ASYMMETRIC RESPONSES IN SYMMETRICAL STRUCUTRES

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Master of Science

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by

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The undersigned, appointed by the dean of the Graduate School, have examined the thesis entitled

ASYMMETRIC RESPONSES IN SYMMETRICAL STRUCUTRES

presented by Adithya Ramamurthy,

a candidate for the degree of Master of Science and hereby certify that, in their opinion, its worthy of acceptance

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ABSTRACT

Symmetries can be attributed to physical states or physical laws. A symmetry can be exact, approximate or broken. Exact symmetry means that the symmetry is unconditionally valid, approximate means its valid in certain conditions, broken symmetry can be interpreted in various ways. This thesis emphasizes on the interesting as well as distinct results of miniscule asymmetric disturbance and its effects on the symmetry and structures. Even miniscule difference in the magnitude of symmetric forces acting on symmetrical structures can result in an asymmetric response and remarkable difference in the buckling point of a structure. This might be a major contributing factor for failure in structures, may it be structures used in large complex assemblies or in Micro Electro Mechanical systems.

A truss element may be quintessential to prove symmetry breaking and its remarkable effects due to miniscule anomaly in symmetry. If we consider truss structures alone, the results are intriguing; in the past there have been numerous instances of failure of truss structures of various sizes and in different applications. In many of such instances causes of failure were attributed to load exceeding critical load. A thorough study on asymmetric responses show that buckling load drops down for an asymmetric response when compared to the symmetric response for the same material properties and geometrical boundary conditions. Hence studying the asymmetric responses of structures can help designing better components resulting in less number of failures

Chapter 1

Introduction

1.1 Deformation Analysis

In this era of rapid technological advancements and evolving production, safety and integrity of structures are of vital importance. Numerous processes and software packages are in use to evaluate the behavior of structural components. Structural components are designed to withstand various kinds of loads and effects. In accordance with their properties, structural components are manufactured to handle large magnitude of force and their application range from buildings to automotive and aircraft sector and hence the margin for failure is very small. Stress analysis is carried out in design of structures of all dimensions. It can range from aircrafts, mechanical components, micro electro mechanical systems, bridges, dams or any other structure. Conventionally the input data for analyzing stresses in a component are the geometrical properties of the component, constraints attached and the forces that are acting on it. The output gives us the magnitude of stresses acting on all parts and joints and the deformation caused as a result of those stresses. In modern era, there arises a need for accurate finite element formulation and analysis of stresses in components in order to maintain safety and proper functioning the component. There are numerous computer aided technologies that provide tremendous accuracy and ease of operation to analyze stress and deformation in components but the task accurate analysis requires proper understanding of the mechanics of materials and good mechanical aptitude.

1.2 Stress strain curve

Structural components undergo deformation when subjected to loading. Deformation is a process of change in shape and size of a structure upon loading. The nature of deformation depends on the type of material, size and geometry of the object, and the forces applied. The various types of deformation are:

- Elastic deformation
- Plastic deformation
- Metal fatigue
- Compressive failure
- Fracture

The occurrence of above phases of deformation are dependent on several factors and based on the phase of deformation, the material will be validated to see if it is strong enough to perform, needs repair or if it should be disposed of. Usually plastic deformation of a material subjects it to an irreversible structural mode. Our study mainly deals with truss structures and most of the truss structures are highly flexible which can undergo large deformations and rotations within the elastic limit. The basic idea of the deformation of a structure can be inferred from a Stress vs Strain curve which is as shown below in figure 1.1

