THE PROCESSES AND PRODUCTS OF FIRST AND SECOND GRADE TEACHERS' INTERPRETATIONS OF STUDENTS' UNDERSTANDING OF PLACE VALUE

A dissertation presented to

the faculty of the Graduate School

at the University of Missouri

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

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by

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JULY 2014

The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

THE PROCESSES AND PRODUCTS OF FIRST AND SECOND GRADE TEACHERS'

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presented by Tiffany R. K. Hill,

a candidate for the degree of doctor of philosophy, and hereby certify that, in their opinion, it is worthy of acceptance.

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DEDICATION

For my dad,
who answers each call, listens, and then makes life seem simpler.

For my mom,
whose pride spills over to complete strangers.

For my brother,
who reminds me to 'live a little.'

For my husband,
whose support made this possible,
who was committed to this as if it were his own,
and whose presence makes life better.

ACKNOWLEDGEMENTS

I am here because of a number of people who have supported me and helped me to develop as a teacher and a researcher. These individuals have demonstrated for me what it means to be an educator, and I will forever be appreciative of each of them.

To start, I thank the members of my committee. Your support helped me to think more deeply and more carefully about my research, and in doing so you helped me to become a better researcher. I appreciate your contribution to this work.

I am especially appreciative of Dr. John Lannin, who served as the chair of my dissertation committee and as my advisor throughout the doctoral program. I remember well our phone conversation four years ago, when I stood in my elementary classroom before school and listened to you describe this program. I am fortunate to have had your support each day since. I appreciate that you are always questioning and wondering, and as a result always learning. I would like to think that those qualities are reflected in me now.

To close, I would like to thank a handful of remarkable women who have served as models and mentors:

- Mrs. Norma Vlieger, who in third grade made me want to become a teacher, and who taught me to find the good in life. This important lesson has not been forgotten, and has been helpful throughout this program.
- Dr. Ronilou Garrison, who made me want to become a teacher educator, and whose courses made me passionate about mathematics and science education. You pushed me further than I thought I was capable, and in doing so helped me to see my potential.

- Mrs. Linda Fryatt, who is the mentor that all teachers deserve. I will forever be grateful for your presence that first year. Our conversations helped me to be more thoughtful and reflective about practice, and as a result made me a better teacher.
- Mrs. Teresa Waters, who is one of the most outstanding teachers I have witnessed and whose passion for children is unmatched. I am fortunate that our classrooms shared the same hall, and that I was able to learn from your example.
- Dr. Deborah Hanuscin, who welcomed me and mentored me from the start my doctoral studies. I am in awe when I watch you teach, and am fortunate to have studied under you.
- Dr. Delinda van Garderen, who involved me in her research and her teaching. Your approach as a mentor made me feel like an equal and a valued member of the team, but at the same time helped me to see those areas where I needed to develop. I appreciate this perfect balance.
- Dr. Jeanine Hastings, who has and continues to welcome all questions as I transition from classroom teacher to teacher educator. Your mentorship is appreciated, and serves as an example of the long-lasting support I hope to offer the students I teach.
- Dr. Selina Sanchez, whose lessons have carried me through the final stages of these doctoral studies. I am so grateful that we met when we did.

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ABSTRACT

Consensus exists in mathematics education that effective *classroom assessment* is an essential component of effective practice; however, an important part of classroom assessment, teacher interpretation of student artifacts, is not often discussed. The purpose of this interpretative qualitative research study was to examine teachers' interpretations of student artifacts. In particular, I sought to understand: (a) the personal resources teachers used to construct their interpretations, and (b) the products of teacher interpretations, which included inferences teachers constructed about students and the instructional responses teachers planned. Toward these ends, nine *experienced* and *professionally active* teachers participated in two interviews. The first interview was semi-structured and focused on the participants' professional experiences, conceptions of assessment, and assessment practices. The second interview was task-based and involved teachers in the interpretation of student artifacts collected from second grade students. The results indicate that teachers applied to the act of interpretation their: (a) conceptions of levels of student performance, (b) expectations for student performance, (c) awareness of common student difficulties, and (d) awareness and understanding of mathematical ideas. Teachers' interpretations of *what occurred* and *student mathematical understanding* were sometimes varied for the same student artifact, and sometimes involved statements about *what occurred* that were inconsistent with or could not be observed in the artifact. Teachers' planned instructional responses were related to their interpretations, and also differed for the same student artifact. These results provide support for the conclusion that professional development related to classroom assessment should address the interpretation, with instruction focused on personal resources used in the act of interpretation and the importance of accurate interpretations.

CHAPTER ONE: INTRODUCTION

Consensus exists in mathematics education that effective classroom assessment is an essential component of effective practice (National Council of Teachers of Mathematics [NCTM], 2000; National Research Council [NRC], 2000, 2001). Used to describe those "formal and informal procedures that teachers use in an effort to make inferences about what their student know and can do" (Popham, 2009, p. 6), the term *classroom assessment* encompasses both formative and summative forms of assessment. In both forms, classroom assessment demands that teachers understand student conceptions, and because student conceptions are unseen and unheard, the usefulness of classroom assessment rests on accurate *teacher interpretation*. Nevertheless, teacher interpretation is an aspect of classroom assessment that is not often studied or discussed in the literature (Even & Wallach, 2003).

This research responds to this omission, and focuses on inferences drawn as a part of the formative assessment process because of its potential to influence student achievement. The focus on formative assessment is reflective of recent reforms in mathematics education that communicate the importance of formative assessment (NCTM, 2000; NRC, 2000, 2001), and that therefore recommend methods of instruction that are *adaptive* and *responsive* to students and their conceptions of mathematics (Empson & Jacobs, 2008; Sherin, Jacobs, & Philipp, 2011). At the core of these reforms is considerable research that indicates formative assessment practices have a substantial, positive impact on student achievement (Black & William, 1998; Kingston & Nash, 2011). However, it is believed that the potential positive impact of formative assessment rests, in part, on accurate teacher interpretation.

In order of occurrence, teacher interpretation is an intermediate step in the formative assessment process—one that occurs after a teacher collects information from the student but before a teacher responds to the student based on his or her inferences about them (Bell & Cowie, 2001). As such, both limited research (Jacobs, Lamb, & Philipp, 2010; Son, 2010) and common sense support the notion that inaccurate interpretation could be detrimental to the potential positive impact of formative assessment on student achievement. Heritage, Kim, Vendlinski, and Herman (2009) reiterate this point in their observation that "inappropriate inference... can lead to errors in what the instructional next steps will be, with the result that the teacher and the learner fail to close the gap" (p. 24) between what the child understands and what the teacher intends for the child to understand. In addition, inaccurate interpretation also results in the loss of the time and effort the teacher used to develop and administer the assessment that was intended for formative purposes.

Because of their critical importance, accurate inferences are essential; however, the *act of interpretation* is complex. In fact, multiple researchers note it is difficult to make sense of student work and to construct an accurate inference (Ball, 1993, 1997, 2001; Schifter, 2001), despite the fact that it is often presented as an "unproblematic" aspect of formative assessment (Even & Wallach, 2003, p. 322). In fact, it is the perceived simplicity of the act of interpretation that results in teacher preparation and development experiences that focus on teacher abilities to collect assessment data, rather than on teacher abilities to interpret and respond to that data. The unfortunate assumption embedded in these efforts is often that "varied information from multiple sources would allow the teacher to formulate more valid inferences" (Wallach & Even, 2005; p. 396).

However, researchers express concern at the oversimplification of this assumption, and have contended that while improvement in the collection of assessment data often results in more robust evidence, robust evidence does not result in valid inferences on all occasions (Even, 2005; Wallach & Even, 2005).

In observation of teachers in the act of interpretation, multiple problems arise in the effort to draw inferences about what a student does and does not understand about mathematics. Ball (1993, 1997) indicated these problems often arise because students do not represent their conceptions in statements that are understandable to an adult listener, as students converse from a perspective that is different from that of an adult. To add, children cannot often represent their conceptions with detailed explanation, so teachers must reason about responses on which students cannot elaborate. Of those student responses that appear correct, embedded ideas can be underdeveloped or incorrect, and of those student responses that appear incorrect, embedded ideas can be sensible and useful to future instruction (Ball, 2001; Schifter, 2001). As a result, teachers must be cautious in their consideration of all student responses, but teacher educators must also better prepare and develop teachers for this critical and complex aspect of formative assessment practice (Crespo, 2000; Even, 2005; Jacobs et al., 2010; Phillip et al., 2007; Schack et al., 2013; Wallach & Even, 2005).

Despite the importance of interpretation in the assessment process, few studies have examined teacher interpretation of assessment data. More than a decade ago, Morgan and Watson (2002) called for additional research related to this important topic. At the time, the researchers noted that, "little research on assessment in mathematics education has focused on the teacher-assessor as an active constructor of knowledge about students' mathematical knowledge" (p. 85). To date, and more than a decade later, Morgan and Watson (2002) are cited as one of a handful of studies related to teacher interpretation of assessment data in the most recent *Handbook of Research on Classroom Assessment* (McMillan, 2013). In their chapter, Schneider, Egan, and Julian (2013) describe the research related to the topic as sparse, and call for additional research. This research is designed to provide additional insight into teacher interpretation of student responses to formative assessment.

Purpose Statement

The purpose of this research is to examine the processes for and products of experienced, professionally active elementary teachers' interpretation of student responses to formative assessments. In particular, my research aims to understand the personal resources teachers use to construct their interpretations. In addition, this study was designed to understand the product of teacher interpretations, which include both the inferences teachers construct about what students do and do not understand about mathematics, and planned instructional responses in consideration of those inferences.

Justification for the Research

This research was pursued to address multiple gaps in two aspects of the literature on classroom assessment. First, the research addresses the lesser studied of the two areas of literature related to *teacher interpretation* in classroom assessments, which is described as *teacher inference*. The justification for the emphasis on *teacher inference* is described in the first subsection below. Second, the research addresses multiple gaps in the literature on *teacher inference*, and these are described in the second subsection below. That said, a more complete review of the literature related to *teacher interpretation* that focuses on *inference* can be found in the second chapter.

Research on teacher interpretation. A review of the classroom assessment literature reveals that much of the literature related to *teacher interpretation* focuses on the teacher and his or her *judgment* of student achievement, not his or her *inference* about what a student does or does not understand about mathematics. Different from the *inference* literature, the *judgment* literature is related to teacher effort to measure and communicate student achievement, often with marks or scores. In fact, the topic of *teacher judgment* is considered to be one of the most studied topics in education research to date (Brookhart, 2011, 2013; Parkes, 2013). That said, literature related to this topic does not encompass all that must be understood about *teacher interpretation*. That is, it does not consider the teacher and his or her inferences about what a student does or does not understand about mathematics.

In contrast from the *judgment* literature, this research focuses on *inferences*. A less-often studied aspect of the classroom assessment literature related to *teacher interpretation*, the *inference* literature is selected in part because it better fits with the nature of interpretations that are needed for effective formative assessment. That is, formative assessment requires interpretations that are specific to what the student does and does not understand about mathematics, not that measure and summarize student achievement. As such, my research is a response to an aspect of *teacher interpretation* literature that is not often considered—one that is believed to be more consistent with formative assessment practices than what is common in the literature at this time.

Research on teacher inference. Of the literature related to *teacher interpretation* with a focus on *inference*, two different emphases can be observed—one specific to the teacher and his or her *processes* in the act of interpretation, and one specific to the *products* of the act of interpretation. Processes are related to *how* a teacher reasons about a student artifact, and *which* personal resources he or she draws on to arrive at an interpretation. In contrast, products describe *what* results from these processes, such as inferences about what a student does or does not understand about mathematics, and the instruction the teacher plans in response to those inferences. However, of the studies reviewed, few focus on both the processes for and product of interpretation for the same teacher participants. In contrast, this research is designed to examine both processes for and products of teacher interpretation, in an effort to provide a more *holistic account* of the act of interpretation for the teachers studied (Creswell, 2007; Hatch, 2002).

In addition, this research is focused on those teachers who are underrepresented in the literature reviewed. In particular, of those studies reviewed, far more consider preservice than inservice teachers, and of the few studies that have started to focus on both the products and products of interpretation involved preservice teachers (e.g., Son, 2010; Son & Sinclair, 2010). Because of their focus on preservice teachers, these studies provide rich information about the difficulties preservice teachers encounter in the act of interpretation, and demonstrate that those difficulties often result in inaccurate or unhelpful products. That said, far less is understood about inservice teachers in the act of interpretation, and the product of interpretation. Whereas some inservice teachers demonstrate difficulties similar to preservice teachers, at least some more experienced teachers demonstrate elements of expertise (Jacobs et al., 2010). However, more research is needed to make specific claims about these teachers, and possible connections between their processes and products.

For this reason, my research examined the processes for and products of inservice teachers in the act of interpretation, and focuses on those who are experienced and active in their professional development are studied. That is, those teachers who have a rich base of experiences in professional development and with children from which they can draw on in the act of interpretation (Morgan & Watson, 2002). This focus is selected to move the research from the certain difficulties associated with this element of practice for preservice teachers, and toward those teachers who demonstrate expertise. Furthermore, the focus attempts to provide a more robust description of these teachers that is present in the current literature, since both the processes and products are considered.

If the research demonstrates that experienced, active teachers face challenges that are similar to preservice teachers, this finding will reinforce recommendations that teacher education and development address these issues early and often (Hiebert, Morris, Berk, & Jansen, 2007; Wilson & Berne, 1999). In addition, this study has the potential to reveal elements of desirable practice related to interpretation in these teachers, as such descriptions provide a model of elements of desirable practice for teacher educators and less developed teachers (Schneider et al., 2013). Furthermore, the research reveals those personal resources called upon in desirable practice, which can inform teacher educators of important concepts that must be incorporated in instruction (Jacobs et al., 2010).

Research Questions

Three related research questions are proposed to drive the research. These include one question related to processes for interpretation and two questions related to products of interpretation:

1. *Process*. What personal resources and prior conceptions do experienced, professionally active first and second grade teachers' use to construct interpretations of students' mathematical understanding from artifacts of student thinking?
2. *Product*. What interpretations of students' mathematical understanding do experienced, professionally active first and second grade teachers construct from artifacts of student thinking?
3. *Product*. What do experienced, professionally active first and second grade teachers plan in response to interpretations of students' mathematical understanding?

Theoretical Framework

Simon (2009) identifies variations of constructivism as common background theories in mathematics education research. These constructivist theories are situated with other cognitive perspectives on how people learn, and adopt a focus on the conceptions of the learner (Greeno, Collins, & Resnick, 1996). When these perspectives are applied to mathematics education research, the learner is defined as one who learns mathematics; however, applied to this mathematics *teacher education* research, the learner is the teacher, since this research focuses on what he or she learns about students from the written and oral responses provided. This perspective of *teacher as learner* allows for the application of a constructivist perspective to the research, and therefore focuses the research on the conceptions of teachers in the act of interpretation.

In the section that follows, the similarities and differences across the multiple forms of constructivism are described. From the review of these multiple forms of constructivism, social constructivism is isolated as the particular perspective adopted in the research, and a justification for this decision is provided. Further, the specific aspects of the adopted form of social constructivism that are applied to the research are described, and their implications for the research are considered.

Multiple constructivist perspectives. Multiple variations of constructivism (i.e., simple, radical, social) are described in the literature (Steffe & Gale, 1995). The common, central tenet of most of these forms of constructivism is the idea that we "construct our knowledge of our world from our perceptions and experiences" (Simon, 1995, p. 115). The use of the term *construct* serves as a metaphor that relates the construction of mental structures to the construction of material structures observed in architects and carpenters (Ernest, 2010). In turn, to learn is to build, and that which is learned is the combined product of the sense one made from his or her experience, which was "assimilated on the basis of prior knowledge" (Simon, 2000, p. 231).

Also consistent with most forms of constructivism is the principle that humans do not have access to an objective truth (Simon, 1995). Instead, individuals construct multiple realities, and the role of the researcher is to understand those realities. Absent of access to an objective truth, these constructed realities cannot therefore be measured for their truth value, but can be considered to the extent that each are viable (von Glasersfeld, 1974). The term *viable* refers to the extent to which conceptions fit or are consistent with experience. As a whole, the perspective advances a notion that the world is "experienceable but not knowable in an ultimate sense" (Ernest, 1996, p. 341).

In contrast, the idea on which the two most prominent forms of constructivism (i.e., radical, social) differ is the place in which construction is assumed to occur; that is, either in the mind of the individual or in the space between individuals (Ernest, 2010). Because the focus of this research is on the processes for and products of teacher interpretation, one could assume the topic itself points toward a constructivist perspective that is more individualistic; however, I instead adopt a form of constructivism that allows for a focus on the individual, but does not discount the important role of the social on the individual. The rationale for this particular decision is provided below, but not before a description of constructivist perspectives that differ on this point are described to provide a context for the decision.

To be certain, a narrow focus on the individual with limited consideration of the social is most often associated with *radical constructivism* (von Glasersfeld, 1974, 1995). However, the radical constructivist perspective has received criticism because of its limited focus on the individual and his or her conceptions without consideration of social aspects that contribute to the construction of conceptions (e.g., Lerman, 1994; 1996). In response to these criticisms, some radical constructivists have attempted to draw attention to components of the radical constructivist perspective that consider the influence of the social on the individual and the individual on the social, but still contend construction occurs in the mind of the individual (Steffe & Thompson, 2000).

Ernest (1994a) asserts that all effort to incorporate the individual and social in the process of construction fall under the umbrella term *social constructivism*, if not in name, at least in definition. He describes two different forms of social constructivism to account for the differences observed in the multiple definitions of the term. The first are social constructivist perspectives that "start from a radical constructivist position and add social aspects" (p. 66). These perspectives prioritize the *individual*, with some consideration of the influence of the social on the individual or on the social construction of conceptions. The second are social constructivist perspectives that maintain shared emphases on the individual and their interaction with others. These perspectives prioritize the *individual in conversation*, and the shared constructions that result from social interaction (Ernest, 1996).

The perspective adopted here is consistent with the first of these two forms of social constructivism, and therefore shares some similarities with radical constructivism. In particular, the adopted perspective assumes construction occurs intraindividual; however, the perspective does not discount the influence of social interaction on the individual and his or her construction (Steffe & Thompson, 2000). In fact, it is a form of social interaction that allows for individual teachers to construct models to represent student conceptions in the described research. In the next two sections, I expand on the social and individual aspects of the act of interpretation related to the research, and describe elements of the adopted social constructivist perspective that focus and inform the described research.

Social aspects of the act of interpretation. The adopted social constructivist perspective supports the notion that an observer (e.g., a teacher of a student, a researcher of a teacher) can construct a model to represent the conceptions of an observed student (Even & Tirosh, 2008). However, without direct access to the conceptions of an observed, the act of interpretation is dependent on an observed who must *publish* his or her conceptions (Ernest, 1991). As it is used here, the term *publish* refers to the actions of a person, which could include the words he or she writes, that make public aspects of his or her conceptions (Ernest, 1991; Cobb, 1996). Because such publication is a prerequisite of interpretation, some form of social interaction is assumed in all acts of interpretation.

In the classroom, occurrences when students share their conceptions, and the teacher interprets those conceptions are a common form of the social interaction described above. However, a modified form of this social interaction occurs in this research study when the teacher interprets students' published conceptions that were captured for the purpose of the research. In both cases, the social interaction prompts the intraindividual construction of mental models about the student and his or her conceptions (Steffe & Thompson, 2000). These intraindividual aspects of the act of interpretation are the focus of the described research, and are therefore discussed below.

Individual aspects of the act of interpretation. In spite of an awareness that the social realm makes the construction of representations of others' conceptions possible, the focus of the research is on individual teachers' processes and products of model construction. Applied to the research, the adopted social constructivist perspective led me to a focus on two aspects of the individual in the act of interpretation. These include the *personal resources* applied in the act of interpretation and *mental models* that result from the act of interpretation, both of which are described below.

Personal resources. The adopted social constructivist perspective assumes learners construct mental models in consideration of their immediate experiences and personal resources. For the teachers described in this research, their immediate experiences with students' published conceptions were similar because each was presented with the same artifacts; however, the personal resources each applies to those experiences were different. The term *personal resources* refers to those conceptions that are internal to the teacher, and include aspects of his or her *knowledge* and *beliefs* (Morgan & Watson, 2002). These personal resources are both the product of previous constructions, and the "building blocks" of future constructions (Ernest, 2010, p. 39).

From the first research question, I aim to understand the personal resources that teachers use in the act of interpretation. The embedded assumption that underlies this consideration is that because personal resources are different for individuals, the models the teachers construct for the same student can be different—both from one another and from the first-order model of the student (Simon, 2000). That said, to understand these differences, we must also understand the nature of the personal resources used.

Mental models. The term *mental models* refers to those internal conceptions the learner constructs to make sense of an experience or observation (Steffe, 1991). As such, mental models are the product of previous construction, but as mentioned above, can also be used in future constructions (Ernest, 2010). Applied to the research, the mental models of interest are those the teacher constructs about the mathematical conceptions of the children whose responses are presented to them. That said, the models an observer constructs are assumed to be a *representation* of the external, published conceptions of the observed, not a *replication* of his or her internal conceptions.

The difference between representation and replication has significant implications for the research, and stems in part from the assumption embedded in most variations of constructivism that "no certain knowledge is attainable" (Ernest, 1994b, p. 344). Instead, all constructed models are assessed on whether or not each fit with the experiences one has had, but need not match all possible experiences, or an objective truth (von

Glasersfeld, 1974), as noted above. Applied to the research, this means all constructed models, which include those of the researcher, are assumed to be falsifiable if an individual has more information or different experiences.

Reflected in the first research question, these ideas lead the researcher to consider the process in which these mental models are constructed, at which point his or her personal resources will be considered, and the nature of those mental models. Whether or not the teacher uses the mental models he or she constructs about the child in coordination with his or her personal resources to plan a response to the child is the product of the third research question.

Conceptual Framework

In consideration of the theoretical perspective adopted, I approached the research with assumptions about the multiple occurrences of knowledge construction in the research. In this section I present and describe two models that make explicit these assumptions. The first of these models communicates the multiple individuals who represent their conceptions or interpret the conceptions of others so that conclusions can be drawn about teachers' processes and products of interpretation. The second of these models below communicates the events that surround the act of teacher interpretation from artifacts, or the process wherein a teacher constructs his or her knowledge of students' mathematical conceptions.

The multiple constructions of mental models to represent others. In the research, multiple individuals (i.e., students, teachers) share their conceptions, and multiple others (i.e., teachers, researcher) interpret and construct models to represent those published conceptions. To account for instances when teachers create models to represent the conceptions of students, Leslie Steffe and his colleagues (e.g., Steffe, 1991, 1995; Steffe & Thompson, 2000) described two different levels of models that are constructed in a teaching and learning situation. That is, the models students construct about mathematics and the models teachers construct about students. Wilson, Lee, and Hollebrands (2011) added this as a third level to account for the models a researcher constructs in his or her studies of a teaching and learning situation.

These multiple levels of model construction, all anticipated in the research, are represented in Figure 1.1, and described in the sections below.

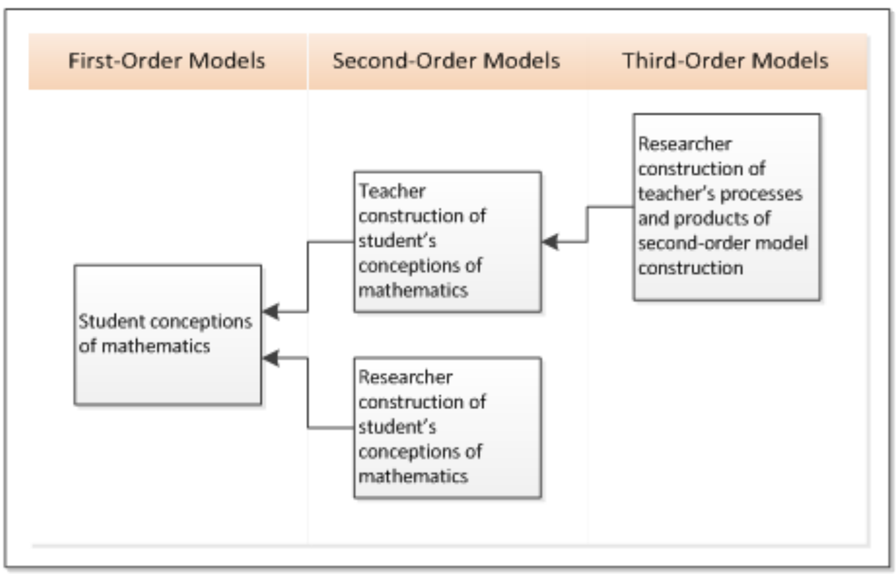

Figure 1.1. Occurrences of model construction in the study. Modified from Wilson et al. (2011).

The model is adapted from Wilson et al. (2011). Most adaptations were minor, and made in an effort to improve the fit of the description of model components with this particular research. As an example, instead of a label that named the first-order model as students' mathematical and technological conceptions, the term *technological* was removed because of the sole focus in this research on students' mathematical conceptions. The one instance when the adaptation was more substantial and an element was added to the model, is described in the related section below, titled *researcher construction of second-order models*.

Student construction of first-order models. At the foundational level, the researcher assumes learners construct their mathematical understanding. These constructions are called first-order models (Steffe, von Glasersfeld, Richards, & Cobb, 1983), and are described as "the knowledge the individual constructs to organize, comprehend, and control his or her experience" (Steffe & Thompson, 2000, p. 205). In the process of teaching and learning, these first-order models are published through verbal and written language and student actions (Cobb, 1996; Ernest, 1991).

Teacher construction of second-order models. At the point that learners share their first-order models in the form of verbal and written language, it is the responsibility of teachers to interpret those models. The teacher must therefore interact with the learner (Steffe, 1991, 1995; Steffe & Thompson, 2000) in order to "build… a model of each child's mathematical knowledge" (Steffe, 1991, p. 171). This model takes the form of a teacher interpretation about what that student does or does not understand, and is called a second-order model (Steffe et al., 1983).

Researcher construction of second-order models. In the research, I formed second-order models of the first-order models of children. Not included in the representation and discussion of Wilson et al. (2011), this element is added because of its presence and influence on the research. At times, these second-order models served as a point of comparison with the third-order models described below. Effort was made to make these occurrences explicit.

Researcher construction of third-order models. The aim of this study was to understand the processes through which teachers construct second-order models, and the nature of those second-order models. Wilson et al. (2011) therefore introduce the term third-order model to describe the constructions of the researcher that represent the second-order models of teachers. According to Wilson et al. (2011), "as researchers, we construct third-order models to describe our observations of teachers' observations of students' thinking" (p. 41).

A conceptual model of teachers' construction of second-order models. The processes and products of teacher construction of second-order models were the focus of the research. Figure 1.2 presents a model that represents the theoretical assumptions about these processes and products.

