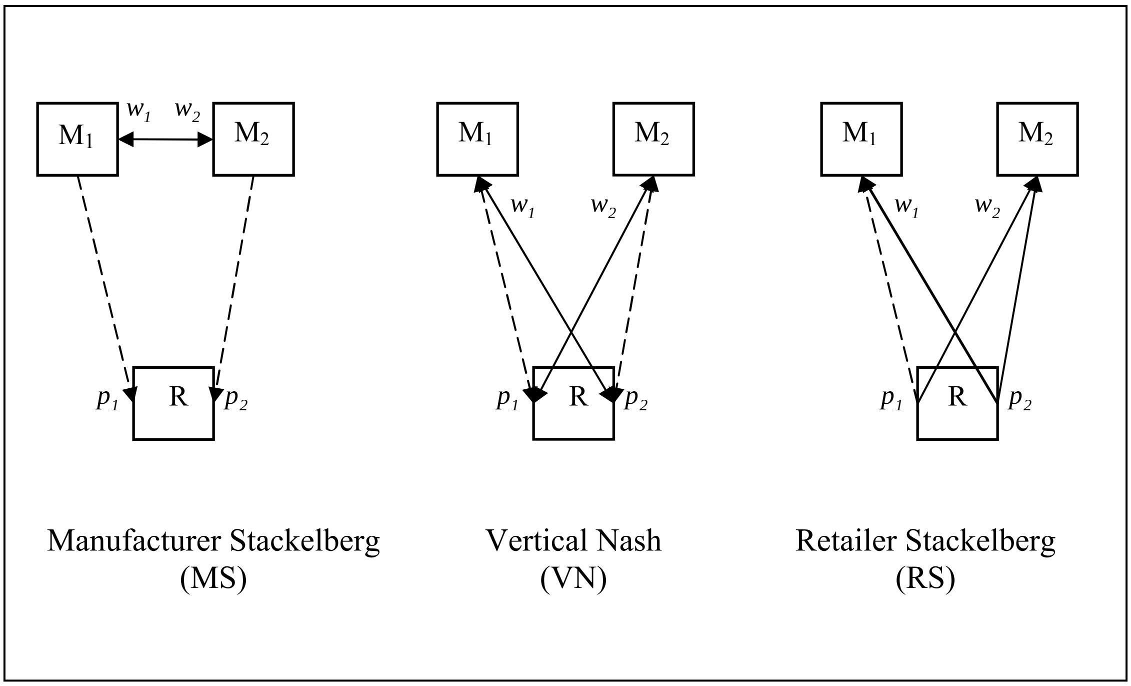

Figure 2.1.1 Choi's Game Rule

Additionally, he examines his model with not only linear demand but also nonlinear demand. He shows the effects of product differentiation and costs based on each player's profits.

Cohen, Agrawal, Agrawal and Raman [6] 1995

They analyze distribution strategies in the industrial paper and plastic industry and save $446 million for the industry through redistribution. They suggest manufacturers use middlemen (i.e., redistributors) in order to mitigate direct price competition at the manufacturer level. If manufacturers sell products to middlemen, they sell them to their customers which are distributors. In this case, they charge lower price to them as compared with manufacturers. It is profitable to distributors, so demand increases. It boosts sales of middlemen and consequently profits manufacturers. Therefore, using middlemen is advantageous to all players in the distribution channel.

Purohit [26] 1997

He takes an example of U. S. automobile industry to suggest various channel structures. He analyzes three structures: 1) separate channel (rental agencies rent cars and dealers sell cars); 2) overlapping channel (rental agencies sell some of their rental cars); and 3) buyback channel (manufacturers buy rental cars from rental agencies and sell them through dealers). He concludes that manufacturers make the most profit from the overlapping channel, and the separate channel does not pay very well. For this reason, manufacturers jump into the retail market with the direct sale strategy. This decision gives manufacturers the benefit because it makes retailers compete with one another.

Fingleton [11] 1997

He notices that many manufacturers change their distribution channels from using middlemen to direct selling. The case of Dell computer [21] is watched by his research. While some manufacturers are distressed with choosing middlemen as retailers, the others shift their channels from using middlemen to direct selling. By examining the competition among middlemen and direct selling, he finds that direct selling mitigates covering of intermediaries. Additionally, he shows that the increasing competition of direct selling gives a positive effect on the market.

Trivedi [30] 1998

He introduces the idea of "store substitutability". It comes from McGuire and Staelin [23], which suggests the notion "substitutability". Store substitutability implies price sensitivity between two stores. He assumes three cases of distribution channels like McGuire and Staelin [23] and obtains results as a special case.

Blasubramanian [2] 1998

His research deals with the competition of the direct versus the retail. He points out that retailers are now faced with the entry of direct sellers. The rival of competition is not only conventional retailers but also direct sellers. He also considers the duty of information in the multiple-channel market and finds that maladjusted products to the direct channel cause reducing price to both direct and retail sellers.

Lynch and Ariely [22] 2000

The interest of their research is that they experiment with 72 participants. They consist of students and staffs, who are asked to buy some wine from either of two online shops. Their shopping patterns indicate consumer behavior. Experiment results are tested by price, quality and store-comparability. Their research has a limitation that they assume only two online stores as retailers. They concentrate on the change of price sensitivity and market share between the two stores.

Kadiyali, Chintagunta and Vilcassim [20] 2000

The most important concept of their research is “power”. They suppose that retailers have more bargaining power than manufacturers in these days. They extend the research of Choi [5] to generalize the model and test it through real issues like juice product category and tuna category. Consequently, they find retailers have substantial pricing power for the brands.

Hendershott and Zhang [14] 2001

The idea of their research is similar to the research we are conducting. They compare direct sale and indirect sale. Especially, direct on-line sale is suggested as a method of direct sale like the research of Lynch and Ariely [22]. They find that in direct sale, manufacturers may boost sales by price discrimination, but they need to allow for many returns. They assert that if direct sale is competitive to intermediary sale then it produces good results in social welfare because of reducing price. Direct sale is accordingly affirmative parts in market. They also determine the proper products for direct on-line sale. These products are characterized by less immediacy and less bulk. Their insistence seems related to Avon products Inc., which shift their sale from direct sale to direct on-line sale [1].

Shaffer and Zettelmeyer [28] 2002

They show a relationship between third-party information and manufacturer's profits. Third-party information means consumer's preference, which is made from internet or media. We easily understand that manufacturers benefit from positive information about their products. However, they show that manufacturer's benefit from positive information about rival's products if it pertains to core consumers of rival. In addition, retailers benefit from positive information about products if it pertains to all consumers. Theses conclusions have a very wide application.

2.3 Channel Coordination

"Channel coordination encompasses the extent to which channel member activities are in alignment and are mutually beneficial."『Bell, Wang and Padmanabhan[3](p.5)』

With study of channel structure, the study of coordination, which promotes sales with all players together and sharing profits, has been another agony to manufacturers. Manufacturers make great effort to harmonize all players in distribution channels, especially retailers, to find a good device for maximizing profits.

Jeuland and Shugan [19] 1983

They call researchers attention to the channel coordination. They show that the channel coordination is advantageous to all players in the distribution channel. However, if the division of profit between manufacturers and retailers lacks charity then the channel coordination is impracticable because they are often independent entities. Hence, they propose a quantity discount schedule to settle this subject.

McGuire and Staelin [24] 1986

They extend previous research, McGuire and Staelin [2], by exploring additional channel strategies. Though they agree with Jeuland and Shugan [19] on the importance of channel coordination, their idea is somewhat different. They characterize coordination mechanisms with two-parts tariffs, which are a fixed fee plus a per unit price.

Moorthy [25] 1987

His research is interesting in that he argues against the result of Jeuland and Shugan [19]. He is doubtful of the effectiveness of a quantity discount because of its complexity. Additionally, he asserts that it may violate the Robinson-Patman Act. He proposes two-part tariffs and franchise fees as alternatives to allocate profits. He emphasizes if the products are sold to the retailer at the manufacturer's marginal cost then channel coordination is achieved.

Gerstner and Hess [12] 1995

They are very emphatic on the importance of coordination. They use the notation "pull pricing" as a contrary concept of push pricing. Push pricing is a wholesale price discount. This idea is based on the assumption that manufacturers have a power in setting price. A coupon is a good example of pull pricing. It is very effective when manufacturers give some incentives to intermediaries and target price conscious customers. They are clear that manufacturers perform channel coordination through pull price promotion.

Ingene and Parry [17] 1995

They have expressed their doubts to the channel coordination. They are suspicious whether manufacturers would actually maximize their own profits by coordinating the channel. They prove that if manufacturers make intermediaries compete with one another then there is no need to pay attention to the coordination.

Desiraju and Moorthy [9] 1997

They highlight that retailers have better information about customer demand than manufacturers. They find the real business environments quite different from what Jeuland and Shugan [19] and Moorthy [25] have suggested. Since intermediaries today check the demand condition with automated electronic scanner checkout systems, they can get more information than manufacturers. It causes an inequality of information between intermediaries and manufacturers. Ultimately, they propose several guides to close the gap of information.

Ingene and Parry [18] 1998

They extend their research, Ingene and Parry [17], through deriving the optimal wholesale price for manufacturers. It is shown that retailer fixed cost bears on an optimal wholesale price and a fixed fee.

Corbett and Kaarmarkar [7] 2001

They consider an entry decision and a post-entry competition. They support that price decreases according to increasing entrants by examination of their model. They also define the outcome of the vertical integration in different market types. When a manufacturer has a monopoly on market, vertical integration always increases its profit. On the other hand, if manufacturers have an oligopoly on the market then total profit is decreased by competition.

Chiang, Chhajed and Hess [4] 2003

They notice that many manufacturers change their channel distribution by engaging in a direct sale. They show that a direct channel is helpful to increase manufacturer's profit. According to what they insist, a direct sale intimidates retailers into the price reduction and spurs the customer demands in a retail market. It leads a manufacturer to increase its profit. In other words, a direct sale plays an important part in the stem of high price and makes both manufacturer and retailer more profitable.

2.4 Summary

Up to the present, manufacturers have persevered in their efforts to improve profits. It consequently leads that many researchers have endeavored to suggest the optimal distribution channel strategy. Though the ways of study are various, all efforts have the same goal, maximizing profits. However, the literature shows that a distribution channel is decided by not only the character of goods or service but also the condition of business environments.

As above reviewed, research about a distribution channel is divided into a structure and a coordination. In channel structure, research analyzes channel conflicts. The main idea is to provoke intermediaries to compete in the market and this promotes manufacturers' profits. This explains why many manufacturers are rushing into the retail market to raise competition these days. In channel coordination, the most interesting issues are harmonizing all players to boost sales and portioning out profits to them. There are several suggestions for them like a wholesale discount, a two-parts tariffs and a pull pricing, etc. Additionally, there is a new trial to integrate a channel structure and coordination. Gupta and Loulou [13] insist that channel equilibrium is accomplished by two facts: differentiation and costs.

This paper follows the stream of channel structure. As most research deals with single product manufacturers, this paper also simplifies the models. In addition, the manufacturer competes with intermediaries by controlling the wholesale quantity and price. As the research of Chiang, Chhajed and Hess [4], this paper considers the manufacturer's use of the direct sale strategy with customers. Our paper will also compare a mixed channel strategy to a direct sale strategy. Therefore, the optimal distribution channel is proposed through comparing them.

CHAPTER 3

MODEL DEVELOPMENT AND ANALYSIS - FIXED RETAIL PRICE CASE

3.1 Problem Description

It is assumed that there are three kinds of basic channels of distribution: a direct sale, an indirect sale and a mixed sale as shown figure 3.1. However, we consider only the direct sale and the mixed sale in this chapter, because many companies recently adopted the direct sale [21], and the mixed sale is the traditional channel in opposition to the indirect sale, which is too simple and too carefree with customer demands.

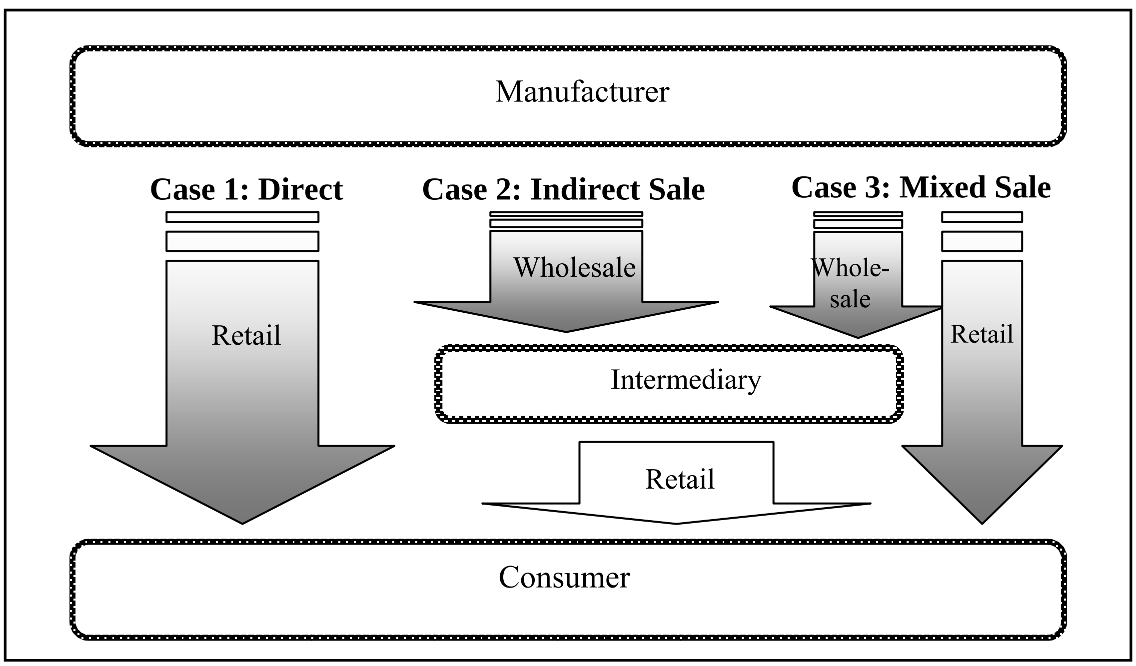

Figure 3.1 Three Cases; direct sale, indirect sale and mixed sale

In this chapter, a manufacturer who produces a single product is discussed as a subject of investigation. It is assumed that the manufacturer has two kinds of distribution channels: one is a direct manufacturer-consumer sale and the other is an indirect sale through an intermediary such as a wholesaler. The manufacturer-consumer direct sale often occurs through online stores or manufacturer-owned outlets. Manufacturers of today commonly have on-line stores, because on-line sale saves time and effort of customers thanks to the progress of internet and delivery system. Since on-line stores are easily accessible and payable, customers do not need to go shopping. This new system of shopping increases sales to manufacturers. There are many instances of companies. Especially, as Hendershott and Zhang [14] point out that the proper products for direct on-line sale is characterized by less immediacy and less bulk. We can easily find an example of this with Avon products Inc., which shifts their sale from direct to direct on-line [1].

We consider two strategies of a manufacturer who is interested in pursuing manufacturer-consumer direct sale. One is using only the direct sale and the other is using both the direct sale and conventional sale through an intermediary. More specifically, the manufacturer first sells some items to the intermediary based on the sales contract and then sells the rest of them to customers directly under the mixed sale strategy. See figure 3.2 for the graphical representation of these two strategies.

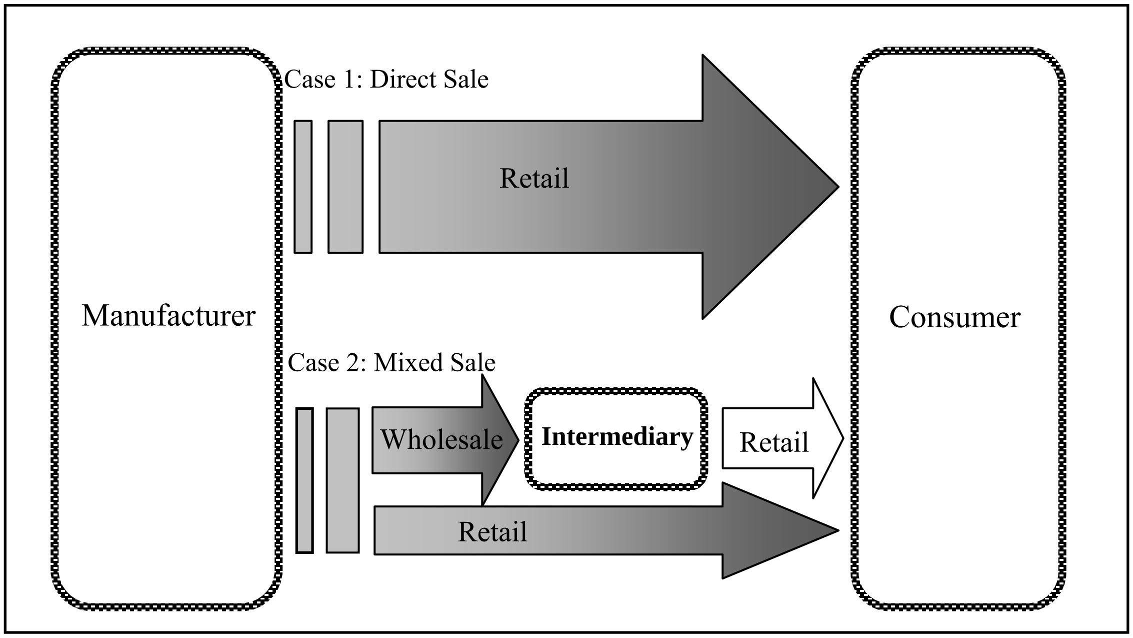

Figure 3.2 Two Cases; direct sale and mixed sale

Our model can describe other similar scenarios. For example, consider a manufacturer, who produces the same products under two brands. The manufacturer sells own brand for the direct sale or makes an OEM contract with buyer for the mixed sale. OEM is very popular with a high-technology industry as we see an example from Dell Computer Inc. [16]. Many big enterprises use OEM to curtail production cost.
OEM has a disadvantage to manufacturers in that if sales of manufacturer depend too much on an OEM contractor, like a big enterprise, then a relationship is easily controlled by the big enterprise. In spite of this, it still has more advantages than disadvantages with small-business manufacturers. It is because manufacturers can learn high technological knowledge and boost sales using sales power of big enterprises. In addition, a market can be divided into domestic and international. A manufacturer sells directly for the domestic market or sells some to the overseas

intermediary for the international market. International sale guarantees stable profits to manufacturers regardless of demands, relaxes restrictions on imports, and reduces disapproval for an unknown company. We find an example from Festiva, which has been sold since 1987 in the U. S. A. Festiva was designed by Mazda Motors of Japan and built by Kia Motors of Korea. Kia Motors sells with a brand Kia Pride in Korea, and Ford Motors sells with a brand Ford Festiva. [35] In case of Kia Motors, Kia uses Ford as an intermediary to let U.S. customers know its manufacturing technique before aggressive marketing in the U. S. A. Figure 3.2 shows how our model is applied to various scenarios.