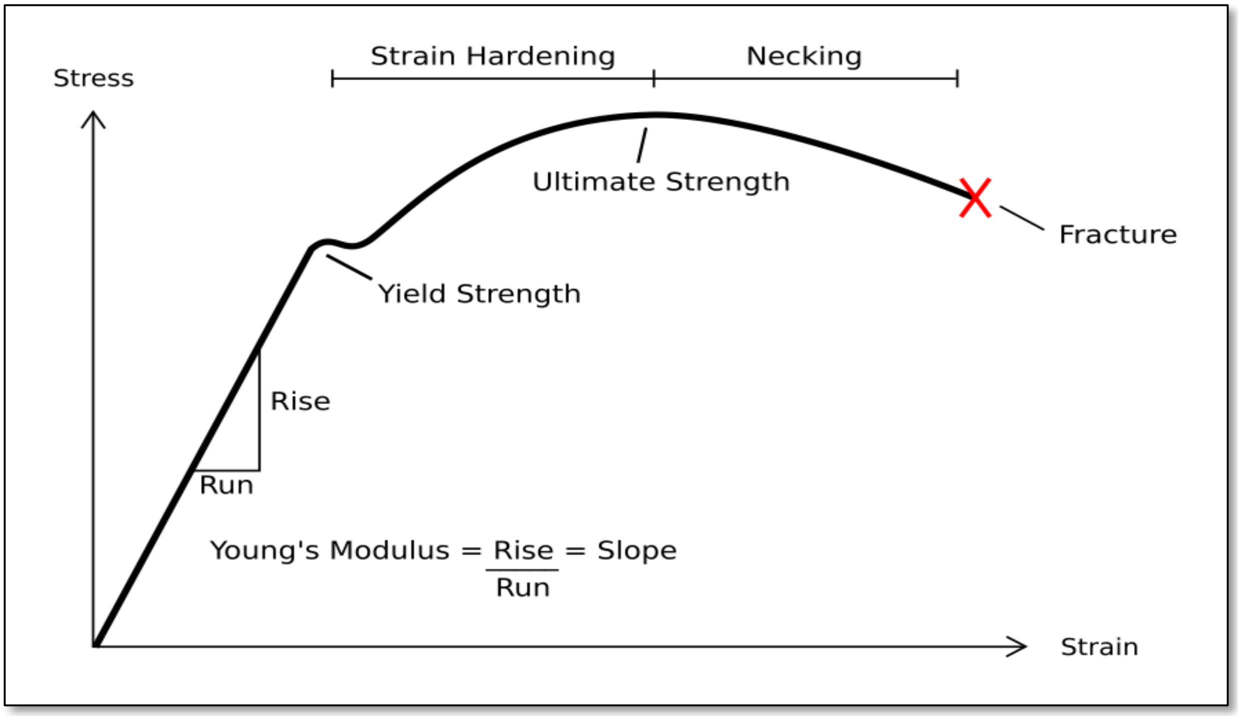

Figure1. 1 Stress Strain Curve

1.2.1 Elastic deformation

Elastic deformation is frequent in Elastomers and rubber wherein if the forces are removed, the structure retains its original shape. Every structure goes through this kind of deformation and then that transforms into plastic deformation or a fracture. The linear elastic deformation is governed by the Universal Hooke's law which states

$$\sigma = E\epsilon \tag{1.1}$$

Where σ is the applied stress, E is the material constant called as Young's modulus and ϵ is the resulting strain. The slope of the stress strain curve is strictly limited to the elastic range. The elastic range ends when the material reaches its yield strength.

1.2.2 Yield strength of a material

If a perpendicular is drawn from the Yield point shown in figure above to the Y-axis, the yield strength can be described as the stress a material can without permanent deformation. It is a point after which the material becomes to plastically deform and can't go back to its original shape. The yield strength of the material somewhere falls between the ultimate strength and the proportional limit.

1.2.3 Plastic deformation

It is an irreversible deformation phase. It can be inferred that plastic deformation will first undergo elastic deformation (which is reversible) and this is the reason that the material can retain part of its original shape. The plastic deformation of the material starts when the yield strength of the material is reached and goes on till a permanent fracture has occurred in the material. In more technical terms, plastic deformation involves breaking of limited number of atomic bonds by the movement of dislocations. In the above figure, the plastic deformation is characterized by a strain hardening area, necking region and finally fracture and this is applicable to tensile stresses. Since the nature of deformation is rigorous, soft thermoplastics have rather large plastic deformation range as do ductile metals such as copper, silver and gold. Steel does have a large plastic deformation range but the effect is rather small in cast iron.