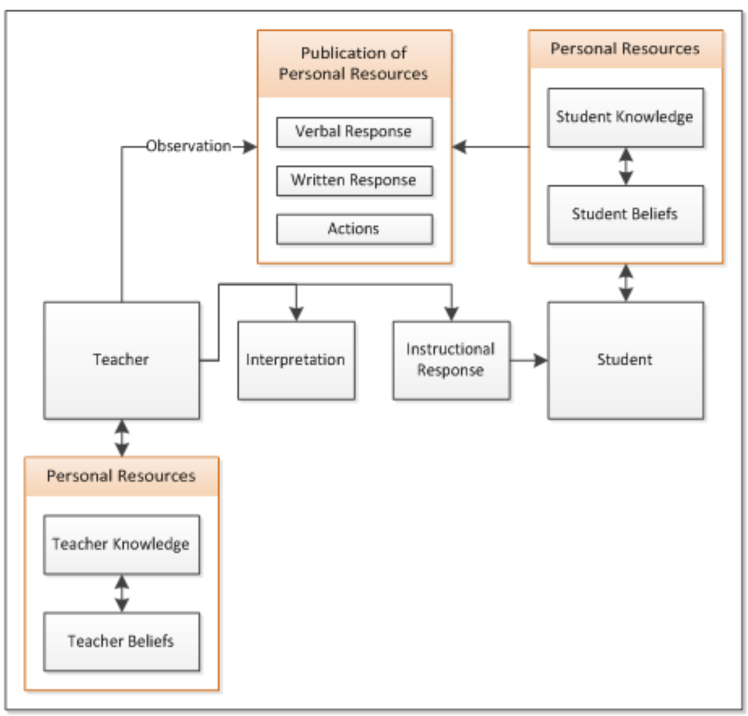

Figure 1.2. A representation of teachers' construction of second-order models. Modified from Watson (2000).

The model is modified from Watson (2000), who developed the model to delineate the "components of the practices of teachers, as assessors of students' mathematics in the normal course of their work" (p. 70). It is presented here to relate these theoretical assumptions to the research questions described above.

Because the model represents teacher construction of second-order models, or his or her interpretations of the student, the teacher is considered the "central actor in [the] model" (Watson, 2000, p. 81). As a result, the level of detail in the model related to teacher construction of second-order models is represented in far more depth than the implied student construction of first-order models. The elements related to the student that are represented in the model are his or her ***knowledge*** and ***beliefs***. These ***personal resources*** of the student are the objects of teacher interpretation.

In order for the teacher to arrive at interpretations about the student, the student must publish his or her personal resources in some observable form, which is represented in the model as a student behavior. These behaviors could be observed in what the student states, writes, makes, or does (Griffin, 2007), and are presented to the teacher participants in the research in the form of an artifact. The teacher must interact with the artifact in the process of interpretation. The first research question considers the ***personal resources*** the teacher calls on to formulate the interpretation.

The second-order model the teacher constructs is the product of the above processes, and is represented on the model as his or her ***interpretation***. The ***instructional response*** is considered to be another product, and is also influenced from both teacher's ***personal resources*** and ***interpretation***. The arrow from the ***response*** to the ***student*** communicates the assumed influence of planned response on the student. The teacher's

interpretation is the focus of the second research question, and his or her ***instructional response*** is the focus of the third research question.

CHAPTER TWO: REVIEW OF THE LITERATURE

In this chapter, I review the literature on teachers' processes for and products of the interpretation of student artifacts, focusing on those studies in which inference, not judgment is a product of interpretation. I first describe the methods used to select studies with this focus. In the sections that follow, I describe the literature related to how teachers reason about student artifacts, and the literature on the personal resources teachers draw on in the act of interpretation. Last, research related to the nature of the products of teacher interpretations is reviewed in two sections to compare studies of preservice and inservice teachers. After each section and at the end of the chapter, an overview of the reviewed literature is provided with a discussion of how this research responds to and builds on the reviewed literature.

Selection of Studies for the Review

To locate studies for this review, I started with a search of literature published from 2000 to 2013 in mathematics education and teacher education journals. These journals included: *Action in Teacher Education, Educational Studies in Mathematics, Journal for Research in Mathematics Education, Journal of Mathematics Teacher Education, Journal of Teacher Education, Studying Teacher Education, Mathematics Education Research Journal, Mathematics Teacher Educator, Mathematics Teacher Education and Development, Teacher Education Quarterly, Teaching and Teacher Education, Teaching Education, The Elementary School Journal,* and *The Teacher Educator, ZDM: The International Journal of Mathematics Education*. From the articles identified in this search, bibliographic references were also used in the identification of additional literature.

Studies were selected for the review that focused, at least in part, on preservice and inservice teachers' abilities to make sense of the student from the written artifacts he or she produced, his or her video-recorded oral responses, or his or her transcribed oral responses. Studies that involved teachers in consideration of videos and transcripts were included even if the video or transcript contained details of the teaching and learning situation, if the teacher attention was directed toward the student and what he or she understood. However, literature was not included in the review if teacher attention was directed toward the interpretation of cases, or if teachers were involved in lesson studies or video clubs. This literature was rejected on the premise that the focus of these studies was on teachers' abilities to make sense of teaching and learning, an emphasis that is broader than the student and his or her thinking.

Teacher Processes in the Act of Interpretation

The term *processes* is used to describe how a teacher reasons in the act of interpretation in order to arrive at the products of his or her interpretation. For that reason, these studies do not often focus on what the teacher interprets, but instead how he or she arrives at his or her interpretation and planned instructional response. These studies often involve observation of teachers as artifacts are considered, but are rare in the literature at this point. However, the studies described below note similar processes for both preservice and inservice teachers as artifacts are studied. That is, in the act of interpretation both preservice and inservice teachers describe the student response and provide their opinion of what was described (Wallach & Even, 2005; Wilson et al., 2011). Furthermore, both preservice and inservice teachers appear to compare the observed student response to an aspect of their personal resources, but personal resource differs for each.

In their research, Wallach and Even (2005) examined a case of an experienced fourth grade teacher who was presented with a video artifact of two of her students who were asked to solve a mathematical task. In part, the researchers aimed to describe the processes of the teacher as she interpreted the artifact without the influence of researcher prompts. The researchers noted four different processes in the behaviors of the inservice teacher studied, which included: *describing*, *explaining*, *assessing*, and *justifying*. Whereas *justifying* occurred when the teacher defended her interpretations, the processes of *explaining*, *describing*, and *assessing* comprised what the researchers called the *act of interpretation* (Wallach & Even, 2005).

Close attention to the occurrences of *explaining*, *describing*, and *assessing* reveal differences in the three behaviors. *Describing* includes occurrences when the teacher restated the words or named the actions of a student, but removed all possible inference from the description. In contrast, *explaining* involved occurrences when the teacher expanded on her description with inferred "ideas about the students' thoughts, reasoning, knowledge, and assumptions" (Wallach & Even, 2005, p. 401) that he or she believes influenced student talk or action. Different from this, *assessing* occurred when the teacher assigned judgment to the student talk or action, such as when she makes a claim about the effectiveness of a method a student used. Therefore, *assessing* involves judgment of a student response, whereas *explaining* involves claims about student conceptions that contribute to that response.

The last of the four processes Wallach and Even (2005) described was called *justifying*, and occurred when the teacher justified “her assessment of, or the meaning she attributes to, students’ talk and actions” (p. 401). The researchers noted this act was different from the others because the statements occurred when the teacher provided a rationale for her interpretations. A teacher can defend him or herself with reference to the student response or action that served as the source of evidence to support his or her interpretation, or with reference to his or her *personal resources* that influenced interpretation, such as components of his or her knowledge or beliefs (Wallach & Even, 2005). These are described below as those conceptions internal to the teacher that can be drawn on to inform the act of interpretation.

Whereas Wallach and Even (2005) studied an experienced classroom teacher, Wilson et al. (2011) studied 16 preservice teachers in the act of interpretation. These preservice teachers were enrolled in a mathematics methods course and as a component of that course studied the responses of middle school students who responded to various statistics tasks. The researchers observed four processes consistent across preservice teachers in their effort to make sense of student responses, which included: *describing*, *inferring*, *comparing*, and *restructuring*. Both *describing* and *inferring* are consistent with processes noted in Wallach and Even (2005); however, *comparing* and *restructuring* are distinctive of the preservice teachers studied in Wilson et al. (2011), and are therefore described further below.

The process of *comparing* was used to describe occurrences when preservice teachers likened their recent experiences as a learner of the content with what could be observed of the student from the artifact (Wilson et al., 2011). The researchers posited this process was the product of the preservice teachers' "limited experiences with students or knowledge of research-based images of prototypical responses" (p. 54). That is, where inservice teachers could draw on their *personal resources* based on their *knowledge of children's mathematics* (Even & Tirosh, 2008), preservice teachers were assumed to be without such resources and were assumed to have instead drawn on their most relevant experiences as students in mathematics classrooms.

The final process Wilson et al. (2011) described was termed *restructuring*, and describes occurrences when preservice teachers shifted their personal conceptions of the content or of students as a result of the consideration of student artifacts. The act of *restructuring* reflects research that indicates teacher interaction with artifacts can support the development of aspects of teachers' content and pedagogical content knowledge (e.g., Francisco & Maher, 2011; Kazemi & Franke, 2004; Phillip et al., 2007). In this chapter, such personal conceptions referred to *personal resources*, and are discussed in the next section in relation to their influence on the act of interpretation.

Summary of literature on teacher processes. In review, the processes of preservice teachers and inservice teachers demonstrate some similarities and differences. In observation, both groups of teachers *described* the student response, and then provided their judgment of what had been described and a claim about what the student understood (i.e., *explaining*, *assessing*, and *inferring*). Preservice teachers *compared* student responses to their own experiences as a student, while the inservice teacher *justified* her interpretations with her conceptions of teaching, learning, and the content, which included his or her approach to the mathematics problem presented. Specific to preservice teachers, the act of *restructuring* described occurrences when the preservice teachers adjusted their conceptions or personal resources in consideration of what was learned from the artifacts.

Personal Resources in the Act of Interpretation

Reflective of constructivist perspectives, researcher consideration of *personal resources* supports the notion that teachers draw on their internal conceptions *in the process of interpretation* to construct products of the act of interpretation (Even & Wallach, 2004). Morgan and Watson (2002) first used the term *resources* to describe the internal conceptions teachers used in the act of interpretation, and later Even and Wallach (2003) used the phrase *hearing through* to describe their observation that teachers "*hear through* various personal factors or screens" (p. 493). Despite variation in terms used, a consensus exists that teacher interpretation is influenced from that which is internal to him or her, and here the term *personal resources* is used to reflect those internal influences.

A review of the literature reveals an empirical basis for two broad aspects of personal resources. These include: (a) teachers' relationships with and conceptions of the students assessed and (b) teachers' conceptions of mathematics and the mathematics content. Outside of these aspects of personal resources, theoretical perspectives have identified other personal resources; however, these claims are not the result of observation of teachers in the act of interpretation. In two different sections that follow, both the literature that is empirical and theoretical in nature are described.

Relationships with and conceptions of the student. Wallach and Even (2005) found in their observation of Ruth, an experienced classroom teacher who studied video-recorded responses collected from two students, that her interpretations were influenced from her: (a) prior experiences with the students, (b) concern and care for the students, and (c) conceptions of the mathematical task and its solution. Whereas the latter of these three influences is related to the influence of the teacher and her conceptions about mathematics, the first two are related to the teacher and her relationship with and perspective of the student. That is, her conceptions of their "abilities, characters, dispositions, motivations, [and] collaboration skills" (p. 407), and her pleasure associated with their success.

Morgan and Watson (2002) also noted the influence of the teacher and his or her *conceptions of the student* in the process of interpretation. In particular, these researchers noted that an experienced teacher who studied a student across multiple course sessions often arrived at conclusions about the student and her abilities that were "*comparative* to what she had done before" (p. 92). Furthermore, his overall impressions of her mathematical abilities were slow to shift in consideration of evidence that was inconsistent with his earlier impressions. As a result, those without prior experiences with the student often arrived at different interpretations of the student than the teacher, despite the fact that the same evidence was considered.

These observations are consistent with the research of Watson (1999), who noted that the use of prior information about the student to interpret new information is a habit with which teachers are aware. In fact, Watson (1999) observed from her interviews with 30 inservice teachers about their informal assessment practices that teachers often admitted "they react differently to different pupils doing the same thing because they had formed judgments about the capability in advance" (p. 109). However, without observation of these teachers in the act of interpretation, Watson (1999) noted the extent to which these personal resources influenced the products of interpretation is unknown, and whether or not "real changes in learning behavior will be interpreted as such from a teacher who has a strong opinion of a pupil" (p. 109) is unclear.

Conceptions of mathematics content. The last of the resources Wallach and Even (2005) noted, and the second of the two sorts of personal resources discussed in this chapter, are related to the teacher and his or her conceptions of mathematics and mathematics content. In particular, Wallach and Even (2005) noted that Ruth, the experienced classroom teacher described above, interpreted the student response to the problem in consideration of her solution to the problem, in spite of the fact that there were multiple correct solutions. Further, when her students arrived at a correct solution that differed from her own, she was unsure whether or not the student response was correct, and was unable to determine whether or not it was correct within the constraints of the interview. The researchers attributed this to her limited *conception of the problem and its solution*, which is related to what she understands about the mathematical ideas assessed.

Morgan and Watson (2002) studied three inservice teachers as each interpreted the same student artifacts. The student artifacts contained two different responses to a task that assessed student abilities to solve problems, such as their abilities to *make and monitor decisions*, *communicate about mathematics*, and *develop skills of mathematical reasoning*. Of the two responses considered, one was a standard, correct response often observed from students, and the other was a non-standard, correct response. The researchers noted just one of the three inservice teachers noted the correctness of the non-standard response. In contrast, the other two teachers were unaware of the correctness of the response, but made claims that the student was “unstable” and that his approach was “unstructured and experimental” (p. 98). The researchers pointed to the teachers’ limited *content awareness* as the factor that inhibited accurate interpretation.

In another example, Heid, Blume, Zbiek, and Edwards (1999) shared the case of classroom teacher who interviewed a student about her ideas related to quadratic functions, and then studied her responses in a *scenario interview* in which the researcher asked the teacher about her interpretation as she watched the video-recorded interview. In consideration of one student’s response, the researchers identified the teacher’s “unstable understanding of the concept” (p. 232) as a factor that limited her abilities to determine the correctness of the student response. In observation, the teacher could recite definitions of mathematical ideas assessed, but had difficulties in the assessment of these ideas applied in the items administered. In particular, she could state that a quadratic function should be symmetric, but was unable to determine whether a student-provided curve was that of a quadratic function, despite the fact that it was asymmetric.

In observation, similar results have been noted in studies of preservice teachers. For example, Van Dooren, Verschaffell, and Onghena (2002) examined preservice teachers’ responses to various contextual word problems, and their evaluations of student responses to some of those same problems. Of the word problems presented, these researchers contended that an expert would most often answer half of them with an *algebraic solution* and half with an *arithmetical solution*. The researchers found that of those preservice teachers studied, about half of the elementary preservice teachers were “unable to solve the most difficult problems from the word problem test because they had not mastered algebra and were unable... to apply one of the arithmetic methods” (p. 347).

As a result, those preservice teachers also had difficulties in their evaluation of student responses in which an *algebraic solution* was used.

Similar to Van Dooren et al. (2002), Son and Crespo (2009) compared various preservice teachers, and noted that elementary preservice teachers' limited content knowledge influenced their abilities to interpret student responses. In particular, Son and Crespo (2009) presented preservice teachers with an artifact on which a student used a novel, non-traditional approach to solve a mathematical task that involved the division of fractions. As in the other studies examined, the researchers found preservice teachers' understanding of the content limited their abilities to determine whether or not the student approach was valid, generalizable, or efficient, and therefore limited subsequent abilities to infer student understanding and plan an instruction in response to their interpretation.

Other Personal Resources. In addition to the personal resources described above, other personal resources are described in the literature; however, much of this literature is theoretical in nature (e.g., Even, 2005; Even & Wallach, 2003; 2004; Walkerdine, 1988), or has not used empirical consideration of teachers' articulations *in the act of interpretation* to arrive at their claims about teachers' personal resources (e.g., Watson, 1999). Most studies are described further below; however, concerns arise about the specific claims that can be made from this literature based on the research methods used.

As an example, Even and Wallach (Even, 2005; Even & Wallach, 2003, Wallach & Even, 2005) have written numerous theoretical papers to illustrate the difference between a teacher *hearing through* his or her personal resources and a *deficient perspective* of his or her personal resources. To illustrate the difference, the researchers use selected descriptions of teachers from the literature to demonstrate instances of *hearing through*. These examples indicate that what a teacher hears from student responses can be influenced from: (a) their plans for the student outcomes from the lesson, (b) their conceptions about student development and common student errors related to a particular mathematical idea, and (c) their beliefs about the value of the verbal and written responses of students (Even, 2005; Even & Wallach, 2004). That said, no evidence indicates the researchers arrived at these personal resources from empirical methods.

In another example, Watson (1999) found when teachers were asked to describe how he or she comes to understand what one knows and can do in mathematics, teachers themselves allude to personal resources that influence their interpretation. In particular, Watson (1999) noted teacher interpretation are influenced by: (a) their beliefs about whether or not students can learn and develop, (b) their preferences for different approaches to the mathematics, (c) their beliefs about how mathematics should be communicated, and (d) their past experiences as a mathematics student. However, different from the research described above, Watson (1999) relies of teachers' self-reports of their assessment and interpretation practices to arrive at her claims, not observation of teachers in the act of interpretation.

Summary of literature on personal resources. The literature from studies in which teachers were observed in the act of interpretation point toward the influence of two aspects of personal resources that include: (a) teachers' relationships with and conceptions of the student and (b) teachers' conceptions of the mathematics content. The first of these involves his or her care and concern for the student (Wallach & Even, 2005), and his or her prior beliefs about what the student does and does not understand based on prior experiences with the student (Morgan & Watson, 2002). The second of these involves the influence of what the teacher understands about the mathematics content assessed (Heid et al., 1999; Morgan & Watson, 2002; Son & Crespo, 2009; Van Dooren et al., 2002; Wallach & Even, 2005).

In addition to these two aspects, theoretical work (Even, 2005; Even & Wallach, 2003, 2005) and a study with research design concerns (Watson, 1999) point toward other personal resources that are called on in the act of interpretation. For example, these include personal resources that teachers draw on, including (a) their experiences with and awareness of other children and their development and difficulties, and (b) their beliefs and preferences for how mathematical problems should be approached and communicated. However, additional research in which teachers are observed *in* the act of interpretation are needed to provide empirical evidence for these personal resources.

Products of the Act of Interpretation

The term *product* is used to describe outcomes of the act of interpretation. These outcomes can include both the teachers' inferences about the student and the teachers' instructional response planned in consideration of those inferences. As a result, studies in this section focus on the *nature* or *accurateness* of these products, and little on how the teacher arrived at those conclusions. Questions related to this aspect are often answered in consideration of teachers' written description of their inferences or instructional response, but are sometimes answered from observation of teachers in the act of interpretation, and the latter of these two methods can sometimes also reveal information about *processes* of interpretation. That said, studies with a central focus on the products of interpretation are described below, and are separated in three sections in consideration of the teachers studied.

Products of the act of interpretation: Preservice teachers. The products of preservice teachers' initial attempts to interpret artifacts are often centered on the correctness of student responses (Crespo, 2000; Otero, 2006). Hines and McMahon (2005) noted that preservice teachers who were asked to sort multiple student responses to proportional reasoning tasks according to their sophistication often associated the correctness of the response with sophistication, without consideration of the methods the student used to arrive at his or her response. Of those preservice teachers who did consider student methods, Hines and McMahon (2005) noted that mathematical sophistication was most often associated with student use of particular mathematical procedures, the use of multiple methods, and a clear description of those methods.

Preservice teacher attention to and favor toward particular mathematical procedures and the clear description of those methods is consistent with the results of other studies as well (Morris, 2006; Sleep & Boerst, 2012; Spitzer et al., 2011; Van Dooren et al., 2002). For example, in consideration of multiple student responses to problems that involved the comparison of two or more fractions, Spitzer et al. (2011) noted that preservice teachers preferred a student response in which the student "listed the steps taken to find a common denominator, converted two fractions into equivalent fractions with that denominator, and then compared them to obtain the correct answer" (Spitzer et al., 2011, p. 81). Further, as compared to other student responses, preservice teachers often considered this response as evidence of attainment of a *conceptual mathematical objective*.

In addition, Spitzer et al. (2011) found that before any intervention preservice teachers often attributed aspects of student responses that were unrelated to the mathematical objective as evidence of their attainment of that objective. For example, preservice teachers who were prompted to determine whether or not a student could use and understand a common denominator approach to compare fractions often considered student responses unrelated to the objective as evidence that the student understood the objective. In particular, preservice teachers often claimed the use of a pictorial representation, the use of a calculator to convert and compare decimals, and a reference to the fractional parts of dollars was sufficient evidence to make a claim about student abilities to use and understand a common denominator approach to compare fractions.

After an intervention that involved instruction in the methods course about the evidence needed to make a claim about student achievement of a mathematical objective, Spitzer et al. (2011) noted a significant increase in the number of preservice teachers who interpreted the three approaches described above as evidence the student *did not understand* the mathematical objective. That said, the researchers expressed further concern about this claim, as the three approaches neither confirm nor deny that the students met the objective. Instead, the preferred interpretation was that *no claim* could be made about student attainment of the objective; however, this response was not often observed. As a result, the researchers raised concerns about the potential for preservice teachers to claim that a student does or does not understand a mathematical objective without sufficient evidence to support either claim.

Other researchers (Sleep & Boerst, 2012) have further questioned preservice teacher abilities to determine what constitutes sufficient evidence for interpretation. In their research on preservice teachers who interviewed and interpreted student responses from those interviews, Sleep and Boerst (2012) found that preservice teacher inferences about students were supported with sufficient evidence less than one quarter of the time, and just about half of the evidence provided was consistent with what the child said or did. Of those inconsistencies between the preservice teachers' inferences and the evidence provided, preservice teachers often provided an assertion that was *too broad* for the evidence described. In other words, preservice teachers often made a claim that extended outside of "what [could] be inferred from the given evidence" (p. 1044).

Similar to the results of the studies described above, Sleep and Boerst (2012) found that assertions that were *too broad* were often the result of preservice teacher use of a procedural approach as evidence of a claim about conceptual understanding. For example, when a student added a zero to .8 to make .80, and then compared .54 and .80 with the justification "because 80 is better" (p. 1045), the preservice teacher called this evidence the student understood place value and the "magnitude of whole numbers and decimals to the tenths and hundredths" (p. 1045). In contrast, the researchers stated the response was no more than evidence the student could use a procedural approach to compare two decimals with a different number of digits, and posited one cannot make a claim about his or her "place value knowledge" (p. 1044). The researchers therefore question the soundness of the preservice teacher interpretation, and call the assertion *too broad*.

Son and Sinclair (Son, 2010; Son & Sinclair, 2010) conducted two studies in which preservice teachers' interpretation and instructional response were examined when student artifacts contained two or more errors, and similar to the studies previously described, the results revealed that preservice teachers often confused what it means to understand *conceptual* and *procedural* aspects of a mathematical idea. In particular, the researchers noted that half of preservice teachers presented with student responses that contained conceptual-based errors interpreted the error as the result of her *actions*, not her awareness of the *properties* of the mathematical ideas assessed (Son, 2010). For example, in consideration an artifact on which the student used the *difference* in the length and width of one rectangle to find a missing length in a second, similar rectangle, preservice teachers pointed to the selection of an incorrect procedure as the source of the student error. In contrast, the researchers identified the source of the error as a limited understanding of concept of *similarity*, which was believed to contribute to the procedural error.

In consideration of the nature of preservice teachers' instructional responses, both studies (Son, 2010; Son & Sinclair, 2010) identified three common instructional responses to student errors. In order from most to least common, these included: (a) a *broad, general* response, (b) a *return to the basics* response, and (c) a *reminder* response. A *broad, general* response involved an effort to reteach the concepts and procedures; however, the response was not specific to the student and his or her error. In contrast, a *reminder* involved an effort to remind the student of the correct concept or procedure, or of prior instruction. Last, a *return to the basics* response involved an effort to introduce an easier problem. Irrespective of the response, it was more common for preservice teachers to plan to *show and tell* in their response than to *give or ask* in their response, which was interpreted as evidence that preservice teachers "prefer to deliver information rather than listen to their students" (Son, 2010, p. 62).

To close the section on the products of preservice teachers interpretations, I describe the results of Schack et al. (2013) whose research provides one of the most comprehensive studies about the products of preservice teacher interpretations to date. These researchers studied 94 preservice teachers in mathematics methods courses at three different universities. In consideration of the *professional noticing* of the preservice teachers studied, the researchers considered preservice teacher abilities to *describe*, *interpret*, and *respond to* the mathematical thinking of this student response captured with video. In the video, the student response was provided to a problem in which he was asked to compare 11 shells with an uncovered group of seven bears to determine how many more shells there are than bears. In response, the student counted each of the seven bears and then raised one finger per count until he reached 11, and then provided four as his answer.

Shack et al. (2013) noted that prior to intervention in the methods course, 20 of 94 preservice teachers studied provided an inaccurate description of the student approach to the problem. The researchers identified three themes in inaccurate descriptions, which included: *operational presumptions*, *purporting evidence that did not occur*, and *cognitive interpretation*. In particular, *operational presumptions* involved a claim about the use of operations that that was unsupported, such as a claim that the student “subtracted 11-7” (p. 387). In contrast, *purporting evidence that did not occur* involved a claim about a student action that did not occur, such as a claim that the student “counted back from 11 to 7” (p. 347). Last, the theme *cognitive interpretation* involved a claim about the internal conceptions of the student, which is not the purpose of the act of description, but is the purpose of the act of interpretation. For example, a claim that the student "lacks a sense of cardinality" involves interpretation, and does not describe what the student did or said in response to the prompt.

Similar to the difficulties the preservice teachers faced in their effort to *describe* student responses, preservice teachers often demonstrated difficulties in their effort to *interpret* student responses. In particular, the researchers observed 21 of 94 preservice teachers who made an accurate interpretation of the student, prior to formal instruction related to interpretation. In fact, just over half of preservice teachers made an inaccurate interpretation. Inaccurate interpretations often contained claims about the student and what he understood about subtraction. For example, one preservice teacher stated,

> [The student] knew that since the teacher said he had too many shells he had to do subtraction. He also knew that because the teacher said leftover he had to do subtraction, or see what the difference was. The child understood key words and phrases and understood how to take away to get the right answer. He used the bigger number and took away using the smaller number and realized that 11-7=4. (Shack et al., 2013, p. 340)

However, such claims are not supported with evidence from the video, since the student counted from one to the lesser of the two numbers presented, and then used his fingers to keep track of the count from the lesser number to the greater number in order to determine *how many more*.

In consideration of preservice teacher abilities to use their interpretation to inform an instructional response, Schack et al. (2013) noted that only 13 of 94 preservice teachers described an appropriate instructional response that was based in evidence from the interpretation before related instruction in the teacher education course. For example, one preservice teacher stated she would support the student described above in the adoption of "other [methods] of getting [the] answer using subtraction" (p. 341). Her response, as an example, is based on an *operational presumption* of the best method to solve the problem (Schack et al., 2013). Further, it involved the instruction related to a different approach as opposed to effort to build on an understand the depth of what the student understands and is able to do with the approach he selected.

