

COMPLEX AND ALMOST-COMPLEX STRUCTURES ON SIX DIMENSIONAL MANIFOLDS

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ABSTRACT

We investigate the properties of hypothetical exotic complex structures on three dimensional complex projective space $\mathbb{C}P^3$. This is motivated by the long standing question in differential geometry of whether or not the six sphere S^6 admits an integrable almost-complex structure. An affirmative answer to this question would imply the existence of many exotic complex structures on $\mathbb{C}P^3$. It is known that $\mathbb{C}P^3$ admits many topologically different almost-complex structures, but it is unknown whether or not $\mathbb{C}P^3$ admits an integrable almost-complex structure other than the standard Kähler structure. In this manuscript we give lower bounds on the Hodge numbers of hypothetical exotic structures on $\mathbb{C}P^3$ and a necessary condition for the Frölicher spectral sequence to degenerate at the second level. We also give topological constraints on the classes of hypothetical exotic complex structures which are \mathbb{C}^* -symmetric. We give restrictions on the fixed point sets of such \mathbb{C}^* actions.