1.3 Structures

In engineering terms, a structure is a combination of bodies in space which forms a structural system capable of supporting loads. A structural system usually consists of various load carrying elemental structures. The assumptions pertaining to physical characteristics of elemental structures determine the category or group in which the load carrying elemental structure belongs to. If the elemental structures were to be categorized in terms of geometries and load carrying capabilities, they can be divided into six groups, they are:

- Cables
- Bars
- Beams
- Membranes
- Plates and
- Shells

Our study mainly deals with bars and beams. Bars are one-dimensional structures that sustain extensional, compressional and torsional loads. In this way, a few basic assumptions can be based according to the force acting. If the bar is only subjected to longitudinal tensile loads, it is called as a rod. Rods are one of the most commonly used primary elemental structure. Many of the common mega structures use rod as a primary load carrying element.

Beams are structures which have the capability of withstanding load primarily by resisting bending. Bending moment is nothing but the reaction of the structural element when the external force applied causes it to bend. Beams are capable of sustaining extension, compression, bending, flexure and twisting loads. Basically strings, bars and rods are special cases of general beams.

1.4 Boundary conditions and their significance

Boundary conditions are set of parameters that limit the motion like translation and rotation in a structure. While structures are being analyzed, boundary conditions play a key role in the outcome of the analysis. The failure load, behavior and various other outcomes depends a lot on how the structure is being constrained. Boundary conditions are applied at either the surface or a point and must be considered carefully since over or under constrained models can yield erroneous results. In present day structures, there could be massive assemblies of components which are connected in a complex manner and in these cases; realistic boundary conditions should be applied for results.

Basic components of structural mechanics include Load-deformation curve, a constitutive law, equilibrium equations and equations of motions with boundary and initial conditions. The equilibrium equations state the balancing of internal stresses, body forces and inertial forces of a differential material particle. The constraints referred in Finite element Analysis are the boundary conditions. In terms of pure mathematics; a boundary condition is the condition required to be satisfied at all or part of the boundary or the region in which a set of differential equation is to be solved. The finite element method distinguishes between essential and natural boundary conditions. Essential (geometric) boundary conditions are the boundary conditions that affect the independent motions or the degrees of freedom and are imposed on the primary variable like displacement. The natural boundary conditions do not directly affect the independent motions and are imposed on secondary variable like forces and tractions. Each supporting structure is designed so that it can support or transfer a specific type of load. Supports can be chosen based on the following factors

- Purpose of the structure
- Number of sub-structures
- Type of material being used
- Dimensional properties of the structure
- Type of the structure etc.,

1.4.1 Types of essential boundary conditions

Structures can be constrained in a few ways; the most commonly used are hinged, roller and fixed constraint. The hinged support is a type of support which allows moment but restricts vertical or horizontal movement. Hinged supports are most commonly seen in doors. A hinge allows a certain degree of rotation and this enables the door to open and close but restricts any sought of a vertical or horizontal motion. Hinges are included in large structures to reduce or eliminate the transfer of bending forces between structural components. Hence constraining a structure tactically can have advantages like low maintenance cost and higher structural integrity. The fixed supports resist any sought of moment in addition to movement along any axis. This means that the fixed support resists tension, compression and moment. The roller supports are widely used in many applications and has many unique benefits. The roller supports are free to rotate and translate along the surface upon which the roller rests. The resulting force is always a single force that is perpendicular and away from the surface. The roller cannot provide resistance to lateral forces other than a perfectly vertical load, as soon as the load exceeds the structure will roll away in response.

