TWO ESSAYS IN ANALYZING DELEGATED PORTFOLIO MANAGEMENT RELATIONSHIPS THROUGH RELATIVE PORTFOLIO MEASURES

A Dissertation

presented to

the Faculty of the Graduate School

at the University of Missouri-Columbia

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

by

DAVID L STOWE

Dr. Dan W. French, Dissertation Supervisor

JULY 2014

The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

TWO ESSAYS IN ANALYZING DELEGATED PORTFOLIO

MANAGEMENT RELATIONSHIPS THROUGH

RELATIVE PORTFOLIO MEASURES

presented by David L. Stowe,

a candidate for the degree of doctor of philosophy,

and hereby certify that, in their opinion, it is worthy of acceptance.

Professor Dan W. French

Professor Xuemin (Sterling) Yan

Professor Michael O'Doherty

Professor Ken W. Shaw

To my wife Melissa and my daughters Isabelle and Eleanore

- without your love and sacrifice this would not have been possible.

To my parents John and Linda Stowe

- thank you for your inspiration and encouragement.

ACKNOWLEDGEMENTS

My special thanks go to my dissertation chairman, Dan W. French whose support, flexibility, and direction were instrumental in permitting me the opportunity to write in this topic area. Special thanks also go to Michael O'Doherty for the time he spent, above and beyond his role as a committee member, providing helpful feedback and advice on multiple occasions. Additionally, I would like to specifically thank Xuemin Yan and Ken Shaw for the extra time they spent assisting me along the way. I would also like to thank Grace Hao, John Howe, Steve Ferris, Shastri Sandy, John Stansfield, Michael Taksar, Allanus Tsoi, and Chunchi Wu for their support and advice.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACADEMIC ABSTRACT

In my first essay, I demonstrate how the Cremers and Petajisto (2009) Active Share measure can be re-parameterized into the standard portfolio parameters we typically see in other portfolio management studies, namely betas and standard deviations. This demonstrates that Active Share is not very different than the measures we traditionally use to study portfolio management. One of the parameters that results from the re-parameterization is a measure of the risk of the manager's active bets, the volatility of the implied hedge position relative to the benchmark. This parameter is equally as strong as Active Share in predicting excess performance and helps give a better economic understanding of why Active Share exhibits predictive power. Active Share and this implied hedge measure are like a confidence and information problem.

In my second essay, I use the idea of benchmark relative investment optimization as outlined in Roll (1992). These portfolios are sub-optimal but they can be better than the alternative, i.e. better than the portfolios that the principals could build themselves. I outline the conditions under which delegated managers increase the principal's utility. Additionally, if implemented properly, tracking error constraints, Jorion (2003) and beta constraints, Roll (1992), can force the delegated manager to buy a more efficient portfolio than the benchmark. Thus, even though relative utility maximization is sub-optimal, if the delegated manager is more skillful than the principal in portfolio construction, delegated portfolio management is still likely preferred to naively holding the benchmark.

CHAPTER 1. DISSERTATION SUMMARY

I. Introduction

Delegated portfolio management is an extremely broad topic. The fundamental idea in studying delegated portfolio management is to study the effects of allocating decisions in a manner that separates the portfolio decision-making from the beneficiary's interests. Most delegated portfolio management literature is concentrated in the broad classification of investments research but also central to this idea is the inherent principal/agent relationship created by the delegation. The principal/agent connection gives a lot of the research in delegated portfolio management a link to research in corporate finance and in particular corporate governance, incentives, contracting, and control. Obviously, the contracts and incentives can both directly and indirectly cause the delegated manager to behave in a manner not completely consistent with that of the principal.

A large portion of this research is focused on a relatively narrow definition of delegation. For example the CIO of an investment operation like a pension or endowment can hire an external manager to manage all or part of the assets. This more narrow definition is usually referred to as decentralized investment management or multistage investment management in the literature. An exceptionally large amount of the money in the financial markets is managed in this way. This is particularly evident when you consider that the entire investment company universe, which includes both open end and closed end mutual funds, is explicitly defined in this manner. Less obvious but just as vast a universe of investment managers is the directed or institutional investment management business, which for all intents and purposes is essentially the same as a mutual fund relationship albeit designed for more sophisticated investors (pension funds, endowments, etc.). With a looser definition, the private wealth management business could be grouped into this category as well.

Although a large portion of the money in the financial markets is managed in this manner, taking a much more nuanced definition of delegated portfolio management opens it up to an even much larger universe of implicit and explicit contracts in investment decision-making. For example, any time a separation exists between the final beneficiary and the individual making an investment decision, the agent has the potential to make decisions that are not completely in the best interests of the principal, even if the agent's intentions are in the principal's best interests. In this context, there is also a separation between portfolio manager and an analyst on the same investment portfolio. There is even a separation between an internal, non-delegated, manager, where all of the investment decision-making is done in house, and the beneficiaries of the funds, as in the case of an endowment. It is evident that close to all investment decisions could be classified as delegated decisions if you sufficiently loosen the definition.

Given that this topic spans the universe of financial research from Investment Theory, Linear Asset Pricing, Portfolio Management, Economics, Corporate Governance, etc., the aspect of this field I have chosen to study is delegated manager portfolio choice given the incentive problem. That is, I am studying some of the metrics and parameters we use to choose between different delegated managers, whether those metrics are useful, and how to improve the relationship through controls and constraints. In order for delegation to be efficient, we must believe that external managers can improve upon the portfolio that a principal could build himself without the delegation, otherwise we would not rationally delegate. The standard decision-making tool is some form of model or parameter that gives us an indication of whether the delegated manager can outperform some benchmark, implicit or explicit, pre-specified or not.

My first essay builds on a metric introduced into the academic literature by Cremers and Petajisto (2009). This paper has been fairly well cited in top academic journals since its publication. Their primary finding is that their measure, Active Share, is predictive of future relative performance. This finding drives their main conclusion that Active Share's predictive power supports the existence of skill in active management. It is a small step to conclude that this measure could perhaps be used to predict which managers have the ability to outperform a benchmark. In my first essay, I decompose this measure into two, multiplicative components and show Active Share's relationship to more traditional portfolio risk metrics. Additionally, this measure is not independent of systematic risk but the level of systematic risk inherent in the measure does not seem to bias the predictive power. Therefore, the conclusion of whether this measure is actually representative of manager skill is in question. It could very well be just a reflection of something more fundamental, something already known.

My second essay builds on a long line of portfolio management research that is based on Markowitz (1952). I study the optimality of the relative performance incentive on portfolio efficiency for the principal in Markowitz's mean/variance space. The relative performance incentive is a structural assumption that defines how a delegated manager behaves and is not necessarily consistent with the behavior of the principal. Many academics have tried to tackle the problem of whether this type of delegation can be efficient with the first true challenge to the problem coming from Sharpe (1981). Since then, under differing sets of assumptions, the problem has both asserted efficiency, and also proven non-efficiency depending on the specific assumptions. I approach the problem differently than many of my predecessors by using the relative performance incentive and Roll (1992) mean-tracking error volatility space, and my results indicate that although increasing efficiency is largely uncertain, constraints to tracking error and beta can improve efficiency and in certain cases replicate any portfolio that the principal would like the agent to build on his behalf.

The balance of this introduction proceeds as follows. Section II describes essay 1 and the general process I use to decompose Active Share into its component parts. This exposes the relationship between Active Share and tracking error (and thus systematic risk). Section III describes essay 2 and the utility problem and primary economic assumptions used to model the relative investment incentive in the space of absolute mean-variance space. Finally, Section IV describes the prospect of publishing in portfolio management and in particular extensions of these ideas. There seems to be a good amount of very interesting future research in the field of delegated portfolio management and there are direct extensions of the research in this dissertation to keep me busy for a number of years to come.

II. Essay 1 - Active Share and Tracking Error

The concept of Active Share was introduced to the academic literature in Cremers and Petajisto (2009). This measure represents the portion of a portfolio that deviates from its benchmark and is measured in terms of the weight differences of the holdings between the portfolio and the benchmark. It can be expressed as follows:

$$\text{Active Share} = AS_p = \frac{1}{2}\sum_{i=1}^{N}\left|w_{p,i} - w_{b,i}\right|, \tag{1}$$

where $w_{p,i}$ and $w_{b,i}$ represent the weights of asset *i* in the portfolio and the benchmark respectively. Active Share is predictive of future excess performance. Portfolios with higher active share tend to outperform their benchmarks and active portfolios with lower active share tend to underperform. This evidence is used to support the conclusion that some active managers have skill. Active Share is introduced as a measure of the risk of active management and placed in direct contrast to Tracking Error Volatility.

Tracking Error Volatility is a long established tool used in evaluating active managers and can be used as both an implicit and explicit risk constraint controlling the behavior of active managers. It is usually defined as follows:

$$\text{Tracking Error} = TE_p = \sigma\left(r_{p,t} - r_{b,t}\right), \tag{2}$$

where $\sigma(\ldots)$ is the standard deviation function and $r_{p,t}$ and $r_{b,t}$ represent the returns in time *t* of the portfolio and the benchmark respectively. Evidence on the implementation of Tracking Error as a risk tool in active management can be found in Roll (1992), Grinold and Kahn (1999), Jorion (2003), and many additional sources. According to

Cremers and Petajisto (2009), tracking error alone does not produce the same predictive effects as Active Share.

In this essay, my primary contribution is in building the functional relationship between the concepts of Active Share and Tracking Error. The reconciling term is the standard deviation of the hedge portfolio whose weights, $w_{h,i}$, are defined as follows:

$$w_{h,i} = \frac{w_{p,i} - w_{b,i}}{\frac{1}{2}\sum_{i=1}^{N}|w_{p,i} - w_{b,i}|} = \frac{w_{p,i} - w_{b,i}}{AS_p}. \tag{3}$$

The hedge portfolio h, with weights $w_{h,i}$, has a very important economic interpretation. Both the long side and the short side of this portfolio have weights that sum to 1, thus both the long and short exposures in this sense are investable. Also, it is the portfolio that the investment manager would buy to differentiate from the benchmark index. Active Share is just the weight that the manager invests in this hedge portfolio.

Using the above construction for portfolio h, I can show that the reconciling factor between Active Share and Tracking Error is the standard deviation of this hedge portfolio, σ_h. That is:

$$AS_p \sigma_h = TE_p. \tag{4}$$

This expression represents a decomposition of Tracking Error, the risky deviations a portfolio makes from its benchmark, as a product of the volatility of the hedge portfolio, essentially the risk of the delegated manager's information, and the Active Share, the weight (or confidence) the delegated manager applies to his/her information.

Given that TE_p is not predictive of either future relative or absolute return, and AS_p is predictive, an important distinction can be made about whether it is actually AS_p directly creating the pricing effect, σ_h directly creating the pricing effect and it showing up indirectly in AS_p, or a combination of both. It seems illogical that merely deviating from one's benchmark by generating Active Share should create value, but as a proxy for confidence perhaps it does. However, there must be some information that Active Share is capturing to create value. That information is encapsulated in the hedge portfolio thus the direct skill of an active manager should be primarily attributable to this information. In this context, Active Share is only a secondary identifier or a proxy for the actual information. Therefore, the purpose of this essay is two-fold:

1. To expose Active Share's functional relationship to Tracking Error
2. To determine whether the predictive power of Active Share in encompassed by the level of Active Share or by the risk in the hedge portfolio.

III. Essay 2 - Tracking Error Volatility Optimization

The concept of benchmark relative risk, usually expressed as tracking error, is almost as old as Modern Portfolio Theory itself. In a world dominated by Delegated Portfolio Management, oftentimes the primary incentive that separates the agents from the principals is the agent's reliance on an implicit or explicit relative performance incentive. If the principal already holds, or has the ability to manage, his optimal portfolio, then delegation of the portfolio management responsibilities should not happen. This is because any additional constraint imposed by the principal/agent relationship will necessarily create a portfolio that is at maximum of equal preference. Therefore, the first assumption that needs to be made in a delegated portfolio management relationship is whether there exists a portfolio with higher utility than the one the principal can build himself.

Let p represent the best portfolio the principal can build, a the portfolio that the agent builds, and m the optimal portfolio given the principal's utility. If $U(\ldots)$ is the utility function of the principal then we know:

$$U(p) < U(m), \text{ and } U(a) \leq U(m). \tag{5}$$

However for the delegation to be rational, we have to also expect that:

$$E[U(a)] > E[U(p)]. \tag{6}$$

It is unfortunately not always the case that $U(a) > U(p)$. In fact, the more risk averse the principal, the less likely it is that the agent will actually increase utility. Additionally, the more efficient the principle's original portfolio, the less likely it is that the agent will increase utility.

These propositions become particularly apparent when we consider a standard utility relationship where preference is increasing in returns and decreasing in risk. If we frame this problem in the context of mean-variance portfolio optimization, the standard procedure is to maximize utility given the satisfaction of a set of parametric equations that define the efficient boundary, or the envelope, upon which lies portfolio m. However, the relative performance incentive encourages the external manager to optimize over relative returns rather than absolute returns which has the consequence of the agent also considering risk measures in relative risk rather than absolute risk.

As a simple example, we can analyze the effects of the agent optimizing a quadratic utility function over the standard deviation of relative returns, the tracking error volatility, rather than the typical constraint of optimizing over the standard deviation of absolute returns. The choice this type of decision-making leads the agent to buy a portfolio that has a higher expected return than the portfolio that the agent would manage themselves, *a*, but at the same time a higher standard deviation of absolute returns. This leaves the agent's resulting portfolio in the ambiguous space where it is the structure of the utility function, at particular the relative levels of risk aversion, which determines whether the agent did indeed increase utility for the principal.

The balance of this essay explores the Roll (1992) TEV (Tracking Error Volatility) space and its theoretical implications of the relative performance incentive with a particular interest in how it affects optimal utility when translated back to mean-variance space. Therefore, the purpose of this essay is to address the following:

1. Show how the Agent's utility function generates the TEV frontier.
2. Demonstrate how relative decision-making, albeit inefficient, could lead to portfolios preferred by the principal.
3. Provide support for the constraints to tracking error and beta and show how these constraints can assure utility improvements.

There are five main contributions of this essay to the academic literature on portfolio management. The first contribution of this essay is in analytically connecting the Roll (1992) TEV frontier to the delegated utility optimization problem. I demonstrate, with quadratic utility, how the optimization process in relative returns causes the delegated managers to invest on the TEV frontier. In fact, the TEV frontier is the critical path delegated managers optimize upon given changes to their risk aversion levels.

The second contribution comes in calculating the optimal Beta and TEV constraints based on the definitions of these constraints in the Roll (1992) and Jorion (2003). I demonstrate that if the agent's opportunity set can be estimated then a utility optimizing principal can assign a TEV or a Beta constraint to not only assure utility improvement but also maximize his utility given the agent's incentives.

The third contribution comes in connecting the Roll (1992) Beta constraint and the Jorion (2003) TEV constraint. The combination of these two constraints and my setup for the agent's optimization process yields a unique solution in mean-variance space. I show that if a principal can estimate the agent's opportunity set then he can assign a Beta/TEV constraint combination that produces any portfolio in mean-variance space. Essentially, the principal can force the agent to optimize the principal's utility instead of the agent's utility, removing all of the inefficiency of the delegated incentive.

The Fourth contribution is in demonstrating the connection between constraining on agent risk aversion and constraining on TEV. Many authors that solve delegated portfolio management problems rely on the knowledge of and ability to constrain agent risk aversion. However, this parameter is rather illusive and a criticism of this literature is abundant. We just cannot reliably estimate agent risk aversion. TEV on the other hand is measurable and directly observable, at least ex-post. I show that a principal constraining an agent on TEV causes the agent to choose a portfolio equivalent to one he would have chosen if he had been constrained on agent risk aversion.

Lastly, I discuss, rather arbitrarily, how even if the agent's opportunity set is unknown, mild constraints to Beta and TEV still likely lead to utility improvements for the principal. This lends support to the anecdotal proposition that these constraints are popular and wildly used in the investment industry. This paper extends a relatively scarce set of literature but a highly important topic in portfolio management and in particular delegated portfolio management. Given the prevalence of delegated contracting in the investment industry, and that these contracts can create some rather drastic agency problems, the ability of principals to reduce their risks and costs is very important.

IV. Summary and Extensions

In recent years, portfolio management research has only been mildly popular in the top finance journals. Every year there are usually a handful of very good articles, probably on the order of 10, published in these journals. Personally, I find this topic area fascinating and enjoy reconciling the models we use in portfolio management with many of the other topics we study in finance. In fact, most of the successful portfolio management literature being published today ties portfolio management together with other interesting and usually more popular topic areas in the broader finance literature. The two essays in this dissertation are just two of those ideas. Below I have outlined some other connections that are worth studying that have only been mildly covered. These are broad ideas and each has multiple extensions built in.

First, if it is true that managers can be shown to have skill then this may imply that Active Share be evidence of the pricing of "idiosyncratic risk." Although this is often taken with a fair amount of skepticism, such as in Bali et. al. (2005), the existence of excess return pricing in a broad market factor model would be strong support. My suspicion is that if excess return is priced on the idiosyncratic factor in my model, is it merely because the benchmark is inefficient and the idiosyncratic factor is picking up real systematic exposure to the true market portfolio. Active Share could help disentangle some of these factors and allow us to get a cleaner look at systematic vs. non-systematic risk.

The fundamental law of active management, Grinold and Kahn (2000), says that the information ratio is related to the productivity of the information, IC, and the square root of the number of bets. The information ratio is the excess return over the tracking error and tracking error is active share times the risk of the hedge. This makes the slope of the models in Active Share comparable to the fundamental law of active management. This connection could be studied either by considering it as manager skill, as both Cremers and Petajisto and Grinold and Kahn consider it, or by exploring the systematic exposure of both measures.

Since active share is akin to the portion of the portfolio in which you deviate from the index, we can consider this portion scalable. Essentially, we can buy more and sell more of the hedge to further lever the position and create more active share. Why did the manager stop where he or she stopped? Why not lever further? In reality the implied long only constraint would prevent this but theoretically the hedge could expand. In this context, the active share number is really a confidence number in the information the manager had about the hedge portfolio. Since the risk of the hedge is the risk of the manager's information, and the active share is the confidence in that information, active share can be considered in concert with the risk of the hedge as a confidence and information problem. This is related to Grinold and Kahn (2000) as well.

The problem of active share and appropriate levels of active share begs the connection with agency and fund governance. This has been studied from the corporate side in papers such as Ferris and Yan (2009), Lakonishok, Shleifer, and Vishny (1994), and Dybvig, Farnsworth, and Carpenter (2010). The framework of relative risk, and in particular measures of active share and tracking error, are ripe with metrics that can be used to measure the cost of agency, the level of interest misalignments, or the benefits/detriments of delegated management and monitoring. These metrics are mostly unknown and unstudied in the realm of governance and connecting the worlds of corporate governance and delegated portfolio management through the analysis of relative portfolio pricing metrics can provide a rich stream of research.

Substituting the decomposition of Tracking Error into the Active Share decomposition yields an equation for Active Share that clearly shows Active Share is dependent upon portfolio beta:

$$AS_p = \frac{1}{\sigma_h}\left(\left(\beta_p - 1\right)^2 \sigma_b^2 + \sigma_\varepsilon^2\right)^{1/2} \tag{7}$$

This equation decomposes Active Share into its component Systematic and Idiosyncratic exposures. Although σ_h is also dependent on β_p, for the moment I will ignore that dependence. Solving for β_p yields the following:

$$\beta_p = 1 \pm \left(\frac{\left(AS_p \sigma_h\right)^2 - \sigma_\varepsilon^2}{\sigma_b^2} \right)^{1/2}. \tag{8}$$

Alternative methods to estimate portfolio beta typically involve using a long time series of historical data as in the market model estimation from above or simultaneously estimating $n(n+1)/2$ covariances and applying the same weight vector used to calculate the Active Share, $w_p - w_b$, to imply a long historical time series. The benefit of calculating beta using this method is that the Active Share is a stock variable, and the only historical data needed is used to estimate three variances, a parameter that can be estimated with relative accuracy even in the presence of non-stationary historical data. This method suggests that a beta calculated in this way might be able to provide information for pricing that is more difficult to estimate when using more standard measurement techniques.

The potential for this the agent TEV optimization constraint to alter aggregate equilibrium in the financial markets is possible and could be explored and conditions for this constraint to not affect the equilibrium could be outlined. This could potentially be an explanation for the seemingly inexplicable amount of momentum in cross-sectional, asset pricing regressions. Perhaps the aggregate, macroeconomic equilibrium movement can be traced to better take advantage of momentum, something that is very difficult to build a viable investment strategy around.

The second essay studies the problem of portfolio delegation from the perspective of a single principal hiring a single delegated agent. This problem can be and has been extended in other research to include a more realistic multi-delegated-manager scenario.

The constraints I apply to assure utility improvements have different implications in a multi manager scenario but could still be used to assure both utility improvement and global optimality. Additionally, another constraint has to be employed in the multi-manager scenario to prevent systematic factor cannibalization between external managers.

CHAPTER 2. ACTIVE SHARE AND THE IMPLIED HEDGE

I. Introduction

Active portfolio management is a hotly debated and controversial topic. The research is both supportive and critical of a portfolio manager's ability to "beat an index" and significant effort has been made by academics and practitioners alike to both prove and disprove the idea of skill in active management. The evidence supporting skill in active management is not particularly robust. In the aggregate, investment managers must underperform their respective benchmarks by their transactions costs and management expenses.[1] With few exceptions, it is generally accepted that there is no magic formula to achieve excess performance through active management. Nonetheless, a large amount of money is managed by professional investment managers on behalf of clients, and the evidence strongly suggests that the external investment manager's primary goal is outperformance of a specified benchmark[2], whether that goal is explicit or implicit.

A recent measure, Active Share, introduced into the academic literature in Cremers and Petajisto (2009), provides evidence that there is indeed skill in active management. This measure is essentially the percent of the holdings in a portfolio that differentiates from the holdings or constituents of a benchmark. It is calculated as follows:

---

[1] Sharpe (1991) and French (2008) strongly assert this viewpoint.

[2] See Treynor and Black (1973) for an early discussion of this.

$$\text{Active Share} = \frac{1}{2}\sum_{i=1}^{n}\left|w_{p,i} - w_{b,i}\right|, \tag{1}$$

where $w_{p,i}$ and $w_{b,i}$ are the portfolio or constituent weights of asset *i* in the portfolio or the benchmark respectively, and *n* is the number of securities held. Cremers and Petajisto (2009) finds that Active Share is predictive of excess returns and that there is a positive relationship. That is, portfolio managers that take more Active Share tend to outperform their benchmarks by more than managers that have low Active Share. This result is confirmed through the post 2008 financial crisis period in Petajisto (2013).

As a measure of benchmark relative decision-making or risk, Active Share is compared and contrasted to tracking error volatility, which has become the favored measure of active risk over the last 20 or so years. Tracking error is the standard deviation of difference in returns between a portfolio and its benchmark. This measure can be contrasted with the standard deviation of the raw or absolute returns as in the sense of Markowitz (1952), which has long been the standard to measure total portfolio risk. Roll (1992) does an analysts of tracking error volatility in mean-variance space under the assumption that the tracking error volatility frontier is the efficient set upon which delegated managers will optimize. Further evidence that tracking error is considered the standard measure of portfolio manager risk can be found in other works such as Jorion (2003), Alexander and Baptisa (2010) and Bertrand (2010), which analyze allocation decisions in the style of modern portfolio theory with constraints to relative benchmarking and tracking error volatility. There is also an interesting extension of the Modigliani and Modigliani (1997) M-2 measure in which Muralidhar (2000) uses tracking error volatility to expand the M-2 measure into M-3 for the purpose of justifying risk-adjusted performance.

When attempting to find a metric to measure skill in the aggregate or of the average investment portfolio, an unbiased statistic of the aggregate skill must net to zero[3]. In the case of metrics that do seem to predict performance, it is important to identify the source of the opposing, counterbalancing effect. As in the case of portfolio beta, the portfolios with beta greater than one are balanced by the portfolios with beta less than one. Essentially, the market cap weighted sum of all portfolio betas should sum to one. When we find factors that are priced, whether that be in absolute or in relative performance, and they are not the standard risk factors that we are typically accustomed to analyzing, we should approach those factors critically until a determination can be made about exactly what we are looking at.

If we consider the portfolio weights and a covariance matrix of stocks, the tracking error[4] can be calculated as follows:

$$\text{Tracking Error} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}(w_{p,i} - w_{b,i})\,(w_{p,j} - w_{b,j})\sigma_i\sigma_j\rho_{i,j}}, \tag{2}$$

where σ_i is the standard deviation of returns from security *i*, and $\rho_{i,j}$ is the correlation between security *i* and security *j*. Tracking error's connection with Active Share is

[3] This is an application of Sharpe's (1991) arithmetic of active management.

[4] By tracking error I technically mean tracking error volatility. Other authors follow Grinold and Kahn (1999) in defining tracking error as the root mean square of the difference in returns. My definition is the standard deviation of the difference in returns. However, it is common practice to just refer to tracking error volatility as tracking error.

immediately detectable since both measures are a function of $\left(w_{p,i} - w_{b,i}\right)$. However, even given the relationship between tracking error and Active Share, Cremers and Petajisto (2009) finds that whereas Active Share is predictive of relative returns, tracking error is not. At least the evidence is exceedingly weak when compared with the strong evidence for Active Share. Therefore, there is something significant about the difference between these two measures that is causing the effect. In fact, there is a simple, functional relationship between these two metrics such that:

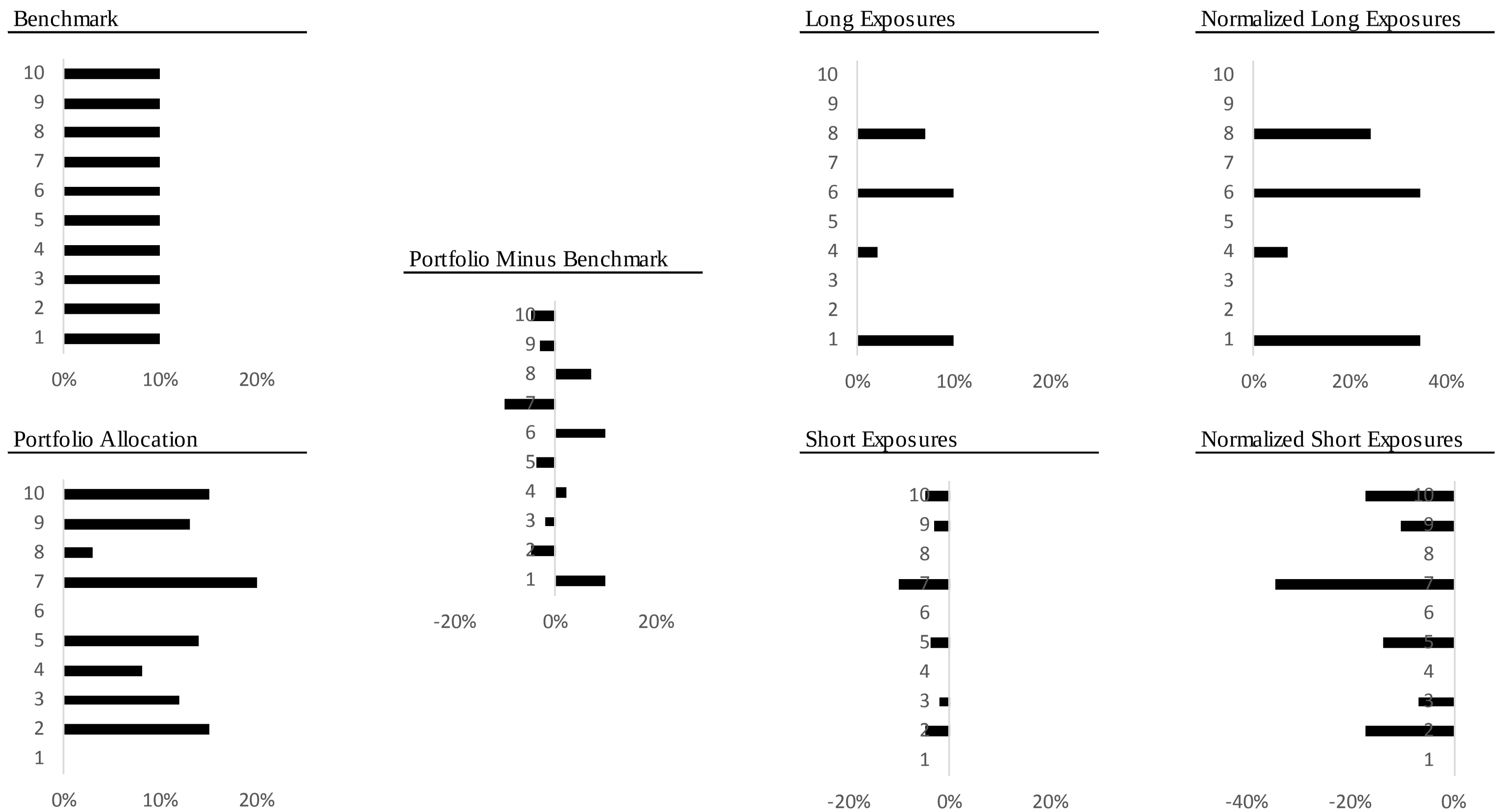
$$\text{Active Share } \times \sigma_H = \text{Tracking Error.} \tag{3}$$

I will derive and define σ_H in Section 3 of this essay, but this measure is essentially an investible, "market neutral" hedge portfolio, 100% long and 100% short, that reflects the active bets of the portfolio manager, and it can be calculated as follows:

$$\sigma_H = \sqrt{\frac{2}{\sum_{i=1}^{n}\left|w_{p,i} - w_{b,i}\right|}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(w_{p,i} - w_{b,i}\right)\left(w_{p,j} - w_{b,j}\right)\sigma_i\sigma_j\rho_{i,j}} \tag{4}$$

A graphical example of the formation of this hedge portfolio can be seen in Figure 1. In this figure, the active portfolio and the benchmark portfolio weight vectors are shown using bar graphs on the left of the figure and as you work across the figure from left to right the relative portfolio is formed and then normalized to create the hedge portfolio weight vector.