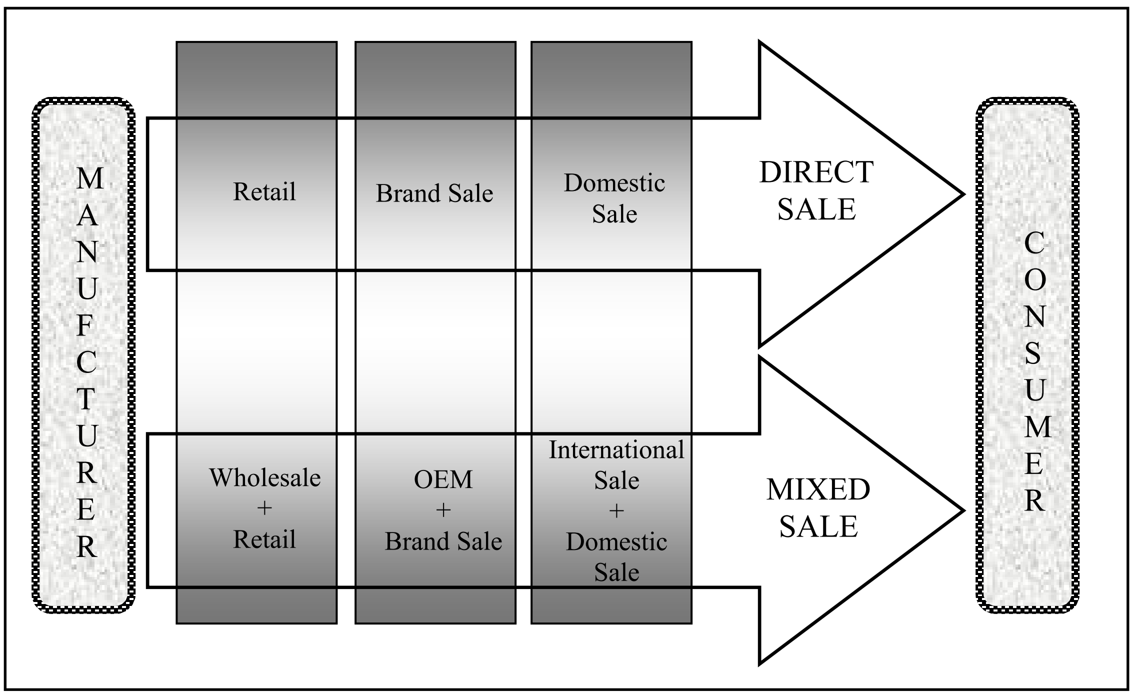

Figure 3.3 Extensions of the model

The chapter is organized as follows. First, mathematical models are formulated for two cases, which are direct sale and mixed sale. Since we assume that salvage, wholesale and retail prices are fixed, it is shown that the model of mixed sale is concave in terms of the wholesale quantity. Next, the optimal wholesale quantity is proposed for mixed sale strategy. The ultimate object is making a decision between the direct sale and the mixed sale, so that the condition of optimal strategic decision is determined. We show that if the difference between wholesale price and salvage price is large enough then the direct sale strategy is more profitable to the manufacturer and if the difference is small then the mixed channel strategy is more profitable. We also find a decision point to change the channel. The chapter concludes with the summary of analysis.

3.2 Formulation

In this section, we develop mathematical models that describe two strategies of the manufacturer such as direct sale and mixed sale. We shall use the following notations.

D_1 : Wholesale Quantity through mixed sale

p_0 : Retail price per item through direct sale and mixed sale

p_1 : Wholesale price per item through mixed sale

s : Salvage price per item through direct sale and mixed sale

c : Production cost per item of a manufacturer

Q : Production quantity of a manufacturer

$\phi(x)$: Customer demand function

$g(0)$: Expected profit function for direct sale

$f(D_1)$: Expected profit function for mixed sale

Assumptions

1. This research considers a single product with one manufacturer.
2. The manufacturer distributes products by centralized system.
3. A reorder is not allowed.
4. It is assumed that c , s , Q , p_0 , p_1 is fixed. Hence, the manufacturer controls only the wholesale quantity.
5. Customer demand is not affected by wholesaler's market, so the manufacturer has same customer demand regardless of distribution types.
6. When the manufacturer transacts business with intermediaries, the number of retailers can be either one or more. However, if the manufacturer deals with more than one retailer then they should be treated equally. That is, the manufacturer offers same wholesale price to them. This follows the Robinson-Patman Act and has an effect that there is one intermediary.
7. Shortage cost is not considered, so if the customer demand is higher than the production quantity then the manufacturer is satisfied with the sale of the whole quantity and does not suffer a financial loss from overproduction.
8. Intuitively, the manufacturer offers lower price to the intermediary than the consumer.
9. A salvage price can be less than zero, because a disposal costs a great deal according to circumstances. However, it is always lower than the wholesale price. Hence, the relationship among the retail price, the wholesale price and the salvage price is $p_0 \geq p_1 \geq s$.

10. The manufacturer can sell products to the intermediary within its whole production quantity. Therefore, the production quantity is always larger than or equal to the wholesale quantity, $Q \geq D_1$.

11. For the case of mixed sale, the manufacturer has a power to control the wholesale quantity.

For the case of only direct sale, the expected profit of a manufacturer consists of the sales profits and salvage costs. (APPENDIX) Using $g(0)$ to represent the profit of a manufacturer, we have

$$\begin{aligned} g(0) &= \int_0^Q \left[p_0 x + s(Q-x)\right]\phi(x)dx + \int_Q^\infty p_0 Q\phi(x)dx - cQ \\ &= \int_0^Q (p_0 - s)x\phi(x)dx + (p_0 - c)Q - (p_0 - s)Q\Phi(Q) \end{aligned} \tag{1}$$

On the other hand, the expected profit function of the mixed sale is more complicated. Since the manufacturer sells some amounts of products to the intermediary first, the wholesale profit is maintained regardless of customer demands. Then, the direct sale profit and the salvage cost are added to the wholesale profit. If we assume that $f(D_1)$ represents the profit of a manufacturer when D_1 items are sold to the intermediary, then we have

$$\begin{aligned} f(D_1) &= p_1 D_1 + \int_0^{Q-D_1} \left[p_0 x + s(Q - D_1 - x)\right]\phi(x)dx + \int_{Q-D_1}^\infty p_0 (Q - D_1)\phi(x)dx - cQ \\ &= p_1 D_1 + \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx + (Q - D_1)p_0 - (p_0 - s)(Q - D_1)\Phi(Q - D_1) - cQ \end{aligned} \tag{2}$$

Finally, we formulate the expected profit function for the direct sale strategy, $g(0)$ and the expected profit function for the mixed sale strategy, $f(D_1)$.

3.3 Analysis

In this section, the previous models are analyzed by deriving the optimal wholesale quantity, D_1 to maximize the profit function for the mixed sale on the assumption that the direct sale price is fixed.

Lemma 3.1 $f(D_1)$ is concave in D_1.

Proof Observe that equation (2) is expanded as

$$
\begin{aligned}
f(D_1) = {} & p_1 D_1 + \int_0^{Q-D_1} p_0 x\phi(x)dx + \int_0^{Q-D_1} sQ\phi(x)dx - \int_0^{Q-D_1} sD_1\phi(x)dx - \int_0^{Q-D_1} sx\phi(x)dx \\
& + \int_{Q-D_1}^{\infty} p_0 Q\phi(x)dx - \int_{Q-D_1}^{\infty} p_0 D_1\phi(x)dx - cQ
\end{aligned}
$$

Then, it is differentiated as follows.

$$
\begin{aligned}
\frac{\partial f(D_1)}{\partial D_1} = {} & p_1 + (Q-D_1)p_0\phi(Q-D_1) - sQ\phi(Q-D_1) - [-sD_1\phi(Q-D_1) + \int_0^{Q-D_1} sQ\phi(x)dx] \\
& + (Q-D_1)s\phi(Q-D_1) + p_0 Q\phi(Q-D_1) - [p_0 D_1\phi(Q-D_1) + \int_{Q-D_1}^{\infty} p_0\phi(x)dx] \\
= {} & p_1 - \int_0^{Q-D_1} sQ\phi(x)dx - \int_{Q-D_1}^{\infty} p_0\phi(x)dx \\
& + (-p_0 Q + p_0 D_1 - sQ + sD_1 + sQ - sD_1 + p_0 Q - p_0 D_1)\phi(Q-D_1) \\
= {} & p_1 + [-s\Phi(Q-D_1)] - [p_0 - p_0\Phi(Q-D_1)] \\
= {} & p_1 - p_0 + (p_0 - s)\Phi(Q-D_1)
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
\frac{\partial^2 f(D_1)}{\partial D_1{}^2} &= s\phi(Q-D_1) - p_0\phi(Q-D_1) \\
&= -(p_0 - s)\phi(Q-D_1)
\end{aligned}
\tag{4}
$$

Since $p_0 \geq s$, $\frac{\partial^2 f(D_1)}{\partial D_1{}^2} \leq 0$ and $f(D_1)$ is concave. This completes the proof.

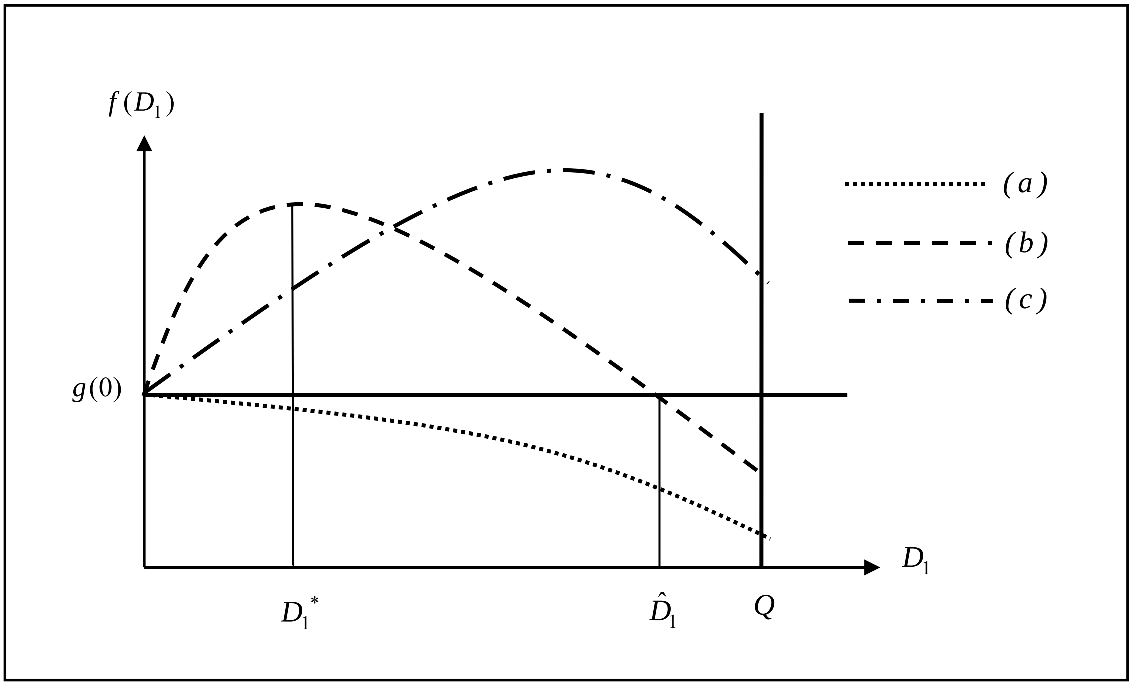

Figure 3.4 Expected graphs for the mixed sale function

Based on the concavity of $f(D_1)$, we can consider three different cases depicted in figure 3.4. Note that $g(0)$ is constant and $g(0) = f(0)$. As seen, $g(0)$ is always larger than $f(D_1)$ in case (a). That is, the direct sale only strategy is always better. On the other hand, the mixed sale strategy is always better than the direct sale only in case (c). In case (b), the mixed sale strategy is better than the direct sale only when $D_1 < \hat{D}_1$. The optimal wholesale quantity in this case is also determined to be D_1^*. It is natural for the manufacturer to seek the best channel strategy to maximize the profit. Since the retail and wholesale prices are fixed in this model, the most profitable strategy is determined by the optimal wholesale quantity D_1^*, which is identified later in this section.

Theorem 3.1

1) If $\Phi(Q) \le \dfrac{p_0 - p_1}{p_0 - s}$ then the direct sale only strategy is always better than the mixed sale strategy.

2) If $\dfrac{\int_0^Q \Phi(x)dx}{Q} < \dfrac{p_0 - p_1}{p_0 - s} < \Phi(Q)$ then

the direct sale is better when $D_1 \ge \hat{D}_1 = \left(\dfrac{p_0 - s}{p_0 - p_1}\right)\int_{Q-\hat{D}_1}^{Q} \Phi(x)dx$,

the mixed sale is better when $D_1 < \hat{D}_1 = \left(\dfrac{p_0 - s}{p_0 - p_1}\right)\int_{Q-\hat{D}_1}^{Q} \Phi(x)dx$.

3) If $\dfrac{\int_0^Q \Phi(x)dx}{Q} \ge \dfrac{p_0 - p_1}{P_0 - s}$ then the mixed sale is better than the direct sale.

Proof It is known that the function increases if $\dfrac{\partial f(D_1)}{\partial D_1} = p_1 - p_0 + (p_0 - s)\Phi(Q - D_1) > 0$

at the point of $D_1 = 0$. Hence, we know that

if $\dfrac{p_0 - p_1}{p_0 - s} < \Phi(Q)$ then the graph will be (a), i.e. $f(D_1)$ decreases at $D_1 = 0$,

if $\dfrac{p_0 - p_1}{p_0 - s} \ge \Phi(Q)$ then the graph will be (b) or (c), i.e. $f(D_1)$ increases at $D_1 = 0$.

Then, let's assume that $D_1 = Q$ to divide the graphs, (b) and (c). If $D_1 = Q$ then the profit of $f(D_1) = f(Q)$ and it is comparable to $g(0)$.

Recall (2),

$$f(D_1) = p_1 D_1 + \int_0^{Q-D_1}(p_0 - s)x\phi(x)dx + (Q - D_1)p_0 - (p_0 - s)(Q - D_1)\Phi(Q - D_1) - cQ$$

If Q substitutes for D_1, (4) changes to $f(Q) = p_1 Q$.

Recall (1), $g(0) = \int_0^Q (p_0 - s)x\phi(x)dx + (p_0 - c)Q - (p_0 - s)Q\Phi(Q)$

Now, they are compared by

$$g(0) : f(Q) = \int_0^Q (p_0 - s)x\phi(x)dx + (p_0 - c)Q - (p_0 - s)Q\Phi(Q)Q : (p_1 - c)Q$$

This shows whether the graph (a), (c) or (b).

Only the graph (b) has the smaller profit than $g(0)$ when $f(D_1) = f(Q)$.

$$g(0) - f(Q)$$
$$= \int_0^Q (p_0 - s)x\phi(x)dx + p_0 Q - (p_0 - s)Q\Phi(Q) - p_1 Q$$
$$= \int_0^Q (p_0 - s)x\phi(x)dx + (p_0 - p_1)Q + (p_0 - s)\Phi(Q)$$
$$= (p_0 - p_1)Q + (p_0 - s)[\int_0^Q x\phi(x)dx - Q\Phi(Q)]$$
$$= (p_0 - p_1)Q + (p_0 - s)[\int_0^Q x\phi(x)dx - Q\int_0^Q x\phi(x)dx]$$
$$= (p_0 - p_1)Q + (p_0 - s)[\int_0^Q (x - Q)\phi(x)dx]$$
$$= (p_0 - p_1)Q - (p_0 - s)[\int_0^Q (Q - x)\phi(x)dx]$$
$$= (p_0 - p_1)Q - (p_0 - s)[\int_0^Q (Q - x)\phi(x)dx]$$
$$= (p_0 - p_1)Q - (p_0 - s)\int_0^Q \Phi(x)dx$$

In this equation, $\int_0^Q (Q - x)\phi(x)dx = \int_0^Q \Phi(x)dx$ because

$$\begin{aligned} \int_0^Q (Q - x)\phi(x)dx &= \Phi(x)(Q - x)\Big|_0^Q - \int_0^Q \Phi(x)(-1)dx \\ &= \left[\Phi(Q)(Q - Q) - \Phi(0)(0 - x)\right] + \int_0^Q \Phi(x)dx \\ &= \int_0^Q \Phi(x)dx \end{aligned}$$

When $g(0) - f(Q) = (p_0 - p_1)Q - (p_0 - s)\int_0^Q \Phi(x)dx > 0$, $f(D_1)$ is lower than $g(0)$ at the point of $D_1 = Q$. Hence, we understand that

if $\dfrac{p_0 - p_1}{p_0 - s} > \dfrac{\int_0^Q \Phi(x)dx}{Q}$ then the graph will be (a) or (b),

i.e. $f(Q)$ is smaller than $g(0)$

if $\frac{p_0 - p_1}{p_0 - s} \leq \frac{\int_0^Q \Phi(x)dx}{Q}$ then the graph will be (c),

i.e. $f(Q)$ is larger than $g(0)$.

Consequently, it is simplified by as follows.

If $\frac{p_0 - p_1}{p_0 - s} < \Phi(Q)$ and $\frac{p_0 - p_1}{p_0 - s} > \frac{\int_0^Q \Phi(x)dx}{Q}$ then the graph (a), which is the direct sale is always better than the mixed sale,

i.e. $f(D_1)$ decreases at $D_1 = 0$ and $f(Q)$ is smaller than $g(0)$.

If $\frac{p_0 - p_1}{p_0 - s} \geq \Phi(Q)$ and $\frac{p_0 - p_1}{p_0 - s} > \frac{\int_0^Q \Phi(x)dx}{Q}$ then the graph is (b),

i.e. $f(D_1)$ increases at $D_1 = 0$ and $f(Q)$ is smaller than $g(0)$.

If $\frac{p_0 - p_1}{p_0 - s} \geq \Phi(Q)$ and $\frac{p_0 - p_1}{p_0 - s} \leq \frac{\int_0^Q \Phi(x)dx}{Q}$ then the graph is (c), which is the mixed sale that is always better than the direct sale,

i.e. $f(D_1)$ increases at $D_1 = 0$ and $f(Q)$ is larger than $g(0)$.

Hence, the condition for the graphs is as follows. Figure 3.5 shows the condition graphically.