1.5 Symmetry breaking

Symmetry for structures strictly restricts it to be invariant to transformation with respect to its original shape and dimension. Symmetries basically can be observed through geometrical transformations. Symmetry breaking can mean different things under different circumstances. Symmetry breaking doesn't necessarily mean absence of symmetry; it rather means that the final state of symmetry is characterized by a lower symmetry compared to the original state. In more theoretical terms this means, the symmetry is transformed to one of its subgroups. The deformed configurations which signify the transformation in symmetry signifies exactly how the transformation in symmetry takes place. If we consider a simple example of a simple truss member as shown below in figure 1.2

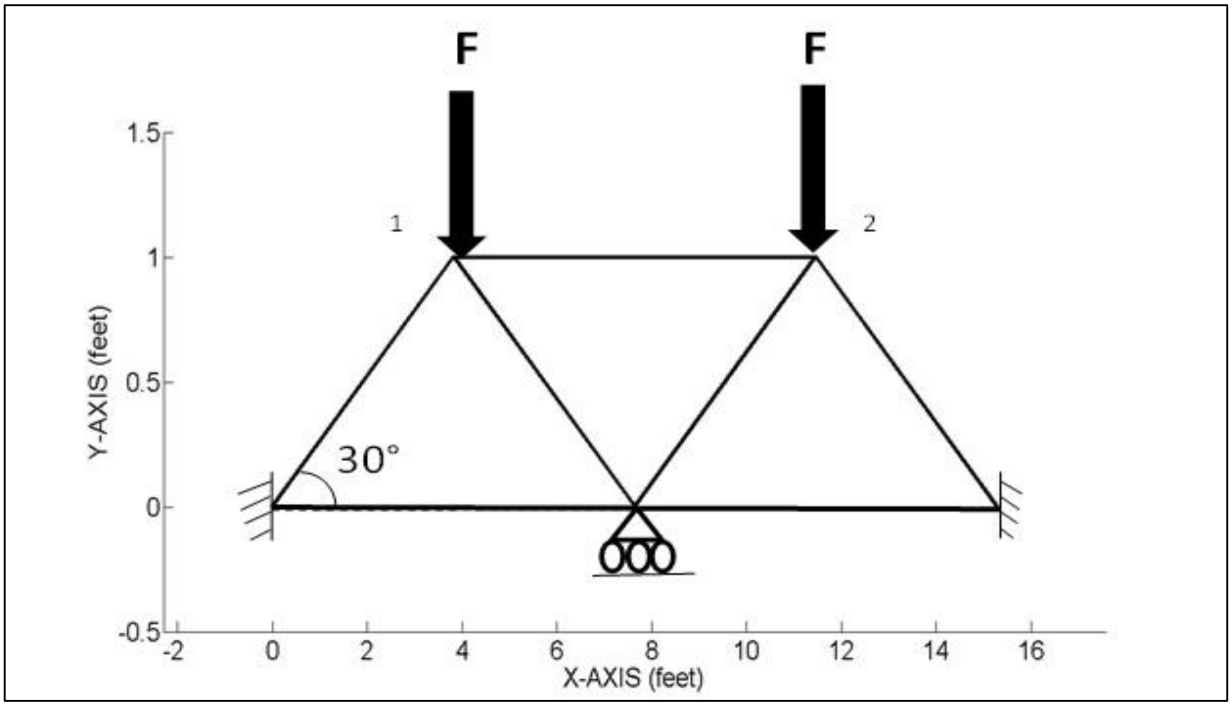

Figure1. 2Simple seven bar truss with symmetric loading

The structure is hinged at both ends and is provided with a roller in between. This is one of the most commonly used boundary conditions in truss experimentation. Such setups can be most frequently seen in bridges and in aircraft wing spars where the truss structures lay horizontal and provide support. The angle of inclination of the truss structure to the horizontal is 30 degree. There are two forces acting on the vertex i.e. node 1 and node 2.

The material properties of the structure are shown below in table 1.1

Table1. 1 Geometrical & Material properties

Poisson's ratio	0.33
Mass density	15.28 slug/ft^3 or 8050 kg/m^3
Area of each structure	2.3 e-5 square feet
Young's modulus	200 GPa (4.17e9 lbs./ft^2)

The load deformation curve which signifies the deformation of the structure according to load applied is as shown below.