Figure 1
Graphical Example of the Formation of the Hedge Portfolio



The two bar charts on the left represend hypothetical weights to 10 different securities. The one on top represents the benchmark with equal weights to all securities and the one on the bottom a hypothetical mutual fund with differing weights to those securities. The first chart fron the left are the weights of a position created by subtracting the benchmark weights from the portfolio weights. Next, all of the positions where the portfolo is long agains the benchmark are allocated to the long exposures vector and all those where the portfolio is short against the benchmark are allocated to the short exposures portfolio. Notable, the sum of the exposures in each of these vectors is equal to the Active Share. Lastly, both the long and short exposure vectors are normalized creating two new portfolios both of which have weights that sum to 1. When combined, these two portfolios make the hedge portfolio.

Cremers and Petajisto (2009) actually use a different measure of tracking error that assumes the benchmark relative systematic risk, β_p, of every portfolio is equal to 1. This measure of tracking error is more popularly thought of as the standard deviation of the error term in a linear regression between the portfolio and the benchmark, σ_ε. The tracking error definition from above has a parameterization that illustrates this phenomenon well:

$$\text{Tracking Error}_p = \sqrt{\left(\beta_p - 1\right)^2 \sigma_b^2 + \sigma_\varepsilon^2}. \tag{5}$$

It is obvious from this formula that as β_p goes to 1, the tracking error goes to σ_ε. In the context of the difference in weights vector, $\left(w_{p,i} - w_{b,i}\right)$, σ_ε can be written as follows:

$$\sigma_\varepsilon = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \left(w_{p,i} - \beta_p w_{b,i}\right) \left(w_{p,j} - \beta_p w_{b,j}\right) \sigma_i \sigma_j \rho_{i,j}}. \tag{6}$$

Thus the similarity between Active Share and σ_ε is also apparent. σ_ε is simply the tracking error where the benchmark weights are scaled by β_p. It is important to note that Active Share and Beta are not independent of each other. As beta deviates from 1, Active Share increases. However, this phenomenon was controlled for in Cremers and Petajisto (2009) and examined in this essay as well and the level of systematic risk does not seem to affect Active Share's predictive ability. Essentially, Active Share is not biased by the systematic risk of β_p. However, the existence of this β_p possibly makes tracking error volatility, as opposed to the error regression volatility , a more appropriate measure for comparison.

In this paper, I derive a representation for Active Share that connects it theoretically to tracking error volatility. This re-parameterization suggests that it is essentially an adjusted tracking error, adjusted by σ_H. Active Share and σ_H are just reflections of each other around tracking error, a very popular measure of relative risk in delegated portfolio management. Active Share's predictive power seems at least partially related to the information already contained within tracking error. In particular, some of Active Share's pricing effect is necessarily attributable to the implied hedge volatility, σ_H. However, a substantial and economically significant part of Active Share is still unaccounted for in the hedge volatility and would typically still be considered representative of manager skill. The significance of both parameters, σ_H and Active Share, gives a better picture of how individual portfolio managers obtain and use both benchmark differentiation, i.e. deviations in weights to generate excess return, and the potential for taking advantage of their specific information.

The balance of the paper proceeds as follows. Section II includes a review of the relevant research and outlines the contribution of this project. Section III is a detailed, constructive derivation of the re-parameterization into standard portfolio parameters and a decomposition of Active Share into its component systematic and non-systematic parts. Section IV is a description of the data used. Section V describes the empirical methods and outlines the results of tests used to assert consistency of the model presented in Section III. Section VI discusses extensions and concludes the paper.

II. Background and Contribution

The history of benchmark relative measures of risk is almost as old as modern portfolio theory. This is evident in two particular ratios related to tracking error and σ_ε. Both are reminiscent of the Sharpe Ratio, which is originally introduced in Sharpe (1966) but revisited in Sharpe (1994). Rather than representing raw returns over standard deviations, the newer measures are indicative of relative performance over relative risk. The first of which is commonly referred to as the information ratio and is the benchmark excess return over the tracking error volatility:

$$\text{Information Ratio} = \frac{r_p - r_b}{te_p}, \tag{7}$$

where te_p is the tracking error volatility of the portfolio. This ratio is merely a Sharpe-style ratio for relative returns since the tracking error is just the standard deviation of relative returns or benchmark excess returns. It is difficult to determine who coined the term "information ratio", but a good description of this ratio is found in Grinold and Kahn (1999).

The second of these two ratios, the Appraisal Ratio, is similar to the Information Ratio but is oftentimes confused with it. Considering the regression of a portfolio's returns on its benchmark, the Appraisal Ratio is the Jensen's Alpha, which is the intercept term from a regression of the portfolio on its benchmark and is described in Jensen (1968), divided by the standard deviation of the regression error:

$$\text{Appraisal Ratio} = \frac{\alpha_p}{\sigma_\varepsilon}, \tag{8}$$

where α_p is Jensen's Alpha. Compared to the Information Ratio, the Appraisal Ratio has the benefit of not being biased by the presence of benchmark relative systematic risk, or beta. Both, however, are examples of early relative risk measures in in context of portfolio theory. Other definitions of the appraisal ratio have been used in the past (in fact the original definition was very different) but this one encompasses the modern usage of this term.

Active Share in particular, as well as tracking error, are highly related to the mathematical notion of a p-norms. Essentially, Active Share is the 1-norm or the "length" of the difference in weight vector, $(w_{p,i} - w_{b,i})$, divided by 2. A 2-norm calculation would provide the true length of the vector in two dimensional space and the squared difference terms make the formula look much more like a variance or standard deviation (or more importantly a tracking error) albeit without the covariance matrix[5]:

$$\|\mathbf{w_p} - \mathbf{w_b}\|_2 = \sqrt{\sum_{i=1}^{n} (w_{p,i} - w_{b,i})^2}\,. \tag{9}$$

This measure is very similar to one devised by Kacperczyk, Sialm, and Zheng (2005) called the Industry Concentration Index and is similar to a Herfindahl–Hirschman Index:

$$\text{Industry Concentration Index} = \sum_{i=1}^{n} (w_{p,i} - w_{b,i})^2. \tag{10}$$

This measure differs in that, other than the obvious lack of the square root, the counter on the sum is a counter for the industry rather than the security. Cremers and Petajisto

[5] Actually, this measure is equivalent to a standard deviation measure when covariance matrix equal to the identity matrix.

(2009) compare the analysis of this measure to that of Active Share, and to a measure of Active Share calculated on the industry level rather than the security level, and find that it is just as significant in pricing regressions however when included together, Active Share tends to dominate. When they analyze the squared calculation at the security level, they find no significance whatsoever. Using a sample of Australian firms however, Brand, Brown, and Gallagher (2005) calculate this exact measure and call it the divergence index. They find that their divergence index predicts fund performance.

Another interesting permutation of Active Share was analyzed in Jiang, Verbeek, and Wang (2011), which creates a cross-sectional average of the relative weights on individual securities. They call their measure Deviations from Benchmark or *DFB* and it is defined as follows:

$$DFB_{i,t} = \sum_{p=1}^{n_i} \left(w_{p,i,t} - w_{b,i,t} \right) / n_{i,} \tag{11}$$

where n_i is the number of funds that hold security *i*. In this context, the authors are trying to measure the average distance a typical fund differentiates from the benchmark rather than the specific amount any particular fund differentiates. Their measure, obviously related to Active Share, also produces positive pricing effects thus is potentially also a candidate to support the idea of manager skill. With their measure they also find that this skill is particularly evident around earnings announcements.

Although the examples above look a lot like Active Share, there is a whole other classification of measures that tries to reveal similar information. Kacperczyk, Sialm, and Zheng (2008) measure what they call the return gap. This is the return difference between a portfolio comprised of the point in time holdings extrapolated forward and the actual returns the portfolio experienced, presumably due to changes in the underlying holdings since the report date of the holdings. This return gap is important and used in Puckett and Yan (2011). They find that these interim portfolio changes seem to be what drives manager outperformance, not end-of-period holdings.

Amihud and Goyenko (2013) use a fund's R^2, generated using a multifactor regression analysis, as a measure of selectivity and assert that lower R^2 significantly predicts better performance. Low R^2 is similar in this manner to high Active Share in that it allows for greater manager selectivity and control of the portfolio, i.e. less reliance on the benchmark. In fact, the authors contrast their measure of selectivity to Active Share and imply strongly that they both measure the same information. Given the relative difficulty of obtaining all of the relevant information required to calculate Active Share on a large sample of mutual funds, and the relative ease in obtaining, for example, return data, R^2 produces just as strong results with less effort. Active Share and R^2 are mechanically and functionally related and their connection is due to their relationships to both tracking error and the implied hedge volatility. In another active management skill story, Kacperczyk and Seru (2007) show that managers who are sensitive to public information (RPI) are less likely to outperform. Their RPI measure is simply an R^2 measure that attempts to identify benchmark differentiation.

Many authors advocate for a method of portfolio construction that emphasizes dependence on relative performance, both in risk and return, simply because optimization in that space removes the obligation that risk aversion in absolute space be considered.

However, risk aversion in relative space is still essential. For evidence of this see Becker et. al. (1999) and Grinold and Kahn (2000). A good example of how this is developed can also be found in Stutzer (2004). It also suggests how we can avoid paradoxes such as Roll's critique, Roll (1977). However, anecdotes of this analysis can be found in many highly cited financial papers such as Gompers and Metrik (2001), Pastor and Stambaugh (2002a,b), Berk and Green (2004), and Cetin (2006). Even very recent papers provide conclusions based on the assumption of consistency of the relative performance optimization. Examples of these are Cuoco and Kaniel (2011) and Li and Tiwari (2009), which further suggests that principals are supportive of this relative optimization in their deliberate alignment of incentive contracts with the solutions to a relative optimization problem.

However, the real problem with relative measures of risk, such as tracking error, regression error variance, and Active Share, is that the traditional systematic priced risk factors are either symmetric around the horizontal axis[6], or explicitly eliminated from the measure. Essentially, increases in a traditional systematic factor, such as beta, above the beta of the relative index are equally offset by decreases. A beta of 1.5 will produce the same level of relative risk as a beta of 0.5. An outline of this phenomenon can be found in Jorion (2003) and is hinted at in Roll (1992). Pricing models with measures of relative risk can be thought of as agnostic to the regular systematic factors. Therefore, if pricing is found on relative risk factors, we can be assured that the pricing is not attributable to systematic factors, assuming the relative risk factor is not biased in systematic risk.

[6] See Figure 2 for a simple demonstration of this with Active Share.

The first contribution of this essay is in connecting Active Share mathematically to these other methods through a simple manipulation of the active bets a manager takes. In this manner, Active Share is really just a different way to look at the information we have always used when considering and analyzing problems in relative portfolio management. Through this relationship, the standard metrics we have used to measure relative or delegated portfolio management throughout time are intimately connected to Active Share all through the measure I call the volatility of the hedge. This measure, derived constructively in the next section and gives hints at the economic interpretation of Active Share and measures like it.

When attributing tracking error into the product of the volatility of the hedge and Active Share, Active Share unambiguously becomes the parameter associated with a manager’s confidence level in the information implied by the manager’s active hedge against the benchmark. Active Share’s predictive effects are thus consistent with the idea of manager skill. But on the other side of the coin, the implied hedge volatility also contains information about manager skill. It is the risk of the manager’s information. In this context, it is just another form of attribution. It is an attribution of manager confidence and information risk. In this context, it can be connected to literature in risk aversion and signal precision, such as in Admati et. al. (1986).

III. A Constructive Decomposition of Active Share

Active Share, as introduced to the academic literature in Cremers and Petajisto (2009), is an intuitive and simple metric for evaluating benchmark deviation. In the context that it is introduced, it is by definition the percent of the fund's holdings that deviate from the benchmark. However, it comes in a difficult and unfamiliar functional form. Active Share is defined as:

$$Active\ Share = as_p = \frac{1}{2}\sum_{i=1}^{n}\left|w_{p,i} - w_{b,i}\right| = \frac{1}{2}\left|\mathbf{w_p} - \mathbf{w_b}\right|_{\mathbf{1}} \tag{12}$$

$w_{p,i}$ and $w_{b,i}$ are the portfolio weights of asset *i* in the fund and in the benchmark. $\left|\mathbf{w_p} - \mathbf{w_b}\right|_{\mathbf{1}}$ is the 1-norm of the difference in the weight vectors and is simply another notation for the absolute value of the summation over *i*. Although this calculation is simple and intuitive, it is difficult to visualize in the context of how we typically study portfolio theory and construction. Namely, it would be more convenient if we could see what Active Share looks like parameterized using more standard portfolio math parameters, standard deviation, correlation, beta, etc.

The key to the re-parameterization is in the intuition behind the original equation. If we consider the benchmark as the base position and make a 100% investment into the benchmark, Active Share is the percent invested into each of a long and short portfolio that make up the hedge portfolio used to reweight the benchmark to that of the fund in question. Thus, I prove the following statement:

III.A Theorem: TE = AS × σH

Tracking error volatility is the product of Active Share and the volatility of a 100% long, 100% short hedge fund (the volatility of the implied hedge). Essentially, $TE = AS \times \sigma_H$ when:

$TE = [(\mathbf{w_p} - \mathbf{w_b})' \,\boldsymbol{\Omega}\, (\mathbf{w_p} - \mathbf{w_b})]^{½}$

$AS = ½ \sum |\, \mathrm{w}_{p,i} - \mathrm{w}_{b,i} \,| = ½\| \mathbf{w_p} - \mathbf{w_b} \|$

$\sigma_H = [\mathbf{w_H}' \,\boldsymbol{\Omega}\, \mathbf{w_H}]^{½}$

Proof:

Suppose $\mathbf{w_p}$ and $\mathbf{w_b}$ represent the $n \times 1$ weight vectors for portfolio p and benchmark b respectively such that $\mathbf{w_p} \neq \mathbf{w_b}$ and:

$$\mathbf{w_p}'\mathbf{1} = 1 \quad \text{and} \quad \mathbf{w_b}'\mathbf{1} = 1 \tag{t.1}$$

Consider the following difference-in-weights vector, $\mathbf{x_H}$:

$$\mathbf{x_H} = \mathbf{w_p} - \mathbf{w_b} \tag{t.2}$$

Note that by (t.1) and (t.2) we have:

$$\mathbf{x_H}'\mathbf{1} = (\mathbf{w_p} - \mathbf{w_b})'\mathbf{1} = \mathbf{w_p}'\mathbf{1} - \mathbf{w_b}'\mathbf{1} = 1 - 1 = 0 \tag{t.3}$$

Therefore, $\mathbf{x_H}$ is a zero weight hedge portfolio. Define two new vectors $\mathbf{x_L}$ and $\mathbf{x_S}$ so that $\mathbf{x_H}$ is split into its long and short exposures:

$$\text{Let } \mathbf{x_L} = \mathbf{x_H} \text{ when } \mathbf{x_H} > 0 \text{ otherwise } \mathbf{x_L} = 0. \tag{t.4}$$

$$\text{Let } \mathbf{x_S} = -\,\mathbf{x_H} \text{ when } \mathbf{x_H} < 0 \text{ otherwise } \mathbf{x_S} = 0. \tag{t.5}$$

$$\mathbf{x_H} = \mathbf{x_L} - \mathbf{x_S} \tag{t.6}$$

By (t.3) and (t.6) we have:

$$\mathbf{x_H}'\mathbf{1} = (\mathbf{x_L} - \mathbf{x_S})'\mathbf{1} = \mathbf{x_L}'\mathbf{1} - \mathbf{x_S}'\mathbf{1} = 0 \tag{t.7}$$

Thus by (t.7) we have:

$$\mathbf{x_L}'\mathbf{1} = \mathbf{x_S}'\mathbf{1} \tag{t.8}$$

The lengths of each side of the hedge portfolio, $\mathbf{x_H}$, are equal. Define two new vectors $\mathbf{w_L}$ and $\mathbf{w_S}$ by normalizing $\mathbf{x_L}$ and $\mathbf{x_S}$ as follows:

$$\mathbf{w_L} = (\mathbf{x_L}'\mathbf{1})^{-1}\,\mathbf{x_L} \tag{t.9}$$

$$\mathbf{w_S} = (\mathbf{x_S}'\mathbf{1})^{-1}\,\mathbf{x_S} \tag{t.10}$$

Note that both $\mathbf{w_L}$ and $\mathbf{w_S}$ sum to 1:

$$\mathbf{w_L}'\mathbf{1} = (\mathbf{x_L}'\mathbf{1})^{-1}\,\mathbf{x_L}'\mathbf{1} = 1 \tag{t.11}$$

$$\mathbf{w_S}'\mathbf{1} = (\mathbf{x_S}'\mathbf{1})^{-1}\,\mathbf{x_S}'\mathbf{1} = 1 \tag{t.12}$$

$$\mathbf{w_L}'\mathbf{1} = \mathbf{w_S}'\mathbf{1} \tag{t.13}$$

Both portfolio L and portfolio S are 100% weight vectors and hypothetically investable. Additionally, their weights are also equal. Next, combine these vectors to create portfolio H:

$$\mathbf{w_H} = \mathbf{w_L} - \mathbf{w_S} \tag{t.14}$$

By (t.11), (t.12), and (t.14):

$$\mathbf{w_H}'\mathbf{1} = (\mathbf{w_L} - \mathbf{w_S})\,'\mathbf{1} = \mathbf{w_L}'\mathbf{1} - \mathbf{w_S}'\mathbf{1} = 1 - 1 = 0 \tag{t.15}$$

Thus by (t.13) and (t.15), portfolio H is a zero weight hedge portfolio where the long and short side of the hedge are both weighted to 100%, i.e. it is a 100% long, 100% short hedge portfolio. From (t.6), (t.9), (t.10), (t.13), and (t.14), the following can be shown:

$$\mathbf{x_H} = \mathbf{x_L} - \mathbf{x_S} = (\mathbf{x_L}'\mathbf{1})\mathbf{w_L} + (\mathbf{x_S}'\mathbf{1})\mathbf{w_S} = (\mathbf{x_L}'\mathbf{1})(\,\mathbf{w_L} - \mathbf{w_S}) = (\mathbf{x_L}'\mathbf{1})\mathbf{w_H} \tag{t.16}$$

Since $\mathbf{x_L}$ and $\mathbf{x_S}$ are strictly non-negative the expressions $(\mathbf{x_L}'\mathbf{1})$ and $(\mathbf{x_S}'\mathbf{1})$ can be restated in terms of a 1-norm, defined as $\|\,\mathbf{x}\,\| = \sum |\,x_i\,|$:

$$(\mathbf{x_L}'\mathbf{1}) = \|\,\mathbf{x_L}\,\| \tag{t.17}$$

$$(\mathbf{x_S}'\mathbf{1}) = \|\,\mathbf{x_S}\,\| \tag{t.18}$$

Again from (t.6), (t.17), and (t.18), if I take the 1-norm length of $\mathbf{x_H}$, the following is true:

$$\|\,\mathbf{x_H}\,\| = \|\,\mathbf{x_L} - \mathbf{x_S}\,\| = \|\,\mathbf{x_L}\,\| + \|\,\mathbf{x_S}\,\| = 2\,(\mathbf{x_L}'\mathbf{1}) \tag{t.19}$$

And from (t.16) and (t.19), the scalar that relates $\mathbf{x_H}$ to $\mathbf{w_H}$ becomes apparent. It is just one half of the 1-norm length of $\mathbf{x_H}$:

$$\mathbf{x_H} = \tfrac{1}{2}\| \mathbf{x_H} \| \, \mathbf{w_H} \tag{t.20}$$

However, one half the 1-norm length of $\mathbf{x_H}$ is just the Cremers and Petajisto (2009) Active Share, *AS*. From (t.2) and the definition of the 1-norm:

$$\tfrac{1}{2}\| \mathbf{x_H} \| = \tfrac{1}{2}\| \mathbf{w_p} - \mathbf{w_b} \| = \tfrac{1}{2} \sum | w_{p,i} - w_{b,i} | = AS \tag{t.21}$$

From (t.2), (t.20) and (t.21), the relationship between the portfolio weight vector, $\mathbf{w_p}$, the benchmark weight vector, $\mathbf{w_b}$, and the implied hedge weight vector, $\mathbf{w_H}$, is as follows:

$$\mathbf{w_p} - \mathbf{w_b} = \mathbf{x_H} = \tfrac{1}{2}\| \mathbf{x_H} \| \, \mathbf{w_H} = AS \times \mathbf{w_H} \tag{t.22}$$

Applying the portfolio variance function, where $\mathbf{\Omega}$ is the $n \times n$ covariance matrix, to the relationship in (t.22) it can be shown that:

$$\mathbf{w_p} - \mathbf{w_b} = AS \times \mathbf{w_H}$$

$$(\mathbf{w_p} - \mathbf{w_b})' \, \mathbf{\Omega} \, (\mathbf{w_p} - \mathbf{w_b}) = AS^2 \times \mathbf{w_H}' \, \mathbf{\Omega} \, \mathbf{w_H}$$

$$[(\mathbf{w_p} - \mathbf{w_b})' \, \mathbf{\Omega} \, (\mathbf{w_p} - \mathbf{w_b})]^{½} = AS \times [\mathbf{w_H}' \, \mathbf{\Omega} \, \mathbf{w_H}]^{½} \tag{t.23}$$

And, by the definitions provided in the statement of the theorem, (t.23) shows that tracking error volatility is the product of Active Share and the volatility of a 100% long, 100% short hedge fund (the volatility of the implied hedge). Essentially:

$$TE = AS \times \sigma_H \qquad \square \tag{t.24}$$

III.B Extensions of the identity

Stated another way, the active share is equal to the fund's tracking error divided by the standard deviation of the risk of the implied hedge position.

$$AS_p = \frac{TE_p}{\sigma_H} \tag{13}$$

This expression leads to a very interesting decomposition of active share into its systematic and idiosyncratic components. Consider first the regression of r_b on r_f in the standard market model:

$$r_{p,i} = \alpha_p + \beta_p r_{b,i} + \varepsilon_{p,i} \tag{14}$$

A popular expression for tracking error is formulated using the parameters from the above regression expression:

$$te_p^2 = \left(1 - \beta_p\right)^2 \sigma_b^2 + \sigma_\varepsilon^2 \tag{15}$$

In this context, tracking error is decomposed into its systematic and non-systematic components. Similarly, due to the relationship between tracking error and active share, this decomposition can be used to express active share as a combination of systematic and non-systematic components.

$$AS_p^2 = \left(1 - \beta_p\right)^2 \frac{\sigma_b^2}{\sigma_H^2} + \frac{\sigma_\varepsilon^2}{\sigma_H^2} \tag{16}$$

Given the functional relationship above, it is obvious that deviations in active share (active share greater than zero) are generated by both systematic deviations (benchmark relative) and non-systematic deviations from the benchmark index. Therefore, analysis of the uses of active share as a risk measure for pricing purposes must recognize that part of the deviation is due to systematic risk.

Before talking about relative return, we have to provide a framework to in which to study it in the same manner as we did for risk above. Since we rely on the market model in the previous section to parameterize active share, this follow-up is appropriate. Therefore, consider the market model for fund p:

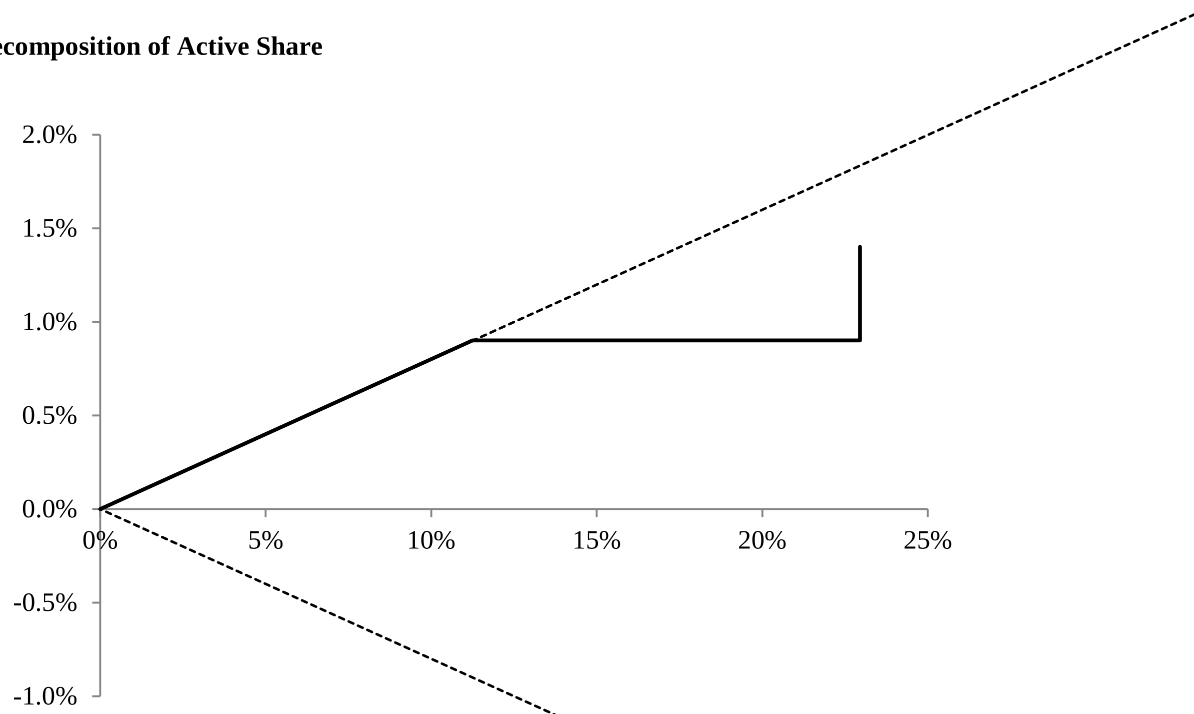
$$r_p = \alpha_p + \beta_p r_b \tag{17}$$

We define excess return as the return of the portfolio minus the benchmark and adjusting the market model appropriately yields the following relationship:

$$r_p - r_b = \alpha_p + (\beta_p - 1) r_b \tag{18}$$

Represented in this specification, on the left, is the excess return. On the right are α_p, the return attributable to idiosyncratic risk, and $(\beta_p - 1) r_b$, the return attributable to systematic risk. Figure 2 illustrates this decomposition on a graph.

