

# HIERARCHICAL NONLINEAR, MULTIVARIATE, AND SPATIALLY-DEPENDENT TIME-FREQUENCY FUNCTIONAL MODELS

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## ABSTRACT

Notions of time and frequency are important constituents of most scientific inquiries, providing complimentary information. In an era of "big data," methodology for analyzing functional and/or image data is increasingly important. This dissertation develops methodology at the cross-section of time-frequency analysis and functional data and consists of three distinct, but related, contributions. First, we propose nonparametric methodology for nonlinear multivariate time-frequency functional data. In particular, we consider polynomial nonlinear functional data models that accommodate higher dimensional functional covariates, including time-frequency images, along with their interactions. The necessary dimension reduction for model estimation proceeds through carefully chosen basis expansions (empirical orthogonal functions) and feature-extraction stochastic search variable selection (SSVS). Properties of the methodology are examined through an extensive simulation study. Finally, we illustrate the approach through an application that attempts to characterize spawning behavior of shovelnose sturgeon in terms of high-density depth and temperature profiles. The second contribution proposes model-based time-frequency estimation through Bayesian lattice filter time-varying autoregressive models. In this context, we take a fully Bayesian approach and allow both the autoregressive coefficients and innovation variance to vary over time. Importantly, our model is estimated within the partial autocorrelation domain (i.e., through the partial autocorrelation coefficients). Additionally, all of the full conditional distributions required for our algorithm are of

standard form and thus can be easily implemented using a Gibbs sampler. Further, as a by-product of the lattice filter recursions, our approach avoids undesirable matrix inversions. As such, estimation is computationally efficient and stable. We conduct a comprehensive simulation study that compares our method with other competing methods and find that, in most cases, our approach performs superior in terms of average squared error between the estimated and true time-varying spectral density. Lastly, we demonstrate our methodology through several real case studies. The final project of the dissertation develops models that accommodate spatially dependent functional responses with spatially dependent image predictors. The methodology is motivated by a soil science study that seeks to model spatially correlated water content functionals as a function of electro-conductivity images. The water content curves are measured at different locations within the study field and at various depths, whereas the electro-conductivity images are spatially referenced images of wavelength by depth. Estimation is facilitated by taking a Bayesian approach, where the necessary dimension reduction for model implementation proceed using basis function expansions along with SSVS. Finally, the methodology is illustrated through an application to our motivating data.