PRICING TO MARKET
WHEN EXCHANGE RATE CHANGES
AND OUTPUT LEVEL MATTERS

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This dissertation clarifies the definition of Pricing to Market (PTM) and formalizes the definition. PTM in this paper focuses on the relative movements of prices in different markets, conditional on exchange rate movements. Therefore the degree of Exchange Rate Pass-Through (ERPT) itself is not important to determine the existence of PTM. But the relative degree of ERPT in different markets is crucial to conclude whether there is PTM behavior or not.

This dissertation provides the bilateral trade model that determines the output levels of two countries endogenously relative to the exchange rate and the cost of production. And it considers the output level is the reason of PTM in the model economy. The model also suggests a standard to identify the existence of PTM based on the definition of PTM.

I conclude that an incomplete ERPT is prevalent in the markets. Invoice currency has an effect on ERPT and PTM: Exporters pass through more exchange rate movements
to the export price -- higher degree of ERPT and the relative price difference between the domestic and the foreign market is larger -- higher degree of PTM when they invoice the export price in their own currency than they do in the buyers’ currency.

In addition, I find that domestic producers produce more output levels and consumers buy more domestic goods in a country if the domestic seller and the exporter charge the same price and the price increases as the exchange rate fluctuates. I also show that a country has a surplus in the trade balance if the country’s currency depreciates.

In the two-period model, I tried to examine the effect of extended time period on the output level, the price, ERPT and PTM. A firm’s output level, price, ERPT and PTM are affected by both its own discount factor and the other firm’s discount factor. I find that second-period ERPT is higher than the first-period ERPT. On the other hand, the effect of the extended time period on the PTM behavior is indeterminate in the model economy.
CHAPTER 1
INTRODUCTION

With the breakdown of the Bretton Woods Agreement, there has been a large increase in exchange rate volatility. Large U.S. dollar appreciations against the Japanese yen were observed, for example, in 1978-1984, 1988-1989, 1995-1998 and 2000-2002. The dollar depreciated sharply against the yen in the 1975-1978, 1985-1988, 1990-1995, 1998-2000 and 2002-2004. In the international market, the exchange rate movement matters since it affects financial markets, the price of traded goods and the trade balance. The United States has been experiencing a trade deficit since 1971. It has been drastically increasing since 1997 and was recorded at about 724 billion dollars in 2005. In theory, the dollar depreciation allows the U.S. exporters to decrease the local-currency price in the foreign market and to sell more products, holding everything else constant. At the same time, it makes the foreign exporters charge higher dollar prices in the U.S. market and helps the U.S. consumers to buy fewer imports. As a result, the dollar depreciation improves the trade balance for the United States. However, the United States could not reduce the large deficit despite the relatively long period of the dollar’s depreciation since 1990. The U.S. economists paid attention to the import price and export price movements as the exchange rate varied. The U.S. dollar price of imports from Japan, for instance, did not decrease as much as the exchange rate changes during the period in which the dollar

2 Data Source: U. S. Census Bureau, Foreign Trade Division, http://www.census.gov/foreign-trade/statistics/historical/gands.txt
3 Here, of course, we assume that the price elasticity of demand in the United States and in the foreign countries is relatively elastic. Thus the total revenue increases if the price decreases. And, the total revenue decreases if the price increases.
appreciated. Nor did it increase as much during the depreciation period (Froot and Klemperer, 1989; Marston, 1990; Knetter, 1993). The most glaring case, perhaps, is that some Japanese exporters decreased their price in the United States when there was a 34 percent depreciation of the dollar against the yen between 1994 and 1995 (Goldberg and Knetter, 1997). Furthermore, researchers present evidence that the foreign exporters do price discriminate even more in the United States market than other markets as the exchange rate fluctuates, so that an import price in the U.S. responds less to the exchange rate changes compared to other countries (Hooper and Mann, 1989; Goldberg and Knetter, 1997). These phenomena also piqued economists’ interest in explaining why the exporters exhibit these pricing behaviors against the backdrop of the exchange rate movements.

Krugman (1987) introduced the concept of Pricing to Market (PTM) in order to explain the relative price differences in different international goods markets when the exchange rate varies. This paper follows Krugman’s (1987) definition and will formalize PTM. I also investigate the existence of exporters’ PTM behavior using the model.

1.1 Pricing to Market: A Definition

I define PTM in this paper as follows: PTM is an exporter’s behavior such that it charges relatively different prices in different markets in response to exchange rate fluctuations. As such, PTM focuses on the relative movements of the export price in different markets, conditional on exchange rate movements.

For instance, let us suppose that a producer sells a product in foreign countries labeled A and B. The producer charges the export price in both foreign countries in
response to exchange rate movements. If the producer changes the export price in country A by the same proportion as the export price in country B relative to an exchange rate movement then there is no PTM behavior. Note that this definition does not depend on the export prices in countries A and B passing through the exchange rate movements. Given that there is an exchange rate increase, for example, if the proportion of the export price changes is very different in countries A and B, then the evidence indicates PTM behavior exists. Therefore, PTM behavior is not affected by whether the export price reflects the full proportion of the exchange rate movement or not. In other words, compared to the literature, PTM differs in that the degree of the exchange rate pass-through to the price is not important. Rather, it is the similarity of the degree of the pass-through in different markets that is crucial.

The following is an equation that formalizes the notion of PTM. The numerator represents the exchange rate pass-through to the export price in country A and the denominator is the exchange rate pass-through in country B.

\[
\frac{\% \Delta \text{ in the export price in country A}}{\% \Delta \text{ in the exchange rate}} = \frac{\% \Delta \text{ in the export price in country B}}{\% \Delta \text{ in the exchange rate}} = \lambda \tag{1.1}
\]

If \( \lambda = 1 \) then there is no PTM by an exporter. However, \( \lambda \neq 1 \) implies that PTM exists.

To illustrate this point, suppose that the percentage change in the export price in country A is 1 percent and the percentage change in the exchange rate between the exporter’s country and country A is 2 percent, so that the exchange rate pass-through to
the export price in country A is 0.5 -- the numerator of the equation. Further, suppose that
the percentage change in the export price in country B is 3 percent and the percentage
change in the exchange rate between the exporter’s country and country B is 6 percent;
then the exchange rate pass-through to the export price in country B is 0.5 -- the
denominator of the equation. In this example, \( \lambda = 1 \) and there is no PTM by the exporter
in countries A and B. In other words, if the exchange rate pass-through in countries A and
B are exactly the same, then PTM is not present. And we could have no PTM even
though we have incomplete exchange rate pass-through in the markets.

To illustrate the relationship between PTM and complete exchange rate pass-
through, I consider another specific example. Suppose the denominator of equation (1.1)
is one. Then, the numerator has to be one in order to conclude that there is no PTM in
these countries. With one for the numerator or denominator, there is complete exchange
rate pass-through to the export price in the country. This illustration makes the difference
clear; in two countries, if both follow complete exchange rate pass-through, then there is
no PTM. However, no PTM does not always imply complete exchange rate pass-through.

In conclusion, equation (1.1) formalizes the notion of PTM. In this dissertation, I
will present a model in which the parameter space identifies whether PTM is satisfied or
not.

1.2 Alternative Definitions of PTM

Researchers frequently implicitly or explicitly treat PTM as equivalent to
incomplete Exchange Rate Pass-Through (ERPT). For example, Froot and Klemperer
(1989) considered that PTM occurs when exporters increase the export price in dollars
during the period of the temporary appreciation of the dollar, instead of decreasing the
and Goldberg and Knetter (1997) have called pricing behavior PTM if the exporters did
not pass through all the exchange rate changes to the export price; more specifically, if
the exporters use markups to reduce the effect of the exchange rate changes on the export
prices.

Identifying what is PTM is the fundamental task. Then one can proceed to
examine the existence of PTM and to account for the reasons that PTM behavior exists
among exporters.

1.3 Roles of PTM in International Markets

Why does PTM matter? Why do we need a clear definition of PTM? What kinds
of advantages do we get from studying PTM?

There are four insights gained from applying a single definition of PTM. First of
all, it becomes easier to identify PTM with a single approach. We can avoid any
confusion that is associated with various or ambiguous definitions of PTM.

Second, we will better understand the characteristics of the international goods
market. It is easy to think that there is no PTM if exporters do not have any market power
over the price. However, the definition of PTM in this dissertation addresses that we
could have no PTM even though exporters have market power. For example, let us say
that exporter K’s exchange rate pass-through in country A is 0.1 and the exchange rate
pass-through in country B is 0.1 so $\lambda = 1$. And exporter Y’s exchange rate pass-through
in country A is 1 and the exchange rate pass-through in country B is 1 thus $\lambda = 1$ as well.

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4 This dissertation presents the definitions of PTM in other research more specifically later in chapter 2.
In this example, the degree of PTM for exporter K and exporter Y are the same; there is no PTM by these two exporters in both countries, A and B. Exporter Y passes through all the exchange rate movements to the export price in countries A and B and did not use any markup to mute the exchange rate fluctuations. On the other hand, exporter K exhibits market power in countries A and B passing through a low degree of ERPT in these countries. Thus, it is possible that no PTM occurs even though the exporters do have market power and markups because they can have the same low degree of ERPT in the markets as we have in the example above.

Third, PTM gives us information about the patterns of exporters’ pricing behavior as the exchange rate changes in various international markets. If there are relative price differences in different markets as the exchange rate fluctuates, it is useful to know how different the price is and how persistent the difference is. A single definition of PTM provides a better perspective because it allows us to measure the relative price changes in the markets and the degree of PTM -- $\lambda$.

Fourth, we may better understand the price movements of the goods in some countries. Since PTM measures the relative price differences, the choice of countries that the exporter sells the product to is important when we investigate the existence of PTM. It is possible that the same producer exercises PTM in countries A and B, for example, but not in countries A and C for the same product. Thus, the single definition of PTM makes it possible to get country-specific or industry-specific information about the price differences and the price movements.
This dissertation makes the following contributions to the existing research: First, the paper clarifies the situation that we can consider as the existence of exporters’ PTM behavior.

Second, I examine PTM behavior by specifying a model economy. My aim is to determine output levels endogenously as a function of the exchange rate and unit cost of production instead of taking the output levels exogenously as the reason of PTM. In this way, I extend the basic model by Froot and Klemperer (1989).

Third, I show that both equilibrium domestic price and export price depend on the exchange rate and the cost of production in the model economy. This approach extends the work in which only export prices vary according to the exchange rate movements. The chief implication is that the exchange rate elasticity of domestic price and the exchange rate elasticity of export price are directly derived in the model. Therefore, it is also possible to directly measure the relative price changes as the exchange rate varies in the different markets.

Fourth, I develop a standard that identifies the existence of PTM behavior displayed by exporters. As defined above, the standard is a ratio of the exchange rate elasticity of a country’s export price to the exchange rate elasticity of the domestic price. Here, I use the export and domestic prices determined in the model economy. The ratio offers a better measure of the relative price movements in the foreign market and the domestic market as the exchange rate fluctuates. The elasticity of the export price or that of the domestic price with respect to the exchange rate changes measures the percentage changes of the prices in response to the percentage changes of the exchange rates. In
other words, the ratio measures the relative price movements related to the exchange rate changes in the domestic market and the foreign market.

Fifth, I present a different result from existing research about the relationship between the invoice currency and ERPT (Bacchetta and Wincoop, 2002), and between the invoice currency and PTM (Gil-Pareja, 2003 and Bleaney, 1997). I found that exporters pass through more exchange-rate movements to the export price when they charge the export price in their own currency than they do in the buyers’ currency. The relative price difference between the domestic and the foreign market is larger -- higher degree of PTM when they invoice the export price in their own currency than they do in the buyers’ currency.

Sixth, I extend the model into a two-period model and investigate the influence of extended time periods on the output level, the export and domestic prices, ERPT and PTM. A firm’s output level, price, ERPT and PTM are affected by both its own discount factor and the other firm’s discount factor. I found that firms pass through more exchange rate movements to the price when they have more time horizon to consider; second-period ERPT is higher than the first-period ERPT. However, the effect of extended time horizon on the PTM behavior is ambiguous in the model economy. I address the conditions of each firm’s discount factor that entail a higher future output level and higher price than the present output level and price.

The rest of the dissertation is as follows: Chapter 2 of the dissertation reviews the relevant literature related to PTM. Chapter 3 specifies the one-period model economy. The model provides the condition that measures ERPT and PTM behavior. In Chapter 4, I develop a two-period model and present the effect of extended time horizon on the output
level, the price, ERPT and PTM. Chapter 5 summarizes the preliminary findings of the models and suggests forward consideration for future studies.
CHAPTER 2
LITERATURE REVIEW

I will review the relevant literature behind Pricing to Market (PTM) in this chapter. First of all, I want to point out the differences among Law of One Price (LOP), Purchasing Power Parity (PPP), Exchange Rate Pass-Through (ERPT) and Pricing to Market (PTM) in order to better understand PTM. Secondly, I review the definitions of PTM in other papers to make clear the differences between those authors’ definitions and the definition of PTM that I use in this paper. Third, different reasons for PTM will be reviewed. Fourth, I present a review of the existing empirical work on PTM.

2.1 Different Terms in International Trade

According to the Law of One Price (LOP), the price of a good in the United States should be the same as the price of the product in other countries in the world after we account for the exchange rate. Of course, the theory assumes that there is no transactions cost.

If the LOP holds as the exchange rate changes for the products in two countries -- for instance, the United States and Japan -- we can say that we have Purchasing Power Parity (PPP) between these two countries.

In the case of Exchange Rate Pass-Through (ERPT), pass-through measures the proportion of the exchange rate changes that is reflected in the export prices. So, the full proportion of the exchange rate pass-through is required to make LOP and PPP hold as the exchange rate changes. Indeed, many papers tested whether the LOP or PPP holds
using data for many products and industries (Isard, 1977; Engel and Rogers, 1996; Rogoff, 1996; Rush and Husted, 1985 and Patel, 1990). Generally, the findings show that the LOP or PPP does not hold in the real world and the result of the test varies across industries and countries. Even admitting the existence of the transactions cost, transactions cost does not account for the differences in product prices across countries as the exchange rate changes violating the LOP and PPP.

Currently, many economists agree that it is not a surprising fact that exporters do not pass through the exchange rate changes to the export prices completely as the exchange rate varies (Frankel, Parsley and Wei, 2005; Lee, 1997; Menon, 1996; Gron and Swenson, 1996; Hooper and Mann, 1989; Froot and Klemperer, 1989 and Krugman, 1987). Pricing to Market (PTM) extends exporter’s pricing behavior beyond incomplete ERPT and violation of LOP and PPP in the international goods market. In other words, in PTM, it does not matter whether the export price reflects the full proportion of the exchange rate changes or not. However, PTM focuses on an exporter’s price discrimination in different markets in response to the exchange rate fluctuations. For instance, if an exporter passes through 20 percent of the exchange rate changes to the export price of the product in country A and also passes through 20 percent of the exchange rate changes to the export price in country B, then there is no PTM behavior. On the other hand, ERPT is not complete and the LOP and PPP do not hold in this case. Namely, PTM values the relative movements of the export price in different markets as the exchange rate varies.
2.2 Pricing to Market: Krugman’s Definition

Krugman (1987) introduced the concept of exporter’s PTM behavior and I prefer to follow his definition of PTM. What he means by PTM is as follows:

…the import price appears to fall “too little,” yet we would not want to call this a case of pricing to market. By contrast, suppose that BMW decides for some reason to keep both its dollar prices in the United States and its mark prices in Germany constant. In this case the price of BMWs will certainly not fall as much as the dollar rises, but that is not the distinctive point. What would be striking would be that prices of autos in Germany and prices of German exports to France would fall relative to export prices to the United States. Indeed, if the prices diverged far enough, there would be an incentive for individuals to bypass BMW’s distribution channels and creates a gray market. This is the situation that I have in mind when discussing pricing to market.\(^5\)

In short, Krugman (1987) emphasizes that the relative price differences in different markets as the exchange rate fluctuates are important in order to call the situation as PTM. The export price itself that is not reflecting the full proportion of the exchange rate changes will not be considered as the case of the PTM.

2.3 Other Definitions of PTM

It is easy to use PTM together with incomplete exchange rate pass-through (ERPT) or violating the law of one price (LOP) implicitly without clarifying what is PTM. Most of the papers reviewed here use their own definition of PTM implicitly or explicitly. We can divide those definitions of PTM in other papers into two categories.

First, most of the papers define the PTM as the situation that the export price does not reflect the full proportion of the exchange rate changes. Thus PTM equals the incomplete ERPT in those papers. Certainly, that is different from the definition of PTM in this paper and in Krugman’s (1987) paper. Ghosh and Wolf (1994) is a good example.

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They consider that PTM occurs when the export prices do not compensate the nominal exchange rate movements and that PTM behavior induces incomplete ERPT. Thus, they basically equate PTM and incomplete ERPT. Froot and Klemperer (1989) also implicitly use the concept of PTM and incomplete ERPT to the export price as the same terms. The authors regard the pricing behavior as PTM when the exporters set the export price perversely to the exchange rate movements. For example, according to the authors, PTM occurs when exporters increase the export price in dollars during the period of the temporary appreciation of the dollar, instead of decreasing the dollar price. I will, however, consider that perverse price setting itself is not the PTM behavior in this paper as long as the exporter sets that perverse price setting on other markets as well.

Indeed, Froot and Klemperer’s (1989) definition is appropriate if we assume that the domestic price is not changing at all according to the exchange rate changes. With this assumption, if the exporter passes through the full proportion of the exchange rate changes to the export price, then there will be no PTM. Moreover, the PPP is achieved and ERPT is completed between the domestic market and the foreign market. So, in this case, we can treat ERPT and PTM as the same thing. On the other hand, if the exporter does not pass through the full proportion of the exchange rate fluctuations or if the direction of export price movement is not corresponding with our expectation as the exchange rate changes, PTM occurs between these two countries. Again, in this situation, incomplete ERPT tells us that there is PTM. However, I could not find any of this assumption or explanation in the paper.

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6 In the model I present later in this dissertation, both the domestic price and the export price vary based on the exchange rate as well as the cost of the production. I will suggest the ratio of the exchange-rate elasticity of export price to the exchange-rate elasticity of domestic price in order to examine whether there is PTM or not. Furthermore, the elasticity of the ratio represents ERPT to the domestic price and ERPT to the export price.
Similarly, Knetter (1989) defines PTM as exporters’ “destination-specific adjustment of markups” in different markets as exchange rate changes. The author calls it PTM if the exporters did not pass through all the exchange rate changes to the export price -- more specifically, if the exporters use markups to reduce the effect of the exchange rate changes on the export prices. Falk and Falk (2000) follow Knetter’s (1989) definition of PTM. Knetter (1993) also identifies the concept of “local currency price stability (LCPS)” in order to explain the exporter’s PTM behavior. According to Knetter (1993), exporters prefer to stabilize local currency prices instead of getting constant markups as the exchange rate changes and he calls this LCPS. Exporters, for instance, need to increase the local currency price when buyers’ currency is depreciated against sellers’ currency. The exporters, however, want to have the constant export price thus they do not increase the local currency price by decreasing the markups that they can earn. Thus, the author considers LCPS as a case of the exporters’ PTM behavior.

In the second category, researchers use an unclear or a general definition of PTM. Marston (1990) calls the pricing behavior PTM when exporters decrease their own currency export price in the foreign market as their own currency appreciates in order to prevent too much increase in the buyers’ currency price of the goods. Interestingly, Betts and Devereux (2000) identify the PTM as follows: “Some firms segment markets by country, and set prices in local currency of sale, a practice we refer to as pricing-to-market (PTM).” The authors develop a general equilibrium\(^7\) model that determines the exchange rate endogenously. They conclude that there will be no PTM if there is no price stickiness. In addition, PPP and LOP will be achieved without any price stickiness. Also,

\(^7\) Most of the research reviewed here is based on a partial equilibrium model to explain exporters’ PTM behavior and I will also develop the model in this paper in partial equilibrium perspective.
the authors remark that the exchange rate is more volatile when there is PTM. Lastly, Gil-Pareja (2003) mentions PTM generally as “the existence of international price discrimination induced by exchange rate movements”. The paper does not identify the kind of price discrimination.

2.4 Reasons for PTM

The basic assumption economists make when research considers PTM is that the exporters have some market power in the international goods market, therefore exporters can adjust the prices in the different markets as the exchange rate changes.