Figure 2
Graphical Decomposition of Active Share



This figure shows graphically how active share is decomposed and attributed between systematic and idiosyncratic risk. Active Share is on the horizontal axis and Excess return in on the vertical axis. The dotted line represents systematic deviations in benchmark relative beta and the solid line an example of the deviations of an example mutual fund. It is evident that if a fund deviates from its benchmark exposure either by increasing or decreasing benchmark beta, large deviations in active share are possible.

IV. Data

Following Cremers and Petajisto (2009), I use the universe of mutual funds for this study. Given the number of products available and the percentage of the total market capitalization managed in these portfolios, it is clear that the market is mostly comprised and therefore probably also driven by delegated managers, like mutual funds. According to the Investment Company Institute[7], as of December 2011 there were $12,968 billion invested in 16,506 registered investment companies. The scope and breadth of external management strongly suggests that these externally managed portfolios make up a substantial, representative sample of the market. Therefore, when attempting to analyze market pricing phenomena, it seems appropriate to use the cross section of mutual funds as a representative sample for the population of delegated investment managers. However, the sample if necessarily incomplete as hedge funds, pension funds, trusts, and separate accounts are also not trivial. But, it should be representative.

This paper relies on five major data sources, CRSP, Thomson Reuters, Petajisto, Compustat, and Morningstar. First, return data on the set of mutual funds is required and obtained from CRSP. When data on mutual fund holdings are needed, I use Thomson Reuters although this information is also available in CRSP. The primary reason to use the holdings of mutual funds is to calculate Active Share. However, I have defaulted to using the Active Share calculations already provided by Antti Petajisto on his website[8] because the list is relatively comprehensive. This also follows other studies that have

---

[7] http://www.icifactbook.org/

[8] http://www.petajisto.net/data.html

used the Active Share measure in research, e.g. Amihud and Goyenko (2013). Another essential source of data used to calculate Active Share is index constituency. Access to the S&P index constituents is available in Compustat. To the extent possible, Active Shares were recalculated independently and confirmed for the three major S&P indices. The lack of availability for the remainder of the index constituencies is not a concern. Lastly, I use Morningstar to obtain index returns for all of the indices in the dataset.

To begin constructing the dataset, I started with the list of funds and statement dates provided by Petajisto on his website. This list is then matched to CRSP and Compustat to obtain fund characteristics, returns, and risk parameters. The list is matched with Morningstar to obtain index returns. From the combined Morningstar and CRSP data, the relative risk parameters can be calculated. After matching and calculating all of the relevant parameters, the data is filtered. I first remove all funds that identify as an index fund or as an enhanced index fund. The purpose of this study is to test active management and these funds by definition do not qualify. Next, I filter out, somewhat arbitrarily, funds that have exceptionally strange characteristics. Essentially this includes removing large outliers that look like incorrectly measured observations. Lastly, I filter out all funds that have Active Shares < 5%, or annualized tracking errors < 0.1%, to account for closet indexers. The final sample yields 61,857 fund-time observations. The number of observations by time is summarized before the filter in Table A1. All filters together, including removing self-identified index funds, reduced the dataset from an original matched 73,317 observations.

Table A1
Number of Funds by Month and Year of Statement Date

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
1990						303	17	8	268	20	9	300	925
1991	20	11	298	28	14	314	15	12	261	25	13	340	1351
1992	16	14	312	28	15	373	15	18	206	22	9	239	1267
1993	9	13	374	35	26	431	18	33	355	87	42	380	1803
1994	37	49	282	97	60	392	34	54	320	134	60	370	1889
1995	40	57	297	151	78	398	46	61	336	155	72	376	2067
1996	43	42	329	117	64	510	33	51	453	156	74	531	2403
1997	44	70	507	208	91	555	53	70	460	209	84	557	2908
1998	58	79	595	158	83	638	46	49	745	94	32	846	3423
1999	49	38	683	88	33	801	62	44	810	55	18	1005	3686
2000	37	45	876	95	43	1006	24	40	919	68	35	1105	4293
2001	28	44	850	67	43	964	44	35	1055	70	48	1027	4275
2002	34	41	1129	86	56	1179	49	47	1003	69	31	1118	4842
2003	50	44	1028	125	38	1220	88	54	1143	209	72	1129	5200
2004	124	121	1022	191	113	1065	270	164	1162	320	190	1040	5782
2005	275	189	1065	266	225	1076	311	244	1029	249	96	1011	6036
2006	318	163	1061	328	173	1065	319	182	1054	386	208	899	6156
2007	332	187	900	339	196	921	286	163	1010	318	203	973	5828
2008	327	159	994	280	159	923	235	177	1124	258	112	945	5693
2009	191	87	952	254	137	908	268	145	548				3490
Total	2032	1453	13554	2941	1647	15042	2233	1651	14261	2904	1408	14191	

This table includes the unfiltered dataset for all 73,317 observations categorized by Active Share months and years.

The final dataset is summarized in Table 1. This table includes the summary statistics for all 61,857 observations in the dataset with active share statement dates ranging from 12/31/1990 through 09/30/2009. The top two entries, activeshare_min and trackingerror_min, are the values as reported on Antti Petajisto's website at www.petajisto.net. The remainder of the data points come in the following format:

##mxPAR

The number before each calculation, ##, represents the number of months, m, over which the calculation is made. The x is stands for the time period of the calculation and equals either p, a, or c. It signifies that the calculation is made either ex-<u>a</u>nte, ex-<u>p</u>ost, or <u>c</u>entered on the Active Share statement date. The final PAR is the parameter under consideration.

Beta is the portfolio's benchmark relative beta,

TE is the benchmark relative tracking error volatility,

SE is the standard deviation of the regression error from market model regression,

SB is the standard deviation of the benchmark,

SP is the standard deviation of the Portfolio,

SH is the standard deviation of the 100% long/short benchmark relative hedge,

BAA is the benchmark adjusted alpha,

ER is the excess return measured PR – BR,

PR is the portfolio return, and

BR is the benchmark return.

Table 1
Summary Statistics

	Mean	Standard Deviation	5th %	25th %	50th %	75th %	95th %
			Panel A: Risk Statistics				
activeshare_min	0.7804	0.1529	0.4960	0.6770	0.8090	0.9100	0.9680
trackingerror_min	0.0661	0.0423	0.0221	0.0385	0.0558	0.0809	0.1464
12maBeta	0.9697	0.2298	0.6248	0.8447	0.9626	1.0804	1.3407
12maTE	0.0612	0.0435	0.0188	0.0334	0.0499	0.0750	0.1425
12maSE	0.0525	0.0355	0.0167	0.0293	0.0435	0.0646	0.1182
12maSB	0.1516	0.0731	0.0636	0.0925	0.1392	0.1915	0.3069
12maSP	0.1577	0.0803	0.0662	0.0993	0.1399	0.1942	0.3126
12maSH	0.0774	0.0509	0.0294	0.0452	0.0635	0.0931	0.1721
12mpBeta	0.9703	0.2240	0.6324	0.8498	0.9642	1.0767	1.3321
12mpTE	0.0601	0.0428	0.0185	0.0328	0.0490	0.0737	0.1393
12mpSE	0.0516	0.0351	0.0164	0.0288	0.0426	0.0635	0.1168
12mpSB	0.1551	0.0744	0.0654	0.0934	0.1430	0.1969	0.3136
12mpSP	0.1607	0.0806	0.0668	0.1010	0.1439	0.1995	0.3152
12mpSH	0.0758	0.0494	0.0293	0.0447	0.0624	0.0909	0.1675
6mpBeta	0.9721	0.2898	0.5514	0.8212	0.9596	1.1018	1.4357
6mpTE	0.0551	0.0428	0.0146	0.0281	0.0439	0.0680	0.1334
6mpSE	0.0438	0.0336	0.0113	0.0224	0.0351	0.0548	0.1051
6mpSB	0.1418	0.0755	0.0558	0.0868	0.1237	0.1744	0.3092
6mpSP	0.1468	0.0824	0.0549	0.0902	0.1271	0.1793	0.3112
6mpSH	0.0694	0.0497	0.0224	0.0385	0.0564	0.0844	0.1595
			Panel B: Performance Statistics				
12maBAA	0.18%	10.27%	-13.90%	-4.43%	-0.27%	4.08%	15.16%
12maER	0.01%	10.86%	-14.19%	-4.92%	-0.67%	3.75%	15.89%
12maBR	7.52%	21.18%	-32.29%	-4.68%	10.86%	21.13%	38.75%
12maPR	7.53%	23.02%	-33.54%	-4.77%	9.92%	20.39%	40.86%
1mpER	-0.12%	2.26%	-3.59%	-1.15%	-0.11%	0.91%	3.32%
1mpBR	0.61%	5.46%	-8.37%	-2.45%	1.16%	4.06%	8.37%
1mpPR	0.49%	5.66%	-8.91%	-2.57%	0.94%	3.91%	8.72%
3mpER	-0.07%	4.17%	-6.15%	-1.96%	-0.18%	1.66%	6.23%
3mpBR	1.92%	10.20%	-17.41%	-3.14%	2.77%	7.49%	17.58%
3mpPR	1.85%	10.80%	-17.78%	-3.40%	2.51%	7.85%	18.14%
6mpBAA	-0.03%	6.46%	-9.54%	-2.95%	-0.11%	2.76%	9.42%
6mpER	-0.15%	6.36%	-9.07%	-3.02%	-0.37%	2.35%	9.20%
6mpBR	3.99%	14.75%	-26.93%	-3.48%	5.97%	11.94%	25.45%
6mpPR	3.84%	15.71%	-26.78%	-3.78%	5.59%	12.22%	26.30%
12mpBAA	-0.02%	9.62%	-13.36%	-4.43%	-0.40%	3.79%	14.04%
12mpER	-0.33%	10.18%	-14.14%	-4.96%	-0.83%	3.43%	14.52%
12mpBR	7.99%	22.02%	-34.28%	-5.38%	11.04%	21.48%	41.77%
12mpPR	7.67%	23.51%	-34.65%	-5.13%	9.97%	20.66%	43.65%

This table includes the summary statistics for all 61,857 observations in the dataset with active share statement dates ranging from 12/31/1990 through 09/30/2009. The top two entries, activeshare_min and trackingerror_min, are the values as reported on Antti Petajisto's website at www.petajisto.net. The remainder of the data comes from CRSP and Morningstar and matched with the Petajisto data. The number before each calculation represents the number of months over which the calculation is made and the p, a, or c represent whether the calculation was made ex-ante, ex-post, or centered on the Active Share statement date. T-statistics are in parentheses.

The standard deviation risk statistics are all annualized but the return numbers are raw returns over the corresponding number of months.

The average Active Share in the sample is 78.04%. This is not particularly skewed except for the extreme left tail. The sample average tracking error, depending on how it is calculated, is around 6% and this number is highly positively skewed. The average value of the implied hedge volatility is around 7.5% and this number is also highly skewed. The average Beta in the sample is less than 1 at 0.9703. In the return statistics, the negative average excess returns reflect the expectation that the average manager underperforms the benchmark by expenses. In particular, looking at 12mpER, this average underperformance seems to be about 0.33%. The distribution of ER and BAA seems mostly symmetric.

Table 2 includes correlation statistics. The first thing to note is that the original measure of trackingerror_min (which is actually the volatility of the regression error, an SE measure) from Cremers and Petajisto (2009) is highly correlated to both my measures TE and SE from the 12ma and 12mp period. I also have these statistics calculated for the 12mc period and the centered period has even higher correlations, as should be expected. The other highlighted cells represent the correlation between the SE and TE calculation. Recall that these two measures are highly related, particularly when Beta is close to 1. The boxes are around the correlations that would signify persistence in the risk measures. The 12ma period represents the 12 months immediately preceding the 12mp period. These correlations are very high and although I haven't provided any robust evidence for this persistence, it seems initially likely. The SH measure is substantially higher in correlation when compared to the tracking error numbers than is Active Share. Active Share still is rather correlated but to a lesser degree.

Table 2
Correlation Table for Risk Statistics

	activeshare_min	trackingerror_min	12maBeta	12maTE	12maSE	12maSH	12mpBeta	12mpTE	12mpSE	12mpSH
activeshare_min	1	0.4022	-0.0324	0.3757	0.3853	0.1021	-0.0497	0.3956	0.4056	0.1294
trackingerror_min	0.4022	1	0.0139	0.8267	0.7701	0.7560	0.0090	0.6736	0.6041	0.5989
12maBeta	-0.0324	0.0139	1	0.0536	0.1364	0.0627	0.4899	0.0606	0.1172	0.0746
12maTE	0.3757	0.8267	0.0536	1	0.9323	0.9384	0.0683	0.6570	0.5977	0.5851
12maSE	0.3853	0.7701	0.1364	0.9323	1	0.8667	0.1286	0.6519	0.6326	0.5769
12maSH	0.1021	0.7560	0.0627	0.9384	0.8667	1	0.0868	0.5713	0.5054	0.6006
12mpBeta	-0.0497	0.0090	0.4899	0.0683	0.1286	0.0868	1	0.0446	0.1154	0.0637
12mpTE	0.3956	0.6736	0.0606	0.6570	0.6519	0.5713	0.0446	1	0.9339	0.9398
12mpSE	0.4056	0.6041	0.1172	0.5977	0.6326	0.5054	0.1154	0.9339	1	0.8676
12mpSH	0.1294	0.5989	0.0746	0.5851	0.5769	0.6006	0.0637	0.9398	0.8676	1

This table includes correlation statistics for all 61,857 observations in the dataset with active share statement dates ranging from 12/31/1990 through 09/30/2009. The top two entries, activeshare_min and trackingerror_min, are the values as reported on Antti Petajisto's website at www.petajisto.net. The remainder of the data comes from CRSP and Morningstar and matched with the Petajisto data. The highlighted cells are emphasized to identify consistency with two of the concerns with this paper. First, Cremer and Petajisto's calculation of tracking error, although slightly different than the ones in this paper, have very high correlations with my calculations. Also, the correlation between tracking error and regression standard error is extremely high. The boxes highlight the persistence of the risk parameters from one period to the next. All of the risk statistics seem, at least in correlation, highly persistent. T-statistics are in parentheses.

In general, this final dataset seems representative of a reasonable sample of actively managed portfolios. It is also reflective of the samples from other papers such as Kacperczyk, Sialm, and Zheng (2008), Amihud and Goyenko (2013), and of course Cremers and Petajisto (2009). Given the performance statistics, at least on the surface, it doesn't seem that the data is biased by survivorship and the distribution of the risk parameters are within reasonable thresholds for magnitude and distribution.

V. Date Analysis and Model Justification

V.A The Determinants of Active Share

A major component and conclusion of Cremers and Petajisto (2009) is that a substantial portion of Active Share is described by tracking error. As shown in section III, there is a fundamental and mathematically factual relationship that makes this relationship true. In fact, the expression:

$$TE = AS\,\sigma_H \tag{19}$$

is an identity, at least for every individual fund/time observation where the difference between the two parameters is merely the volatility of the implied hedge.

This identity doesn't make the cross sectional regression of one component on the other irrelevant however. Take for instance the average value of SH from table 1, 0.0774. When regressing Tracking error as the independent variable on Active Share as the dependent variable should yield a cross sectional average regression parameter reflective of the inverse of the average SH, about 12.9. In table 3, this regression, specification (2), yields a parameter of 1.3205 suggesting that there is some significant

Table 3
The Determinants of Active Share and the Volatility of the Implied Hedge

Panel A: Regressions with Tracking Error Volatility

	activeshare_min		12maTE		12maSH		12maTE	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	N/A	0.6996	N/A	-0.0222	N/A	0.0103	N/A	-0.0009
		(711.2203)		(26.3247)		(84.0938)		(8.3742)
12maTE	8.9144	1.3205			1.2085	1.0971		
	(387.9639)	(100.8285)			(1216.7348)	(675.2632)		
activeshare_min			0.0795	0.1069				
			(387.9639)	(100.8285)				
12maSH							0.7943	0.8026
							(1216.7348)	(675.2632)
N	61857	61857	61857	61857	61857	61857	61857	61857
R2	0.7087	0.1565	0.7087	0.1565	0.9599	0.8805	0.9599	0.8805

Panel B: Regressions with the Regression Standard Error

	activeshare_min		12maSE		12maSH		12maSE	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	N/A	0.6933	N/A	-0.0173	N/A	0.0122	N/A	0.0057
		(684.5689)		(25.2686)		(67.2289)		(43.6280)
12maSE	10.7247	1.6598			1.4020	1.2420		
	(409.0891)	(103.8412)			(840.8160)	(432.1168)		
activeshare_min			0.0681	0.0894				
			(409.0891)	(103.8412)				
12maSH							0.6559	0.6048
							(840.8160)	(432.1168)
N	61857	61857	61857	61857	61857	61857	61857	61857
R2	0.7301	0.1484	0.7301	0.1484	0.9195	0.7512	0.9195	0.7512

This table includes all observations with active share statement dates ranging from 12/31/1990 through 09/30/2009. This table mirrors the active share determination table from Cremers and Petajisto (2009) with the addition of regressing on the implied hedge volatility. Additionally, recognizing the functional relationship between Active Share, Tracking Error Volatility, and the Implied Hedge Volatility, this analysis flips the dependent and independent variables as well as regressing with a zero constant. Given the theoretical relationship between these three parameters, these simple regressions give insight into whether these parameters can be measured cross-sectionally. T-statistics are in parentheses.

misfit or misspecification in the model. However, when forcing the intercept to 0, the parameter jumps to 8.8144, more in line with our expectation about the theoretical value of this parameter.

Rather than regress in this direction, the theoretical model suggests that perhaps we should regress SE on TE as in specifications (3) and (4). Note that the value of the parameter in specification (3) is 0.0795, which is pretty close to the average value from table 1. This is repeated for SH and TE, attempting to estimate Active Share in the cross section and the values are 0.7943 in specification (7) and 0.8026 in specification (8). Compared to an average value of 0.7804 in table 1, this is incredibly accurate. Statistical tests to show that the cross-sectional regressed values are different than the averages are highly significant but one cannot argue that they are at least in the same economic ballpark. The results of these regressions are highly consistent with the construction from section 3 of this essay. To some extent, this is not surprising given the results from Cremers and Petajisto (2009), but the tests here are more theoretically appropriate for measuring determination.

V.B The Pricing of Active Share in Relative Performance

One of the major conclusions of Cremers and Petajisto (2009) is that Active Share is predictive of excess performance. Additionally, tracking error seems not particularly predictive. The functional relationship in this paper shows that if Active Share has predictive power and tracking error does not, then the implied hedge volatility must also have predictive power, and in the opposite direction.

Table 4
Predictive Regressions of the 6 Month Benchmark Adjusted Alphas Following Active Share Statement Dates

	6mpBAA									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Intercept	-0.0117	-0.0005	-0.0117	-0.0017	-0.0014	0.0248	-0.0033	0.0027	-0.00565	0.0029
	(8.6629)	(1.1552)	(8.4676)	(3.7075)	(3.1215)	(22.0710)	(5.5453)	(4.6701)	(2.1597)	(0.9179)
activeshare_min	0.0146		0.0147						0.0093	-0.0022
	(8.6190)		(8.5941)						(5.0018)	(0.7344)
12maSH		0.0035	-0.0010							-0.1181
		(0.6832)	(0.1973)							(4.9826)
12maTE				0.0226					-0.0865	0.0531
				(3.7917)					(4.8750)	(1.6009)
12maSE					0.0223				0.2043	0.2118
					(3.0483)				(8.5577)	(8.8568)
12maBeta						-0.0258			-0.0052	-0.0047
						(22.9358)			(2.3111)	(2.0700)
12maSB							0.0200		0.1569	0.1627
							(5.6432)		(11.2582)	(11.6374)
12maSP								-0.0187	-0.1651	-0.1699
								(5.7829)	(10.9828)	(11.2800)
N	61857	61857	61857	61857	61857	61857	61857	61857	61857	61857
R2	0.0012	0.0000	0.0012	0.0002	0.0002	0.0084	0.0005	0.0005	0.0118	0.0121

The dependent variable in these regressions is the 6 month trailing benchmark adjusted alpha, following Active Share statement dates. All of the independent variables are calculated over the 12 preceding months, except for Active Share which is a stock variable, and thus provide a look at the predictive nature of the risk parameters. These tests are full sample, cross-sectional, time-series regressions but are filtered as per the directions in Section 4. T-statistics are in parentheses.

Table 4 is mostly reflective of the data used for these tests in Cremers and Petajisto (2009). The dependent variable is 6 month trailing benchmark adjusted alphas. Active Share is highly significant in predicting 6 month trailing benchmark adjusted alpha. For every 1% increase in active share you get about 1.5 bps in excess performance over the next 6 months. Other risk measures are included for robustness. In fact, it seems that the SH is completely insignificant. In specification (9), Active Share holds its level of significance even in the presence of many other potential related risk factors. What is surprising is that in specification (10), when SH is reintroduced into the regression, is cannibalizes all of the explanatory power of both TE and Active Share. It has a significant effect of -0.1181 thus for a 1% increase in SH, you lose about 12bps of risk-adjusted performance for the trailing 6 month period.

Table 5 reproduces this analysis using only the trailing 1 month returns. Additionally, I have removed the risk-adjusted component of the relative return and this table represents only the raw excess return, PR – BR. Referring to specification (10) again, over the following 1 month, for a 1% increase in SH you lose 2.1bps of excess performance, which is non-coincidently about 12bps / 6. Moving left in the table doesn't bode well for Active Share's significance. For the trailing 1 month, Active Share does particularly poorly explaining the excess performance and SH seems to work well in every specification in which it is included. It is possible that depending on the period in which you are interested, either Active Share or SH could both be useful in predicting excess return.

Table 5
Predictive Regressions of the 1 Month Excess Return Following Active Share Statement Dates

	1mpER									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Intercept	-0.0010	-0.0005	-0.0005	-0.0007	-0.0008	-0.0059	-0.0001	-0.0006	-0.00169	-0.00012
	(2.0179)	(3.2537)	(1.0565)	(4.3045)	(4.9135)	(14.6774)	(0.6399)	(2.9714)	(1.8324)	(0.1082)
activeshare_min	-0.0003		0.0000						0.0006	-0.0015
	(0.5660)		(0.0662)						(0.9106)	(1.4554)
12maSH		-0.0087	-0.0087							-0.0217
		(4.9001)	(4.8678)							(2.6057)
12maTE				-0.0088					-0.0097	0.0160
				(4.2307)					(1.5494)	(1.3724)
12maSE					-0.0080				0.0032	0.0046
					(3.1197)				(0.3831)	(0.5468)
12maBeta						0.0048			0.0011	0.0012
						(11.9563)			(1.4456)	(1.5690)
12maSB							-0.0071		-0.0080	-0.0069
							(5.7477)		(1.6336)	(1.4103)
12maSP								-0.0039	0.0034	0.0025
								(3.4830)	(0.6348)	(0.4677)
N	61857	61857	61857	61857	61857	61857	61857	61857	61857	61857
R2	0.0000	0.0004	0.0004	0.0003	0.0002	0.0023	0.0005	0.0002	0.0009	0.0010

The dependent variable in these regressions is the 1 month trailing benchmark excess return, calculated PR - BR, following Active Share statement dates. All of the independent variables are calculated over the 12 preceding months, except for Active Share which is a stock variable, and thus provide a look at the predictive nature of the risk parameters. These tests are full sample, cross-sectional, time-series regressions but are filtered as per the directions in Section 4. T-statistics are in parentheses.

Active Share does not seem to perform very when the metric of interest is non-risk adjusted excess performance. Table 6 regresses the major risk parameters of interest, Active Share, SH, TE, and Beta, on trailing excess returns for 1, 3, 6, and 12 months. SH seems to predict excess performance pretty well across every specification and Active share falters, except in the specifications of 6mpER and 12mpER when SH is not included in the regression. It seems that in the presence of SH and TE, TE is cannibalizing the significance of Active Share. This should not be surprising given the functional relationship between these variables. In general, it can be stated that SH tends to perform better in the short term and AS in the long term. I suspect this is due to Active Share being more highly persistent that SH. However, in the presence of TE and SH, Active Share seems to lose its role as a predictor of excess return.

Rather than considering different time periods of return, Table 7 looks at differing measures of absolute and excess performance. Controlling for TE and Beta, SH gives predictable and significant results and those results are meaningful for both ER and BAA. Active Share's results are significant but not as strong and as I have shown before, mixed when controlling for SH. Given this evidence, Active Share is a very useful measure for determining the potential for skill in active management, meaning that it is predictive of excess returns. However, when considering SH, Active Share's contribution is overshadowed slightly and SH seems to be the driving force behind the pricing power.

Another interesting fact in Table 7 is the relationship between these risk parameters and both Benchmark and Portfolio Return, PR and BR. High Active Share seems to signify high future market performance, and symmetrically, low SH also seems

Table 6
The Trend in Predictive Power of Active Share and Implied Hedge Volatility over Trailing Excess Returns

	1mpER			3mpER			6mpER			12mpER		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	-0.0030	-0.0023	-0.0012	-0.0029	-0.0026	-0.0003	-0.0067	-0.0034	0.0025	-0.0250	-0.0154	-0.0104
	(4.8815)	(5.5722)	(1.2931)	(2.5403)	(3.3733)	(0.1599)	(3.8462)	(2.9499)	(0.9730)	(8.9189)	(8.3309)	(2.5589)
activeshare_min	0.0009		-0.0014	0.0003		-0.0029	0.0038		-0.0075	0.0116		-0.0064
	(1.3666)		(1.3899)	(0.2891)		(1.5406)	(2.1194)		(2.5965)	(4.0237)		(1.3757)
12maSH		-0.0148	-0.0238		-0.0153	-0.0337		-0.0695	-0.1168		-0.1453	-0.1854
		(2.8570)	(2.8683)		(1.5988)	(2.2014)		(4.7829)	(5.0126)		(6.2456)	(4.9704)
12maTE	-0.0105	0.0068	0.0186	0.0375	0.0546	0.0787	0.0631	0.1444	0.2063	0.1185	0.2933	0.3457
	(4.6554)	(1.1332)	(1.7896)	(8.9837)	(4.8994)	(4.0986)	(9.9350)	(8.5017)	(7.0496)	(11.6601)	(10.7885)	(7.3828)
12maBeta	0.0018	0.0018	0.0018	-0.0003	-0.0003	-0.0004	-0.0017	-0.0016	-0.0018	0.0056	0.0056	0.0055
	(4.6418)	(4.6712)	(4.6021)	(0.4530)	(0.4126)	(0.4834)	(1.5035)	(1.4543)	(1.5729)	(3.1273)	(3.1259)	(3.0590)
N	61857	61857	61857	61857	61857	61857	61857	61857	61857	61857	61857	61857
R2	0.0007	0.0008	0.0008	0.0016	0.0016	0.0016	0.0023	0.0026	0.0027	0.0037	0.0041	0.0041

The dependent variable in these tests spans from 1 month to 12 month trailing benchmark excess returns, calculated PR - BR, following Active Share statement dates. All of the independent variables are calculated over the 12 preceding months, except for Active Share which is a stock variable, and thus provide a look at the predictive nature of the risk parameters. These tests are full sample, cross-sectional, time-series regressions but are filtered as per the directions in Section 4. T-statistics are in parentheses.