After instruction related to description, interpretation, and instructional response in the mathematics methods course, increases in preservice teachers' abilities related to all three components of assessment practice were observed, and these increases were consistent for preservice teachers at all three research sites, which included three different universities with three different methods instructors (Schack et al., 2013). The most substantial development was observed in preservice teachers' abilities to plan an instructional response; however, this was also the component of the act of interpretation on which preservice teachers scored lowest before instruction in the methods course. As a whole, these results indicate that preservice teachers can develop their abilities as it relates to these three components of assessment practice, at least for a narrow scope of mathematical ideas, in a reasonable period of time.

Products of the act of interpretation: Comparison of preservice and inservice teachers. One of the most robust sources of information about preservice and inservice teachers' products of interpretation is the research of Jacobs et al. (2010). Similar to Schack et al. (2013), Jacobs et al. (2010) measured the abilities of a teacher to describe, interpret, and respond to artifacts. The artifacts presented included student responses to two whole number tasks, one captured in video and the other in written form. In all, the researchers studied 36 preservice teachers and 95 inservice teachers. Of those inservice teachers, 31 had not attended professional development related to the act of interpretation, 31 attended related professional development for two years, and 33 attended related professional development for four years. All inservice teachers had at least four years of teaching experience.

In terms of teacher abilities to describe a student response to a problem, Jacobs et al. (2010) noted 42% of preservice teachers and 65% of inservice teachers with no relevant professional development experience provided sufficient detail of a student response. Teachers who did not meet the standard of sufficient detail provided general details about the student response or described his or her uncertainties about the student response. To demonstrate the difference between teacher responses that were considered detailed, and those that were not, the researchers provided two examples. The problem presented to the student whose artifact the teachers examined was: *Todd has 6 bags of M&M's. Each bag has 43 M&M's. How many M&M's does Todd have?* A teacher whose response was of sufficient detail stated,

> [She] made 6 circles with the number 43 in each one. Then she combined every 2 circles by adding the 10s together and then adding the 1s together for each pair. Next, she added the 10s (80+80) and the 1s (6+6) for the first 4 circles. After adding 160+12 to equal 172, she needed to add 86. Knowing that 80+20=100 (a familiar #), she took 20 from the 70 to get to 100. Then she figured she needed to add the 52 left from the 172. What she forgot about was the 6 left from the 86. That's why her answer is off by 6. (Jacobs et al., 2010, p. 183)

In contrast, a teacher who provided general details of the same student response stated, "[She] wrote 43 down 6 times, then added them together in groups of 2. Then added those answers together to come up with her final answer" (Jacobs et al., 2010, p. 183). Although the latter of these two responses is accurate, it is incomplete, and therefore lacks important details that should inform an interpretation of what the student does and does not understand.

In terms of teacher interpretation of what a student does and does not understand, Jacobs et al. (2010) found that while 47% of preservice teachers and 61% of inservice teachers without professional development provided a *limited* interpretation of students' mathematical understanding, no preservice teachers and just 16% of inservice teachers without professional development provided a *robust* interpretation. Teachers who provided a *robust* interpretation connected the details of what the student did to what he or she must then understand. For example, in response to a problem that asked students to determine the number of children in a class of 19 that ordered cold lunch, if seven of the children ordered hot lunch, a teacher with *robust* interpretation noted,

> The first pair understands the problem is a subtraction problem by writing a number sentence that showed 19-7=□. They did not need to count out 19 and take away 7 to get 12. They simply used their fingers to count backwards from 19. They seem to have good number sense. (Jacobs et al., 2010, p. 185)

In contrast, teachers who provided a *limited* interpretation made broad, but not specific claims about the students, and those claims often lacked close connection to what the child did. For example, one teacher with a *limited* interpretation noted, "These children understand subtraction and addition—and which to choose when presented with a problem. They know how to write a number sentence" (Jacobs et al., 2010, p. 186). As was observed in Sleep and Boerst (2012), *limited* interpretations often contain broad claims about what the student does and does not understand that are both inaccurate and unhelpful in the preparation of future instruction.

In terms of the teacher abilities to plan an instructional response, Jacobs et al. (2010) noted that 86% of preservice teachers and 74% of inservice teachers without related professional development planned an instructional response without consideration of their interpretation, or in consideration of an interpretation that was inaccurate. The researchers described these responses as *limited*. For example, when presented with the written artifacts of three different students, a preservice teacher with a *limited* response stated that she would increase the value of the numbers presented in a word problem for all three of the students, as if to indicate all three students had the same abilities and required the same response.

> I think I would give them the same problem using a 100 number, such as 6 bags of 154 M&M's. All of these kids know how to break numbers into 10's plus 1's. I think they are ready to go to the next step and look at 100's, 10's, plus 1's. (Jacobs et al., 2010, p. 189).

In contrast, teachers with a *robust* interpretation produced three different instructional responses that were specific to their descriptions and interpretation of student responses. For example, for the same three students one teacher noted that she would use the same problem, but select different values for each student. For one student, she stated she would select numbers that she believed would promote a more efficient approach, and for another student stated that she would select similar values to the first problem to see if her error in calculations would be consistent across similar problems (Jacobs et al., 2010).

Jacobs et al. (2010) observed that teachers' skills in describing and interpreting improved for teachers with experience in the classroom and with focused professional development support. In fact, significant differences were observed between the preservice teachers and inservice teachers with teaching experience, even if those teachers had not participated in related professional development. That said, significant differences were observed between inservice teachers with and without related professional development experience for the component skills of description and interpretation. In contrast, teachers' abilities to plan an instructional response did not increase with teaching experience alone, but instead required professional development support (Jacobs et al., 2010). This observation is consistent with other studies of inservice teachers (Heritage et al., 2009) that indicate effort to plan an instructional response is more difficult than to infer what the student does and does not understand.

Products of the act of interpretation: Inservice teachers. Similar to Jacobs et al. (2010), studies of teams of inservice teachers demonstrate preservice teacher difficulties in the effort to describe the details of student responses. In particular, Kazemi and Franke (2004) noted that when a team of ten inservice teachers was asked to "determine how their students had solved a problem" (p. 216) from the written work samples each had collected from their students, the teachers provided detail student approaches that resulted in correct, not incorrect responses. Kazemi and Franke posited this was the result of teachers that had not watched or heard students solve the tasks; however, these observations are consistent with prior research that has indicated even when teachers are provided opportunities to watch videos of students, the teachers "did not see the subtle distinctions between and among children's solution [methods]" (Whitenack, Knipping, Novinger, Coutts, & Standifer, 2000, p. 116) before professional development that aimed toward this end.

That said, the results of these studies demonstrate that, in time and with support, teachers can improve their abilities to describe the details of a student approach. As noted above, Jacobs et al. (2010) found that both classroom teaching experience and two or more years of professional development experiences related to student mathematical thinking were associated with significant gains in teachers' abilities to provide the details of the strategies student use. Others (Kazemi & Franke, 2004; Whitenack et al., 2000) have observed improvements in teachers' abilities to describe after participation in professional development programs of shorter duration, when the sole focus of the program was on teachers' abilities to interpret artifacts. These improvements were characterized as increased attention to student strategies, improved abilities to detail these strategies, and an increased appreciation for the complexities of strategies used (Kazemi & Franke, 2004).

Summary of literature on products of the act of interpretation. The research related to products of preservice teachers' interpretations paints a somewhat discouraging picture of their abilities. In observation, preservice teachers often focus on correctness of responses over student methods (Crespo, 2000; Hines & McMahon, 2005; Otero, 2006;), and when focused on methods often prefer traditional, procedural approaches (Hines & McMahon, 2005; Morris, 2006; Sleep & Boerst, 2012; Spitzer et al., 2011). Further, preservice teacher claims about students are not often supported with evidence from the student response (Sleep & Boerst, 2012), and instructional responses are not often specific to particular student difficulties (Schack et al., 2013; Son, 2010; Son & Sinclair, 2010). However, the research provides evidence that these abilities can develop in time and with support, at least for some aspects of this element of practice (Schack et al., 2013; Sleep & Boerst, 2012).

In comparison, research related to inservice teachers' products of the act of interpretation reveal some similarities in difficulties, but also reveals that inservice teachers' products of interpretation are more accurate than preservice teachers. Similar to preservice teachers, inservice teachers do not often see the subtleties in student responses (Whitenack et al., 2000), and omit important details in their effort to recount student responses (Jacobs et al., 2010; Kazemi & Franke, 2004). However, despite these difficulties, inservice teachers provided more detail and interpreted student responses, even without related professional development experience (Jacobs et al., 2010). Of the multiple components of the act of interpretation, effort to plan an instructional response appears to be the most difficult (Jacobs et al., 2010; Heritage et al., 2009), and does not improve with classroom experience alone (Jacobs et al., 2010).

Far more is understood about the products of preservice teachers' interpretations than the products of inservice teachers' interpretations. In fact, the review reveals that little research has been conducted on individual inservice teachers and the products of their interpretations; instead, much of what is understood about inservice teachers is from observation of teams of inservice teachers (Kazemi & Franke, 2004; Whitenack et al., 2000). Much of what can be learned about individual preservice teachers is the result of a few studies (e.g., Heritage, 2009; Jacobs et al., 2010). Of these studies, only one focuses on the nature of the products of the act of interpretation (Jacobs et al., 2010), whereas the other maintains a sole focus on the accurateness of the products of the act of interpretation, which are of limited usefulness to teacher educators.

Summary of the Reviewed Literature

In review, we find that the literature on how teachers reason about artifacts and the personal resources they draw on is limited, but also separate from the literature related to the products of the act of interpretation. That is, few studies cross the divide and provide information about processes and products of interpretation for the same teachers. In response, this research attempts to consider both the processes and the products of interpretation in an effort to provide a more *holistic account* of the act of interpretation for the teachers studied (Creswell, 2007; Hatch, 2002). Considered as a whole, it is posited that consideration of both processes and products will allow one to better describe those processes that are associated with accurate, useful products, and therefore provide further information about the means towards these ends.

From the literature, it becomes clear that preservice teachers are the focus of those studies that have started to blur the lines between attention to processes and products of interpretation (e.g., Son, 2010; Son & Sinclair, 2010), and these studies provide little useful information about productive processes that will point toward accurate products. Instead, these studies point toward a handful of difficulties in the act of interpretation that point toward inaccurate interpretations and unsupported instructional responses. For this reason, this research examines the processes and products of inservice teachers, and considers those teachers who are experienced and active in their professional development. This research was designed to gain insight into those teachers who could demonstrate improved abilities with this element of practice.

In addition, this research contributes to the limited research that details the nature of the products of inservice teachers' interpretation. Specifically, this research study allows us to better understand commonalities in the inferences and instructional responses of teachers who have experience and are active in their professional development, a population that was not considered in the literature reviewed. This particular population is considered important since these teachers are representative of a substantial portion of the population who have often not had professional development related to the act of interpretation, but still practiced over time the act of interpretation as a component of their practices in the classroom.

CHAPTER THREE: RESEARCH METHODS

The purpose of this research study was to understand the interpretations and instructional responses that experienced and active teachers construct as a result of their consideration of student artifacts, and the personal resources these teachers used to construct those interpretations and instructional responses. In this chapter, I describe and defend an approach and methods that are consistent with the research purpose described above, and that addressed the research questions. These research questions are:

1. *Process*. What personal resources and prior conceptions do experienced, professionally active first and second grade teachers' use to construct interpretations of students' mathematical understanding from artifacts of student thinking?
2. *Product*. What interpretations of students' mathematical understanding do experienced, professionally active first and second grade teachers construct from artifacts of student thinking?
3. *Product*. What do experienced, professionally active first and second grade teachers plan in response to interpretations of students' mathematical understanding?

The chapter is presented in five sections, which include a discussion of: (a) selection of the research approach, (b) participants, (c) data collection, (d) data analysis, and (e) trustworthiness.

Selection of the Research Approach

Merriam (2009) described *qualitative research* as a term that labels all research interested in "how meaning is constructed" (p. 24), or the *process* of knowledge construction, and "how people make sense of their lives and their worlds" (p. 24), or the *product* of knowledge construction. The definition alludes to two important elements of qualitative research. The first is related to epistemological assumptions embedded within the use of the term *constructed*, which will be discussed below. The second is related to a focus on the *process* and *product* of knowledge construction (Bogdan & Biklen, 2007) in order to provide a holistic account of the studied (Creswell, 2007; Hatch, 2002). Consistent with these definitions, this research aimed to achieve a holistic account of both a process and product of construction, and therefore justifies a qualitative approach.

The adoption of a qualitative approach demands that the researcher consider what sort of qualitative approach aligns with the research purpose and questions (Creswell, 2007; Merriam, 2009). Particular approaches to qualitative research (e.g., narrative, phenomenological) add one or more dimensions to the broad definition of qualitative research. The added dimension or dimensions make the approach distinct from other approaches, although each are still situated under the umbrella of qualitative research (Merriam, 2009). However, Merriam noted that it is common to see qualitative studies in education that do not add a dimension that situates them within a particular approach, but instead adopt a *basic, interpretive* approach to qualitative research.

In consideration of this observation, Merriam (2009) labeled those qualitative studies to which additional dimensions are not added as basic, interpretative qualitative studies. Similar to the purpose of this research, the purposes of these studies are to "*understand* how people make sense of their lives and their experiences" (p. 23). These studies often share five characteristics, which Merriam (2009) suggested should inform researchers that adopt this approach.

- Focus on meaning, understanding, process
- Purposeful sample
- Data collection via interviews, observations, documents
- Data analysis is inductive and comparative
- Findings are richly descriptive and presented as themes/categories (Merriam, 2009, p. 38)

The first of these five characteristics is consistent with the purpose of this research. That is, the research questions adopted focus on the participants' *processes* of interpretation and their understanding of students based on those interpretations. For this reason, the first characteristic is consistent with my research study. The final four characteristics, also consistent with this research, are expanded upon in the description of the research methods below.

However, before I proceed to the description of the research methods, I believe it is important to note the use of a interpretative qualitative approach does not mean the research is atheoretical, or without epistemological foundation, as others have indicated is the case for *generic qualitative research* (Caelli, Ray, & Mill, 2003). Instead, Merriam (2009) indicated constructionism provides the foundation for the research she labels as interpretative qualitative studies, and argued the assumptions embedded in this paradigm, in conjunction with the adopted theoretical or conceptual frameworks, inform the design of the research as it relates to the five characteristics described above.

Merriam (2009) defines the term *constructionism* with reference to Crotty (1998). In his most complete definition of the term, Crotty (1998) describes constructionism as:

> … the view that all knowledge, and therefore all meaningful reality as such, is contingent upon human practices, being constructed in and out of interaction between human beings and their world, and developed and transmitted within an essentially a social context. (Crotty, 1998, p. 42)

In definition, constructionism appears similar to the term *social constructivism*, as was presented in the first chapter. In fact, scholars note the terms *constructionism*, *social constructionism*, *constructivism*, and *social constructivism* are often interchanged (Gergen & Gergen, 2009; Lincoln & Guba, 2000). Even Crotty (1998) who is critical of the interchangeable use of these terms refers to constructivism as a form of constructionism. As such, the adopted perspective of social constructivism described in the first chapter is considered consistent with the tenants of basic, interpretative qualitative studies, and will therefore inform all aspects of research design.

To provide a review from the first chapter, the ontological and epistemological assumptions embedded within the adopted perspective assume knowledge construction occurs in individuals and groups of individuals (Ernest, 1994a). Relativist in nature, the perspective rejects an absolute truth, and seeks to understand the multiple realities of individuals. Even without a single, absolute truth, knowledge can develop or accumulate over time as individuals and groups of individuals construct and negotiate "more informed and sophisticated reconstructions" (Lincoln & Guba, 2000, p. 166) in response to new experiences and prior conceptions.

Applied to this research study, I aimed to construct models that described participant processes and products of construction from their interaction with artifacts. The models I constructed are, in fact, representations of the models the participants constructed to describe students. I have, as constructivist researchers must (Constantino, 2008), used interviews and observations of participants in interviews to construct these models. The methods used to collect these data and draw interpretations from these data are described below.

Participants

The term *purposeful sampling* (Creswell, 2007; Merriam, 2009; Patton, 2002) refers to the planned, intentional selection of research participants to inform the research questions. *Criterion sampling*, one of multiple purposeful sampling strategies, refers to the selection of participants that meet specific criteria based on the research purpose and questions (Miles & Huberman, 1994). The selection of participants based on specific criteria is the approach that was used in this research. The two criteria selected established that the teacher must: (a) have five or more years of teaching experience with first or second grade children, and (b) be professionally active, as identified by an administrator or instructional coach.

The decision to select teachers with five or more years of teaching experience was made in consideration of research specific to teachers' interpretation of assessment data, that indicates that four or more years of classroom teaching experience contributes to statistically better interpretation abilities than are observed in preservice teachers without teaching experience (Jacobs et al., 2010). This particular amount of time is also supported in results of research outside of teacher assessment practices, which demonstrate that, on average, five to seven years of experience is required to achieve "high levels of skill" (Berliner, 2004, p. 14) as a teacher. That said, both research and common sense support the notion that experiences must be relevant in order to contribute to abilities (Berliner, 1986). For that reason, the criterion is written to focus on those teachers whose experiences are with students of the same age as those whose artifacts are presented.

The final criterion, related to the identification of teachers who are professionally active, is added in response to research specific to teacher interpretation of assessment data (Jacobs et al., 2010) and general consensus (Berliner, 1986) that experience alone is not a sufficient predictor of teachers' abilities. Instead, Berliner (2004) notes that, in order to improve their abilities, teachers must "work hard at it" (p. 14). For that reason, the final criterion was added to select those teachers who, as it appears to those who observe them, put effort toward their professional growth. The phrase *professionally active* is used to describe teachers that:

> … pursue opportunities for growth outside of the requirement of his or her contract. These opportunities might be formal, such as involvement in: professional development inside or outside of the district, building- or district-level committees, or professional organizations. These opportunities might also be less formal, such as regular engagement with professional literature.

This definition was intended to help administrators and instructional coaches select teachers who, based on their overall sense of the teacher, were committed to their professional growth.

A total of nine teachers from two different school districts participated in the research. A description of: (a) these school districts, (b) the procedures for the identification of potential participants in these districts, and (c) the selected participants are provided below.

District information. Two different school districts provided approval to solicit participation from teachers in their districts. Both school districts are located in the Midwestern United States. District A served students in a metropolitan area and a suburb of that area, and District B served students in a suburb of the same metropolitan area. Both District A and District B were state accredited at the time of the research was conducted.

At the time of data collection, District A served about 14,000 students from kindergarten to twelfth grade, of which 73.7% of students were White, 16.6% were Black, and 5.1% were Hispanic. In District A, 68.5% of students were eligible for Free or Reduced-Price Lunch. Teachers in District A had an average of 11.5 years of experience, and 70.7% held an advanced degree.

In contrast, at the time of data collection, District B served about 11,000 students from kindergarten to twelfth grade, of which 83.9% of students were White and 5.7% were Black. In District B, 21.1% of students were eligible for Free or Reduced-Price Lunch. Teachers in District B had an average of 11.7 years of experience, and 73.2% held an advanced degree.

Identification of participants. The identification of potential participants was achieved with similar procedures in both districts. In District A, the district-level coordinator of instructional coaches identified teachers who met the criteria described above, and in District B, the district-level coordinator of assessment and evaluation and principals identified teachers that met the criteria described previously. A total of 25 potential participants were identified in District A and 35 in District B. Each potential participant was contacted in an email that described the project. This email was followed with a more complete description of the project that was delivered to the school. A total of nine participants responded and later consented to participate in the project.

Participant information. Of the nine participants, three were from District A and six were from District B. All participants were female. At the time of data collection, five teachers were placed in first grade, and four teachers were placed in second grade. All nine participants held an advanced degree.

The participants had between seven and 27 years of experience, with an average of 16 years of experience. As was established in the selection criteria, all teachers had taught first or second grade for five or more years. Three teachers had more than 10 years of experience in first grade, one more than 20 years of experience in first grade, and one more than 10 years of experience in second grade. For more detailed information about participant experience, see Table 3.1.

Table 3.1

Years of experience per teacher participant

Participant	Current Grade	Years of Experience		
		First	Second	Total
AM	1	5	0	12
BA	2	0	5	7
BC	2	2	7	10
IA	2	0	5	11
KP	1	13	0	13
LG	1	11	0	15
LP	2	0	10	27
RR	1	23	0	25
SB	1	11	0	24
Average	-	7.2	3.0	16.0

Data Collection

Two interviews were used to collect data. These included: (a) a semi-structured interview about the participant and his or her professional experiences, conceptions of assessment, and assessment practices (see Appendix A); and (b) a task-based interview with tasks centered on student work samples and the interpretation of student thinking based on those samples (see Appendix B). Table 3.2 shows the alignment between and research questions and interview questions (Anfara, Brown, & Mangione, 2002). A description of both interviews and the data collected from each follow.

Table 3.2

Alignment of research question and interview questions

Research Questions	**Interview Questions**[1]
RQ1. *Process*. What personal resources and prior conceptions do experienced, professionally active first and second grade teachers' use to construct interpretations of students' mathematical understanding from artifacts of student thinking?	SS1; SS2; SS3; SS4; TB1; TB2.3[2]
RQ2. *Product*. What interpretations of students' mathematical understanding do experienced, professionally active first and second grade teachers construct from artifacts of student thinking?	TB2; TB2.1; TB2.2
RQ3. *Product*. What do experienced, professionally active first and second grade teachers plan in response to interpretations of students' mathematical understanding?	TB2.4; TB2.5

[1] Interview questions from semi-structured interview labeled SS and task-based interview labeled TB.
[2] All interview questions have the potential to reveal aspects of personal resources; however, those interview questions that were written to elicit personal resources are listed in the table.

Semi-Structured Interview. The term *semi-structured interview* is a label that describes interviews that are designed in consideration of particular research purposes and questions, but allow the researcher choice in the administration of the interview. This choice could be related to "how and in what sequence questions are asked, and in whether and how particular areas might be followed up and developed with different interviewees" (Mason, 2004, p. 1021). Because of this choice, the semi-structured interview is more flexible than a structured interview, but more structured than an open interview.

Design of the semi-structured interview. A semi-structured interview protocol (see Appendix A) was used to collect data from each participant in an initial interview that occurred in one session that lasted 60 minutes, with about 45 minutes allocated to the interview itself and about 15 minutes allocated to assembling and disassembling materials. The initial interview questions determined the prior professional experiences of the participant. Subsequent questions addressed his or her conceptions and practices related to assessment in the mathematics classroom, with specific questions about artifact collection and interpretation. The final questions assessed teacher comfort and confidence as related to the act of interpretation.

Data collected from the semi-structured interview. Two digital recorders were used to audio-record the interview with participant permission. In addition, the researcher recorded brief notes during the interview on the interview protocol, if needed. These notes most often focused on aspects of the interview that would have been missed if audio had been the sole source of data collection (e.g., actions, reference to objects). I expanded on these brief notes after the session, if needed.

Handwritten researcher notes were scanned to a digital form. All data were saved in both its original and digital form. Digital data was saved on two hard drives, as recommended for purposes of data preservation (Corti, 2008).

Task-Based Interview. Different from the interview described above, the *task-based interview*, a type of clinical interview, involved the participant in interaction with tasks that the researcher preselected in consideration of the research purpose (Goldin, 2000). Common in mathematics education research, the purpose of the task-based interview is often to understand participant "processes of addressing mathematical tasks, rather than just on the patterns of correct and incorrect answers in the result they produce" (Goldin, 2000, p. 520). To understand these processes, researchers observe participant behaviors and their descriptions of their thought processes in order to draw conclusions about his or her "mathematical thinking, learning, and/or problem solving" (Goldin, 2000, p. 520).

Much the same as mathematical tasks have been applied to mathematics education research, pedagogical tasks were applied to this mathematics *teacher education* research. In specific, the tasks administered included the presentation of an artifact of student thinking and a request for the teacher to interpret students' mathematical thinking from what was demonstrated on the artifact. From observation of the teacher as he or she considered the artifact, the researcher considered the processes through which experienced teachers constructed interpretations of students' mathematical thinking from written and video-recorded artifacts, and the resultant products of that interpretation.

Design of the task-based interview. The task-based interview occurred in one session with each session lasting about 90 minutes, with 75 minutes allocated for the interview itself and 15 minutes allocated for assembling and disassembling materials.

The task-based interview started with instructions that described the nature of the responses desired of participants (see Appendix C). These instructions were modified from protocols used in research that aimed to elicit participant thoughts while solving mathematical tasks (e.g., Ericsson & Simon, 1993; Koichu & Harel, 2007), and aimed to promote the "concurrent verbalization of heeded thoughts… rather than a retrospective report" (Ericsson & Simon, 1993, p. 373). However, requests for retrospect responses also occurred in the effort to best understand how the participant arrived at an interpretation and instructional response, and the nature of that interpretation and instructional response.

Three different types of tasks were presented to participants including: (a) mathematical tasks, (b) interpretation of student response tasks, and (c) sort of student response tasks (Nickerson & Masarik, 2010). These three types of tasks are described below. The specific prompts and interview questions associated with each of these three types of tasks can be found in Appendix B.

Mathematical tasks. Before I introduced a student artifact, I presented the mathematical task to which the student responded. The presentation of the mathematical task allowed the participant to become familiar with the mathematical task before he or she was asked to reason about a student response to that task. After the mathematical task was presented, I asked the teacher to think aloud as she reasoned about the task. If not articulated in the think aloud, the participants were prompted to consider: (a) the solution to the task, (b) possible student responses, (c) possible student challenges, and (d) the purpose of the task.

Interpretation of student response tasks. Student responses to mathematical tasks were presented to the participant in video form that was sometimes included written work. All teachers viewed each video artifact at least once, and were allowed to re-watch the video as many times as desired. In consideration of the artifact, the participants were asked to consider the student response as though he or she were the teacher of the student, and think aloud as he or she reasoned about the student response. If not articulated in the think aloud, I prompted the participant to consider her interpretation of the student and her thinking or understanding. Specific follow-up questions prompted the participant to consider: (a) strengths and weaknesses in the student response, (b) specific claims that could be made about the student, and (b) a planned response to the student.

Sort of student response tasks. For some of the student artifacts presented, the participants were asked to sort two or more student responses according to which she would be more or less concerned about if she were the students' teacher. The teacher was prompted to think aloud as she completed this task, and to explain his or her decisions after the task was complete. If not articulated in the think aloud, the participant was prompted to consider: (a) what criteria were used to sort students, (b) how one or more student responses compare to that criterion, and (c) how she selected the criterion.

Selection of student artifacts for the task-based interview. Student artifacts are the product of student responses to instructional and assessment tasks related to various number and operations constructs (Lannin & van Garderen, 2009). The student responses were collected as data for the purposes of a separate research project[1] that aimed to understand the development of student understanding related to various number and operations constructs for students in first and second grades. In particular, the research pursued two goals:

1. To identify similarities and differences among learning trajectories in number and operation for struggling and typically developing learners.
2. To determine the cognitive barriers that impede learning and what instructional tasks facilitate learning to overcome the obstacles.

To achieve these goals, data were collected from both struggling and typically developing learners, and data from both groups of learners are utilized in the research.

Multiple factors guided the selection of artifacts from the data set, including: (a) the number and operations construct assessed, (b) the correctness of the student response, and (c) the nature of student approach or justification. For the purpose of artifact selection, student approaches and justifications were examined in consideration of the extent to which each was valid, generalizable, and efficient. Taken together, these dimensions provided a framework to assist in the intentional selection of artifacts that would draw out different aspects of teachers' processes and products of interpretation. The three broad components of the framework are described below.