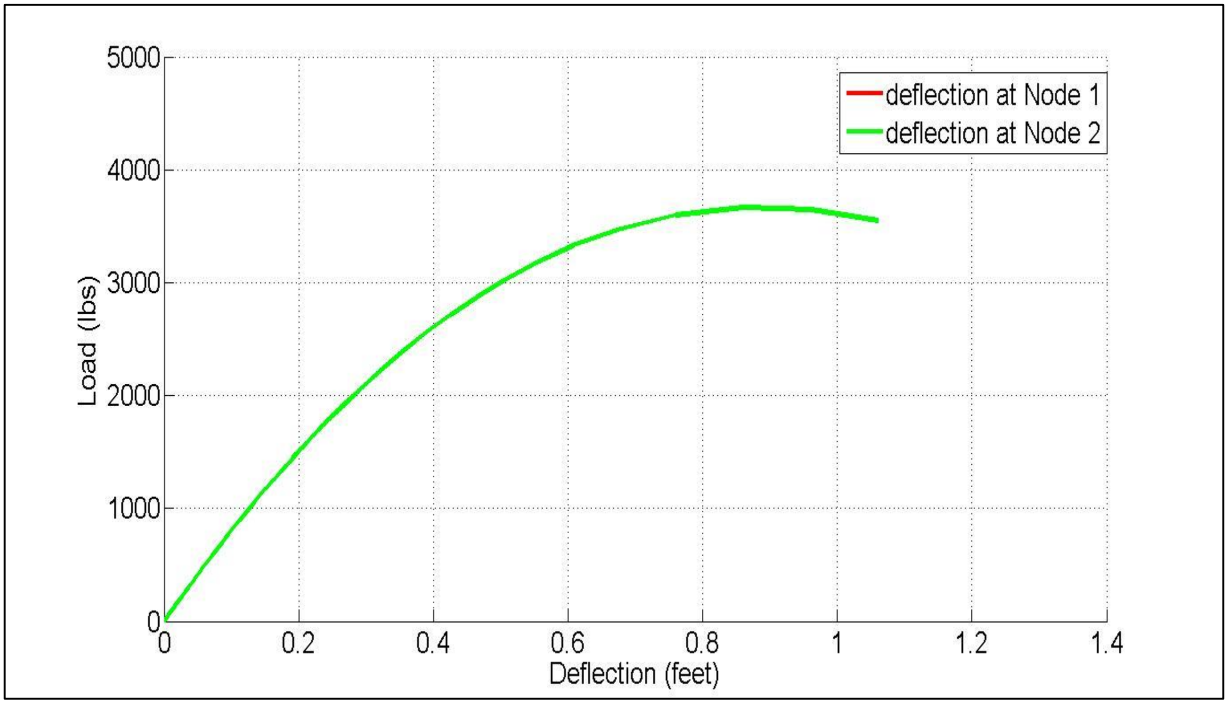

Figure1. 3 Load deformation curve

The Load deformation curve is generated by plotting the deformation values of respective nodes along the x-axis and the load values along the y-axis. The Load deformation curve for the vertices points overlap exactly since the forces acting are same in magnitude. In other words the identical forces acting helps maintain the symmetry and roller in between doesn't move. Now if the force on the 2nd node is reduced by small amount of 1% compared to the 1st node as shown in the figure below and keeping all the other geometrical properties same as the previous case.