Table 7
The Predictive Power of Active Share and Implied Hedge Volatility over Different Measures of Trailing Return

	6mpPR			6mpER			6mpBAA			6mpBR		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	-0.007276	0.0669943	0.0451842	-0.0067	-0.0034	0.0025	0.0145223	0.0240641	0.0218567	-0.000539	0.0703952	0.0427149
	(1.6861)	(23.6082)	(7.2339)	(3.8462)	(2.9499)	(0.9730)	(8.1867)	(20.6108)	(8.5039)	(0.1331)	(26.4450)	(7.2909)
activeshare_min	0.0925		0.0279	0.0038		-0.0075	0.0119		0.0028	0.0887		0.0354
	(20.8090)		(3.9195)	(2.1194)		(2.5965)	(6.4876)		(0.9641)	(21.2736)		(5.3035)
12maSH		-0.8415	-0.6657		-0.0695	-0.1168		-0.1109	-0.0931		-0.7720	-0.5489
		(23.5187)	(11.6048)		(4.7829)	(5.0126)		(7.5310)	(3.9430)		(23.0006)	(10.2012)
12maTE	-0.2298	0.8159	0.5860	0.0631	0.1444	0.2063	0.0143	0.1516	0.1283	-0.2929	0.6715	0.3798
	(14.6893)	(19.5135)	(8.1371)	(9.9350)	(8.5017)	(7.0496)	(2.2192)	(8.8128)	(4.3307)	(19.9657)	(17.1203)	(5.6218)
12maBeta	-0.0129	-0.0138	-0.0133	-0.0017	-0.0016	-0.0018	-0.0257	-0.0258	-0.0258	-0.0112	-0.0122	-0.0115
	(4.6814)	(5.0323)	(4.8463)	(1.5035)	(1.4543)	(1.5729)	(22.7798)	(22.9039)	(22.8350)	(4.3415)	(4.7348)	(4.4856)
N	61857	61857	61857	61857	61857	61857	61857	61857	61857	61857	61857	61857
R2	0.0084	0.0104	0.0106	0.0023	0.0026	0.0027	0.0095	0.0097	0.0098	0.0105	0.0117	0.0122

The dependent variable in these tests spans different measures of portfolio and benchmark performance over 6 month trailing benchmark relative performance following Active Share statement dates. All of the independent variables are calculated over the 12 preceding months, except for Active Share which is a stock variable, and thus provide a look at the predictive nature of the risk parameters. These tests are full sample, cross-sectional, time-series regressions but are filtered as per the directions in Section 4. T-statistics are in parentheses.

to signify high future market performance. There could potentially be an explanation behind the momentum factor of Carhart (1997) and Jagadeesh and Titman (1993) related to the idea of aggregate Active Share. Any persistent factor over time could be a candidate.

V.C Alpha sorts as a Robustness Check

A major component of Cremers and Petajisto (2009) is that low Active Share portfolios tend to underperform on a risk-adjusted basis in addition to high Active Share funds outperforming. Table 8 reaffirms this result. Low active share funds tend to underperform on the trailing 6 months by 0.31% in raw returns and by 0.22% on a risk-adjusted basis. And, high Active Share funds tend to outperform by 0.26% in raw excess return and by 0.58% in risk adjusted returns. This pattern for the implied hedge volatility is much less obvious. From quintiles 1-5 in SH the relationship seems to be far from monotonic. When broken down by Active Share quintile, the relationship seems in the right direction except in the highest Active Share quintile. This is the case for both the benchmark excess returns and for the benchmark adjusted alphas. Thus, although the Active Share story seems straightforward, there is obviously more going on with SH than can be described in a simple cross-sectional regression.

Another concern, particularly given the relationship between both Active Share and SH to beta, is whether these measures seem to be biased by systematic risk. For Active Share, this has been partially answered by the analysis done in Cremers and Petajisto (2009) since they do alpha sorting on risk-adjusted performance for benchmark

Table 8
6 Month Equal Weighted Alphas Double Sorted on Active Share and the Volatility of the Implied Hedge

		Volatility of the Implied Hedge Quintile						
		1	2	3	4	5	All	5 - 1
		Panel A: Benchmark Excess Returns						
Active Share Quintile	5	-0.26%	-0.30%	-0.13%	0.04%	1.26%	0.26%	1.51%
		(6.6853)	(6.8698)	(2.4644)	(0.5985)	(10.6056)	(3.2219)	(12.1504)
	4	-0.04%	0.01%	0.15%	-0.12%	-0.33%	-0.09%	-0.29%
		(1.1334)	(0.1656)	(2.5945)	(1.7450)	(3.4381)	(1.4236)	(2.8197)
	3	-0.11%	-0.07%	-0.37%	-0.61%	0.02%	-0.36%	0.13%
		(3.3048)	(1.7023)	(7.5969)	(10.4755)	(0.2389)	(6.5541)	(1.5092)
	2	-0.12%	-0.01%	-0.48%	-0.27%	-0.15%	-0.26%	-0.04%
		(4.0995)	(0.2474)	(12.5470)	(5.6447)	(2.3537)	(6.1067)	(0.5318)
	1	-0.12%	-0.33%	-0.28%	-0.18%	-0.25%	-0.31%	-0.13%
		(5.5313)	(13.5681)	(9.8073)	(4.6839)	(5.0863)	(9.5137)	(2.4514)
	All	-0.31%	-0.39%	-0.42%	-0.24%	0.60%	-0.15%	0.91%
		(26.3971)	(25.0229)	(20.7910)	(9.1672)	(14.0312)	(5.9159)	(20.5869)
	5 - 1	-0.14%	0.03%	0.15%	0.22%	1.50%	0.57%	
		(3.1698)	(0.6132)	(2.4809)	(2.6986)	(11.7433)	(6.6022)	
		Panel B: Benchmark Adjusted Alphas						
Active Share Quintile	5	0.12%	0.22%	0.41%	0.56%	1.11%	0.58%	0.98%
		(2.7684)	(4.4262)	(6.9113)	(7.0649)	(9.4254)	(7.1003)	(7.8247)
	4	-0.04%	0.01%	0.15%	-0.12%	-0.33%	-0.07%	-0.29%
		(1.1334)	(0.1656)	(2.5945)	(1.7450)	(3.4381)	(1.1020)	(2.8197)
	3	-0.11%	-0.07%	-0.37%	-0.61%	0.02%	-0.24%	0.13%
		(3.3048)	(1.7023)	(7.5969)	(10.4755)	(0.2389)	(4.3933)	(1.5092)
	2	-0.12%	-0.01%	-0.48%	-0.27%	-0.15%	-0.20%	-0.04%
		(4.0995)	(0.2474)	(12.5470)	(5.6447)	(2.3537)	(4.6422)	(0.5318)
	1	-0.12%	-0.33%	-0.28%	-0.18%	-0.25%	-0.22%	-0.13%
		(5.5313)	(13.5681)	(9.8073)	(4.6839)	(5.0863)	(6.7748)	(2.4514)
	All	-0.08%	-0.04%	-0.10%	-0.09%	0.17%	-0.03%	0.26%
		(6.0322)	(2.2235)	(4.3521)	(3.3213)	(4.2937)	(1.0643)	(6.0222)
	5 - 1	0.24%	0.55%	0.69%	0.74%	1.35%	0.80%	
		(4.8774)	(9.8721)	(10.5060)	(8.3976)	(10.6555)	(9.0932)	

Funds are sorted by two dimensions of risk attributable to active management. The measures of active management are computed as defined in Section 4. Fund returns are NAV returns are the raw returns for the 6 month period immediately following the Active Share statement date. T-statistics are in parentheses.

Table 9
6 Month Equal Weighted Alphas Double Sorted on Active Share and Beta

		Beta Quintiles						
		1	2	3	4	5	All	5 - 1
		Panel A: Benchmark Excess Returns						
Active Share Quintile	5	0.07%	0.07%	0.43%	0.82%	0.20%	0.26%	0.13%
		(0.9326)	(1.0430)	(5.9968)	(10.0299)	(2.2044)	(3.2219)	(1.0673)
	4	0.54%	0.04%	0.09%	0.04%	-0.87%	-0.09%	-1.41%
		(8.8340)	(0.7450)	(1.4614)	(0.5741)	(10.6901)	(1.4236)	(13.8540)
	3	0.15%	-0.37%	-0.23%	-0.21%	-0.50%	-0.36%	-0.65%
		(2.6082)	(7.5898)	(5.0790)	(3.8932)	(8.0294)	(6.5541)	(7.7187)
	2	0.17%	-0.17%	-0.30%	-0.34%	-0.28%	-0.26%	-0.45%
		(3.9224)	(4.6717)	(7.4905)	(7.7039)	(5.7251)	(6.1067)	(6.8800)
	1	0.33%	-0.11%	-0.33%	-0.25%	-0.52%	-0.31%	-0.84%
		(8.2537)	(3.6413)	(10.9295)	(8.4882)	(12.6067)	(9.5137)	(14.8056)
	All	-0.24%	-0.51%	-0.29%	0.14%	0.16%	-0.15%	0.40%
		(8.2936)	(23.7931)	(14.5308)	(6.0780)	(4.9511)	(5.9159)	(9.2700)
	5 - 1	-0.25%	0.18%	0.75%	1.07%	0.72%	0.57%	
		(2.9111)	(2.3946)	(9.7590)	(12.3312)	(7.2051)	(6.6022)	
		Panel B: Benchmark Adjusted Alphas						
Active Share Quintile	5	1.51%	0.87%	0.71%	0.60%	-0.99%	0.58%	-2.50%
		(19.5585)	(11.7536)	(10.0302)	(7.1088)	(10.2921)	(7.1003)	(20.2896)
	4	0.54%	0.04%	0.09%	0.04%	-0.87%	-0.07%	-1.41%
		(8.8340)	(0.7450)	(1.4614)	(0.5741)	(10.6901)	(1.1020)	(13.8540)
	3	0.15%	-0.37%	-0.23%	-0.21%	-0.50%	-0.24%	-0.65%
		(2.6082)	(7.5898)	(5.0790)	(3.8932)	(8.0294)	(4.3933)	(7.7187)
	2	0.17%	-0.17%	-0.30%	-0.34%	-0.28%	-0.20%	-0.45%
		(3.9224)	(4.6717)	(7.4905)	(7.7039)	(5.7251)	(4.6422)	(6.8800)
	1	0.33%	-0.11%	-0.33%	-0.25%	-0.52%	-0.22%	-0.84%
		(8.2537)	(3.6413)	(10.9295)	(8.4882)	(12.6067)	(6.7748)	(14.8056)
	All	0.68%	0.03%	-0.10%	-0.09%	-0.67%	-0.03%	-1.35%
		(24.7286)	(1.3490)	(4.6281)	(3.6688)	(20.5498)	(1.0643)	(31.6825)
	5 - 1	1.19%	0.98%	1.03%	0.86%	-0.47%	0.80%	
		(13.6588)	(12.2603)	(13.4943)	(9.5255)	(4.5017)	(9.0932)	

Funds are sorted by two dimensions of risk attributable to active management. The measures of active management are computed as defined in Section 4. Fund returns are NAV returns are the raw returns for the 6 month period immediately following the Active Share statement date. T-statistics are in parentheses.

Table 10
6 Month Equal Weighted Alphas Double Sorted on the Volatility of the Implied Hedge and Beta

Volatility of the Implied Hedge Quintile		Beta Quintiles 1	2	3	4	5	All	5 - 1
		Panel A: Benchmark Excess Returns						
	5	0.65%	0.34%	0.37%	1.81%	0.28%	0.60%	-0.36%
		(7.1027)	(3.8869)	(3.9835)	(19.0482)	(2.8251)	(6.2750)	(2.6593)
	4	0.15%	-0.14%	0.11%	0.00%	-0.44%	-0.24%	-0.58%
		(2.5458)	(2.5074)	(1.8033)	(0.0734)	(6.4862)	(4.0997)	(6.5731)
	3	0.68%	-0.28%	-0.14%	-0.23%	-0.48%	-0.42%	-1.16%
		(15.0970)	(5.6859)	(2.8825)	(4.3533)	(9.7164)	(9.2980)	(17.3396)
	2	0.69%	0.10%	-0.26%	-0.28%	-0.37%	-0.39%	-1.07%
		(18.3362)	(2.7368)	(6.7271)	(6.8842)	(8.8986)	(11.1906)	(18.8925)
	1	0.48%	0.13%	-0.16%	-0.25%	-0.31%	-0.31%	-0.79%
		(14.6187)	(4.1961)	(5.4312)	(8.6925)	(8.6624)	(11.8052)	(16.3100)
	All	-0.24%	-0.51%	-0.29%	0.14%	0.16%	-0.15%	0.40%
		(8.2936)	(23.7931)	(14.5308)	(6.0780)	(4.9511)	(5.9159)	(9.2700)
	5 - 1	0.16%	0.21%	0.53%	2.07%	0.59%	0.91%	
		(1.6809)	(2.2068)	(5.4525)	(20.7589)	(5.5339)	(9.2068)	
		Panel B: Benchmark Adjusted Alphas						
	5	1.06%	0.55%	0.42%	0.85%	-1.12%	0.17%	-2.18%
		(13.0013)	(6.3321)	(4.8643)	(9.1688)	(11.5103)	(1.9202)	(17.1757)
	4	0.15%	-0.14%	0.11%	0.00%	-0.44%	-0.09%	-0.58%
		(2.5458)	(2.5074)	(1.8033)	(0.0734)	(6.4862)	(1.4853)	(6.5731)
	3	0.68%	-0.28%	-0.14%	-0.23%	-0.48%	-0.10%	-1.16%
		(15.0970)	(5.6859)	(2.8825)	(4.3533)	(9.7164)	(1.9463)	(17.3396)
	2	0.69%	0.10%	-0.26%	-0.28%	-0.37%	-0.04%	-1.07%
		(18.3362)	(2.7368)	(6.7271)	(6.8842)	(8.8986)	(0.9944)	(18.8925)
	1	0.48%	0.13%	-0.16%	-0.25%	-0.31%	-0.08%	-0.79%
		(14.6187)	(4.1961)	(5.4312)	(8.6925)	(8.6624)	(2.6977)	(16.3100)
	All	0.68%	0.03%	-0.10%	-0.09%	-0.67%	-0.03%	-1.35%
		(24.7286)	(1.3490)	(4.6281)	(3.6688)	(20.5498)	(1.0643)	(31.6825)
	5 - 1	0.58%	0.42%	0.58%	1.10%	-0.81%	0.26%	
		(6.5504)	(4.5043)	(6.3804)	(11.3623)	(7.8449)	(2.6932)	

Funds are sorted by two dimensions of risk attributable to active management. The measures of active management are computed as defined in Section 4. Fund returns are NAV returns are the raw returns for the 6 month period immediately following the Active Share statement date. T-statistics are in parentheses.

relative Alphas and four factor alphas[9]. However, for comparison refer to Table 9. This is a double sort of Active Share and Beta. This table attempts to answer the question about whether active share and beta have a relationship. Except for a few exceptions, Active Share seems to retain its property of outperformance in the highest quintile and underperformance in the lowest quintile under both specifications of the relative performance. Beta on the other hand is particularly mixed. As we would expect, in raw excess return, high beta outperforms low beta but in benchmark adjusted alphas, is strongly significantly underperforms. This is consistent with the cross sectional regressions from the previous part.

Table 10 is a double sort of implied hedge volatility on Beta. Just as with the sorts of SH and Active Share, SH seems to behave rather strangely in the highest quintile. Whereas it behaves as expected in the lower 4 quintiles, the highest quintile SH actually outperforms rather than underperforms. Beta's characteristics are similar to the results from Table 9 in that the lowest quintile outperforms the highest quintile all around. In aggregate, this analysis is consistent with the regressions but the strange result in the 5^{th} SH quintile probably deserves further consideration.

VI. Conclusion

Active Share was introduced to the academic literature by Cremers and Petajisto (2009). Active Share takes a difficult functional form and they show that it is predictive of excess return. In this paper, I re-parameterized Active Share into a more familiar and

[9] See Carhart (1997) and Fama and French (1993)

usable analytical form. This form includes the metrics we are accustomed to analyzing in portfolio analysis. I compared and contrasted the predictive power of Active Share with its new counterpart, the implied hedge volatility. This attribution shows that Active Share and the implied hedge volatility are reflections of each other around tracking error. And, if one has predictive power in excess return, the other probably does as well. I follow this theoretical attribution and re-parameterization with empirical analysis to both reaffirm the predictive effects of Active Share and to compare the predictive effects of Active Share with that of the implied hedge volatility. Active Share dominates in benchmark adjusted alpha style relative return prediction but the implied hedge volatility tends to dominate when looking at raw excess returns. Additionally, as you extend the time period of estimation, Active Share's predictive power strengthens. However, the implied hedge volatility is significant throughout all time periods (but perhaps diminishes around 12 months).

CHAPTER 3. TEV OPTIMIZATION AND UTILITY

I. Introduction

Delegated contracting in investment management is the most popular and predominant form of management in the modern investment industry. This delegated portfolio management creates a classical incentive problem where the additional incentives that affect the agent, the delegated manager, are not completely consistent with the incentives of principal. It is generally accepted and consistently asserted through theory that this delegated incentive leads to sub-optimal, less-good, portfolios. This is particularly true when comparing the portfolio that the principal would build, given the same level of expertise and opportunity, to the portfolio the agent would build given the delegated incentives. If the portfolio choice problem is thought of as an optimization problem, then it is not difficult to imagine that the delegated incentives create binding constraints that can at best generate portfolios that are equally preferred to the unconstrained choice. Most likely the constraints will lead to sub-optimal portfolios. This sub-optimality is often considered as borderline irrational in the literature and is used as an argument either for passive investment management or for the inefficiency of using benchmarks portfolios as performance measures. However, given the pervasiveness of delegated contracting in the investment industry, it is difficult to think that so many professionals have it wrong. The more important question is, “Do the predominant delegated incentives lead to better portfolios than the principals could build themselves since they have less skill and opportunity?”

Portfolio theory and asset pricing were developed under the assumption that the agent in the delegated relationship is a perfect agent whose incentives are perfectly aligned with those of the principal. Seminal work such as Markowitz (1952), Sharpe (1964), and Lintner (1965) paved the path for most modern investment research but like most foundational research, simplifying assumptions are made that potentially distort reality. It is evident that researchers and practitioners alike were concerned with the delegated incentive problem from the early life of investment theory but not many had done work critiquing how the relative incentive affected the theoretical models. An early example of an influential paper that discusses the delegated portfolio management situation in depth is Treynor and Black (1973). Although this paper is much more about reconciling inefficiency in markets, exploited through security selection, with an equilibrium look at an efficient market, the structure of how researchers approach the delegated incentive problem derives from Treynor and Black's approach in considering security selection versus the market portfolio, a benchmark.

One of the earliest criticisms and direct recognitions of the unrepresentativeness of Markowitz mean-variance space and subsequently the CAPM came from Professor Sharpe himself in Sharpe (1981). Sharpe hypothesizes that it is highly unlikely that mean-variance efficiency can be obtained in a decentralized portfolio management setting where the investment decision-making responsibilities are delegated to an agent, in particular multiple agents. He proposes the foundation of a delegated incentive problem in multiple external agents and is skeptical of an optimal solution. Elton and Gruber (2004) solve the problem, albeit with some very narrow constraints, suggesting that delegated portfolio management could reach optimality. However, vanBisburgen, Brandt, and Koijen (2008) revisits the problem relaxing the assumption about the certainty of the agents' risk appetite and concludes that serious inefficiency exists without this assumption. Blake et. al. (2013) uses the BBK framework and applies it to the delegated relationships in the pension industry and, among many other things, shows that the delegated incentive is pervasive in professional investment management.

Although Sharpe (1981) may have motivated the study of delegated portfolio management, another seminal paper in the area is Bhattacharya and Pfleiderer (1985). This paper set the stage for studying the delegated incentive as a principal/agent problem. In this paper they model a utility relationship between the principal and the agent. Their model implies the same conclusions of the other research, that it is unlikely that the delegation can reach optimality in utility. A direct follow-up to this work, Admati and Pfleiderer (1997), assumes that the motivating factor behind the delegated incentive involves a conditional optimization and a benchmark that is the solution to the Markowitz optimization problem given the limited set of information. Their conclusion is again consistent with the other literature on the subject; the conditional optimization leads to necessary sub-optimality in portfolio choice.

This is the same conclusion reached in Roll (1992) albeit by a slightly different construction. Roll builds a framework much more closely related to Markowitz (1952) but instead of investment managers optimizing in mean/variance space, they optimize in mean and tracking error variance, the relative variance. This leads to a frontier, the TEV (Tracking Error Volatility) frontier that passes through the benchmark portfolio. The dependence on a benchmark as a relative incentive is the connection between Admati and Pfleiderer (1997) and Roll (1992). The overriding implication of Roll's construction is that agents optimize utility in mean and tracking error volatility, not in mean and variance. If we assume that the principal derives utility in mean-variance space and that the relative incentive causes the delegated agent to derive utility in mean-TEV space, then the broad implication of Roll's TEV frontier optimization is that we need to study the transformation of the agent's utility from a relative optimization to the principal's utility in absolute optimization.

If, for example, we assume that the portfolio used to benchmark the external manager is the principal's best option, then we can consider the principal's utility curve going through that benchmark as the highest utility the principal can obtain given his information. The delegation to an external manager, as long as the principal's utility curve is not tangent to his opportunity set, the TEV frontier, at the benchmark, necessarily has a utility increasing deviation for the principal as long as the agent has better information or more skill than the principal and proper constraints are applied. The contribution of this essay is in directly extending Roll's framework by recognizing that the relative optimization problem can produce higher utility for the principal. Although it is true that the principal, given all the information and skill of the agent, would build a different and better portfolio, except for a very specific and unlikely case, the agent can still create higher utility for the principal than the default benchmark, even given the inefficiency of relative optimization.

The closest application of a utility problem directly to Roll's TEV frontier is in Bertrand (2010). In this paper he considers the problem of a fixed risk aversion constraint in mean/variance and its ability to generate preferred portfolios. My assumption is slightly different that Bertrand's in that managers cannot, in my framework, be constrained on risk aversion in mean and variance but only in risk aversion in mean and TEV. Although these spaces are related, they are different enough that they lead to very different conclusions. Based on my assumptions, Bertrand's iso-risk aversion curves are the path that a principal would like to follow, not the path the delegated manager actually follows. In section II, I look at the space of mean-TEV and translate it back to mean-variance. It is a direct application of Roll's construction with a relative utility overlay. The path an unconstrained relative optimizer would follow is far inferior to the efficient frontier and equivalent to the TEV frontier at the agent's portfolio choice given changes in the agent's risk aversion level. However it is still likely that this path increases utility for the principal, at least over some controllable range of possibilities. Section III considers the relationship of principal utility in the space of mean and variance given the principal's best alternative, investing in the benchmark, to the TEV frontier in mean-variance. I show that the delegated manager can almost universally increase the principal's utility.

Another important paper that stems directly from Roll (1992) is Jorion (2003), in which Jorion considers the frontier in mean variance space given a constraint to tracking error variance. This is an important problem in practice because in examples of real world delegated investment management, oftentimes external managers are constrained by a tracking error bound. In a similar fashion to Roll, Jorion concludes that the tracking error bound most likely incentivizes agents to take more variance than the benchmark in portfolio selection and that this constraint should be used with caution when applied to external managers. Using my relative utility framework, I show in section IV of this paper that when given only a tracking error constraint, principals can control the risk level of the external manager to guarantee that the portfolio selected by an agent will increase the principal's utility. If implemented properly, the tracking error constraint placed by principals on external managers is rational.

Another pervasive constraint applied to external managers in the industry is the constraint related to style drift. We can calculate beta relative to the underlying benchmark and consider deviations from factor sensitivity of 1 to be deviations due to style drift. Although this constraint is controversial in the industry, it is necessary for many of the theoretical conclusions in active portfolio management, most notably the Fundamental Law of Active Management, which is summarized well in Grinold and Kahn (2000). Roll (1992), in addition to deriving the TEV frontier, also considers a number of other problems, one being constraining benchmark relative beta. His conclusion is that the beta constraint generates a necessarily superior frontier to the TEV frontier in the region over which external managers are likely to optimize. I look at this beta constraint in the context of my relative utility overlay in section V and conclude much the same with similar caveats. However, the most interesting result of analyzing the beta constraint is that it always has the potential to increase the principal's utility. There are a couple scenarios under the tracking error constraint that, however unlikely, could cause the delegated manager to act against the interests of the principal. As long as the benchmark is above the global minimum variance portfolio in return, the beta constraint always has the potential to increase principal utility, particularly if the benchmark has a higher expected return that the minimum variance portfolio. Thus, just like the tracking error constraint, constraining external managers on beta is also rational.

The beta constraint itself is not enough to guarantee a superior portfolio from delegated management either. It is quite possible for the external manager, given low enough levels of risk aversion, to build a portfolio less efficient than without the constraint. Additionally, the tracking error constraint, however rational and possibly utility increasing, still lies on a frontier below that of a constrained beta frontier. It seems natural that the combination of these two constraints could force a delegated manager to invest on a frontier superior to the TEV frontier and additionally assure the principal that the external manager will not invest in a portfolio risky enough to erode utility. In section VI, I analyze the inclusion of both the tracking error constraint and the beta constraint in controlling a delegated agent. These constraints, if implemented properly, necessarily cause the delegated manager to choose a portfolio that increases utility for the principal. I use these constraints to develop a benchmarking strategy to force external managers to not only build a portfolio that increases utility, but to build a portfolio that maximizes the principal's utility given the agents opportunity set. Essentially, if the parameters are known, setting the tracking error and beta constraints to the levels of the desired portfolio, even the global optimal portfolio for the principal, incentivizes the agent to buy that exact portfolio. These two constraints are enough to pin the delegated manager to an exact location within investment opportunity set. They guarantee not only a unique but a superior portfolio.

The conclusion of this work is that if we believe delegated investment managers to have skill and expertise, then it is not only rational but preferred to delegate our investment management responsibilities to those individuals, even given that they are relative TEV optimizers. The big caveat here is the assumption that the external manager really does have the ability to outperform a benchmark. Two of the most popular risk control metrics in the industry, beta constraints and tracking error constraints, can be used to pin the external manager to an exact spot in mean-variance space. If the principal understands his own utility in this space, and has an idea of what the external manager's skill and capabilities are, then this spot is a necessary utility increase for the principal. The delegated portfolio choice, when delegated to a skillful agent and under the tracking error and beta constraints, is always better than the portfolio a principal could build himself.

II. The TEV Frontier and the Agent's Objective Function

The Markowitz mean-variance optimization problem involves minimizing the variance of a portfolio of securities. There are many well-known solutions to the Markowitz problem but perhaps the most popular is considering the problem as a quadratic programming problem. Below are the variable definitions and the problem's statement and solution. Matrices and vectors are in boldface.