Krugman (1987) contributed the basic idea behind PTM empirically and theoretically but did not offer a rigorous treatment to explain PTM. After the introduction of the PTM, many efforts have been made to explain the reason for exporters’ PTM behavior by providing theoretical models and testing the existence of PTM empirically. Froot and Klemperer (1989), Kasa (1992), Knetter (1989), Knetter (1993), Marston (1990) and Goldberg and Knetter (1997) et al. are some of the good examples of the efforts.

Gil-Pareja (2003) points out that there could be two major categories of reasons for PTM. Firstly, PTM occurs because of exporters’ strategic price discrimination. Exporters set different prices in different markets strategically in order to get more profits as the exchange rate moves. For instance, if we consider elasticity of demand, market share, adjustment costs and expectations of the exchange rate movements as the reasons for PTM then exporters’ strategic price discrimination is the explanation for the PTM.
Secondly, PTM occurs because of price rigidity. We observe PTM phenomena because of the time lag between the exchange rate fluctuations and exporters’ setting export prices based on the exchange rate movements. Thus, in this case, exporters do not really intend to have the price differences in different markets but actually will have the price differences in those markets when the exchange rate varies. The long-term contract practice (Bergin and Feenstra, 2001), the market segmentation or the choice of invoice currency could make the price-rigidity problem even worse in the international goods markets.

The major trends in explaining the reasons for PTM rely on the exporters’ strategic price discrimination, such as demand side explanation and supply side explanation. Knetter (1989) and Knetter (1993) reflect those explanations: the effect of exchange rate changes on export prices will be caused by changes in marginal cost of production (supply side effect of exchange rate changes) and changes in markups which depend on the convexity of demand schedule (demand side effect of the exchange rate changes).

Additionally, some economists, such as Krugman (1987), Froot and Klemperer (1989) and Kasa (1992), apply these two sides of explanation further involving exporters’ expectation of the exchange rate fluctuations: temporary or permanent exchange rate changes.
2.4.1 Market Segmentation

Market segmentation as a cause of PTM suggests that it is not so surprising to have different prices in different markets if the markets are separated and we have different costs to provide the goods and different currencies in those markets.

Goldberg and Knetter (1997) try to define the meaning of market segmentation and integration. The authors indicate that the market is segmented geographically if the consumers need to pay significant transactions cost or if there are trade disturbances caused by cultural differences, different requirements for the products or tariffs and so on. They agree that the international goods market is segmented and the segmented market creates market power for the sellers. And that market power gives the sellers the opportunity to charge different prices for the same products in different countries. The authors also consider that the source of the segmentation and market power could be trade barriers that are raised by “country- and industry-specific” reasons, border effect and distance effect created by geographical country borders and the distance between countries. 8

It is therefore possible that the sellers have more incentive to commit price discrimination in these segmented markets to get more profits, especially when the exchange rate fluctuation is enormous.

Engel and Rogers (1996) assert that market segmentation is one of the reasons for violating LOP, and provide an interesting result that not only the border but both distance and the border are important to explain the relative price movements among countries. The authors found that exporters do take into account the availability of substitutes and

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8 Such as tariff, trade frictions, transactions cost and different requirements for the products with different societal customs in different countries, etc.
competitive suppliers within some distance when they set the prices. Thus distance and the border take a role in exporters’ price setting, causing relative price differences in different markets. The authors also propose that exporters adjust their prices more frequently if they have more competitors in a nearby location.

In other words, according to the authors, we observe the failure of LOP because the international goods markets are segmented by border and distance. Therefore the international market does not have many competitors and substitutes nearby and does not have a big incentive to adjust the price often. Thus, the price stickiness created by the market segmentation makes the price differences even worse.

Moreover, Gil-Pareja (2003) found that the EU automobile market is a segmented market even after the “single market programme”, and that the existence of market segmentation factors likely plays a role as the force of price differences in the international market.

2.4.2 Invoice Currency

It is known that usually exporters predetermine export prices, do not change the export price frequently, and set the price in the sellers’ currency (Page, 1981 and Godburg and Knetter, 1989). It makes sense to assume that the selection of invoice currency could play an important role in PTM behavior as the exchange rate fluctuates.

Gil-Pareja (2003) criticizes the fact that many papers ignore the role of invoice currency in PTM phenomena. He argues that the choice of invoice currency has even more effect on PTM and ERPT when the exporters do not change the export prices frequently based on the exchange rate movements. Giovannini (1988) argues that the
currency choice between the destination currency and the exporters’ currency is crucial to the price differences in different markets, especially when the exporters preset the export price. The author suggests that expectation of the exchange rate movements decides the expected profits, and that the expectation about the profits influences exporter’s pricing behavior such as invoice choices. Inversely, the choice of export price currency has a different impact on exporters’ profit based on the relationship between the exchange rate and the profit. In particular, in the model, Giovannini (1988) proves that “If profits are a concave function of the exchange rate, setting export price in foreign currency leads to higher expected profits. If profits are a convex function of the exchange rate, setting export price in home currency leads to higher expected profits.”  

It is common that exporters state prices in exporters’ currency when they sell products in foreign countries. Japanese exporters, however, frequently invoice in dollars and exhibit severe PTM behaviors (Page, 1981 and Goldberg and Knetter, 1997). So, Bleaney (1997) pays attention to the fact that many Japanese exporters use the U.S. dollar as the invoice currency not only for the products to the U.S. but also for the products to other foreign markets. Because of this fact, Japanese exporters’ pricing behaviors are affected by the real value of the dollar and the real value of the yen. He points out Japanese exporters’ choice of the invoice currency as one of the reasons for the price differences between the foreign market and the domestic market when the exchange rate varies.

After comparing the price difference between the dollar export price and the domestic price that Japanese exporters charge as the exchange rate changes, Bleaney (1997) concludes that in the short run, the price difference between the foreign market

and the domestic market is relatively big when the yen appreciates against all other currencies. However, it is relatively small when the dollar appreciates against other currencies other than the yen. According to the author, this is because Japanese exporters decrease the dollar export price more when there is yen appreciation against other currencies than they do when there is dollar appreciation. Bleaney (1997) calls it “dollar-invoicing effect.” In other words, Japanese exporters change the dollar export price less in foreign markets if there is real dollar value change than they do if there is real yen value change against other currencies. The dollar-invoicing effect could occur because of the time lag between the dollar price setting and delivery of the product. Moreover, there is the dollar-invoicing effect in the short run but not in the long run. Therefore, the PTM is not affected by the choice of invoicing currency in the long run.

Gil-Pareja (2003) also investigates the role of invoice currency in PTM behavior. He tested European car producers’ PTM behavior related to choice of the invoice currency after “the single market programme”. Unfortunately, Gil-Pareja (2003) could not find strong evidence to support the important role of invoice currency to explain PTM. And he concludes that the different choice of invoice currency is not a factor to explain the PTM phenomenon in the EU car market.

In contrast, Bacchetta and Wincoop (2002) found a strong relationship between the choice of invoice currency and ERPT. If exporters impose the export price in terms of buyer’s currency then there is zero ERPT. However, if exporters charge the export price in terms of seller’s currency then there is complete ERPT. And exporters choose the invoice currency based on the size of market share and how different the product is.
exporters have larger market share and more differentiated goods, they tend to choose exporter’s currency instead of buyer’s currency.

Magee (1973) analyzes how the choice of currency on contracts affects the trade balance. It is an interesting analysis, especially when we consider America’s growing trade deficit since 1971. The author presumes that there is a time lag between making contract and shipping the product. So, if there is exchange rate movement after the contract is made and before the product is shipped, the exchange rate fluctuation could have influence on the trade balance. The exchange rate represents the amount of dollar per unit of foreign currency. For simplicity, Magee considers the situation that the dollar depreciated. Magee introduces four possible cases, as follow, to examine the relationship between the invoice currency and the trade balance: first, U.S. exports in foreign currency and U.S. imports in dollars; second, U.S. exports in foreign currency and U.S. imports in foreign currency; third, U.S. exports in dollars and U.S. imports in dollars; and fourth, U.S. exports in dollars and U.S. imports in foreign currency. For example, if the U.S. producer exports the product charging the price expressed in foreign currency and the U.S. consumers buy imports in terms of dollars, then if the dollar depreciates, the value of the U.S. exports will be increased as the U.S. producers receive the same amount of revenue in foreign currency by selling their goods but get more revenue in dollars after the dollar depreciation. On the other hand, the value of the U.S. imports will stay the same in terms of dollars because consumers pay in dollars. As a result, U.S. trade balance rises as the dollar depreciates in this case.
2.4.3 Demand Side Explanation

The common story of the demand side cause of PTM is a price discrimination based on the changes of price elasticity of demand. Knetter (1989) and Knetter (1993) emphasize the relationship among the exchange rate, the changes of price elasticity of demand, Local Currency Price Stability (LCPS) and markups. If buyers’ elasticity of demand is increasing as price increases, then exporters will not increase local currency price as much as we expected in a very competitive model as buyers’ currency depreciates because exporters prefer not to face higher elasticity of demand. As the result, exporters keep the local currency price stable, namely allow LCPS and the decrease in their markups. Of course, we assume that marginal cost is constant.

On the other hand, if buyers’ elasticity of demand is decreasing as price increases, exporters will increase local currency price more than they do in the case of increasing elasticity of demand but less than they do in a competitive model as buyers’ currency depreciates. This is because exporters will face decrease in the elasticity of demand as they increase the price. It means that the consumers are less responsive to the price changes as the price increases. As the result, exporters allow the increase in their markups in this case by allowing LCPS. 10

Finally, as Froot and Klemperer (1989) also mention the exchange rate changes would be fully passed through to export prices in the case of constant elasticity of demand, and there is no LCPS if elasticity of demand is constant in Ketter (1993).

Furthermore, Krugman (1987) points out the importance of both reputation and time lag in studying the effect of price on demand. He argues that: it will take time to see

10 As I explain earlier in this chapter, Knetter (1993) is treating LCPS as the same as exports’ PTM behavior.
the effect of export price changes on demand when the price changes due to exchange rate changes. In addition, Krugman assumes that there exist trade-offs between today’s profit and future market share, meaning if exporters want to get more market share in the future, it is necessary to sacrifice today’s high profit.

On the consumer’s side, Krugman assumes that, first of all, consumers decide whether they will enter a particular goods market or not, considering both the price and the expected price, where the expected price depends on the firm’s reputation. They then choose whether to buy the good, as well as the quantity they want to buy if they decide to buy. The number of consumers that actually enter a firm’s market depends on the expected price of the good. After consumers decide to enter a goods market, a firm will have a less elastic demand curve,\(^{11}\) and will face an incentive to change the actual price from the expected price of consumers; if an exporter follows this incentive, however, it will lose market share because consumers will not believe the firm in the future. Thus, an exporter will have a smaller market share, and a smaller profit, in the future.

As a result, an export firm must first take into account whether exchange rate movements are temporary or not. Secondly, an exporter considers the trade-off between today’s profit and the market share in the future when he sets the export price as the exchange rate varies.

Froot and Klemperer (1989) also start with the assumption that there exist trade-offs between today’s profit and future market share. Thus, a foreign exporter invests in market share using the current export price. According to the authors, when a foreign exporter believes that an appreciation of the dollar is temporary, there could be an inter-temporal substitution, that is, an exporter will prefer to get a high current dollar profit

\(^{11}\) Because of brand switching costs and network externalities, etc.
rather than invest in market share for the future by increasing, rather than decreasing, the dollar export price in the U.S. market. However, in the case of permanent dollar appreciation, an exporter will prefer to get a larger market share in the U.S. market in the future by decreasing the current dollar export price, rather than take the current dollar profit expressed in the exporter’s currency. In other words, the second-period profit depends on first-period market share that is determined by the current price. Since first-period market share determines second-period market demand\textsuperscript{12} and second-period market demand decides second-period profit, Froot and Klemperer (1989) set the current market share as one of the independent variables in the model and introduce “cost effects” and “real interest rate effects” to investigate the effect of the exchange rate changes on the export price.

Formally, the domestic firm’s discounted future profit, denoted by a superscript $D$, expressed in dollars, is:

$$\pi^D = \pi_1^D(p^D, p^F) + \lambda^D \pi_2^D\left(\sigma^D(p^D, p^F), e_2\right),$$

where the total profits are the sum of the first- and second-period profits, $\pi_1^D$ and $\pi_2^D$ respectively. Note that $p^D, p^F$ denotes the domestic firm’s and the foreign firm’s first-period prices, respectively. Second-period profits depend on the first- and second-period market share. $\lambda^D$ is a discount factor, $\sigma^D$ is first-period market share, expressed as a function of the domestic and foreign price of a good, and $e_2$ is the number of the foreign

\textsuperscript{12} The authors say that second-period demand depends on first-period market shares because of consumers’ brand-switching costs, brand loyalty and network externalities. See Froot and Klemperer (1989), p. 639 and Klemperer (1987).
currency over a dollar in the second period. The idea is that the first-period investment in market share can affect second-period demand.

The foreign firm’s total discounted future profit, denoted by $\pi^F$, expressed in its own currency, is:

$$
\pi^F = e_1 \pi_1^F \left( p^D, p^F, \epsilon_1 \right) + \lambda^F e_2 \pi_2^F \left( \sigma \left( p^D, p^F \right), \epsilon_2 \right).
$$

(2.2)

The discount factor is given by:

$$
\lambda^i = \frac{\beta}{1 + r^i}, \quad i = D, F,
$$

(2.3)

where $\beta, r^i$ is the duration of the second period relative to the first, and interest rate in the $i$ country, respectively.

The authors hold the interest rate constant in the foreign firm’s home market, therefore hold $\lambda^F$ constant as follows:

$$
\lambda^D = \lambda^F \frac{e_2}{e_1} = \lambda
$$

(2.4)

The assumptions are given by$^{13}$:

i) F has constant marginal costs, $\gamma^F$ in terms of foreign currency and $c_1^F = \frac{\gamma^F}{e_1}$,

$$
c_2^F = \frac{\gamma^F}{e_2}
$$

in terms of dollars.

ii) The exchange rate and the aggregate price level are given exogenously.

iii) $\frac{\partial \sigma^i}{\partial p^i} < 0$, $\frac{\partial \pi_1^i}{\partial \sigma^i} > 0$, $\frac{\partial \pi_1^i}{\partial p^i} > 0$, $i = D, F$.

Froot and Klemperer find firm \( i \)’s price in reduced form from taking total differentiation of the first order condition of each country’s profit function with respect to \( c_i^F, c_2^F \) or \( \lambda \) is:

\[
p^i = p^i \left( c_i^F, c_2^F, \lambda \right),
\]

(2.5)

“the effect of a proportional change in the period-t exchange rate” is:

\[
\frac{dP^i}{de_i} = \left( \frac{\partial P^i}{\partial c_i^F} \right) \left( \frac{dc_i^F}{de_i} \right) + \left( \frac{\partial P^i}{\partial \lambda} \right) \left( \frac{d\lambda}{de_i} \right)
\]

(2.6)

\[
= \text{cost effects } + \text{real interest rate effects.}
\]

The important contribution of this paper is that the authors divide the effect of exchange rate changes on price of goods into cost effects and real interest rate effects when market share matters.

Those cost effects and real interest rate effects can be specified into first-period and second-period as follows: “the effect of current, temporary and proportional appreciation of the domestic currency” is:

\[
\frac{dP^i}{de_1} = -c_i^F \left( \frac{\partial P^i}{\partial c_i^F} \right) - \lambda \left( \frac{\partial P^i}{\partial \lambda} \right)
\]

(2.7)

\[
= \text{cost effects } + \text{real interest rate effects.}
\]

“the effect of a future proportional appreciation” is:

\[
\frac{dP^i}{de_2} = -c_2^F \left( \frac{\partial P^i}{\partial c_2^F} \right) + \lambda \left( \frac{\partial P^i}{\partial \lambda} \right)
\]

(2.8)

\[
= \text{cost effects } + \text{real interest rate effects.}
\]
“the effect of a permanent percentage change in the exchange rate (i.e., \( \text{de}_1 = \text{de}_2 \))” is:

\[
\frac{dP'}{de_1} + \frac{dP'}{de_2} = -c_1^F \left( \frac{\partial P'}{\partial c_1^F} \right) - c_2^F \left( \frac{\partial P'}{\partial c_2^F} \right).
\]  

(2.9)

**Result 1 (Froot and Klemperer):** Cost effects on the price changes; according to the authors, cost effects force \( P^F \) to decrease, as the expectation of future exchange rate increases and cost effects on \( P^D \) is indeterminate.

**Result 2 (Froot and Klemperer):** An appreciation of the dollar decreases F’s dollar costs, \( c_2^F = \frac{\gamma^F}{e_2} \). Because of the decrease in dollar cost in the future, marginal value of first-period market share is increasing and that increased value of the market share will increase second-period profit, \( \left( \frac{\partial \pi^F}{\partial \sigma^F} \right) \). Hence, firm F will decrease \( P^F \) to invest in market share and to get higher second-period profit when there is a decrease in second-period dollar cost as the exchange rate increases. Thus, \( \left( \frac{\partial P^F}{\partial e_1} \right) > 0 \Rightarrow -c_2^F \left( \frac{\partial P'}{\partial c_2^F} \right) < 0 \).

**Result 3 (Froot and Klemperer):** Real interest rate effects on prices changes; real interest effects make \( P' \) rise as the exchange rate increases temporarily. From equation (2.4), \( \lambda \) decreases with an increase in \( e_1 \). The lower \( \lambda \), discount factor, means the higher cost of the second-period investment, so firm F prefers to invest less in the second-period by getting less market share in first-period and getting less first-period profit by increasing current price. As the result, the marginal value of price in first-period, \( \left( \frac{\partial \pi^F}{\partial P'} \right) \), decreases.

Therefore, \( \left( \frac{\partial P'}{\partial \lambda} \right) < 0 \Rightarrow \lambda \left( \frac{\partial P'}{\partial \lambda} \right) < 0 \) or \( -\lambda \left( \frac{\partial P'}{\partial \lambda} \right) > 0 \).
**Result 4 (Froot and Klemperer):** Temporary dollar appreciation and exchange rate pass-through and PTM; from equation (2.7), we have cost effects, \( -c_{i}^{F} \left( \frac{\partial P^{i}}{\partial c_{i}^{F}} \right) < 0 \) and real interest rate effects, \( -\lambda \left( \frac{\partial P^{i}}{\partial \lambda} \right) > 0 \), thus if real interest rate effects dominate cost effects under temporary dollar appreciation, then the price of goods will increase instead of decreasing. Therefore, this model can explain hysteresis of import prices in the United States in 1980s with exchange rate changes. In this case, authors consider that PTM also occurs.

**Result 5 (Froot and Klemperer):** Temporary and permanent exchange rate changes and exchange rate pass-through; when we compare equations (2.7) and (2.9), a foreign firm lowers its dollar price more when it expects that the exchange rate change is permanent than it does when it expects that the exchange rate is temporary. This is because there are only negative cost effects in the case of permanent exchange rate changes in equation (2.9) and there are negative cost effects and positive real interest rate effects in the case of temporary exchange rate changes in equation (2.7).

In summary, according to Froot and Klemperer (1989), when a foreign exporter believes that an appreciation of the dollar is temporary, the appreciation of the dollar increases the value of the current dollar profits expressed in the exporter’s currency. Thus, an exporter will prefer to get a high current dollar profit rather than invest in market share for the future. An exporter will increase the dollar export price \( P^{F} \) in the U.S. market in the situation of the temporary appreciation of the dollar, instead of decreasing the dollar price. However, in the case of permanent dollar appreciation, an exporter will prefer to get a larger market share in the U.S. market in the future rather
than take the current dollar profit expressed in the exporter’s currency. Therefore, an exporter will decrease the current export price \( P^f \) as quoted by the dollar.

Krugman (1989) also introduces a two-period model to conduct a dynamic analysis of PTM as well but he could not get a clear conclusion about the temporary and permanent effect on export prices. The profit function -- expressed in terms of the foreign firm’s currency -- of a firm that sells a commodity in the U.S. market in two periods is:

\[
\left( e_1 P_1 - c^* \right) D_1(P_1) + \beta \left( e_2 P_2 - c^* \right) D_2(P_1, P_2),
\]

where \( D_1(P_1), D_2(P_2) \) is a first- and second-period demand respectively, \( \frac{c^*}{e_1}, \frac{c^*}{e_2} \) is the foreign firm’s marginal cost in dollars, and \( \beta \) is a discount factor. In Krugman (1989), the first-period demand function depends on the first-period price but second-period demand depends on the first-period price and the second-period price. \( e_1 \) is a foreign firm’s currency price over a dollar.