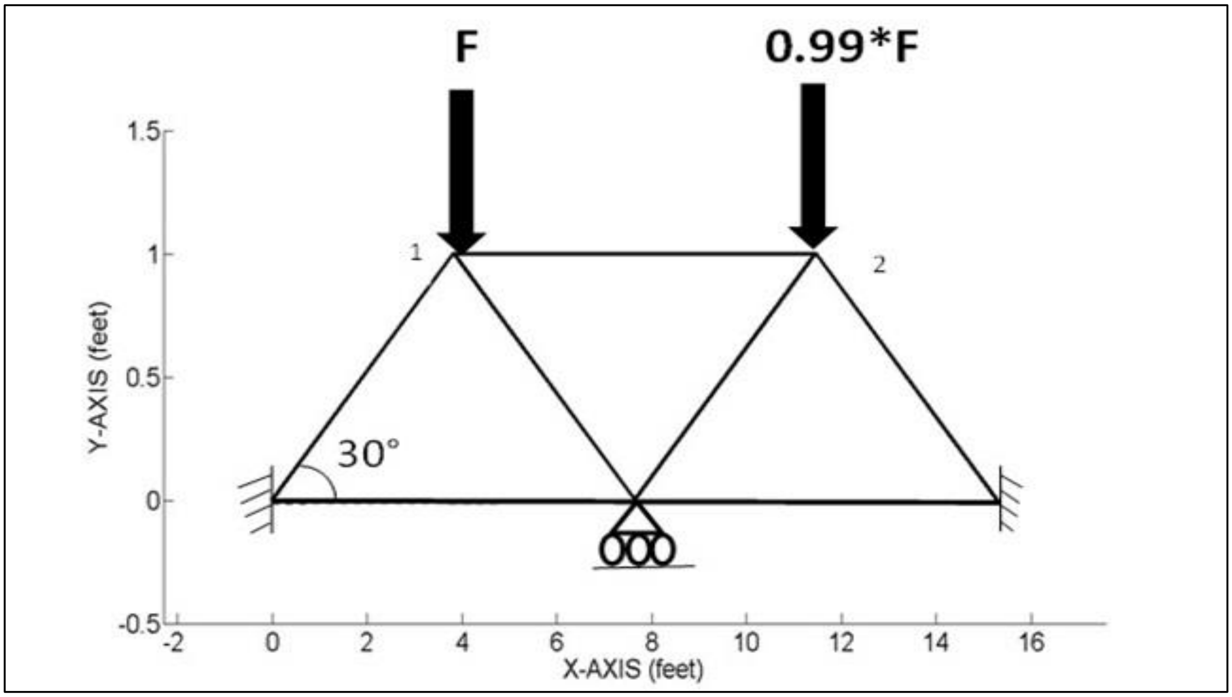

Figure1. 4 Simple seven bar truss with asymmetric loading

The load deformation curve will be as shown below.