$\mathbf{w}$	-	$n \times 1$ the weight vector of the portfolio
$\mathbf{\Omega}$	-	$n \times n$ covariance matric (symmetric and positive definite)
$\mathbf{r}$	-	$n \times 1$ vector of returns
$\mathbf{1}$	-	$n \times 1$ vector of 1's

The problem is to minimize the variance as a function of return given that the weight vector sums to 100%:

$$\min_{\mathbf{w}} \mathbf{w}'\mathbf{\Omega}\mathbf{w} \quad s.t.\ \mathbf{w}'\mathbf{1} = 1\ \ and\ \mathbf{w}'\mathbf{r} = r \tag{1}$$

We can set up the Lagrangian as follows:

$$\mathrm{L}(\mathbf{w}, \lambda_1, \lambda_r) = \mathbf{w}'\mathbf{\Omega}\mathbf{w} - \lambda_1 (\mathbf{w}'\mathbf{1} - 1) - \lambda_r (\mathbf{w}'\mathbf{r} - r) \tag{2}$$

Next we differentiate and set the results to $\mathbf{0}$:

$$\begin{gathered} 2\mathbf{\Omega}\mathbf{w} - \lambda_1\mathbf{1} - \lambda_r\mathbf{r} = \mathbf{0} \\ \mathbf{w}'\mathbf{1} = 1 \\ \mathbf{w}'\mathbf{r} = r \end{gathered} \tag{3}$$

Solving this system for **w** yields the following: (notation such as $[\mathbf{1} \quad \mathbf{r}]$ is an $n \times 2$ augmented matrix where the first column is all 1's and the second is the return vector; also, the last matrix in this notation is a 2×1 vector with the constants 1 and *r*):

$$\mathbf{w} = \mathbf{\Omega}^{-1}[\mathbf{1} \quad \mathbf{r}]\left[[\mathbf{1} \quad \mathbf{r}]'\mathbf{\Omega}^{-1}[\mathbf{1} \quad \mathbf{r}]\right]^{-1}\begin{bmatrix} 1 \\ r \end{bmatrix} \tag{4}$$

This weight vector can be plugged back into variance function and the parabola of the minimum variance set, i.e. the efficient frontier, can be expressed with the variance of the portfolio, σ^2, as a function of the return constant, *r*. In essence, this is the equation for the efficient frontier.

$$\sigma^2 = \mathbf{w}'\boldsymbol{\Omega}\mathbf{w} = \begin{bmatrix} 1 \\ r \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r}]'\boldsymbol{\Omega}^{-\mathbf{1}}[\mathbf{1} \quad \mathbf{r}]\right]^{-\mathbf{1}} \begin{bmatrix} 1 \\ r \end{bmatrix} \tag{5}$$

The Tracking Error Variance or TEV Frontier from Roll (1992) can be derived similarly but with a slightly different objective function. If the weight vector for the benchmark is denoted as **b**, then Roll's framework requires minimizing tracking error variance by choosing **w**. The problem proceeds similarly but with a slight complication because of the differenced weight vector (**w** – **b**). Below is a statement of the problem and the subsequent Lagrangian:

$$\min_{\mathbf{w}} T^2 = \min_{\mathbf{w}} (\mathbf{w}-\mathbf{b})'\boldsymbol{\Omega}(\mathbf{w}-\mathbf{b}) \quad s.t.\ \mathbf{w}'\mathbf{1} = 1 \ and\ \mathbf{w}'\mathbf{r} = r \tag{6}$$

$$\mathrm{L}(\mathbf{w}, \lambda_1, \lambda_r) = (\mathbf{w}-\mathbf{b})'\boldsymbol{\Omega}(\mathbf{w}-\mathbf{b}) - \lambda_1(\mathbf{w}'\mathbf{1}-1) - \lambda_r(\mathbf{w}'\mathbf{r}-r) \tag{7}$$

Below is the system of simultaneous equations to be solved. It is the same as the original problem but with the extra term, 2**Ωb**:

$$\begin{gathered} 2\boldsymbol{\Omega}\mathbf{w} - 2\boldsymbol{\Omega}\mathbf{b} - \lambda_1\mathbf{1} - \lambda_r\mathbf{r} = \mathbf{0} \\ \mathbf{w}'\mathbf{1} = 1 \\ \mathbf{w}'\mathbf{r} = r \end{gathered} \tag{8}$$

In a similar form to what was derived for the variance minimization, in the case of tracking error variance we choose the following weight vector where $r_b = \mathbf{b}'\mathbf{r}$.

$$\mathbf{w} = \mathbf{b} + \boldsymbol{\Omega}^{-\mathbf{1}}[\mathbf{1} \quad \mathbf{r}]\left[[\mathbf{1} \quad \mathbf{r}]'\boldsymbol{\Omega}^{-\mathbf{1}}[\mathbf{1} \quad \mathbf{r}]\right]^{-\mathbf{1}} \begin{bmatrix} 0 \\ r - r_b \end{bmatrix} \tag{9}$$

This weight vector can be plugged bask into the variance function and an expression for the parabola of the TEV frontier as a function of r can be obtained. If we define σ_b^2 = **b**'**Ωb** then:

$$\sigma^2 = \mathbf{w}'\boldsymbol{\Omega}\mathbf{w} = \sigma_b^2 + \begin{bmatrix} 0 \\ r - r_b \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r}]'\boldsymbol{\Omega}^{-1}[\mathbf{1} \quad \mathbf{r}] \right]^{-1} \begin{bmatrix} 0 \\ r - r_b \end{bmatrix} \tag{10}$$

$$+ 2\begin{bmatrix} 1 \\ r_b \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r}]'\boldsymbol{\Omega}^{-1}[\mathbf{1} \quad \mathbf{r}] \right]^{-1} \begin{bmatrix} 0 \\ r - r_b \end{bmatrix}$$

The relationship between the efficient frontier from (5) and the TEV frontier from (10) is illustrated and analyzed thoroughly in Roll (1992). Roll's primary conclusion is that the TEV frontier is less optimal than the efficient frontier whenever the benchmark portfolio is not on the efficient frontier. This is illustrated in Figure 1, Panel A. The TEV frontier is to the right of the efficient frontier because optimization on relative tracking error variance rather than absolute variance is less efficient. The horizontal axis in this figure is standard deviation. However, it is probably more useful to look at this diagram with tracking error volatility on the horizontal axis. This graph, with tracking error on the horizontal axis, is depicted in Figure 1 panel B and the equations for the curves are derived as follows.

First recall that the tracking error variance, T^2, is defined as follows:

$$T^2 = (\mathbf{w} - \mathbf{b})'\boldsymbol{\Omega}(\mathbf{w} - \mathbf{b}) \tag{11}$$

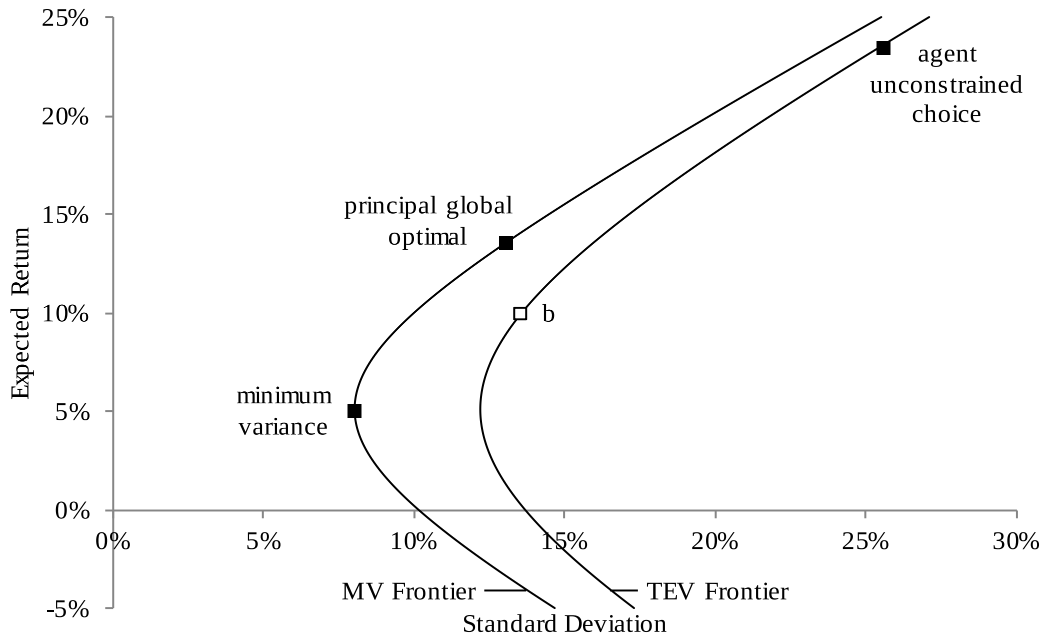
For the TEV frontier, we simply take the weight vector from (9) and move **b** to the left hand side.

$$(\mathbf{w} - \mathbf{b}) = \boldsymbol{\Omega}^{-1}[\mathbf{1} \quad \mathbf{r}] \left[[\mathbf{1} \quad \mathbf{r}]'\boldsymbol{\Omega}^{-1}[\mathbf{1} \quad \mathbf{r}] \right]^{-1} \begin{bmatrix} 0 \\ r - r_b \end{bmatrix} \tag{12}$$

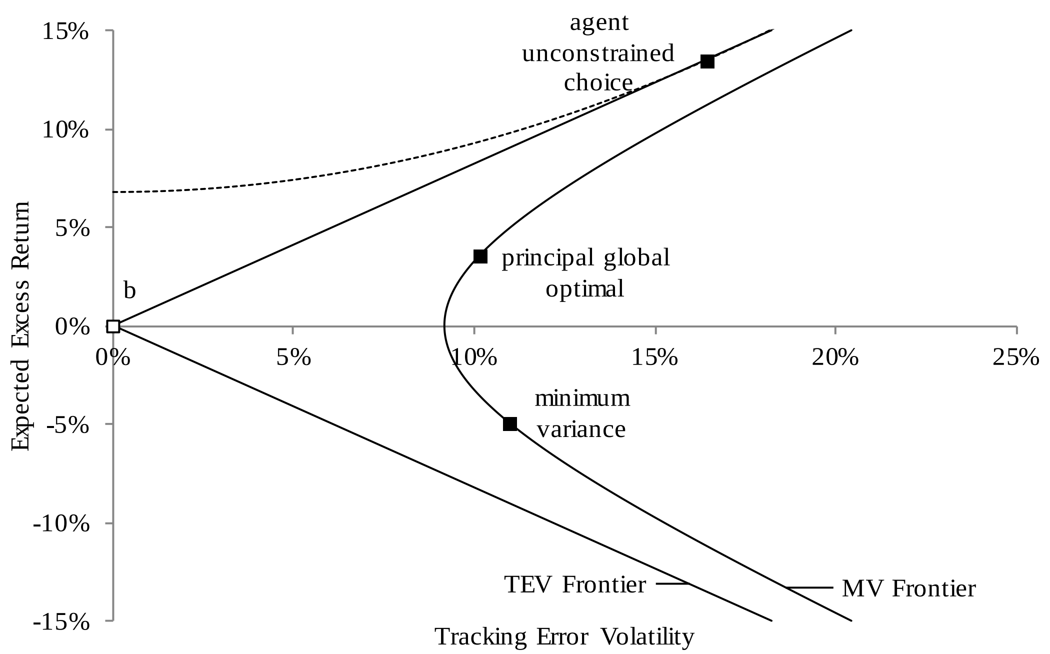
Plugging this vector into the tracking error variance equation yields the following parabola where T^2 is a function of r. This is the TEV frontier in the space of mean and tracking error variance rather than in mean and variance. This is, not coincidentally, just one of the terms from equation (10).

Figure 1
The TEV Frontier and the Agent's Objective Function

Panel A: Mean-Variance Space



Panel B: Mean-TEV Space



This figure shows how an agent, when maximizing utility in mean-TEV space, chooses a portfolio along the TEV frontier in Panel B. This portfolio is translated back into mean-variance space, in Panel A, and as is evident, the portfolio lies along the sub-optimal TEV frontier in that space as well.

$$T^2 = \begin{bmatrix} 0 \\ r - r_b \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r}]' \mathbf{\Omega}^{-1} [\mathbf{1} \quad \mathbf{r}] \right]^{-\mathbf{1}} \begin{bmatrix} 0 \\ r - r_b \end{bmatrix} \tag{13}$$

Given the weight vector for the efficient frontier from equation (4), we can adjust it by differencing it with the benchmark as follows. Note that a $\mathbf{b}$ was just subtracted from both sides.

$$(\mathbf{w} - \mathbf{b}) = \mathbf{\Omega}^{-1} [\mathbf{1} \quad \mathbf{r}] \left[[\mathbf{1} \quad \mathbf{r}]' \mathbf{\Omega}^{-1} [\mathbf{1} \quad \mathbf{r}] \right]^{-\mathbf{1}} \begin{bmatrix} 1 \\ r \end{bmatrix} - \mathbf{b} \tag{14}$$

Plugging this vector into the tracking error variance equation yields the following parabola. This is the equation for the efficient frontier when plotted in the space of tracking error variance.

$$\begin{aligned} T^2 = \begin{bmatrix} 1 \\ r \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r}]' \mathbf{\Omega}^{-1} [\mathbf{1} \quad \mathbf{r}] \right]^{-\mathbf{1}} \begin{bmatrix} 1 \\ r \end{bmatrix} + \sigma_b^2 \\ - 2 \begin{bmatrix} 1 \\ r_b \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r}]' \mathbf{\Omega}^{-1} [\mathbf{1} \quad \mathbf{r}] \right]^{-\mathbf{1}} \begin{bmatrix} 1 \\ r \end{bmatrix} \end{aligned} \tag{15}$$

The mean variance frontier, when mapped to tracking error variance space is to the right of the TEV frontier just like the TEV frontier is to the right of the efficient frontier when in the space of absolute variance. This is the phenomenon illustrated in Figure 1, Panel B.

One of the most important implications of the TEV frontier is the implication about how delegated managers choose portfolios given a relative performance incentive. That is, they optimize return relative to tracking error rather than variance or standard deviation. This implication also directly asserts that agents will optimize a utility function that is parameterized with tracking error volatility as the risk measure rather than standard deviation. No matter the exact structure of the utility relationship, utility and preference in relative space can be thought of exactly like it is thought of in absolute space. The preference set is a closed, convex set and increases to the "northwest" just as it does in mean-variance. Also, the iso-utility curve is upward sloping with a non-negative second derivative. The iso-utility curve associated with optimizing utility in relative space sits exactly tangent to the TEV frontier in relative space and this is illustrated in Figure 1 Panel B. For the purposes of illustration, I derive all of the utility relationships using quadratic utility but it should be evident that any properly formed utility function generally obeys all of the rules of the quadratic utility for the purposes of this essay.

Suppose the utility function for the agent is as follows where θ is the coefficient of risk aversion:

$$U = r - \theta\, T^2 = \mathbf{w}'\mathbf{r} - \theta(\mathbf{w} - \mathbf{b})'\boldsymbol{\Omega}(\mathbf{w} - \mathbf{b}) \tag{16}$$

The problem is to maximize utility subject to the constraint on the weight vector.

$$\max_{\mathbf{w}} U = \max_{\mathbf{w}} \ \mathbf{w}'\mathbf{r} - \theta(\mathbf{w} - \mathbf{b})'\boldsymbol{\Omega}(\mathbf{w} - \mathbf{b}) \quad s.t.\ \mathbf{w}'\mathbf{1} = 1 \tag{17}$$

$$\mathrm{L}(\mathbf{w}, \lambda_1) = \mathbf{w}'\mathbf{r} - \theta(\mathbf{w} - \mathbf{b})'\boldsymbol{\Omega}(\mathbf{w} - \mathbf{b}) - \lambda_1\,(\mathbf{w}'\mathbf{1} - 1) \tag{18}$$

Differentiating the Lagrangian and finding the critical value yields the following solution for the optimal weight vector:

$$\mathbf{w} = \frac{1}{2\theta}\boldsymbol{\Omega}^{-1}\left(\mathbf{r} - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\mathbf{1}\right) + \mathbf{b} \tag{19}$$

The portfolio representing this optimal weight vector is depicted in Figure 1, Panels A and B, with two different levels of risk aversion. In Panel A, it is necessarily along the TEV frontier above the benchmark portfolio and it is notable that it not possible for this portfolio to be on the efficient frontier unless the benchmark is also on the efficient frontier. As the agent's risk aversion coefficient decreases, the agent chooses a portfolio further and further up the TEV frontier. In Panel B, this portfolio is the optimal allocation and along the TEV frontier. Also depicted in Panel B is the iso-utility curve associated with this portfolio's level of utility. It can be back-solved simply by rearranging the utility function from (16) and plugging in the weight vector from (19), and it is expressed as follows:

$$\begin{aligned} r = U + \theta\, T^2 &= \mathbf{w}'\mathbf{r} - \theta(\mathbf{w}-\mathbf{b})'\boldsymbol{\Omega}(\mathbf{w}-\mathbf{b}) + \theta\, T^2 \\ &= r_b + \frac{1}{4\theta}\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) + \theta\, T^2 \end{aligned} \tag{20}$$

Delegated managers that are incentivized by a relative return incentive will optimize along the TEV frontier and maximize relative utility in the space of mean and tracking error volatility. This gives rise to the utility relationship as depicted in (20). This is the agent's iso-utility curve and in general it is inconsistent with the process of maximizing utility in mean/variance space, the space in which the principal derives utility.

Table 1 calculates the quadratic utility deviations for the principal under differing scenarios. Panel A depicts the utility increase or decrease for the principal from unconstrained delegated contracting. The delegated performance incentive has the potential to increase utility for the principal but this is far from certain. Given the levels of utility I use in the figures for this paper, the utility decreases by 5.15% through delegation. Panel B shows the utility depreciation from the principal's global optimal portfolio. This is necessarily non-positive. In this paper, the agent chooses a portfolio that 9.25% less optimal in utility than the principal's global optimal. This example is

Table 1

Utility Deviations from the Unconstrained Delegated Portfolio Management Performance Incentive

Panel A: Principal's Utility Deviations From Benchmark

		Principal Risk Aversion Coefficient									
		6	5.5	5	4.5	4	3.5	3	2.5	2	1.5
Agent Risk Aversion	6	-2.09%	-1.45%	-0.80%	-0.16%	**0.48%**	**1.13%**	**1.77%**	**2.42%**	**3.06%**	**3.70%**
	5.5	-2.56%	-1.84%	-1.11%	-0.38%	**0.34%**	**1.07%**	**1.79%**	**2.52%**	**3.24%**	**3.97%**
	5	-3.19%	-2.36%	-1.53%	-0.70%	**0.13%**	**0.96%**	**1.79%**	**2.62%**	**3.45%**	**4.28%**
	4.5	-4.04%	-3.08%	-2.12%	-1.15%	-0.19%	**0.77%**	**1.74%**	**2.70%**	**3.66%**	**4.63%**
	4	-5.25%	-4.11%	-2.97%	-1.83%	-0.68%	**0.46%**	**1.60%**	**2.74%**	**3.88%**	**5.03%**
	3.5	-7.04%	-5.65%	-4.25%	-2.86%	-1.47%	-0.08%	**1.31%**	**2.70%**	**4.09%**	**5.49%**
	3	-9.82%	-8.06%	-6.30%	-4.55%	-2.79%	-1.03%	**0.73%**	**2.48%**	**4.24%**	**6.00%**
	2.5	-14.49%	-12.15%	-9.82%	-7.48%	-5.15%	-2.82%	-0.48%	**1.85%**	**4.19%**	**6.52%**
	2	-23.18%	-19.84%	-16.50%	-13.16%	-9.82%	-6.48%	-3.14%	**0.20%**	**3.54%**	**6.88%**
	1.5	-42.18%	-36.79%	-31.39%	-26.00%	-20.61%	-15.21%	-9.82%	-4.43%	**0.97%**	**6.36%**

Panel B: Principal's Utility Deviations from the Global Optimal Portfolio

		Principal Risk Aversion Coefficient									
		6	5.5	5	4.5	4	3.5	3	2.5	2	1.5
Agent Risk Aversion	6	-7.18%	-6.19%	-5.26%	-4.39%	-3.62%	-2.98%	-2.54%	-2.43%	-2.88%	-4.45%
	5.5	-7.65%	-6.58%	-5.57%	-4.62%	-3.76%	-3.04%	-2.52%	-2.33%	-2.69%	-4.19%
	5	-8.28%	-7.10%	-5.98%	-4.93%	-3.97%	-3.15%	-2.53%	-2.23%	-2.49%	-3.88%
	4.5	-9.13%	-7.83%	-6.57%	-5.39%	-4.29%	-3.34%	-2.58%	-2.15%	-2.27%	-3.53%
	4	-10.34%	-8.86%	-7.42%	-6.06%	-4.79%	-3.65%	-2.72%	-2.10%	-2.05%	-3.13%
	3.5	-12.13%	-10.39%	-8.71%	-7.10%	-5.57%	-4.19%	-3.00%	-2.14%	-1.84%	-2.67%
	3	-14.91%	-12.81%	-10.76%	-8.78%	-6.89%	-5.14%	-3.59%	-2.36%	-1.70%	-2.16%
	2.5	-19.58%	-16.90%	-14.27%	-11.72%	-9.25%	-6.93%	-4.80%	-2.99%	-1.75%	-1.63%
	2	-28.27%	-24.59%	-20.96%	-17.39%	-13.92%	-10.59%	-7.45%	-4.64%	-2.39%	-1.27%
	1.5	-47.27%	-41.53%	-35.85%	-30.23%	-24.71%	-19.32%	-14.14%	-9.27%	-4.97%	-1.80%

This table calculates the quatradic utility deviations for the principal under differing scenarios. Panel A depicts the utility increase or decrease for the principal from unconstrained delegated contracting. The delegated performance incentive has the potential to increase utility for the principal but this is far from certain. Given the levels of utility I use in the figures for this paper, the utility decreases by 5.15% through delegation. Panel B shows the utility depreciation from the principal's global optimal portfolio. This is necessarily non-positive. In this paper, the agent chooses a portfolio that 9.25% less optimal in utility than the principal's global optimal.

consistent with what was shown above. Although the agent has the potential to increase utility above that of the benchmark, unconstrained there is no guarantee for utility improvement.

III. The Principal's Utility and Improvements Along the TEV Frontier

As is conventional in portfolio management literature, I assume that the principal investor derives utility in mean-variance space. Thus his goal is to maximize utility subject to the constraint of the mean-variance efficient frontier. The premise of this essay is that principals delegate portfolio management responsibilities because they lack the ability or opportunity to build efficient portfolios themselves. The principal expects the delegated manager to build a portfolio better than the principal could build. Therefore the most logical basis for determining whether an agent improves upon the principal's utility is to compare the agent portfolio choice to whatever the principal could build given his limited skill level and opportunity set. If we suppose that the principal benchmarks the external manager against his best alternative, and incentivizes the agent relative to this benchmark (implicitly or explicitly), then this necessitates that the principal's utility curve pass through that benchmark portfolio. In the previous section we showed how the agent derives utility to create the TEV frontier. The relative positioning and interaction between these two curves, the TEV frontier and the principal's iso-utility curve, is what determines whether the agent can indeed improve the principal's utility given the delegated performance incentive.

Just as before when discussing the agent's utility, as long as the principal's utility is well formed, upward sloping with a non-negative second derivative in mean-variance space, the exact expression of the utility relationship is irrelevant to the conclusions of this essay. Utility is increasing for the principal to the "northwest" and the preference set is closed and convex. Notably, the opportunity set underneath the TEV envelope in mean-variance space is also a closed convex set. As discussed previously, these two sets intersect at least once, at portfolio **b**, which lies on the boundary of both sets. Unless the TEV frontier and the utility curve are exactly tangent at the benchmark portfolio, then the opportunity for utility improvement exists because in this framework there would be an overlap between the sets. There are four interesting cases of how these two sets could intersect, and these cases affect how the principal should constrain the external manager to assure utility improvement. The first three cases involve the slope of the sets at the intersection point, **b,** assuming **b** is above the minimum variance point. The slope of the utility curve can be steeper, flatter, or the same as the TEV frontier at this point. The fourth case is when the intersection happens given **b** is below the minimum variance portfolio. These cases are analyzed heuristically in Figure 2 and analytically with derivations in quadratic utility.

The steepness of the utility relationship is equivalent to the level of risk aversion. The higher the coefficient of risk aversion is, the steeper is the utility relationship in mean and variance. Below is an example of the utility function of the principal and note that is it parameterized in mean and variance. θ is the coefficient of risk aversion. And once again, quadratic utility is used in this example for simplicity in expression. In general, all of the assertions and conclusions in this essay are true no matter the exact form of the utility function. Thus, suppose principals maximize utility by choosing **w** given the following relationship:

$$U = r - \theta\,\sigma^2 = \mathbf{w}'\mathbf{r} - \theta\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} \tag{21}$$

If the principal had the ability and opportunity to maximize this utility function given the universe of all investment opportunities, he would choose a portfolio by maximizing this function, unconstrained as follows:

$$\max_{\mathbf{w}} U = \max_{\mathbf{w}}\ \mathbf{w}'\mathbf{r} - \theta\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} \quad s.t.\ \mathbf{w}'\mathbf{1} = 1 \tag{22}$$

$$\mathrm{L}(\mathbf{w}, \lambda_1) = \mathbf{w}'\mathbf{r} - \theta\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} - \lambda_1\,(\mathbf{w}'\mathbf{1} - 1) \tag{23}$$

Differentiating the Lagrangian and finding the critical value yields the following result.

$$\mathbf{w} = \frac{1}{2\theta}\boldsymbol{\Omega}^{-1}\left(\mathbf{r} - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1} - 2\theta}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\mathbf{1}\right) \tag{24}$$

This is the weight vector of the portfolio that the principal would ideally like to hold given the agent's opportunity set. Applying this vector to the utility relationship in (21) reveals the maximum iso-utility curve given the constraint of the efficient frontier.

$$\begin{aligned} r = U + \theta\,\sigma^2 &= \mathbf{w}'\mathbf{r} - \theta\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} + \theta\,\sigma^2 \\ &= \frac{1}{4\theta}\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1} - 2\theta)^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) + \theta\,\sigma^2 \end{aligned} \tag{25}$$

This level of utility is practically unreachable however because of the principal's lack of ability and opportunity. I assumed earlier that the principal can select a portfolio **b** that maximizes utility given his constrained skillset. If this is the portfolio on which the principal measures and incentivizes the agent, then this is the same **b** from the TEV analysis in section II. The utility of portfolio **b** and the iso-utility curve associated with the utility of portfolio **b** are expressed as follows:

$$U = \mathbf{b}'\mathbf{r} - \theta\mathbf{b}'\boldsymbol{\Omega}\mathbf{b} = r_b - \theta\, \sigma_b^2 \tag{26}$$

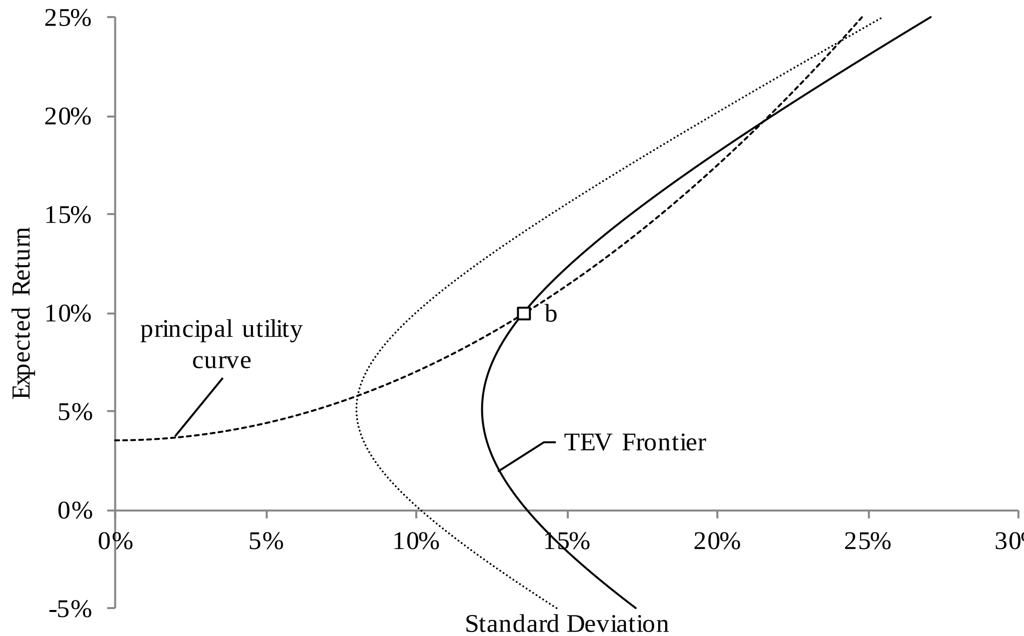
$$r = U + \theta\, \sigma^2 = r_b - \theta\, \sigma_b^2 + \theta\, \sigma^2 = r_b + \theta\, (\sigma^2 - \sigma_b^2) \tag{27}$$

Thus, we are concerned about the interaction between the curve from (27) and the curve from (10), the TEV frontier.