To examine the effect of the expectation of exchange rate changes -- temporary or permanent -- on the export price, the author compares two cases, the effect of the \( e_1 \) only on \( P_1 \) -- when the exporter expects the exchange rate change is temporary -- and the effect of the \( e_1 \) and \( e_2 \) on \( P_1 \) -- when the exporter expects the exchange rate change to be permanent. Krugman questions: which view -- that is, whether the exchange rate movements are temporary or permanent -- will decrease the current export price more? He concludes that the impact of temporary and permanent exchange rate on the export prices is ambiguous.
2.4.4 Supply Side Explanation

For simplicity, most papers assume that marginal cost of production is constant. As the exchange rate varies, they investigate the effect that such movements will have on export prices. Knetter (1989) follows the same assumption, but admits that changes in the marginal cost of production occur over time. He attempts to distinguish changes in marginal cost induced over time from fluctuations in the markups caused by exchange rate changes empirically, in order to measure the net markup fluctuations that represent exporters’ price discrimination across the destination markets as the exchange rate varies.

Krugman (1987) and Kasa (1992) identify the adjustment cost that is another factor. Kasa (1992) argues that the adjustment cost model explains an exporter’s PTM more precisely than the demand-side analysis, which relies on the shape of the demand curve, market share, switching costs, brand loyalty, etc. in order to explain PTM, since adjustment costs is a more direct and intuitive factor affecting an export price in response to exchange rate movements.

Both authors suggest that the adjustment cost could be the additional factor that results because exporters expand overseas quickly or contract their product quantities. More specifically, Krugman (1987) identifies that marketing and distribution costs are more reasonable to consider as adjustment costs rather than transportation costs.

Krugman (1987) and Kasa (1992) note that if a firm decreases the export price as measured in buyers’ currency as much as buyers’ currency appreciation against exporter’s currency, the firm will face an increase in demand in a foreign export market. However, increasing the supply immediately causes adjustment costs for a firm. Therefore, if a firm believes that the exchange rate change is temporary, it will maintain
the export price, or decrease the export price less than that implied by the exchange rate appreciation. Thus, a firm will not face a large increase in demand, allowing it to avoid a large supply increase, along with the endogenous change in adjustment costs. It is not worth paying adjustment costs to meet the temporary demand increase if the exchange rate movements are temporary. In this instance, a firm pursues PTM behavior if it believes the exchange rate fluctuation is temporary, using its static markups to smooth the temporary exchange rate changes.

In the case of permanent dollar appreciation, however, a firm will decrease the dollar export price and pay adjustment costs to increase the supply. Thus, the maintenance of an exporter’s PTM behavior is determined not only by how long the exchange rate fluctuation has lasted, but also by the exporter’s expectation of the persistence of exchange rate fluctuations.

Overall, Froot and Klemperer (1989) and Kasa (1992) propose dynamic analyses of PTM. Indeed, each sets of results share a similar spirit: a temporary appreciation of the dollar has a smaller effect on the export price than a permanent appreciation of the dollar. Thus, the exporter’s PTM behavior can be observed when there is an expectation of temporary exchange rate changes.

Similar to the literature on adjustment costs, Ghosh and Wolf (1994) view menu cost as the reason that accounts for PTM and for incomplete ERPT. If a frequent price change incurs significant menu costs, the exporters will not reflect the exchange rate movements on the export price completely.

In contrast, Baldwin (1988) insists that the effect of the temporary exchange rate changes could have a permanent effect through the structure changes of economy on
import prices; more specifically, “hysteresis\textsuperscript{14} in import prices.” The structural changes will be induced since a large appreciation of domestic currency will decrease the foreign cost in terms of domestic currency and will attract other firms to enter the industry and the firms will face more competition.\textsuperscript{15} Competition will lower the import price. So, even though the temporary appreciation is eliminated, firms will keep the price lower than they would have.

2.5 Empirical Test of PTM

The most serious problem in testing PTM explanations empirically is that there is lack of data available to measure a country’s export and domestic prices across industries. Alternatively, it is hard to get the data that measure export prices of the same product in different export markets. Most commonly, unit values of aggregate price indices are used to do the empirical analysis, even though it cannot represent the exact export price or domestic price of a commodity.

Knetter (1993) chose large economies as the destination of exports to get more observations and to obtain more accurate unit value data -- total value of exports divided by the total quantity. He intended to get industries in which more than one of the four countries exports the product to other countries in the sample. This is difficult, however, because the industry classification code is hard to match across the countries. Knetter (1989) uses detailed seven-digit industry export data measured in unit values. In Froot and Klemperer (1989), the authors use highly disaggregated bilateral export unit value

\textsuperscript{14} By this term, the author means, for example, that even though the temporary exchange rate shock is eliminated, the import price will not go back to the original level.

\textsuperscript{15} Knetter (1994) concluded that he could not find strong evidence that there was increased competition in the U.S. market as the dollar appreciated in large amount in the 1980s.
data from the U.N. to get a measure of the changes in export prices. Kasa (1992) compares the import price differences of seven German products\textsuperscript{16} within the U.S. and Canada from 1978 to 1987. He presents evidence suggesting that the temporary component in the exchange rate increases the frequency of PTM. The author uses 120 monthly observations produced by the customs agencies in the U.S. and Canada to test the model. He argues that the data set contains narrowly defined commodities.\textsuperscript{17} He finds that the German export prices in the U.S. increased more than in Canada. The dollar appreciated against the Deutschmark; moreover, German exporters considered adjustment costs to be an important factor in both the U.S. and Canada, except in the sewing machine and shoe markets.

Marston (1990) uses data from the Bank of Japan, which contains export and domestic prices for disaggregated products to estimate PTM in the case of Japanese manufacturing firms in the 1980s. The author shows how the firms in selected industries\textsuperscript{18} responded to changes in the real exchange rate by varying their export prices relative to prices for the domestic market. He found strong evidence of the Japanese firms’ PTM behavior as the real exchange rate changes in the U.S. market across 17 industries. Two notable exceptions are the small truck and camera industries. In addition, the Japanese firms responded less to the exchange rate change in their price when the yen depreciated than when the yen appreciated. Also, the Japanese firms’ PTM behavior helped them protect their competitive position in the U.S. market.

\textsuperscript{16} Cars, wine, beer, industrial sewing machines, suits, brass rods and leather footwear. p. 18. The authors chose these commodities because of the available data stability.

\textsuperscript{17} Also, the author provided the problems of these data. p. 16.

\textsuperscript{18} Such as transport and tractor equipment, consumer goods.
Krugman (1987) uses the method below to measure, for example, German exporter’s PTM behavior empirically:

\[
\frac{\text{Germany's export price in the U.S.} - \text{Germany's export price in other countries}}{\text{Germany's export price in the U.S.} - \text{Other countries' export price in the U.S.}}
\]

He uses the denominator to measure the real exchange rate changes. If Krugman’s ratio equals zero, there is no PTM. Otherwise, the author concludes that there is exporters’ PTM behavior. Krugman uses unit values of exports in 1980 and 1983 to create the price index in order to test the criteria of PTM above. The result shows 0.39 in the machinery and transport equipment industries and he concludes that there is PTM behavior by German exporters in the US market relative to other countries. The data shows, however, that there is no PTM behavior in other German export industries to the U.S.

Froot and Klemperer (1989) test the extent to which expectations of future exchange rate changes affected the degree of PTM in the 1980s. The authors focused on the measurement of the expected exchange rate changes, as well as on the relationship between the expected exchange rate changes and PTM in the U.S and other countries. To measure the degree of PTM, they considered foreign exporters’ profit margins as real exchange rates changed. According to the authors, the trends found in foreign exporters’ profit margins almost follow that of the real value of the dollar, meaning there is perverse movement of export price in the U.S. as the real exchange rate changes: in other words, as dollar value increases, the dollar import price increases and foreign exporters’ profit margin increases as well with constant marginal cost assumption.

In order to measure expected exchange rate changes,\textsuperscript{19} Froot and Klemperer uses interest rate differentials and survey data.\textsuperscript{20} The authors argue that these data demonstrate

\textsuperscript{19} See more about exchange rate expectations in Ito (1990).
that exporters viewed the exchange rate changes in the 1980s as a temporary phenomenon. If exporters expected the appreciation of the dollar in the 1980s to be temporary, then according to the market-share model of Froot and Klemperer, exporters should have taken high-profit margins from the temporary exchange rate changes. The authors performed ordinary least squares (OLS) using these interest-rate differentials and survey data to test their model.

\[ \Delta p_{i,t}^{UK,US} - \Delta p_{i,t}^{UK,JA} = \beta_1 \Delta e_{i,t}^{US,JA} + \beta_2 \Delta e_{i,t}^{US,JA} + \varepsilon_{i,t}^{US,JA}, \]

where \( i \) is the industry, \( \Delta e_{i,t}^{US,JA} \) is the change from period \( t-1 \) to \( t \) in the log of the real dollar/yen rate. \( \Delta E_i \left( \Delta e_{i,t+1}^{US,JA} \right) \) and the percentage-point change from period \( t-1 \) to \( t \) in expected depreciation of the real dollar/yen rate over the following period.\(^{21}\)

The authors argue that \( \Delta p_{i,t}^{UK,US} - \Delta p_{i,t}^{UK,JA} \), the differential effect of exchange rate changes on foreign exporters’ prices in different markets, gives a more precise result of PTM from expected depreciations.

Using the empirical test, Froot and Klemperer conclude that there is no PTM and perfect exchange rate pass-through is evident when there is permanent exchange rate change. They could not find evidence to support the notion that PTM depends on the expectation of future dollar depreciation. In short, they conclude that the evidence on PTM is fragile.

Knetter (1993) introduces the following empirical framework to study PTM behavior in several different export markets, using the annual value and quantity of exports in the U.S., the U.K., Japan, and Germany:

\[ \Delta p_n = \theta_i + \beta_i \Delta x_n + \varepsilon_u, \]

where \( p \) is the log of the export price and \( x \) is the log of the destination-specific exchange rate -- the number of buyer’s currency over a seller’s currency. He estimates \( \theta_i \) and \( \beta_i \). The author considers that \( \theta_i \) will vary over time by changes in exporters’ marginal costs.

In the case of a competitive market, \( \beta_i = 0 \) shows that a change in currency values would be fully passed through to the buyer and no LCPS would exist. In contrast, with \( \beta_i < 0 \), the evidence is consistent with LCPS. In other words, depreciation of buyers’ currency decreases sellers’ markup and appreciation decreases sellers’ markup. Finally, with \( \beta_i > 0 \), the evidence is consistent with the notion that destination-specific markup adjustment amplifies the effect of the exchange rate changes on the price in units of the buyer’s currency.

However, if the exporter is a monopolist, the value of \( \beta \) is determined by the convexity of the demand schedule in the destination market: for example, \( \beta_i = 0 \) means constant elasticity with respect to price and no LCPS; \( \beta_i < 0 \) indicates less convex demand and the existence of LCPS. Therefore, from the value of \( \beta_i \), we can infer how responsive the consumers are to various price levels in the goods market as well.

Three important empirical results of the paper by Knetter (1993) are: First, the market in which the exporters sell the products is not important in determining the existence of PTM behavior. Actually, Japanese exporters did apply the same PTM behavior to the U.S. market as with other markets. Second, the exporters of the four countries (U.S., U.K., Germany and Japan) in the same industry exhibit similar PTM as
the exchange rate changes. Thus, Knetter (1993) concludes that industry is important to explaining the pattern of PTM behavior. Third, after mixing all the industries, the exporters in the U.S. and U.K. exploit different PTM according to the particular industry to which the exporters belong. However, those in Germany and Japan exhibit PTM no matter what the industry. For all the industries of each country, the degree of markup-adjustment as the exchange rate fluctuates is about 36 percent for German and British exporters, about 48 percent for Japanese exporters and zero percent for US exporters.

Knetter (1989) tries to separate the marginal cost changes from the markup changes empirically in order to measure more precise markup changes as the exchange rate changes and to examine the exporters’ price discrimination according to the destination markets. The author uses a “fixed-effects regression model”:

$$\ln P_{it} = \theta_t + \lambda_i + \beta_i \ln S_{it} + U_{it},$$

where $\theta_t$ = time effect, $\lambda_i$ = country effect, $U_{it}$ = regression disturbance,

where time effect indicates the price changes that all the countries have in each period. Country effect is country-specific changes as exchange rate changes measuring the markups.

The author reaches similar conclusions to those in Knetter (1993); U.S. exporters are not responding through price changes according to the exchange rate changes sensitively. However, the paper provides strong evidence that German exporters do not reflect exchange rate changes to dollar prices in the U.S. market.
CHAPTER 3
THE MODEL

In the model economy, the firms choose output levels as the exchange rate changes, taking the market demand as given. And then the market demand determines the price in a country based on the output levels. I measure the impact that movements in the exchange rate have on the equilibrium price. Initially, I focus on the existence of the PTM. More specifically, I derive the conditions under which PTM exists. Basically, I presume that the output level is the factor that can account for why exporters behave according to PTM. Output level is determined endogenously in the model, depending on the exchange rate and the cost of production that maximizes the exporters’ profit that is expressed in its own currency. Output level will affect the price of the goods because it enters into the inverse demand function. The implication is that prices are affected by the exchange rate through the output level. In other words, the profit-maximizing prices in the domestic market and in the foreign market contain the exchange rate, as well as the cost of production. Thus, it is possible to directly obtain the exchange-rate elasticity of the domestic price and the exchange-rate elasticity of the export price. I use the ratio of the exchange-rate elasticity of the export price to that of the domestic price in order to investigate the existence of exporters’ PTM behavior between the domestic market and the foreign market.

The model economy gives means of understanding the existence of PTM and provides an explicit method to test the existence of PTM with the reduced form of the exchange rate and the cost of production.
3.1 Assumptions

There are two countries, denoted A and B, and each country has one firm. For simplicity, I refer to these as firm A and firm B, respectively. Each firm produces one good and the goods are similar; more specifically, goods A and B are near-perfect substitutes. This is the two-way trade model. Both firms sell their products in both their own domestic market and foreign market. Firms charge the local-currency price in the foreign market. The consumers in both countries have the same preferences for the goods. We assume that each firm faces a linear demand curve for the product in both the domestic and the foreign market and unit cost of production. Each firm produces the goods only in the domestic market, excluding the case of outsourcing production in other countries. The exchange rate is given exogenously. These two firms maximize their own currency value of the profits.

3.2 The Model

Firm A maximizes the profits -- expressed in its own currency -- as follows:

\[
\max \quad \pi^A = P^A q^{AD} + eP^B q^{AB} - C^{AD} q^{AD} - C^{AB} q^{AB} \\
= \left[ a - q^{AD} - q^{BA} \right] q^{AD} + e \left[ a - q^{BD} - q^{AB} \right] q^{AB} - C^{AD} q^{AD} - C^{AB} q^{AB} \tag{1}
\]

Firm B maximizes the profits -- expressed in its own currency -- as follows:

\[
\max \quad \pi^B = P^B q^{BD} + \frac{1}{e} P^A q^{BA} - C^{BD} q^{BD} - C^{BA} q^{BA} \\
= \left[ a - q^{BD} - q^{AB} \right] q^{BD} + \frac{1}{e} \left[ a - q^{AD} - q^{BA} \right] q^{BA} - C^{BD} q^{BD} - C^{BA} q^{BA} \tag{2}
\]
where \( P^i \) \((i = A, B)\) denotes the price of the products that are sold in country \( i \) and is an inverse demand function of \( Q^i \) \((i = A, B)\). For simplicity, I assume that the demand curve, which each firm faces in both its domestic and foreign market, is a linear demand curve, and that consumers have the same preferences across countries:

\[
P^A = a - Q^A
\]
\[
P^B = a - Q^B, \quad a \text{ in both equations is constant and } a > 0.
\]

Firms A and B charge the price of \( P^A \) in country A to sell the product. \( P^A \) is expressed in country A’s currency. Firms A and B charge the price of \( P^B \) in country B to sell the product. \( P^B \) is in terms of country B’s currency. Thus, each firm imposes the local-currency price as an export price. In other words, firm A charges \( P^A \) in the domestic market in its own currency and \( P^B \) in the foreign market (country B) in country B’s currency. Firm B charges \( P^B \) in the domestic market in its own currency and \( P^A \) in the foreign market (country A) in country A’s currency. \( Q^i \) is the total quantity of the goods supplied by firm A and B to country \( i \) \((i = A, B)\). The total quantity supplied to country A is \( Q^A \):

\[
Q^A = q^{AD} + q^{BA}.
\]
\( q^{AD} \) is the quantity of the product produced by firm A and sold in the domestic market.

\( q^{BA} \) is the quantity of the product produced by firm B and sold in country A. In the case of country B, the total quantity supplied to country B is \( Q^B \):

\[
Q^B = q^{BD} + q^{AB}.
\]

Similarly, \( q^{BD} \) is the quantity of the output produced by firm B and sold in the domestic market. \( q^{AB} \) is the quantity of the good produced by firm A and sold in country B. Therefore, we can rewrite \( P^A \) and \( P^B \) as follows:

\[
P^A = a - Q^A = a - q^{AD} - q^{BA}, \quad \text{where } Q^A = q^{AD} + q^{BA}, \quad a > 0
\]

\[
P^B = a - Q^B = a - q^{BD} - q^{AB}, \quad \text{where } Q^B = q^{BD} + q^{AB}, \quad a > 0.
\]

\( P^A \) is the function of the output level of \( q^{AD} \) and \( q^{BA} \) and \( P^B \) relies on the output level of \( q^{BD} \) and \( q^{AB} \). The nominal exchange rate is denoted by \( e \) and given exogenously. It represents the units of country A’s currency per unit of country B’s currency:

\[
e = \frac{A's \ currency}{B's \ currency}.
\]

Firm A charges \( P^A \) and sells the quantity of \( q^{AD} \) in the domestic market; thus \( P^A q^{AD} \) is the revenue that firm A gets from the domestic market. Firm A charges \( P^B \) and
sells $q_{AB}^{AB}$ in the foreign market, so $eP_B^B q_{AB}^{AB}$ is the revenue that firm A will earn in the foreign market in terms of country A’s currency. As the result, the total revenue that firm A can achieve in terms of its own currency is $P_A^A q_{AD}^{AD} + eP_B^B q_{AB}^{AB}$. And, as firm B charges $P_B^B$ and sells the quantity of $q_{BD}^{BD}$ in the domestic market, firm B will earn the revenue $P_B^B q_{BD}^{BD}$ in the domestic market. Firm B charges $P_A^A$ and sells the quantity of $q_{BA}^{BA}$ in the foreign market; therefore, firm B will get the revenue, $\frac{1}{e} P_A^A q_{BA}^{BA}$ from the foreign market expressed in country B’s currency. In consequence, the total possible revenue for firm B in terms of its own currency is $P_B^B q_{BD}^{BD} + \frac{1}{e} P_A^A q_{BA}^{BA}$.

The costs of the production for each firm are expressed in terms of its own currency, assuming as well that each firm produces the goods in the domestic market excluding the case of outsourcing production in other countries. $C_{AD}^{AD}$ is the unit cost to produce $q_{AD}^{AD}$, and $C_{AB}^{AB}$ is the unit cost to produce $q_{AB}^{AB}$. The model assumes that $C_{AD}^{AD}$ and $C_{AB}^{AB}$ are different because selling the product in a foreign country requires costs, such as transportations cost. $C_{AT}^{AT}$ is the total cost to firm A to produce $q_{AD}^{AD}$ and $q_{AB}^{AB}$. The unit cost of $C_{BD}^{BD}$ for country B to produce $q_{BD}^{BD}$ and $C_{BA}^{BA}$ to produce $q_{BA}^{BA}$ are different. $C_{BT}^{BT}$ is the total cost to firm B to produce $q_{BD}^{BD}$ and $q_{BA}^{BA}$. However, the total cost of $C_{AT}^{AT}$ and $C_{BT}^{BT}$ are the same and I designate this as $C$:

\[
C_{AT}^{AT} = C_{AD}^{AD} q_{AD}^{AD} + C_{AB}^{AB} q_{AB}^{AB}, \quad C_{AD}^{AD} \neq C_{AB}^{AB}, \quad C_{AD}^{AD}, C_{AB}^{AB} > 0 \tag{6}
\]

\[
C_{BT}^{BT} = C_{BD}^{BD} q_{BD}^{BD} + C_{BA}^{BA} q_{BA}^{BA}, \quad C_{BD}^{BD} \neq C_{BA}^{BA}, \quad C_{BD}^{BD}, C_{BA}^{BA} > 0 \tag{7}
\]
\[ C^{AT} = C^{BT} = C. \]  \hspace{1cm} (8)

From equations (1) and (2), let us take the first-order conditions in order to get the equilibrium prices and quantities for countries A and B.