Graph (a): direct sale $\Phi(Q) \leq \frac{p_0 - p_1}{p_0 - s}$

Graph (b): direct sale /mixed sale $\frac{\int_0^Q \Phi(x)dx}{Q} < \frac{p_0 - p_1}{p_0 - s} < \Phi(Q)$

Graph (c): mixed sale $\frac{p_0 - p_1}{p_0 - s} \leq \frac{\int_0^Q \Phi(x)dx}{Q}$

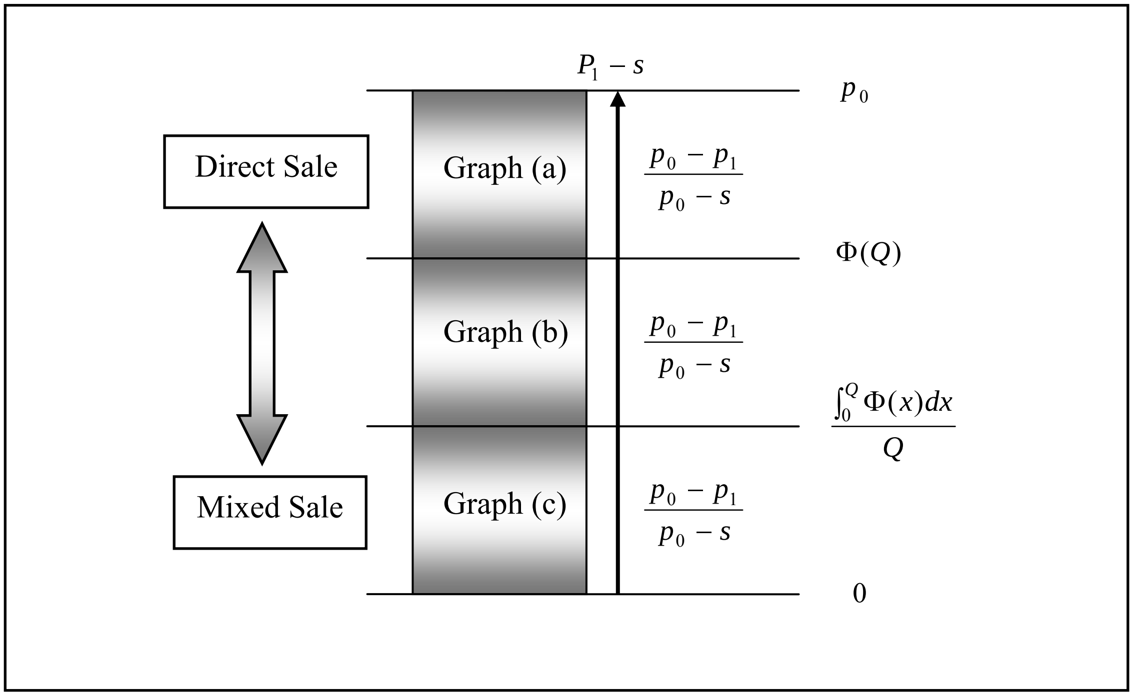

Figure 3.5 The condition for the mixed sale graphs

Managerial Insights

- Since we assume that the salvage, wholesale and retail prices are fixed, it is shown that the graph for the channel decision is determined by the difference between the wholesale price p_1 and the salvage price s.
- If the difference between p_1 and s is small then $\frac{p_0 - p_1}{p_0 - s}$ is large and the graph will be (a), which means the manufacturer should distribute products through the direct sale. If $\frac{p_0 - p_1}{p_0 - s}$ is low then the graph (c) is drawn and the manufacturer should maintain only the mixed channel.
- As the difference between p_1 and s increases, the channel strategy changes from the direct sale to mixed sale. Since the wholesale profits gratify the

manufacturer's stable income regardless of demand, if s is too small to compare to p_1 then the direct sale is risky. On the other hand, if the difference is not much then the direct sale is worth a try because the manufacturer may expect much profit from direct sale.

Therefore, this condition is acceptable to suggest a decision of the distribution channel.

Now, the point of intersection $\hat{D}_1$ should be defined for the case of graph (b). In figure 3.4, we can find the point that the channel strategy is changed from mixed sale strategy to direct sale strategy. We represent it as $\hat{D}_1$. The expected profit from the mixed sale strategy is higher than the expected profit from the direct sale strategy until $\hat{D}_1$, but it is reversed. Since $\hat{D}_1$ is the point of intersection between $g(0)$ and $f(D_1)$, the profits of two functions are same at $\hat{D}_1$. Hence, $\hat{D}_1$ can be obtained from solving $g(0) = f\left(D_1\right)$. Let us assume that $g(0) = f\left(D_1\right)$. From (1) and (2),

$$\begin{aligned} g(0) &= \int_0^Q \left[p_0 x + s(Q-x)\right]\phi(x)dx + \int_Q^\infty p_0 Q\phi(x)dx - cQ \\ &= \int_0^Q (p_0 - s)x\phi(x)dx + (p_0 - c)Q - (p_0 - s)Q\Phi(Q) \end{aligned} \quad (1)$$

$$\begin{aligned} f(D_1) &= p_1 D_1 + \int_0^{Q-D_1} \left[p_0 x + s(Q - D_1 - x)\right]\phi(x)dx + \int_{Q-D_1}^\infty p_0 (Q - D_1)\phi(x)dx - cQ \\ &= p_1 D_1 + \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx + (Q - D_1)p_0 - (p_0 - s)(Q - D_1)\Phi(Q - D_1) - cQ \end{aligned} \quad (2)$$

$$g(0) = f(D_1) = \int_0^Q (p_0 - s)x\phi(x)dx + p_0 Q - (p_0 - s)Q\Phi(Q) - cQ$$

$$= p_1 D_1 + \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx + (Q - D_1)p_0 - (p_0 - s)(Q - D_1)\Phi(Q - D_1) - cQ$$

$$\begin{aligned}
&g(0) - f(D_1) \\
&= \int_0^Q (p_0 - s)x\phi(x)dx + p_0 Q - (p_0 - s)Q\Phi(Q) \\
&- p_1 D_1 - \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx - (Q - D_1)p_0 + (p_0 - s)(Q - D_1)\Phi(Q - D_1) \\
&= p_0 Q - p_1 D_1 - p_0 Q + p_0 D_1 \\
&+ (p_0 - s)\left[\int_0^Q x\phi(x)dx - \Phi(Q) - \int_0^{Q-D_1} x\phi(x)dx + (Q - D_1)\Phi(Q - D_1)\right] \\
&= (p_0 - p_1)D_1 + (p_0 - s)\left[\int_0^Q x\phi(x)dx - \int_0^Q Q\phi(x)dx - \int_0^{Q-D_1} x\phi(x)dx + \int_0^{Q-D_1} (Q - D_1)\phi(x)dx\right] \\
&= (p_0 - p_1)D_1 + (p_0 - s)\left[-\int_0^Q (Q - x)\phi(x)dx + \int_0^{Q-D_1} ((Q - D_1) - x)\Phi(x)dx\right] \\
&= (p_0 - p_1)D_1 + (p_0 - s)\left[-\int_0^Q \Phi(x)dx + \int_0^{Q-D_1} \Phi(x)dx\right] \\
&= (p_0 - p_1)D_1 + (p_0 - s)\left[-\int_{Q-D_1}^Q \Phi(x)dx\right] \\
&= (p_0 - p_1)D_1 - (p_0 - s)\int_{Q-D_1}^Q \Phi(x)dx = 0
\end{aligned}$$

From $g(0) - f(D_1) = (p_0 - p_1)D_1 - (p_0 - s)\int_{Q-D_1}^{Q_1} \Phi(x)dx = 0$,

$\hat{D}_1$ is derived as $\hat{D}_1 = \left(\dfrac{p_0 - s}{p_0 - p_1}\right)\int_{Q-\hat{D}_1}^Q \Phi(x)dx$ and $\dfrac{\int_{Q-\hat{D}_1}^Q \Phi(x)dx}{\hat{D}_1} = \dfrac{p_0 - p_1}{p_0 - s}$.

$$\hat{D}_1 = \left(\frac{p_0 - s}{p_0 - p_1}\right)\int_{Q-\hat{D}_1}^Q \Phi(x)dx. \qquad (5)$$

This proves the turning point of distribution channel.

Theorem 3.2 For the cases of the graph (b) and (c) in theorem 3.1,

$$D_1^* = Q - \Phi^{-1}\left(\frac{p_0 - p_1}{p_0 - s}\right)$$

Proof Because the expected profit function of the mixed sale is concave in D_1,

solving $\dfrac{\partial f(D_1)}{\partial D_1} = 0$ yields the optimal wholesale quantity D_1^*.

From (5), $\frac{\partial f(D_1)}{\partial D_1} = p_1 - p_0 + (p_0 - s)\Phi(Q - D_1) = 0$.

$$\Phi(Q - D_1) = \left(\frac{p_0 - p_1}{p_0 - s}\right)$$

$$Q - D_1 = \Phi^{-1}\left(\frac{p_0 - p_1}{p_0 - s}\right)$$

$$D_1^* = Q - \Phi^{-1}\left(\frac{p_0 - p_1}{p_0 - s}\right) \qquad (6)$$

Managerial Insights

- Since Q is the production quantity and D_1 is the wholesale quantity, $Q - D_1$ is the amount of products sold directly to customers. Since $\frac{p_0 - p_1}{p_0 - s} = 1 - \left(\frac{p_1 - s}{p_0 - s}\right)$, the optimal quantity of $Q - D_1$ decreases with the wholesale price p_1 and increases with the retail price p_0. If p_1 is low and p_0 is high then it is profitable to retain a large amount of the product for the direct sale because the manufacturer may increase profit from the direct sale. On the other hand, if p_1 is high and p_0 is low then the manufacturer needs to utilize the wholesale as much as possible because the wholesale quantity guarantees the fixed profit regardless of customer demands.
- The optimal $Q - D_1$ increases with the salvage price s. If the manufacturer overproduces goods then the rest of sales is disposed with s. In this case, if s

is low then it generates a potentially larger loss when the customer demand is small. However, the relatively large s reduces the potential loss, so the manufacturer needs to retain a large amount of product for the direct sale to increase the expected profit.

- D_1^* is production quantity minus expected sales volume from direct sale strategy. If the manufacturer sets aside expected sales volume in retail market out of production quantity and wholesales the remainder then the manufacturer's profit is maximized because we assume the customer demand in retail market is not affected by the wholesaler's market.
- The expected sales volume from direct sale strategy is determined by

$$\Phi(x) = \frac{p_0 - p_1}{p_0 - s} \quad \Rightarrow \quad x = \Phi^{-1}\left(\frac{p_0 - p_1}{p_0 - s}\right).$$

Therefore, as retail price and salvage price increase, $\frac{p_0 - p_1}{p_0 - s}$ increases and the expected sales volume from direct sale strategy also increases. In other words, if wholesale price increases and salvage price decreases then $\frac{p_0 - p_1}{p_0 - s}$ decreases and expected sales volume from direct sale also decreases, but D_1^* increases.

Let us summarize our results based on the graph (a), (b) and (c) in figure 3.4 and figure 3.5, and identify the best channel for maximizing the manufacturer's profit in table 3.1. Since all prices and production quantity are fixed, only D_1^* is determined. As seen, we select the best channel through computing $\frac{p_0 - p_1}{p_0 - s}$ and $\hat{D}_1$.

As $\frac{p_0 - p_1}{p_0 - s}$ increases, the optimal selling channel moves from mixed sale strategy to direct sale strategy. If p_1 and s increase then $\frac{p_0 - p_1}{p_0 - s}$ decreases, so $\frac{p_0 - p_1}{p_0 - s}$ can be understood as the loss rate that comes from the wholesale.

Graph	Standard Judgment	Best Channel	Optimal D_1^*
(a)	$\Phi(Q) \le \frac{p_0 - p_1}{p_0 - s}$	direct sale	$D_1^* = 0$
(b)	$\frac{\int_0^Q \Phi(x)dx}{Q} < \frac{p_0 - p_1}{p_0 - s} < \Phi(Q)$	$\hat{D}_1 = \left(\frac{p_0 - s}{p_0 - p_1}\right)\int_{Q-\hat{D}}^{Q} \Phi(x)dx$ if $D_1 \ge \hat{D}_1$, direct sale if $D_1 < \hat{D}_1$, mixed sale	if $D_1 \ge \hat{D}_1$, $D_1^* = \hat{D}_1$ if $D_1 < \hat{D}_1$, $D_1^* = Q - \Phi^{-1}\left(\frac{p_0 - p_1}{p_0 - s}\right)$
(c)	$\frac{p_0 - p_1}{p_0 - s} \le \frac{\int_0^Q \Phi(x)dx}{Q}$	mixed sale	$D_1^* = Q - \Phi^{-1}\left(\frac{p_0 - p_1}{p_0 - s}\right)$

Table3.1 Explanation for the each case

Additionally, $\Phi(Q)$ is the rate that the manufacturer sells whole production quantity in retail market. Hence, if $\Phi(Q) \le \frac{p_0 - p_1}{p_0 - s}$, the expected sale rate in terms of production quantity is less than the loss rate from wholesale, the manufacturer should sell products through direct sale only strategy.

On the other hand, $\int_0^Q \Phi(x)dx$ is the expected value from direct sale, so $\frac{\int_0^Q \Phi(x)dx}{Q}$ is the expected value for each item. If $\frac{p_0 - p_1}{p_0 - s} \le \frac{\int_0^Q \Phi(x)dx}{Q}$, the loss rate from mixed sale is less than the expected value from direct sale for each item, then the manufacturer should sell products through mixed sale only strategy. Hence, D_1^* is the amounts products quantity minus the expected sales volume in retail market,

$$D_1^* = Q - \Phi^{-1}\left(\frac{p_0 - p_1}{p_0 - s}\right).$$

Though the channel decision is decided by various factors such as business environments, substitution of products, and competitors, we sort the products only according to the retail price, wholesale price and salvage price. The character of direct sale products is a high retail price and a high salvage price like cars, jewelry, cosmetics and furniture. The character of mixed sale products is a high wholesale price and a low salvage price like vegetables, dairy products and electronics.

3.4 Summary

In this chapter, we characterized the wholesale quantity, D_1, that shows a optimal channel distribution maximizing the profit for the manufacturer as a strategic variable based on a fixed retail price.

A manufacturer who produces a single product considers two kinds of distribution channel: direct and mixed sale strategy. The manufacturer's profit function is formed by considering all other factors: the production price, the wholesale price, the retail price, the salvage price, the wholesale quantity, the production quantity, and the customer demand. We assume that all prices are fixed, so the model is analyzed through controlling the wholesale quantity to suggest the best distribution channel. The expected profit function, $f(D_1)$, is concave in D_1, and the condition to choose the distribution channel is determined. If $\frac{p_0 - p_1}{p_0 - s}$ is large enough then the direct sale is more profitable, but if it is small enough then the mixed sale is more profitable. That is, the characters of products appropriate for the mixed sale include high p_1 and low s. On the other hand, products appropriate for the direct sale only has high p_0 and high s. In addition, the point of intersection, which is the turning point to change the distribution channel from the mixed sale to the direct sale is determined as $\hat{D}_1 = \left(\frac{p_0 - s}{p_0 - p_1}\right) \cdot \int_{Q-\hat{D}}^{Q} \Phi(x)dx$. If $D_1 > \hat{D}_1$, the distribution channel strategy should be changed from the mixed channel to the direct channel. Simultaneously, D_1^* is also described as $D_1^* = Q - \Phi^{-1}\left(\frac{p_0 - p_1}{p_0 - s}\right)$, which means that the optimal wholesale quantity is the rest of customer demand out of production quantity.

CHAPTER 4

MODEL DEVELOPMENT AND ANALYSIS - FIXED RETAIL PRICE CASE WITH UNIFORMLY DISTRIBUTED CUSTOMER DEMAND FUNCTION

4.1 Problem Description

In the previous chapter, we developed a mathematical model for the direct sale strategy and the mixed sale strategy using the general customer demand function $\phi(x)$. In this chapter, we analyze the previous mathematical models with the assumption that the customer demand function $\phi(x)$ follows a uniform distribution. Further more, the uniform distributions is bounded by two functions, $a(p_0)$ and $b(p_0)$, which are general but depend on the retail price p_0. The uniformly distributed customer demand used by many researchers including Wadhwa, John, and Gandhi [33]. Simplifies the mathematical model and allows us to further analyze the problem. When the customer demand function distributes uniformly on some range, two functions make the shape of range. Therefore, we can assume that $\phi(x)$ consists of two functions, $a(p_0)$ and $b(p_0)$, and the range of customer demands extends from $a(p_0)$ to $b(p_0)$. Since we already have mathematical models for the direct sale strategy and the mixed sale strategy, we convert these models, $g(0)$ and $f(D_1)$, into new forms through substituting

$\frac{1}{b(p_0)-a(p_0)}$ for $\phi(x)$. Then we determine the best channel strategy to maximize the manufacturer's profit and define the optimal wholesale quantity. Finally, the numerical experiments are conducted to verify the result.

4.2 Formulation

Based on the following assumption, the previous mathematical models are transformed to the new forms, which include uniformly distributed customer demand function.

Assumption

1. Customer demand functions $a(p_0)$ and $b(p_0)$ are not defined, but $b(p_0)$ is always larger or equal than $a(p_0)$. Hence, $a(p_0) \le x \le b(p_0)$.
2. The possible retail quantity for the mixed sale, $Q - D_1$, is within the limit of $[a(p_0), b(p_0)]$. Hence, $a(p_0) \le Q - D_1 \le b(p_0)$.
3. $\phi(x)$ is uniformly distributed customer demand function on interval $[a(p_0), b(p_0)]$.

Since $\phi(x)$ is uniformly distributed customer demand function on interval $[a(p_0), b(p_0)]$, we substitute $\frac{1}{b(p_0)-a(p_0)}$ for $\phi(x)$. Hence, the manufacturer's profit functions transform from (1), (2) to (7), (8). (Appendix)

$$
\begin{aligned}
g(0) &= \int_0^Q [p_0 x + s(Q-x)]\phi(x)dx + \int_Q^\infty p_0 Q\phi(x)dx - cQ \\
&= \int_0^Q (p_0 - s)x\phi(x)dx + (p_0 - c)Q - (p_0 - s)Q\Phi(Q) \\
&= \frac{(s-p_0)[Q-a(p_0)]^2 + 2Q(p_0-c)[b(p_0)-a(p_0)]}{2[b(p_0)-a(p_0)]} \qquad (7) \\
&= (p_0 - c)Q + \frac{(s-p_0)[Q-a(p_0)]^2}{2[b(p_0)-a(p_0)]}
\end{aligned}
$$

$$
\begin{aligned}
f(D_1) &= p_1 D_1 + \int_0^{Q-D_1} [p_0 x + s(Q-D_1-x)]\phi(x)dx + \int_{Q-D_1}^\infty p_0(Q-D_1)\phi(x)dx - cQ \\
&= p_1 D_1 + \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx + (Q-D_1)p_0 - (p_0-s)(Q-D_1)\Phi(Q-D_1) - cQ \qquad (8) \\
&= (p_0 - c)Q - (p_0 - p_1)D_1 - \frac{(p_0-s)[Q-D_1-a(p_0)]^2}{2[b(p_0)-a(p_0)]}
\end{aligned}
$$

Now, we have newly transformed mathematical models for the direct and mixed sale strategy.

4.3 Analysis

In this section, we show that $f(D_1)$ is concave in D_1 when $\phi(x)$ is uniformly distributed. Additionally, the optimal wholesale quantity D_1^* is determined for this case.

Lemma 4.1 $f(D_1)$ is concave in D_1 when $\phi(x) = \dfrac{1}{b(p_0)-a(p_0)}$.

Proof $f(D_1)$ is transformed from (2) to (8) based on $\phi(x) = \dfrac{1}{b(p_0)-a(p_0)}$.