The number and operations construct(s) assessed. All artifacts selected involved students in an assessment or instructional task that addressed mathematical ideas related to place value. That is, tasks were related to the properties and structure of our base-ten

[1] *A Study of Struggling Learner's Knowledge and Development for Number and Operations* (NSF Award No. 0918060).

number system, and focused on “two-digit number[s] to represent amounts of tens and ones” (National Governors Association Center for Best Practices, Council of Chief State School Officers [CCSSO], 2010, p. 15). This focus, which is narrower than the multiple tasks related to number and operation available for selection, was the result of effort to select a mathematical focus that was common and relevant to both first and second grades (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), and therefore familiar to teachers with experience in those grades.

Correctness of the student response. As it is used in the framework, the term *student response* refers to the answer the student provides to the prompt. Student responses can either be correct or incorrect, although incorrect responses can be incorrect for various reasons (e.g., student didn’t respond to the actual prompt, student didn’t understand the mathematical content). Effort was made to select artifacts that differ on this dimension, in part, because of research that indicates preservice teachers’ often attend more to the correctness of the response than other elements of the artifact (e.g., Crespo, 2000; Otero, 2006), such as the approach or justification.

The nature of the student approach or justification. The term *approach* refers to how the student formulated his or her response to the task, which can be observed from his or her behaviors on video as he or she reasons about the task, or inferred based on his or her written products. In contrast, the term *justification* refers to how the student defended or explained his or her approach and response, either in spoken or written words. Taken together, there is consensus that both the student approach and justification are more helpful in the act of interpretation than the correctness of student responses (Sowder, 2007). Therefore, effort was made to select artifacts that differed in approach or justification. Further, the inclusion of this dimension is reflective of research that indicates preservice (e.g., Schack et al., 2013) and inservice (e.g., Jacobs et al., 2010; Whitenack et al., 2000) teachers do not often see the subtleties in student approaches and justifications, despite their usefulness in the act of interpretation.

In the selection of artifacts, I considered three dimensions on which student approaches or justifications can differ. These dimensions examine the extent to which student replies are: *valid*, *efficient*, and *generalizable*. These three dimensions arose in the work of Cambell, Rowan, and Suarez (1998), who used each to inform interpretations of students' nontraditional, invented algorithms. In definition, *valid* refers to whether or not the student ideas have a sound basis in accepted mathematical ideas. The term *efficient* refers to the extent to which the student approach of justification takes a reasonable amount of time to execute, and is often related to the relative sophistication of the approach. The term *generalizable* refers to the extent to which the approach or justification "can be applied to the full range of problems of the type being solved" (Cambell et al., 1998, p. 53).

Two approaches to task design. In consideration of the three different task types (i.e., mathematical, interpretation, and sort) and the three different dimensions on which artifacts should be considered (i.e., construct assessed, response correctness, and nature of approach and justification), I present in this section two different approaches to task design, and the tasks included in the task-based interview that are associated with each. The first approach involves the design of tasks that focus on an individual student and his or her understanding, and the second approach involves the design of tasks that focus on multiple students and their understanding.

Interpretation of individual student responses to two or more tasks. Two tasks were included in the task-based interview that featured one student and his or her response to two different sets of tasks, both of which assessed mathematical ideas related to place value. The artifacts presented were similar in the *construct* or mathematical ideas assessed, but differed in the *correctness of responses* and *nature of the student approach or justification* (see Figure 3.1 for a sample).

Task 3.1	
Interviewer:	Make 56 with cubes.
Student:	[Student touches five ten sticks, one touch per count, and then pull six ones. On table: ||||| ......]
Interviewer:	Using what you have out, make 66.
Student:	[Student touches each of five tens.] I need another one of these [points to ten-sticks]. [On table: |||||| ......]
Interviewer:	Using what you have out, make 46.
Student:	[Student places left hand on four ten-sticks, and removes two ten-sticks with right hand. On table: |||| ....]
Task 3.2	
Interviewer:	[Represents 24 with Unifix cubes || ....] How many cubes do I have?
Student:	24, see 10, 20, 4. 24.
Interviewer:	[Represents 34 with Unifix cubes || .... |] How many cubes do I have?
Student:	24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34. [Counts each block in last tens stick]
Interviewer:	[Represents 54 with Unifix cubes || .... |||] How many cubes do I have?
Student:	[On fingers] 24, 25, 26, 27... 54. 54. [Counts each block in last three tens sticks.]

Figure 3.1. Transcript of student artifacts presented for interpretation of an individual student response to two or more tasks, from Task 3.1 and 3.2.

These tasks used the prompts and interview questions associated with *Mathematical Tasks* and *Interpretation of Student Response Tasks* (see Appendix B). That is, the teachers were asked to consider the mathematical task, and interpret and respond to the student artifact. For details of the two tasks on the task-based interview that used this approach, and the student artifacts associated with them, see Appendix D and Appendix E.

Interpretation and sort of multiple student responses to the same task. One task included in the interview involved five student responses to the same contextualized place value problem. Therefore, the artifacts presented were identical in the *construct* or mathematical ideas assessed, but differed on at least one of the other two dimensions. In particular, the set of five student responses included: (a) three *responses* that were correct and two responses that were incorrect, and (b) five *approaches* and *justifications* that were different, but sets of two or more students that contained similarities. For example, two student artifacts were presented in which the student used a drawing to represent the problem, but responses differed in correctness and their drawings differed (see Figure 3.2 for a description).

Interviewer Prompt
Josh has 4 bags with 10 toys in each bag. He also has 7 extra toys. How many toys does Josh have?

Task 2.4
Student: So, he has 4 bags.
[Student draws four boxes in one row with the number 10 written in each. Student draws fifth box in second row with the number 7 written in it. Student writes, "He has _ toys." Student fills in blank with 47.]

Task 2.5
[After interviewer reads, student writes. On paper: 10+7=17]

Student: I did this in first grade the tens plus a number would equal something. A teen number. So, 17.
[Student is asked to solve with another method. Student draws 4 circles. Student puts 10 dots in each circle. Student draws 7 balloon-like objects. Student writes 10, 20, 30, 40, 41, 42... 47 above objects on drawing. Student is asked which answer is correct. Student points to 17.]

Figure 3.2. Transcripts of two student artifacts presented for interpretation and sort of multiple student responses to the same task, from Task 2.4 and 2.5.

These tasks used the prompts and interview questions associated with *Mathematical Tasks*, *Interpretation of Student Response Task*, and *Sort of Student Responses Task* (see Appendix B). That is, the teachers were asked to consider the mathematical task, interpret and respond to each of the student artifacts, and sort the artifacts in order of the students he or she would be least to most concerned about. For details of the task on the task-based interview that used this approach, and the student artifacts associated with it, see Appendix F.

Data Collected from Task-Based Interviews. One audio recorder and one video recorder were used to record the interview with participant permission. I focused the video recorder on the immediate space and objects around the participant (e.g., table, written student products, manipulatives). In addition, I took brief written notes during the task-based interview and expanded on those notes after the interview session when needed. These notes focused on those aspects of the interview that could be missed if video had been the sole source of data collection (Lesh & Lehrer, 2000). If the participant wrote on the artifacts or on paper made available to him or her, those documents were also collected and scanned.

Notes taken in the session were handwritten and scanned, and the expanded notes were produced with a word processor. All data was saved in both its original and digital form. Digital data was saved on two hard drives, as recommended for purposes of data preservation (Corti, 2008).

Data Analysis

Inductive and comparative methods were used to analyze these data (Boeije, 2002; Corbin & Strauss, 2008; Merriam, 2009). These methods are consistent with the recommendations of Merriam (2009) for basic, interpretative qualitative studies.

In this section, I describe the sequence of events for the analysis of the data based on these methods, with reference to the specific analytic procedures used. The sequence of events was organized into three parts, which included: (a) the preparation of the data, (b) the coding of the data, and (c) the interpretation of the data. It should be noted that although the process appears linear with isolated steps that lead to completion, overlap occurred in the steps and elements of the process were recursive. I have tried to make these occurrences explicit in the description below.

Level one: Preparing the data.

Transcription of the data. Each audio- or video-record was transcribed verbatim, and references to participant actions were made based on researcher discretion (Clement, 2000). To encompass both of these elements, transcripts include the words of the researcher and the participant in the form of a script. In addition, notes of what could be viewed, but not heard in brackets were included within the script.

A professional transcription service was used to transcribe the audio of most, but not all interviews. If the service was used, I reviewed each transcript produced to check for potential inaccuracies. Individuals who worked for the transcription service were provided with an audio-, not video-recorded version of each interview. For that reason, I added notes about participant actions to the transcripts as I watched the video-record of the interview.

Effort was made at this step to include on the transcript important details that could be observed, but not what could be inferred (Goldin, 2000). Therefore, reference to inferences about participant "thinking, reasoning, cognitive processes, internal representations, meanings, knowledge structures, schemata, affective or emotional states" (Goldin, 2000, p. 527) were not included in the transcript. However, researcher inferences were recorded in *marginal remarks* and *researcher memos* (Miles & Huberman, 1994) that are each described further below.

Development of marginal remarks and researcher memos. After each interview was transcribed, I read each transcript multiple times in order to understand and reflect on what could be learned from the data in consideration of research questions. As I read the transcripts, I added *marginal remarks* (Miles & Huberman, 1994) to the transcript. Marginal remarks are described as those reflections, observations, and inferences the researcher adds to the margins of the transcript, which serve to document the thought processes of the researcher at a point in time, and could serve to prompt a *researcher memo* (Miles & Huberman, 1994).

Researcher memos were created throughout the entire research process. The purposes of these memos were to document researcher reflections about "… what has been learned, the insights [the transcript] provides, and the leads these suggest about future action" (Ely, 1991, p. 80). Leads included the modification of the order of interview questions and researcher interview behaviors (Bogdan & Biklen, 2007), and the creation of labels used to code the data (Miles, Huberman, & Saldaña, 2014). Most researcher memos were created with a word processor; however, handwritten notes that documented researcher reflections in the research process were scanned and added to the document with researcher memos.

Level two: Coding the data.

Development and refinement of the code list. To begin the coding process, I applied three different *tags* to the data that reflected the research questions. These tags labeled sections of the data related to: the personal resources a teacher used as he or she reasoned about the artifact [**PR**], the teacher's interpretations of the student [**INT]**, and the teacher's planned instructional response [**RES**]. Data not associated with one of these three tags were marked as other [**O**].

To start the process of code creation, I read the data associated with each of the tags. As I read the data, I created marginal remarks and researcher memos to develop an *initial list of codes* for the data associated with each tag. As the initial list of codes was applied to data, additions to, deletions from, and refinements to the code list were made to improve the fit of the codes with the data (Miles et al., 2014).

To assist in the development of the code list, a process of *within-code comparison* was used to compare all instances in the transcripts where the same code was applied to determine "different properties and dimensions of the code" (Corbin & Strauss, 2008, p. 74). This process sometimes resulted in more robust definitions of codes and sometimes revealed the need for a new code to fit subtle differences in the data retrieved. For instance, the initial code list contained a code related to the claims a teacher made about a student that were related to mathematics [**CM**]; however, consideration of the data associated with the code revealed important differences within these claims. That is, some involved interpretations or claims about *what occurred* **[CEV]** and some involved interpretations or claims about *student mathematical understanding* [**CM**]. As such, two codes were created to reflect these differences and the data was recoded.

In addition, a process of *across-code comparison* was used when I believed two or more codes were similar or overlapped. In this case, data associated with two or more similar codes were retrieved and the text compared to determine that differences across codes are “bold and clear” (Patton, 2002, p. 465). When this process revealed overlap in two or more codes, the codes were collapsed. As such, this process of comparison was often the means in which the code list was condensed. For instance, the initial code list contained a code to reflect instances when the teachers provided a statement about how the student thought about the problem or what he or she did in his or her mind [**CMET**]; however, it became clear that these data often overlapped with teacher interpretations of *what occurred* and *student mathematical understanding*, and were recoded to reflect the more appropriate of these two codes.

Characteristics of the code list. The final code list was comprised of three categories of codes that were associated with each of the research questions and the original tags [**PR/INT/RES**]. Each of these three categories of codes contained multiple levels of codes. To provide an example, the final code list related to teachers' interpretation of the student [**INT**] is described in this section.

In the end, all data related to teachers' interpretations [**INT**] were coded with one of five first-level codes. These included interpretations or claims about *confidence* [**CCON**], *speed* [**CS**], *listening and attention* [**CL**], and *mathematics*. Interpretations about mathematics were assigned one of two different first-level codes, one related to *what occurred* in the video as it related to mathematics [**CEV**] and the other related to *student mathematical understanding* [**CM**]. When it was clear that the teacher interpretation of the student was positive or negative, a ‘+’ or ‘–’ was also applied to each of these first-level codes. When it was unclear whether the interpretation of the student was positive or negative, a ‘+/–’ was applied.

The data associated with each of the first-level codes related to mathematics [**CEV/CM**] were also coded with a subcode that described the mathematics or mathematical idea referenced. These subcodes included more than 25 subcodes that were referred to as the *mathematics subcodes*, and were applied to all data in which mathematics or mathematical ideas were discussed. For example, one mathematical subcode was related to students' abilities to determine that a ten-stick is the same as 10 blocks without a count [**15_10**], and this subcode could have been applied to all data that referenced this mathematical idea. For instance, if a teacher stated that the student understood that a ten-stick is 10 blocks without a count, it would have been applied to data tagged as interpretation **[INT**]. That is, this data would have been tagged [**INT**] and coded [**CM**: **15_10+**]

In contrast, if a teacher intended to tell the child that a ten-stick was the same as 10 blocks as a part of her instructional response, the subcode would have been applied to data tagged as instructional response [**RES**]. The statement would have also been coded with a first-level code from the code list related to teachers' planned instructional responses. In this case, the first-level code would have been related to the teacher's decision to tell or transfer information from the teacher to the student **[TELL**]. That is, this data would have been tagged [**RES**] and coded [**TELL**: **15_10**]. The use of the same mathematics subcodes was essential to the effort to transform the codes to assertions, which will be discussed in the next subsection, **Level three: Interpret the coded data.**

Reliability of the coded data. A second coder was involved in coding data collected from three participants. I selected these three participants in consideration of two factors. First, I selected participant responses that I found both more and less difficult to code. Second, I selected participant responses that I perceived to be different from one another in their personal resources, interpretations, and planned instructional responses. In the end, the second coder and I coded data from at least one participant together, and she coded data collected from two participants in isolation. The details of the process are described below.

To start the process, the second coder and I used the tentative code list that I had created to code data collected from at least one participant. We approached the data collected from this participant in multiple phases, to allow the participant time to become familiar with small pieces of the code list in each phase. In this phase, we coded together, reading aloud and discussing the data and codes. As we coded, we also considered the appropriateness of the code list. These discussions resulted in modifications to the code list to improve the fit of the codes with the data. At that time, definitions of multiple codes were clarified, and codes were collapsed to eliminate overlap. For example, two codes related to *student mathematical understanding* [**CM**] that attempted to differentiate between claims made that were and were not situated in or specific to the mathematical task were condensed because differences in the two codes were determined to be unclear.

The second coder coded at least 20 pages of transcribed data from two different participants in isolation. She approached these data in multiple, small sections, and agreement was calculated and discussions occurred after each section was coded. This process allowed us to monitor agreement throughout the process, discuss trends in disagreement, and consider whether or not those disagreements warranted modification to the definition of codes or the code list. In the end, 83.16% agreement was reached overall, and all disagreements were resolved through discussion. I then applied the modified code list to the remaining data.

Level three: Interpreting the coded data.

The research results are communicated in **Chapter Four** through nine claims that respond to the research questions. These claims are assertions about the products of teacher interaction with the student artifacts, and the personal resources applied to arrive at those products (Miles et al., 2014). As such, these claims describe the third-order models that I constructed to represent teachers' second-order models and their construction of those models (see Figure 1.2). Two means were used to arrive at these assertions. These included: (a) the use of patterns across codes to develop claims, and (b) the use of matrices to develop claims.

Use of patterns across codes to develop Claims. For some claims, multiple first-level codes were combined to arrive at the claim (Anfara, Brown, & Mangoine, 2002; Miles et al., 2014). For example, first-level codes related to teachers' interpretations about *confidence* [**CCON**], *speed* [**CS**], *listening and attention* [**CL**] were combined to arrive at the claim that teachers' interpretations involved more than attention to mathematics. As such, **Claim Five**: **Teachers constructed interpretations without a mathematical focus** was developed. In another example, multiple first-level codes related to the student approaches and responses that teachers anticipated and their ideas about of those approaches and responses were used to arrive at **Claim One: Teachers applied their conceptions of levels of student performance to the act of interpretation**. However, matrices similar to those described below helped determine commonalities in these levels across teachers for each student artifact, and deepened what was understood about the claim. These matrices are described in more detail below.

Use of matrices to develop claims. Matrices were used to arrive at or to support almost all of the claims presented (Miles et al., 2014; Patton, 2002). These data presentations (i.e., retrieved codes associated with particular tags, retrieved data associated with particular codes) revealed patterns in and variation across participants that informed these claims. For example, matrices were created to arrive at **Claim Seven: Teachers constructed interpretations that were varied for the same student artifact**. To arrive at this claim, two different matrices were used to compare: (a) teachers' interpretations of *what occurred* [**CEV**] for the same student artifact and (b) teachers' interpretations of *student mathematical understanding* [**CM**] for the same student artifact.

Figure 3.3 provides a part of a matrix I created to compare teachers' interpretations of *student mathematical understanding* [**CM**] for Task 2.2.

Teacher	**Interpretation [INT]** [**CM:** mathematics subcode]
CA	**CM:** 14+ **CM:** 16+ **CM:** 4EXP+ **CM:** 40MULT+
CC	**CM:** 16+ **CM:** 4EXP- **CM:** 40MULT+
JA	**CM:** 14+ **CM:** 16+ **CM:** 4EXP- **CM:** 40MULT+ **CM:** 15TENS+

Figure 3.3. Matrix created to compare teachers' interpretations of *what the student understood about mathematics* from Task 2.2.

This portion of the matrix helped me see that all three of these teachers' interpretations involved an observation that the student was able to solve the problem in his or her head [**CM: 16+**] and that she understood multiplication or an aspect of multiplication [**CM: 40MULT+**]. In addition, two teachers made a general, positive claim about the students' understanding of mathematics [**CM: 14+**]. Further, one teacher noted that the student was able to explain his or her approach [**CM: 4EXP+**] and two stated the student was not able to explain his or her approach [**CM: 4EXP-**].

Matrices similar to the one featured in Figure 3.3 were developed from the state in which it is displayed in order to better understand the data. For example, the data associated with each of these mathematics subcodes was pulled to consider subtle similarities and differences between data associated with the same code. In addition, these matrices were later developed to include: (a) all interpretations in the interpretations column [**CCON/CS/CL/CM/CEV]** and related subcodes, and (b) a third column that included teachers planned instructional responses [**RES**] and related first-level codes and subcodes. These additions allowed me to better understand the relationship between a teacher's interpretations and her planned instructional response, and allowed me to arrive at **Claim Eight: Teachers planned instructional responses that were related to their interpretation**.

Refinement of claims through negative case analysis. After claims were determined, I compared them to the raw data in search of instances that disconfirmed the claims. This process served as the final moment of comparison (Boeije, 2002). Similar to *negative case analysis* (Lincoln & Guba, 1985), the approach improved the fit of the claims with the data, and served to produce interpretations that were credible. Inconsistencies observed in the data for particular teachers or student artifacts were noted in the description of each claim. That is, each claim characterized what is representative of *most* teachers, and exceptions observed for particular teachers are described in the presentation of evidence to support the claim (see **Chapter Four**).

Trustworthiness

Multiple strategies were used to establish trustworthiness in the research. Each of these strategies aimed to address the criteria Lincoln and Guba (1985) indicate are needed to establish trustworthiness in qualitative research. These strategies are described below.

Thick description. The use of rich, thick description was utilized to allow the reader to determine whether or not the research is transferable to his or her situation (Lincoln & Guba, 1985). Ponterotto (2006) suggested these descriptions be present in multiple sections of the written product of the research, including the methods (i.e., participants, procedures), results, and discussion. In the presentation of the results, the researcher used direct quotations from participants as evidence to deepen descriptions (Merriam, 2009).

Peer debriefing. In an effort to promote the credible interpretation of the data, the researcher debriefed with a peer about “substantive, methodological, legal, ethical, or other relevant matters” (Lincoln & Guba, 1985). The peer selected was a doctoral student in science education at the time that the research was conducted. This doctoral student had prior qualitative research experiences related to various areas of education, including teacher preparation and development. Sessions were scheduled with this individual once every two weeks, and additional sessions were scheduled as needed. As Lincoln and

Guba (1985) recommend, the peer was a person outside the doctoral committee, but a person who had relevant experiences and expertise. The researcher produced written memos of these sessions, which were embedded in the reflexive journal.

Reflexive journal. Lincoln and Guba (1985) indicate a reflexive journal can serve to establish all four criteria for trustworthiness in qualitative research. For that reason, the researcher maintained a reflexive journal. The journal contains researcher reflections, and included information about "schedule and logistics, insights, and reasons for methodological decisions" (Erlandson, Harris, Skipper, & Allen, 1993, p. 143). The journal also serves as a component of an audit trail, together with other materials from the research (e.g., raw data, data reduction products), should an audit be conducted (Lincoln & Guba, 1985).

Review of the Chapter

In the chapter, I present a detailed description of the research design and methods that were used to address the research questions. To review, the research utilized an interpretative qualitative approach. Two interviews were used to collect data from participants. These included: (a) a semi-structured interview about the participant and his or her professional experiences, conceptions of assessment, and assessment practices; and (b) a task-based interview with tasks centered on student work samples and the interpretation of student thinking based on those samples. Inductive and comparative methods were used to analyze data collected, and results are presented in the form of claims.

CHAPTER FOUR: RESULTS

In this chapter, I present the results of the research in three sections, each of which is related to a research question. These research questions are:

1. *Process*. What personal resources and prior conceptions do experienced, professionally active first and second grade teachers use to construct interpretations of students' mathematical understanding from artifacts of student thinking?
2. *Product*. What interpretations of students' mathematical understanding do experienced, professionally active first and second grade teachers construct from artifacts of student thinking?
3. *Product*. What do experienced, professionally active first and second grade teachers plan in response to interpretations of students' mathematical understanding?

In each section, claims related to the research questions are described. To conclude the chapter, the results are summarized.

Research Question One: Personal Resources

In this section, I describe four claims related to the first research question. These include:

Claim One: Teachers applied their conceptions of levels of student performance to the act of interpretation.

Claim Two: Teachers applied their expectations for student performance to the act of interpretation.

Claim Three: Teachers applied their awareness of common student difficulties to the act of interpretation.

Claim Four: Teachers drew on their current awareness and understanding of mathematical ideas in the act of interpretation.

Evidence to support each these are provided in the sections that correspond with each claim below.

Claim One: Teachers applied their conceptions of levels of student performance to the act of interpretation. Before student artifacts were shared, teachers were prompted to describe possible student approaches and responses to the mathematical tasks that were represented in the student artifacts. These prompts revealed teacher awareness of multiple student approaches and responses for most mathematical tasks. In addition, these prompts revealed that teachers often associated the approaches and responses that he or she anticipated with various *levels of student performance*. That is, teachers often ordered the multiple student approaches and responses that were anticipated from least to most sophisticated. I refer to these ordered approaches and responses as the teachers' *level scheme*. Teachers often referenced their level schemes later in the act of interpretation, and as such demonstrated the influence of the level scheme on the act of interpretation.

The approaches and responses that teachers anticipated for the mathematical task associated with Task 1.1 provides an example of a level scheme. In this mathematical task, a student was asked to determine whether one ten-stick and four blocks [| ....] was the same as 14 blocks. In consideration of the mathematical task, 8 of 9 teachers anticipated two to three different correct student approaches that could be used to determine that one ten-stick and four blocks was the same as fourteen blocks, and teachers described these approaches as representative of different levels of student performance. For example, Ms. Pope noted three levels,

> Well, beginning students are going to *count each one*, and they might even have to take it apart and actually see it spread and count it, and the more advanced students are going to *know that's a 10 and say 10 and count on*, and then the further along are just going to be able to *picture it and not have to count*. (Task-Based Interview, Task 1.1)

All teachers who articulated a level scheme for this particular mathematical task included in their level scheme at least two of the three levels Ms. Pope described. That is, teachers anticipated that students would *count all*, *count on from ten*, or state *'fourteen blocks' but not count*. Similar to Ms. Pope, other teachers compared these approaches in terms of their relative sophistication.

It was evident that teachers drew on this aspect of their personal resources in the act of interpretation, since level schemes were often referenced in their interpretations. In these instances, teachers compared student performance to their level scheme. For example, in response to the student artifact associated with Task 1.1, Ms. Pope stated,

> Well, she knew it right away. She didn't have to *count it out*, I noticed that... I mean, she just seems to have a good grasp of place value. She knew right away. There wasn't a lot of thought. She just was able to *answer right away that this makes 14* [motions to one ten-stick and four ones]. (Task-Based Interview, Task 1.1)

In her interpretation, Ms. Pope noted an aspect of student performance that she did not observe; that is, she stated the student did not *count*. While it is unclear whether she intended to communicate that the student did not *count all*, *count on*, or both, she articulates that the student did not demonstrate the least advanced of the levels that she described and attained the most advanced of the levels she described. As demonstrated in her response, reference to levels that were not observed in the student artifact occurred across all teachers who articulated a level scheme for this particular mathematical task, and was common practice across other mathematical tasks for which a level scheme was articulated.

Teachers sometimes included in their level scheme incorrect approaches or responses that preceded a correct response. For the mathematical task described above, two teachers anticipated an error that involved the student identification of the ten-stick and the set of four blocks as *two* sets of objects, not fourteen blocks (see Table 4.1). For these teachers, the scope of levels of student performance included all student approaches and responses the teachers anticipated as probable, both correct and incorrect.

Table 4.1

Anticipated incorrect student response for the mathematical task associated with Task 1.1

Teacher	**Response**
Ms. Allen	I also have a student that would probably say that was *two because he sees two different items* [touches ten-stick and then touches ones]. (Task-Based Interview, Task 1.1)
Ms. Carson	I could see them counting so they would consider this [touches a ten-stick] one block. So, they would be counting each individual one. A student that didn't get it right, I would see them like doing one and then two, so *two blocks since there's two separate ones*, would be another possibility, too. (Task-Based Interview, Task 1.1)

Teacher awareness of incorrect approaches and responses is further discussed in **Claim Three**.

Level schemes sometimes differed across teachers for the same mathematical task. The mathematical task associated with Task 3.1 provides an example of such differences. In a portion of this mathematical task, the student was prompted to represent 56 with blocks, and then add to the 56 blocks to represent 66 blocks. Table 4.2 describes the level schemes of those teachers who articulated one for this portion of this particular mathematical task.