The first-order conditions with respect to \( q^{AD} \), \( q^{AB} \), \( q^{BD} \) and \( q^{BA} \) are respectively:

\[
\frac{\partial \pi^A}{\partial q^{AD}} = a - 2q^{AD} - q^{BA} - C^{AD} = 0, \quad q^{AD} = \frac{a - q^{BA} - C^{AD}}{2e}, \]  \hspace{1cm} (9)

\[
\frac{\partial \pi^A}{\partial q^{AB}} = e(a - q^{BD}) - 2eq^{AB} - eC^{AB} = 0, \quad q^{AB} = \frac{a - q^{BD} - eC^{AB}}{2e}, \]  \hspace{1cm} (10)

\[
\frac{\partial \pi^B}{\partial q^{BD}} = a - 2q^{BD} - q^{AB} - C^{BD} = 0, \quad q^{BD} = \frac{a - q^{AB} - C^{BD}}{2e}, \]  \hspace{1cm} (11)

\[
\frac{\partial \pi^B}{\partial q^{BA}} = \frac{1}{e} (a - q^{AD}) - \frac{2}{e} q^{BA} - eC^{BA} = 0, \quad q^{BA} = \frac{a - q^{AD} - eC^{BA}}{2e}. \]  \hspace{1cm} (12)

The above four equations show the profit-maximizing conditions -- marginal revenue equals marginal cost -- when each firm A and B produces. From the four equations, equilibrium quantities are derived in both countries:

\[
q^{AD*} = \frac{a}{3} + \frac{eC^{BA}}{3} - \frac{2C^{AD}}{3}, \quad \text{if } a + eC^{BA} > 2C^{AD}, \quad q^{AD*} > 0 \]  \hspace{1cm} (13)

\[
q^{AB*} = \frac{a}{3} + \frac{C^{BD}}{3} - \frac{2C^{AB}}{3e}, \quad \text{if } e(a + C^{BD}) > 2C^{AB}, \quad q^{AB*} > 0 \]  \hspace{1cm} (14)
\[ q^{BD^*} = \frac{a}{3} + \frac{C^{AB}}{3e} - \frac{2C^{BD}}{3}, \quad \text{if } ae + C^{AB} > 2eC^{BD}, \quad q^{BD^*} > 0 \]  

\[ q^{BA^*} = \frac{a}{3} + \frac{C^{AD}}{3} - \frac{2eC^{BA}}{3}, \quad \text{if } a + C^{AD} > 2eC^{BA}, \quad q^{BA^*} > 0. \]  

Each firm’s output level in the domestic market and the foreign market relies on the exchange rate and not only its own cost of production but also the other firm’s production cost. Firm A’s domestic output level \((q^{AD^*})\), for example, is determined by the exchange rate \((e)\), firm A’s cost of production for the goods in country A \((C^{AD})\) and firm B’s cost of production for the exports to country A \((C^{BA})\). \(q^{AD^*}\) is an increasing function in \(C^{BA}\) and exchange rate, and is a decreasing function in \(C^{AD}\). It means that firm A can sell more products in the domestic market if firm B has higher cost of exports to country A and if country A’s currency depreciates. As well, firm A sells fewer products if it has higher cost of goods for the domestic market.

In equation (14), firm A’s optimal output level in the foreign market \((q^{AB^*})\) relies on firm A’s unit cost of producing \(q^{AB^*}\) to sell in country B \((C^{AB})\) and firm B’s unit cost of producing \(q^{BD^*}\) to sell in the domestic market \((C^{BD})\). \(q^{AB^*}\) is an increasing function in \(C^{BD}\) and the exchange rate, and is a decreasing function in \(C^{AB}\). It implies that firm A can sell more goods in the foreign market (country B) if firm B’s cost of producing goods for the domestic market is higher and if country A’s currency depreciates. Firm A sells fewer goods if it has higher cost of exports to the foreign country.

Using equations (9) and (12), we can draw the firms’ best response curve choosing the equilibrium output level in country A. Equation (9) represents firm A’s best
response curve in the domestic market and equation (12) is firm B’s best response curve in country A. In country B, the firms also react to each other to choose the best output level with given costs and the exchange rate. Equation (11) presents firm B’s best reaction curve in the domestic market. Equation (10) is firm A’s best reaction curve in country B. The figures below show the equilibrium output levels in country A and in country B respectively:

Figure 3.1
Firms’ best response curve in country A
In Figure 3.1, we have the equilibrium at the intersection of two curves since we have the conditions in equations (13) and (16) in order to get $q^{AD^*} > 0$ and $q^{BA^*} > 0$ respectively.\textsuperscript{22} In Figure 3.2, we also have the equilibrium at the intersection of two curves because of the conditions in equations (14) and (15) to get $q^{AB^*} > 0$ and $q^{BD^*} > 0$ respectively.

From equations (3), (13) and (16), the equilibrium price of the output that firms A and B charge in country A in terms of country A’s currency is:

\textsuperscript{22} On the vertical axis, $a - eC^{BA^*} > \frac{a - C^{AD}}{2}$ is valid. If we rewrite this inequality, we have $2a - 2eC^{BA^*} - a + C^{AD} > 0$ or $a + C^{AD} > 2eC^{BA^*}$ which is the condition we have in equation (16). And on the horizontal axis, $\frac{a - eC^{BA^*}}{2} < a - C^{AD}$ is effective. If we rearrange this inequality, we have $a + eC^{BA^*} > 2C^{AD}$ and it is the condition we assumed in equation (13). In Figure 3.2, we can reach the same conclusion for the vertical axis because of the condition in equation (14), and for the horizontal axis because of the condition in equation (15).
From equations (4), (14) and (15), the equilibrium price of the output that firms A and B charge in country B expressed in country B’s currency is:

\[
P^{B*} = a - q^{BD*} - q^{AB*} = \frac{a}{3} + \frac{C^{BD}}{3} + \frac{C^{AB}}{3e}.
\]  

(18)

In equations (17) and (18), the advantage of this model is that the exchange rate has effect on both domestic price and the export price for each country, instead of only the export price for each country. In addition, the prices in the countries depend on not only the domestic firm’s cost of production but also the exporter’s cost of production.

The equilibrium price of the product that firm A receives in the foreign market (country B) expressed in country A’s currency as the exchange rate fluctuates is:

\[
EP^{B*} = \frac{ae}{3} + \frac{eC^{BD}}{3} + \frac{C^{AB}}{3}.
\]  

(19)

The equilibrium price of the output that firm B gets in the foreign market (country A) in terms of country B’s currency as the exchange rate changes is:

\[
P^{A*} = \frac{a}{3e} + \frac{C^{AD}}{3e} + \frac{C^{BA}}{3}.
\]  

(20)
As a result, the optimal output level and the prices in both a domestic and a foreign market are derived endogenously to maximize each firm’s profit and are determined by the exchange rate and unit costs in the model.

Next, let us identify the relationship between the exchange rate and the prices, and the relationship between the exchange rate and the output levels.

**Proposition 1:** The price in country A charged by firm A and firm B is positively related to movements in the exchange rate; the price in country B charged by firm A and firm B is negatively related.

Proof:

\[
\frac{\partial P^A}{\partial e} = \frac{C^{BA}}{3} > 0, \quad (21)
\]

\[
\frac{\partial P^B}{\partial e} = -\frac{C^{AB}}{3e^2} < 0. \quad (22)
\]

Not surprisingly, if the exchange rate increases (country A’s currency depreciates), firm B increases the export price in country A. And firm A charges a higher price in the domestic market as well. The price charged in country A \( (P^A) \) by both firms is dependent on country B’s unit cost of producing \( q^{BA} \) \( (C^{BA}) \) according to equation (21). This implies that firm B (the exporter in country A) sets the export price based on its unit cost of producing the output level in country A, \( q^{BA} \), and firm A follows the price that firm B imposes in country A. In fact, it is not so hard to observe these pricing behaviors among firms if the market is under imperfect competition. And, obviously, we can say
that each firm takes into account the exchange rate changes when it sets the domestic price and the local currency export price.

According to equation (22), firm A charges a lower price in country B if the exchange rate increases (country B’s currency appreciates). Firm B also sets a lower price in the domestic market. The price charged in country B \( P_{BA^*} \) by both firms is determined by country A’s unit cost of producing \( q_{AB} \) \( (C_{AB}^*) \) and the exchange rate. So, in country B, firm A as an exporter sets a lower price and the domestic firm B imposes the same price as firm A charges.

In other words, if the exchange rate increases (country B’s currency appreciates), firm A increases the price in the domestic market and decreases the price in the foreign market. Firm B imposes a lower price in the domestic market and higher price in the foreign market as the exchange rate increases (country A’s currency depreciates).

What is the effect of the exchange rate changes on the output levels?

**Proposition 2:** In country A, firm A’s output level increases but firm B’s output level decreases when both firms increase the price as the exchange rate increases (country A’s currency depreciates); in country B, firm B’s output level decreases but firm A’s output level increases when both firms decrease the price as the exchange rate increases (country B’s currency depreciates).

**Proof:**

\[
\frac{\partial q_{AD^*}}{\partial e} = \frac{C_{BA}^*}{3} > 0, \quad (23)
\]

\[
\frac{\partial q_{BA^*}}{\partial e} = -\frac{2C_{BA}^*}{3} < 0 \quad (24)
\]
Both firm A and firm B increase the price in country A as country A’s currency devalues but it brings a different effect on the output level for each firm. We have the result that firm A’s output level in the domestic market increases and firm B’s output level in country A decreases when the price increases as exchange rate increases (country A’s currency depreciates). It implies that domestic producers produce more output levels and consumers buy more domestic goods in country A if the domestic seller and the exporter charge the same price and the price increases as the exchange rate fluctuates.

From equations (25) and (26), in country B, firm B will get a smaller output level but firm A will get a bigger output level when both firms impose a lower price as the exchange rate increases (country B’s currency is more valuable). This means that exporters produce more output levels and consumers buy more exports in country B if the domestic seller and the exporter charge the same price and the price decreases as the exchange rate moves.

Overall, consumers in both countries buy the domestic goods more if the domestic producer and the foreign producer impose the same price and the price increases as the exchange rate fluctuates. In contrast, consumers buy the exports more if the domestic producer and the foreign producer charge the same price and the price is lower than before.
What is the implication for the trade balances between these countries then?

**Proposition 3:** If a country’s currency depreciates, the country has a surplus in the trade balance with the other country; if a country’s currency appreciates, we have an indeterminate result in the country’s trade balance with the other country.

**Proof:**

**Country A:**

\[
\begin{align*}
\frac{\partial (eP^B)}{\partial e} &= a + \frac{C_{BD}}{3} > 0, \\
\frac{\partial q^{AB}}{\partial e} &= \frac{2C_{AB}}{3e^2} > 0, \\
\frac{\partial P^A}{\partial e} &= \frac{C_{BA}}{3} > 0, \\
\frac{\partial q^{BA}}{\partial e} &= -\frac{2C_{BA}}{3} < 0.
\end{align*}
\]

**Country B:**

\[
\begin{align*}
\frac{\partial \left(\frac{P^A}{e}\right)}{\partial e} &= -\frac{a}{3e^2} - \frac{C_{AD}}{3e^2} < 0, \\
\frac{\partial q^{BA}}{\partial e} &= -\frac{2C_{BA}}{3} < 0, \\
\frac{\partial P^B}{\partial e} &= -\frac{C_{AB}}{3e^2} < 0, \\
\frac{\partial q^{AB}}{\partial e} &= \frac{2C_{AB}}{3e^2} > 0.
\end{align*}
\]
Let us consider the export side first for country A: the export price that firm A receives expressed by country A’s currency \((eP^{A*})\) increases as country A’s currency becomes less valuable. Additionally, the output level of firm A in country B \((q^{4B*})\) grows bigger as country A’s currency depreciates. Therefore the total revenue of the exports that firm A receives in terms of its own currency increases as country A’s currency depreciates. On the import side, the local-currency import price that firm B charges in country A \((P^{A*})\) increases. However, the output level of the import in country A \((q^{B,A*})\) decreases. If we compare equations (29) and (30), the decrease in the quantity is greater than the increase in the price. Thus the value of imports that country A pays falls as country A’s currency depreciates. As the result, country A has a surplus in the trade balance when country A’s currency depreciates -- the exchange rate increases. Generally speaking, if country A’s price elasticity of demand for the imports is elastic, the total amount of money that country A pays will decrease. Therefore country A has a surplus for certain in the trade balance. In contrast, if country A’s price elasticity of demand for the imports is inelastic, the total amount of money that country A pays in terms of its own currency will increase. In this case, both the value of exports and imports increases so we have an ambiguous result about the trade balance. If the increase in the value of exports is greater than that of imports, country A has a surplus. And if the increase in the value of exports is smaller than that of imports, country A has a deficit.

From equations (31), (32), (33) and (34), the export price that firm B receives in terms of its own currency decreases as country B’s currency is appreciated -- the exchange rate increases. At the same time, the output level that firm B has in country A \((q^{B,A*})\) grows smaller as country B’s currency appreciates. Thus the total revenue of the
exports that firm B earns in terms of its own currency decreases as country B’s currency becomes more valuable. On the import side of country B, its local-currency import price ($P^{B*}$) that firm A imposes decreases. However, the output level of the imports in country B ($q^{AB*}$) increases. If we compare equations (33) and (34), the increase in the quantity is greater than the decrease in the price. Hence the value of imports that country B pays rises as country B’s currency appreciates. As the result, we have an ambiguous result in the trade balance for country B when country B’s currency appreciates. It is because we have a decrease in the value of exports and an increase in the value of imports and we do not know which one is greater than the other. On the other hand, we conclude that country B has a surplus when its own currency depreciates.

3.3 Exchange Rate Pass-Through

Is there any exchange rate pass-through?

\[
\frac{\partial P^{A*}}{\partial e} \times \frac{e}{P^{A*}} = \left( \frac{C^{BA}}{3} \right) \times \left( \frac{e}{a + \frac{C^{AD}}{3} + \frac{eC^{RA}}{3}} \right) = \frac{eC^{RA}}{a + C^{AD} + eC^{RA}} < 1. \tag{35}
\]

$P^{A*}$ is the local-currency price that firms A and B charge in country A. Firm A imposes $P^{A*}$ in the domestic market and firm B charges $P^{A*}$ in country A as the export price. In order to examine the degree of ERPT in country A, I want to determine the exchange-rate elasticity of the price in country A as shown above in equation (35). It is less than 1, indicating that there is an incomplete ERPT in the country. Less than 1 for the
elasticity also means that the percentage change in the price in country A is less than the percentage change in the exchange rate. So firms A and B do not pass through the full proportion of the exchange rate movements into the domestic price in the case of firm A and into the export price in the case of firm B.

We can also measure the ERPT in country B as follows:

\[
\frac{\hat{e}P^{B*}}{\hat{e}e} \times \frac{e}{P^{B*}} = \left(-\frac{C^{AB}}{3e^2}\right) \times \left(\frac{e}{ea + eC^{BD} + C^{AB}}\right) = \frac{C^{AB}}{ea + eC^{BD} + C^{AB}} < 1.
\] (36)

In country B, firm A charges as the local-currency export price and firm B imposes as the domestic price. The exchange-rate elasticity of the price in country B is incomplete as well. It implies that firms A and B pass through only part of the exchange rate movements into the price in the country.

In both markets, we have incomplete ERPT for the domestic price and the export price. And the result of incomplete ERPT is consistent with the common observations in the international goods market.

Moreover, let us calculate the exchange-rate elasticity of firm A’s export price that is expressed by its own currency as given below. Firm A’s exchange rate pass-through is incomplete:

\[
\frac{\hat{e}eP^{B*}}{\hat{e}e} \times \frac{e}{ep^{B*}} = \left(\frac{a + C^{BD}}{3}\right) \times \left(\frac{e}{ae + eC^{BD} + C^{AB}}\right) = \frac{e(a + C^{BD})}{ae + eC^{BD} + C^{AB}} < 1.
\] (37)
The exchange-rate elasticity of firm B’s export price that is expressed by its own currency is as follows. The exchange rate pass-through is incomplete:

\[
\frac{\partial \left( \frac{P_{A_1^*}}{e} \right)}{\partial e} \times \frac{e}{P_{A_1^*}^e} = \left( -\frac{a}{3e^2} - \frac{C^{AD}}{3e^2} \right) \times \left( \frac{e}{a + \frac{C^{AD}}{3e} + \frac{C^{BA}}{3}} \right) = -\frac{a + C^{AD}}{a + C^{AD} + eC^{BA}} < 1.
\]

(38)

We discover further implications about exporter’s invoice choice and the ERPT as follows.

**Proposition 4:** The degree of ERPT is higher when the exporter invoices in its own currency than when it invoices in the buyer’s currency.

**Proof:**

\[
\left\{ \frac{\partial P_{A_2^*}}{\partial e} \times \frac{e}{P_{A_2^*}^e} = -\frac{C^{AB}}{ea + eC^{BD} + C^{AB}} \right\} < \left\{ \frac{\partial eP_{A_2^*}}{\partial e} \times \frac{e}{ep_{A_2^*}^e} = \frac{e(a + C^{BD})}{ae + eC^{BD} + C^{AB}} \right\},
\]

(39)

\[
\left\{ \frac{\partial P_{A_1^*}}{\partial e} \times \frac{e}{P_{A_1^*}^e} = \frac{eC^{BA}}{a + C^{AD} + eC^{BA}} \right\} < \left\{ \frac{\partial \left( \frac{P_{A_1^*}}{e} \right)}{\partial e} \times \frac{e}{\left( \frac{P_{A_1^*}}{e} \right)} = \left( -\frac{a + C^{AD}}{a + C^{AD} + eC^{BA}} \right) \right\}.
\]

(40)

In the case of firm A, we derive inequality (39) from equations (36) and (37). Firm A’s exchange rate pass-through to the export price when the export price is
measured by its own currency, $\frac{e(a + C^{BD})}{ae + eC^{BD} + C^{AB}}$, is greater than the exchange rate pass-through to the export price when the export price is expressed by the buyer’s currency, 

$$\left| \frac{C^{AB}}{ea + eC^{BD} + C^{AB}} \right|.$$ 

This is true because of the condition, $e(a + C^{BD}) > 2C^{AB}$, in equation (14) in order to make firm A sell more than zero quantity of the good in country B ($q^{AB*} > 0$). If $e(a + C^{BD}) > 2C^{AB}$ or $\frac{e(a + C^{BD})}{2} > C^{AB}$ is valid from equation (14), then $e(a + C^{BD}) > C^{AB}$ is binding as well in inequality (39). Thus, firm A’s degree of ERPT is higher when it sets the export price in its own currency rather than in the buyer’s currency.

We have the result of inequality (40) based on equations (35), (38) and (16) for firm B. Firm B’s exchange rate pass-through to the export price when the export price is measured by its own currency, 

$$\left| -\frac{a + C^{AD}}{a + C^{AD} + eC^{BA}} \right|,$$ 

is greater than the exchange rate pass-through to the local-currency export price, $\frac{eC^{BA}}{a + C^{AD} + eC^{BA}}$. Firm B has the condition of $a + C^{AD} > 2eC^{BA}$ in equation (16) in order to get more than zero quantity of the good in country A ($q^{BA*} > 0$). If $a + C^{AD} > 2eC^{BA}$ or $\frac{a + C^{AD}}{2} > eC^{BA}$ from equation (16), then 

clearly the inequality, $a + C^{AD} > eC^{BA}$, is effective in inequality (40). Therefore we can conclude that firm B’s degree of ERPT is higher when it sets the export price in its own currency rather than in the buyer’s currency.
3.4 On PTM

Now I will examine whether there is PTM behavior by each firm or not applying the definition of the PTM: the relative price change in the domestic and the foreign market as exchange rate changes matters in determining the existence of PTM. If firm A’s export price in country B reflects the same proportion of the exchange rate movements as the firm A’s domestic price does, then there is no PTM. Otherwise, there exists firm A’s PTM behavior between the domestic and the foreign market as the exchange rate moves.