From (8), $f(D_1)$ is differentiated to as follows.

$$f(D_1) = (p_0 - c)Q - (p_0 - p_1)D_1 - \frac{(p_0 - s)\left[Q - D_1 - a(p_0)\right]^2}{2\left[b(p_0) - a(p_0)\right]}$$

$$\begin{aligned}
\frac{\partial f(D_1)}{\partial D_1} &= p_1 \\
&\quad + \frac{1}{2}\left\{\frac{\left[(s - p_0)\left[(Q - D_1) - a(p_0)\right]^2 + 2(Q - D_1)p_0\left[b(p_0) - a(p_0)\right]\right]' \cdot \left[b(p_0) - a(p_0)\right]}{\left[b(p_0) - a(p_0)\right]^2}\right\} \\
&\quad - \frac{1}{2}\left\{\frac{\left[(s - p_0)\left[(Q - D_1) - a(p_0)\right]^2 + 2(Q - D_1)p_0\left[b(p_0) - a(p_0)\right]\right] \cdot \left[b(p_0) - a(p_0)\right]'}{\left[b(p_0) - a(p_0)\right]^2}\right\} \\
&= p_1 + \frac{\left[-2(s - p_0)\left[(Q - D_1) - a(p_0)\right] - 2p_0\left[b(p_0) - a(p_0)\right]\right] \cdot \left[b(p_0) - a(p_0)\right]}{2\left[b(p_0) - a(p_0)\right]^2} \\
&= p_1 - \frac{2(s - p_0)\left[(Q - D_1) - a(p_0)\right] - 2p_0\left[b(p_0) - a(p_0)\right]}{2\left[b(p_0) - a(p_0)\right]} \\
&= p_1 - p_0 + \frac{(p_0 - s)\left[Q - D_1 - a(p_0)\right]}{\left[b(p_0) - a(p_0)\right]}
\end{aligned} \tag{9}$$

$$\frac{\partial^2 f(D_1)}{\partial D_1{}^2} = \frac{s - p_0}{\left[b(p_0) - a(p_0)\right]} < 0 \tag{10}$$

Since we assume that $b(p_0) \geq a(p_0)$ and $p_0 \geq s$, $\frac{\partial^2 f(D_1)}{\partial D_1{}^2} \leq 0$. Hence, $f(D_1)$ is concave in D_1 when $\phi(x) = \frac{1}{b(p_0) - a(p_0)}$.

Theorem 4.1 $D_1^* = Q - a(p_0) - \left(\frac{p_0 - p_1}{p_0 - s}\right)\left[b(p_0) - a(p_0)\right]$ when $\phi(x) = \frac{1}{b(p_0) - a(p_0)}$.

Proof Since $f(D_1)$ is concave, solving $\frac{\partial f(D_1)}{\partial D_1} = 0$ deceives $D_1{}^*$.

$$\frac{\partial f(D_1)}{\partial D_1} = p_1 - p_0 + \frac{(p_0 - s)\left[Q - D_1 - a(p_0)\right]}{\left[b(p_0) - a(p_0)\right]} = 0$$

$$(p_0 - s)[Q - D_1 - a(p_0)] = (p_0 - p_1)[b(p_0) - a(p_0)]$$

$$[Q - D_1 - a(p_0)] = \frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}$$

Hence, $D_1^* = Q - a(p_0) - \left(\dfrac{p_0 - p_1}{p_0 - s}\right)[b(p_0) - a(p_0)]$. (11)

This ends the proof and we clarify the optimal wholesale quantity for uniformly distributed customer demand function.

4.4 Numerical Experiment

We validate our theoretical results using numerical examples and identify the best channel strategy.

Example 1

Assume that the manufacturer produces MP3 Player. The production quantity is 5000 per quarter and the production cost is \$70 per unit. Let us assume that the retail price is \$220 per unit and the wholesale price is \$130 per unit. The salvage price is \$110 per unit and customer demands distribute U [2000, 6000] which means $a(p_0) = 2000$ and $b(p_0) = 6000$. Then, the expected profit from the direct sale strategy is \$626,250. In addition, the optimal wholesale quantity is zero and the expected maximum profit from the mixed sale strategy is the same as from the direct sale strategy. Since the expected profit from the mixed sale strategy decreases as the wholesale quantity increases, the manufacturer should use the direct sale strategy. On the other hand, let us assume that all conditions are the same except the wholesale price is \$200 per unit. Then, the expected profit from the direct sale strategy is the same, \$626,250. However, when the optimal wholesale quantity is changed to 2272 then the expected maximum profit from the mixed sale strategy is \$697,273. In this case, it is the most profitable strategy that the manufacturer sells 2272 of the products to the intermediary and sells 2728 to the customer directly. Table 4.1 summarizes these results.

Q	p_0	c	p_1	s	Demands	Loss Rate	D_1^*	$g(0)$	$f(D_1^*)$	Gain (%)
5000	220	70	130	110	U[2000,6000]	0.82	0	626250	626250	0
			200			0.18	2272		697273	11.34

Table 4.1 Result of Example1

Consider the table 4.1. As we observed the relationships among retail price, wholesale price, and salvage price, the loss rate shows what channel should be selected. If we analyze table 4.1 using $\frac{p_0 - p_1}{p_0 - s} = 1 - \frac{p_1 - s}{p_0 - s}$,

- As the retail price and salvage price increase, the loss rate from the mixed sale, $\frac{p_0 - p_1}{p_0 - s}$ increases and the direct sale strategy is more profitable.
- As the wholesale price increases and salvage price decreases, the loss rate from the mixed sale, $\frac{p_0 - p_1}{p_0 - s}$ decreases and the mixed sale strategy is more profitable.

We represent these conclusions with figure 4.1 as follows.

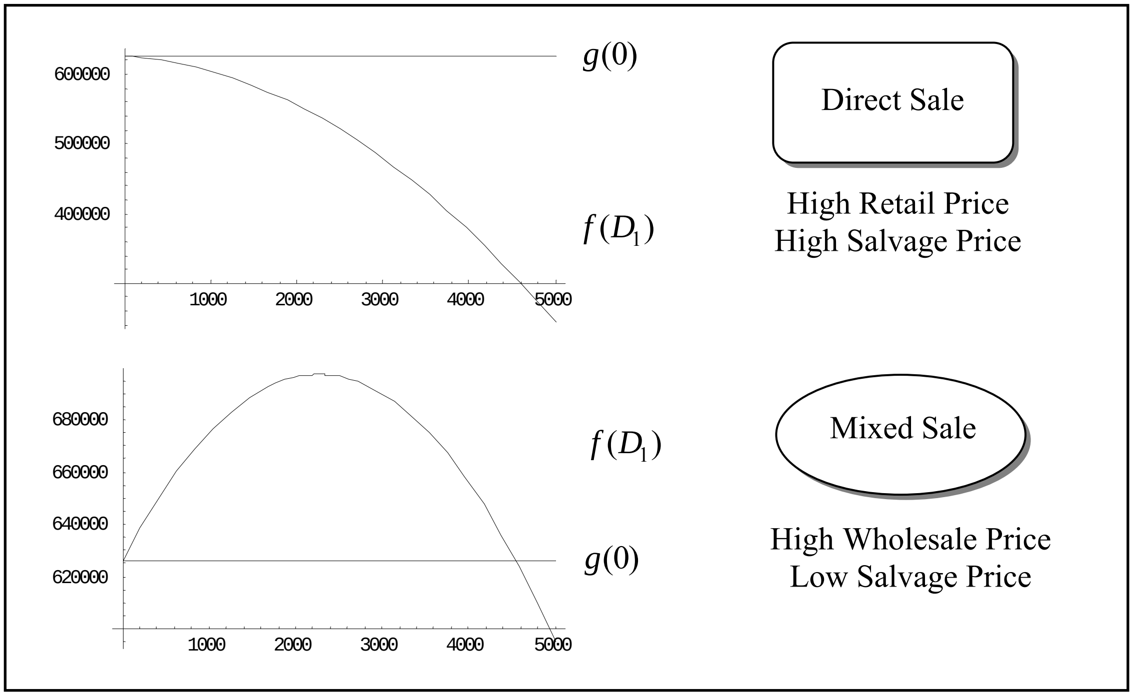

Figure 4.1 Analysis of example1

Until now, we verify relationship among p_0, p_1 and s. Now, we examine the relationships among all other factors.

Example 2

Based on the example 1, a specific experiment is conducted to know how much each factor affects the manufacturer's profit. In this case, six variables are considered: production quantity, retail price, production cost, wholesale price, salvage price and customer demand. These variables are tested through computing for three values, which are high, middle, and low. Table 4.2 shows all conditions. We compute the expected profit from the direct sale strategy and the mixed sale strategy with all middle values at first, then compute them with high and low values in order. Computations are carried out like the example 1 and the optimal wholesale quantity is calculated on interval $[0,Q]$ by Mathematica. (Appendix) Table 4.3 summarizes the results. As seen, the gain profits from the mixed sale strategy when compared to the direct sale strategy increases as wholesale quantity and wholesale price increase and retail price and salvage price decrease. Therefore, if wholesale price increases and salvage price decreases, then the gain increases. If retail price and salvage price increases, then the gain decreases. Hence, the manufacturer should maintain only direct sale strategy if there is high retail price, salvage price and customer demand. The manufacturer should utilize the mixed sale strategy if there is high wholesale quantity and optimal wholesale quantity.

Value	Alias	A	B	C	D	E	F
	Variables	Q	p_0	c	p_1	s	Demands
High(+)		6000	260	110	240	140	U[3000,9000]
Middle		5000	250	100	230	130	U[2000,7000]
Low(-)		4000	240	90	220	120	U[1000,5000]

Table 4.2 Data for example2

NO	Alias	Q	p_0	c	p_1	s	Demands	D_1^*	$g(0)$	$f(D_1^*)$	Gain%
1	0	5000	250	100	230	130	U[2000,7000]	2167	642000	698333	7.69
2	A+	6000	250	100	230	130	U[2000,7000]	3167	708000	828333	17.00
3	A-	4000	250	100	230	130	U[2000,7000]	1167	552000	568333	2.96
4	B+	5000	260	100	230	130	U[2000,7000]	1846	683000	727308	6.49
5	B-	5000	240	100	230	130	U[2000,7000]	2545	601000	672273	11.86
6	C+	5000	250	110	230	130	U[2000,7000]	2167	592000	648333	9.52
7	C-	5000	250	90	230	130	U[2000,7000]	2167	692000	748333	8.14
8	D+	5000	250	100	240	130	U[2000,7000]	2583	642000	722083	12.47
9	D-	5000	250	100	220	130	U[2000,7000]	1750	642000	678750	5.72
10	E+	5000	250	100	230	140	U[2000,7000]	2091	651000	699091	7.39
11	E-	5000	250	100	230	120	U[2000,7000]	2231	633000	697692	10.22
12	F+	5000	250	100	230	130	U[3000,9000]	3333	710000	720000	1.41
13	F-	5000	250	100	230	130	U[1000,5000]	1000	510000	676667	32.68

Table 4.3 Result of Example 2

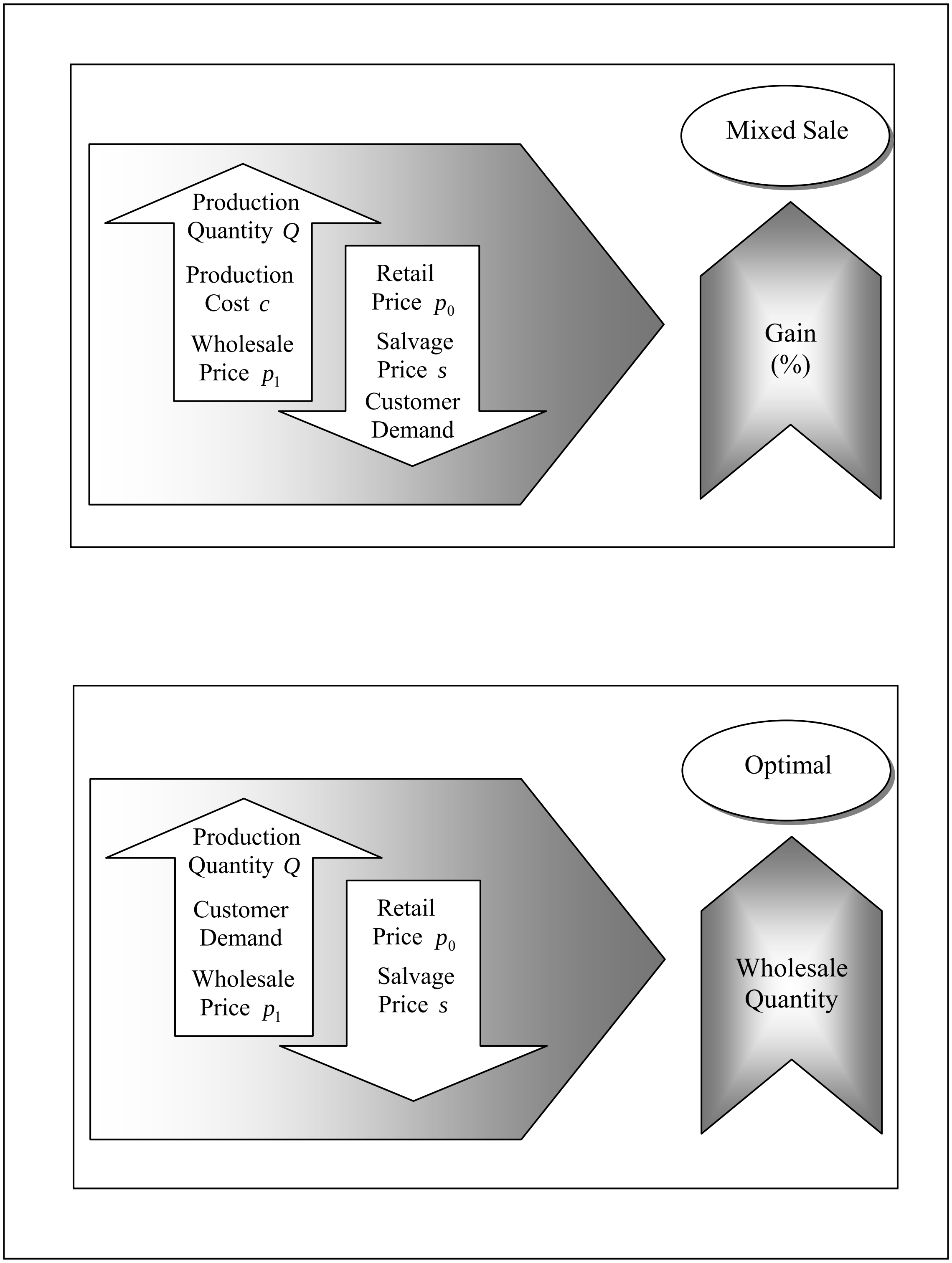

Figure 4.2 Interpretation of Example 2

Several conclusions are drawn as shown in figure 4.2 from the interpretation of table 4.3.

- Data NO 1 shows the result when all variables have middle value, so it is a basic value to compare other experiments.
- Production quantity has positive effect on the gain and the wholesale quantity, so if Q increases then the gain and the wholesale quantity increase. Since D_1^* is production quantity minus customer demand, it increases as much as Q increases. Therefore, $f(D_1^*)$ is more profitable than $g(0)$ if the manufacturer produces more products.
- Retail price has negative effect on the gain and the wholesale quantity, so if p_0 increases then the gain and the wholesale quantity decrease. If p_0 increases, then the difference between p_0 and p_1 increases and the decision for the channel strategy moves from mixed sale to direct sale.
- Production cost has positive effect on the gain, so if c increases then the gain also increases. However, Production cost does not affect the wholesale quantity. Though c has no effect on D_1^* and a negative effect on $g(0)$ and $f(D_1^*)$, the gain increases as c increases because $g(0)$ is more affected than $f(D_1^*)$.
- Wholesale price has positive effect on the gain and the wholesale quantity, so if p_1 increases then the gain and the wholesale quantity increase. It is because p_1 has no effect on $g(0)$ and negative effect on $f(D_1^*)$.

- Salvage price has negative effect on the gain and the wholesale quantity, so if s increases then the gain and the wholesale quantity decrease. It is because $g(0)$ is more affected than $f(D_1^*)$, though s has positive effect on $g(0)$ and $f(D_1^*)$.
- Customer demand has a negative effect on the gain, so if the interval of customer demand is high then the gain decreases. Likewise, if the interval of customer demand is high, then D_1^* is small. It shows that if customer demand is high then direct sale strategy is more profitable.
- Consequently, as D_1^* increases, the gain also increases except the case of c and customer demand.
- The sum of customer demand and D_1^* is Q. It means that the manufacturer's maximized profit is realized when the manufacturer wholesales the rest of customer demand exactly and retails the amounts of customer demand fully.

Managerial Insights

- If the product is high retail price and high salvage price, then the manufacturer should use the direct sale strategy. If the product is high wholesale price and low salvage price, then the manufacturer should use the mixed sale strategy.
- When the manufacturer utilizes only the mixed sale strategy, as the production quantity and the wholesale price increase, the profit increases through wholesaling much quantity. It means that the mixed sale strategy is proper in

this environment to maximize the profit. In addition, the wholesale quantity also increases because this environment guarantees maximum profit.

- However, as the retail price and the salvage price increase, the profit decreases through wholesale less quantity. Though the mixed sale strategy is still more profitable, the manufacturer earns a smaller gain from the mixed sale strategy if the character of product is closer to the direct sale strategy.

4.5 Summary

In the previous chapter, we defined the optimal conditions for the direct sale strategy and the mixed sale strategy. The character of the direct sale products is a high retail price and a high salvage price, and the character of the mixed sale products is a high wholesale price and a low salvage price. However, a general customer demand function was considered. In this chapter, the customer demand function was defined as uniformly distributed between $a(p_0)$ and $b(p_0)$. Since $g(0)$ is a constant, independent from D_1, only $f(D_1)$ is shown concave. Then D_1^* is determined and the condition for the channel decision between the direct sale strategy and the mixed sale strategy is proposed. The numerical experiment supports the result. The direct sale strategy is more profitable when the product has high retail price and high salvage, and the mixed sale strategy is more profitable when the product has high wholesale price and low salvage price, confirming the results of the previous chapter. In addition, we experiment with the case that the manufacturer's maximum profit occurs from the mixed sale strategy. In this example, each factor's effect on the gain and the wholesale quantity is verified. We show that as production quantity and the wholesale price increase, and the retail price and the salvage price decrease, the manufacturer should utilize the mixed sale strategy and increase the wholesale quantity. Consequently, we confirm the result of the previous chapter and specify it with the case of uniformly distributed customer demand function.

CHAPTER 5

MODEL DEVELOPMENT AND ANALYSIS– UNFIXED RETAIL PRICE CASE WITH LINEARY BOUNDED

5.1 Problem Description

The chapter considers a decision of the distribution channel, which is the same as the previous chapters, but discusses the unfixed retail price case in opposition to the fixed retail price case. Since the production price, the wholesale price, and the salvage price are mostly decided by external factors, we assume that the manufacturer controls only the wholesale quantity and the retail price. In fact, the wholesale price is decided by a negotiation with an intermediary in real business. Especially, it happens when the manufacturer has transactions with a big seller like Wal-Mart. Therefore, it is reasonable to consider only the wholesale quantity and the retail price for the strategic points of manufacturers.

