Expectation of p-norm of random matrices with heavy tails

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ABSTRACT

The p-norm ($p > 2$) of a random matrix whose entries are gaussian, subgaussian and log concave have been studied previously. We conjecture the following generalization of the above results for heavy tailed random matrices:

Conjecture 0.1. *Let $p > 2$. Fix $n > 0$. $A = (X_{ij}), i = 1, 2, ..., n, j = 1, 2, ..., N,$ be a random matrix whose entries are independent random variables with bounded $2p^{th}$ moments, where $N \geqslant n^{\frac{p}{2}}$. Then, with high probability, $\forall x \in \mathbb{S}^{n-1}$:*
$cN^{1/p} \leqslant \|Ax\|_p \leqslant CN^{1/p}$, for some constants, $c, C > 0$.

We establish the following upper bound, generalising the work of [Latala05]:

Theorem 0.2. *Let $p > 2$. Fix $n > 0$. $A = (X_{ij}), i = 1, 2, ..., n, j = 1, 2, ..., N,$ be a random matrix whose entries are independent, identically distributed random variables with bounded $2p^{th}$ moments, where $N \geqslant n^{p-1}$. Then, with high probability, (close to 1):$\forall x \in \mathbb{S}^{n-1}$:*

$$\|Ax\|_p \leqslant CN^{1/p} \log N \left[\log(\log N) \log^2(\log(\log N)) \right]^{1-1/p}$$

for some constant C, depending on p and the pth and $2p$th moments of the random variable. We also get a similar upper bound when the entries of the random matrix are independent random variables with bounded $2p$th moment and are not necessarily identically distributed.