3.4.1 Firm A’s Local-Currency Export Price vs. Firm A’s Domestic Price

Following the definition I presented in chapter 2, I want to identify the ratio of the exchange-rate elasticity of firm A’s export price to the exchange-rate elasticity of firm A’s domestic price. It measures the relative price changes in the domestic and the foreign market as the exchange rate fluctuates and allows us to investigate the existence of PTM behavior by firm A. We can get the ratio from the equation (35) and (36) as given:

\[
\frac{\frac{\partial P^{B*}}{\partial e} \times \frac{e}{P^{B*}}}{\frac{\partial P^{A*}}{\partial e} \times \frac{e}{P^{A*}}} = \left(\frac{\frac{-C^{AB}}{ea + eC^{BD} + C^{AB}}}{\frac{eC^{BA}}{a + C^{AD} + eC^{BA}}}\right) = \frac{-C^{AB}(a + C^{AD} + eC^{BA})}{eC^{BA}(ea + eC^{BD} + C^{AB})}, \tag{41}
\]

If \(\left|\frac{-C^{AB}(a + C^{AD} + eC^{BA})}{eC^{BA}(ea + eC^{BD} + C^{AB})}\right| = 1\) then, there is no PTM by firm A. \(\tag{42}\)
If
\[
\left| \frac{-C_{AB}^{BA}(a + C_{AD}^{BA} + eC_{BD}^{BA})}{eC_{BD}^{BA}(ea + eC_{BD}^{BA} + C_{AB}^{BA})} \right| > 1 \quad \text{or} \quad \left| \frac{-C_{AB}^{BA}(a + C_{AD}^{BA} + eC_{BD}^{BA})}{eC_{BD}^{BA}(ea + eC_{BD}^{BA} + C_{AB}^{BA})} \right| < 1,
\]
firm A’s PTM behavior exists. \hfill (43)

More specifically, in the case of equation (42), the export price of firm A in country B decreases in exactly the same way as the domestic price in country A increases as the exchange rate increases. In this case, again, how much exchange rate change passes through to the export price and the domestic price does not matter in determining the existence of PTM. Therefore, we can say that there are no differences in prices as the exchange rate changes according to the market -- the domestic and the foreign market -- and there is no PTM.

On the other hand, in inequality (43), when the exchange rate increases (country B’s currency value increases), the decrease in firm A’s local-currency export price in country B relative to the exchange rate increase is greater than the increase in the domestic price in country A relative to the exchange rate increase (the numerator of (43) is greater than the denominator). Or the decrease in firm A’s local-currency export price in country B relative to the exchange rate increase is smaller than the increase in the domestic price in country A relative to the exchange rate increase (the numerator of (43) is smaller than the denominator). How much exchange rate passes through to the export price and the domestic price does not matter to the determination of the existence of PTM. The result in inequality (43) gives us the evidence of firm A’s PTM behavior as the exchange rate changes.
3.4.2 Firm B’s Local-Currency Export Price vs. Firm B’s Domestic Price

Similarly, we have the ratio below from the equations (35) and (36) in the case of firm B and use it to determine the existence of PTM behavior:

\[
\frac{\partial P^A}{\partial e} \times \frac{e}{P^A} = \frac{\frac{eC_{BA}}{a + C^{AD} + eC_{BA}}}{\frac{C_{AB}}{-ea + eC_{BD} + C_{AB}}} = \frac{eC_{BA}(ea + eC_{BD} + C_{AB})}{-C_{AB}(a + C^{AD} + eC_{BA})},
\]

(44)

If \(\frac{eC_{BA}(ea + eC_{BD} + C_{AB})}{-C_{AB}(a + C^{AD} + eC_{BA})} = 1\), there is no firm B’s PTM behavior. \hspace{1cm} (45)

If \(\frac{eC_{BA}(ea + eC_{BD} + C_{AB})}{-C_{AB}(a + C^{AD} + eC_{BA})} > 1\) or \(\frac{eC_{BA}(ea + eC_{BD} + C_{AB})}{-C_{AB}(a + C^{AD} + eC_{BA})} < 1\), then firm B’s PTM exists between the domestic and the foreign market. \hspace{1cm} (46)

Therefore, from inequalities (41) and (44), we know that if one firm practices PTM behavior, the other firm also exhibits PTM between the domestic and the foreign market since the two equations are reciprocals of each other.

3.5 PTM and Invoice Currency

So far, we have assumed that the firms impose export prices in buyer’s currency when we investigate PTM. What would happen to the PTM if the firms set the export prices in their own currency? Is there any effect of the invoice currency on the PTM?
In order to answer the question, it would be useful to examine the PTM when the exporters set the export price in their own currency. Let us assume that the export price is expressed by the seller’s currency in the foreign market.

Based on the new assumption about the invoice price of the exports, firm A charges $P^A$ in the domestic market and $eP^B$ in the foreign market (country B). Firm B charges $P^B$ in the domestic market and $\frac{P^A}{e}$ in country A. So, $P^i$ denotes the price of the product that country $i$ charges in the domestic market (country $i$) and is an inverse demand function of $Q^i$ ($i = A, B$). And, as we had before, $Q^i$ is the total quantity of the goods supplied by firms A and B to country $i$ ($i = A, B$). Thus, nothing has changed in equations (1) and (2) to maximize each firm’s profit in terms of its own currency and in other assumptions we made before.

Now, let us investigate PTM by getting the ratio of the exchange-rate elasticity of the export price when the firms set the invoice currency in terms of their own currency to the exchange-rate elasticity of the domestic price for each country in this case.

3.5.1 Firm A’s Export Price in Terms of its Own Currency vs. Firm A’s Domestic Price

We have the ratio of the exchange-rate elasticity of firm A’s export price in terms of country A’s currency to the exchange-rate elasticity of country A’s domestic price for firm A from the equations (35) and (37) as given:
the exchange-rate elasticity of firm A's export price in terms of country A's currency
the exchange-rate elasticity of firm A's domestic price

\[
\frac{\partial p^B}{\partial e} \times \frac{e}{e} = \frac{e(a + C^{BD})}{ae + eC^{BD} + C^{AB}} = \frac{(a + C^{BD})(a + C^{AD} + eC^{BA})}{C^{BA}(ae + eC^{BD} + C^{AB})},
\]

(47)

If \( \left| \frac{(a + C^{BD})(a + C^{AD} + eC^{BA})}{C^{BA}(ae + eC^{BD} + C^{AB})} \right| = 1 \), there is no PTM by firm A. \( \text{(48)} \)

If \( \left| \frac{(a + C^{BD})(a + C^{AD} + eC^{BA})}{C^{BA}(ae + eC^{BD} + C^{AB})} \right| > 1 \) or \( \left| \frac{(a + C^{BD})(a + C^{AD} + eC^{BA})}{C^{BA}(ae + eC^{BD} + C^{AB})} \right| < 1 \) then, firm A’s PTM behavior exists. \( \text{(49)} \)

In the case of equation (48), much exchange rate passes through firm A’s export price in terms of country A’s currency increasing exactly as much as the domestic price in country A increases as the exchange rate increases.\(^{23}\) In this case, again, how much exchange rate change passes through to the export price and the domestic price does not matter in determining the existence of PTM. As the result, we conclude that there is no PTM.

However, in inequality (49), when the exchange rate increases, country B’s currency value increases, firm A’s export price in terms of its own currency increases more than the domestic price in country A increases, so we have greater than 1 for the

\(^{23}\) If the firms set the export price in the buyer’s currency, the export price decreases as the buyer’s currency appreciates. We can confirm that from equation (33) in the case of firm A. However, if the firms set the export price in its own currency, the export price increases as the buyer’s currency appreciates. Equation (27) indicates it in the case of firm A.
ratio; or, firm A’s export price in terms of country A’s currency increases less than the domestic price in country A increases, thus the ratio is less than 1. The degree of exchange rate pass-through to the export price and the domestic price is not important to determine the existence of PTM. This result gives us the evidence of firm A’s PTM behavior as the exchange rate varies.

3.5.2 Firm B’s Export Price in Terms of its Own Currency vs. Firm B’s Domestic Price

We use the ratio below from the equations (36) and (38) in the case of firm B to determine the existence of PTM behavior:

\[
\frac{\partial \left( \frac{P^*_e}{e} \right)}{\partial e} \times e = \frac{\partial P^*_B}{\partial e} \times e \quad \text{the exchange-rate elasticity of firm B's export price in terms of country B's currency}
\]

\[
\frac{\partial \left( \frac{e}{P^*_A} \right)}{\partial e} = \frac{-\frac{a + C^AD}{a + C^AD + eC^BA}}{-\frac{C^AB}{ea + eC^BD + C^AB}} = \frac{(a + C^AD)(ea + eC^BD + C^AB)}{C^AB \left(a + C^AD + eC^BA\right)},
\]

If \( \left| \frac{(a + C^AD)(ea + eC^BD + C^AB)}{C^AB \left(a + C^AD + eC^BA\right)} \right| = 1 \), there is no firm B’s PTM behavior. \( (51) \)

If \( \left| \frac{(a + C^AD)(ea + eC^BD + C^AB)}{C^AB \left(a + C^AD + eC^BA\right)} \right| > 1 \) or \( \left| \frac{(a + C^AD)(ea + eC^BD + C^AB)}{C^AB \left(a + C^AD + eC^BA\right)} \right| < 1 \), firm B practices PTM behavior. \( (52) \)
3.5.3 The Effect of The Invoice Currency on PTM

We can figure out the effect of the invoice currency on PTM for firm A by comparing equations (41) and (47). Equation (41) represents the situation that firm A has the export price expressed by the buyer’s currency. We take the absolute value of the equation when examining PTM.\(^{24}\) Let us rewrite the equation as follows:

\[
\frac{\partial P^B}{\partial e} \times \frac{e}{P^B} = \frac{\partial e}{\partial P^B} \times \frac{e}{P^B} = \left| -C^{AB}_e \left( a + C^{AD}_e + eC^{BA}_e \right) \right| - \left| eC^{BA}_e \left( ea + eC^{BD}_e + C^{AB}_e \right) \right|\]

Equation (47) indicates the case that firm A charges the export in terms of its own currency. We can rewrite it as given:

\[
\frac{\partial eP^B}{\partial e} \times \frac{e}{eP^B} = \frac{\partial P^B}{\partial P^B} \times \frac{e}{P^B} = \left( a + C^{BD}_e \right) \left( a + C^{AD}_e + eC^{BA}_e \right) \left( ea + eC^{BD}_e + C^{AB}_e \right)\]

We do not know whether there exists PTM behavior from equations (53) and (54) because we cannot figure out whether these ratios are exactly 1 or greater than 1 or not.

\(^{24}\) The sign of the ratio shows the decrease or increase in the prices in the markets and the absolute value of the ratio indicates the amount of relative price movements. The PTM is influenced by the value of the relative price movements instead of the direction of the price movements. Therefore we take the absolute value of the ratio to determine whether the ratio is greater than 1 or not and to get the degree of PTM.
with the given information. However, if there is any PTM behavior by firm A, we can say in which situation the firm presents a more severe PTM by comparing the values of equations (53) and (54). If we have a greater number for the ratio then it implies that the firm practices a more severe PTM. So let us compare the value of equations (53) and (54) to determine in which case firm A exhibits a higher degree of PTM between the domestic and the foreign market.

**Proposition 5:** If there is any PTM behavior by firm A, the firm presents a more severe PTM between the domestic and the foreign market when it imposes the export price in terms of its own currency rather than it does in terms of the local-currency export price.

**Proof:**

\[
\begin{align*}
\frac{\partial P_{B}^{*}}{\partial e} \times \frac{e}{P_{B}^{*}} &= \left[ \frac{-C_{AB}^{*}}{e} \times \frac{(a + C_{AD}^{*} + eC_{BA}^{*})}{C_{BD}^{*}(ea + eC_{BD}^{*} + C_{AB}^{*})} \right] \\
\frac{\partial P_{A}^{*}}{\partial e} \times \frac{e}{P_{A}^{*}} &= \left[ \frac{\partial eP_{B}^{*}}{\partial e} \times \frac{e}{eP_{B}^{*}} \times \frac{(a + C_{BD}^{*} + eC_{BA}^{*})}{(ea + eC_{BD}^{*} + C_{AB}^{*})} \right] \leq \left( a + C_{BD}^{*} \right) \times \frac{(a + C_{AD}^{*} + eC_{BA}^{*})}{C_{BD}^{*}(ea + eC_{BD}^{*} + C_{AB}^{*})}
\end{align*}
\]

(55)

Basically, we compare the values of \( \frac{C_{AB}^{*}}{e} \) and \( (a + C_{BD}^{*}) \) from equations (53) and (54). In addition, we have the condition, \( e(a + C_{BD}^{*}) > 2C_{AB}^{*} \) from equation (14), in order to make firm A sell more than zero quantity of products in country B \( (q_{AB}^{*} > 0) \). We need this condition since we do not want to have the situation that firm B provides all the
products in country B. So firms A and B will share the output level in country B with this condition. If we rewrite equation (14), we have the following inequality:

$$\frac{C_{AB}}{e} < \frac{(a + C_{BD})}{2}.$$  Therefore we find that the inequality, $$\frac{C_{AB}}{e} < (a + C_{BD})$$, is valid. It means that equation (54) is greater than equation (53). In other words, firm A has a higher degree of PTM between the domestic and the foreign market when it imposes the export price in terms of its own currency. So we can conclude that the relative price difference between these two markets is greater when firm A has the export price in terms of its own currency on the contract than it does in terms of the buyer’s currency. Therefore (55) is valid as well.

How about firm B? Compare equations (44) and (50) to get the answer. Equation (44) shows that firm B charges the export price in the local-currency price. If we rearrange it, we have the following equation:

$$\frac{\partial P^A}{\partial e} \times \frac{e}{P^A} = \frac{eC_{BA}(ea + eC_{BD} + C_{AB})}{-C_{AB}(a + C_{AD} + eC_{BA})}$$

$$= \left[(-eC_{BA}) \times \frac{(ea + eC_{BD} + C_{AB})}{C_{AB}(a + C_{AD} + eC_{BA})}\right] \quad (56)$$

Equation (50) presents the case that firm B has its own currency as the export price on the contract. If we rearrange it, we have the equation as given:
\[
\frac{\partial \left( \frac{P^A}{e} \right)}{\partial e} \times \frac{e}{P^A} = \frac{(a + eC^{BD} + C^{AB})}{C^{AB} \left( a + C^{AD} + eC^{BA} \right)}
\]

Proposition 6: If there is any PTM behavior by firm B, the firm presents a more severe PTM between the domestic and the foreign market when it imposes the export price in terms of its own currency rather than it does in terms of the local-currency export price.

Proof:

\[
\left\{ \frac{\partial P^A}{\partial e} \times \frac{e}{P^A} \right\} = \left\{ -eC^{BA} \times \frac{(ea + eC^{BD} + C^{AB})}{C^{AB} (a + C^{AD} + eC^{BA})} \right\}
\]

Let us compare the values, \( eC^{BA} \) and \((a + C^{AD})\) from equations (56) and (57) to figure out which case has a higher degree of PTM. So we need the condition,

\[ a + C^{AD} > 2eC^{BA} \]

from equation (16). This condition guarantees that firm B provides more than zero quantity of the good in country A \( (q^{BA} > 0) \). If you rearrange the
inequality, we get \( eC^B_A < \frac{a + C^{AD}}{2} \). Hence, certainly we have the result that \( eC^B_A < a + C^{AD} \). It represents that the value of equation (57) is greater than equation (56). And inequality (58) is valid. Thus we conclude that, if any PTM is presented by firm B, the firm exhibits a more severe PTM behavior between the domestic and the foreign market when it charges the export price in its own currency rather than in the buyer’s currency.

3.6 Price Sensitivity to The Exchange Rate Movements in Country A

If we assume that the exporters present its own currency as the export price on the contract to sell the goods in the foreign market, the model tells us which firm (the domestic producer or the exporter) is more price sensitive to the exchange rate changes in each country.

Let me take the equations (35) and (38) and get the ratio of these two equations as follows in order to answer the question raised above:

the exchange-rate elasticity of firm B's export price in terms of country B's currency
the exchange-rate elasticity of firm A's domestic price

\[
\frac{\partial \left( \frac{P^x_A}{e} \right)}{\partial e} \times \frac{e}{P^x_A} = \frac{\partial P^x_A}{\partial e} \times \frac{e}{P^x_A} = \frac{a + C^{AD}}{a + C^{AD} + eC^B_A} = \frac{eC^B_A}{a + C^{AD} + eC^B_A} = - \frac{(a + C^{AD})(a + C^{AD} + eC^B_A)}{eC^B_A(a + C^{AD} + eC^B_A)} = - \left| \frac{a + C^{AD}}{eC^B_A} \right| > 1
\]
From the equation (16), we have \[ \frac{a + C^{AD}}{2eC^{BA}} > 1 \] in order to get \( q^{BA*} > 0 \). Therefore, we know that the equation (59) is greater than 1 as well: \[ -\frac{a + C^{AD}}{eC^{BA}} > 1. \] This result represents that firm B’s export price in terms of its own currency decreases more than firm A’s domestic price increases as the exchange rate increases -- as country B’s currency values more. In other words, if we assume that firm B sets the export price in its own currency, firm B (the exporter) is more price responsive to the exchange rate movements than firm A (the domestic firm) does in country A.\(^{25}\)

3.7 Price Sensitivity to The Exchange Rate Movements in Country B

From equations (36) and (37), we obtain the ratio of the equations as follows:

\[
\frac{\partial P^{B*}}{\partial P} \times \frac{e^{B*}}{P^{B*}} = \frac{\left( e\left( a + C^{BD} \right) \right)}{ea + eC^{BD} + C^{AB}}
\]

\[
= -\frac{e\left( a + C^{BD} \right)}{C^{AB}\left( ea + eC^{BD} + C^{AB} \right)} = -\frac{e\left( a + C^{BD} \right)}{C^{AB}} > 1
\]

\( ^{25} \) We can also take the reciprocal of equation (59) to prove that firm B (the exporter in country A) is more price sensitive to the exchange rate fluctuations than firm A (the domestic firm) does in country A. In this case, we have the result that is less than 1 for the reciprocal of equation (59), though.
We have \[ \frac{e(a + C^{BD})}{2C^{AB}} > 1 \] from the equation (14) in order to have \( q^{AB} > 0 \). Hence we can say that the equation (60) is greater than 1 as well: \[ \frac{e(a + C^{BD})}{C^{AB}} > 1 \]. This means that firm A’s export price in terms of its own currency increases more than firm B’s domestic price decreases as the exchange rate increases -- as country B’s currency appreciates. And firm A (the exporter) is more price responsive in terms of its own currency to the exchange rate changes than firm B (the domestic firm) is in country B.

**Proposition 7:** The price of the export in each country is more responsive to the exchange rate movements than the price of the domestic good when exporters charge the export price in terms of their own currency.

*Proof:* (59) in country A and (60) in country B are valid as shown above.

### 3.8 Summary

The specified model provides endogenously determined output level related to the cost of production and the exchange rate. I find that domestic producers produce more output levels and consumers buy more domestic goods in a country if the domestic seller and the exporter charge the same price and the price increases as the exchange rate fluctuates. However, exporters produce more output levels and consumers buy more exports in a country if the domestic seller and the exporter charge the same price and the price decreases as the exchange rate moves.
I illustrate the condition, which tells you whether the PTM exists or not. And we find the relationship between PTM and the invoice currency that is different from existing research; according to the model, if there is any PTM behavior by each firm, the firm presents a more severe PTM between the domestic and the foreign market when it imposes the export price in terms of its own currency than it does in terms of the local-currency export price.

The result about ERPT is consistent with other papers; there exists incomplete exchange rate pass-through to the export price and to the domestic price. However, exporters pass through more exchange-rate movements to the export price when they invoice the export price in its own currency than they do in the buyer’s currency.

When I compare changes in the export price denominated in the exporter’s currency and changes in the price of the domestic good as the exchange rate varies, I find that the export price is more responsive to the exchange rate movements than the price of the domestic good in a country.