In the previous chapter, we assumed that the customer demand function $\phi(x)$, which is uniformly distributed, is bounded by two functions, $a(p_0)$ and $b(p_0)$. In this chapter, we further assume that $a(p_0)$ and $b(p_0)$ are linear functions. Just as Dolan and Simon [10] utilized the decreasing linear function in their example of the electronic power tool sales volume, we use the linear function to determine customer demand. Then we find the optimal wholesale quantity and retail price by a numerical experiment. Finally, we suggest the best channel to maximize the manufacturer's profits.

5.2 Formulation

We reformed the mathematical models for the uniformly distributed customer demand function and determined the optimal wholesale quantity D_1^* for the mixed sale strategy. In this section, we reform $g(0)$ and $f(D_1)$ based on the following assumption.

Assumptions

1. Customer demand functions $a(p_0)$ and $b(p_0)$ are linear functions.
2. $a(p_0) = c - ap_0$, $b(p_0) = d - bp_0$ and $a, b, c, d \geq 0$, $d \geq c$.

Therefore, if $p_0 = 0$ then customer demand functions are $a(p_0) = c$, $b(p_0) = d$, that is, customer demands distribute uniformly on interval $[c, d]$. In addition, they decrease as p_0 increases with slopes a, b. Figure 5.1 describes this assumption well.

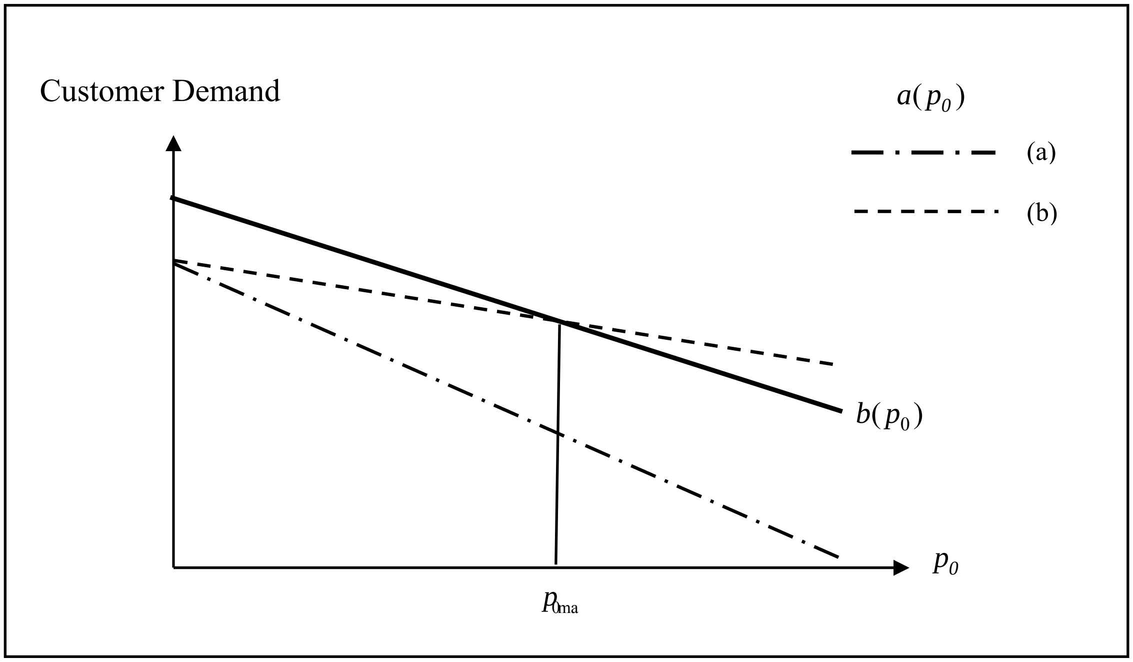

Figure 5.1 Customer demand for the case of linear functions

For the case of graph (a), the graph shows that the lower demand function $a(p_0)$ decreases much more than the higher demand function $b(p_0)$, so slopes are $a \geq b$. It means that if p_0 increases, then the lower demand function is more affected than the higher demand function. This happens when the products are luxuries or there are many other substitutes. In one case, if $b(p_0)$ is parallel with $a(p_0)$, then customer demands decrease as same slope $a = b$ and $p_{0\max}$ occurs when $a(p_0) = 0$.

The graph (b) represents that the lower demand function $a(p_0)$ decreases less than the higher demand function $b(p_0)$, so slopes are $a < b$. It means that the lower demand function is not affected by p_0 as much as the higher demand function. This happens when the products are necessities or there are fewer substitutes. In one case, if the lower demand function is $a(p_0) = c$, then the lower demand is stable regardless of p_0.

Additionally, the point of intersection between $a(p_0)$ and $b(p_0)$ occurs at $p_{0\max}$, which is the largest possible value of p_0.

Since we consider not only the wholesale quantity D_1, but also the retail price p_0 in this chapter, we change $g(0)$ and $f(D_1^*)$ to $g(p_0)$ and $f(D_1^*, p_0)$. From (7) and (8),

$$g(p_0) = (p_0 - c)Q + \frac{(s - p_0)[Q - a(p_0)]^2}{2[b(p_0) - a(p_0)]} \tag{12}$$

$$f(D_1^*, p_0) = (p_0 - c)Q - (p_0 - p_1)D_1 - \frac{(p_0 - s)[Q - D_1 - a(p_0)]^2}{2[b(p_0) - a(p_0)]} \tag{13}$$

D_1^* was verified in the previous chapter, so $f(D_1^*)$ is derived from (2) as follows. (APPENDIX) Recall (11),

$$D_1^* = Q - a(p_0) - \left(\frac{p_0 - p_1}{p_0 - s}\right)[b(p_0) - a(p_0)] \tag{11}$$

$$f(D_1^*, p_0) = (p_1 - c)Q + (p_0 - p_1) \cdot a(p_0) + \frac{(p_0 - p_1)^2 [b(p_0) - a(p_0)]}{2(p_0 - s)} \tag{14}$$

Now, we have new mathematical models for searching the optimal retail price.

5.3 Analysis

Until now, the mathematical models were changed through several steps. In this section, we show that $g(p_0)$ and $f(D_1^*, p_0)$ are concave in p_0 when $a(p_0) = c - ap_0$ and $b(p_0) = d - bp_0$.

Lemma 5.1 $g(p_0)$ is concave in p_0 when $a(p_0) = c - ap_0$, $b(p_0) = d - bp_0$.

Proof From (12), we differentiate $g(p_0)$ in p_0 to define the concavity.

(APPENDIX)

$$g(p_0) = (p_0 - c)Q + \frac{(s - p_0)[Q - a(p_0)]^2}{2[b(p_0) - a(p_0)]} \tag{12}$$

$$\begin{aligned}\frac{\partial g(p_0)}{\partial p_0} = Q &- \frac{[Q - a(p_0)]^2 + 2a(p_0)'(s - p_0)[Q - a(p_0)]}{2[b(p_0) - a(p_0)]} \\ &+ \frac{(s - p_0)[b(p_0)' - a(p_0)'][Q - a(p_0)]^2}{2[b(p_0) - a(p_0)]^2}\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\partial^2 g(p_0)}{\partial {p_0}^2} = &\frac{(s - p_0)\{2a(p_0)'^2 - a(p_0)''[Q - a(p_0)]\}}{[b(p_0) - a(p_0)]} \\ &+ \frac{(s - p_0)[Q - a(p_0)]^2 [b(p_0)'' - a(p_0)'']}{2[b(p_0) - a(p_0)]^2} \\ &- \frac{(s - p_0)[Q - a(p_0)]^2 [b(p_0)' - a(p_0)']}{[b(p_0) - a(p_0)]^3}\end{aligned} \tag{16}$$

Since $a(p_0) = c - ap_0$, $b(p_0) = d - bp_0$ and $a, b \geq 0$, $d \geq c$, $a(p_0)' = -a$, $b(p_0)' = -b$, $a(p_0)'' = b(p_0)'' = 0$. Then, equations are converted as follows.

$$\begin{aligned}\frac{\partial g(p_0)}{\partial p_0} = Q &- \frac{[Q - c + ap\ \ ]^2 - 2a(s - p_0)[Q - c + ap_0]}{2[d - c - (b - a)p_0]} \\ &- \frac{(s - p_0)[Q - c + ap_0](b - a)}{2[d - c - (b - a)p_0]^2}\end{aligned} \tag{17}$$

$$\frac{\partial^2 g(p_0)}{\partial {p_0}^2} = \frac{2a^2(s - p_0)}{[d - c - (b - a)p_0]} + \frac{(s - p_0)[Q - c - ap_0]^2(b - a)}{[d - c - (b - a)p_0]^3} \tag{18}$$

From (18), $\frac{2a^2(s - p_0)}{[d - c - (b - a)p_0]} < 0$ and $\frac{(s - p_0)[Q - c - ap_0]^2(b - a)}{[d - c - (b - a)p_0]^3} < 0$,

because $p_0 \geq s$, $b(p_0) > a(p_0)$, and $a(p_0) > 0$.

Therefore, $\frac{\partial^2 g(p_0)}{\partial p_0{}^2} < 0$ and $g(p_0)$ is concave.

Lemma 5.2 $f(D_1{}^*, p_0)$ is concave in p_0 when $a(p_0) = c - ap_0$, $b(p_0) = d - bp_0$.

Proof Using the same method as of the previous proof, we differentiate $f(D_1{}^*, p_0)$ in p_0 to define the concavity. (APPENDIX)

$$f(D_1{}^*, p_0) = (p_1 - c)Q + (p_0 - p_1)\cdot a(p_0) + \frac{(p_0 - p_1)^2[b(p_0) - a(p_0)]}{2(p_0 - s)} \tag{14}$$

$$\begin{aligned}\frac{\partial f(D_1{}^*, p_0)}{\partial p_0} &= a(p_0) + (p_0 - p_1)\cdot a(p_0)' + \frac{(p_0 - p_1)(p_0 - 2s + p_1)}{2(p_0 - s)^2}[b(p_0) - a(p_0)] \\ &+ \frac{(p_0 - p_1)^2}{2(p_0 - s)}[b(p_0)' - a(p_0)']\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial^2 f(D_1{}^*, p_0)}{\partial p_0{}^2} &= 2\cdot a(p_0)' + (p_0 - p_1)a(p_0)'' + \frac{(p_1 - s)^2}{(p_0 - s)^3}[b(p_0) - a(p_0)] \\ &+ \frac{(p_0 - p_1)(p_0 - 2s + p_1)}{(p_0 - s)^2}[b(p_0)' - a(p_0)'] + \frac{(p_0 - p_1)^2}{2(p_0 - s)}[b(p_0)'' - a(p_0)'']\end{aligned} \tag{20}$$

Since $a(p_0) = c - ap_0$, $b(p_0) = d - bp_0$ and $a, b \ge 0$, $d \ge c$, $a(p_0)' = -a$, $b(p_0)' = -b$, $a(p_0)'' = b(p_0)'' = 0$. Then, equations are converted as follows.

$$\frac{\partial f(D_1{}^*, p_0)}{\partial p_0} = c - 2ap_0 + p_1 + \frac{1}{2(p_0 - s)^2}\left\{\begin{array}{l} -2(b-a)p_0{}^3 \\ +[(b-a)(3s + 2p_1) + (d-c)]p_0{}^2 \\ -[4sp_1(b-a) + 2s(d-c)]p_0 \\ +sp_1{}^2(b-a) + (2sp_1 - p_1{}^2)(d-c) \end{array}\right\} \tag{21}$$

$$\frac{\partial^2 f(D_1{}^*, p_0)}{\partial p_0{}^2} = \frac{(p_1 - s)^2}{(p_0 - s)^3}[d - c - (b-a)s] - (a+b) \tag{22}$$

From (22), $\frac{\partial^2 f(D_1^*, p_0)}{\partial p_0^{\,2}} = \frac{(p_1 - s)^2}{(p_0 - s)^3}\left[d - c - (b-a)s\right] - (a+b)$

$\frac{(p_1 - s)^2}{(p_0 - s)^3}$ is always positive increasing function, because $p_0 \geq p_1 \geq s$.

The other values are constants. Therefore, if $\frac{(p_1 - s)^2}{(p_0 - s)^3} \leq \frac{a+b}{d - c - (b-a)s}$ then

$f(D_1^*, p_0)$ is concave.

In (21), $-(b+a) \leq 0$, because $a, b \geq 0$. Additionally, the minimal value of p_0,

$p_{0\min} = p_1$ because $p_0 \geq p_1 \geq s$. Let us assume that $p_0 = p_1$. Then, (21) is

changed as

$$
\begin{aligned}
\frac{\partial f(D_1^*, p_0)}{\partial p_0} = c - ap_0 - a(p_0 - p_1) + \frac{(p_0 - p_1)(p_0 - 2s + p_1)}{2(p_0 - s)^2}\left[d - c - (b-a)p_0\right] \\
-\frac{(p_0 - p_1)^2}{2(p_0 - s)}(b-a) \\
= c - ap_0
\end{aligned}
$$

Since $c - ap_0 = a(p_0) \geq 0$, $f(D_1^*, p_0)$ increases at the starting point $p_0 = p_1$ in terms of p_0. Hence, the function of manufacturer's profit is depicted as figure 5.2.

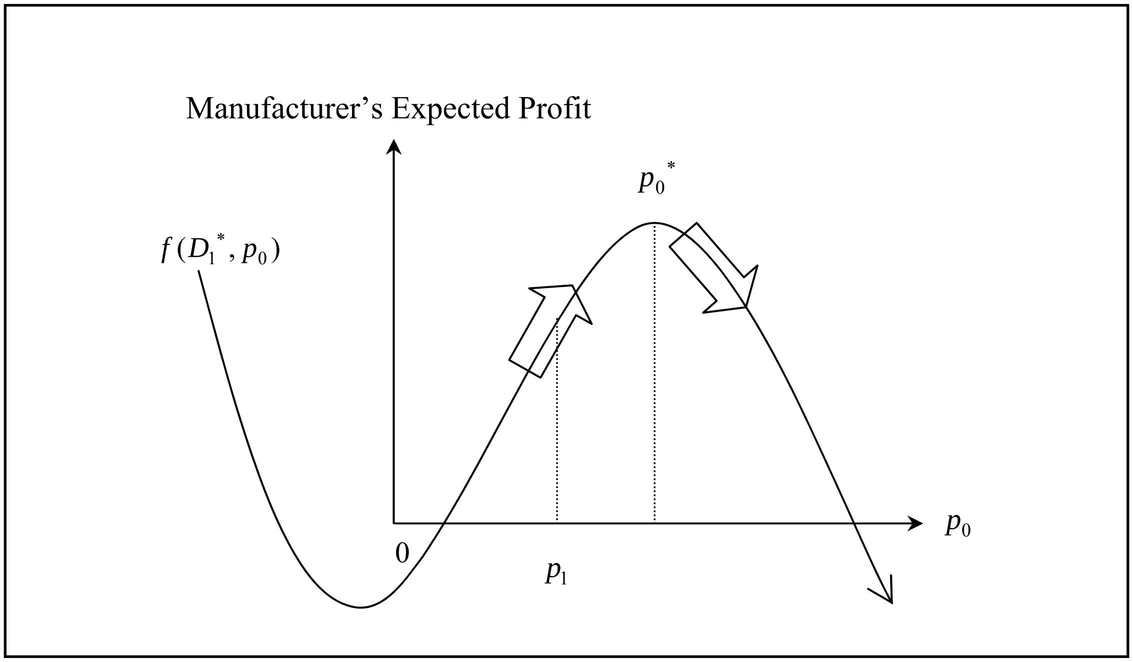

Figure 5.2 the expected function of the manufacturer's profit in terms of p_0

Though the graph is a cubic curve, it is restricted within narrow limits because $p_0 \geq p_1$. We know that $p_{0\min} = p_1$, and $p_{0\max}$ is determined by the relationship between a and b. Therefore, we confirm that p_0 is determined in a quadric and concave function between p_1 and $p_{0\max}$ in figure 5.2.

Theorem 5.1 For the mixed sale strategy, when $a(p_0) = c - ap_0$, $b(p_0) = d - bp_0$

1) If $a \geq b$ then $p_1 \leq p_0^* \leq \dfrac{c}{a}$.

2) If $a < b$ then $p_1 \leq p_0^* \leq \dfrac{d-c}{b-a}$.

Proof The following figures show the relationship between the decreasing slope of customer demands and the retail price.

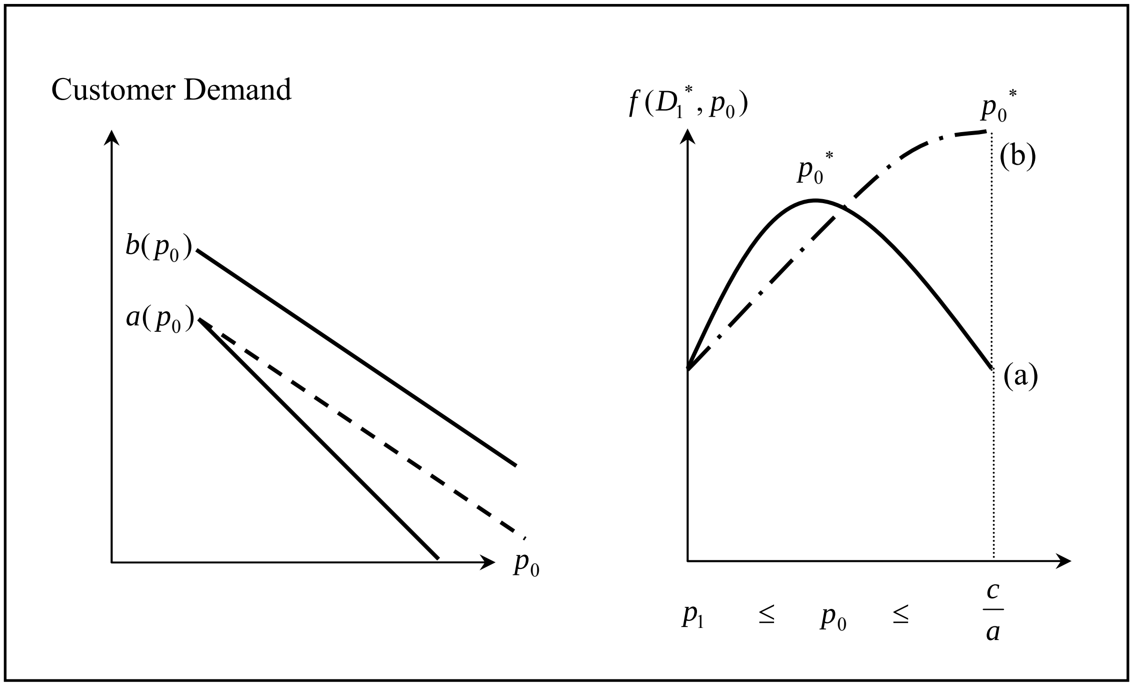

Figure 5.3 Expected customer demand and manufacturer's profit: the case of $a \geq b$

In figure 5.3, when $a \geq b$, $a(p_0) < b(p_0)$ and customer demand can not be less than zero. Hence, p_0^* can exist between $p_{0\min} = p_1$ and $p_{0\max} = \frac{c}{a}$ like (a), or can be $p_0^* = p_{0\max} = \frac{c}{a}$ like (b).