Table 4.2

Level scheme for the mathematical task associated with Task 3.1

Approach [from least to most sophisticated]	**Teacher**					
	BA	**BC**	**IA**	**KS**	**LG**	**LP**
Recount all						
Recount all [e.g., 1, 2, 3... 64, 65, 66]					•	
Recount all [e.g., 10, 20, 30... 60... 64, 65, 66]	•					•
Count on/back [e.g., 57, 58, ... 66]	•	•	•	•	•	
Add/remove ten-stick(s) without count	•	•	•	•	•	•

Notice that all teachers represented in Table 4.2 described a level that involved the addition of a ten-stick without a count, and all teachers indicated this was their preferred approach. That said, consensus on the student approach or response that teachers preferred was not observed for all mathematical tasks, and is further discussed in **Claim Two**. Further, notice that the teachers represented in Table 4.2 were inconsistent in their description of those approaches that were less preferred, despite a pattern across teachers that includes three approaches that develop from least to most sophisticated.

Teachers drew on their level schemes in the act of interpretation, whether or not their levels were consistent. For example, Ms. Spain applied her level scheme (see column KS in Table 4.2) to her consideration of a student artifact that corresponded to the mathematical task in Task 3.1. In the student artifact, the student appeared to *recount* the five tens from the 56 blocks before he added a ten-stick.

> Especially when he made sixty-six, he had to *go back and count by tens* [touches table six times], he wasn't able just to *add the ten*, but that's just kind of checking his work too, or maybe remembering what it was. I don't know. (Task-Based Interview, Task 3.1)

In her statement, she noticed that the student did not demonstrate the more advanced of the two levels she described, but that he *recounted all*, a level she did not include in her level scheme before she observed the student. However, she compared the student response to the levels she referenced when she described her plans for an instructional response,

> I would probably, just probably throw out and have him do it one more time with different numbers to kind of see if he had to always *go back and count ten, twenty, thirty, forty* [touches table four times, one count per touch], or if he was able to *kind of just know fifty-six or count up ten more…* (Task-Based Interview, Task 3.1)

As such, Ms. Spain planned to administer additional mathematical tasks to determine which level the student would demonstrate. In particular, she wanted to understand if he could demonstrate one of the two levels she referenced when she first considered the mathematical task.

In their semi-structured interviews, 6 of 9 teachers alluded to their awareness of various levels of student performance and its usefulness in the consideration of student artifacts. For example, when asked to describe what she considers when she looks at student artifacts, Ms. Baehr noted,

> I'm looking to see if they have the correct answer, of course. And, if they don't have the correct answer, or if they do have the correct answer, I usually look to see how they arrived at it. We teach using a number line. We've taught using fingers, we've talked about touch point math, we've talked about cubes out, and then just plain old have it memorized. So, I will look at their paper while they're taking the test and I will realize and make mental notes or paper notes who uses their fingers, who uses manipulatives, who has it already memorized, who draws dots on their paper—all the different forms to write that correct answer. That shows you what level they're at. (Semi-Structured Interview, Task 3.6)

Whereas Ms. Baehr did not compare the relative sophistication of the approaches she described, she indicates that some approaches are more sophisticated than others. Much the same, Ms. Graham noted in response to a similar prompt,

> Well, some of them have the facts memorized, some of them count on, some of them use their fingers, some of them need to draw a picture, some of them need cubes or some form of a manipulative. It's because they're all at different stages... Just to see how they arrived at their final response, was it a real primitive, you know they needed the concrete, or are they, have they moved to abstract. (Semi-Structured Interview, Task 3.6)

Notice that Ms. Graham indicates that the levels she described differ in sophistication, at least in part, based in whether or not the approach involves a concrete model. Whereas these examples differ in content and the levels described due to the open nature of the semi-structured interview, both provide additional support for the observation that teachers approach at least some student artifacts mindful of levels of student performance.

In all, the use of a level scheme was observed on 3 of the 5 mathematical tasks presented. A level scheme was not evident for the mathematical task associated with Task 1.2 or the contextual situation associated with Task 2. In response to Task 2, teachers anticipated multiple approaches and responses for the contextual situation, but it was often unclear whether or not those approaches and responses were perceived as indicative of different levels of student performance. In contrast, a level scheme was not observed for the mathematical task associated with Task 1.2 because no teacher anticipated more than one approach or response.

The reason that multiple approaches or responses were not anticipated for the mathematical task associated with Task 1.2 is unclear. Table 4.3 provides examples of the approaches and responses that three teachers anticipated for this particular mathematical task, each of which is representative of approaches that other teachers shared.

Table 4.3

Examples of anticipated student approaches and responses for the mathematical task associated with Task 1.2

Teacher	Response
Ms. Pope	I think they might be able to show you, you know come up and show you what 14 tens would look like... We do a lot of drawing of tens and ones, too, so they could maybe draw it and show the difference, show what 14 tens would look like versus 14 ones. (Task-Based Interview, Task 1.2)
Ms. Spain	They could just say that there's only one ten-stick [touches ten stick]. They could just look at it and say it. (Task-Based Interview, Task 1.2)
Ms. Graham	Well, ... to me, they would just say this is one 10 [lifts ten-stick]. Just be able to recognize this is one 10. That it's one set of 10. (Task-Based Interview, Task 1.2)

For this particular task, all teachers provided one anticipated correct response and no anticipated incorrect responses. The absence of teachers' anticipated incorrect responses are discussed in **Claim Three**.

Claim Two: Teachers applied their expectations for student performance to the act of interpretation. A related aspect of teachers' personal resources that were called on in the act of interpretation involved their ideas about the student approaches or responses that were *preferred* from or *expected* of students at a particular point in time. In some instances these approaches and responses were shared as the most sophisticated of the levels the teachers described, but at other times were shared separate from a level scheme. In either form, teacher *expectations for student performance* sometimes differed across teachers for the same mathematical task, but informed their interpretation and planned instructional response nonetheless.

The responses teachers provided to the mathematical task associated with Task 3.2 is an example of the differences observed in teacher expectations for the same mathematical task. In consideration of this mathematical task, most teachers described a level scheme; however, the level described as most preferred was inconsistent across teachers. For instance, when the student was prompted to name the total number of blocks after two ten-sticks were added to the right of two ten-sticks, four ones, and one ten-stick [||.... | → ||.... |||], Ms. Baehr described three levels,

> A high student would look at this and say, '10, 20, 30, 40, 50' in their head for 54. The middle student would put all your ten-sticks together and separate your ones, 10, 20, 30, 40, 50, 1, 2, 3, 4. The low student would probably still have to count

> them one by one. But, you wouldn't have very many of those left, maybe one or two. (Task-Based Interview, Task 3.2)

Notice in her response that Ms. Baehr expects that advanced students would *recount all* in order to provide a response. This expectation is one that others shared. In particular, four levels that involved a recount reoccurred across teachers. These included: (a) recount all in ones (1, 2, 3.... 54); (b) recount all in tens and then ones (10, 20, 30, 40, 50... 54); (c) recount all in tens, ones, and then tens (10, 20, 21, 22, 23, 24, 34, 44, 54); and (d) the movement of the tens to left of the ones before a recount occurs.

In contrast, other teachers stated that students should not need to recount all of the blocks after a ten-stick or ten-sticks were added, and did not include this approach as the most advanced in their level scheme. Instead, these teachers preferred that the student *count on* from what was on the table. For example, Ms. Carson noted,

> Hopefully they would say 34, and then 44, 54... Being able to remember what number they were on before, so how much they had before and then being able to add those tens on to it, I think would be the most difficult task, them not going all the way back to the beginning every time and being able to build upon where they were. (Task-Based Interview, Task 3.2)

Similar to Ms. Carson, multiple teachers described a preferred approach in which the student *counted on* in tens. As such, their preferred approach was different, and more sophisticated, than the preferred approach Ms. Baehr and others described.

The student artifact that was associated with Task 3.2 included a student approach in which the student counted on by ones, not tens. In their description of the student approach, teachers' interpretation of what the student did in response to the mathematical task did not often differ for this particular mathematical task, but their statements about what the student did not do reflected their preferred approach. For example, Ms. Spain, a teacher who preferred that students *recount*, stated,

> I would probably be a little concerned about, especially from a second grade standpoint, I think I might be a little concerned about his approach to that. To me, it showed me *he didn't really have that 10, 20…* [touches table once per count], being able to solve things quickly. (Task-Based Interview, Task 3.2)

In contrast, Ms. Pope, a teacher who preferred that students *count on* in tens, stated,

> He had to count on and I thought he would be able to just add on, I thought he had it. I would say that he is able to, when given a number, he is able to count it out, but *he is not able to… count on in the tens*, he has to actually count it out [points to each individual cube in a ten-stick], he's not to the point where he can just visually see that you change the number in the tens. (Task-Based Interview, Task 3.2)

In both responses, we see that the teachers' preferred approach seemed to influence their interpretations. That is, what teachers stated the student did not do was related to how she preferred the student approach the mathematical task.

Teacher expectations for student performance were evident in their interpretations even when a clear level scheme was not articulated, and when the student approaches and responses observed were outside of their articulated level scheme. For example, a level scheme was not revealed for Task 2, but teacher expectations were revealed in the act of interpretation, and those expectations influenced the act of interpretation. The mathematical task administered to students in Task 2 involved a contextual problem in which the student was prompted to determine the total number of objects one would have if he or she had four groups of ten objects and seven additional objects. As a part of her approach, Beth, a second grader in the middle of second grade, wrote on her paper: *4x10=40*, *7-0=7*, and *47*.

In consideration of this student response, most teachers shared a sense that the first part of Beth's response [*4x10=40*] demonstrated a mathematical idea that is outside of what is expected of a second grade student. For example, Ms. Afton noted that multiplication was not addressed in the second grade curriculum,

> I think she was a little more advanced because I've taught third grade, so I know where they're coming from, and starting out with a foundation of no multiplication. And I've also taught second grade enough to know now that we teach multiplication, but we don't call it multiplication. So, we do all these little fabulous circles and tally marks, then we put things in groups. But they're not actually able to write out that four times ten equals 40, and do that all in their head. Like most of my kids, when they leave second grade, they're still drawing circles and tally marks. So, if I have a student that can do that [points to *4x10=40*], they're typically a more advanced student. (Task-Based Interview, Task 2.2)

Similar to the comments above, Ms. Allen noted,

> She has the concept of multiplication [points to *4x10=40*] and not repeated addition or skip counting. She just knew it was multiplication... You know, I would say that she definitely is very proficient, and for a second grader very advanced in her math facts. In knowing multiplication, and in understanding very

> quickly and very easily all mentally what it was asking, which is what I would expect of a much older student, to be able to do that. So, she clearly is very number smart, I mean she really, really got it... (Task-Based Interview, Task 2.2)

Both teachers seem to draw on a sense of what a student should be able to understand about or do related to a mathematical idea at a particular point in time, and both teachers compared the student response to that expectation.

In contrast from the teachers described above, some teachers were not certain of their expectations for student performance for this particular mathematical task, or other mathematical tasks that involved students who were not in the grade in which they had the most experience. For instance, Ms. Manfield, a teacher who had not taught second grade, seemed hesitant about whether or not Beth's use of multiplication was common. She noted, "Well, she understands multiplication, and I don't know that in the middle of second grade that is terribly normal" (Task-Based Interview, Task 2.2). In consideration of a different student artifact for the same contextual situation, Ms. Manfield articulated uncertainty about appropriate expectations for second grade students, "I don't know exactly what's going on in second grade at this time of year..." (Task-Based Interview, Task 2.2).

Much the same as Ms. Manfield, Ms. Spain, a teacher who had not taught second grade, noted after she considered all student responses to the same contextual word problem,

> ... it's hard for me to know what second graders are expected to know. I'm not exactly sure what, as first graders... with the last little girl, I might have been a little more concerned but she had the basis there. So, on one hand, I wouldn't be

> concerned because I would think, okay, she's really getting it, but from a second grade standpoint, I would see that that might be concerning because she did have to draw ten dots in each bag and she did have to put the 10, 20, 30, 40, and count on to 41, 42... I don't know if she would have to do that because she was just showing us that or if that's how she had to solve it. That would concern me a little bit if she had to actually go step by step by step to get to the number forty-seven, as a second grade teacher, I think that would concern me. But, as a first grade teacher, I would say 'wow.' She is, especially mid-year, I'd think, okay, she's got the concept down we're right where we should be and we're doing okay. It will come, is kind of how I would look at that. (Task-Based Interview, Task 2.5)

In her statement, Ms. Spain indicated that she has a sense of expectations for student performance for first grade students, but not for second grade students. Further, she pointed out that this aspect of her personal resources limits her interpretation.

The notion that teachers draw on a sense of where a student should be at a particular point in time was also supported in the semi-structured interviews; however, the extent to which teachers were explicit about the use of this personal resource differed across teachers. For example, when asked to describe what she considers when she looks at student artifacts, Ms. Baehr noted,

> What I'm looking for to get a 3 would be... 4 plus 3 equals 7 [motions 4 + 3 = 7]. That's what I'm looking for as the completed problem... So, I'm looking at what strategies they're using to do that. And later as the year goes on, like in third and fourth quarter, if they're still just drawing pictures instead of writing an equation, then I know that they're starting to struggle. In fourth quarter, definitely they

should all write an equation without drawing pictures and things. (Semi-Structured Interview, Task 3.6)

Notice that Ms. Baehr described a particular example from her classroom, and in the example provides additional evidence for the claim that teachers are mindful of where a student should be at a particular point in time in their consideration of student artifacts. In contrast, other teachers did not provide a specific example, but described their overall sense of where a student should be as a source of support for their interpretations. For example, when asked what was simple for her as it relates to the interpretation of student artifacts, Ms. Afton noted,

> And it also helps that I've been teaching second grade consistently. So, I know exactly what I'm supposed to be teaching and then looking for, and I've done it so many times now that I am really, fully understanding... So, I'm pretty confident that I understand what the kids are doing, and where they're coming from. And it helps that I'm really good friends with the first grade teachers and coming down from third grade, I know where they need to go. (Semi-Structured Interview, Task 3.9.1)

In her statement, Ms. Afton seems to describe her awareness of what a student should understand about mathematics in second grade, but also a broader awareness of the transition from first through third grade.

Claim Three: Teachers applied their awareness of common student difficulties to the act of interpretation. Multiple aspects of teachers' *awareness of common student difficulties* influenced the act of interpretation. Teachers often communicated this awareness in their discussion of: (a) correct student approaches that were less preferred, and (b) incorrect student approaches and responses. The former involved references to common student difficulties that teachers' associated with a lower level approach but a correct response, and the latter involved references to common student difficulties that teachers' associated with an incorrect approach or response. Both means of communicating these ideas are presented in this section to describe this aspect of teachers' personal resources, and the connection between them and the act of interpretation.

Common student difficulties associated with correct, but less preferred approaches. Teachers described correct, but less preferred approaches in their discussion of their level schemes (see **Claim One**). In their discussion of these lower levels, teachers often noted that common student difficulties contributed to or were associated with lower level approaches, and this aspect of their discussion revealed their awareness of common student difficulties associated with a particular mathematical task. Teachers' descriptions of these difficulties within their discussions of correct, but less preferred approaches are the focus of this section.

For example, recall that most teachers shared a common level scheme for the mathematical task associated with Task 3.1 (see Table 4.2) that prompted students to build 56 and then add and remove tens to build other quantities (e.g., 66, 46). In their articulation of this level scheme, most teachers also articulated an awareness that the subtraction component of the mathematical task would be more difficult for children than the addition component, and as such teachers anticipated that a lower level approach would be more often observed for the subtraction component of the problem than for the addition component. For example, Ms. Afton noted,

> That [the subtraction] is going to be, I think, the struggle for kids. They would know hopefully to *take two [tens] away*, but I think subtraction is more difficult for them. I think you're going to start getting kids doing a lot more *counting* and taking off individual cubes. (Task-Based Interview, Task 3.1)

Ms. Carson shared this perspective,

> I can see some kids being really puzzled and not knowing how to do that. And I could see some counting back – actually I could see a majority of my kids knowing what to do, but *counting back by ones*. So, 66, then 65, 64, all the way back to 46. And they might get off a little bit... But, I would want them just to be able to know to *take away two tens*... Just for my students every year, it seems like they can add, but... it always seems like subtraction is so much harder than addition every year. (Task-Based Interview, Task .31)

As such, both teachers maintained that common student difficulties around subtraction would influence the level of student performance. That is, the teachers indicated that students would be more apt to *count back* (see Table 4.2) for the aspect of the mathematical task that involved subtraction than he or she would *count on* for an aspect of the mathematical task that involved addition.

For this mathematical task and others, teachers applied their ideas about common student difficulties to the act of interpretation. For example, in consideration of a student artifact in which the student responded to the mathematical task in Task 3.1 with the removal of a ten-stick *without a count*, Ms. Carson noted,

> Then, I thought it was interesting when he subtracted, he easily took those two away, so quickly, and I thought that was the hardest part... I think that's I good. I

> was impressed that he was able to do that so quickly. (Task-Based Interview, Task 3.1)

Notice that Ms. Carson seems to have arrived at a positive interpretation of the student, in part, because she considered the student approach to be representative of an advanced level of performance for a component of the mathematical tasks that she considered difficult for students.

Common student difficulties associated with incorrect responses. In addition to communicating their ideas about common student difficulties in their discussion of correct, but less preferred approaches, teachers also shared their awareness of common student difficulties in their discussion of incorrect approaches and responses. As noted above (see Table 4.1), incorrect student approaches and responses were sometimes presented as a component of a level scheme. However, incorrect responses were also presented as separate from a level scheme when one was not described for a particular mathematical task, and some incorrect approaches and responses were not anticipated, but were referred to as common after the teachers considered a student artifact.

For example, various incorrect approaches were anticipated for the contextual situation in Task 2. Recall that this mathematical task prompted students to determine the total number of objects in four groups of 10 objects and seven extra objects. For this mathematical task, all but two teachers anticipated an incorrect approach that involved the addition of the three numbers presented in the contextual situation, and three teachers anticipated an incorrect approach that involved the addition of two numbers presented in the contextual situation. Ms. Manfield predicted both,

> So, I can see some of the lower kids, I can see them adding four plus ten here [touches four and ten in text of word problem], and then not knowing what to do with the seven, or going ahead and adding the seven in [touches four, ten, and seven in text of the problem]... (Task-Based Interview, Task 2)

The reoccurrence of these two incorrect approaches across teachers indicated that teachers shared some awareness that these incorrect approaches were common. In fact, others described the *common* nature of such approaches (see Table 4.4).

Table 4.4

Common student difficulties with the mathematical task associated with Task 2

Teacher	**Response**
Ms. Afton	I think they would add the 4 plus the 10 plus the 7. Like they don't take the time to understand and to really draw it out – like he has 4 actual bags and each one has 10. So, it's not just 4. I don't think they get it. Like they don't visualize it and I don't know if that's because they're still learning to read, or it's a higher level skill and they're not there yet, or if it's genetic – like I really don't know, but I don't think they interpret the problem the right way. (Task-Based Interview, Task 2)
Ms. Allen	Well, my speedy kids are just going to add it together, they'll come up with 21, you know that's the beginning strategy that we teach them in word problems, to look at the numbers and look for those subtraction words, or how many would be an addition word, and so lots of kids don't pay attention to the actual concept they just do numbers, so I wouldn't be surprised at all if they answer 21. (Task-Based Interview, Task 2)
Ms. Spain	The thing that would be challenging, I think, is to understand that the four bags had ten toys in each bag, so that they had to count by tens. That they wouldn't throw that four in there. Sometimes they just see numbers; they see the four, they see the ten, and they see the seven. They don't really read and understand that this [points to the ten in the text of problem] is telling us how many are in each bag. (Task-Based Interview, Task 2)

In contrast from the mathematical task above, other mathematical tasks produced few anticipated incorrect approaches and responses, and one produced no anticipated incorrect approaches or responses. For this particular mathematical task, some teachers noted that the student artifact demonstrated aspects of common student difficulties associated with the mathematical task after it was observed, but none anticipated or described them before it was observed. Of those teachers who noted the common student difficulties after consideration of the student artifact, one noted the incorrect approach was common and the other noted the incorrect response was common. That said, notable differences were observed in the nature of these two teachers' responses, which are described further below.

To provide context, the student artifact that teachers observed involved a mathematical task in which the student was asked to determine whether a ten-stick and four ones [l ....] were the same as fourteen ten-sticks. In response, the student pulled out four ten-sticks, and as each was pulled said, "10, 20, 30, 40." She then described the number of blocks on the table, but did not respond to the prompt about the number of ten-sticks. In response, Ms. Pope noted,

> Yeah, she really doesn't have a good foundation in place value, because she can't, and I see this a lot, the kids can't, [she can't] go back and forth from seeing this as a 10 or counting it 1, 2, 3, they have to count it as a 10 and they don't see that you can switch your counting depending on what the problem is. (Task-Based Interview, Task 1.2)

As a part of her instructional response, Ms. Pope planned to demonstrate for the student the difference between the two units she described in her interpretation.

> I think I would have tried to teach her. I would say that is awesome that you know that this is a 10 and you can count by tens, but I asked you to do something

different this time. I asked you to count the number of ten-sticks that you had out there. And I think I would, you know, grab some and start practicing, this is 1 ten-stick, 2 ten-sticks, and your friend thinks that you had 14 ten-sticks... I mean, hopefully, we would get to that point sooner or later for her to be able to verbalize, 'Oh, I see what I did wrong now.' (Task-Based Interview, Task 1.2)

As such, Ms. Pope noted the approach was both incorrect and common, and planned an instructional response that addresses the incorrect approach.

Ms. Afton seemed to recall that student difficulties with the mathematical idea related to the mathematical task were common for her students also. She stated,

> I totally forgot about that, but when we did teach them, it was hard for them to conceptualize that like 14 of these [touches ten stick] would be more than like the number 14, like you would have over 100. So, I remember the first lesson we taught, we all kind of panicked. Then as we retaught it and retaught it and got more and more manipulatives, the kids started to catch on. But right away, they were all doing the same thing, and even my lower students would still—I don't know what it is. It's like a block in their brain, like they're not developmentally ready for that yet, like it doesn't make sense to them to get that – that 14, they'd have to have 14 of these [touches ten-stick], but that would not mean the number 14, that would mean something different. So, I even had students where I'm like, 'No, you have to count like 10.' So, we would get our 14 out, and then we would have to count 10, 20, 30, 40, and some of them are still only able to count the 10, 20, 30, 40 and not conceptualize that this right here, right away I know if there's

14 it makes that number. So, yeah, flashbacks of first quarter. (Task-Based Interview, Task 1.2)

However, her response differed from that of Ms. Pope because Ms. Afton did not communicate a problem with the approach the student used, despite the fact that she claimed to understand that difficulties related to this mathematical idea are common. In fact, Ms. Afton described an instructional approach in which she modeled for her students the same count that the student used in the student artifact—a count that is not consistent with the unit identified in the mathematical task. As such, the response Ms. Afton provided seemed to demonstrate her developing ideas related to this particular mathematical idea, which will be discussed in **Claim Five**.

An implicit connection existed between teacher awareness of common student difficulties observed in incorrect approaches and responses and the act of interpretation. This connection is implicit because, unlike instances when teachers used their levels to support the act of interpretation, teachers did not often refer to their anticipated incorrect response when it was noted in a student artifact, and never stated that their awareness of common student difficulties helped them to see such difficulties in a response or approach. However, a notable pattern arose across the data that shows that these teachers never failed to mention the presence of a student approach or response that reflected the common student difficulties they anticipated before consideration of the artifact, but these teachers often failed to mention aspects of approaches and responses that were representative of common student difficulties described in the literature when they had not anticipated or described them as common. As such, some support exists for the conclusion that this aspect of teachers' personal resources informed and sometimes limited the act of interpretation.

The notion that teachers' awareness of common student difficulties informed the act of interpretation was also supported in statements collected from 5 of 9 teachers in the semi-structured interview. While teachers were not asked whether or not their awareness of common student difficulties was useful in the act of interpretation, some teachers described their awareness as useful to the act of interpretation. In particular, Ms. Carson noted,

> And over the years, not that all your kids are the same, but you kind of know what to look for as you're teaching certain things. Like, I know this is where kids can get caught up with this skill. I know this is where kids can get caught up with that skill. So, I think that kind of helps me too, to kind of know what I'm looking for as well. (Semi-Structured Interview, Task 3.6).

Much the same, when asked about her confidence as it relates to the assessment of students, Ms. Afton noted,

> Now that I've been doing this for a long time, I can do it very quickly because most times, the kids make the same mistakes year after year. It's like oh, okay. You've come in with this misconception, or somehow you've made this. So, I think it's a little bit easier for me now than it was five years ago. (Semi-Structured Interview, Task 3.9)

Notice that both teachers seem to indicate that their awareness of common student difficulties, derived from their observations of students in their classrooms, supports their interpretations. In particular, this awareness informs what Ms. Carson *looks for* and makes assessment *easier* for Ms. Afton.

Claim Four: Teachers drew on their current awareness and understanding of mathematical ideas in the act of interpretation. For most of the mathematical tasks presented, which all involved mathematical ideas related to first and second grade standards (CCSSO, 2010), the connection between teacher understanding of the mathematical content and the act of interpretation was unclear; that is, it did not appear to impact the act of interpretation. This is perhaps because teachers understood the content, and for that reason it did not limit their interpretations. However, a mathematical idea embedded within the mathematical task associated with Task 1.2 was the sole exception to this observation, and therefore warrants consideration in future research. As such, it will be discussed further in **Chapter Five**.

At this point, it seems important to note that teachers' limited awareness of the unit as an important mathematical idea and their understanding of that mathematical idea limited their interpretations and instructional responses for the student artifact that involved student difficulties with the unit. This assertion is demonstrated in response of Ms. Afton (described above), who seemed to indicate that the best means to determine whether or not four ten-sticks and four blocks was 14 ten-sticks would be to count the *value* of the ten-sticks (e.g., 10 blocks, 20 blocks, 30 blocks...), but not the *number* of ten-sticks. In fact, most teachers' interpretations were similar to Ms. Afton in that no concern was articulated when the student counted the *value* of the ten-stick (see Table 4.5).

Table 4.5

Examples of positive comments about student artifact from the mathematical task associated with Task 1.2

Teacher	Response
Ms. Allen	I mean, it looked like she was on the right path for a second, she was going to count, you know, 10, 20, 30... (Task-Based Interview, Task 1.2)
Ms. Manfield	That was kind of interesting because she started out really good. When she stopped at the four, so forty. (Task-Based Interview, Task 1.2)
Ms. Spain	At first I thought she was going to do it. She was grabbing those tens, so I thought oh, she's going to get 14 of them. I would want to hear her count by tens all the way up to 140, or 200 or whatever, because she kept stopping. (Task-Based Interview, Task 1.2)

Further, and similar to Ms. Afton, Ms. Graham noted that she would promote an approach that involved the *value* of the ten-sticks in her planned instructional response.

> If you showed her how to make 140, I think she would, I think she'd get that... But, I'd say, you know, just slow down and show me, see if you can count out. You've established this is 40 here [motions as if four tens are on the counter], you know, and show her. And say, what do you need to do to get to 140? You know, you're counting them out, 10, 20, 30... (Task-Based Interview, Task 1.2)

In contrast to these interpretations and instructional responses, consideration of other teachers' responses to this same student artifact demonstrate a deeper understanding of the unit. In particular, Ms. Pope and Ms. Rand noted the student did not attend to the unit identified in the prompt because she counted the *value* of ten-sticks instead of the *number* of ten-sticks (e.g., 1 ten-stick, 2 ten-sticks, 3 ten-sticks). As such, these teachers seem to articulate a weakness in the same aspect of the student artifact that other teachers seemed to praise and promote (see Table 4.5).