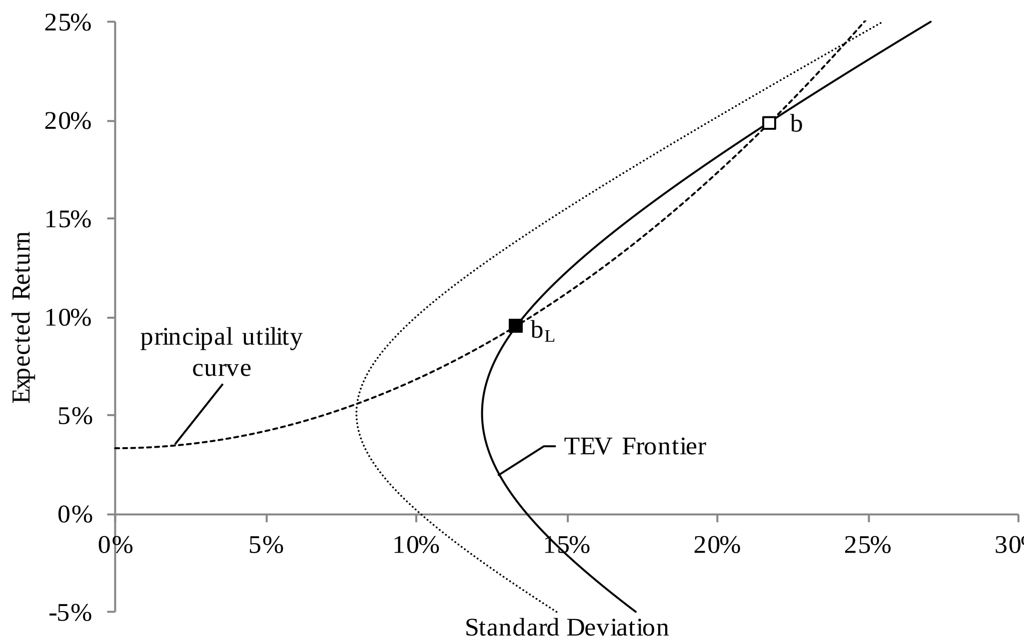
Figure 2, Panel A shows the most likely (or at least the most convenient) scenario for this relationship. In this case, the slope of the TEV frontier is steeper than the slope of the principal's utility curve at **b**. Recall from section II and also from Roll (1992) that an agent optimizing based on a relative incentive will choose a portfolio by moving along the TEV frontier up from the benchmark portfolio. In this case, since the utility curve has a flatter slope than the TEV frontier at the intersection point **b**, the preference set, the set above the iso-utility curve, overlaps with the opportunity set, the set underneath the TEV envelope, and every point in the intersection is a utility increase for the principal. In particular, as the agent's risk aversion level decreases, he differentiates from the benchmark and moves up the curve into this preferred space. Eventually however, the agent's risk aversion level could get so low that his optimization process pushes the portfolio back out of the preferred space. Therefore, if the agent is allowed to act on his own unconstrained utility, it is still likely that he will build a portfolio that decreases the principal's utility even though he could have increased it by buying a portfolio within the preferred region. I will discuss a method to constrain the agent in the next section but

Figure 2
Different Scenarios of how the TEV Frontier can Interact with Principal Utility

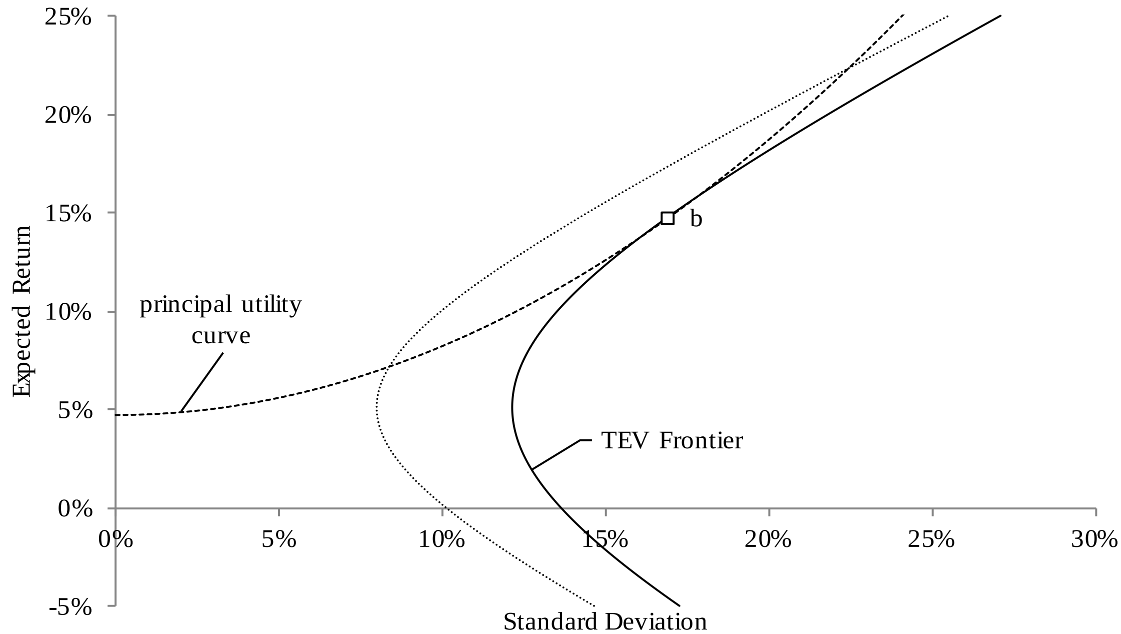
Panel A: TEV Frontier Steeper than Utility Curve at b



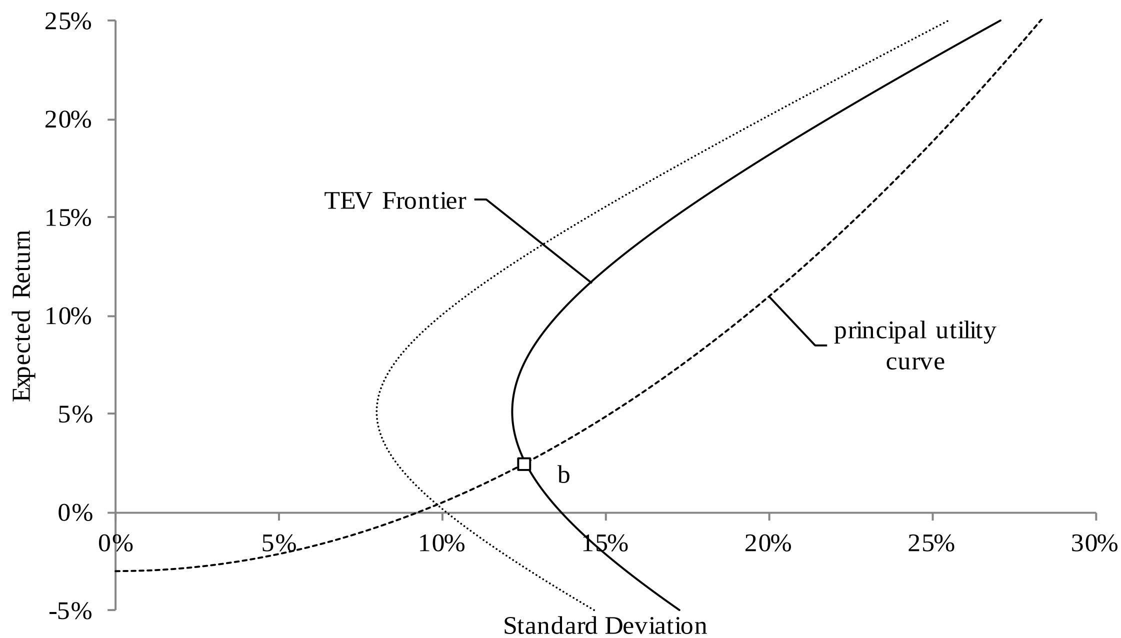
Panel B: TEV Frontier Flatter than Utility Curve at b



Panel C: TEV Frontier Steeper tangent to Utility Curve at b



Panel D: Benchmark portfolio, b, Below the Minimum Variance Portfolio



This figure depicts the 4 cases of interaction between the TEV frontier and the principal's iso-utility curve. The utility curve of the principal passes through the benchmark, b, and this is the point the principal is trying to improve upon through delegation. The MV frontier is shown as a lightly dotted line. Except for the case of Panel C, there is always a utility improvement for the principal along the TEV frontier.

what should be evident at this point is that delegated portfolio management very likely could increase principal utility.

Figure 2, Panel B shows the opposite scenario to Panel A. In this case, the principal's utility curve is steeper than the TEV frontier at **b**. Although a space still exists in the intersection of the preference set and the opportunity set, this space is on the wrong side of **b**. The process the agent follows when incentivized with the relative return inventive will push the portfolio choice away from the preferred region rather than into it. This may seem like a dire situation, but there is a simple transformation in this space that allows the relative incentive to continue to be used to the principal's advantage. First we should recognize that the principal's utility must have a flatter slope at **b** than the line segment that connects portfolio **b** to the origin. This is true only if we assume that the origin could be a potential long investment opportunity for the principal. If the line segment is not steeper than the utility curve then there exists a portfolio on the line segment with a higher utility than portfolio **b**, and this violates the assumption behind portfolio **b**. That is, portfolio **b** must be the highest utility portfolio available to the principal given his skill level and opportunity set. This condition assures us that this line segment intersects the TEV frontier below the set where the preference set intersects with the opportunity set. If we define this lower intersection point as point $\mathbf{b}_L$, then we have transformed the Case B problem into a Case A problem. If the delegated manager is now benchmarked against $\mathbf{b}_L$, then we are back into a situation where the delegated manager can create a utility increase under the right constraints. Care must be taken in this situation however because the principal needs to worry not only about the agent taking too much risk but also not taking enough risk. Deviations up the TEV frontier from $\mathbf{b}_L$ must cross a threshold before the preferred region is reached.

Figure 2, Panel C depicts the situation when the utility curve is exactly tangent to the TEV frontier at **b**. Given that the intersection is a single point, there is no region in which a preferred deviation would be made by a delegated manager incentivized by a relative return incentive alone. This truly is a dire situation for delegated management. In this scenario, it would be irrational to delegate portfolio management responsibilities to an unconstrained agent because the principal already holds the best possible portfolio given the delegated performance incentive. This situation is most comparable to the situation analyzed in Admati and Pfleiderer (1997). They find, among other things, that when a delegated manager is incentivized by a relative performance incentive, given an efficiently allocated benchmark, there is no possibility for utility improvement through delegation. This would indicate that the principal should choose not to delegate. However, in the Roll (1992) framework, which I am using in this essay, there is a way to constrain the external manager to necessarily improve utility even in this dire situation. This involves the beta constraint proposed by Roll and it is applicable to all three cases presented thus far. The beta constraint is analyzed in this paper in sections V and VI.

Figure 2, Panel D is the case when the benchmark portfolio, **b**, lies below the minimum variance portfolio and equivalently also below the minimum point on the TEV frontier. This is a special case of Case A. The reason it is special is because no matter the slope of the utility curve at **b**, deviations up from **b** along the TEV frontier always increase utility. As favorable as this situation seems, at least anecdotally, I consider this situation highly unlikely but not impossible. In order for this to occur, however, there would have to be massive inefficiency in the investment set available to the principal. Thus, although this case is intellectually curious, it is probably also practically impossible.

As depicted in Figure 2, it is highly likely that given the interaction between the principal's utility curve and the TEV frontier, that benchmarking and the implied relative return incentives could potentially create a utility increase for the principal. However, care must be taken in this relationship to ensure that the agent does not take too much risk as to exceed the region in which a preferred portfolio could be chosen. Thus at this point the framework is set to analyze the proposed constraints on external managers. How can we force the external manager to buy a portfolio within the preferred region?

IV. Constraining the Agent's Risk Appetite using a TEV Constraint

To control the behavior of external managers, other papers have assumed a direct constraint on agent risk aversion, such as Bertrand (2010). However, risk aversion is the parameter that defines the behavior of the individual and trying to forcibly constrain (or extend) an agent's risk level is akin to trying to constrain a law of nature. A principal could filter potential delegated managers based on his perceived notion of the agent's risk aversion but implicit in this filtering is a situation that creates more uncertainty, i.e. more risk, due to the potential for asymmetric information. Additionally, there is no good way to either ex-ante or ex-post measure an agent's risk aversion level. It can be proxied ex-post but would require a substantial amount of data. Practically however, if it could be done, constraining on agent risk aversion would peg the external manager to the optimal point for the principal along the TEV frontier. Constraining risk aversion would only allow the delegated manager to move so far up the frontier until the point where the principal's utility is optimized. But, why use the something so elusive when a conventional measure exists that is more tangible, measurable, and serves the same purpose: tracking error volatility.

Jorion (2003) considers the problem of maximizing (and minimizing) return given a tracking error constraint in mean-variance space. Equivalently we could minimize (and maximize) standard deviation or variance given a tracking error constraint. If the problem is set up in this fashion, on variance, it looks like the following:

$$\min_{\mathbf{w}} \mathbf{w}'\mathbf{\Omega}\mathbf{w} \quad s.t.\ \mathbf{w}'\mathbf{1} = 1 \ \ \mathbf{w}'\mathbf{r} = r \ and \ (\mathbf{w}-\mathbf{b})'\mathbf{\Omega}(\mathbf{w}-\mathbf{b}) = T^2 \tag{28}$$

And the Lagrangian can be set up like this:

$$\begin{aligned} \mathrm{L}(\mathbf{w}, \lambda_1, \lambda_r, \lambda_{T^2}) \\ &= \mathbf{w}'\mathbf{\Omega}\mathbf{w} - \lambda_1(\mathbf{w}'\mathbf{1} - 1) - \lambda_r(\mathbf{w}'\mathbf{r} - r) \\ &- \lambda_{T^2}((\mathbf{w}-\mathbf{b})'\mathbf{\Omega}(\mathbf{w}-\mathbf{b}) - T^2) \end{aligned} \tag{29}$$

Differentiating yields the following set of simultaneous equations.

$$\begin{gathered} 2\mathbf{\Omega}\mathbf{w} - \lambda_1\mathbf{1} - \lambda_r\mathbf{r} - \lambda_{T^2}(2\mathbf{\Omega}\mathbf{w} - 2\mathbf{\Omega}\mathbf{b}) = \mathbf{0} \\ \mathbf{w}'\mathbf{1} = 1 \\ \mathbf{w}'\mathbf{r} = r \\ (\mathbf{w}-\mathbf{b})'\mathbf{\Omega}(\mathbf{w}-\mathbf{b}) = T^2 \end{gathered} \tag{30}$$

Solving this system for **w** yields the following weight vector:

$$\mathbf{w} = \mathbf{b} + \left(\frac{\lambda_1 \mathbf{1} + \lambda_r \mathbf{r} - 2\lambda_{T^2} \boldsymbol{\Omega} \mathbf{b}}{2(1-\lambda_{T^2})}\right) \tag{31}$$

And applying this weight vector to the variance calculation yields the following ellipse in mean-variance space. Jorion calls this the constant TEV frontier.

$$\begin{aligned} &\begin{bmatrix} \sigma^2 - \sigma_b^2 - T^2 \\ r - r_b \end{bmatrix}' \begin{bmatrix} \mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \dfrac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}} & -2\left(r_b - \dfrac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) \\ -2\left(r_b - \dfrac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) & 4\left(\sigma_b^2 - \dfrac{1}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) \end{bmatrix} \begin{bmatrix} \sigma^2 - \sigma_b^2 - T^2 \\ r - r_b \end{bmatrix} \\ &- T^2 \begin{vmatrix} \mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \dfrac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}} & -2\left(r_b - \dfrac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) \\ -2\left(r_b - \dfrac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) & 4\left(\sigma_b^2 - \dfrac{1}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) \end{vmatrix} = 0 \end{aligned} \tag{32}$$

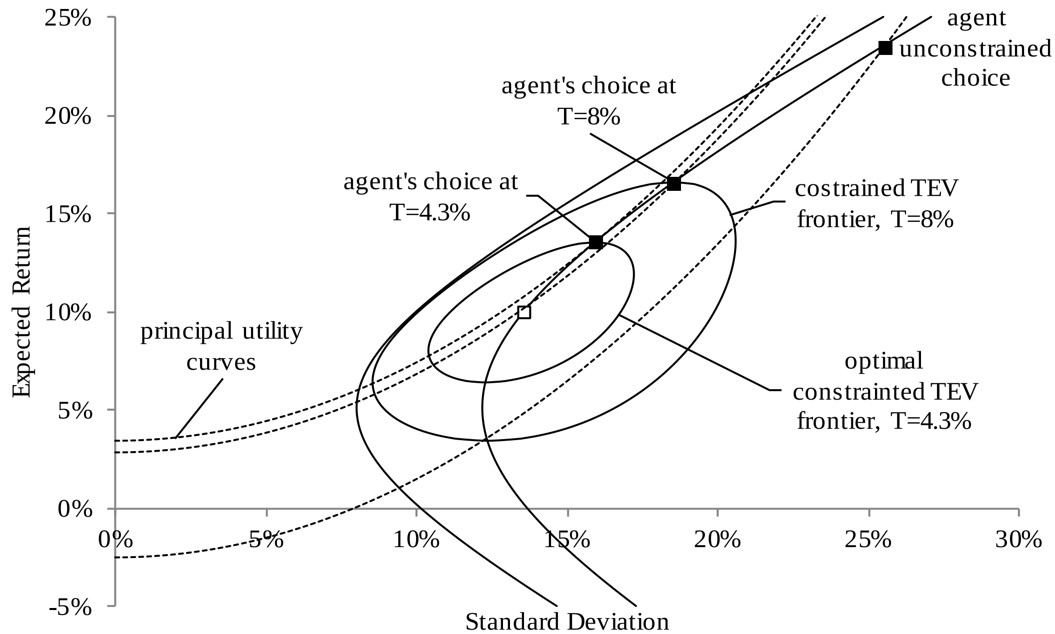
The notation |…| is the determinant of the matrix. This ellipse is illustrated in Figure 3, Panel A. Note that this is an ellipse in mean and variance and this figure is in mean and standard deviation, so the ellipse is slightly distorted. This ellipse grows and shrinks as the constant, T^2, is increased and decreased. However, since this is an ellipse, it is only defined over a limited region in r. As Jorion (2003) shows, this ellipse reaches its maximum and minimum values in return along the TEV frontier at the following levels, given a fixed T:

$$r = r_b \pm T\sqrt{\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)} \tag{33}$$

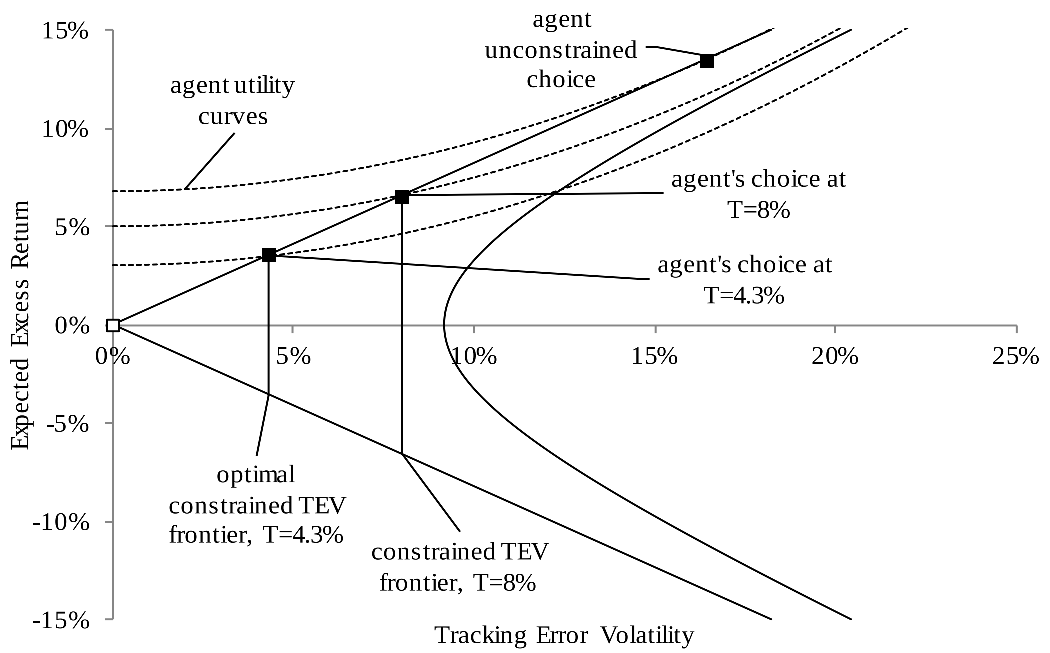
This expression is actually equivalent to equation (13) from Section II of this essay, the TEV frontier in mean and tracking error volatility. Equation (13) is just solved for the return. The constant TEV frontier is graphed with the TEV frontier in relative space in Figure 3, Panel B. The constant TEV frontier is just a vertical line connecting the top of

Figure 3
Constraining the Agent's Risk Appetite using a Tracking Error Constraint

Panel A: Mean-Variance Space



Panel B: Mean-TEV Space



This figure shows that constraining an agent on tracking error causes the agent to chose a portfolio further down the TEV frontier. This decreases agent utility but has the potential to increase principal utility. Depicted in this figure are an arbitrary constraint of T=8% and the principal's optimal constraint of about 4.3%. Note that the principal's utility curve is tangent to the TEV frontier at the optimal tracking error constraint.

the curve to bottom of the curve and it should be evident from this figure that the maximum and minimum must be reached along the frontier.

The expression with the radical from (33) is the slope of the line extending from the intercept and is also the maximum possible information ratio given the return vector and the covariance matrix. This is an important constant and it even appeared in equation (20) from this essay. Many other authors, stemming from Merton (1972), define this constant by giving it a name (usually *d*). The equivalence between constraining in tracking error volatility and agent risk aversion is also evident in equation (33)'s similarity to equation (19), the external manager's optimal choice given a level of risk aversion. If we take the transpose of equation (19) and multiply it by the return vector, we get the following:

$$\mathbf{w}'\mathbf{r} = \mathbf{b}'\mathbf{r} + \frac{1}{2\theta}\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{r}\right) \tag{34}$$

$$r = r_b + \frac{1}{2\theta}\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) \tag{35}$$

And, by setting equation (35) equal to equation (33), equivalence between the agent's quadratic utility risk aversion coefficient, θ, and the tracking error volatility, T, can be obtained:

$$T = \frac{1}{2\theta}\sqrt{\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^{2}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)} \tag{36}$$

This shows that constraining on tracking error is a practical way of constraining delegated managers to act as if their risk aversion had been constrained. The constraint does not change the agent's level of risk aversion. It merely forces the external manager to buy a portfolio as if he had a higher level of risk aversion.

What is obvious when considering the utility maximization problem of the agent, equation (16), is that if tracking error is constrained to be fixed, then the agent's utility maximization problem just becomes a return maximization problem: maximize *r* given a fixed *T*.

$$\max_{\mathbf{w}} U = \max_{\mathbf{w}} \mathbf{w}'\mathbf{r} - \theta(\mathbf{w}-\mathbf{b})'\boldsymbol{\Omega}(\mathbf{w}-\mathbf{b}) \ , \tag{37}$$

$$s.t. \ \mathbf{w}'\mathbf{1} = 1 \ and \ (\mathbf{w}-\mathbf{b})'\boldsymbol{\Omega}(\mathbf{w}-\mathbf{b}) = T^2$$

Solving this problem will yield the following weight vector but equation (36) could also be substituted into equation (19) to arrive at the same result:

$$\mathbf{w} = T\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)^{-1/2}\boldsymbol{\Omega}^{-1}\left(\mathbf{r} - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\mathbf{1}\right) + \mathbf{b} \tag{38}$$

Note that this equation is independent of the agent's risk aversion parameter. Equivalently, the minimization problem for (37) could be solved and it yields a similar solution to (38) except that the first term is negative.

The return for this weight vector is already given in equation (35) and the tracking error is fixed. This could be verified by plugging the weight vector in (38) into the tracking error equation (11). The combination of this return and tracking error are plotted in Figure 3, Panels A and B. Additionally, the utility curve associated with this portfolio is plotted alongside the agent's maximal utility curve. This constraint is necessarily utility decreasing for the agent. The opposite is true for the principal however. Since the principal cares mostly about the variance of this weight vector (38), plugging this into the variance equation yields the following:

$$\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} = T^2 + \sigma_b^2 + 2T\left(r_b - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)^{-1/2} \tag{39}$$

This is the variance of the portfolio above the benchmark on the TEV frontier as a function of tracking error, T. Similarly, the portfolio below the benchmark on the TEV frontier at the same level of tracking error has a variance with a similar equation but with one small difference: the third term in (39) is negative.

Given that the delegated manager will choose a portfolio on the TEV frontier, a principal can maximize his own utility relative to the TEV frontier by choosing the appropriate tracking error bound to apply to an external manager. Since the return and standard deviation can be parameterized in T, then we simply maximize the principal's utility function (21) with respect to T by substituting in equations (33) and (39) for the return and variance.

$$\begin{aligned}\max_T U &= \max_T \ \mathbf{w}'\mathbf{r} - \theta\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} \\ &= \max_T \ r_b + T\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)^{1/2} \\ &\quad - \theta\left(T^2 + \sigma_b^2\right. \\ &\quad \left.+ 2T\left(r_b - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)^{-1/2}\right)\end{aligned} \tag{40}$$

And the solution is:

$$T = \frac{1}{2\theta}\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)^{1/2} - \left(r_b - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1})^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)^{-1/2} \tag{41}$$

This is the tracking error bound a principal with quadratic utility should choose to force the delegated manager to buy a portfolio that maximizes the principal's utility subject to the constraint of the TEV frontier. If this bound is chosen, and the agent abides by the constraint, then as is depicted in Figure 3, Panel A, the principal's utility curve associated with this constraint is tangent to the TEV frontier at the intersection of the frontier with the constant TEV ellipse. This level of utility is preferred to the one associated with holding the benchmark albeit still less preferred than the one associated with the principal's global optimization problem.