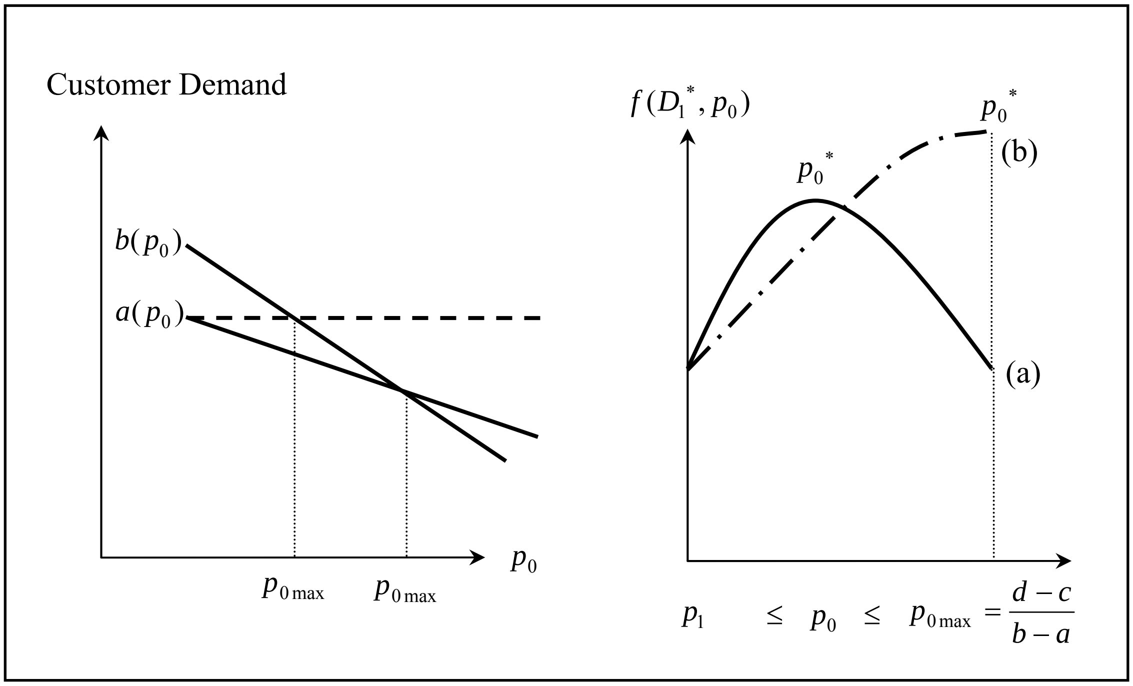

Figure 5.4 Expected customer demand and manufacturer's profit: the case of $a < b$

In figure 5.4, when $a < b$, $p_{0\max}$ happens at the point of intersection between $a(p_0)$ and $b(p_0)$. Since $p_{0\max} = a(p_0) = b(p_0) = c - ap_0 = d - bp_0$, $p_{0\max} = \frac{d-c}{b-a}$.

Hence, p_0^* exists between $p_{0\min} = p_1$ and $p_{0\max} = \frac{d-c}{b-a}$ like (a), or can be $p_0^* = \frac{d-c}{b-a}$ like (b).

The following figure shows the possible relationship between the direct sale strategy and the mixed sale strategy.

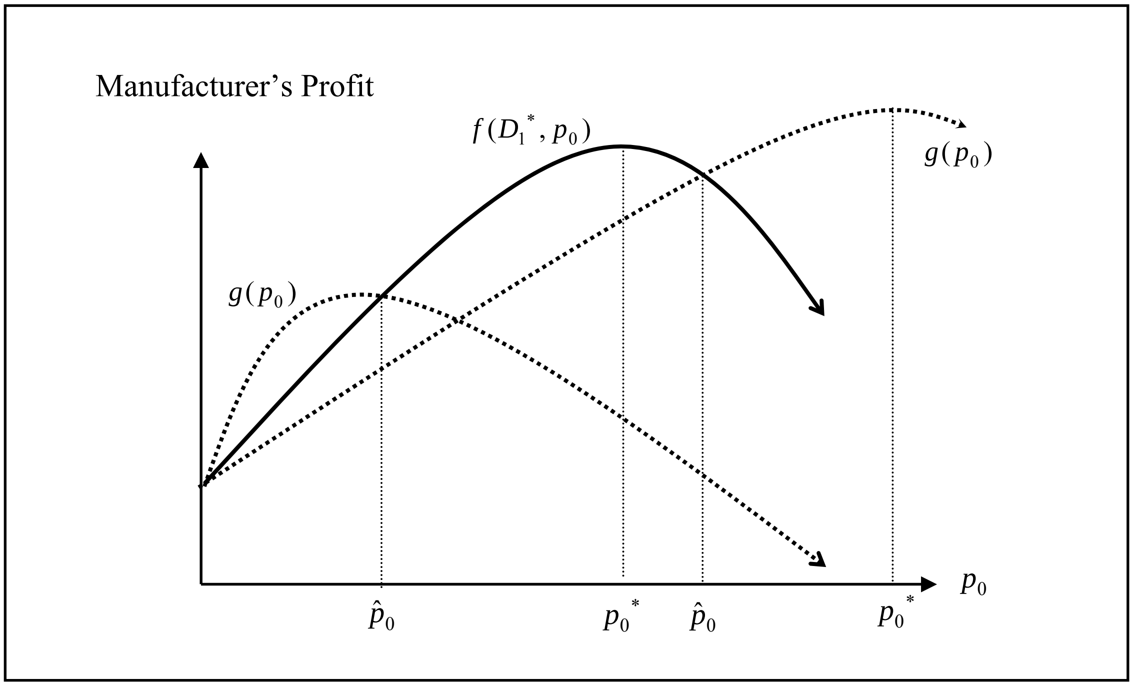

Figure 5.5 Manufacturer's expected profit 1

Since we verified that both $g(p_0)$ and $f(D_1^*, p_0)$ are concave between p_1 to $p_{0\max}$, figure 5.5 can be drawn. If $g(p_0)$ is always smaller than $f(D_1^*, p_0)$, i.e. the mixed sale strategy is always more profitable than the direct sale strategy, then $\hat{p}_0$ does not exist. However, as we draw, it may happen $g(p_0)$ is more profitable than $f(D_1^*, p_0)$ until we reach some point of retail price $\hat{p}_0$, and then $f(D_1^*, p_0)$ is more profitable. In this case, the manufacturer should utilize the direct sale strategy until the retail price $\hat{p}_0$, and then utilize the mixed sale strategy. On the other hand, it may happen $f(D_1^*, p_0)$ is more profitable than $g(p_0)$ until we reach some point of retail price $\hat{p}_0$, and then $g(p_0)$ is more profitable. In both cases, $\hat{p}_0$ is the turning point to convert the channel strategy. To figure out which channel should be utilized first, we should compare the expected profit between p_1 and $\hat{p}_0$ from $g(p_0)$ and $f(D_1^*, p_0)$.

Theorem 5.2 $\hat{p}_0 = p_1 + \dfrac{[Q - a(p_0)](p_1 - s)}{[b(p_0) - Q]}$ when $b(p_0) > a(p_0)$.

Proof Since $\hat{p}_0$ is the point of intersection between $g(0)$ and $f(D_1^*, p_0)$, the profits of the two functions are the same at $\hat{p}_0$. Hence, $\hat{p}_0$ is derived using Mathematica when $g(p_0) = f(D_1^*, p_0)$. (Appendix) Recall (12) and (14),

$$g(p_0) = (p_0 - c)Q + \frac{(s - p_0)[Q - a(p_0)]^2}{2[b(p_0) - a(p_0)]} \tag{12}$$

$$f(D_1^*, p_0) = (p_1 - c)Q + (p_0 - p_1) \cdot a(p_0) + \frac{(p_0 - p_1)^2 [b(p_0) - a(p_0)]}{2(p_0 - s)} \tag{14}$$

From $g(0) - f(D_1) = \dfrac{(p_0 - s)^2 [Q - a(p_0)]^2 + (p_0 - p_1)^2 [b(p_0) - a(p_0)]^2}{2[Q - a(p_0)][b(p_0) - a(p_0)]} = 0$

$$g(0) - f(D_1) = (p_0 - p_1)[Q - a(p_0)] + \frac{(p_0 - p_1)^2 [b(p_0) - a(p_0)]^2 - (p_0 - s)^2 [Q - a(p_0)]^2}{2(p_0 - s)[b(p_0) - a(p_0)]} = 0$$

$\hat{p}_0$ is derived as $\hat{p}_0 = \dfrac{[b(p_0) - a(p_0)]p_1 - [Q - a(p_0)]s}{[b(p_0) - Q]} = p_1 + \dfrac{[Q - a(p_0)](p_1 - s)}{[b(p_0) - Q]}$ (23)

This proves the turning point of distribution channel.

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- $[Q - a(p_0)]$ is the difference between the production quantity and the minimum customer demand. $[b(p_0) - Q]$ is the difference between the maximum customer demand and the production quantity. Therefore, $\frac{[Q-a(p_0)]}{[b(p_0)-Q]}$ is the expected sale rate. Since we assume $a(p_0) \le Q \le b(p_0)$, if the manufacturer produces a lot of production quantity, then the expected sale rate increases.

- $\hat{p}_0$ is obtained from the sale rate multiplied by the difference between the wholesale price and the salvage price. Therefore, as the wholesale price increases and the salvage price decrease, $\hat{p}_0$ increases.
- It means that if the direct sale strategy is more profitable than the mixed sale strategy, then the turning point of channel strategy increases. Therefore, we find that $\hat{p}_0$ exists only when the manufacturer should use the direct sale strategy first.
- Consequently, we know that the manufacturer should utilize the direct sale strategy until the price $\hat{p}_0$ as shown figure 5.6.

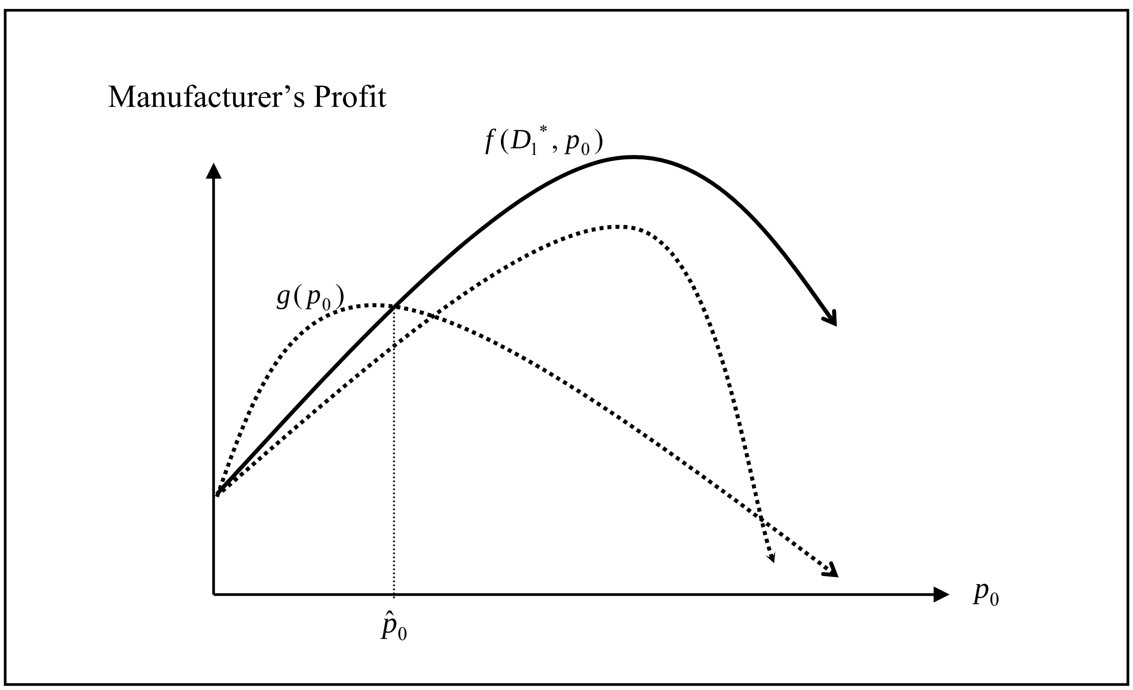


Figure 5.6 Manufacturer's expected profit 2

Until now, we show that $g(p_0)$ and $f(D_1^*, p_0)$ are concave in p_0 on the assumption that customer demand function consists with uniformly distributed two linear functions $a(p_0)$ and $b(p_0)$. However, a numerical experiment should be practiced to find the optimal retail price.

5.4 Experiment

In this section, a numerical experiment is practiced to prove the previous subsection. The experiment is processed as the method of the previous chapter. The experiment is conducted with two cases: $a \geq b$ and $a < b$. The optimal wholesale quantity, the optimal retail price, and the best channel are proposed.

Example 3

Assume that the manufacturer produces MP3 Player as the examples of the previous chapter. The production quantity is 4000 per quarter, the production cost is \$80 per unit, the wholesale price is \$160, and the salvage price is \$120. When customer demand functions, $a(p_0)$ and $b(p_0)$, are linear functions, we can suppose two cases. If the lower demand function decreases faster than the higher demand function, $a(p_0) = 5600 - 10p_0$ and $b(p_0) = 6000 - 7p_0$, then the optimal wholesale quantity is 1141, the optimal retail price is \$417, and manufacturer's maximum profit is \$871,091. If the higher demand function decreases faster than the lower demand function, $a(p_0) = 5600 - 10p_0$ and $b(p_0) = 6000 - 11p_0$, then the optimal wholesale quantity is 1934, the optimal retail price is $\$357$, and manufacturer's maximal profit is $\$723{,}431$. Theses results are summarized in table 5.1 as follows.

$a:b$	$a(p_0)$	$b(p_0)$	Best Strategy	D_1^*	p_0^*	Maximal Profit
$a \geq b$	$5600 - 10p_0$	$6000 - 7p_0$	Mixed Sale	1141	417	871091
$a < b$		$6000 - 11p_0$		1934	357	723431

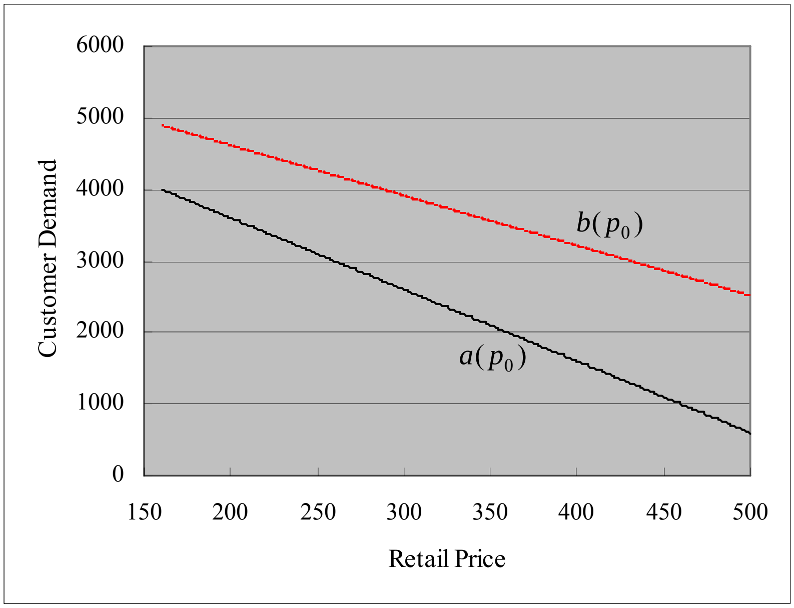
Table 5.1 Result of example 1

Table 5.1 is also described as figure 5.7 and 5.8, which represent the cases: $a \geq b$ and $a < b$.

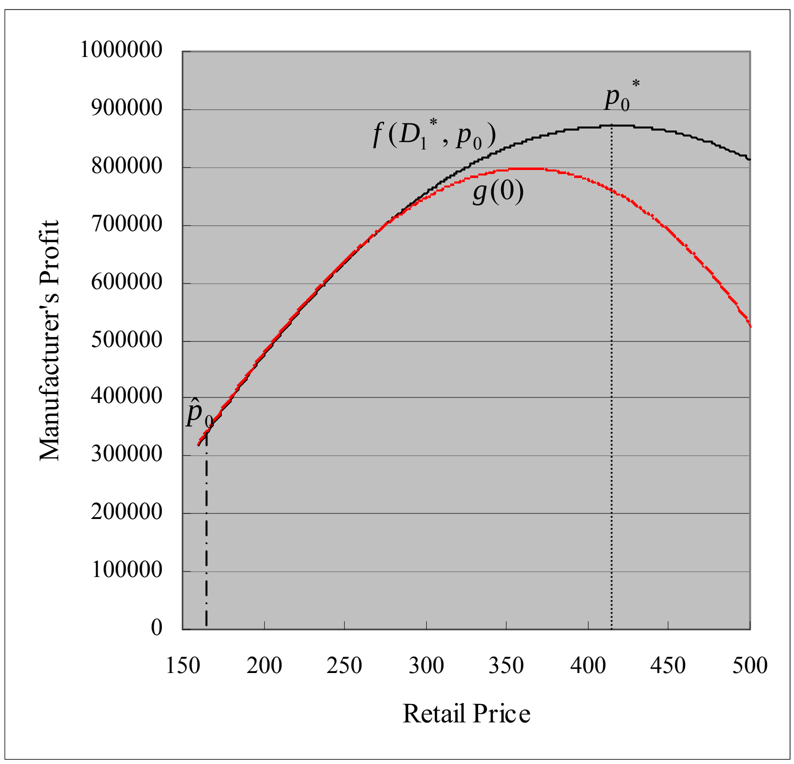
If $a \geq b$, $a(p_0) = 5600 - 10p_0$ and $b(p_0) = 6000 - 7p_0$ then $p_1 \leq p_0^* \leq \frac{c}{a}$. Since $p_1 = \$160$ and $\frac{c}{a} = \frac{5600}{10} = 560$, $\$160 \leq p_0^* \leq \$560$. In this case, $p_0^* = \$417$ and the manufacturer maximizes the profit from the mixed sale strategy, so $f(D_1^*, p_0^*) = \$871{,}091$. Table 5.2 shows the summary of this case.