Research Question Two: Interpretations

In this section, I describe three claims related to the second research question. These include:

Claim Five: Teachers constructed interpretations were not related to mathematics.

Claim Six: Teachers constructed interpretations of what occurred in the student artifact that were different from or could not be supported in the student artifact.

Claim Seven: Teachers constructed interpretations that were varied for the same student artifact.

Evidence to support each these are provided in the sections that correspond with each claim below.

Claim Five: Teachers constructed interpretations were not related to mathematics. In addition to interpretations with a mathematical focus, which are discussed below, aspects of teacher interpretations were also related to the: (a) *speed* with which the student responded, (b) *confidence* the student demonstrated, and (c) *attention* the student paid to the mathematical problem as it was introduced. These three sorts of non-mathematical interpretations are described below.

Statements about speed. All but two teachers made at least one statement about the speed with which at least one student solved a mathematical task. These statements were positive on almost all occasions, and most often noted instances when students were *quick* with their approach or response. For example, multiple teachers noted the speed with which Beth provided her initial response to the contextual problem in Task 2.2 (see Table 4.6).

Table 4.6

Interpretations related to speed for the second student artifact related to the contextual problem in Task 2.2

Teacher	**Interpretation**
Ms. Carson	The fact that she did it mentally so quickly was very impressive. (Task-Based Interview, Task 2.2)
Ms. Allen	So, she is like some of my little girls in here that could just look at it and know, like they're very quick. (Task-Based Interview, Task 2.2)
Ms. Spain	She was able to do it quickly in her head. (Task-Based Interview, Task 2.2)

Each of these examples included a statement about the approach the student used to solve the problem, in addition to a statement about the speed with which she provided her response. Furthermore, each contributed to a broad, positive statement about the response Beth provided.

Teachers also commented on instances when the student was *not quick*, but such statements were not observed as often. For example, Ms. Allen noted, "It wasn't automatic, I did notice that, he had to think about it for a minute. I mean he was able to do it, but you could see him hesitate..." (Task-Based Interview, Task 3.2). She continued later in the interview,

> He is coming up with the right answers. I would just like for him to be able to do it more automatically, more quickly, you know. And trusting that, and seeing a pattern that, it's not counting, that ten more is 56, or you know 56 to 66 and not having to count, because I think that pause he was probably counting in his head

> or figuring it out, instead of remembering the pattern. (Task-Based Interview, Task 3.2)

In this case, the teacher assumed the lack of speed is indicative of an approach that she did not prefer; that is, an *add on* instead of *count on* approach to determine ten more, which is related to **Claim One**. As such, a quick or automatic approach was preferred.

Teacher perception of a lack of speed was not considered undesirable all of the time, and this was most often the case when teachers associated speed with thought. For example, Ms. Carson noted, “I like that she stopped and she really thought about it for a little bit” (Task-Based Interview, Task 1.2). In this instance, lack of speed seemed to mean to the teacher that the student thought about his approach or response, and it was therefore perceived well. Likewise, other teachers who seemed to associate speed and thought perceived speed as undesirable when the student response was incorrect. For instance, Ms. Baehr noted, “He went too quick, he really didn’t think about it” (Task-Based Interview, Task 2.1).

In the end, teacher perception of speed appeared to be associated with their preference for the student approach and accurateness of the student response. That is, if a preferred approach was used or the response was accurate, speed was positive, but if a preferred approach was not used or the response was inaccurate, speed was negative. As such, perception of speed differed per student artifact, not per teacher.

Statements about confidence. Five of 9 teachers studied made a statement about student *confidence* in their interpretation of one or more students, and almost all of these statements referenced the presence, not absence of student confidence. Further, statements about student confidence occurred whether or not the approach was preferred or the response was correct. For example, in observation of Ben's incorrect approach and response to the contextual problem in Task 2.1, Ms. Afton and Ms. Baehr both noted his confidence (see Table 4.7).

Table 4.7

Interpretations related to confidence for the first student artifact related to the contextual problem in Task 2

Teacher	Interpretation
Ms. Afton	I think he was confident. (Task-Based Interview, Task 2.1)
Ms. Baehr	Ben is very sure of himself, he knew exactly that he was solving that the correct way; he did not doubt at all that he was doing it correctly. He is very sure of himself. (Task-Based Interview, Task 2.1)

Notice the statements in Table 4.7 did not include explicit reference to those observable behaviors that teachers used as indicators of confidence, and as such the source of the interpretations are unclear.

Statements about attention. All but one teacher made at least one statement about student *attention* or his or her effort to *listen* to the teacher; however, over half of the statements about attention were made in observation of Ben, who used an incorrect approach to solve the contextual word problem in Task 2 (see Table 4.8).

Table 4.8

Interpretations related to attention for the first student artifact related to the contextual problem in Task 2.1

Teacher	Interpretation
Ms. Spain	That he really wasn't listening and just kind of thought he knew it. That's what I got from that. He really, you know, wasn't listening. The teacher even repeated and he still came up with that. (Task-Based Interview, Task 2.1)
Ms. Pope	He didn't listen to the whole problem, he started the numbers and started writing (Task-Based Interview, Task 2.1)
Ms. Rand	He probably did the same thing I did, didn't read it, or didn't listen. He heard 'bags' and he heard ten and he heard seven. He didn't pay much attention to the bags. (Task-Based Interview, Task 2.1)

While some teachers, like Ms. Spain and Ms. Pope, followed the responses above with a statement that one could not interpret student mathematical understanding *because* the student did not listen, others did not communicate these reservations. Ms. Rand provided two interpretations, one assuming the student listened and the other assuming the student did not listen.

> Well, if he was truly not listening, he may be a pretty good student, but he just didn't listen correctly... because if he heard 10 and he heard 7, he's thinking of putting them together... Or [if he was listening] he just doesn't understand groups, four groups of 10. (Task-Based Interview, Task 2.1)

While Ben's approach and response was incorrect, not all teacher statements about attention were observed for student artifacts that included an incorrect response. Teachers described a number of observable behaviors that he or she associated with attention. These included observations of (a) whether or not the student listened to the whole problem before he or she started to write, (b) whether or not the student asked for clarification, and (c) whether or not the student was able to repeat the problem. Examples of each of these are provided in Table 4.9. That said, for at least some references to attention, the observable behavior that prompted the statement was unclear.

Table 4.9

Observable behaviors associated with claims about attention

Teacher	**Indicator**	**Response**
Ms. Manfield	Listens then writes	He put plus before he even heard the question. He didn't hear enough to know that that was an addition problem.
Ms. Pope	Seeks clarification	And she asked him for clarification. I liked that.
Ms. Spain	Repeats problem	He was able to repeat the problem too.

Claim Six: Teachers constructed interpretations of what occurred in the student artifact that were inconsistent with or could not be confirmed in the student artifact. Two forms of teacher interpretations with a mathematical focus were observed in the data. The first of these involved teacher statements about *what occurred* in the student artifact, and these interpretations described what the student said or did in consideration of what was observed in the student artifact. Seven teachers made at least one statement about what occurred in a student artifact that was inconsistent with the student artifact, or that could not be confirmed or disconfirmed in the student artifact. These inconsistent and unsupported interpretations of what occurred are described in this section.

Interpretations about what occurred that were inconsistent with the student artifact. Four of 9 teachers studied provided at least one interpretation of what occurred that was inconsistent with the student artifact. That is, these teachers stated that the student said or did something that never occurred in the student artifact. Teachers' interpretations of the student artifact associated with Task 1.1 provides an example of the statements about what occurred that were inconsistent with the student artifact. To serve as a point of comparison, the student artifact is provided, presented here in the form of a transcript that details the video teachers observed (see Figure 4.1).

Interviewer:	**So, we put this [touches ten-stick] and this [touches stick of 4] in front of a kid, and we asked them what they saw, and they said they saw 14 blocks. Do you agree?**
Student:	[Nods head in affirmation.] Uh-huh.
Interviewer:	**Why do you agree?**
Student:	Since this is the tens [touches ten-stick], and they are in the ones right now. And this is fourteen, since there are 4 [holds up 4], then there is this teen here [touches ten-stick].
Interviewer:	**How much is this [touches four ones]?**
Student:	Four.
Interviewer:	**How much is this [touches ten-stick]?**
Student:	10.
Interviewer:	**And you said all of this together makes how much?**
Student:	14.

Figure 4.1. Transcript of Student Artifact for Task 1.1.

In consideration of this student artifact, three teachers made statements about what occurred that was inconsistent with the student artifact (see Table 4.10). While these inconsistencies are subtle, each teacher response indicates that the teacher believed the student said or did something that never occurred in the student artifact.

Table 4.10

Statements inconsistent with student artifact for Task 1.1

Teacher	**Interpretation**
Ms. Allen	She was able to identify place value correctly, that this [touches ten-stick] is the tens *and this [touches four ones] is the ones*. (Task-Based Interview, Task 1.1)
Ms. Graham	She pointed to this one [touches ten-stick] and she said this is your *teens*, so she knew, she took that over in her mind and she put it over in the tens [points pen to the right]. (Task-Based Interview, Task 1.1)
Ms. Manfield	...she said *teens place*. That would be something I would want to correct because when we have 14, that is fine, but how about when you have 24? So, we want her to know that that's the tens place, because there are 10 here, and they're in bundles of 10. Then, we count them by tens. (Task-Based Interview, Task 1.1)

For example, Ms. Allen noted that the student stated the ten-stick was *in the tens* and the four ones were *in the ones*, but the student did not state this. Much the same, the student never stated that the tens are in the *teens* as Ms. Graham indicated, nor did the student state the tens are in the *teens place* as Ms. Manfield stated. Instead, the student stated that the ten-stick "is a teen."

Interpretations about what occurred that could not be confirmed or disconfirmed in the student artifact. Different from interpretations about what occurred that are described above, six teachers made at least one interpretation about what occurred in a student artifact that could not be confirmed or disconfirmed with the student artifact. To provide an example of such statements, consider two teachers' interpretations of the student approach and response provided for the student artifact associated with Task 3.1. In this student artifact, the student responded to a mathematical task that prompted him to build 56 blocks, then use the 56 blocks to build 66 blocks and then 46 blocks. In response to the student artifact, two teachers described *how* the student counted to arrive at his solution; however, a count was not audible in the student artifact (see able 4.11).

Table 4.11

Teacher statements about the student artifact for Task 3.1 that could not be confirmed or disconfirmed

Teacher	**Interpretation**
Ms. Allen	He had to *count some of them individually*. He was counting a little bit the ones cubes instead of taking away, like subtracting the 10 to get it. (Task-Based Interview, Task 3.1)
Ms. Baehr	It was interesting because he was counting by tens. I couldn't hear him, but it looked like in his head he was counting 10, 20, 30, 40, 50. So, I think he was in his head first counting by tens. So, he started out good counting by tens then added on the ones at the end. (Task-Based Interview, Task 3.1)

Notice that Ms. Allen stated the student *counted individual cubes*, and used that observation to support an interpretation that the student was not at the most advanced level within her level scheme. In contrast, Ms. Baehr noted that a count could not be observed, but still asserted that the student counted in his head *by tens*. Because both of these are statements were about occurrences that were internal to the student, the statements could neither be confirmed nor disconfirmed.

Much the same, three teachers made statements about Ben's approach and response to the contextual task associated with Task 2. Recall that the contextual situation prompted the student to determine the number of blocks in four groups of ten objects and seven additional objects. In response, Ben wrote on his paper *10+7=17* and then said, "Ten plus seven equals seventeen." Table 4.12 provides examples of these teachers observation of this student approach and response.

Table 4.12

Teacher statements about the contextual task that could not be confirmed or disconfirmed

Teacher	**Interpretation**
Ms. Allen	He completely *didn't hear the first part*. He was gone.
Ms. Afton	He *heard the 4* but I don't think he quite knew what to do with it so *he just ignored*... since he didn't know what to do with the four, *that's not what he was comfortable* with so he just went with the part he knew... (Task-Based Interview, Task 2.1)
Ms. Rand	He *heard bags and he heard 10 and he heard seven*. He didn't pay much attention to the bags. (Task-Based Interview, Task 2.1)

Notice that Ms. Allen stated that Ben *did not hear the first part* of the contextual situation that referenced four groups, whereas Ms. Afton and Ms. Rand stated *he did hear* the same part. As a result of these differences in interpretations about what occurred, Ms. Afton used her interpretation to claim that the student *ignored* and was not *comfortable* with the part of the problem that involved four groups—a claim that is not consistent with Ms. Allen's interpretation of what occurred. In fact, from the artifact alone we cannot confirm or disconfirm whether or not the student heard all aspects of the contextual situation when it was read to him.

Claim Seven: Teachers constructed interpretations that were varied for the same student artifact. Teachers' interpretations often varied for the same student artifact; however, the amount of variation across teacher interpretations differed for each artifact. For most student artifacts, variation across teacher interpretations was notable. In this section, I discuss a student artifact that prompted some trends but some variation in both forms of teacher interpretation, and at the end of the section compare interpretations associated with that artifact to other artifacts teachers considered. The decision to compare and share interpretations that resulted from one student artifact was made in order to provide the reader with a rich example of the variation observed.

To explain the differences that occurred across teachers' interpretations, three sections are presented below. The first two sections demonstrate the differences observed in each of the two forms of interpretations; that is, the first section describes variation in teachers' interpretations of what occurred and the second section describes teacher interpretations of *student mathematical understanding*. In the third section, variation noted in teachers' interpretations for different student artifacts is discussed. To prepare to read the first and second sections, the reader can find a transcript of the student artifact discussed in these sections in Figure 4.1.

Variation in interpretations of what occurred. The first form of interpretation involved teacher statements about what occurred in the student artifact. While this aspect of interpretation seems to be no more than a description of what is observable, it is an aspect of teachers' interpretations that sometimes varied across teachers. While some of this variation was the result of the inclusion of teacher interpretations of what occurred that were inconsistent with or could not be confirmed in the student artifact (see **Claim Six**), this was not the sole reason for such variation. In addition, teachers sometimes articulated interpretations of what occurred that were consistent with the artifact, but that other teachers omitted from their interpretations. As such, these omissions also contributed to variation across teachers' interpretations of what occurred. The first problem in Task 1 is used to demonstrate both the trends and variation observed in teachers' interpretations of what occurred.

For the mathematical task associated with Task 1.1, the most notable trends in teachers' interpretations of what occurred in the student artifact involved statements about what the student did or did not do to prove there were 14 blocks. In particular, 8 of 9 teachers noted the student did not count to determine that a ten-stick contained 10 blocks, 5 of 9 teachers noted the student did not count to determine there were four ones, and 7 of 9 teachers noted that the student did not count to determine there were 14 blocks in all. Ms. Carson noted all three of these aspects of the student response,

> I liked how she didn't count them individually... I would say that she definitely can visually tell that this is [motions toward ten-stick and four blocks], when she has this in front of her, she can visually tell that there is a 10 and there are four, and there are 14 altogether.

In all, 8 of 9 teachers noted at least one of the three aspects of the student response described above. Recall from **Claim One** that these statements about what did not occur were reflective of the levels of student performance that teachers described.

In addition, a weaker trend occurred in teacher statements about aspects of the student response that demonstrated incorrect or inaccurate use of mathematical terms. In particular, 4 of 9 teachers made reference to an occurrence when the student stated *tens are in the ones*, and 6 of 9 teachers made reference to an occurrence when the student stated the ten-stick was *a teen*. Two teachers noted both of these aspects of the student response, and all but one teacher noted at least one of these two aspects of the student response. Ms. Carson noted both,

> I think I would maybe clarify why she called that a teen, like if she means ten. You know what I mean? Like she said this is the four and this is the teen. And I mean I understand why she said that, putting that number together. She said that this [touches ten-stick] was 10 and it is in the ones. So, her saying that it's in the ones, not understanding that it's the tens place value. (Task-Based Interview, Task 1.1)

Notice these references did not include verbatim accounts of the student statements, and as such sometimes involved description of the student statements that were inconsistent with the student artifact and further contributed to variation in interpretations. For example, Ms. Carson stated the student said, "that this [touches ten-stick] was *ten* and it is in the ones," which was inconsistent with the actual student statement, "this is the *tens*, and they are in the ones right now." These discrepancies, some of which are subtle, are described in **Claim Six**.

Each of the trends described above reflect instances when about half of the teachers studied noted the same aspect of the student approach or response. As such, it seems important to note that what is presented as a *trend* still often means that multiple teachers did not include in their interpretations of what occurred the same aspect of the student artifact. For example, 5 of 9 teachers did not note that the student stated the *tens are in the ones*. While we cannot assume that this omission or others means that the teacher did not notice the aspect of the student artifact, since he or she could have noticed it but decided not to describe it, it does seem important to note no aspect of this particular student artifact was noted by all of the teachers studied.

Variation in interpretations of student mathematical understanding. The most notable variation across teachers' interpretations was observed in their interpretations of student mathematical understanding. In fact, this variation was even observed in the interpretations of teachers who noted the same aspect of the student artifact in their description of what occurred, and as such no certain claims can be made about the influence of teacher interpretations of what occurred on teacher interpretations of student mathematical understanding. To provide an example of the inconsistencies in teachers' interpretations of student mathematical understanding for teachers who referenced the same aspect of the student artifact, the interpretations Ms. Carson and Ms. Graham provided for the student artifact associated with Task 1.1 are compared below.

Note that these two teachers were selected as a point of comparison because both noted all aspects of the student response described as trends in the section about ***Variation in interpretations of what Occurred***. That is, both teachers noted that the student did not need to count: (a) the ten-stick to determine it contained ten blocks, (b) the ones to determine there were four, or (c) the 14 blocks to determine there were 14 blocks. Further, both teachers referenced occurrences when the student stated *tens are in* *the ones* and called the ten-stick *a teen*. However, these teachers' interpretations of student mathematical understanding varied, which is demonstrated below.

For example, consider both teachers' interpretations of student mathematical understanding that corresponded with their reference to the aspect of the student artifact in which the student called a ten-stick *a teen* (Table 4.13). As their interpretations are considered, recall that Ms. Graham claimed the student said the ten-sticks are *teens*, which was different from the actual student statement that the ten-stick is *a teen* (see **Claim Six**).

Table 4.13

Teacher interpretation of student mathematical understanding related to aspect of student response in which student stated ten-stick is a teen

Teacher	**Interpretation**
Ms. Carson	Like she said this is the four and this is the teen. And I mean I understand why she said that, putting that number together. But just kind of to clarify what that was – I don't know. (Task-Based Interview, Task 1.1)
Ms. Graham	She pointed to this one [touches ten-stick] and she said this is your teens, so she knew, she took that over in her mind and she put it over in the tens [points pen to the right], just automatically put it in that category. (Task-Based Interview, Task 1.1)

Notice that Ms. Carson did not use this aspect of the student artifact to inform her interpretation of student mathematical understanding, but instead used it to inform a planned instructional response that would help her to better arrive at an interpretation. As such, her interpretation of what occurred led to a decision to withhold an interpretation of student mathematical understanding, and the absence of an interpretation of student mathematical understanding is as important to note as the presence of one. In contrast, Ms. Graham used this aspect of the response to inform her interpretation that the student understands the ten-stick is associated with *the tens*.

Compared to other teachers who referenced this same aspect of the student response, Ms. Carson responded similarly to Ms. Manfield and Ms. Spain who both used what was observed to inform their planned instructional response, but did not provide an interpretation outside of an observation that the response was incorrect. For example, Ms. Spain noted,

> Then I liked the way she called it a teen, she said here's a teen.... The only thing I would be [concerned about was] when she called it a teen, I thought that was kind of cute, but I might reinforce that there is not a concept called teen, that is just a number we use. (Task-Based Interview, Task 1.1)

Notice that different from Ms. Carson, Ms. Spain noted that the aspect of the response was incorrect, and planned to correct the student in their instructional response. Teachers' planned instructional responses and the consistencies and inconsistencies across them will be described further in **Claim Eight**.

In contrast from these teachers, Ms. Allen associated the aspect of the student artifact in which the student called the ten-stick *a teen* with a component of her personal resources related to **Claim Three**. That is, Ms. Allen maintained that *language* is an aspect of mathematics that is difficult for students, and she associated this aspect of the student artifact to these common student difficulties. In particular, Ms. Allen noted,

> She knew it, but just the language of it, you know like, well this is tens [touches ten-stick] and so it's a teen, and the '4.' It is very second grade, where they get it, but they don't quite have the true language of it... I just think the language of the

different place value [is difficult for her], I think that's what she was going for, like the tens and the ones, but she said the teens, because this is four- [touches for ones] teen [touches ten-stick]. (Task-Based Interview, Task 1.1)

Notice that Ms. Allen's interpretation of student mathematical understanding started with a statement that the student *knew it*. From her response, it is unclear what Ms. Allen believed the student understood, but that she believed the student did not understand the *language* of place value. As such, her interpretation differed from the other teachers who noted this same aspect of the student response.

Ms. Carson and Ms. Graham also arrived at different interpretations of student mathematical understanding from their reference to the aspect of the student artifact in which the student stated the *tens are in the ones* (Table. 4.14).

Table 4.14

Teacher interpretation of student mathematical understanding related to aspect of student response in which student stated tens are in the ones

Teacher	**Interpretation**
Ms. Carson	Okay, so she doesn't understand actual place value... she doesn't actually know that this is the tens [motions over tens] and this would be the ones [motions over ones]... because she said... that this was 10, and it's in the ones. So, her saying that it's in the ones, not understanding that it's the tens place value. (Task-Based Interview, Task 1.1)
Ms. Graham	She did say, these are in the ones at first [points to ten-stick], and I think what she meant was that if you have ten ones then you have a 10... she knew it, she jumped over the fact that they don't have to be separate for her know that they could be separate, at one time they were separate before someone put them together. You know what I mean? She knew that. That's what strikes me (Task-Based Interview, Task 1.1)

In her response, Ms. Carson used this aspect of the student artifact to inform her interpretation that the student does not understand ten-sticks are *tens* and individual blocks are *ones*. In contrast, Ms. Graham interpreted the same aspect of the student artifact to mean that the student understood that the individual blocks in the ten-stick *could be separate* and were at one time. As such, the interpretations are not just different in their content, but different in the fact that one involves a negative interpretation and the other a positive interpretation.

The interpretations of Ms. Carson and Ms. Graham are different still from the other two teachers who referenced this same aspect of the student response. For example, Ms. Baehr noted that the aspect of the student response was *confusing*, but followed that statement with a general, positive interpretation of student mathematical understanding. In particular, Ms. Baehr stated,

> She was a little confused in her explanation because I think she said these [point to ten-stick] were ones whereas, we teach the place value, the tens place, the ones place, and use place mats and separate it out. That kind of was a little confusing, but she had a pretty good grasp of it I would say. (Task-Based Interview, Task 1.1)

Ms. Afton also arrived at a positive interpretation of the student because she believed that the student later corrected herself as it related to this aspect of the student artifact. Ms. Afton noted,

> I think she called these ones [touches ten-stick]. Then she corrected after the explanation. She said it again, and she was able to identify the place value correctly that this is the tens [touches ten stick] and this is the ones [touches four ones]. (Task-Based Interview, Task 1.1)

While Ms. Afton stated that the student corrected her error in what can be assumed from her response was a statement that the ten-stick was in the *tens* and the individual cubes in the *ones*, such a correction is not observed in the student artifact, and this component of her interpretation is therefore inconsistent with the artifact. Note that because Ms. Afton seems to believe something occurred in the video that did not, her interpretation of student mathematical understanding is the opposite of Ms. Carson (see Table 4.14).

Ms. Carson and Ms. Graham both concluded their interpretations of this student artifact with statements about what the student did and did not understand about place value and other related mathematical ideas. For Ms. Carson, this component of her interpretation was carried forward from her observation of the student's statement that the *tens were in the ones*, but for Ms. Graham, statements about these mathematical ideas did not appear in earlier aspects of her interpretation (Table 4.15).

Table 4.15

Final teacher interpretations of student mathematical understanding for the student artifact presented for the mathematical task associated with Task 1.1

Teacher	**Interpretation**
Ms. Carson	When it comes to actually knowing the actual place value of a number, so I think with her knowing, if you have 10, that's the tens place [she does not understand]. (Task-Based Interview, Task 1.1)
Ms. Graham	She seems to understand place value at least tens and ones pretty solid, at this point it appears. (Task-Based Interview, Task 1.1)

Notice that the two responses appear opposite, as one teacher stated that the student *understands* and the other that student *does not understand* place value. However, the responses lack the clarification needed to understand what the teachers mean when the term *place value* is used. That is, Ms. Carson stated the student does not understand the *ten is in the tens place*, but Ms. Graham stated the student understands *tens and ones*. The differences and possible overlap between these two phrases are unclear.

Table 4.16 compares all teachers who provided an interpretation of student mathematical understanding related to this artifact that was about place value and mathematical ideas related to place value.

Table 4.16

Teacher interpretations of student mathematical understanding for the student artifact presented for the mathematical task associated with Task 1.1 related to place value and other mathematical ideas

Interpretation	**Teacher**					
	BA	**BC**	**KS**	**LG**	**LP**	**SB**
Understands place value	+	$-^{Q}$	+	$+^{Q}$	+	
Understands tens and ones	+					+
Understands tens place and ones place	+	−				
Understands that after nine there are tens*			+			

Note: + and the - indicate a positive or negative interpretation; Q indicates the statement was qualified with a description of particular component of place value; * indicates aspect of response that cannot be confirmed or disconfirmed with student artifact (see **Claim Six**).

Notice that three teachers are not referenced in Table 4.16 as these teachers did not provide an interpretation that was related to place value. Instead, teachers who were not represented in Table 4.16 arrived at a general, positive interpretation of student mathematical understanding. For example, Ms. Spain noted, "Other than that, she had a good concept. She did really well,' and Ms. Baehr noted, "I would say she pretty much does understand" (Task-Based Interview, Task 1.1).

Occurrences of differences in teacher interpretations across student artifacts. In consideration of teacher interpretations of all student artifacts, it is important to note their interpretations of what occurred and of student mathematical understanding varied for most all student artifacts presented, including those associated with Tasks 1.1, 1.2, 2.5, 3.1, and 3.2. These differences were most notable for Task 1.2 that prompted the student to consider whether one ten-stick and four blocks was the same as 14 ten-sticks, which contained common student difficulties with which teachers seemed unfamiliar (see **Claim Four**) and mathematical ideas that seemed unfamiliar to most teachers (see **Claim Five**). That said, student artifacts resulted in more consistent interpretations than others when the video associated with the student artifact was short in duration and when students used a common mathematical approach.

The second student artifact considered for the contextual problem in Task 2 provides an example of a student artifact that prompted consistent teacher interpretations. To help the reader to recall this student artifact, Figure 4.2 includes a transcript of the video and description of the written artifact that were presented to the teacher for interpretation.

Interviewer:	**Josh has 4 bags with 10 toys in each bag. He also has 7 extra toys. How many toys does Josh have?**
Student:	47 Because 40... 4 bags and 10 toys in each one is like [student starts to write] 4 times 10, and I know 4 times 10 equals 40. And I know 7 minus 0 equals 0 equals 7. So, 47.
On Paper:	*4 x 10 = 40; 7 – 0 = 7; 47*

Figure 4.2. Transcript of Student Artifact for Task 2.2.