Table 2 calculates the quadratic utility deviations for the principal under differing scenarios of agent and principal risk aversion. Panel A depicts the utility increase or decrease for the principal from a fixed tracking error constraint of 8%. The delegated performance incentive with a tracking error constraint has the potential to increase utility for the principal if the constraint is chosen appropriately. Given the levels of utility I use in the figures for this paper, the utility increases by 0.20% through delegation with an 8% constraint. Panel B shows the utility increase from the benchmark given an optimal tracking error constraint; this is necessarily non-negative. Also depicted is that the level of utility increase is independent of agent risk aversion. Essentially, values down each

Table 2
Utility Deviations from TEV Constrained Delegated Portfolio Management

Panel A: Principal's Utility Deviations from the Benchmark given a Fixed Constraint of 8%

		Principal Risk Aversion Coefficient									
		6	5.5	5	4.5	4	3.5	3	2.5	2	1.5
Agent Risk Aversion	6	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	5.5	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	5	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	4.5	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	4	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	3.5	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	3	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	2.5	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	2	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%
	1.5	-2.99%	-2.20%	-1.40%	-0.60%	0.20%	1.00%	1.79%	2.59%	3.39%	4.19%

Panel B: Principal's Utility Deviations from the Benchmark given an Optimal Tracking Error Constraint

		Principal Risk Aversion Coefficient									
		6	5.5	5	4.5	4	3.5	3	2.5	2	1.5
Agent Risk Aversion	6	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	5.5	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	5	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	4.5	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	4	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	3.5	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	3	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	2.5	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	2	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%
	1.5	0.05%	0.12%	0.25%	0.45%	0.74%	1.17%	1.80%	2.74%	4.26%	6.90%

This table calculates the quadratic utility deviations for the principal under differing scenarios. Panel A depicts the utility increase or decrease for the principal from a fixed tracking error constraint of 8%. The delegated performance incentive with a tracking error constraint has the potential to increase utility for the principal if the constraint is chosen appropriately. Given the levels of utility I use in the figures for this paper, the utility increases by 0.20% through delegation and an 8% constraint. Panel B shows the utility increase from the benchmark given an optimal tracking error constraint; this is necessarily non-negative. Also depicted is that the level of utility increase is independent of agent risk aversion.

column is identical. Recall that the tracking error constraint substitutes for the agent's risk aversion level.

V. Constraining Benchmark Relative Beta

In addition to the tracking error constraint, another pervasive constraint applied in the modern investment industry is the benchmark relative beta constraint. The colloquial way to consider a beta constraint is to put a limitation on "style drift" but the beta constraint is about much more than just the technical definition of style drift. Most delegated investment management is done in two stages: a strategic allocation stage, where the asset class weights (the beta or factor sensitivities) are set, then the tactical asset allocation is made to improve performance over each factor. When hiring an external manager to manage all or part of an allocation to a particular asset class, the principal is mostly concerned with whether the agent manages the factor sensitivity appropriately. If an agent creates factor sensitivity different than $\beta = 1$ against the benchmark, this hurts the optimality of the strategic allocation either by altering the agent's mandated factor sensitivity and therefore the effective weight of that allocation in the overall portfolio, or though the agent cannibalizing factor sensitivity from a different delegated manager. However, even in the case of a single delegated manager, the beta constraint seems justified. Agents are being hired to use their skill to the benefit of the principal. As per our assumptions, the principal already has the skill to buy and therefore lever the factor sensitivity of the benchmark, at least downward. Thus some form of beta constraint seems rational when trying to control delegated managers.

Many external managers will argue that the beta constraint limits their ability to generate returns for the principal. And, just like the tracking error constraint, the beta constraint necessarily reduces the utility of the agent but has the potential to increase utility for the principal. Roll (1992) analyzes the beta constraint and shows that when the benchmark is above the minimum variance portfolio in return the beta constrained TEV frontier is superior to the unconstrained TEV frontier for some finite region above the benchmark portfolio. We can set up the minimization problem as follows.

$$\min_{\mathbf{w}}(\mathbf{w}-\mathbf{b})'\boldsymbol{\Omega}(\mathbf{w}-\mathbf{b}) \quad s.t.\ \mathbf{w}'\mathbf{1} = 1\ \mathbf{w}'\mathbf{r} = r\ and\ \mathbf{w}'\boldsymbol{\Omega}\mathbf{b} = \beta\sigma_b^2 \qquad (42)$$

This problem is actually set up with a constraint to covariance, $\mathbf{w}'\boldsymbol{\Omega}\mathbf{b}$, rather than beta but since the variance of the relative benchmark is a constant, constraining on beta and constraining on covariance are equivalent. Also, it should be noted that under the constraint on beta, or covariance, the minimization problem on tracking error and the minimization problem on variance are also equivalent since the tracking error is now a function of the variance of the portfolio and two constants, covariance and the variance of the benchmark. Thus the following problem could be solved for **w** and an identical solution would be obtained.

$$\min_{\mathbf{w}} \mathbf{w}'\boldsymbol{\Omega}\mathbf{w} \quad s.t.\ \mathbf{w}'\mathbf{1} = 1\ \mathbf{w}'\mathbf{r} = r\ and\ \mathbf{w}'\boldsymbol{\Omega}\mathbf{b} = \beta\sigma_b^2 \qquad (43)$$

Unlike the situation with unconstrained TEV optimization, under the condition of a beta constraint, the delegated manager and the principal are, at least, now optimizing against the same curve. Below is the Lagrangian based on equation (43)

$$\mathrm{L}(\mathbf{w},\, \lambda_1,\, \lambda_r,\, \lambda_\beta) \tag{44}$$
$$= \mathbf{w}'\boldsymbol{\Omega}\mathbf{w} \,-\, \lambda_1\,(\mathbf{w}'\mathbf{1} - 1) \,-\, \lambda_r\,(\mathbf{w}'\mathbf{r} - r)$$
$$-\, \lambda_\beta(\mathbf{w}'\boldsymbol{\Omega}\mathbf{b} - \beta\sigma_b^2)$$

Differentiating yields the following system of equations:

$$2\boldsymbol{\Omega}\mathbf{w} \,-\, \lambda_1\mathbf{1} \,-\, \lambda_r\mathbf{r} - \lambda_\beta\boldsymbol{\Omega}\mathbf{b} = \mathbf{0}$$
$$\mathbf{w}'\mathbf{1} = 1 \tag{45}$$
$$\mathbf{w}'\mathbf{r} = r$$
$$\mathbf{w}'\boldsymbol{\Omega}\mathbf{b} = \beta\sigma_b^2$$

Solving this system yields the following weight vector:

$$\mathbf{w} = \boldsymbol{\Omega}^{-\mathbf{1}}[\mathbf{1} \quad \mathbf{r} \quad \boldsymbol{\Omega}\mathbf{b}]\Big[[\mathbf{1} \quad \mathbf{r} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-\mathbf{1}}[\mathbf{1} \quad \mathbf{r} \quad \boldsymbol{\Omega}\mathbf{b}]\Big]^{-\mathbf{1}} \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix} \tag{46}$$

And the equation for the parabola in mean-variance space is derived by plugging the weight vector from (46) into the variance equation as follows. This curve is depicted in Figure 4, Panel A for the cases $\beta = 1$, $\beta > 1$, and $\beta < 1$.

$$\sigma^2 = \mathbf{w}'\boldsymbol{\Omega}\mathbf{w} = \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix}' \Big[[\mathbf{1} \quad \mathbf{r} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-\mathbf{1}}[\mathbf{1} \quad \mathbf{r} \quad \boldsymbol{\Omega}\mathbf{b}]\Big]^{-\mathbf{1}} \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix} \tag{47}$$

The equation for the curve in relative mean-tracking error space can be computed by first differencing the weight vector in (46) with **b**:

$$(\mathbf{w} - \mathbf{b}) = \boldsymbol{\Omega}^{-\mathbf{1}}[\mathbf{1} \quad \mathbf{r} \quad \boldsymbol{\Omega}\mathbf{b}]\Big[[\mathbf{1} \quad \mathbf{r} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-\mathbf{1}}[\mathbf{1} \quad \mathbf{r} \quad \boldsymbol{\Omega}\mathbf{b}]\Big]^{-\mathbf{1}} \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix} - \mathbf{b} \tag{48}$$

Next, the differenced vector in equation (48) can be plugged into the tracking error variance equation to yield the following:

$$T^2 = \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r} \quad \mathbf{\Omega b}]' \mathbf{\Omega}^{-1} [\mathbf{1} \quad \mathbf{r} \quad \mathbf{\Omega b}] \right]^{-1} \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix} + \sigma_b^2$$

$$- 2 \begin{bmatrix} 1 \\ r_b \\ \sigma_b^2 \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r} \quad \mathbf{\Omega b}]' \mathbf{\Omega}^{-1} [\mathbf{1} \quad \mathbf{r} \quad \mathbf{\Omega b}] \right]^{-1} \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix} \qquad (49)$$

$$= \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix}' \left[[\mathbf{1} \quad \mathbf{r} \quad \mathbf{\Omega b}]' \mathbf{\Omega}^{-1} [\mathbf{1} \quad \mathbf{r} \quad \mathbf{\Omega b}] \right]^{-1} \begin{bmatrix} 1 \\ r \\ \beta\sigma_b^2 \end{bmatrix}$$

$$+ (1 - 2\beta)\sigma_b^2$$

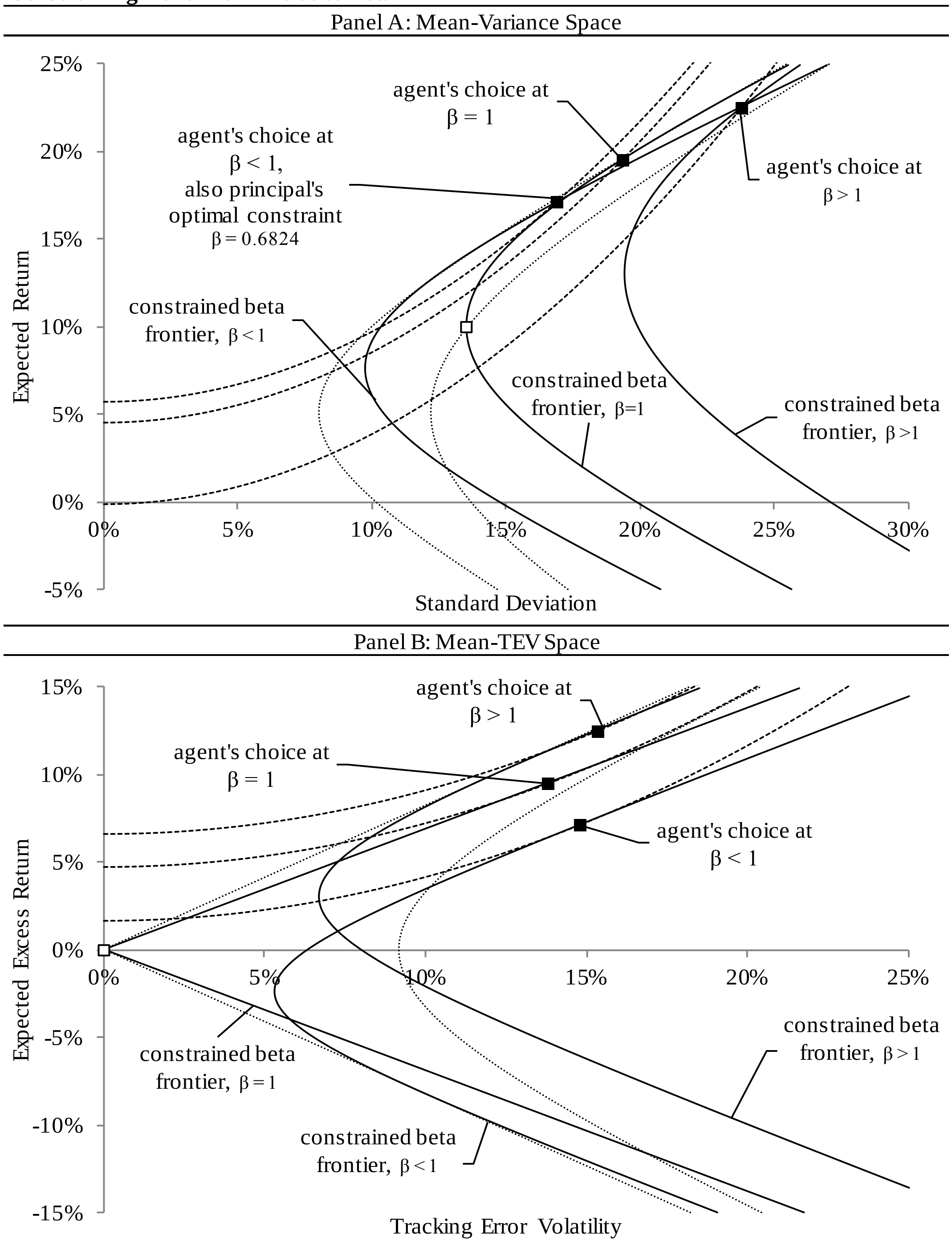
This curve is depicted in Figure 4, Panel B also for three cases, $\beta = 1$, $\beta > 1$, and $\beta < 1$. Note that these curves are all at best as good at the TEV frontier in preference and a principal requiring a beta constraint will most likely require the agent to suffer a utility deterioration. The weight vector the external manager will choose can be derived by maximizing his utility function however this time with the addition of a beta constraint.

Just as with the derivation of the beta constrained TEV frontier, maximizing the agent's utility function parameterized in tracking error and maximizing the principal's utility function parameterized in variance yields the same choice vector, given the same value for θ, because the beta, or covariance, constraint makes $\mathbf{w'\Omega w}$ the only free variable in the risk calculation. However, there is no reason why the risk aversion parameter would be the same so it is still likely an agent and a principal would choose a different portfolio. Below is the utility to be optimized, parameterized in variance.

$$\max_{\mathbf{w}} U = \max_{\mathbf{w}} \mathbf{w'r} - \theta \mathbf{w'\Omega w} \quad s.t.\ \mathbf{w'1} = 1 \ and \ \mathbf{w'\Omega b} = \beta\sigma_b^2 \qquad (50)$$

$$\mathrm{L}(\mathbf{w}, \lambda_1, \lambda_\beta) = \mathbf{w'r} - \theta \mathbf{w'\Omega w} - \lambda_1 (\mathbf{w'1} - 1) - \lambda_\beta (\mathbf{w'\Omega b} - \beta\sigma_b^2) \qquad (51)$$

Figure 4
Constraining Benchmark Relative Beta



This figure demonstrates how the agent's portfolio choice changes with the benchmark relative beta constraint. In general, the agent's utility is increasing in beta. However, there is an optimal beta constraint for the principal. This constraint is depicted as the β < 1 curve. Not coincidently, this is the beta of the principal's global optimal portfolio.

Differentiating the Lagrangian, finding the critical value and solving for **w** yields the following vector:

$$\mathbf{w} = \frac{1}{2\theta}\mathbf{\Omega^{-1}}\Bigg[\mathbf{r} - [\mathbf{1} \quad \mathbf{\Omega b}]\big[[\mathbf{1} \quad \mathbf{\Omega b}]'\mathbf{\Omega}^{-1}[\mathbf{1} \quad \mathbf{\Omega b}]\big]^{-1}\Bigg[[\mathbf{1} \quad \mathbf{\Omega b}]'\mathbf{\Omega}^{-1}\mathbf{r} - 2\theta\begin{bmatrix}1\\ \beta\sigma_b^2\end{bmatrix}\Bigg]\Bigg] \tag{52}$$

This is the weight vector for the portfolio that the principal and the agent would choose given a risk aversion level of θ and a beta constraint of β.

The idea of the principal choosing an optimal beta constraint is slightly more ambiguous than the principal's choice of a tracking error constraint. The tracking error problem eliminated the need to estimate the agent's risk aversion, but the risk aversion level is still a prominent parameter in the beta constrained TEV frontier as is evident in equation (47). The delegated manager's weight vector in (52) could be used in the portfolio variance and return functions, plugged into the principal's utility function, and that function could be solved for the critical value on beta to arrive at the optimal beta. But, there is an easier way to discover this value by inspection. Recognize that since the optimization happens on the same curve in absolute and relative space, if the principal and agent have the same value for their individual risk aversion constants, the principal could constrain the agent based on the beta of the global optimal portfolio and the agent would buy the global optimal portfolio. However, it is more likely that their risk aversion levels differ. But, this is not a concern. When you plug the weight vector from (52) into the principal's utility function and differentiate on beta, the agent's risk aversion constants cancel in the only remaining terms. Thus, the optimal beta constraint is independent of the agent's risk aversion level. And, since we already discovered it given equal risk aversion levels, it must be equal to this value, the beta of the global optimal portfolio. Therefore, to obtain the optimal beta constraint, all one needs is to go back to the portfolio choice made in the principal's utility given the principal's risk aversion level of θ, and calculate the beta of the optimal choice portfolio. Beta is calculated as follows:

$$\beta = \frac{1}{\sigma_b^2}\mathbf{w}'\boldsymbol{\Omega}\mathbf{b} \tag{53}$$

Thus, we simply plug-in the weight vector from equation (24) and the optimal beta is calculated:

$$\beta = \frac{1}{2\theta\sigma_b^2}\left(\mathbf{r}'\mathbf{b} - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1} - 2\theta}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\mathbf{1}'\mathbf{b}\right) = \frac{1}{2\theta\sigma_b^2}\left(r_b - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1} - 2\theta}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) \tag{54}$$

This result could be verified by going through the standard process of maximizing a utility function described in the last paragraph. This optimal beta constrained TEV curve is depicted in Figure 4, Panels A and B and in this case is represented by the curve where $\beta < 1$. As is evident from the figure, the iso-utility curves are plotted for an agent with a higher risk appetite than the principal. The agent's utility curve under a beta constraint is always below his optimal utility curve and therefore this is a utility decrease for the agent. However, as depicted in the figure, this constraint could be a possible utility increase for the principal.

Table 3 calculates the quadratic utility deviations for the principal under differing scenarios. Panel A depicts the utility increase or decrease for the principal from a fixed

Table 3

Utility Deviations from Beta Constrained Delegated Portfolio Management

Panel A: Principal's Utility Deviations from the Benchmark given a Fixed Beta Constraint of 1

		Principal Risk Aversion Coefficient									
		6	5.5	5	4.5	4	3.5	3	2.5	2	1.5
Agent Risk Aversion	6	1.98%	2.14%	2.31%	2.47%	2.64%	2.80%	2.97%	3.13%	3.30%	3.46%
	5.5	1.96%	2.16%	2.35%	2.55%	2.75%	2.94%	3.14%	3.34%	3.53%	3.73%
	5	1.90%	2.14%	2.37%	2.61%	2.85%	3.09%	3.32%	3.56%	3.80%	4.04%
	4.5	1.76%	2.05%	2.34%	2.64%	2.93%	3.22%	3.52%	3.81%	4.10%	4.40%
	4	1.48%	1.85%	2.23%	2.60%	2.97%	3.34%	3.71%	4.08%	4.45%	4.82%
	3.5	0.97%	1.45%	1.94%	2.42%	2.91%	3.39%	3.88%	4.36%	4.85%	5.33%
	3	0.00%	0.66%	1.32%	1.98%	2.64%	3.30%	3.96%	4.62%	5.28%	5.94%
	2.5	-1.90%	-0.95%	0.00%	0.95%	1.90%	2.85%	3.80%	4.75%	5.70%	6.65%
	2	-5.94%	-4.45%	-2.97%	-1.48%	0.00%	1.48%	2.97%	4.45%	5.94%	7.42%
	1.5	-15.83%	-13.19%	-10.55%	-7.91%	-5.28%	-2.64%	0.00%	2.64%	5.28%	7.91%

Panel B: Principal's Utility Deviations from the Benchmark given an Optimal Beta Constraint

		Principal Risk Aversion Coefficient									
		6	5.5	5	4.5	4	3.5	3	2.5	2	1.5
Agent Risk Aversion	6	5.09%	4.73%	4.39%	4.07%	3.77%	3.52%	3.33%	3.23%	3.30%	3.70%
	5.5	5.07%	4.75%	4.44%	4.15%	3.88%	3.66%	3.50%	3.43%	3.53%	3.97%
	5	5.01%	4.72%	4.46%	4.21%	3.99%	3.80%	3.68%	3.66%	3.80%	4.28%
	4.5	4.87%	4.64%	4.43%	4.23%	4.07%	3.94%	3.88%	3.91%	4.11%	4.64%
	4	4.59%	4.44%	4.31%	4.19%	4.10%	4.06%	4.07%	4.18%	4.45%	5.06%
	3.5	4.08%	4.04%	4.02%	4.02%	4.04%	4.11%	4.24%	4.46%	4.85%	5.57%
	3	3.11%	3.25%	3.40%	3.57%	3.77%	4.02%	4.32%	4.71%	5.28%	6.18%
	2.5	1.21%	1.64%	2.08%	2.54%	3.04%	3.57%	4.16%	4.84%	5.70%	6.89%
	2	-2.83%	-1.86%	-0.89%	0.11%	1.14%	2.20%	3.33%	4.55%	5.94%	7.66%
	1.5	-12.72%	-10.60%	-8.47%	-6.32%	-4.14%	-1.92%	0.36%	2.73%	5.28%	8.16%

This table calculates the quadratic utility deviations for the principal under differing scenarios. Panel A depicts the utility increase or decrease for the principal from a fixed Beta constraint of 1. The delegated performance incentive with a beta constraint has the potential to increase utility for the principal if the constraint is chosen appropriately. Given the levels of utility I use in the figures for this paper, the utility increases by 1.90% through delegation and a beta of 1. Panel B shows the utility increase from the benchmark given an optimal beta constraint; this is necessarily better than Panel A. Note that the level of utility increase is substantial and relatively certain under reasonable levels of risk aversion.

Beta constraint of 1. The delegated performance incentive with a beta constraint has the potential to increase utility for the principal if the constraint is chosen appropriately. Given the levels of utility I use in the figures for this paper, the utility increases by 1.90% through delegation and a beta of 1. Although utility increase is relatively certain, if the agent's risk aversion level is low enough, he can still destroy utility under a beta constraint. Panel B shows the utility increase from the benchmark given an optimal beta constraint; this is necessarily better than Panel A. Essentially, cell my cell the utility level is higher in Panel B than in Panel A. Note that just like Panel A, the level of utility increase is substantial and relatively certain under reasonable levels of risk aversion, but still not guaranteed.

VI. Maximizing Principal Utility by Constraining TEV and Beta

In section IV, I discussed controlling the level of risk an external manager takes by using a tracking error constraint, similar to Jorion (2003). This constrains the delegated manager to purchase a portfolio that optimizes principal utility despite the agent's actual level of risk aversion. In section V, I discussed using a beta constraint to encourage the external manager to invest on a frontier superior (at least for some finite region) to the TEV frontier, similar to Roll (1992). Both of these constraints have the ability to increase the principal's utility and have the unfortunate downside of decreasing agent utility. The combination of these two constraints should allow the principal to potentially increase utility to a greater extent than either constraint can accomplish alone. The inclusion of both a beta and tracking error constraint limits the set of possible portfolios to either 2, 1, or 0 real solutions in both mean-variance and mean-TEV space. The evidence that there are at most 2 solutions to these intersections is more technically understood from an analysis of equation (49). This is an equation parameterized in T, β, and r. T and β are constrained to be fixed, and since the equation is quadratic in r, it has a maximum of 2 solutions.

Since the values for T and β are derived from the same data, there are combinations of these calculations that are not feasible in a real sense based on the return vector and the covariance matrix, thus the option of zero solutions is a definite possibility. Also as was discussed in the analysis of the agent's portfolio choice along the TEV frontier, if there are two solutions to the optimization problem, one of the solutions always dominates the other solution in the agent's portfolio choice based on his utility function. Thus the proper use of a beta and TEV constraint, as long as the combination is consistent, could direct the external manager to buy any portfolio underneath the envelope of the efficient frontier. "Any portfolio" obviously includes the principal's global optimal portfolio. Therefore, a principal would calculate the optimal constraints by maximizing his global utility function, unconstrained, then merely calculate the beta and tracking error of this portfolio and provide those constraints to the agent. Again, this would force the agent to buy the principal's global optimal portfolio.

The optimal beta constraint was calculated in the previous section and recall that this constraint is not dependent on the risk aversion level of the agent. Thus, the optimal beta constraint is always as is given in equation (54). Additionally, the tracking error constraint requirement made the agent's risk aversion irrelevant. However, just as with controlling agent risk level along the TEV frontier by increasing and decreasing tracking error, the principal controls the agent's risk level along the beta constrained frontier with a tracking error constraint. As the tracking error constraint is increased, the agent moves further and further up the constrained beta frontier until the point where the constraint is reached, at the optimal portfolio. The tracking error variance of the global optimal portfolio can be calculated by using the weight vector of the principal's optimal portfolio choice from equation (24) and substituting it into the definition of tracking error variance:

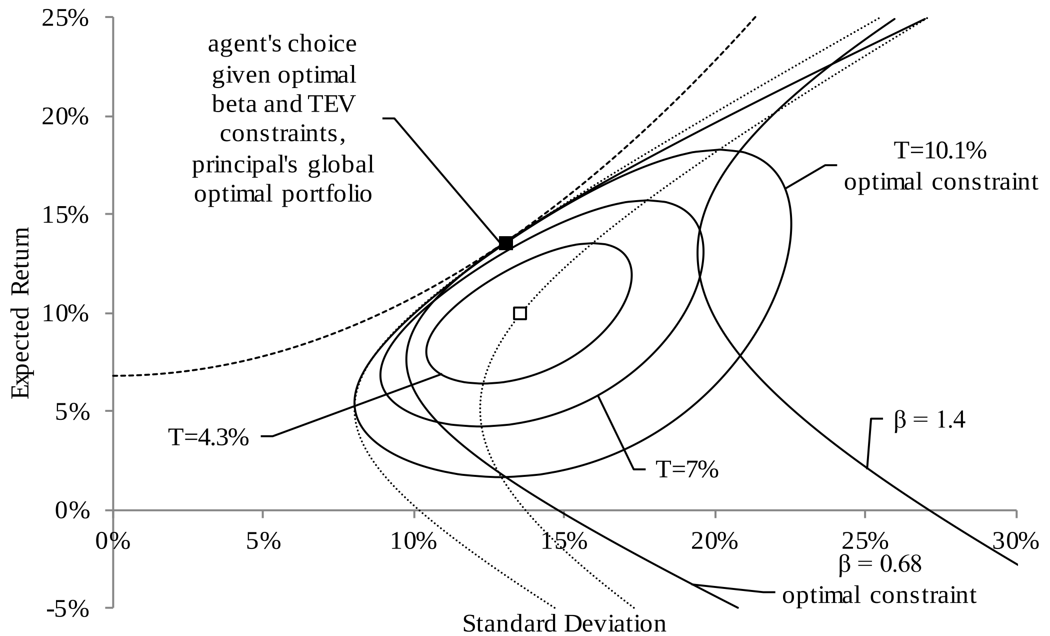
$$T^2 = \frac{1}{2\theta}\left(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{r} - 2\left(\frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1} - 2\theta}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right)\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1} + \frac{(\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1} - 2\theta)^2}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) + \sigma_b^2 - \frac{1}{\theta}\left(r_b - \frac{\mathbf{r}'\boldsymbol{\Omega}^{-1}\mathbf{1} - 2\theta}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}\right) \tag{55}$$

This is the tracking error of the principal's global optimal portfolio given an optimization on the agent's information. In this context, the tracking error constraint is definitely a function of the principal's risk aversion coefficient.