$a \geq b$				
$a(p_0)$	$b(p_0)$	p_0^*	$g(p_0)$	$f(D_1^*, p_0^*)$
4000	4880	160	320000	320000
1430	3081	417	753919	871091
0	2080	560	227692	698182

Table 5.2 Result of example 1: the case of $a \geq b$

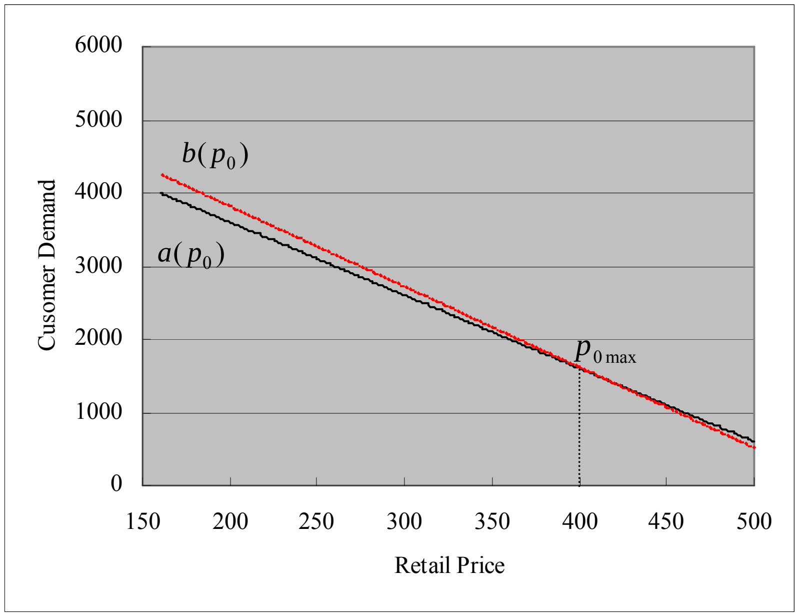


The retail price vs. The customer demands

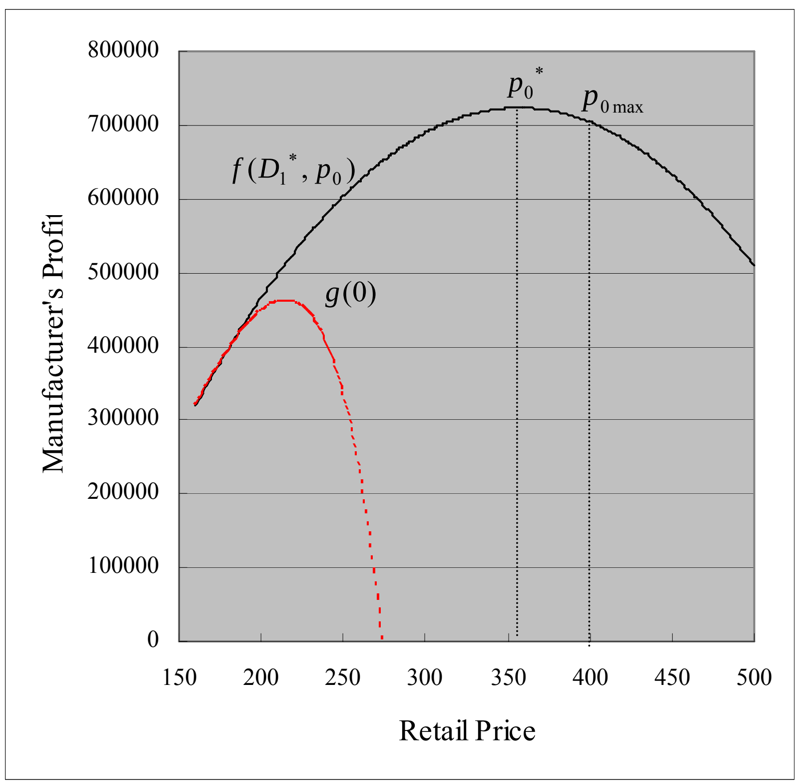


The retail price vs. The manufacturer's profit

Figure 5.7 Result of example 1: the case of $a \geq b$



The retail price vs. The customer demands



The retail price vs. The manufacturer's profit

Figure 5.8 Result of example 1: the case of $a < b$

If $a < b$, $a(p_0) = 5600 - 10p_0$ and $b(p_0) = 6000 - 11p_0$ then $p_1 \le p_0^* \le \frac{d-c}{b-a}$. Since $p_1 = \$160$ and $\frac{d-c}{b-a} = \frac{6000-5600}{11-10} = 400$, $\$160 \le p_0^* \le \$400$. In this case, the mixed sale strategy is always more profitable than the direct sale, and $p_0^* = \$357$ and $f(D_1^*, p_0^*) = \$723{,}431$. Table 5.3 shows the summary of this case.

$a < b$				
$a(p_0)$	$b(p_0)$	p_0^*	$g(p_0)$	$f(D_1^*, p_0^*)$
4000	4240	160	320000	320000
2030	2073	357	0	723431
1600	1600	400	0	704000

Table 5.3 Result of example 1: the case of $a < b$

5.5 Summary

We modeled and analyzed the fixed retail price case in the previous chapters. In this chapter, a variable retail price case with two linear functions is used to find the optimal retail price. Two functions which are uniformly distributed in $\phi(x)$, are characterized as $a(p_0) = c - ap_0$, $b(p_0) = d - bp_0$. Then, the mathematical models are transformed to $g(p_0)$ and $f(D_1^*, p_0)$, in order to represent the variability of retail price p_0. Based on new models, we show $g(p_0)$ and $f(D_1^*, p_0)$ are concave between p_1 to $p_{0\max}$. Especially, we prove that $f(D_1^*, p_0)$ is unimodal for the feasible range although it is not concave. Additionally, the feasible range for p_0^* is verified according to the relationship between $a(p_0)$ and $b(p_0)$. If $a \geq b$, the lower customer demand function decreases faster than the higher customer demand function, then the optimal retail price exists $p_1 \leq p_0^* \leq \frac{c}{a}$. If $a < b$, i.e., when the higher customer demand function decreases faster than the lower customer demand function, the optimal retail price exists $p_1 \leq p_0^* \leq \frac{d-c}{b-a}$. Since the channel strategy should be converted when $g(p_0)$ and $f(D_1^*, p_0)$ are intersected, we clarify $\hat{p}_0$ as $\hat{p}_0 = p_1 + \frac{[Q - a(p_0)](p_1 - s)}{[b(p_0) - Q]}$. We determine p_0^* and suggest the best channel to maximize the manufacturer's profit through the numerical experiments. Finally, we find that if the product is luxuries and there are many substitutes then the direct sale is proper, and if the product is necessities and there are few substitutes then the mixed sale strategy is more profitable.

CHAPTER 6

CONCLUSION

6.1 Conclusion

The competition has deepened in the market and manufacturers have searched for ways to increase their competitive power. This has lead to the development of production and the study of sale strategy. The proper sale is important as much as effective production, because the profit can differ according to the sale strategy as well as be controlled. Many engineers and economists have studied channel structure and channel coordination. This research focuses on the channel structure; especially profit maximization through the right decision of distribution channels. We consider three types of distribution channel: the direct sale strategy; the indirect sale strategy; the mixed sale strategy. This research proposes the best decision of distribution channel based on several conditions. The conditions include the character of products and the type of customer demand. The mathematical models are developed for the case of the fixed retail price. It is assumed that the manufacturer can control the wholesale quantity. Based on concavity of the expected profit function of the mixed channel strategy, several conditions for the decision of channel structure are proposed. We show that if the product has the high retail price and the high salvage price then the direct sale is proper, and if the product has the high wholesale price and the low salvage price then the mixed sale strategy is more profitable than the direct sale strategy. In addition, we identify the optimal wholesale quantity, that maximizes the expected profit of a manufacturer. These basic results are defined to be adapted for the case of uniformly distributed customer demand. We assume that the customer demand function consisting of two functions that are uniformly distributed. The mathematical models are transformed and tested for the concavity. Since the new models are concave, the optimal wholesale quantity is determined and tested by a numerical experiment. The experiment shows that the character for channel decision is reasonable. In addition, we find the optimal wholesale quantity decreases as the retail price and the salvage price increase. Then, we transform the mathematical model again with the optimal wholesale quantity and linear customer demand functions for searching the optimal retail price. We assume that the uniformly distributed customer demand functions are decreasing linear functions. The concavity of the models are shown, and the numerical experiment supports the result and determines the optimal wholesale quantity and the optimal retail price. The optimal wholesale quantity increases as the production quantity and the wholesale price increase, and decreases as the retail price and the salvage price increase. The optimal retail price is determined in limited range according to the relation between two customer demand functions through the numerical experiment.

In conclusion, this research suggests which distribution channel should be adapted depending on the business environments. In addition, the research determines the optimal wholesale quantity and the optimal retail price for maximizing the manufacturer's profit.

6.2 Future Research

The future research can be suggested in two ways. First of all, the research can be examined with other forms. There are several assumptions to limit this research. Customer demand function is examined only for uniform distribution. In addition, two functions, which are uniformly distributed customer functions, are limited as linear functions. For the future research, exponential functions or other types of functions can be used to reflect complicated and various customer demand. Moreover, indirect sale strategy can be easily compared together so that the channel decision can be determined among direct sale strategy, mixed sale strategy, and indirect sale strategy, under various situations. Secondly, other more realistic and complicated scenarios can be considered. For example, customer may freely more between the direct purchase from a manufacturer and indirect purchase through a retailer. It means that the manufacturer should consider the rate of customers transfer to optimize the profit. Based on the rate of customer transfers and stock availability, the manufacturer can have many opportunities to utilize the different sales channel strategies. In conclusion, this research can be extended through adapting other types of functions and scenarios.

APPENDIX

1. The Simplification of the Expected Profit functions

The expected profit for the direct sale is

$$
\begin{aligned}
g(0) &= \int_0^Q \left[p_0 x + s(Q-x)\right]\phi(x)dx + \int_Q^\infty p_0 Q\phi(x)dx - cQ \\
&= \int_0^Q p_0 x\phi(x)dx + \int_0^Q sQ\phi(x)dx - \int_0^Q sx\phi(x)dx + \int_Q^\infty p_0 Q\phi(x)dx - cQ \\
&= \int_0^Q p_0 x\phi(x)dx + \left[sQ\Phi(Q) - sQ\Phi(0)\right] - \int_0^Q sx\phi(x)dx + \left[p_0 Q\Phi(\infty) - p_0 Q\Phi(Q)\right] - cQ \\
&= \int_0^Q (p_0 - s)x\phi(x)dx + sQ\Phi(Q) + (p_0 - c)Q - p_0 Q\Phi(Q) \\
&= \int_0^Q (p_0 - s)x\phi(x)dx + (p_0 - c)Q - (p_0 - s)Q\Phi(Q)
\end{aligned}
$$

The expected profit for the mixed sale is

$$
\begin{aligned}
f(D_1) &= p_1 D_1 + \int_0^{Q-D_1} \left[p_0 x + s((Q-D_1) - x)\right]\phi(x)dx + \int_{Q-D_1}^\infty p_0 (Q - D_1)\phi(x)dx - cQ \\
&= \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx + p_1 D_1 + \int_0^{Q-D_1} s(Q - D_1)\phi(x)dx + \int_{Q-D_1}^\infty p_0 (Q - D_1)\phi(x)dx - cQ \\
&= p_1 D_1 + \int_0^{Q-D_1} p_0 x\phi(x)dx + \int_0^{Q-D_1} sQ\phi(x)dx - \int_0^{Q-D_1} sD_1\phi(x)dx - \int_0^{Q-D_1} sx\phi(x)dx \\
&\quad + \int_{Q-D_1}^\infty p_0 Q\phi(x)dx - \int_{Q-D_1}^\infty p_0 D_1\phi(x)dx - cQ \\
&= p_1 D_1 + \int_0^{Q-D_1} p_0 x\phi(x)dx + \left[sQ\Phi(Q-D_1) - sQ\Phi(0)\right] - \left[sD_1\Phi(Q-D_1) - sD_1\Phi(0)\right] \\
&\quad - \int_0^{Q-D_1} sx\phi(x)dx - cQ + \left[p_0 Q\Phi(\infty) - p_0 Q\Phi(Q-D_1)\right] - \left[p_0 D_1\Phi(\infty) - p_0 D_1\Phi(Q-D_1)\right] \\
&= p_1 D_1 + \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx + sQ\Phi(Q-D_1) - sD_1\Phi(Q-D_1) - cQ \\
&\quad + p_0 Q - p_0 Q\Phi(Q-D_1) - p_0 D_1 + p_0 D_1\Phi(Q-D_1) \\
&= p_1 D_1 + \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx + (Q - D_1)p_0 + (sQ - sD_1 + p_0 Q - p_0 Q)\Phi(Q-D_1) - cQ \\
&= p_1 D_1 + \int_0^{Q-D_1} (p_0 - s)x\phi(x)dx + (Q - D_1)p_0 + (p_0 - s)(Q - D_1)\Phi(Q-D_1) - cQ
\end{aligned}
$$

2. The Conversion of the Expected Profit Functions

When $\frac{1}{b(p_0)-a(p_0)}$ substitutes for the customer demand function $\phi(x)$, $g(0)$ is

$$
\begin{aligned}
g(0) &= \int_{a(p_0)}^{Q}\left[p_0 x + s(Q-x)\right]\phi(x)dx + \int_{Q}^{b(p_0)} p_0 Q\phi(x)dx - cQ \\
&= (p_0 - s)\int_{a(p_0)}^{Q} x\phi(x)dx + s\int_{a(p_0)}^{Q} Q\phi(x)dx + p_0\int_{Q}^{b(p_0)} Q\phi(x)dx - cQ \\
&= \int_0^Q p_0 x\phi(x)dx + \left[sQ\Phi(Q) - sQ\Phi(0)\right] - \int_0^Q sx\phi(x)dx + \left[p_0 Q\Phi(\infty) - p_0 Q\Phi(Q)\right] - cQ \\
&= (p_0 - s)\cdot \left.\frac{x^2}{2\left[(b(p_0)-a(p_0)\right]}\right|_{a(p_0)}^{Q} + s\cdot \left.\frac{Qx}{\left[(b(p_0)-a(p_0)\right]}\right|_{a(p_0)}^{Q} + p_0\cdot \left.\frac{Qx}{\left[(b(p_0)-a(p_0)\right]}\right|_{Q}^{b(p_0)} - cQ \\
&= \frac{(p_0 - s)\left[Q^2 - a(p_0)^2\right]}{2\left[(b(p_0)-a(p_0)\right]} + \frac{s\left[Q^2 - Q\cdot a(p_0)\right] - p_0\left[Q^2 - Q\cdot b(p_0)\right]}{\left[(b(p_0)-a(p_0)\right]} - cQ \\
&= \frac{Q^2 p_0 - p_0\cdot a(p_0)^2 - Q^2 s + s\cdot a(p_0)^2 + 2Q^2 s - 2Qs\cdot a(p_0) - 2Qp_0\cdot b(p_0) - 2Q^2 p_0}{2\left[(b(p_0)-a(p_0)\right]} - cQ \\
&= \frac{-Q^2 p_0 + Q^2 s - p_0\cdot a(p_0)^2 + s\cdot a(p_0)^2 - 2Qs\cdot a(p_0) - 2Qp_0\cdot b(p_0)}{2\left[(b(p_0)-a(p_0)\right]} - cQ \\
&= \frac{(s-p_0)Q^2 + (s-p_0)\cdot a(p_0)^2 - 2Qs\cdot a(p_0) - 2Qp_0\cdot b(p_0)}{2\left[(b(p_0)-a(p_0)\right]} - cQ \\
&= \frac{(s-p_0)\left[Q^2 + a(p_0)^2\right] + 2Q\left[p_0\cdot b(p_0) - s\cdot a(p_0)\right] - 2cQ\left[(b(p_0)-a(p_0)\right]}{2\left[(b(p_0)-a(p_0)\right]} \\
&= \frac{(s-p_0)\left[Q-a(p_0)\right]^2 + 2Q(s-p_0)\cdot a(p_0) + 2Q\left[p_0\cdot b(p_0) - s\cdot a(p_0) - c\cdot b(p_0) + c\cdot a(p_0)\right]}{2\left[(b(p_0)-a(p_0)\right]} \\
&= \frac{(s-p_0)\left[Q-a(p_0)\right]^2 + 2Q\left[p_0\left[(b(p_0)-a(p_0)\right] - c\left[(b(p_0)-a(p_0)\right]\right]}{2\left[(b(p_0)-a(p_0)\right]} \\
&= \frac{(s-p_0)\left[Q-a(p_0)\right]^2 + 2Q(p_0-c)\left[(b(p_0)-a(p_0)\right]}{2\left[(b(p_0)-a(p_0)\right]} \\
&= (p_0 - c)Q + \frac{(s-p_0)\left[Q-a(p_0)\right]^2}{2\left[(b(p_0)-a(p_0)\right]}
\end{aligned}
$$

Since $a(p_0) \le Q - D_1 \le b(p_0)$, $f(D_1)$ is simplified as

$$f(D_1) = p_1 D_1 + \int_{a(p_0)}^{Q-D_1} \left[p_0 x + s((Q - D_1) - x) \right] \phi(x) dx + \int_{Q-D_1}^{b(p_0)} p_0 (Q - D_1) \phi(x) dx - cQ$$

$$= p_1 D_1 + (p_0 - s) \int_{a(p_0)}^{Q-D_1} x \phi(x) dx + s \int_{a(p_0)}^{Q-D_1} (Q - D_1) \phi(x) dx + p_0 \int_{Q-D_1}^{b(p_0)} (Q - D_1) \phi(x) dx - cQ$$

$$= p_1 D_1 - cQ + (p_0 - s) \cdot \left. \frac{x^2}{2\left[b(p_0) - a(p_0) \right]} \right|_{a(p_0)}^{Q-D_1}$$

$$+ s \cdot \left. \frac{(Q - D_1) x}{\left[b(p_0) - a(p_0) \right]} \right|_{a(p_0)}^{Q-D_1} + p_0 \cdot \left. \frac{(Q - D_1) x}{\left[b(p_0) - a(p_0) \right]} \right|_{Q-D_1}^{b(p_0)}$$

$$= p_1 D_1 - cQ + \frac{(p_0 - s)\left[(Q - D_1)^2 - a(p_0)^2 \right]}{2\left[b(p_0) - a(p_0) \right]}$$

$$+ \frac{2s\left[(Q - D_1)^2 - (Q - D_1) \cdot a(p_0) \right]}{2\left[b(p_0) - a(p_0) \right]} + \frac{2 p_0 \left[(Q - D_1) \cdot b(p_0) - (Q - D_1)^2 \right]}{2\left[b(p_0) - a(p_0) \right]}$$

$$= p_1 D_1 - cQ + \frac{(p_0 - s)\left[(Q - D_1)^2 - a(p_0)^2 \right]}{2\left[b(p_0) - a(p_0) \right]}$$

$$+ \frac{-(Q - D_1)^2 (p_0 - s) + (Q - D_1)\left[P_0 \cdot b(p_0) - s \cdot a(p_0) \right]}{\left[b(p_0) - a(p_0) \right]}$$

$$= p_1 D_1 - cQ$$

$$+ \frac{(p_0 - s)(Q - D_1)^2 - (p_0 - s) a(p_0)^2 - 2(Q - D_1)^2 (p_0 - s)}{2\left[b(p_0) - a(p_0) \right]}$$

$$+ \frac{2(Q - D_1)\left[P_0 \cdot b(p_0) - s \cdot a(p_0) \right]}{2\left[b(p_0) - a(p_0) \right]}$$

$$= p_1 D_1 - cQ + \frac{(p_0 - s)\left[-(Q - D_1)^2 - a(p_0)^2 \right] + 2(Q - D_1)\left[P_0 \cdot b(p_0) - s \cdot a(p_0) \right]}{2\left[b(p_0) - a(p_0) \right]}$$

$$= p_1 D_1 - cQ - \frac{(p_0 - s)\left[(Q - D_1)^2 - 2(Q - D_1) a(p_0) + 2(Q - D_1) a(p_0) + a(p_0)^2 \right]}{2\left[b(p_0) - a(p_0) \right]}$$

$$+ \frac{2(Q - D_1)\left[P_0 \cdot b(p_0) - s \cdot a(p_0) \right]}{2\left[b(p_0) - a(p_0) \right]}$$

$$= p_1 D_1 - cQ - \frac{(p_0 - s)\left[(Q - D_1) - a(p_0)\right]^2}{2\left[b(p_0) - a(p_0)\right]}$$

$$- \frac{-2(p_0 - s)(Q - D_1)a(p_0) + 2(Q - D_1)\left[P_0 \cdot b(p_0) - s \cdot a(p_0)\right]}{2\left[b(p_0) - a(p_0)\right]}$$

$$= p_1 D_1 - cQ - \frac{(p_0 - s)\left[(Q - D_1) - a(p_0)\right]^2}{2\left[b(p_0) - a(p_0)\right]} + \frac{2(Q - D_1)\left[-(p_0 - s) + P_0 \cdot b(p_0) - s \cdot a(p_0)\right]}{2\left[b(p_0) - a(p_0)\right]}$$

$$= p_1 D_1 - cQ - \frac{(p_0 - s)\left[(Q - D_1) - a(p_0)\right]^2}{2\left[b(p_0) - a(p_0)\right]} + \frac{2 p_0 (Q - D_1)\left[b(p_0) - a(p_0)\right]}{2\left[b(p_0) - a(p_0)\right]}$$

$$= p_1 D_1 - cQ + p_0 (Q - D_1) - \frac{(p_0 - s)\left[(Q - D_1) - a(p_0)\right]^2}{2\left[b(p_0) - a(p_0)\right]}$$

$$= (p_0 - c)Q - (p_0 - p_1)D_1 - \frac{(p_0 - s)\left[(Q - D_1) - a(p_0)\right]^2}{2\left[b(p_0) - a(p_0)\right]}$$