In their interpretations, all but one teacher noted that the student subtracted zero from seven, and all of those teachers noted that they intended to ask the student about this response. As such, all instructional responses were planned to inform an interpretation of student mathematical understanding. In addition, two teachers attributed this aspect of the student response to a claim that the student was not able to describe her process for solving the problem. For example, Ms. Carson noted,

> ... she knows it, but then trying to explain it is like, yeah 4 times 10 is 40 [points to *4x10=40*], but then 7 minus 0 is 7 [teacher shakes head], instead of just saying plus 7 more [teacher motions shape of plus sign over the minus sign in *7-0=0*]. (Task-Based Interview, Task 2.2)

In addition, all teachers noted that the student multiplied four times ten, and 8 of 9 teachers used that observation to support an interpretation that the student has an understanding of or was familiar with multiplication. For example, Ms. Baehr noted, "Definitely the four times ten; she understands multiplication so she didn't have to add ten four times, she knows multiplication" (Task-Based Interview, Task 2.2).

In addition to these interpretations, 5 of 9 teachers provided a general, positive interpretation of the student mathematical understanding. For instance, Ms. Afton noted, "So, I think she's a pretty solid student. I think she has a very good mathematical understanding of what's going on" (Task-Based Interview, Task 2.2). Outside of these trends, some teachers arrived at additional interpretations of student understanding related to place value, her awareness that 4 tens is 40, and her awareness that seven added to 40 is 47. However, none of these interpretations conflicted with the trends in interpretations noted above.

Research Question Three: Instructional Responses

In this section, I describe two claims related to the third research question. These include:

Claim Eight: Teachers planned instructional responses that were related to their interpretation.

Claim Nine: Teachers' planned instructional responses varied for the same student artifact.

Evidence to support each these are provided in the sections that correspond with each claim below.

Claim Eight: Teachers planned instructional responses that were related to their interpretation. The instructional responses teachers planned were related to their interpretations on almost all occasions, for all student artifacts. That is, clear connection existed between one or more aspects of their interpretations and their planned instructional responses. Teachers sometimes articulated this connection themselves, but the connection was often apparent even when they did not mention it. Teachers' responses to the student artifact for Task 1.2 provides an example of the close connection between teachers' interpretations and their planned instructional responses (see Table 4.17). Recall that this mathematical task prompted the student to determine whether or not one ten-stick and four blocks was the same as 14 ten-sticks.

In their consideration of this student artifact, Ms. Pope and Ms. Rand focused their interpretations on the unit identified in the problem and the difference between it and the unit the student used to count in her approach. Notice from the third column in Table 4.17 that both teachers' instructional responses communicated their intention to model for or tell the student the count that corresponds with the unit identified in the problem. As such, the planned instructional response is related to the interpretation. Further, this instructional response is not observed in other teachers, but this is understandable since no other teachers focused their interpretation on the unit.

Table 4.17

Connection between interpretations about the unit and instructional response for Task 1.2

Teacher	Interpretation	Instructional Response
Ms. Pope	She's not able to transfer back and forth in her thinking, in *counting ten-sticks versus the value of a ten-stick*. She's not able to go back and forth. She doesn't see the two different ways. (Task-Based Interview, Task 1.2)	I would say that is awesome that you know that this is a 10 and you can count by tens, but I asked you to do something different this time. I asked you to *count the number of ten-sticks that you had out there*. And I think I would... grab some and start practicing. This is *1 ten-stick, 2 ten-sticks*... (Task-Based Interview, Task 1.2)
Ms. Rand	She didn't know that was one group of 10 [touches ten-stick and raises one finger], two groups of ten [points as if a second ten-stick is on the table], three [points as if a third ten-stick is on the table]... At first I thought she was on track. But then she was going 10, 20, 30. No, *she should have counted 1, 2, 3, 4, 5*... (Task-Based Interview, Task 1.2)	First of all you need to go back and show her what 14 is, and make it very clear to her that this would be a group of 10 [touches ten-stick], 11, 12, 13, 14 [slides to the left one cube per count], and then go and pull out 14 of these [lifts ten-sticks] and then show her this is *one group of 10, another one would be two groups of 10 [points as if a second ten-stick is on the table]*... That's how you could get 14 groups of 10, or ten-sticks. (Task-Based Interview, Task 1.2)

As Ms. Pope and Ms. Rand noted in their interpretation, the student whose artifact is considered counts the value of ten-sticks as a component of her approach. In particular, the student counted four ten-sticks with "10, 20, 30, 40," but then stops. A number of teachers focused on this aspect of the student approach in their interpretation, and arrived at an interpretation that this student had or could have difficulties with *greater numbers* (see column two of Table 4.18 for example).

Table 4.18

Connection between interpretations about the greater numbers and instructional response for Task 1.2

Teacher	Interpretation	Instructional Response
Ms. Spain	I would tell them that she's got a good basic understanding of at least the lower numbers. But, when she got to *adding by tens or getting to the hundreds place*, she still needed some assistance or some help. (Task-Based Interview, Task 1.2)	We probably need to practice this. Maybe practice counting up to higher numbers and some of that... Probably *counting by tens would be helpful*... (Task-Based Interview, Task 1.2)
Ms. Graham	*It seems like she may not be confident over 50*. She was able to pull out the 50 and then she stopped. She overlooked the one [ten-stick], the one 10, at one point and was counting and said it was 40... Now, the issue is, *could she go past 100*, have they worked with that? Have they counted by tens *past 50*? (Task-Based Interview, Task 1.2)	If... you showed her how to make 140, I think she would, I think she'd get that... just slow down and show me, see if you can count out. You've established this is 40 here [motions as if four tens are on the counter], you know, and show her, whatever. And say, what do you need to do to get to 140? You know, you're counting them out, 10, 20, 30 and she may not be able to count, *you could see how high she could count by tens*. I'd let her try to do it, and then I would guide her into finally getting it if necessary. (Task-Based Interview, Task 1.2)

Notice the teachers' interpretations are different in that Ms. Spain stated student difficulties with higher numbers related to the *hundreds*, whereas Ms. Graham stated student difficulties were related to *50*. However, both teachers focused their planned instructional response on counting tens (see column three of Table 4.18). In particular, Ms. Spain intended to practice the count, while Ms. Graham intended to better understand student abilities related to the count. That said, both teachers' planned instructional responses were also connected to the focus of their interpretation.

For this particular student artifact, the most common topic of teachers' interpretations was about an aspect of the student response in which the student stated four ten-sticks and one individual block could be "14" or "41." For example, consider the interpretations of Ms. Allen and Ms. Spain (see column two of Table 4.19 for example).

Table 4.19

Connection between interpretations about the 14 and 41 and instructional response for Task 1.2

Teacher	Interpretation	Instructional Response
Ms. Allen	But it's the place value where she's mixing up the digits, she doesn't truly understand why in 14 the one is here [touches ten-stick] and the four is here [touches four singles]... She said here's, you know, four tens, and then a one, that's 14, too... That right there shows you *she has the numeration of knowing a one and a four, has no concept of their value*. (Task-Based Interview, Task 1.2)	I just think she needs more *work in place value and understanding a digit versus a value*... to understand a one and a five in the ones place are not equal to one and five, but in the tens place... I may just take... tape, and I would have her sit next to me... [draws two horizontal lines, labels rightmost ones and leftmost tens]... I would do this [points out two tens sticks and two singles], and then you know on a white board or on paper, show that that is 2 [points to ten-sticks] and that's 2 [points to singles], but this is 20 and this is worth 2. (Task-Based Interview, Task 1.2)
Ms. Spain	I think part of it was the forty-one. I think she was reversing that in her head to fourteen... That was kind of interesting because she started out really good. When she stopped at the four, so forty, *she might not have the same concepts of 14 and 40 as I would think when she first started* [the first task]. (Task-Based Interview, Task 1.2)	Have her make sure she did understand what she was talking about was 41 not 14.... I would have her count by tens [touches table four times] and then see if she can come up with that it actually was 41. Then, if she says *41, I might go back and say you told me this was the same as fourteen*. Then, I would have her write the numbers, or I would have her write the numbers and say, '*Which one is 14 and which one is 41*?' To see if she's getting that confused in her brain. (Task-Based Interview, Task 1.2)

Notice that both teachers' interpretations differ, though related to the same aspect of the student artifact. That is, Ms. Allen stated the student does not understand the value of a digit is dependent on its placement, and Ms. Spain related the error to her *concept* of 14 and 41. As such, this is another example of variation in teacher interpretations (see **Claim Seven**). However, both teachers use their interpretation to inform a planned instructional response. In particular, Ms. Allen focuses on instruction that is specific to the value of digits, and Ms. Spain on the difference between 14 and 41.

Whereas the examples above demonstrate the relationship between the teachers' *negative* interpretations of a student and their planned instructional responses, *positive* interpretations also informed teachers' planned instructional responses. For example, consider the relationship between Ms. Afton's interpretations of two student artifacts associated with the contextual situation in Task 2. In this contextual situation, the students were asked to determine the total number of objects in four groups of ten objects and seven extra objects. To this problem, one student responded, "47. Because there are 4 tens and 7 ones, and 40 plus 7 equals 47." In consideration of his response, Ms. Afton noted,

> So, he's a pretty smart kid. He has a concept of place value... and addition. Like he can put it all together. He went above – like I don't know that I would ever have seen that in a kid so far in second grade. I mean he didn't even have to ask. He's like it's four tens and seven ones, and you put it together and it's 47. So somehow in his brain, he was able to put all of that stuff together. To me, that's very, 'Okay here's this, then here's this, then here's this. Then these two work

really well together.' But he put all of his concepts together and still made it work. (Task-Based Interview, Task 2.3)

Ms. Afton followed her positive interpretation of his approach and response with a broad instructional response that emphasized the need to *challenge* the student,

> I would think that Henry would, you know, if Henry was in my classroom, he would probably be my teacher. He would kind of be my go-to guy. So I would really have to do some research and get him some challenging – maybe a little bit of a third grade curriculum or something that could help Henry... (Task-Based Interview, Task 2.3)

While the instructional response is broad, there is still clear connection between her positive interpretation and her desire to select an instructional response that would *challenge* the student.

In another example, Ms. Afton's interpretation and instructional response for a different student artifact associated with the same contextual problem is provided below. In the student artifact considered, the student used a pictorial representation to solve the problem. The representation included five squares with the numeral 10 written inside four of the squares and the numeral 7 written inside one square.

> I think her approach is what I would expect from an average second grader. Because this is where I have seen most of my second graders. I think this is the mainstream. This is what we're teaching... I think she's on track for where the average second grader is. I would probably monitor her. This is my expectation for my kiddos right now, and then when we move on to further lesson, then we would keep going with her. I wouldn't push her to challenge her. I think she's just

> okay with maintaining with this point, and then knowing that the next skill that we teach hopefully will be the number model and not drawing the picture, and let's see if she can do that. (Task-Based Interview, Task 2.4)

Notice her response contained both her interpretation and her instructional response. In this response, Ms. Afton interpreted that the student was *where she should be*, and her interpretation informs her decision to *monitor* the student and let the traditional curriculum support the student.

Claim Nine: Teachers' planned instructional responses varied for the same student artifact. For each of the student artifacts discussed, differences were observed in the instructional responses teachers planned. In part, this is related to the variation in teachers' interpretations for the same artifact (see **Claim Seven**) and the close relationship between teachers' interpretations and their planned instructional responses (see **Claim Eight**). Differences in teachers' planned instructional responses included both differences in *what* was addressed and in *how* it was addressed. Examples of both of these differences are discussed in this section.

Variation in what was addressed in the instructional response. Differences were observed in *what* was addressed in the instructional responses teachers planned for the same student artifact. These differences included variation in the content, topic, or focus of the instructional response. The instructional responses teachers planned for the student artifact associated with Task 1.2 (see Figure 4.1) demonstrate such differences. For example, compare teachers' instructional responses for this student artifact as presented in Table 4.17, 4.18, 4.19. Notice that *what* is addressed in the instructional responses presented within each table differs. That is, teachers' instructional responses described in

Table 4.17 were related to the unit, those in Table 4.18 were related to *counting by tens*, and those in Table 4.19 were related to the *value of a digit* and the *value of a number*.

Because some student artifacts prompted more consistent interpretations than others, and because of the direct relationship between interpretations and instructional responses, there were some student artifacts that prompted more consistent planned instructional responses than is demonstrated in the example above. For example, Ben's approach and response to the contextual problem for Task 2 prompted more consistent focus in instructional responses. Recall that teacher interpretations for this student artifact were consistent, as teachers often noted that the student did not listen or attend to the teacher and the problem (see **Claim Five**). In their instructional responses, most teachers focused on listening and attending (see Table 4.20).

Table 4.20

Instructional responses focused on listening and attending for the student artifact for Task 2.1

Teacher	**Instructional Response**
Ms. Carson	I probably would have read that sentence again... and really emphasize the four bags. 'Can you tell me what's missing from your number story?'... *Why didn't you put that in there? Do you not understand it? Did you not hear it?* So, if I gave you another chance to do that, what would you do? (Task-Based Interview, Task 2.1)
Ms. Graham	Probably would have clarified, made sure, I probably would have repeated it and... emphasized, I would have said 4, 4 [in higher tone]... I would have said it again. Now, there are 4. Are you sure you *heard the question correctly*? (Task-Based Interview, Task 2.1)
Ms. Rand	I would just take it step by step. And then even have him put four bags [touches table four times]. I would go back and say listen to the story again. Let's try it again. *Now listen very carefully*, and repeat after me. 'What does Josh have?' He has four bags. 'Okay, he's got four bags. Keep that in your head. You've got four bags. And he has 10 toys in each bag, put 10 dots in each bag.' So, he could see what the part that he missed on that. (Task-Based Interview, Task 2.1)

Notice that while the focus of the instructional response is the same for these teachers, the means teachers use to address the concern does differ across teachers. The methods or means of instruction are discussed in the next section.

Variation in how content was addressed in the instructional response. In addition to differences in *what* was addressed in the instructional response, teachers also differed in *how* it would be addressed. That is, teachers differed in the means in which they would interact with the student to address a particular topic or focus. For example, return to Table 4.20 and read the instructional responses the teachers planned. Note that while all teachers' instructional responses were related to listening and attending to the problem, *how* teachers approached this focus differed. That is, both Ms. Carson and Ms. Graham planned to read the contextual situation and then prompt the student to consider if he *missed something* or *heard the question*. Both teachers seem to provide the student an additional chance to listen in their instructional response.

In contrast, Ms. Rand planned an instructional response in which she went *step by step* to help him solve the problem. This response is similar to others, such as Ms. Manfield, who described an intention to read the problem *part by part*,

> I would go through it sentence by sentence... part by part... I would probably read the whole thing and let's determine, let's look at the end and determine whether we're adding or subtracting, but then I would go through the parts, if he has 4 bags with 10 toys in each bag, how many toys does he have? Now he has 7 extra toys, now how many toys does he have? (Task-Based Interview, Task 2.1)

As such, both of these teachers also reread the contextual situation, but communicated an intention to help the student *unpack* or *break apart* the contextual situation in order to help the student to attend to all parts of the problem. As such, this response and the responses of Ms. Carson and Ms. Graham differed in approach, but were similar in that all were related to *listening* and *attending*.

The most notable differences in *how* teachers planned to respond to students was related to the intended direction of information transfer in the instructional response. For example, above Ms. Carson was clear that she intended to learn more from the student; that is, she intended for information to travel from the student to the teacher. This is supported in her statement, “If I gave you [the student] another chance to do that, what would you do?” Much the same, Ms. Graham also desired to understand if the child could be successful if the contextual situation is repeated. That is, she intended to learn more about the student, from the student. In contrast, for Ms. Manfield and Ms. Rand, the direction of information transfer was less clear, but appeared to move in both directions.

Differences in the direction of information transfer were clearer in the instructional responses teachers planned for the fifth student artifact related to the contextual problem in Task 2. In this student artifact, the student demonstrates two different approaches to the contextual situation. In the first approach, she wrote *10+7=17*, and in the second approach the student drew a representation that involved four groups of ten dots with seven additional objects, and then labeled the objects in the representation with the count, *10, 20, 30, 40, 41, 42, 43, 44, 45, 46, 47*. For this particular student artifact, 7 of 9 teachers planned an instructional response aimed to better understand how the student arrived at her answer (see Table 4.21).

Table 4.21

Instructional response focused on understanding how the student arrived at her answer in Task 2.5

Teacher	**Instructional Response**
Ms. Afton	I'd really like to ask her what made her choose this one [number sentence] and why she's defaulting this one [number sentence]. I really want to understand her thinking, like how she knew to draw this, like what told her to draw this, and why did she default to this one [points toward *10+7=17* in student artifact]? (Task-Based Interview, Task 2.5)
Ms. Spain	I probably would have, when she said this is the right answer [points toward *10+7=17* in student artifact], I would have said, 'Why did you draw this then? Why did you put that there?'... I would have just said... 'Why do you think that's the right answer?' (Task-Based Interview, Task 2.5)
Ms. Pope	I'd ask her to explain why she thinks 10 plus seven is the correct answer. And, I'd have to take it from there, that would be the driving point for my direction that I would go with her is what she told me about that. (Task-Based Interview, Task 2.5)

As such, all of these teachers planned an instructional response that involved information transfer from the student to the teacher.

In addition, 5 of 9 teachers planned to help the student *unpack* or *break apart* the question in order to help her consider at least one of her two approaches. Similar to teachers' efforts to *unpack* or *break apart* with Ben above, the intended direction of information transfer in these responses was less clear, and seemed to move both directions. For example, Ms. Carson noted,

> Then starting with that first sentence again, so kind of breaking it apart, Josh has four bags. 'So, how many bags did you draw?' I drew four. 'And then what does it say?' He has 10. 'So, what did you put in each bag?' Oh, I put 10. So really breaking each of those steps up and looking at those parts. So, doesn't that picture make sense for what that says? (Task-Based Interview, Task 2.5)

Notice that Ms. Carson aimed her instructional response toward supporting the student in seeing the inconsistencies in her second approach and the first approach, but plans to use an instructional response that would allow her to provide support and better understand the student.

Different from both of these common instructional responses, Ms. Rand planned to *tell* the student about the strengths of her second approach, and did not comment on the first approach the student provided.

> I would certainly go back and look at this all over and give her some praise and say wow, this was really great [points toward student representation]. You actually made four bags and you put 10 toys in each bag. And these were all the 7 that weren't in a bag. That was really good. We can see we can count by 10, 20, 30, 40 up to 47. (Task-Based Interview, Task 2.5)

In her response, the intended direction of information appeared to move from the teacher to the student; that is, the teacher planned to tell the student which response was correct, and did not communicate an intention to prompt the student for additional information.

Review of the Chapter

The data collected informed nine separate claims related to the research questions. It was found that these teachers used their *conceptions of levels of student performance*, *expectations for student performance*, and *awareness of common student difficulties* to inform the act of interpretation. Further, it was found that the interpretations teachers' constructed about mathematical understandings involved *interpretations without a mathematical focus*. Teachers constructed *interpretations of what occurred in the student artifact that were different from or could not be supported in the student artifact*, and that overall interpretations of what occurred and of student understanding were *varied for the same student artifact*. In addition, *teachers' instructional responses were related to their interpretation*, and *their planned instructional responses were varied for the same student artifact*.

While each of these claims are presented in this chapter in separate sections, the reader should note the connections and relationship between multiple aspects of these claims. For example, personal resources influenced teachers' interpretations and planned instructional responses, and because of the close connection between teachers' interpretations and planned instructional response, interpretations influenced instructional responses. Effort was made within the chapter to note these important connections, with references to other relevant claims within a particular claim. The implications for these claims and the relationship and connections between these claims will be discussed in **Chapter Five**.

CHAPTER FIVE: DISCUSSION, IMPLICATIONS, AND LIMITATIONS

In the previous chapter, I presented results associated with each of the three research questions. These research questions are:

1. *Process*. What personal resources and prior conceptions do experienced, professionally active first and second grade teachers use to construct interpretations of students' mathematical understanding from artifacts of student thinking?
2. *Product*. What interpretations of students' mathematical understanding do experienced, professionally active first and second grade teachers construct from artifacts of student thinking?
3. *Product*. What do experienced, professionally active first and second grade teachers plan in response to interpretations of students' mathematical understanding?

In this chapter, I discuss the findings presented in the previous chapter in relation to the extant literature and describe their implications for professional development, teacher preparation, and future research. The limitations of the research are also provided.

Discussion

The results presented in the previous chapter communicate the third-order models that I constructed to represent the second-order models teachers constructed and the personal resources involved in their construction of these models. Teachers' second-order models were constructed to represent the first-order models of the students whose artifacts were considered. These occurrences of model construction are reflected in the first of two conceptual models described in **Chapter One** (see Figure 1.1). In this section, I discuss the third-order models that I constructed as a result of the data analysis process.

In particular, I compare the results presented in the previous chapter to the prior research reviewed in **Chapter Two**, and describe how the results affirm and extend what is understood about teachers and their processes for and products of the act of interpretation. Further, I describe how the results presented in the previous chapter develop our understanding of aspects of the conceptual model that represents teachers' construction of second-order models (see Figure 1.2). The discussion is comprised of two sections, that include: (a) teachers' personal resources in the act of interpretation, and (b) teachers' products of the act of interpretation.

Personal resources in the act of interpretation. The first four claims presented in the previous chapter describe aspects of teachers' personal resources (Morgan & Watson, 2002) that were applied to the act of interpretation. These included their: (a) conceptions of levels of student performance, (b) expectations for student performance, (c) awareness of common student difficulties, and (d) awareness and understanding of mathematical ideas. These results are consistent with and inform notions that the internal conceptions of teachers influence the act of interpretation (e.g., Morgan & Watson, 2002; Wallach & Even, 2004); however, these particular personal resources differ in the extent to which each have been described in the literature, and these differences will be discussed in this section.

The discussion of these personal resources is presented in two sections. In the first section, those personal resources related to teacher *conceptions of how students develop and demonstrate understanding of a mathematical idea* are described. In particular, the first three aspects of teachers' personal resources are discussed in this section since these personal resources were often interrelated and share a focus on what teachers understand about students—where the term *students* is used to refer to all students, not the particular student whose artifact is considered, as is often described in the literature (Morgan & Watson, 2002; Wallach & Even, 2005). In the second section, the fourth aspect of teachers' personal resources that involves teacher *awareness and understanding of a mathematical idea* is discussed.

Understanding how students develop and demonstrate understanding of a mathematical idea. The first three aspects of teachers' personal resources are interrelated, and involved their: (a) conceptions of levels of student performance, (b) expectations for student performance, and (c) awareness of common student difficulties. These personal resources are discussed in this section.

Conceptions of levels of student performance. The teachers studied often approached student artifacts with *conceptions of levels of student performance* for the mathematical tasks addressed in the student artifacts, and used level schemes associated with their conceptions to construct products of the act of interpretation. These teachers described numerous student approaches and responses as components of a level scheme, and compared them in terms of their relative sophistication. For example, all but one of the teachers studied described at least two levels of approaches that a student could use to determine that a ten-stick and four blocks [| ....] is the same as fourteen blocks. From least to most sophisticated, these levels involved: *counting all*, *counting on from ten*, or *stating 'fourteen blocks' but not counting*. In addition, some teachers anticipated approaches that would produce an incorrect response, such as a response of '*two*' since there were two sets of objects.

These level schemes included multiple anticipated approaches that prompted both correct and incorrect responses, and as such served as an indicator that the experienced and active teachers studied accounted for far more than correctness of response in their consideration of student artifacts—an observation that is in direct contrast to numerous studies of preservice and novice teachers who often focus on correctness of response (e.g., Crespo, 2000; Hines & McMahon, 2005; Otero, 2006). Instead, the teachers studied were aware that various approaches could prompt correct or incorrect responses, and seemed aware that these approaches could inform their products of the act of interpretation. Consistent with research that indicates the importance of the approaches students use (e.g., Carpenter & Fennema, 1991, 1992; Romberg & Carpenter, 1986), these results point toward an element of these teachers' processes that are consistent with improved practice.

Further, these level schemes are an aspect of teachers' personal resources that are not described in the prior research literature, and are in contrast to the results of studies that describe teacher attention to and consideration of one approach or response as an aspect of their personal resources that limit the act of interpretation (Even, 2005; Even & Wallach, 2004; Wallach & Even, 2005). These level schemes share similarities with research-based progressions in that both describe student development around a particular mathematical idea, and the use of level schemes is consistent with recommendations that teachers approach their practice mindful of student development (Daro, Mosher, & Corcoran, 2011). However, further research is needed to determine the extent to which these level schemes are consistent with research-based progressions, and to consider the relationship between level schemes and accurate products of the act of interpretation.

All said, just 3 of 5 mathematical tasks prompted consistent teacher description and use of a level scheme. In the place of a level scheme, teachers otherwise described just one approach or response for a particular mathematical task, as has been observed in other studies (Even, 2005; Even & Wallach, 2004; Wallach & Even, 2005), or described numerous approaches and responses, but did not compare them in terms of their relative sophistication. As a point of comparison, these occurrences could indicate that the teachers studied share more developed ideas about how students approach, respond, and develop as it relates to some mathematical tasks than others; however, the potential that some mathematical tasks better reveal various levels of student performance should not be overlooked.

Expectations for student performance. The teachers studied often approached student artifacts with *expectations for student performance* related to their preferences for a particular student response and approach; however, the notion that teachers approach the act of interpretation with such expectations or preferences is not new. That is, the results of other studies demonstrate that teachers often have preferences for a particular approach (Morris, 2006; Sleep & Boerst, 2012; Spitzer et al., 2011; Van Dooren et al., 2002) or particular mode of communication (Even, 2005; Even & Wallach, 2004; Watson, 1999) that influence their interpretations. These results build on these observations, and reveal that the experienced and active teachers studied: (a) often described their awareness of where a student should be at a particular point in time as the reason for their expectations, and (b) sometimes differed in their expectations for the same mathematical task.

In particular, the experienced and active teachers studied often described their expectations in terms of their sense of what a student should understand and be able to do in second grade, or at a particular point in second grade. For example, most teachers stated that a response to the contextual problem that involved multiplication exceeded their expectations for a student in second grade. These results parallel the observation that these teachers approached the act of interpretation aware of levels of student performance, as both seem to demonstrate that these teachers were mindful that students develop over time, and that approaches and responses differ as students develop. Further, the results indicate that teacher expectations were not static, but fluid in consideration of their ideas about what is appropriate at a particular point in time, which is consistent with recommendations that teachers be attentive to student development in their assessment practices (Pellegrino, Chudowsky, & Glaser, 2001).

Despite a common rationale for their expectations, teacher expectations often differed. For instance, for the same mathematical task one teacher believed that a student should *recount* blocks before he added to the blocks on the table while others believed a student should *count on*. Because of such differences, the same student approach and response could meet the expectation of one teacher but not another. These results are in contrast to studies that communicate common expectations of preservice teachers on particular mathematical tasks (e.g., Morris, 2006; Spitzer et al., 2011). Further, these observations are of interest since the teachers studied were from the same state and therefore shared common content standards. All said, two teachers themselves alluded to their limited experience teaching the grade from which the student artifact was collected as a factor that influenced their expectations, and further research is needed to better understand the influence of this factor.