In addition, the model allows us to find out some information about the trade balance; a country has a surplus in the trade balance if the country’s currency depreciates.
CHAPTER 4
TWO-PERIOD MODEL

I extend the model in this chapter, developing a two-period version. However, this two-period model does not have any intertemporal link between the first-period profit and the second-period profit. It is because the model has time-additively separable profit specifications that are linear in the demand for the product at that point in time.

How are their decisions different in the first period and the second period? The purpose of the two-period model is to answer the question. I will examine the role of discount factors in the output level, the price, ERPT and PTM. Overall, I focus on showing the differences between the first-period and the second-period output level and the price. I also compare the second-period condition of ERPT and PTM with those in the first period.

4.1 Assumptions

In the two-period model, I assume the following: (i) firms face a two-period time horizon to maximize their total profit in terms of its own currency; and (ii) firms have their own subjective discount factors that apply to the second period. Otherwise the model setup is the same as the one-period setup. In other words, firm A may be more or less patient than firm B. If a firm has a low discount factor, the firm is impatient. It means that the firm prefers to get the value in the present rather than in the future. In contrast, if a firm has a high discount factor, the firm is patient.
Firms decide the best quantity (output level) and the price is determined by the output level separately in each period related to the exchange rate, unit cost and the discount factor they have at that time. Therefore first-period output level itself does not have any direct effect on the second-period output level. This occurs because the output level is determined endogenously each period in the model economy. And, the endogenous output level has an effect on the price.

4.2 Two-period Model

Firm A maximizes the profits in terms of its own currency as follows:

\[
\text{max } \pi^A = P_1^A q_1^{AD} + e_1 P_1^B q_1^{AB} + \beta^A \left[ P_2^A q_2^{AD} + e_2 P_2^B q_2^{AB} \right]
\]
\[
- C_1^{AD} q_1^{AD} - C_1^{AB} q_1^{AB} - C_2^{AD} q_2^{AD} - C_2^{AB} q_2^{AB}
\]
\[
= \left[ a - q_1^{AD} - q_1^{BA} \right] q_1^{AD} + e_1 \left[ a - q_1^{BD} - q_1^{BA} \right] q_1^{BA}
\]
\[
+ \beta^A \left[ a - q_2^{AD} - q_2^{BA} \right] q_2^{AD} + \beta^B e_2 \left[ a - q_2^{BD} - q_2^{BA} \right] q_2^{BA}
\]
\[
- C_1^{AD} q_1^{AD} - C_1^{AB} q_1^{AB} - C_2^{AD} q_2^{AD} - C_2^{AB} q_2^{AB}
\]  

(61)

Firm A maximizes the profits in terms of its own currency as follows:

\[
\text{max } \pi^B = P_1^B q_1^{BD} + \frac{1}{e_1} P_1^A q_1^{BA} + \beta^B \left[ P_2^B q_2^{BD} + \frac{1}{e_2} P_2^A q_2^{BA} \right]
\]
\[
- C_1^{BD} q_1^{BD} - C_1^{BA} q_1^{BA} - C_2^{BD} q_2^{BD} - C_2^{BA} q_2^{BA}
\]
\[
= \left[ a - q_1^{BD} - q_1^{BA} \right] q_1^{BD} + \frac{1}{e_1} \left[ a - q_1^{BD} - q_1^{BA} \right] q_1^{BA}
\]
\[
+ \beta^B \left[ a - q_2^{BD} - q_2^{BA} \right] q_2^{BD} + \beta^B \frac{1}{e_2} \left[ a - q_2^{BD} - q_2^{BA} \right] q_2^{BA}
\]
\[
- C_1^{BD} q_1^{BD} - C_1^{BA} q_1^{BA} - C_2^{BD} q_2^{BD} - C_2^{BA} q_2^{BA}
\]  

(62)
where $P^i_1$ ($i = A, B$) indicates the price of the products that are sold in country $i$ in the first period. $P^i_2$ ($i = A, B$) represents the price of the goods that are sold in country $i$ in the second period. So the subscripts are the period of the time and the superscripts are the country designation. Let $P^i_1$ ($i = A, B$) and $P^i_2$ ($i = A, B$) be the inverse demand function of $Q^i_1$ ($i = A, B$) and $Q^i_2$ ($i = A, B$), respectively. $Q^i$ is total quantity of the goods supplied by firm A and B to country $i$ in each period. So we can rewrite the prices as follows and insert these prices into the equations (61) and (62) in order to get the second part of equations (61) and (62):

$$P^A_1 = a - Q^A_1 = a - q^{AD}_1 - q^{BA}_1, \text{ where } Q^A_1 = q^{AD}_1 + q^{BA}_1, \ a > 0$$  \hspace{1cm} (63)

$$P^A_2 = a - Q^A_2 = a - q^{AD}_2 - q^{BA}_2, \text{ where } Q^A_2 = q^{AD}_2 + q^{BA}_2, \ a > 0$$  \hspace{1cm} (64)

$$P^B_1 = a - Q^B_1 = a - q^{BD}_1 - q^{AB}_1, \text{ where } Q^B_1 = q^{BD}_1 + q^{AB}_1, \ a > 0$$  \hspace{1cm} (65)

$$P^B_2 = a - Q^B_2 = a - q^{BD}_2 - q^{AB}_2, \text{ where } Q^B_2 = q^{BD}_2 + q^{AB}_2, \ a > 0$$  \hspace{1cm} (66)

I assume that the demand curve, which each firm faces in both its domestic and foreign market in each period, is a linear demand curve, and that consumers have the same preferences across countries. Each firm imposes a local-currency price as an export price. Firm A charges $P^A$ in the domestic market in its own currency and $P^B$ in the foreign market (country B) in country B’s currency. Firm B charges $P^B$ in the domestic market in its own currency and $P^A$ in the foreign market (country A) in country A’s currency.
\( \beta^A \) is the discount factor that firm A has in the second period and \( \beta^B \) is firm B’s discount factor in the second period. And, firms A and B have different discount factors so \( \beta^A \) and \( \beta^B \) are not the same. We can define the discount factor as given:

\[
\beta^i = \frac{1}{1+r^i}, \quad r \text{ is the discount rate in each country}, \quad i = A, B, \quad \beta^A \neq \beta^B, \quad 0 < \beta^A, \beta^B < 1
\]

There is a negative relationship between the country’s discount factor and the discount rate. So if the country’s discount rate rises, the discount factor falls, and vice versa. In equations (61) and (62), if a firm has a low discount factor, the firm values the present revenue more than the future revenue. In other words, it discounts the future revenue heavily. On the other hand, if a firm has a high discount factor, the firm discounts the second-period revenue lightly. With \( \beta^A < \beta^B \), the firm A is less patient since it values profits in the first period more than profits in the second period compared to firm B. Firm B values second-period profits more than firm A. Thus firm B will be more patient to get the second-period revenue.

The nominal exchange rate is denoted by \( e \) and given exogenously. It represents the units of country A’s currency per unit of country B’s currency:

\[
e = \frac{A’s \ currency}{B’s \ currency}
\]
The costs of production for each firm are expressed in terms of its own currency.

The unit costs in each period are given as follows:

\[
C_1^{AT} = C_1^{AD} q_1^{AD} + C_1^{AB} q_1^{AB}, \quad C_1^{AD} \neq C_1^{AB}, \quad C_1^{AD}, C_1^{AB} > 0 \quad (69)
\]

\[
C_2^{AT} = C_2^{AD} q_2^{AD} + C_2^{AB} q_2^{AB}, \quad C_2^{AD} \neq C_2^{AB}, \quad C_2^{AD}, C_2^{AB} > 0 \quad (70)
\]

\[
C_1^{BT} = C_1^{BD} q_1^{BD} + C_1^{BA} q_1^{BA}, \quad C_1^{BD} \neq C_1^{BA}, \quad C_1^{BD}, C_1^{BA} > 0 \quad (71)
\]

\[
C_2^{BT} = C_2^{BD} q_2^{BD} + C_2^{BA} q_2^{BA}, \quad C_2^{BD} \neq C_2^{BA}, \quad C_2^{BD}, C_2^{BA} > 0 \quad (72)
\]

\[
C_1^{AT} = C_1^{BT} = C_1, \quad C_1 \text{ is constant and } C_1 > 0 \quad (73)
\]

\[
C_2^{AT} = C_2^{BT} = C_2, \quad C_2 \text{ is constant and } C_2 > 0 \quad (74)
\]

\[
C_1 \neq C_2 \quad (75)
\]

The subscripts indicate the date, \( t = 1, 2 \). In each period, \( C^{AD} \) is the unit cost to produce \( q^{AD} \) and \( C^{AB} \) is the unit cost to produce \( q^{AB} \). The model assumes that \( C^{AD} \) and \( C^{AB} \) are different because selling the product in a foreign country requires costs, such as transportations cost. \( C^{AT} \) is the total cost to firm A to produce \( q^{AD} \) and \( q^{AB} \). The unit cost of \( C^{BD} \) for firm B to produce \( q^{BD} \) and \( C^{BA} \) to produce \( q^{BA} \) are different. \( C^{BT} \) is the total cost to firm B to produce \( q^{BD} \) and \( q^{BA} \). In the first period, the total cost of \( C^{AT} \) and \( C^{BT} \) are the same and I designate this as \( C_1 \). In the second period, the total cost of \( C^{AT} \) and \( C^{BT} \) are the same and given as \( C_2 \). And I assume that \( C_1 \) and \( C_2 \) are different.

From equations (61) and (62), we take the first-order conditions in order to get the equilibrium prices and output levels for countries A and B in each period. The first-order conditions with respect to \( q^{AD} \), \( q^{AB} \), \( q^{BD} \) and \( q^{BA} \) in each period are respectively:
\[
\frac{\partial \pi^A}{\partial q^A_1} = a - 2q^A_1 - C^A_1 = 0, \quad \quad q^A_1 = \frac{a - q^{BA}_1}{2} - \frac{C^A_1}{2} \quad (76)
\]

\[
\frac{\partial \pi^A}{\partial q^B_1} = e_1 \left[ a - q^{BD}_1 - 2q^A_1 \right] - C^A_1 = 0, \quad \quad q^B_1 = \frac{a - q^{BD}_1}{2} - \frac{C^A_1}{2e_1} \quad (77)
\]

\[
\frac{\partial \pi^A}{\partial q^A_2} = \beta^A \left[ a - 2q^A_2 - q^{BA}_1 \right] - C^A_2 = 0, \quad \quad q^A_2 = \frac{a - q^{BA}_2}{2} - \frac{C^A_2}{2\beta^A} \quad (78)
\]

\[
\frac{\partial \pi^A}{\partial q^{AB}_2} = \beta^A e_2 \left[ a - q^{BD}_1 - 2q^{AB}_2 \right] - C^{AB}_2 = 0, \quad \quad q^{AB}_2 = \frac{a - q^{BD}_2}{2} - \frac{C^{AB}_2}{2\beta^A e_2} \quad (79)
\]

\[
\frac{\partial \pi^B}{\partial q^{BD}_1} = a - 2q^{BD}_1 - C^{BD}_1 = 0, \quad \quad q^{BD}_1 = \frac{a - q^{AB}_1}{2} - \frac{C^{BD}_1}{2} \quad (80)
\]

\[
\frac{\partial \pi^B}{\partial q^{BA}_1} = e_1 \left[ a - q^{AD}_1 - 2q^{BA}_1 \right] - C^{BA}_1 = 0, \quad \quad q^{BA}_1 = \frac{a - q^{AD}_1}{2} - \frac{e_1 C^{BA}_1}{2} \quad (81)
\]

\[
\frac{\partial \pi^B}{\partial q^{BD}_2} = \beta^B \left[ a - 2q^{BD}_2 - q^{AB}_1 \right] - C^{BD}_2 = 0, \quad \quad q^{BD}_2 = \frac{a - q^{BD}_2}{2} - \frac{C^{BD}_2}{2\beta^B} \quad (82)
\]

\[
\frac{\partial \pi^B}{\partial q^{BA}_2} = \beta^B e_2 \left[ a - q^{AD}_2 - 2q^{BA}_2 \right] - C^{BA}_2 = 0, \quad \quad q^{BA}_2 = \frac{a - q^{AD}_2}{2} - \frac{e_2 C^{BA}_2}{2\beta^B} \quad (83)
\]

From the eight equations above, equilibrium quantities are derived for each firm in each period as follows:

\[
q^{AD^*}_1 = \frac{a}{3} + \frac{e_1 C^{BA}_1}{3} - \frac{2C^{AD}_1}{3}, \quad \text{if } a + e_1 C^{BA}_1 > 2C^{AD}_1, \quad q^{AD^*}_1 > 0 \quad (84)
\]

\[
q^{AB^*}_1 = \frac{a}{3} + \frac{C^{BD}_1}{3} - \frac{2C^{AB}_1}{3e_1}, \quad \text{if } e_1 \left( a + C^{BD}_1 \right) > 2C^{AB}_1, \quad q^{AB^*}_1 > 0 \quad (85)
\]
\[ q_{2}^{AD^*} = \frac{a}{3} + \frac{e_{2}C_{2}^{BA}}{3\beta^B} - \frac{2C_{2}^{AD}}{3\beta^A}, \quad \text{if } \beta^A(a\beta^B + e_{2}C_{2}^{BA}) > 2\beta^B C_{2}^{AD}, \quad q_{2}^{AD^*} > 0 \]  

(86)

\[ q_{2}^{AB^*} = \frac{a}{3} + \frac{C_{2}^{BD}}{3\beta^B} - \frac{2C_{2}^{AB}}{3\beta^Ae_{2}}, \quad \text{if } \beta^Ae_{2}(a\beta^B + C_{2}^{BD}) > 2\beta^B C_{2}^{AB}, \quad q_{2}^{AB^*} > 0 \]  

(87)

Both firms’ first-period output levels in the domestic and the foreign markets are the same as the one-period model in chapter 3. This occurs because firms determine the output level and the price separately in each period. Each period’s linear demand curve is not related to the other. In other words, when firms decide the first-period output level, second-period output level does not enter into the equilibrium condition. They decide the first-period output level based only on the first-period exchange rate and unit costs. And, in the second period, the firms consider the discount factors as well as the second-period exchange rate and the unit cost when they decide the output levels.

The value of the first-period and the second-period revenue are not the same because of the time difference. So the discount factor alters the value of the second-period revenue into the first-period value. However, the first-period output level and the
second-period output level are related to each other since firms have limited resources. More specifically, if firms want to increase one period’s output level, they should decrease the other period’s output level. There is a trade-off between the quantities in these two periods because resources they can use are limited.

From equations (63), (84) and (89), and (64), (86) and (91), the equilibrium prices of the output that firms A and B charge for each period in country A in terms of country A’s currency are:

\[ P_{1}^{a} = \frac{a}{3} + \frac{e_{1}C_{1}^{BA}}{3} + \frac{C_{1}^{AD}}{3} \]  \hspace{1cm} (92)

\[ P_{2}^{a} = \frac{a}{3} + \frac{e_{2}C_{2}^{BA}}{3\beta^{B}} + \frac{C_{2}^{AD}}{3\beta^{A}} \]  \hspace{1cm} (93)

From equations (65), (85) and (88), and (66), (87) and (90) the equilibrium prices of the output that firms A and B charge in country B expressed in country B’s currency for each period are:

\[ P_{1}^{b} = \frac{a}{3} + \frac{C_{1}^{AB}}{3e_{1}} + \frac{C_{1}^{BD}}{3} \]  \hspace{1cm} (94)

\[ P_{2}^{b} = \frac{a}{3} + \frac{C_{2}^{AB}}{3\beta^{A}e_{2}} + \frac{C_{2}^{BD}}{3\beta^{B}} \]  \hspace{1cm} (95)

The equilibrium prices of the product that firm A receives in each period from the foreign market (country B) expressed in country A’s currency are:
\begin{equation}
\frac{e_1 P_{1B^*}}{3} = \frac{ae_1}{3} + \frac{C_{1AB}}{3} + \frac{e_1 C_{1BD}}{3}
\end{equation}

\begin{equation}
\frac{e_2 P_{2B^*}}{3} = \frac{ae_2}{3} + \frac{C_{2AB}}{3\beta^A} + \frac{e_2 C_{2BD}}{3\beta^B}
\end{equation}

The equilibrium prices of the output that firm B gets in each period from the foreign market (country A) in terms of country B’s currency are:

\begin{equation}
\frac{P_{1A^*}}{e_1} = \frac{a}{3e_1} + \frac{C_{BA}}{3} + \frac{C_{1AD}}{3e_1}
\end{equation}

\begin{equation}
\frac{P_{2A^*}}{e_2} = \frac{a}{3e_2} + \frac{C_{BA}}{3\beta^B} + \frac{C_{2AD}}{3\beta^B e_2}
\end{equation}

I derived the profit-maximizing quantities and prices in a two-period setup. Each first-period’s quantity and price consists of the exchange rate, its own unit cost of production and the other firm’s cost of production. Note that the price and quantity are related to the other firm’s cost as well. In equation (92), for example, \( P_{1A^*} \) is the price that firms A and B charge in country A in terms of country A’s currency. Since \( P_{1A^*} \) represents both firms’ price in country A, it reflects both firms’ cost of production. And in the second-period, the price \( P_{2A^*} \) is discounted by both firms’ discount factor in equation (93). In the case of quantities, firms compete with each other in a market so they consider each other’s cost of the production when they choose their own quantity. Thus, we observe both firms’ unit costs in each output level. And that is why the second-period output levels are discounted by both firms’ discount factor as well.
If the output level and the price are related to the discount factor in the second period, there should be certain relationships among these. That is what we will examine next.

4.3 Output level and Discount Factor

How do changes in the discount factors affect the output levels? In this section, I report on the relationship between the output level and each firm’s discount factor. In equations (86), (87), (90) and (91), note that the size of a firm’s output level is affected not only by its own discount factor but also by the other firm’s discount factor.

Proposition 8: Firm A’s domestic and foreign output levels are positively related to its own discount factor. Firm A’s domestic and foreign output levels are inversely related to firm B’s discount factor.

Proof:

\[
\hat{\partial} q_2^{AD} = \frac{2C_2^{AD}}{3(\beta^A)^2} > 0
\]

(100)

\[
\hat{\partial} q_2^{AB} = \frac{2C_2^{AB}}{3e_2^2(\beta^A)^2} > 0
\]

(101)

\[
\hat{\partial} q_2^{AD} = -\frac{e_2C_2^{BA}}{3(\beta^B)^2} < 0
\]

(102)

\[
\hat{\partial} q_2^{AB} = -\frac{e_2C_2^{BA}}{3(\beta^B)^2} < 0
\]

(103)
Let us consider the effect of firm A’s discount factor on firm A’s domestic output level and the foreign output level. According to equations (100) and (101), there is a positive relationship between $\beta^A$ and $q_{2D}^A$ and between $\beta^A$ and $q_{2B}^A$. So, if firm A is more patient -- that is, $\beta^A$ is higher -- then its domestic output level in the second period ($q_{2D}^A$) rises. And, in this case, its export output level in country B ($q_{2B}^A$) increases as well.

In contrast, from equations (102) and (103), $\beta^B$, country B’s discount factor, has a negative impact on $q_{2D}^A$ and $q_{2B}^A$. Thus when firm B is more patient, firm A decreases both its domestic output level and its foreign output level.

Firms share the customers in a market so if one firm sells more goods, the other firm sells fewer goods in the market. If firm B’s discount factor increases and it is more patient, then firm B increases its second-period output level in country A. Thus, firm A’s second-period output level in the domestic market decreases. Similarly, in this case, firm B increases its second-period output level in the domestic market. So, firm A’s second-period output level in country B decreases as well. That is why firm A’s domestic and foreign output levels are negatively related to firm B’s discount factor.

Next, I show that firm B has a similar relationship between the discount factor and the output level as shown below.

