POTENTIAL THEORY AND HARMONIC ANALYSIS METHODS
FOR QUASILINEAR AND HESSIAN EQUATIONS

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ABSTRACT

The existence problem is solved, and global pointwise estimates of solutions are obtained for quasilinear and Hessian equations of Lane–Emden type, including the following two model problems:

$$-\Delta_p u = u^q + \mu, \qquad F_k[-u] = u^q + \mu, \qquad u \geq 0,$$

on $\mathbb{R}^n$, or on a bounded domain $\Omega \subset \mathbb{R}^n$. Here Δ_p is the p-Laplacian defined by $\Delta_p u = \operatorname{div}(\nabla u|\nabla u|^{p-2})$, and $F_k[u]$ is the k-Hessian defined as the sum of $k \times k$ principal minors of the Hessian matrix D^2u ($k = 1, 2, \ldots, n$); μ is a nonnegative measurable function (or measure) on Ω.

The solvability of these classes of equations in the renormalized (entropy) or viscosity sense has been an open problem even for good data $\mu \in L^s(\Omega)$, $s > 1$. Such results are deduced from our existence criteria with the sharp exponents $s = \frac{n(q-p+1)}{pq}$ for the first equation, and $s = \frac{n(q-k)}{2kq}$ for the second one. Furthermore, a complete characterization of removable singularities for each corresponding homogeneous equation is given as a consequence of our solvability results.