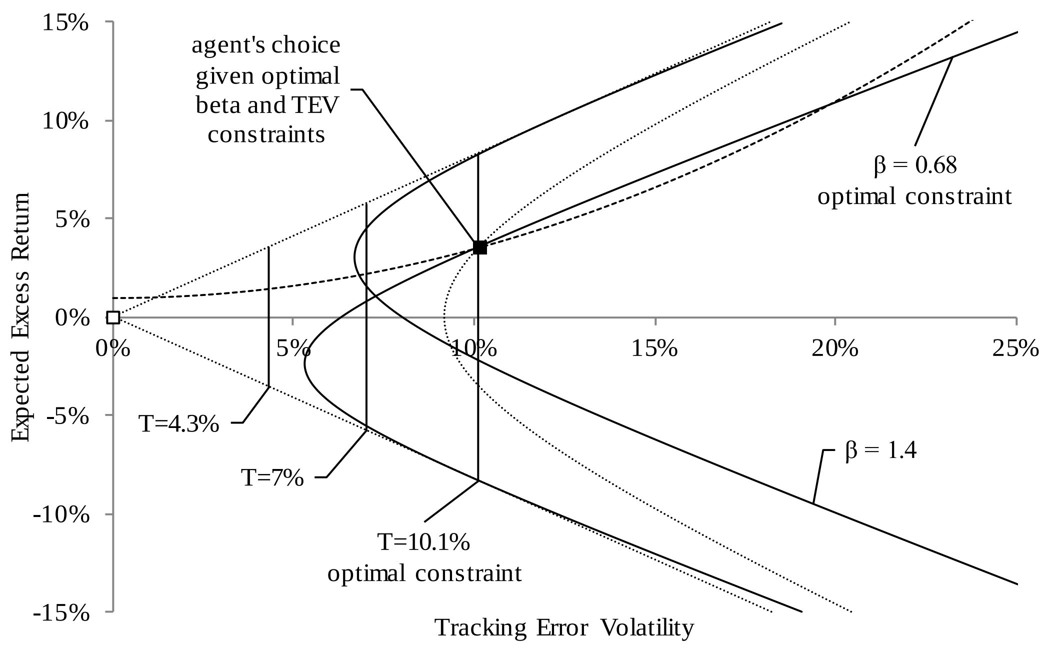
In Figure 5, Panel A and B the curves of the mean-variance frontier, the constant TEV ellipse, and the constant beta frontier are graphed for various scenarios. Additionally, the global maximum portfolio is identified and the associated iso-utility curves are drawn; the principal's utility is depicted in Panel A and the agent's utility is depicted in Panel B. It is evident in the figure that the constant beta frontier can cross the constant TEV ellipse in at most 2 places and also that it is possible for the curves to have no real intersections, indicating a set of inconsistent constraints. The constant beta frontier is also tangent to the mean-variance frontier at exactly one point. If the tracking error and beta constraints are as was defined above, then the curves of the efficient

Figure 5
Maximizing Principal Utility by Constraining Both the Tracking Error and Beta

Panel A: Mean-Variance Space



Panel B: Mean-TEV Space



This figure shows various levels of beta and tracking error constrained frontiers along with the principal's optimal constraints. Given the optimal constraints, the agent will choose the principal's global optimal portfolio. It is evident from the figure that the agent's utility is sub-optimal but that the principal's utility is maximized. The optimally constrained beta and tracking error curves and the efficient frontier are all tangent to the principal's utility curve at the agent's choice portfolio.

frontier, the constant beta frontier, and the constant TEV ellipse are exactly tangent at one point, the global mean-variance optimal portfolio. Thus the intersection of these curves reflects that the constraints incentivize the delegated manager to buy the principal's global optimal portfolio.

However, the constraints associated with the global maximum portfolio may be too loose. For example, in the dataset used to build the figures in this paper, the beta constraint of the global optimal portfolio is about 0.68 and the tracking error volatility constraint is about 10%. These constraints seem rather ridiculous to give an external manager in the context of delegated management. In particular, the concern with a loose tracking error constraint is whether it is even possible to generate that type of tracking error versus a benchmark without buying another uncorrelated systematic risk. This theory implies that all of the differentiation is done with the delegated manager's uncorrelated alpha generating process. Which leads to the second concern: if you benchmark a manager and give him a beta constraint of 0.6, what does the manager do with the other 0.4? If the external manager's uncorrelated, idiosyncratic strategy, their alpha strategy, takes factor exposure to any asset class other than the mandated asset class, then that creates inefficiency in in the principal's overall portfolio.

That however is only the first concern. It is not only probable but highly likely that it is not possible for the principal to identify his global optimal portfolio much less calculate the tracking error variance and beta of the portfolio. This is particularly true considering the set of information he would need to optimize over is the agent's set of information, information for which he would be hiring the delegated manager because the principal does not know this information. As other authors have suggested, using general bounds for these two constraints is probably more realistic. Roll (1992) emphasizes the beta constraint of $\beta = 1$. This constraint has the benefit of a guaranteed utility increase no matter the slope of the principal's utility and is also consistent with the idea of preventing style drift. The tracking error constraint can be imposed to move up the beta constrained frontier. Below I calculate the optimal tracking error constraint given a beta constraint of 1; however, it is likely that a moderate tracking error constraint would also be good enough. Not coincidently, these two constraints, with reasonable values, are exactly the way principals control delegated managers in the investment industry.

Thus, the problem is to optimize principal utility by choosing tracking error given the constraint of $\beta = 1$. We could parameterize the principal's utility as a function of tracking error. Similar to the problem of optimizing agent utility given a tracking error constraint, under the conditions of a fixed tracking error and a fixed beta, optimizing either agent or principal utility is now just a return maximization problem. The relationship between tracking error and variance is a simple one, particularly with $\beta = 1$. The expression is just as follows:

$$\sigma^2 = T^2 + \sigma_b^2 \tag{56}$$

Calculating the level of return associated with the two constraints is a rather complicated expression but could be done by solving equation (49) for r. It could also be done by using equation (52) above, setting beta equal to zero and using the principal's risk aversion instead of the agent's risk aversion.

$$\mathbf{w} = \frac{1}{2\theta}\boldsymbol{\Omega}^{-1}\Bigg[\mathbf{r} - [\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]\big[[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-1}[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]\big]^{-1}\Bigg[[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-1}\mathbf{r} - 2\theta\begin{bmatrix}1\\ \sigma_b^2\end{bmatrix}\Bigg]\Bigg] \tag{57}$$

Plugging this weight vector into the return function yields the following:

$$r = \mathbf{w}'\mathbf{r} = \frac{1}{2\theta}\Bigg[\mathbf{r} - [\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]\big[[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-1}[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]\big]^{-1}\Bigg[[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-1}\mathbf{r} - 2\theta\begin{bmatrix}1\\ \sigma_b^2\end{bmatrix}\Bigg]\Bigg]'\boldsymbol{\Omega}^{-1}\boldsymbol{r} \tag{58}$$

And the optimal constraint to *T* is chosen by applying that weight vector to the tracking error function:

$$T^2 = \frac{1}{(2\theta)^2}\Bigg[\mathbf{r} - [\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]\big[[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-1}[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]\big]^{-1}\Bigg[[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-1}\mathbf{r} - 2\theta\begin{bmatrix}1\\ \sigma_b^2\end{bmatrix}\Bigg]\Bigg]'\boldsymbol{\Omega}^{-1}\Bigg[\mathbf{r} - [\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]\big[[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-1}[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]\big]^{-1}\Bigg[[\mathbf{1} \quad \boldsymbol{\Omega}\mathbf{b}]'\boldsymbol{\Omega}^{-1}\mathbf{r} - 2\theta\begin{bmatrix}1\\ \sigma_b^2\end{bmatrix}\Bigg]\Bigg] \tag{59}$$

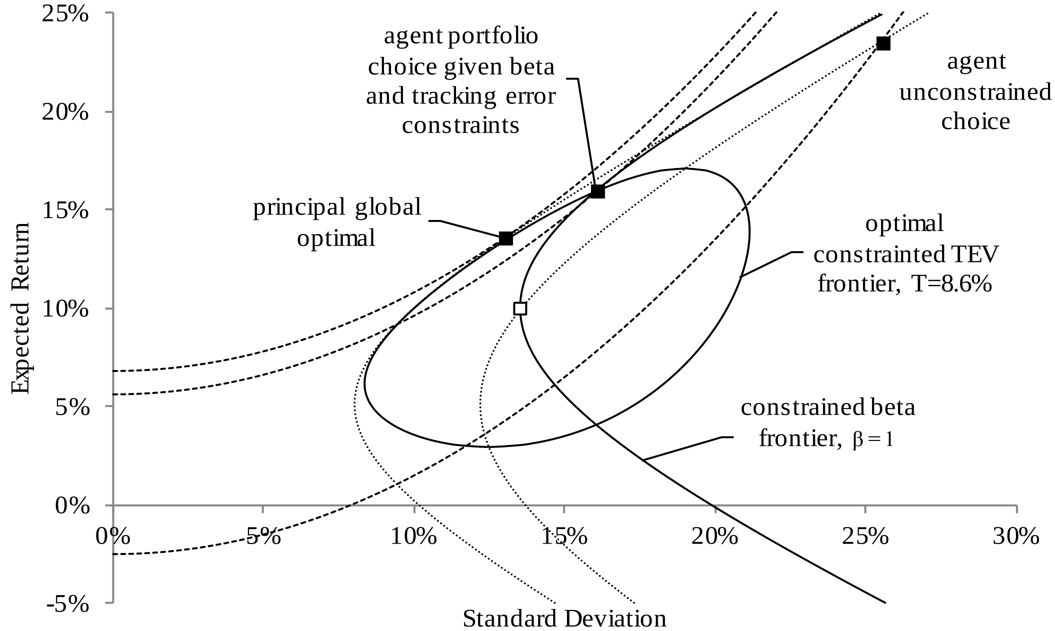
This is the optimal constraint to *T* given a beta of 1 and the agent's optimization process in relative space.

Just for reference, the numerical value of this point in my figures is about 9%. Realistically, this may also be a particularly loose tracking error constraint. Even if a constraint is lower than where the optimal point is chosen, this is still guaranteed to be a utility increase for the principal since any deviation from the benchmark below the optimal point and above the benchmark must be a utility increase. Jorion (2003) agrees with the idea of a general tracking error constraint. His analysis brings up the point of using a tracking error bound to not extend variance too far and I am emphasizing that the bound should be set to maximize utility. But, those two goals consistent with each other once the optimal tracking error bound is reached.

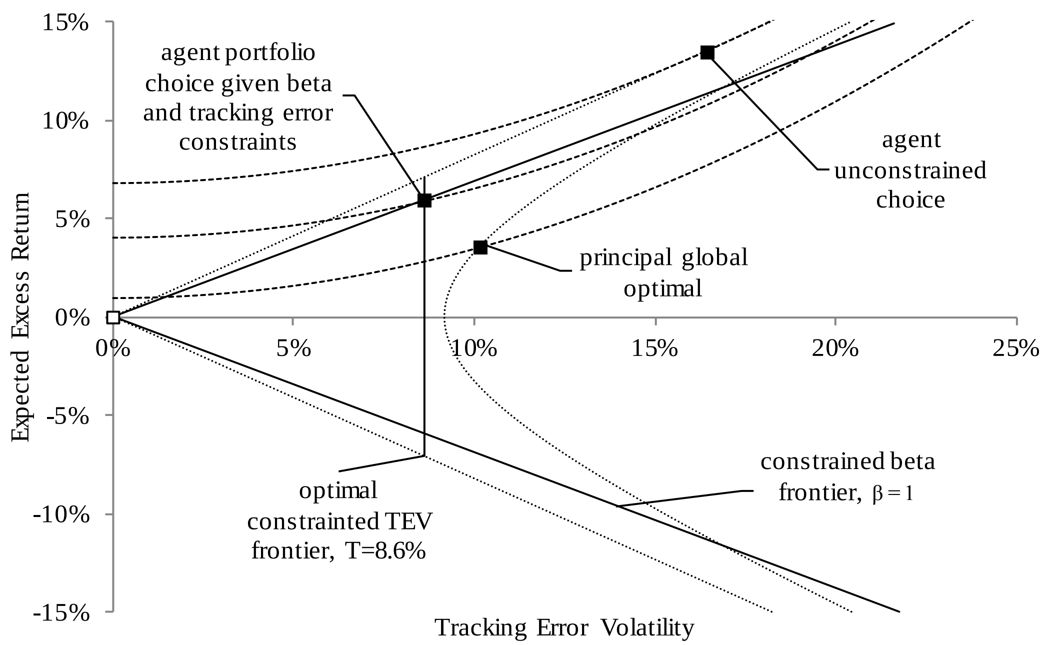
The constraints described above are depicted in Figure 6, Panels A and B. It is evident from the figure that the constrained beta frontier when $\beta = 1$ passes thought the benchmark at its minimum point. The frontier is superior to the TEV frontier over a good portion of its region, and the curve is tangent to the efficient frontier at a point well up the curve from the global efficient portfolio. Increasing constant TEV ellipses eventually at the point of the optimally constrained portfolio are depicted. The principal's iso-utility curves are included in Panel A at the level of the benchmark, the optimally constrained portfolio, and the global optimal portfolio. Utility at the optimally constrained portfolio has increased from the benchmark and the utility curve is tangent to the constrained beta frontier at this point. Additionally, the optimal TEV ellipse passes through the chosen portfolio at this point. The utility of this portfolio is obviously less than the utility of the

Figure 6
Maximizing Principal Utility by Choosing Tracking Error given a Beta Constraint of 1

Panel A: Mean-Variance Space



Panel B: Mean-TEV Space



In this figure, the principal uses a fixed beta constraint of 1 and chooses a tracking error to maximize his utility given the agent's portfolio choice. The constrained TEV ellipse passes through the point at which the principal's utility is tangent to the constrained beta curve. Also depicted are the agent's unconstrained choice and the principal's global optimal portfolios.

global optimal but recall that it is unlikely that the principal knows the location of this portfolio anyway. The agent's utility curves are drawn in Panel B along with the optimization curves and it is apparent that utility in this space decreases for the agent.

VII Conclusion

This paper revisits the problem of Roll (1992) where a delegated investment manager optimizes over tracking error volatility rather than standard deviation. In comparison to mean-variance optimization, TEV optimization creates a frontier that is inferior to the efficient frontier. I show how this directly implies that the agent is optimizing over a utility function that is parameterized in tracking error volatility rather than in variance. This framework is a prominent feature of modern investment management and arises through both a direct and an indirect performance incentive. This performance incentive has been studied by many other authors and they arrive at various conclusions, mostly implying that relative optimization over tracking error is inefficient. Therefore delegated management, because of this inefficiency, is likely to be inferior for a principal even if the delegated manager has more skill and information.

However, I show that the principal's preference is really dependent upon the interaction of the TEV frontier and the principal's utility function. Except for a very unlikely case, the case where the benchmark is perfectly optimal over the agent's information, substantial utility improvements exist for the principal if the delegated agent is properly controlled. Given industry anecdotes related to tracking error constraints and beta constraints, I reanalyze the tracking error constraint from Jorion (2003) and the beta constraint from Roll (1992) in the context of a principal's utility function to show that there are levels of these constraints that likely increase utility for the principal.

In isolation, the tracking error constraint is used to limit the level of standard deviation an unconstrained delegated manager would likely take. Without the tracking error constraint, an agent could buy a portfolio so far up the TEV frontier that it actually decreases the principal's utility therefore making the delegation a bad idea. By imposing this constraint, the principal could ensure that the agent buy a portfolio within the region that increases utility. If "good enough" information is available to the principal, I show how the principal could optimize the level of the tracking error constraint (given quadratic utility) thus, by imposing this optimal TEV constraint, could maximize his utility given the delegated performance incentive. This level of utility is necessarily a utility increase except for the special case mentioned above. I also show how the TEV constraint is effectively just a risk aversion constraint. So, rather than trying to constrain or filter delegated managers directly on their level of risk aversion, tracking error allows principals to force managers to invest as if their risk aversion level was known to the principal. The agent's actual level of risk aversion is an unnecessary element of how to set the tracking error constraint. Only the principal's risk aversion level matters.

Additionally, as Roll (1992) shows, the beta constrained TEV frontier is superior to the unconstrained TEV frontier and therefore is also a guaranteed utility increase for the principal as long as the beta constraint is set appropriately. A beta constraint of 1 always has a positive utility deviation for the principal. Just as an optimal tracking error can be calculated, there is an optimal beta constraint that can be used to maximize the principal's utility. Perhaps surprisingly, this beta constraint is constant despite the level of risk aversion of agent and is, once again, only dependent on the level of the principal's risk aversion. It happens to be equal to the beta of the principal's global optimal portfolio, the portfolio optimized in principal utility over the agent's information. Choosing this beta constraint is necessarily a utility increase for the principal and thus makes a beta constrained portfolio, in the context of delegated portfolio management a good idea.

Given that these two constraints both work well in isolation, it is likely that a combination of these constraints lead to an even better portfolio. Additionally, because of the intersection of the beta constrained frontier with the constant TEV ellipse, and the agent's utility function, these two constraints can be used to pin a delegated manager to exactly one point underneath the envelope of the efficient frontier. Since this includes the principal's global optimal portfolio, setting the tracking error constraint and the beta constraint, which is just the same as the optimal beta constraint in isolation, to the levels of the global optimal portfolio forces the delegated manager to buy the optimal portfolio. However, since it is unlikely that a principal has the information to make this calculation, setting the constraints to reasonable levels are also likely to generate a utility increase for the principal. Following Roll and Jorion, who both suggest considering reasonable levels for these constraints rather than a rigorous calculation, I assume that the beta constraint is set to one, the case with an unambiguous utility increase. Under this assumption, I calculate the optimal tracking error constraint but it should be noted that any reasonable tracking error constraint under this scenario is almost certainly a utility increase for the principal.

Overall, delegated portfolio management is less efficient that doing it yourself. However, the reality of asymmetric information and differences in skill necessitate the delegation of a good portion of the assets that are managed in the modern investment industry. Even given the inefficiency of delegated management, it is still possible to create utility increases for the principal if constraints are properly imposed. In fact, under the scenario built in this essay, it is even possible to constrain the agent to act in a manner identical to that which the principal would act given exactly the same information and skill level. This work is supportive of the idea of delegated portfolio management and shows clearly that it can be an efficient and rational exercise if the delegated managers are properly controlled and not left to maximize their own (agent) utility at the expense of the principal.

Table A1
Numerical Values Used to Build the Figures and Tables

r		b		Ω			
0.06		0		0.01114	0.00639	0.00522	0.00127
0.10		1		0.00639	0.01836	0.00948	0.00843
0.13		0		0.00522	0.00948	0.01779	0.00676
0.04		0		0.00127	0.00843	0.00676	0.01245

Every figure and calculation done in this paper were built with these three matrices. I tried to be a pure as I could in the expressions without redefining variables, as much as possible. These values were chosen somewhat arbitrarily for ease of depiction and exposition. The final examples are mildly representative of a realistic scenario but not actually done with real calculations.

REFERENCES

Admati, Anat, Sudipto Bhattacharya, Paul Pfleiderer, and Stephen Ross, 1986, On timing and selectivity, *Journal of Finance* 41, 715-730.

Admati, Anat R., and Paul Pfleiderer, 1997, Does it all add up? Benchmarks and the compensation of active portfolio managers, *Journal of Business* 70, 323–350.

Alexander, Gordon, and Alexandre Baptisa, 2010, Active portfolio management with benchmarking: A frontier based on alpha, *Journal of Banking and Finance* 34, 2185-2197.

Amihud, Yakov, and Ruslan Goyenko, 2013, Mutual fund's R2 as predictor of performance, *Review of Financial Studies* 26, 667-694.

Barran, Laurent, Olivier Scaillet and Russ Wermers, 2010, False discoveries in mutual fund performance: measuring luck in estimated alphas, *Journal of Finance* 65, 179-216.

Bali, Turan, Nusret Cakici, Xuemin Yan and Zhe Zhang, 2005, Does idiosyncratic risk really matter?, *Journal of Finance* 60, 905-929.

Basak, Suleyman, Alex Shapiro and Lucie Tepla, 2006, Risk management with benchmarking, *Management Science* 52, 542-557.

Bhattacharya, Sudipto, and Paul Pfleiderer, 1985, Delegated portfolio management, *Journal of Economic Theory* 36, 1-25.

Becker, Connie, Wayne Ferson, David Myers, and Michael Schill, 1999, Conditional market timing with benchmark investors, *Journal of Financial Economics* 52, 119-146.

Berk, Jonathan, and Richard Green, 2004, Mutual fund flows and performance in rational markets, *Journal of Political Economy* 112, 1269–1295.

Bertrand, Philippe, 2010, Another Look at Portfolio Optimization under Tracking Error Constraints, *Financial Analysts Journal,* 66, 78-90.

Bhattacharya, Sudipto, and Paul Pfleiderer, 1985, Delegated portfolio management, *Journal of Economic Theory* 36, 1-25.

Binsbergen, Jules, Michael Brandt and Ralph Koijen, 2008, Optimal decentralized investment management, *Journal of Finance* 63, 1849-1895.

Blake, David, Alberto G. Rossi, Allan Timmermann, Ian Tonks, and Russ Wermers. 2013, Decentralized investment management: Evidence from the pension fund industry. *The Journal of Finance* 68, 1133-1178.

Brand, Simone, Stephen Brown, and David Gallagher, 2005, Portfolio concentration and investment manager performance, *International Review of Finance* 5, 149-174.

Carhart, Mark, 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Cetin, Coskun, 2006, Delegated dynamic portfolio management under mean-variance preferences, *Journal of Applied Mathematics and Decision Sciences*, 1-22.

Cremers, Martijn and Antti Petajisto, 2009, How active is your fund manager? A new measure that predicts fund performance, *Review of Financial Studies* 22, 3329-3365.

Cuoco, Domenico and Ron Kaniel, 2011, Equilibrium prices in the presence of delegated portfolio management, *Journal of Financial Economics* 101, 264-296.

Daniel, K., M. Grinblatt, S. Titman, and R. Wermers, 1997. Measuring mutual fund performance with characteristic-based benchmarks. *Journal of Finance*, 52, 1035-1058.

Dybvig, Philip, Heber Farnsworth, and Jennifer Carpenter, 2010, Portfolio performance and agency, *Review of Financial Studies* 23, 1-23.

Elton, Edwin, and Martin Gruber, 2004, Optimum centralized portfolio construction with decentralized portfolio management, *Journal of Financial and Quantitative Analysis* 39, 481-494.

Fama, Eugene and Kenneth French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Ferris, Stephen and Xuemin Yan, 2009, Agency costs, governance, and organizational forms: evidence from the mutual fund industry, *Journal of Banking and Finance* 33, 619-626.

French, Dan, John Groth and James Kolari, 1983, Current investor expectations and better betas, *Journal of Portfolio Management* 10, 12-18.

French, Ken, 2008, Presidential Address: The Cost of Active Investing, *Journal of Finance* 63, 1537-1573.

Goldman, Eitan, and Steve Slezak, 2003, Delegated portfolio management and rational prolonged mispricing, *Journal of Finance* 58, 283-311.

Gompers, Paul and Andrew Metrick, 2001, Institutional investors and equity prices, *Quarterly Journal of Economics* 116, 229-259.

Grinblatt, Mark, and Sheridan Titman, 1989, Mutual fund performance: An analysis of quarterly portfolio holdings, *Journal of Business* 62, 393–416.

Grinold, Richard and Ronald Kahn, 2000, *Active Portfolio Management: A Quantitative Approach for Producing Superior Returns and Reducing Risk*, second edition, New York: McGraw-Hill.

He, Zhigou, and Wei Xiong, 2013, Delegated asset management, investor mandates, and capital immobility, *Journal of Financial Economics* 107, 239-258.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *The Journal of Finance* 48, 65-91.

Jensen, Michael, 1968, The performance of mutual funds in the period 1945-1964, *Journal of Finance* 23, 389-416.

Jiang, Hao, Marno Verbeek, and Yu Wang, 2011, Information content when mutual funds deviate from benchmarks, working paper.

Jorion, Philippe, 2003, Portfolio optimization with tracking-error constraints, *Financial Analysts Journal* 59, 70-82.

Kacperczyk, Marcin, and Amit Seru, 2007, Fund manager use of public information: new evidence on managerial skill, *Journal of Finance* 62, 485-528.

Kacperczyk, Marcin, Clemens Sialm, and Lu Zheng, 2005, On the industry concentration of actively managed equity mutual funds, *Journal of Finance* 60, 1983-2011.

Kacperczyk, Marcin, Clemens Sialm, and Lu Zheng, 2008, Unobserved actions of mutual funds, *Review of Financial Studies* 21, 2379-2419.

Lakonishok, Josef, Andrei Shleifer, and Robert Vishny, 1994, Contrarian investment, extrapolation, and risk, *Journal of Finance* 49, 1541-1578.

Li, C. Wei and Ashish Tiwari, 2009, Incentive contracts in delegated portfolio management, *Review of Financial Studies* 22, 4681-4714.

Lintner, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13-37.

Markowitz, Harry, 1952, Portfolio Selection, *Journal of Finance* 7, 77-91.

Markowitz, Harry, 1959, *Porfolio Selection: Eficient Diversification of Investments*. John Wiley.

Merton, Robert C., 1972, An Analytic Derivation of the Efficient Portfolio Frontier, *Journal of Financial and Quantitative Analysis* 7, 1851-1872.

Modigliani, Franco and Leah Modigliani, 1997, Risk-Adjusted Performance, *Journal of Portfolio Management* 23, 45–54.

Muralidhar, Arun S., 2000, Risk-adjusted performance: The correlation correction, *Financial Analysts Journal*, 63-71.

Ou-Yang, Hui, 2003, Optimal contracts in a continuous-time delegated portfolio management problem, *Review of Financial Studies* 16, 173-208.

Pastor, Lubos, and Robert Stambaugh, 2002a, Mutual fund performance and seemingly unrelated assets, *Journal of Financial Economics* 63, 315–349.

Pastor, Lubos, and Robert Stambaugh, 2002b, Investing in equity mutual funds, *Journal of Financial Economics* 63, 351–380.

Petajisto, Antii, 2013, Active share and mutual fund performance, *Financial Analysts Journal*, forthcoming.

Puckett, Andy, and Xuemin Sterling Yan, 2011, The interim trading skills of institutional investors, *The Journal of Finance* 66, 601-633.

Roll, Richard, 1977, A critique of the asset pricing theory's tests part I: on past and potential testability of the theory, *Journal of Financial Economics* 4, 129-176.

Roll, Richard, 1992, A mean/variance analysis of tracking error, *Journal of Portfolio Management* 18, 13-22.

Samuelson, Paul A., 1958, An exact consumption-loan model of interest with or without the social contrivance of money, *Journal of Political Economy* 66, 467-482.

Sharpe, William, 1964, A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.

Sharpe, William, 1966, Mutual Fund Performance, *Journal of Business* 39, 119–138.

Sharpe,William, 1981, Decentralized investment management, *Journal of Finance* 36, 217–234.

Sharpe, William, 1991, The arithmetic of active management, *Financial Analysts Journal* 47, 7-9.

Sharpe, William, 1994, The Sharpe Ratio, *Journal of Portfolio Management* 21, 49–58.

Stutzer, Michael, 2004, Asset allocation without unobservable parameters, *Financial Analysts Journal* 60, 38-51.

Treynor, Jack, and Fischer Black, 1973, How to use security analysis to improve portfolio selection, *Journal of Business* 46, 66–86.

Wermers, Russ, 2000, Mutual fund performance: An empirical decomposition into stock-picking talent, style, transaction costs, and expenses, *Journal of Finance* 55, 1655–1695.

Yan, Xuemin and Zhe Zhang, 2009, Institutional investors and equity returns: are short-term institutions better informed?, *Review of Financial Studies* 22, 893-924.

VITA

David L. Stowe was born in Miami Florida to John and Linda Stowe. He grew up in Columbia, MO as the son of two university faculty members and graduated from Rock Bridge High School in 1994. He holds a BSBA in Finance and Banking from the University of Missouri (1999), a MS in Finance from the University of Colorado at Denver (2003), and a MS in Applied Mathematics from Ohio University (2009). Prior to joining the PhD program at the University of Missouri, David spent 7 years in the public pension and mutual fund industries with experience in quantitative analysis, investment modeling, and portfolio risk analysis. His primary research interests are in the field of investments: specifically quantitative methods, portfolio management, risk management, and mathematical modeling. He has peer reviewed publications in the Financial Analysts Journal and in the Journal of Financial and Economic Practice. He is also a CFA Charterholder (granted in 2003). David will begin his academic career as an Assistant Professor of Finance at Ohio University in the fall of 2014.