Proposition 9: Firm B’s domestic and foreign output levels are positively related to its own discount factor. Firm B’s domestic and foreign output levels are inversely related to firm A’s discount factor.
Proof: \[
\frac{\partial q_{2}^{BD}}{\partial \beta^B} = \frac{2c_{2}^{BD}}{3\left(\beta^B\right)^2} > 0 \tag{104}
\]
\[
\frac{\partial q_{2}^{BA}}{\partial \beta^A} = \frac{2e_{2}C_{2}^{BA}}{3\left(\beta^A\right)^2} > 0 \tag{105}
\]
\[
\frac{\partial q_{2}^{BD}}{\partial \beta^A} = -\frac{C_{2}^{AB}}{3e_{2}\left(\beta^A\right)^2} < 0 \tag{106}
\]
\[
\frac{\partial q_{2}^{RA}}{\partial \beta^A} = -\frac{C_{2}^{AD}}{3\left(\beta^A\right)^2} < 0 \tag{107}
\]

Equations (104) and (105) show that firm B’s discount factor (\(\beta^B\)) is positively related to its own domestic output level and its export output level. And, equations (106) and (107) indicate that the exporter’s (firm A’s) discount factor (\(\beta^A\)) is negatively related to firm B’s domestic output level and firm B’s output level in country A.

Since the number of consumers in a market is limited, there is a trade-off between one firm’s output level and the other firm’s output level. If firm A’s higher discount factor induces firm A to sell more goods in the domestic market in the second period, firm B sells fewer products in country A. And if firm A has more output level in country B because of its higher discount factor, firm B’s second-period output level in the domestic market falls.

Based on the relationship between the output levels and the discount factor that we found, I want to compare the size of the first-period output level and the second-period output level to examine the effect of discount factor on the output level.
Proposition 10: Suppose $\beta^A - \beta^B$ is positive and increases algebraically, then firm A’s second-period output level is greater than its first-period output level.

Proof: In the case of firm A’s domestic output level,

\[ q_{1\, AD}^{*} = \frac{a}{3} + \frac{e_1 C_{1\, AD}^{RA}}{3} - \frac{2C_{1\, AD}^{AD}}{3}, \quad \text{if } a + e_1 C_{1\, AD}^{RA} > 2C_{1\, AD}^{AD}, \quad q_{1\, AD}^{*} > 0 \quad (108) \]

\[ q_{2\, AD}^{*} = \frac{a}{3} + \frac{e_2 C_{2\, AD}^{RA}}{3\beta^B} - \frac{2C_{2\, AD}^{AD}}{3\beta^A}, \quad \text{if } \beta^A (a \beta^B + e_2 C_{2\, AD}^{RA}) > 2\beta^B C_{2\, AD}^{AD}, \quad q_{2\, AD}^{*} > 0 \quad (109) \]

\[ \frac{\partial q_{2\, AD}^{*}}{\partial \beta^A} = \frac{2C_{2\, AD}^{AD}}{3(\beta^A)^2} > 0 \quad (110) \]

\[ \frac{\partial q_{2\, AD}^{*}}{\partial \beta^B} = -\frac{e_2 C_{2\, AD}^{RA}}{3(\beta^B)^2} < 0 \quad (111) \]

In the case of firm A’s foreign output level,

\[ q_{1\, AB}^{*} = \frac{a}{3} + \frac{C_{1\, BD}^{RA}}{3} - \frac{2C_{1\, AB}^{AD}}{3e_1}, \quad \text{if } e_1 (a + C_{1\, BD}^{RA}) > 2C_{1\, AB}^{AD}, \quad q_{1\, AB}^{*} > 0 \quad (112) \]

\[ q_{2\, AB}^{*} = \frac{a}{3} + \frac{C_{2\, BD}^{RA}}{3\beta^B} - \frac{2C_{2\, AB}^{AD}}{3\beta^A e_2}, \quad \text{if } \beta^A e_2 (a \beta^B + C_{2\, BD}^{RA}) > 2\beta^B C_{2\, AB}^{AD}, \quad q_{2\, AB}^{*} > 0 \quad (113) \]

\[ \frac{\partial q_{2\, AB}^{*}}{\partial \beta^A} = \frac{2C_{2\, AB}^{AD}}{3e_2 (\beta^A)^2} > 0 \quad (114) \]

\[ \frac{\partial q_{2\, AB}^{*}}{\partial \beta^B} = -\frac{e_2 C_{2\, AB}^{RA}}{3(\beta^B)^2} < 0 \quad (115) \]

Let us start with firm A’s domestic output level. By comparing equations (108) and (109), we can show that one period’s domestic output level is bigger than the other.
And firm A’s discount factor and firm B’s discount factor play important roles for the answer to the question. From equation (110), if firm A’s discount factor (\( \beta^A \)) increases, firm A is more patient, so that output levels in the second period are larger, resulting in its domestic output level in the second period. At the same time, in equation (111), if firm B’s discount factor (\( \beta^B \)) falls, firm B is less patient and prefers to sell the products in the first period in country A. So, firm A’s domestic output level rises in the second period as well in this case. As the result, \( \beta^A - \beta^B \) is positive and increases. \( \beta^A \) and \( \beta^B \) make firm A’s domestic output level in the second period (\( q_{2D}^{AD} \)) increase. Thus firm A’s second-period domestic output level will be greater than the first-period output level.

On the other hand, if firm A’s discount factor decreases in equation (110), firm A discounts the future more so it decreases its second-period domestic output level. And, if firm B has higher discount factor in equation (111), firm B discounts the future less and is patient to sell more exports in country A in the second period. Thus firm A’s domestic output level falls in the second period. As the result, \( \beta^A \) and \( \beta^B \) make firm A’s domestic output level in the second period (\( q_{2D}^{AD} \)) decrease. Therefore, firm A’s second-period domestic output level will be smaller than the first-period output level.

Other cases we have are as follows: First, firm A is more patient so it increases its domestic output level in the second period. And firm B is more patient as well, thus firm B increases the second-period output level in country A. Firm B’s higher discount factor means that firm A sells fewer goods in the domestic market in the second period. Therefore, each \( \beta^A \) and \( \beta^B \) leads firm A’s domestic output level in the second period, \( q_{2D}^{AD} \), to the opposite direction of the movement and we do not know whether \( q_{2D}^{AD} \).
increases or decreases. We have ambiguous results and cannot say whether \( q_{2}^{AD} \) is greater than or smaller than the first-period output level. Second, firm A discounts the future more so it decreases its second-period domestic output level. Firm B also discounts the future more so its second-period output level in country A falls. Thus, firm A’s second-period domestic output level increases. Therefore, each \( \beta^A \) and \( \beta^B \) leads firm A’s domestic output level in the second period, \( q_{2}^{AD} \), to the opposite direction of the movement and we do not know whether \( q_{2}^{AD} \) increases or decreases. Thus we cannot say whether firm A’s domestic output level in the second period is greater than or smaller than its domestic output level in the first period.

So far, we have investigated the size of firm A’s domestic output level in the first period and in the second period. Then, what will happen to firm A’s foreign output level in the second period? In the case of firm A’s foreign output level, equation (114) indicates that if \( \beta^A \) rises -- firm A is more patient -- firm A’s second-period foreign output level (\( q_{2}^{AB} \)) rises. At the same time, equation (115) indicates that if \( \beta^B \) falls -- firm B is less patient -- firm B prefers to sell more goods in the first period in its domestic market. So firm A’s second-period foreign output level rises. Therefore, firm A’s second-period foreign output level will increase. As the result, firm A’s second-period foreign output level is greater than the first-period foreign output level.

According to equations (114) and (115), if \( \beta^A \) falls and \( \beta^B \) rises, \( q_{2}^{AB} \) falls. So firm A’s second-period foreign output level is smaller than the first-period output level. In other conditions of discount factors -- both firms’ discount factors move in the same
direction, we have ambiguous results since the movements of $q_{2A}^{AB}$ related to $\beta^A$ and $\beta^B$ are opposite directions.

We have similar results for firm B’s first- and second-period output level.

**Proposition 11:** Suppose $\beta^A - \beta^B$ is positive and increases algebraically, then firm B’s second-period output level is greater than its first-period output level.

**Proof:** In the case of firm B’s domestic output level,

$$q_{1B}^{BD} = \frac{a}{3} + \frac{C_{1A}^{AB}}{3e_1} - \frac{2C_{1B}^{BD}}{3}, \quad \text{if } ae_1 + C_{1A}^{AB} > 2e_1C_{1B}^{BD}, \quad q_{1B}^{BD} > 0 \quad (116)$$

$$q_{2B}^{BD} = \frac{a}{3} + \frac{C_{2A}^{AB}}{3\beta^Ae_2} - \frac{2C_{2B}^{BD}}{3\beta^B}, \quad \text{if } \beta^B (a\beta^Ae_2 + C_{2A}^{AB}) > 2\beta^Ae_2C_{2B}^{BD}, \quad q_{2B}^{BD} > 0 \quad (117)$$

$$\frac{\partial q_{2B}^{BD}}{\partial \beta^B} = \frac{2C_{2B}^{BD}}{3(\beta^B)^2} > 0 \quad (118)$$

$$\frac{\partial q_{2B}^{BD}}{\partial \beta^A} = -\frac{C_{2A}^{AB}}{3e_2(\beta^A)^2} < 0 \quad (119)$$

In the case of firm B’s foreign output level,

$$q_{1B}^{BA} = \frac{a}{3} + \frac{C_{1A}^{AD}}{3} - \frac{2e_1C_{1B}^{BA}}{3}, \quad \text{if } a + C_{1A}^{AD} > 2e_1C_{1B}^{BA}, \quad q_{1B}^{BA} > 0 \quad (120)$$

$$q_{2B}^{BA} = \frac{a}{3} + \frac{C_{2A}^{AD}}{3\beta^A} - \frac{2e_2C_{2B}^{BA}}{3\beta^B}, \quad \text{if } \beta^B (a\beta^A + C_{2A}^{AD}) > 2\beta^Ae_2C_{2B}^{BA}, \quad q_{2B}^{BA} > 0 \quad (121)$$

$$\frac{\partial q_{2B}^{BA}}{\partial \beta^B} = \frac{2e_2C_{2B}^{BA}}{3(\beta^B)^2} > 0 \quad (122)$$
\[
\frac{\partial q_2^{BD}}{\partial \beta^d} = -\frac{C_2^{AD}}{3(\beta^d)^2} < 0
\] (123)

An increase in a discount factor means a firm discounts the future lightly so the firm is more patient to sell the product in the second period in a market, and vice versa. In the domestic market, if \( \beta^B \) rises and \( \beta^d \) falls, for example, \( q_2^{BD} \) rises in equations (118) and (119). So firm B’s second-period domestic output level is greater than the first-period domestic output level. In contrast, if \( \beta^B \) falls and \( \beta^d \) rises, \( q_2^{BD} \) falls from equations (118) and (119). Hence firm B’s second-period domestic output level is smaller than the first-period output level.

In the foreign market (country A), if firm B has a higher discount factor and firm A has a lower discount factor, then firm B’s second-period foreign output level is greater than the first-period output level. However, if firm B has a lower discount factor and firm A has a higher discount factor, firm B will sell a smaller number of goods in the foreign market in the second period than in the first period. In other conditions of discount factors -- both firms’ discount factors move in the same direction, we have ambiguous results.

In this section, we examined how the changes in the discount factor affect the output levels. As the result, we found that if firm A’s discount factor and firm B’s discount factor move in opposite directions -- one of the firms is more patient but the other firm is less patient -- then we can compare the size of the output level in the first period and the second period and conclude which one is larger. On the other hand, if the discount factors move in the same direction -- both firms are more patient or both firms
are less patient -- then we have ambiguous results about the size of the output levels in the second period. Thus we do not know whether the second-period output level is bigger or smaller than the first-period output level in these cases.

4.4 The Price and Discount Factor

Next, what is the effect of the discount factor on the price? According to equations (93) and (95), $P^*_2$ is affected by both firm A’s discount factor, $\beta^A$, and firm B’s discount factor, $\beta^B$, at the same time. Both $\beta^A$ and $\beta^B$ have influence on $P^*_2$ as well.

**Proposition 12**: (i) The local-currency price in country A is negatively related to $\beta^A$ and $\beta^B$; (ii) The local-currency price in country B is negatively related to $\beta^A$ and $\beta^B$.

**Proof:**

\[
\frac{\partial P^A_2}{\partial \beta^A} = -\frac{C^{AD}}{3(\beta^A)^2} < 0
\]  \hfill (124)

\[
\frac{\partial P^A_2}{\partial \beta^B} = -\frac{e^2 C^{BA}}{3(\beta^B)^2} < 0
\]  \hfill (125)

\[
\frac{\partial P^B_2}{\partial \beta^A} = -\frac{C^{AB}}{3e^2 (\beta^A)^2} < 0
\]  \hfill (126)

\[
\frac{\partial P^B_2}{\partial \beta^B} = -\frac{C^{BD}}{3e^2 (\beta^B)^2} < 0
\]  \hfill (127)

If we assume that exporters impose the export price in terms of buyer’s currency, firms A and B charge $P^*_2$ in country A in the second period and both firms charge $P^*_2$. 

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in country B. In equations (124) and (125), firm A’s domestic price and firm B’s export price, \( P_2^{*}\), and in equations (126) and (127), firm B’s domestic price and firm A’s export price, \( P_2^{*}\), decreases when firm A and B are more patient -- firm A’s discount factor and firm B’s discount factor rise. In other words, firms A and B will decrease the second-period price in country A when they are patient and want to sell more goods in the second period. Similarly, in equations (126) and (127), both firms decrease the second-period price in country B when both firms are more patient. There is a trade-off between the price they charge and the quantity they can sell, therefore if firms want to sell more goods they should lower the price. If the discount factor is higher -- it implies that the firm is willing to sell more goods in the second period, then it lowers the price in each market as we found in the equations above.

Then, what kind of discount factors make the second-period price higher or lower than the first-period price?

Proposition 13: Suppose both \( \beta^A \) and \( \beta^B \) are higher, then the second-period prices in country A and B are lower than the first-period prices.

Proof: in country A,

\[
P_1^{*} = \frac{a}{3} + \frac{e^1 C_{1}^{BA}}{3} + \frac{C_{1}^{AD}}{3}
\]

(128)

\[
P_2^{*} = \frac{a}{3} + \frac{e^2 C_{2}^{BA}}{3 \beta^B} + \frac{C_{2}^{AD}}{3 \beta^A}
\]

(129)

\[
\frac{\partial P_2^A}{\partial \beta^A} = -\frac{C_{2}^{AD}}{3(\beta^A)^2} < 0
\]

(130)
\[
\frac{\partial P_{2}^{A}}{\partial \beta^{B}} = -\frac{e_{2}C_{2}^{BA}}{3(\beta^{B})^{2}} < 0 \quad (131)
\]

In country B, \[ P_{1}^{B*} = \frac{a}{3} + \frac{C_{1}^{AB}}{3e_{1}} + \frac{C_{1}^{BD}}{3} \quad (132) \]

\[ P_{2}^{B*} = \frac{a}{3} + \frac{C_{2}^{AB}}{3e_{2}} + \frac{C_{2}^{BD}}{3} \quad (133) \]

\[
\frac{\partial P_{2}^{B}}{\partial \beta^{A}} = -\frac{C_{2}^{AB}}{3e_{2}(\beta^{A})^{2}} < 0 \quad (134)
\]

\[
\frac{\partial P_{2}^{B}}{\partial \beta^{B}} = -\frac{C_{2}^{BD}}{3e_{2}(\beta^{B})^{2}} < 0 \quad (135)
\]

From equations (129) and (133), the prices in countries A and B are dependent on both \( \beta^{A} \) and \( \beta^{B} \). If firms A and B are more patient -- both \( \beta^{A} \) and \( \beta^{B} \) rise -- then both firms lower the future price in the domestic and the foreign market to obtain higher profits. Hence, the second-period prices in both markets are lower than the first-period prices. And if firms A and B are less patient -- both \( \beta^{A} \) and \( \beta^{B} \) falls -- then both firms increase the future price. Thus, the second-period price is higher than the first-period price in both markets.

4.5 Exchange Rate Pass-Through

Is there any exchange rate pass through in the second period?

\[
\frac{\partial P_{2}^{A*}}{\partial e_{2}} \times \frac{e_{2}}{P_{2}^{A*}} = \frac{C_{2}^{BA}}{3\beta^{B}} \times \left( \frac{e_{2}}{3} + \frac{e_{2}C_{2}^{BA}}{3\beta^{B}} + \frac{C_{2}^{AD}}{3\beta^{A}} \right) = \frac{\beta^{A}e_{2}C_{2}^{BA}}{a\beta^{B} + \beta^{A}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}} < 1 \quad (136)
\]
Suppose that exporters charge the export price in terms of the buyer’s currency. Firm A imposes $P^*_2$ in the domestic market and firm B charges $P^*_2$ in country A as the export price in the second period. We take the exchange-rate elasticity of the price in the second period as shown above in equation (136) to investigate the degree of ERPT in country A. It is less than 1 and indicates that there is an incomplete ERPT in the country. Less than 1 for the elasticity also means that the percentage change in the price in country A is less than the percentage change in the exchange rate. So firms A and B do not pass through the full proportion of the exchange-rate movements into the domestic price in the case of firm A and into the export price in the case of firm B. This result is the same as the result of ERPT we had in the one-period model. Thus we conclude that firms do not pass through the exchange rate movement into the prices even though they have an extended time horizon.

We can also measure the ERPT for country B in the second period as follows:

\[
\frac{\partial P^*_{2B}}{\partial e_2} \times e_2 = \left( -\frac{C^A_{2B}}{3\beta^A e_2} \right) \times \frac{e_2}{\left( \frac{a}{3} + \frac{\beta^A C^A_{2B} + C^BD_{2}}{3\beta^B} \right)}
\]

\[
= \left| -\frac{\beta^B C^A_{2B}}{a \beta^A \beta^B e_2 + \beta^B C^A_{2B} + \beta^A e_2 C^BD_{2}} \right| < 1
\]

(137)

$P^*_{2B}$ is the local-currency price that firms A and B charge in the second period in country B. Equation (137) is the exchange-rate elasticity of the price in country B in the second period. It shows the relative price changes as the exchange rate varies. The exchange-rate elasticity of the price in country B is incomplete as well since it is less than
1. It implies that firms A and B pass through only part of the exchange rate movements into the price in the country. In conclusion, the firms exhibit incomplete ERPT in both markets not only in the first period but also in the second period. Therefore, the length of the time period that firms have does not make any differences in firm’s ERPT behavior.

Furthermore, do the firms show the incomplete ERPT behaviors when they set the export price in their own currency as well? Suppose that exporters impose the export price in terms of its own currency, the exchange-rate elasticity of firm A’s export price that is expressed in its own currency as given below. Firm A’s exchange-rate pass through is incomplete:

\[
\frac{\partial e_2 P^{2^*}}{\partial e_2} \times \frac{e_2}{e_2 P^{2^*}} = \left( \frac{a}{3} + \frac{C_{2BD}}{3\beta^B} \right) \times \frac{1}{\frac{ae_2}{3} + \frac{C_{2AB}}{3\beta^A} + \frac{e_2 C_{BD}}{3\beta^B}} \]

\[
= \frac{\beta^A e_2 \left( a\beta^A + C_{2BD} \right)}{a\beta^A \beta^B e_2 + \beta^A e_2 C_{2AB} + \beta^A e_2 C_{BD}} < 1
\]

The exchange-rate elasticity of firm B’s export price that is expressed by its own currency is as follows. The exchange-rate pass through is incomplete:

\[
\frac{\partial \left( \frac{P^{2^*}}{e_2} \right)}{\partial e_2} \times \frac{e_2}{\left( \frac{P^{2^*}}{e_2} \right)} = \left( -\frac{a}{3e_2^2} - \frac{C_{2AD}}{3\beta^A e_2^2} \right) \times \frac{e_2}{\frac{a}{3e_2} + \frac{C_{2BA}}{3\beta^B e_2} + \frac{C_{2AD}}{3\beta^A e_2}} \]

\[
= \left| -\frac{\beta^B \left( a\beta^A + C_{2AD} \right)}{a\beta^A \beta^B e_2 + \beta^A e_2 C_{2BA} + \beta^A e_2 C_{AD}} \right| < 1
\]
Then, can we tell the difference in the degree of ERPT when the firms have a different choice of invoice currency? We find more implications about the exporter’s invoice choice and the ERPT as follows.

**Proposition 14:** The degree of ERPT in the second period is higher when the exporter invoices in its own currency than in the buyer’s currency.