3. The Conversion of $f(D_1^*, p_0)$

Then, the expected profit function $f(D_1^*, p_0)$ with the optimal wholesale quantity D_1^* is

$$
\begin{aligned}
f(D_1^*, p_0) = {} & p_1\left\{Q - a(p_0) - \frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}\right\} \\
& - \frac{(s - p_0)\left\{Q - \left[Q - a(p_0) - \frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}\right] - a(p_0\right\}^2}{2[b(p_0) - a(p_0)]} \\
& + p_0\left\{Q - \left[Q - a(p_0) - \frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}\right]\right\} \\
= {} & p_1Q - p_1 \cdot a(p_0) - p_1\left\{\frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}\right\} \\
& - (s - p_0)\left\{\frac{(p_0 - p_1)^2[b(p_0) - a(p_0)]^2}{(p_0 - s)^2}\right\} + p_0 \cdot a(p_0) + p_0\left\{\frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}\right\} \\
= {} & p_1Q + (p_0 - p_1) \cdot a(p_0) - \frac{(p_0 - p_1)^2[b(p_0) - a(p_0)]}{2(p_0 - s)} \\
& - p_1\left\{\frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}\right\} + p_0\left\{\frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}\right\} \\
= {} & p_1Q + (p_0 - p_1) \cdot a(p_0) \\
& + (p_0 - p_1)\left\{\frac{(p_0 - p_1)[b(p_0) - a(p_0)]}{(p_0 - s)}\right\} - \frac{(p_0 - p_1)^2[b(p_0) - a(p_0)]}{2(p_0 - s)} \\
= {} & p_1Q + (p_0 - p_1) \cdot a(p_0) + \frac{(p_0 - p_1)^2[b(p_0) - a(p_0)]}{2(p_0 - s)}
\end{aligned}
$$

4. The Differentials of $g(p_0)$

We differentiate $g(p_0)$ in p_0.

$$\frac{\partial g(p_0)}{\partial p_0} = \frac{[Q-a(p_0)]^2 Q}{[b(p_0)-a(p_0)]^2} - \frac{[b(p_0)-a(p_0)]\{[Q-a(p_0)]^2 + 2(s-p_0)[Q-a(p_0)]a(p_0)'\}}{2[b(p_0)-a(p_0)]^2}$$
$$+\frac{[b(p_0)'-a(p_0)'](s-p_0)[Q-a(p_0)]^2}{2[b(p_0)-a(p_0)]^2}$$
$$= Q - \frac{[Q-a(p_0)]^2 + 2a(p_0)'(s-p_0)[Q-a(p_0)]}{2[b(p_0)-a(p_0)]}$$
$$+\frac{(s-p_0)[b(p_0)'-a(p_0)'][Q-a(p_0)]^2}{2[b(p_0)-a(p_0)]^2}$$

$$\frac{\partial^2 g(p_0)}{\partial {p_0}^2} = \frac{[X'[b(p_0)-a(p_0)] - X[b(p_0)'-a(p_0)']]}{2[b(p_0)-a(p_0)]^2}$$
$$+\frac{[Y'[b(p_0)-a(p_0)]^2 - 2Y[b(p_0)-a(p_0)][b(p_0)'-a(p_0)']]}{2[b(p_0)-a(p_0)]^4}$$
$$= \frac{A-B}{2[b(p_0)-a(p_0)]^2} + \frac{C-D}{2[b(p_0)-a(p_0)]^4}$$

Let

$$X = [Q-a(p_0)]^2 - 2a(p_0)'(s-p_0)[Q-a(p_0)]$$

$$Y = [b(p_0)'-a(p_0)'](s-p_0)[Q-a(p_0)]^2$$

$$A = 2(s-p_0)\{2a(p_0)'^2 - a(p_0)''[Q-a(p_0)]\}[b(p_0)-a(p_0)]$$

$$B = \{-2[Q-a(p_0)]^2 - 2(s-p_0)[Q-a(p_0)]a(p_0)'\}[b(p_0)'-a(p_0)']$$

$$C = [b(p_0)''-a(p_0)''][b(p_0)-a(p_0)]^2(s-p_0)[Q-a(p_0)]^2$$
$$-[b(p_0)'-a(p_0)'][b(p_0)-a(p_0)]^2\{[Q-a(p_0)]^2 + 2a(p_0)'(s-p_0)[Q-a(p_0)]\}$$

$$D = 2[b(p_0)'-a(p_0)']^2(s-p_0)[Q-a(p_0)]^2[b(p_0)-a(p_0)]$$

$$\frac{\partial^2 g(p_0)}{\partial p_0^{\;2}} = \frac{(s-p_0)\left\{2a(p_0)'^2 - a(p_0)''[Q-a(p_0)]\right\}}{[b(p_0)-a(p_0)]}$$

$$+\frac{(s-p_0)[Q-a(p_0)]^2[b(p_0)''-a(p_0)'']}{2[b(p_0)-a(p_0)]^2}$$

$$-\frac{(s-p_0)[Q-a(p_0)]^2[b(p_0)'-a(p_0)']}{[b(p_0)-a(p_0)]^3}$$

Then, equations are converted with $a(p_0)=c-ap_0$, $b(p_0)=d-bp_0$.

$$\frac{\partial g(p_0)}{\partial p_0} = Q - \frac{[Q-c+ap\ \ ]^2 - 2a(s-p_0)[Q-c+ap_0]}{2[d-c-(b-a)p_0]}$$

$$-\frac{(s-p_0)[Q-c+ap_0](b-a)}{2[d-c-(b-a)p_0]^2}$$

$$\frac{\partial^2 g(p_0)}{\partial p_0^{\;2}} = \frac{2a^2(s-p_0)}{[d-c-(b-a)p_0]} + \frac{(s-p_0)[Q-c-ap_0]^2(b-a)}{[d-c-(b-a)p_0]^3}$$

5. The Differentials of $f(D_1^*, p_0)$

From $f(D_1^*, p_0)$, we derive $\dfrac{\partial f(D_1^*, p_0)}{\partial p_0}$ and $\dfrac{\partial^2 f(D_1^*, p_0)}{\partial {p_0}^2}$.

$$\frac{\partial f(D_1^*, p_0)}{\partial p_0} = a(p_0) + (p_0 - p_1)\cdot a(p_0)' + \frac{1}{2}\left\{\frac{\left\{(p_0 - p_1)^2\left[b(p_0) - a(p_0)\right]\right\}'(p_0 - s)}{(p_0 - s)^2}\right\}$$

$$-\frac{1}{2}\left\{\frac{(p_0 - p_1)^2\left[b(p_0) - a(p_0)\right]}{(p_0 - s)^2}\right\}$$

$$= a(p_0) + (p_0 - p_1)\cdot a(p_0)'$$

$$+\frac{1}{2(p_0 - s)^2}\left\{2(p_0 - p_1)\left[b(p_0) - a(p_0)\right] + (p_0 - p_1)^2\left[b(p_0)' - a(p_0)'\right]\right\}(p_0 - s)$$

$$-\frac{1}{2(p_0 - s)^2}\left\{(p_0 - p_1)^2\left[b(p_0) - a(p_0)\right]\right\}$$

$$= a(p_0) + (p_0 - p_1)\cdot a(p_0)' + \frac{\left[b(p_0) - a(p_0)\right]}{2(p_0 - s)^2}\left\{2(p_0 - p_1)(p_0 - s) - (p_0 - p_1)^2\right\}$$

$$+\frac{\left[b(p_0)' - a(p_0)'\right]}{2(p_0 - s)^2}\left\{(p_0 - p_1)^2(p_0 - s)\right\}$$

$$= a(p_0) + (p_0 - p_1)\cdot a(p_0)' + \frac{\left[b(p_0) - a(p_0)\right]}{2(p_0 - s)^2}\left\{(p_0 - p_1)(p_0 - 2s + p_1)\right\}$$

$$+\frac{\left[b(p_0)' - a(p_0)'\right]}{2(p_0 - s)}(p_0 - p_1)^2$$

$$= a(p_0) + (p_0 - p_1)\cdot a(p_0)' + \frac{(p_0 - p_1)(p_0 - 2s + p_1)}{2(p_0 - s)^2}\left[b(p_0) - a(p_0)\right]$$

$$+\frac{(p_0 - p_1)^2}{2(p_0 - s)}\left[b(p_0)' - a(p_0)'\right]$$

$$\frac{\partial^2 f(D_1^*, p_0)}{\partial p_0^{\,2}} = a(p_0)' + a(p_0)' + (p_0 - p_1)a(p_0)''$$

$$+ \frac{(p_0 - s)^2}{2(p_0 - s)^4}\{(p_0 - 2s + p_1)[b(p_0) - a(p_0)] + (p_0 - p_1)[b(p_0) - a(p_0)]\}$$

$$+ \frac{(p_0 - s)^2}{2(p_0 - s)^4}\{(p_0 - p_1)(p_0 - 2s + p_1)[b(p_0)' - a(p_0)']\}$$

$$- \frac{2(p_0 - p_1)(p_0 - 2s + p_1)[b(p_0) - a(p_0)](p_0 - s)}{2(p_0 - s)^4}$$

$$+ \frac{\{2(p_0 - p_1)[b(p_0)' - a(p_0)'] + (p_0 - p_1)^2[b(p_0)'' - a(p_0)'']\}(p_0 - s)}{2(p_0 - s)^2}$$

$$- \frac{(p_0 - p_1)^2[b(p_0)' - a(p_0)']}{2(p_0 - s)^2}$$

$$= a(p_0)' + a(p_0)' + (p_0 - p_1)a(p_0)''$$

$$+ \frac{[b(p_0) - a(p_0)]}{2(p_0 - s)^4}\left\{(p_0 - s)^2(p_0 - 2s + p_1) + (p_0 - s)^2(p_0 - p_1)\right\}$$

$$- \frac{[b(p_0) - a(p_0)]}{2(p_0 - s)^4}\{2(p_0 - p_1)(p_0 - 2s + p_1)(p_0 - s)\}$$

$$+ \frac{[b(p_0)' - a(p_0)']}{2(p_0 - s)^2}\left\{(p_0 - p_1)(p_0 - 2s + p_1) + 2(p_0 - p_1)(p_0 - s) - (p_0 - p_1)^2\right\}$$

$$+ \frac{[b(p_0)'' - a(p_0)'']}{2(p_0 - s)^2}\left\{(p_0 - p_1)^2(p_0 - s)\right\}$$

$$= a(p_0)' + a(p_0)' + (p_0 - p_1)a(p_0)'' + \frac{[b(p_0) - a(p_0)]}{2(p_0 - s)^4}\left\{2(p_0 - s)(p_1 - s)^2\right\}$$

$$+ \frac{[b(p_0)' - a(p_0)']}{2(p_0 - s)^2}\{2(p_0 - p_1)(p_0 - 2s)\} + \frac{[b(p_0)'' - a(p_0)'']}{2(p_0 - s)^2}\left\{(p_0 - p_1)^2(p_0 - s)\right\}$$

$$= 2 \cdot a(p_0)' + (p_0 - p_1)a(p_0)'' + \frac{(p_1 - s)^2}{(p_0 - s)^3}[b(p_0) - a(p_0)]$$

$$+ \frac{(p_0 - p_1)(p_0 - 2s + p_1)}{(p_0 - s)^2}[b(p_0)' - a(p_0)'] + \frac{(p_0 - p_1)^2}{2(p_0 - s)}[b(p_0)'' - a(p_0)'']$$

7. Mathemarica

Let us assume that $r = p_0,\ w = p_1, A = a(p_0),\ B = b(p_0)$.

- Example 1

```
Clear All
r=220
w=130
s=110
c=70
Q=5000
A=2000
B=6000
 All Clear
 220
 130
 110
 70
 5000
 2000
 6000
```

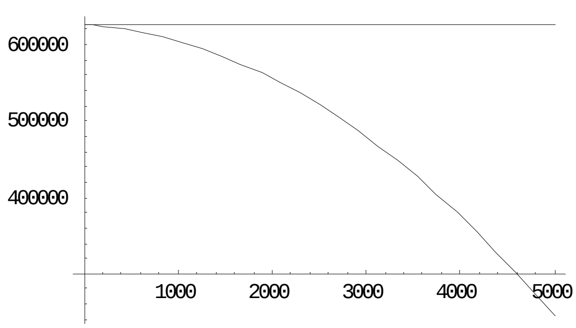
$$g = \frac{(s-r)*(Q-A)^2 + 2Q*(r-c)*(B-A)}{2(B-A)}$$

```
 626250
```

$$\text{FindMaximum}\left[(r-c)*Q-(r-w)*d-\frac{(r-s)*(Q-d-A)^2}{2(B-A)},\ \{d, 0, Q\}\right]$$

```
 {627273.,{d→-272.727}}
```

$$\text{Plot}\left[\left\{g,\ (r-c)*Q-(r-w)*d-\frac{(r-s)*(Q-d-A)^2}{2(B-A)}\right\},\ \{d, 0, Q\}\right]$$




```
 -Graphics-

r=220
w=200
```

```
r=220
w=200
s=110
c=70
Q=5000
A=2000
B=6000
 220
 200
 110
 70
 5000
 2000
 6000
```

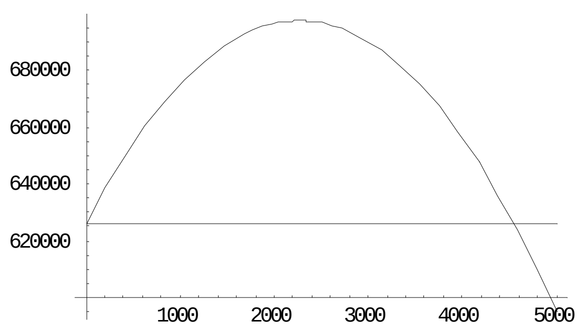
$$g = \frac{(s-r)*(Q-A)^2 + 2Q*(r-c)*(B-A)}{2\,(B-A)}$$

626250

$$\text{FindMaximum}\left[(r-c)*Q-(r-w)*d-\frac{(r-s)*(Q-d-A)^2}{2\,(B-A)},\ \{d,0,Q\}\right]$$

{697273., {d→2272.73}}

$$\text{Plot}\left[\left\{g,\ (r-c)*Q-(r-w)*d-\frac{(r-s)*(Q-d-A)^2}{2\,(B-A)}\right\},\ \{d,0,Q\}\right]$$



-Graphics-

(697273-g)/g*100

$$\frac{142046}{12525}$$

- Example 2

No 1

```
Q=5000
r=250
c=100
w=230
s=130
A=2000
B=7000
 5000
 250
 100
 230
 130
 2000
 7000
```

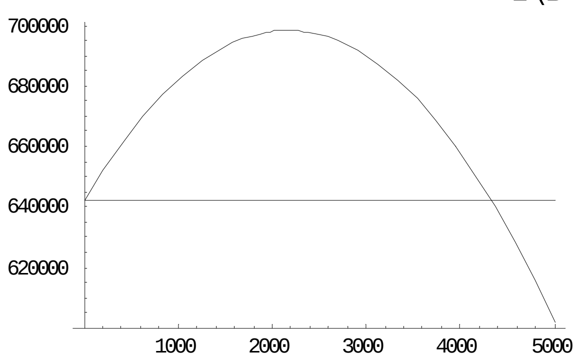
$$g=\frac{(s-r)*(Q-A)^2+2Q*(r-c)*(B-A)}{2(B-A)}$$

642000

$$\text{FindMaximum}\left[(r-c)*Q-(r-w)*d-\frac{(r-s)*(Q-d-A)^2}{2(B-A)},\ \{d,0,Q\}\right]$$

$\{698333.,\{d\to 2166.67\}\}$

$$\text{Plot}\left[\left\{g,\ (r-c)*Q-(r-w)*d-\frac{(r-s)*(Q-d-A)^2}{2(B-A)}\right\},\ \{d,0,Q\}\right]$$



-Graphics-

(698333-g)/g*100

$$\frac{56333}{6420}$$

- Deduction of $\hat{p}_0$

$$\mathbf{g} = (\mathbf{r}-\mathbf{c})*\mathbf{Q}+\frac{(\mathbf{s}-\mathbf{r})*(\mathbf{Q}-\mathbf{A})^2}{\mathbf{2}*(\mathbf{B}-\mathbf{A})}$$

$$Q(-c+r)+\frac{(-A+Q)^2(-r+s)}{2(-A+B)}$$

$$\mathbf{f} = (\mathbf{w}-\mathbf{c})*\mathbf{Q}+(\mathbf{r}-\mathbf{w})*\mathbf{A}+\frac{(\mathbf{r}-\mathbf{w})^2*(\mathbf{B}-\mathbf{A})}{\mathbf{2}*(\mathbf{r}-\mathbf{s})}$$

$$A(r-w)+\frac{(-A+B)(r-w)^2}{2(r-s)}+Q(-c+w)$$

```
Solve[g-f==0,r]
```

$$\left\{\left\{r\to\frac{As-Qs-Aw+Bw}{B-Q}\right\},\left\{r\to\frac{As-Qs-Aw+Bw}{B-Q}\right\}\right\}$$

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