Awareness of common student difficulties. Of these three related aspects of teachers' personal resources, teacher awareness of common student difficulties is often referenced in the literature, where it is described as a personal resource that *limits* or inhibits accurate interpretation (Even, 2005; Even & Wallach, 2004). However, the literature in which it is discussed is theoretical, not empirical in nature, and offers just one instance of a teacher who overlooks an erroneous approach and response that reflects common student difficulties. In contrast, the results presented in the previous chapter provide limited empirical support for the conclusion that teacher awareness of common student difficulties does not just limit teachers' interpretations, but also that the personal resource supports interpretation of what occurred. This claim stems from an observation that teachers often described aspects of a student artifact that involved common student difficulties that were anticipated, but did not often describe aspects of the student artifact that involved common student difficulties that were not anticipated.

For example, most teachers anticipated common student difficulties associated with contextual problems that involved the addition of the numbers presented despite the lack of an additive relationship in the problem, and were consistent in their description of those difficulties when observed in student artifacts. This is in contrast to instances in which teachers did not anticipate common student difficulties related to other mathematical tasks, and later did not describe occurrences of those difficulties when observed in student artifacts. In consideration of this observation, the potential positive impact of this particular resource depends, in part, on teachers whose awareness of common student difficulties is robust and accurate. As such, further research is needed to understand the extent to which the student difficulties teachers describe reflect what is understood from the literature (Even & Tirosh, 2008).

Awareness and Understanding of Mathematical Ideas. Numerous studies (Heid et al., 1999; Morgan & Watson, 2002; Wallach & Even, 2005) have demonstrated that teachers' *understanding of mathematical ideas* limits interpretation when teachers do not understand the content in the depth needed to determine the correctness of student approaches and responses. However, the results presented in the previous chapter differ from these studies in that the teachers studied could determine the correctness of the student approaches and responses, but still demonstrated difficulties related to their *awareness and understanding of a mathematical idea*. In particular, most teachers seemed unaware of the importance of the unit in a mathematical task that prompted students to determine whether a ten-stick and four blocks [| ....] was the same as 14 ten-sticks. To represent 14 ten-sticks, most teachers used their *understanding* that 14 ten-sticks is equivalent to 140 blocks, and counted the value of ten-sticks (10 blocks, 20 blocks...) until she arrived at 140 blocks. Further, most teachers praised and promoted the same approach in students.

The approach these teachers preferred is of concern because multiple mathematical ideas underlie the approach; that is, to use the approach one must understand: (a) the unit, (b) that 14 ten-sticks is equivalent to 140 blocks, and (c) how to count in tens. However, if one understands the unit and can count, he or she can count 14 ten-sticks (1 ten-stick, 2 ten-stick...), and need not understand that 14 ten-sticks are equivalent to 140 blocks. However, the teachers studied did not use this approach themselves, nor describe it as an option for students. From the data available at this time, it seems that teacher preferences for their approach was related to their: (a) limited *awareness* of the unit as an important and often difficult mathematical idea for children or (b) limited *understanding* that ten connected blocks is both one ten-stick and ten blocks. However, additional research is needed to better understand the source of these teachers' tendencies to treat the ten-stick as ten blocks in this particular problem.

Personal Resources in the Construction of Teachers' Second-Order Models. In the sections above, claims related to teachers' personal resources were discussed and compared to the extant literature. These claims allow us to better understand the aspects of teachers' ***personal resources*** that inform the construction of second-order models for the teachers studied, and in turn allow us to better understand components of the conceptual model for the teachers studied (see Figure 1.2). In particular, consistencies between the prior research reviewed and the results presented in the previous chapter provide support for our current awareness of those items that comprise the ***personal resources*** component of the conceptual model. For example, these research results provide additional support for the claims that teachers' *awareness of common student difficulties* (Even, 2005; Even & Wallach, 2004) and *understanding of a mathematical idea* (Heid et al., 1999; Morgan & Watson, 2002; Son & Crespo, 2009; Van Dooren et al., 2002; Wallach & Even, 2005) inform the act of interpretation. In addition, aspects of personal resources that have not been described in the literature but were observed in the teachers studied and presented in the previous chapter deepen and broaden our awareness of those items that comprise the ***personal resources*** component of the conceptual model for the teachers studied. For instance, these results provide initial support for the importance of teachers' *conceptions levels of student performance* and *awareness of mathematical ideas* on the act of interpretation. That is, while these results do not alter the conceptual model, the results do help us to better understand what is involved in a component of the conceptual model for the teachers studied.

Products of the act of interpretation. Despite the observation that some student artifacts prompted consistent teacher interpretations, most student artifacts prompted interpretations that varied across the teachers studied. At least some of this variation was observed in teacher interpretations of what occurred, that sometimes involved statements that were inconsistent with or could not be confirmed in the student artifact. For instance, in consideration a student who referenced the ten-stick as a *teen*, teacher interpretations of what occurred involved statements that asserted the student had referred to the ten-stick as the *teens* and the *teens place*. In addition, instances of consistent interpretations of what occurred often resulted in interpretations of student mathematical understanding that differed across the teachers studied. At times, these interpretations involved statements about what a student did or did not understand about a mathematical idea that differed across teachers, but also led to conflicting teacher interpretations. For example, in consideration of the same student response four teachers stated the student understands place value, and one teacher stated she does not understand place value.

These results are consistent with the observation that interpretation is not simple for even experienced teachers (Jacobs et al., 2010; Wallach & Even, 2005). Further, the results support the conclusion that we should not assume that those who are experienced and active are able to produce consistent, accurate interpretations of student artifacts presented to them. In turn, the results support the notion that recommendations for reform that communicate the importance of formative assessment (NCTM, 2000; NRC, 2000; 2001) and that necessitate methods of instruction that are *adaptive* and *responsive* (Empson & Jacobs, 2008; Sherin, Jacobs, & Philipp, 2011) should not be perceived as *unproblematic* (Even & Wallach, 2003; Little, 2004) since interpretation is an essential component of these practices and is difficult for even experienced and active teachers.

Our attention to interpretation is of utmost importance not just because it appears difficult for teachers, but because the results demonstrate the close connection between the interpretations and instructional responses of the teachers studied—a connection not often observed in other studies of preservice or inservice teachers (Jacobs et al., 2010; Schack et al., 2013). The potential benefit of this connection for the teachers studied is dependent on accurate interpretation, and from the varied interpretations observed we can infer that some of these interpretations are a better fit with the first-order model than others. Of concern then is the observation that these varied and sometimes inaccurate interpretations reach the student in the form of implemented instructional responses.

Morgan and Watson (2002) expressed concern with the reach of these varied interpretations, and describe them as an *issue of equity*. These researchers push us to consider the fairness in the fact that the same *published* conception prompts different interpretations of what occurred and student mathematical understanding across teachers. Whereas these researchers focus their discussion on the impact of inaccurate interpretations of student artifacts collected for summative assessment purposes, this research warrants our consideration of the impact of varied interpretations of student artifacts collected for formative assessment purposes. In particular, the research demonstrates their impact on the classroom experiences of students as a result of the instructional responses that teachers plan, and as such point toward the potential loss of the benefits of formative assessment on student achievement (Black & William, 1998; Kingston & Nash, 2011).

Implications

The findings presented in the previous chapter and discussed above have implications for the professional development, teacher preparation, and for future research related to teacher involvement in the act of interpretation. These implications are discussed in this section.

Implications for professional development. The observation that teachers' interpretations were often varied for the same student artifact implies that some interpretations are a better representation of the student and his or her conceptions than others. As such, teachers similar to those studied must be supported in their effort to construct second-order models that better represent the student and his or her conceptions. The results presented in the previous chapter underscore the importance of this focus for the teachers studied since a close connection between their interpretations and instructional responses was observed. In turn, we can assume that more accurate interpretations will better inform their instructional responses.

To achieve these ends, we must consider the nature of professional development that would best support experienced and active teachers in the effort to produce interpretations that best represent the student and his or her conceptions of mathematics. Whereas the most obvious focus of such professional development is *assessment* and *interpretation*, the results presented in the previous chapter indicate this limited focus is insufficient. Instead, professional development must also focus on those personal resources that teachers use to inform the act of interpretation. Such professional development should focus on topics specific to the mathematics addressed in teachers' classrooms, and relate to their understanding of *students* and the *content*.

In particular, the personal resources identified above point to the importance of professional development designed to assess and build on teacher awareness of *how students develop and demonstrate understanding of a mathematical idea*. For example, this could involve professional development around research-based progressions and trajectories that describe the development of student understanding of particular mathematical ideas over time and common difficulties that students demonstrate in developing that understanding. In conjunction, emphasis on content standards could help teachers become aware of the mathematical ideas children should understand at a particular point in time since these ideas inform interpretations, with discussion around what constitutes understanding.

In addition, the results presented in the previous chapter are similar to the results of other studies (Heid et al., 1999; Morgan & Watson, 2002; Wallach & Even, 2005) that indicate teachers would benefit from professional development that develops their *awareness and understanding of particular mathematical ideas*. For example, it seems the interpretations of the teachers studied would be more consistent and perhaps better reflect the student and his or her conceptions if the teachers were aware of and understood the unit. However, professional development should help teachers not just to understand the content of the mathematical idea itself, but also to understand the importance of the mathematical idea in isolation and in relation to other mathematical ideas.

Implications for teacher preparation. In addition to the need to support inservice teachers in their effort to construct second-order models that better represent the student and his or her conceptions, the results also point toward the need to provide the same support for preservice teachers. This emphasis is of particular importance since the results presented here, in conjunction with the results of other studies, indicate that improved interpretation takes time to develop (e.g., Jacobs et al., 2010), and since teacher education programs provide a place to initiate such development with the support of teacher educators. These recommendations are further supported in the results of studies that indicate preservice teachers can develop their abilities to interpret in the teacher education program (Crespo, 2000; Schack et al., 2013; Sleep & Boerst, 2012).

In particular, teacher educators should focus their initial attention on helping preservice teachers see the importance of the student approach not just the correctness of his or her response, as these experienced and active teachers have demonstrated, but is often difficult for preservice teachers (e.g., Crespo, 2000). Further, teacher educators should begin developing those aspects of teachers' personal resources that are identified here and in other studies as influential to the act of interpretation (e.g., Morgan & Watson, 2002), and help preservice teachers see the usefulness of these personal resources in the act of interpretation. Last, teacher educators should aim to develop in preservice teachers a close connection between their interpretations and instructional responses as was observed in the teachers studied, but is often not observed in preservice teachers (e.g., Schack et al., 2013; Son, 2010).

Implications for future research. The results described in the previous chapter support the notion that teachers use their personal resources in the act of interpretation, and as such personal resources inform the products teachers construct (Even & Wallach, 2004; Morgan & Watson, 2002). Further, the results affirm and develop our awareness of those personal resources that experienced and active teachers use in the act of interpretation. However, the results draw attention to aspects of these teachers' personal resources that are not common in the literature, and provide initial and limited support for their influence on products of the act of interpretation. As such, there are aspects of these teachers' personal resources that appear important for interpretation, but are not well understood at this time.

If we are to develop these personal resources in teachers, we need a more robust description of those personal resources that inform interpretation than is available in the limited and isolated studies that have considered the topic at this point. This description should stem from research that: (a) describes those aspects of personal resources that inform and limit *more accurate* interpretation and (b) identifies those personal resources that are more and less developed in teachers studied. Further, researchers must consider the extent to which *less accurate* interpretation is the product of underdeveloped personal resources, or other factors that are unclear at this time. In addition, these claims must be made in consideration of differences in the teachers studied and the content considered in order to best inform those that prepare and develop teachers.

However, studies consistent with these ends are dependent on researcher abilities to note more and less accurate interpretations, which is a complicated venture that researchers must consider. In fact, most researchers who have described the accurateness of interpretations have done so with a comparison of the researchers' third-order model that represents the teacher to the researchers' second-order model that represents the student (Wilson et al., 2011). As such, conclusions are dependent on two uncertain components, which include assumptions that: (a) the third-order model constructed to represent the teacher is accurate and (b) an assumption that the second-order model of the researcher is accurate and superior. However, we must use caution in the assumption that the researchers' second-order models of students are superior to those of teachers studied, and consider methods that address such assumptions.

Limitations

Multiple limitations should be considered as one reflects on the research findings and their implications. These limitations stem from decisions I made about: (a) the sample, (b) the student artifacts presented to teachers, and (c) the theoretical lens adopted. Each of these decisions and the limitations that result from them are discussed below.

The sample. Nine teachers from two different school districts participated in the research. These teachers were selected to meet specific criteria reflective of the research purpose and questions (Miles & Huberman, 1994). In particular, the participants were selected because each reflected the criteria for experienced and active as described in **Chapter Three**. The decision to select teachers consistent with these criteria allowed me to better understand an understudied population, and the decision to select a limited number of teachers allowed me to collect rich data from each of the participants. However, as a result of these decisions one must use caution in generalizing these claims to all experienced and active teachers since the sample size is small and was not selected to be representative of all teachers who reflect these criteria.

The student artifacts. The student artifacts selected shared a common focus on mathematical ideas related to place value. The selection of student artifacts with this particular mathematical focus was intentional and done in order to: (a) use student artifacts that addressed mathematics content that was familiar to all teachers studied, and (b) better understand teacher interaction with a focused set of mathematical ideas. However, this decision resulted in claims that are specific to teacher interaction with *these* mathematical ideas, which are representative of just a portion of the mathematics content addressed in first and second grades (CCSSO, 2010). As such, caution should also be used in generalizing these claims to teacher interactions with student artifacts that involve different mathematical content.

Further, the student artifacts that I selected involved students unfamiliar to the teachers studied. The decision to use these student artifacts was useful in that it allowed me to: (a) control the characteristics of the student artifacts presented, and (b) compare teachers' processes for and products of interpretation for the same student artifacts. However, in their actual practice, teachers most often consider student artifacts collected from familiar students, and as such claims made are the result of teacher participation in a *simulation* of their actual practice. Because research indicates teachers' relationships with and conceptions of the students influence interpretation (Morgan & Watson, 2002; Wallach & Even, 2005), we must mindful of these results in consideration of this difference.

The theoretical lens. Claims were written to describe the processes in which teachers constructed second-order models, and the characteristics of their second-order models. As such, all claims were the product of researcher constructed third-order models that represent the second-order models of teachers (Wilson et al., 2011). The use of the term *represent* is intentional in this sentence, and is the product of the theoretical framework selected (see **Chapter One**), which articulates the difference between a representation and a replication. That is, the best that can be attained when one attempts to describe another is a representation of their external, published conceptions, and as such a replication of those should not be assumed when one considers these claims.

The third-order models described in the previous chapter were constructed with methods used to optimize the fit of the claims with that which teachers *published*. However, published conceptions were dependent on teachers who were asked to share, but not filter their conceptions. Whereas the best intentions of participants are assumed, the difficulties associated with the articulation of conceptions that most often remain internal must be considered (Ericsson & Simon, 1993); that is, it is probable that these claims are the product of some but not all that was internal of the teacher as she participated in the act of interpretation. All said, the research claims should be interpreted as a limited, but best represent teacher second-order models under the constraints described.

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Appendix A. Semi-Structured Interview Protocol

Interviewee:
Date, time, and location of interview:
Position of interviewee:

During our session today, I'd like to talk with you about your beliefs about assessment and your assessment practices in the mathematics classroom. Keep in mind, if there is a topic you would prefer not to discuss, or if you would like to stop at any time, just let me know. Do you have any questions for me before we begin?

With your permission, I would like to audio record the session so I can remember what we discussed. Do I have your permission to audio record?

1. General Information about Prior Experiences

- *1.1. Please describe your current teaching position.*
- *1.2. Please describe your past teaching experiences.*
  - *1.2.1. How many years have you taught?*
  - *1.2.2. What grades have you taught?*
  - *1.2.3. How many years did you teach each grade?*
- *1.3. Please describe other relevant professional experiences.*

2. Assessment in the Mathematics Classroom

- 2.1. *Tell me about how you assess students in your classroom.*
  - 2.1.1. [Probe for reasons or purposes of assessment described in response to question above.]
- 2.2. *Have you attended professional development related to assessment?* If so:
  - 2.2.1. *Tell me about those professional development experiences.* [Probe for experiences relevant to mathematics.]
- 2.3. *Have you read books/professional literature related to assessment?* If so:
  - 2.3.1. *Tell me about what you have read.* [Probe for books/professional literature relevant to mathematics.]

3. Artifact Collection and Interpretation in the Mathematics Classroom

- *3.1. How do you collect information from students about their thinking related to mathematics?*
- *3.2. How often do you collect information from students about their thinking?*
- *3.3. How do you decide what information you will gather?*
- *3.4. What types of student work do you collect?*
- *3.5. What do you do with the student work that is collected?*
- *3.6. Imagine that we are sitting down to look at student work together. Tell me about what that process would look like for you.*
  - *3.6.1. What would you be thinking about as you look at the student work?*
  - *3.6.2. What would you be looking for in the student work?*
- *3.7. Imagine that we are listening to a student explain his or her mathematical thinking. Tell me about what you would be thinking about as you listen to the student.*
  - *3.7.1. What would you be listening for in the student response?*
- *3.8. Tell me about the usefulness of looking at student work or listening to students.*

3.8.1. When is student work useful to you? Tell me more about that.
3.8.2. When is student work not as useful to you? Tell me more about that.
3.8.3. When is it helpful to listen to students? Tell me more about that.
3.8.4. What is it not as helpful to listen to students? Tell me more about that.

3.9. Describe your confidence as it relates to your ability to assess students.
3.9.1. What is easy about looking at student work?
3.9.2. What is difficult about looking at student work?
3.9.3. What is easy about listening to students' oral responses?
3.9.4. What is difficult about listening to students' oral responses?

4. Examination of student work with colleagues.

4.1. Do you examine look at student work with your grade-level colleagues/other colleagues?
4.1.1. Tell me a little bit more about that.
4.1.2. Why do you look at student work with colleagues?
4.1.3. How often do you look at artifacts with colleagues?
4.1.4. How do you decide what student work samples will be considered?
4.1.5. What is the goal/purpose of these sessions?

Appendix B: Prompts and Interview Questions Per Task Type

1. Mathematical Tasks

I am going to share with you a mathematical task that was administered to a _ grade student. I will first share the task, but not a student response to the task.

After I present the task, please share your thinking with me. Tell me everything you are thinking from the time you first see the task until you feel you understand the task.

[Present student work sample]

Follow-up Questions

1.1. *How would you solve this task?*
1.2. *How might your students solve this task?*
1.3. *Are there any other ways a student might solve this task?*
1.4. *How would you prefer students solve this task? Why?*
1.5. *Would you expect a _ grade student should be able to solve this task? Why or why not?*
1.6. *What challenges might a student have with this task?*
1.7. *What mathematical ideas does this task assess?*

2. Interpretation of Student Response Tasks

I am going to share a student response to the mathematical task that you just considered. The student whose work I will share was a _ grade student when he/she completed the task. I would like for you to consider the student response as though you are the child's teacher, and as you would if had you collected this from the child after a typical day of teaching.

After I present the mathematical task, please share your thinking with me. Tell me everything you are thinking from the time you first see the student response until you feel you understand the child's response.

[Present student work sample]

Follow-up Questions

2.1. *What do you think this child did in response to the task?*
  2.1.1. *How did you know?*
  2.1.2. *What did the child do well?*
  2.1.3. *What didn't the child do well?*
2.2. *What did you learn about this child's understanding of mathematics?*
  2.2.1. *How did you know?*
  2.2.2. *What helped you to arrive at this conclusion?*
2.3. *Have you always [describe specific teacher behavior in act of interpretation]?*
  2.3.1. *If no, when did you start?*

I would like for you to consider what actions you would take now that you have considered the child's response to this task.

Follow-up Questions

2.4. *How might you respond to this child, if you were the child's teacher?*
  2.4.1. *How did you decide this?*

2.4.2. What helped you to arrive at this decision?

2.5. How might you plan instruction for this child?

2.5.1. How did you decide this?

2.5.2. What helped you to arrive at this decision?

2.6. Have you always [describe specific teacher behavior in act of response]?

2.6.1. If no, when did you start?

3. **Sort of Student Responses Tasks**

Here are all of the student responses to the task that you just considered. I would like for you to consider which of these children you would be least to most concerned about based on their responses, if you were the child's teacher. Please sort them in this order.

[Present student work samples]

Follow-up Questions

3.1. How did you decide how to sort these student responses?

3.2. What criteria did you use to sort the responses?

3.3. Why do you use these criteria to sort the responses?

3.4. Why do you think these criteria are important?

3.5. Tell me about how these student responses compare to one another in relation to these criteria.

3.6. Have you always [describe specific teacher behavior in act of interpretation]?

3.6.1. If no, when did you start?

Appendix C: Instructions for Task-Based Interview

0. Instructions

During our session today, I would like you to share your thinking with me. I will ask you to respond to tasks, and we hope to learn about your thinking by hearing your honest responses to these tasks. Keep in mind that if there is a task that you prefer not to respond to, or if you would like to stop the interview, you can let me know. Do you have any question for me?

With your permission, I would like to video record the session so I can remember what you have shared. Do I have your permission to video record?

In response to these tasks, I would like you to tell me everything you are thinking, from the moment your first see the problem or student work. I would like you to talk aloud from the time I present each problem or student work sample until you have finished the task. When you are looking at a problem or student work, you do not need to plan out what should be said. Just act as if you are alone in the room speaking to yourself.

Appendix D. Task-Based Interview: Student Artifact Details for Task One

Artifact Details
Construct Assessed: Place value
Student Response: Initial responses correct, latter responses incorrect, described below
Approach or Justification: Differs per response, described below

Task-Based Interview Details
Task type(s) to be administered to the teacher:

- Mathematical Task
- Interpretation of Student Response Task, once after each set of tasks

Mathematical Task
Do: Represent 14 with Unifix cubes [| ....]
Ask: A student said this is 14 blocks, would you agree with them?
Ask: A student said this is 41 blocks, would you agree with them?

Student Artifacts [Transcript of Video Presented in Interview.]

Task 1.1:
Interviewer: So, we put this [touches ten-stick] and this [touches stick of 4] in front of a kid, and we asked them what they saw, and they said they saw 14 blocks. Do you agree?
Student: [Nods head in affirmation.] Uh-huh.
Interviewer: How do you agree?
Student: Since this is the tens [touches ten-stick], and they are in the ones right now. And this is fourteen, since there are 4 [holds up 4], then there is this teen here [touches stick of 10].
Interviewer: How much is this [touches 4]?
Student: 4.
Interviewer: How much is this [touches 10]?
Student: 10.
Interviewer: And you said all of this together makes how much?
Student: 14.

Task 1.2:
Interviewer: What if they said there are 14 tens sticks?
Student: Well... Oh... [Student pulls 4 sticks of 10 and 4 singles], I mean to do 14, but this is how you make 44 in all.
Interviewer: How does that tell me about if there are 14 tens sticks?
Student: Well, like I said. You take away these three [takes away 3 tens] and you only have this left, so that is 14.
Interviewer: That is 14 tens sticks or 14 blocks?
Student: 14 blocks.
Interviewer: Is this 14 tens sticks?
Student: No. Now I get it. [Student pulls 5 tens sticks and 1 single]. Here is 50, and here is 1, so that is... Wait. [Student pulls from 51 one stick of ten, which results in 41]. So, that is how you make 14 or 41.
Interviewer: So, this could be 14 or 41? Because you told me before that this is 14 [builds one ten and four singles].
Student: Uh-huh. Right here [points to 14] since here is 10 and here is 4.

Interviewer: You told me that this could be 14? How could this be 14?
Student: You just have 40 [points to 4 tens] and you put a 1 [holds one single], so that is 14 or 41.
Interviewer: So, if I have 40 and I put a 1 that is 14 also?
Student: Uh-huh.

Appendix E: Task-Based Interview: Student Artifact Details for Task Three

Artifact Details
Construct Assessed: Place value
Student Response: Initial responses correct, latter responses incorrect, described below
Approach or Justification: Differs per response, described below

Task-Based Interview Details
Task type(s) to be administered to the teacher:
- Mathematical Task
- Interpretation of Student Response Task, once after each set of tasks

Mathematical Task
Ask: Make 56 with cubes.
Ask: Using what you have out, make 66.
Ask: Using what you have out, make 46.

Do: Represent 24 with Unifix cubes [|| ....]
Ask: How many cubes do I have?
Do: Represent 34 with Unifix cubes [|| .... |]
Ask: How many cubes do I have?
Do: Represent 54 with Unifix cubes [|| .... |||]
Ask: How many cubes do I have?

Student Artifacts [Transcript of Video Presented in Interview.]

Task 3.1:
Interviewer: Make 56 with cubes.
Student: [Student touches five ten-sticks, one touch per count, and then pulls six ones. On table: ||||| ......]
Interviewer: Using what you have out, make 66.
Student: [Student touches each of five tens.] I need another one of these [points to ten-sticks]. [On table: |||||| ......]
Interviewer: Using what you have out, make 46.
Student: [Student places left hand on four ten-sticks, and removes two ten-sticks with right hand. On table: |||| ....]

Task 3.2:
Interviewer: [Represents 24 with Unifix cubes || ....] How many cubes do I have?
Student: 24, see 10, 20, 4. 24.
Interviewer: [Represents 34 with Unifix cubes || .... |] How many cubes do I have?
Student: 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34. [Counts each block in last ten-stick]
Interviewer: [Represents 54 with Unifix cubes || .... |||] How many cubes do I have?
Student: [On fingers] 24, 25, 26, 27... 54. 54. [Counts each block in last three ten-sticks.]

Appendix F. Task-Based Interview: Student Artifact Details for Task Two

<u>**Artifact Details**</u>
Construct Assessed: Place value in a contextual word problem
Student Responses: Three correct, two incorrect
Approach or Justification: Differs per response, see below

<u>**Task-Based Interview Details**</u>
Task type(s) to be administered to the teacher:

- Mathematical Task
- Sort of Student Responses Task

<u>**Mathematical Task**</u>
Josh has 4 bags with 10 toys in each bag. He also has 7 extra toys. How many toys does Josh have?

<u>**Student Artifacts**</u>

Task	Student	Correctness	Artifact	Description
2.1	Ben	Incorrect	Video Written	[As interviewer reads, student writes. On paper: 10+7=17] S: Ten plus 7 equals 17.
2.2	Beth	Correct	Video Written	47 Because 40... 4 bags and 10 toys in each one is like [student starts to write] 4 times 10, and I know 4 times 10 equals 40. And I know 7 minus 0 equals 0 equals 7. So, 47. [On paper: 4 x 10 = 40; 7 – 0 = 7; 47]
2.3	Henry	Correct	Video	47. Because there are 4 tens and 7 ones, and 40 plus 7 equals 47.
2.4	Kayla	Correct	Video Written	So, he has 4 bags. [Student draws four boxes in one row with the number 10 written in each. Student draws fifth box in second row with the number 7 written in it. Student writes, “He has _ toys.” Student fills in blank with 47.]
2.5	Mica	Incorrect	Video Written	[After interviewer reads, student writes. On paper: 10+7=17] I did this in first grade the tens plus a number would equal something. A teen number. So, 17. [Student is asked to solve with another method. Student draws 4 circles. Student puts 10 dots in each circle. Student draws 7 balloon-like objects. Student writes 10, 20, 30, 40, 41, 42... 47 above objects on drawing. Student is asked which answer is correct. Student points to 17.]

VITA

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