**Proof:**

**Firm A**

\[
\frac{\partial P_{2}^{B*}}{\partial e_2} \times P_{2}^{B*} = -\frac{\beta^B C_{2}^{AB}}{a \beta^A \beta^B e_2 + \beta^B C_{2}^{AB} + \beta^A e_2 C_{2}^{BD}}
\]

\[
< \frac{\partial P_{2}^{B*}}{\partial e_2} \times \frac{e_2}{e_2 P_{2}^{B*}} = \frac{\beta^A e_2 \left( a \beta^B + C_{2}^{BD} \right)}{a \beta^A \beta^B e_2 + \beta^B C_{2}^{AB} + \beta^A e_2 C_{2}^{BD}}
\]

**Firm B**

\[
\frac{\partial P_{2}^{A*}}{\partial e_2} \times P_{2}^{A*} = -\frac{\beta^A e_2 C_{2}^{BA}}{a \beta^A \beta^B + \beta^A e_2 C_{2}^{BA} + \beta^B C_{2}^{AD}}
\]

\[
< \frac{\partial \left( \frac{P_{2}^{A*}}{e_2} \right)}{\partial e_2} \times \frac{e_2}{\left( \frac{P_{2}^{A*}}{e_2} \right)} = -\frac{\beta^B \left( a \beta^A + C_{2}^{AD} \right)}{a \beta^A \beta^B + \beta^A e_2 C_{2}^{BA} + \beta^A C_{2}^{AD}}
\]

\(P_{2}^{B*}\) is the second-period price in country B charged by firms A and B. It is in terms of country B’s currency. So, in this case, firm A imposes the export price expressed in the buyers’ currency in country B. \(e_2 P_{2}^{B*}\) is the price that firm A charges in country B expressed in its own currency. The first equation in inequality (140) represents ERPT

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when firm A invoices in buyer’s currency to sell the exports in country B. \( P_{2}^{A*} \) is the price in country B charged by firms A and B. It is in terms of country A’s currency. \( \frac{P_{2}^{A*}}{e_{2}} \) is the price that firm B charges in country A expressed in its own currency.

In the case of firm A, we derive inequality (140) from equations (137) and (138). Firm A’s exchange rate pass-through to the export price in the second period when the export price is measured by its own currency, \( \frac{\beta^{4}e_{2}(a\beta^{B} + C_{2}^{BD})}{a\beta^{4}\beta^{B}e_{2} + \beta^{B}C_{2}^{AB} + \beta^{A}e_{2}C_{2}^{BD}} \), is greater than the exchange rate pass-through to the export price when the export price is expressed by the buyer’s currency, \( \left| -\frac{\beta^{B}C_{2}^{AB}}{a\beta^{4}\beta^{B}e_{2} + \beta^{B}C_{2}^{AB} + \beta^{A}e_{2}C_{2}^{BD}} \right| \). It is true because of the condition in equation (87). If \( \beta^{4}e_{2}(a\beta^{B} + C_{2}^{BD}) > 2\beta^{B}C_{2}^{AB} \) is binding in order to get \( q_{2}^{AB*} > 0 \) then \( \beta^{4}e_{2}(a\beta^{B} + C_{2}^{BD}) > \beta^{B}C_{2}^{AB} \) is true. Thus, firm A’s degree of ERPT is higher when it sets the export price in its own currency rather than in the buyer’s currency.

We have inequality (141) based on equations (136), (139) and (91) for firm B in the second period. Firm B’s exchange rate pass-through to the export price when the export price is measured by its own currency, \( \left| -\frac{\beta^{B}(a\beta^{A} + C_{2}^{AD})}{a\beta^{4}\beta^{B}e_{2} + \beta^{A}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}} \right| \), is greater than the ERPT to the local-currency export price, \( \frac{\beta^{A}e_{2}C_{2}^{BA}}{a\beta^{4}\beta^{B}e_{2} + \beta^{A}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}} \). This is because firm B has the condition of \( \beta^{B}(a\beta^{A} + C_{2}^{AD}) > 2\beta^{A}e_{2}C_{2}^{BA} \) in equation (91) in order to get more than zero quantity of the good in country A in the second period.
If \( \beta^B \left( a \beta^A + C_{2}^{AD} \right) > 2 \beta^A e_2 C_{2}^{BA} \) or \( \frac{\beta^B \left( a \beta^A + C_{2}^{AD} \right)}{2} > \beta^A e_2 C_{2}^{BA} \) from equation (91) then the inequality, \( \beta^B \left( a \beta^A + C_{2}^{AD} \right) > \beta^A e_2 C_{2}^{BA} \), is valid. Therefore we conclude that firm B’s degree of ERPT in the second period is higher when it sets the export price in its own currency rather than in the buyer’s currency. Hence we have the same result about the ERPT in the second period as we have in the one-period model.

Do firms have different degrees of ERPT in the first period and the second period because of the discount factors they have in the second period?

**Proposition 15:** The degree of ERPT in the second period is higher than the first period.

**Proof:** In country A,

\[
\left\{ \frac{\partial P_2^*}{\partial e_2} \times \frac{e_2}{P_2^*} = \frac{\beta^A e_2 C_{2}^{BA}}{a \beta^A \beta^B + \beta^A e_2 C_{2}^{BA} + \beta^B C_{2}^{AD}} \right\} > \left\{ \frac{\partial P_1^*}{\partial e_1} \times \frac{e_1}{P_1^*} = \frac{e C_{1}^{BA}}{a + e C_{1}^{BA} + C_{1}^{AD}} \right\}
\]

In country B,

\[
\left\{ \frac{\partial P_2^{*B}}{\partial e_2} \times \frac{e_2}{P_2^{*B}} = \frac{\beta^B C_{2}^{AB}}{a \beta^A \beta^B e_2 + \beta^B C_{2}^{AB} + \beta^A \epsilon_2 C_{2}^{BD}} \right\} > \left\{ \frac{\partial P_1^{*B}}{\partial e_1} \times \frac{e_1}{P_1^{*B}} = \frac{C_{1}^{AB}}{a e_1 + e C_{1}^{AB} + e C_{1}^{BD}} \right\}
\]

Suppose that the exchange rate and the costs are the same in the first period and in the second period to get the pure effect of discount factors on ERPT in the comparison.
Suppose that exporters invoice in the buyer’s currency as the export price in each country as well. In equation (67), we have an assumption, $0 < \beta^A, \beta^B < 1$. And if we multiply a constant number by a number that is greater than zero but less than one, we have a smaller number after the multiplication. If we have a smaller number for the denominator, we have a bigger value of a fraction. Therefore, in inequality (142),

$$\frac{\beta^A e_2 C_{2}^{BA}}{a \beta^A \beta^B + \beta^A e_2 C_{2}^{BA} + \beta^B C_{2}^{AD}}$$

has the biggest number, indicating that firms in country A pass through the most exchange rate fluctuations to the price when they have a different discount factor for each firm in the second period ($\beta^A \neq \beta^B$). We have

$$\frac{e_2 C_{2}^{BA}}{a \beta + e_2 C_{2}^{BA} + C_{2}^{AD}}$$

when we suppose that firms have the same discount factor for the firms in the second period ($\beta^A = \beta^B$). In this case, the ERPT is higher than the first-period ERPT but lower than the case of different discount factors. $\frac{e_1 C_{1}^{BA}}{a + e_1 C_{1}^{BA} + C_{1}^{AD}}$ has a lowest value indicating that firms have the lowest ERPT if they have only one period to consider.

In conclusion, exporters pass through more exchange rate changes when they have longer time periods and different discount factors than they do when they have only one-period to consider. The same is true in the case of country B in inequality (143).

4.6 On PTM

What are the conditions that indicate the existence of PTM in the case of the second period of the two-period model? Are there any differences in the firms’ PTM behaviors if they have more time horizon to consider?
4.6.1 Firm A’s Local-Currency Export Price vs. Firm A’s Domestic Price

When firm A imposes local-currency export price in country B, we have the following equation to measure the relative price change in the domestic and the foreign market as the exchange rate moves in the second period:

\[
\frac{\partial P_{2}^{AB}}{\partial e_{2}} \times \frac{e_{2}}{P_{2}^{AB}} = \frac{-\beta_{2}^{B}C_{2}^{AB}}{a\beta^{A}\beta^{B}e_{2} + \beta^{B}C_{2}^{AB} + \beta^{A}e_{2}C_{2}^{BD}}
\]

\[
\frac{\partial P_{2}^{A}}{\partial e_{2}} \times \frac{e_{2}}{P_{2}^{A}} = \frac{\beta^{A}e_{2}C_{2}^{BA}}{a\beta^{A}\beta^{B}e_{2} + \beta^{B}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}}
\]

\[
\begin{align*}
&= \left\{ \frac{-\left(\beta_{2}^{B}C_{2}^{AB}\right)\left(a\beta^{A}\beta^{B} + \beta^{A}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}\right)}{\left(\beta^{A}e_{2}C_{2}^{BA}\right)\left(a\beta^{A}\beta^{B}e_{2} + \beta^{B}C_{2}^{AB} + \beta^{A}e_{2}C_{2}^{BD}\right)} \right\} \\
&= \left\{ \frac{-\left(\beta_{2}^{B}C_{2}^{AB}\right)\left(a\beta^{A}\beta^{B} + \beta^{A}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}\right)}{\left(\beta^{A}e_{2}C_{2}^{BA}\right)\left(a\beta^{A}\beta^{B}e_{2} + \beta^{B}C_{2}^{AB} + \beta^{A}e_{2}C_{2}^{BD}\right)} \right\}
\end{align*}
\]

If \( \left| \frac{-\left(\beta_{2}^{B}C_{2}^{AB}\right)\left(a\beta^{A}\beta^{B} + \beta^{A}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}\right)}{\left(\beta^{A}e_{2}C_{2}^{BA}\right)\left(a\beta^{A}\beta^{B}e_{2} + \beta^{B}C_{2}^{AB} + \beta^{A}e_{2}C_{2}^{BD}\right)} \right| = 1 \), the relative price changes in the foreign and the domestic market are the same as the exchange rate moves. Therefore, there is no PTM in the second period. (145)

If \( \left| \frac{-\left(\beta_{2}^{B}C_{2}^{AB}\right)\left(a\beta^{A}\beta^{B} + \beta^{A}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}\right)}{\left(\beta^{A}e_{2}C_{2}^{BA}\right)\left(a\beta^{A}\beta^{B}e_{2} + \beta^{B}C_{2}^{AB} + \beta^{A}e_{2}C_{2}^{BD}\right)} \right| > 1 \), or

\( \left| \frac{-\left(\beta_{2}^{B}C_{2}^{AB}\right)\left(a\beta^{A}\beta^{B} + \beta^{A}e_{2}C_{2}^{BA} + \beta^{B}C_{2}^{AD}\right)}{\left(\beta^{A}e_{2}C_{2}^{BA}\right)\left(a\beta^{A}\beta^{B}e_{2} + \beta^{B}C_{2}^{AB} + \beta^{A}e_{2}C_{2}^{BD}\right)} \right| < 1 \), there are relative price differences in the foreign and the domestic market. Thus, PTM occurs by firm A in the second period. (146)
4.6.2 Firm B’s Local-Currency Export Price vs. Firm B’s Domestic Price

When firm B imposes local-currency export price in country A, we have the following equation to measure the relative price change in the domestic and the foreign market as the exchange rate moves in the second period:

\[
\frac{\partial P^{**}_A}{\partial e_2} \times \frac{e_2}{P^{**}_A} = \frac{\beta^A e_2 C^{RA}_2}{a \beta^A \beta^B + \beta^A e_2 C^{RA}_2 + \beta^B C^{AD}_2} - \frac{\beta^B C^{AB}_2}{a \beta^A \beta^B e_2 + \beta^B C^{AB}_2 + \beta^A e_2 C^{BD}_2}
\]

\[
= \left\{ -\left(\frac{\beta^A e_2 C^{RA}_2}{a \beta^A \beta^B + \beta^A e_2 C^{RA}_2 + \beta^B C^{AD}_2}\right)\right\}
\]

(147)

If \( \left| \frac{\beta^A e_2 C^{RA}_2}{a \beta^A \beta^B + \beta^A e_2 C^{RA}_2 + \beta^B C^{AD}_2} \right| = 1 \), the relative price changes in the foreign and the domestic market are the same as the exchange rate varies. Therefore, there is no PTM in the second period.

(148)

If \( \left| \frac{\beta^A e_2 C^{RA}_2}{a \beta^A \beta^B + \beta^A e_2 C^{RA}_2 + \beta^B C^{AD}_2} \right| > 1 \), or

\[
\left| \frac{\beta^A e_2 C^{RA}_2}{a \beta^A \beta^B + \beta^A e_2 C^{RA}_2 + \beta^B C^{AD}_2} \right| < 1
\]

there are relative price differences in the foreign and the domestic market. Thus, PTM occurs by firm B in the second period.

(149)
Then, what is the effect of discount factors on PTM? Do we have a different degree of PTM between the first period and the second period?

\[
\frac{\partial P^{a_k^*}}{\partial e_1} \frac{e_2}{P^{a_k^*}} = \left\{ \frac{\left( \beta^B e^C_{2B} \right) \left( a \beta^A \beta^B e_2 + \beta^B e^C_{2B} + \beta^B C^A_{2B} \right)}{\left( \beta^A e^C_{2A} \right) \left( a \beta^A \beta^B e_2 + \beta^B C^A_{2B} + \beta^A e^C_{2B} \right)} \right\} (150)
\]

\[
\frac{\partial P^{a_k^*}}{\partial e_2} \frac{e_2}{P^{a_k^*}} = \left\{ \frac{C^A_{2B} \left( a \beta + e^C_{2A} + C^A_{2D} \right)}{C^A_{2B} \left( a \beta + e^C_{2A} + C^A_{2D} \right)} \right\} (151)
\]

\[
\frac{\partial P^{a_k^*}}{\partial e_1} \frac{e_1}{P^{a_k^*}} = \left\{ \frac{C^A_{1B} \left( a + e^C_{1A} + C^A_{1D} \right)}{C^A_{1B} \left( a + e^C_{1A} + C^A_{1D} \right)} \right\} (152)
\]

In the case of firm A, if we assume that exporters have local-currency export prices have different discount factors, we have equation (150) as the standard to determine the existence of PTM. On the other hand, if we assume that firms A and B have the same discount factor \( \beta^A = \beta^B \), we have equation (151) as the condition of the existence of PTM. And equation (152) is the condition of PTM existence when firm A has only one period.

We cannot determine which case has a relatively higher degree of PTM. Therefore, the effect of discount factors on PTM is indeterminate. We cannot tell whether exporters show a more severe PTM when they have extended time period or not. The same is true in the case of firm B.
4.7 Summary

In this chapter, I establish a two-period model but it does not have any intertemporal links. The intertemporal aspects are negated by the fact that the model has time-additively separable profit specifications that are linear in the demand for the product at that point in time. I have focused on investigating the difference between the first-period and the second-period output level decisions and the equilibrium prices as the exchange rate moves. A firm’s output level, price, ERPT and PTM are affected by both its own discount factor and the other firm’s discount factor. I address the condition of discount factors that make the second-period output level and the price higher than the first-period output level and price. If one firm’s discount factor is high but the other firm’s discount factor is low -- one firm is more patient and discounts the future lightly but the other firm is less patient and discounts the future heavily, then the more patient firm’s second-period output level in both domestic and the foreign market is greater than its first-period output level. However, when both firms are more patient, the second-period prices in the domestic and the foreign market are lower than the first-period price.

I concluded that ERPT in the second period is incomplete as well as in the first period. However, I found that firms pass through more exchange rate movements to the price when they have more time horizon to consider; second-period ERPT is higher than the first-period ERPT.

I presented the condition of PTM in the second period as well. Each firm’s discount factor is multiplied by every cost and the exchange rate in the numerator and the denominator of the condition. Thus the influence of the discount factor on PTM is ambiguous. We also cannot show the effect of extended time period on PTM behavior.
A clear definition of PTM is important. It helps in measurement of PTM and in the development of empirical work related to PTM. This paper clarifies the situation that we call PTM based on Krugman’s (1987) initial idea. PTM in this dissertation focuses on the relative movements of the export price in different markets, conditional on exchange rate movements. An interesting feature is that the degree of ERPT itself is not important to determine the existence of PTM. But the relative degree of ERPT in different markets is crucial to conclude whether there is PTM behavior or not.

The advantage of the model in this dissertation is as follows: First, the model takes the output level as the motivation of firms’ PTM behavior. Thus output level is determined in the model to maximize the profit rather than take it as given (Froot and Klemperer, 1989). Second, in the model developed in this dissertation, both equilibrium domestic price and export price depend on the exchange rate as well as the cost of production, rather than only export prices vary according to the exchange rate changes. That makes it possible to obtain the exchange-rate elasticity of domestic price and the exchange-rate elasticity of export price directly. Therefore, it is also possible to directly measure the relative price changes as the exchange rate varies in the different markets.

I present a standard to identify the existence of PTM based on the definition of PTM I presented in this dissertation. Thus the standard formalizes the conception of PTM. It is a ratio of the exchange-rate elasticity of a country’s export price to the exchange-rate elasticity of the domestic price using the export and domestic prices
determined in the model. The ratio provides a better measure of the relative price movements in the foreign market and the domestic market as the exchange rate fluctuates. This is because the elasticity of the export price, or that of the domestic price with respect to the exchange rate changes, measures the percentage changes of the prices in response to the percentage changes of the exchange rates.

I report a different result compared to existing research in terms of the relationship between the invoice currency and ERPT (Bacchetta and Wincoop, 2002), and between the invoice currency and PTM (Gil-Pareja, 2003 and Bleaney, 1997). I found that exporters pass through more exchange-rate movements to the export price when they invoice the export price in their own currency than they do in the buyers’ currency. The relative price difference between the domestic and the foreign market is larger -- higher degree of PTM when they invoice the export price in their own currency than they do in the buyers’ currency. However, my finding is similar to the findings in other literature on the degree of ERPT (Frankel, Parsley and Wei, 2005; Lee, 1997; Menon, 1996; Gron and Swenson, 1996; Hooper and Mann, 1989; Froot and Klemperer, 1989 and Krugman, 1987): an incomplete ERPT is prevalent in the markets.

In my comparison of changes in the export price denominated in the exporter’s currency and changes in the price of the domestic good as the exchange rate varies, I found that the export price is more responsive to the exchange rate movements than the price of the domestic good in a country.

In addition, I found that domestic producers produce more output levels and consumers buy more domestic goods in a country if the domestic seller and the exporter charge the same price and the price increases as the exchange rate fluctuates. However,
exporters produce more output levels and consumers buy more exports in a country if the domestic seller and the exporter charge the same price and the price decreases as the exchange rate moves.

I also show that if a country’s currency depreciates, the country has a surplus in the trade balance with the other country. On the other hand, if a country’s currency appreciates, we have an indeterminate result in the country’s trade balance with the other country. In this way, I extended Magee’s (1973) and Kreinin’s (1977) work.

Using the two-period model, I tried to examine the effect of extended time period on the output level, the price, ERPT and PTM. A firm’s output level, price, ERPT and PTM are affected by both its own discount factor and the other firm’s discount factor. I address the conditions of each firm’s discount factor that entail a higher future output level and higher price than the present output level and price. If one firm’s discount factor is high but the other firm’s discount factor is low -- one firm is more patient and discounts the future lightly but the other firm is less patient and discounts the future heavily, then the more patient firm’s second-period output level in both domestic and the foreign market is greater than its first-period output level. However, when both firms are more patient, the second-period prices in the domestic and the foreign market are lower than the first-period price.

Furthermore, I concluded that an incomplete ERPT is prevalent when the firms have extended time period as well as they have only one period. I found, however, that firms pass through more exchange rate movements to the price when they have more time horizon to consider; second-period ERPT is higher than the first-period ERPT. On the
other hand, the effect of the extended time period on the PTM behavior is indeterminate in the model economy.

Further study should focus on developing a general model showing the conditions of PTM existence without limiting the model’s results within certain demand curves or cost functions. In addition, it will be interesting to study a generalized two-period model in which the intertemporal link between the first-period profit and the second-period profit exists. The empirical work to test the model will also be valuable.

This dissertation accepts the exchange rate as given and does not consider the exporter’s expectation of the exchange rate movements in the future as a variable. However, an analysis that includes the expectation factor into the model could provide useful implications to the exchange-rate policy. It will also be valuable to set up a model with heterogeneous goods so that we could focus on firms on operating in an economy in which Bertrand price competition exists. Then the firms choose prices directly. Furthermore, investigating the effect of trade policy on exporter’s PTM behavior will be interesting.
REFERENCE LIST


VITA

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