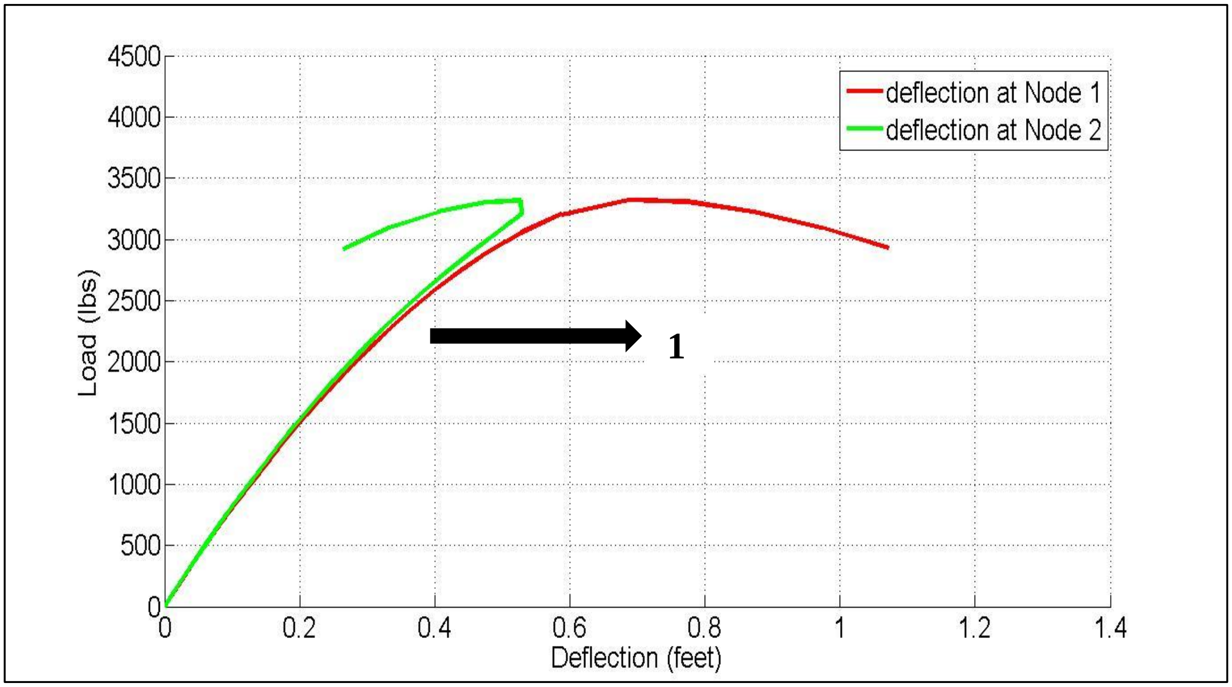

Figure1. 5 Load deformation curve

It can be observed that when the forces are varied by a small amount, the Load vs. Deflection curve differs significantly from the previous case. The point marked as "**1**" shows the branching in the Load vs deflection curve. This is nothing but the breaking of symmetry wherein the load deflection curve doesn't overlap anymore. The symmetry of the structure is not uniform anymore and this can cause a significant dip in the buckling load and affect many other factors.

1.6 Problem statement

Based on the above experimental analysis, it can be inferred that Symmetry breaking is a rather underestimated phenomena which can cause remarkable impacts in the analysis of a structure. Symmetry plays an important role with respect to the laws of nature and they summarize the regularities of the laws that are independent of specific dynamics. The intriguing fact about symmetry breaking is that infinitesimally small fluctuations acting on a system can create significant impacts in the analysis of a structure. These small fluctuations can be varying load, indifferent area of cross section, external forces acting etc. Since no manufacturing process can be complete accuracy and there will defects of some kind, there's always a factor of safety assumed during manufacturing of a component. The factor of safety is the ratio of the maximum strength to the intended load of a structure and normally a structure should be able to withstand a load up to two times its intended load but past instances of failures in structures have shown that there could be several unprecedented factors that could contribute to a structure's failure. The factor of safety should be varied according to the structure's applications like:

- If the components are subjected to repeated shock loading, there should be high factor of safety
- When the stresses acting are unprecedented, the factor of safety will be high and this is also difficult to predict
- When the structure is prone to corrosion, the factor of safety should be high

Therefore a constant monitoring of the impact of symmetry breaking is the need of the hour in the near future. This need is also necessary considering the constant increase in usage of structures in aircrafts, automotive, buildings etc.

Chapter 2

Literature review

2.1 Concept of Symmetry breaking in Aircrafts

There is no exact term called symmetry breaking in Aviation industry but the concept of unequal forces acting on the aircraft structures influencing premature failure exists. I was privileged to meet and work with the Structures team of Southwest Airlines Co. during the course of my internship and went through some very intriguing processes that are followed during the Aircraft maintenance. There are various parameters to be considered during maintenance and following the procedures correctly is of vital importance and requires a specific skill set and a lot of experience.

2.1.1 Basic forces acting on an aircraft

The aircraft experiences a variety of forces during its flight path and while also on the ground. Structural integrity is the major factor in aircraft design and construction. Every aircraft is subjected to structural stress and regardless of the design; every aircraft components must carry, resist or transmit the load applied on it. Stresses are inevitable for any aircraft or any structure for that matter and responding to it by operating under safe limits is the best way to handle stresses in aircrafts. The aircraft body should be durable and strong enough to overcome all the forces and maintain its shape which is crucial. The basic forces on an aircraft are tension, compression and bending.

Tension is the resistance to pull; it is the stress of stretching an object or pulling at its ends. An airplane may experience tremendous amount of tension on the fuselage section when in midair because of the drastic change in temperature and pressure. Tension can be caused when the engine propels the aircraft forward and the wings, tail and fuselage resists the forward movement. The aircraft structure is made up of metal (aluminum) and any metal undergoes expansion or contraction. Even though all of these factors will be considered during the design of the aircraft, the cracks evolving on the aircraft skin are inevitable.

Compression is the inward force, if the forces acting on the aircraft move towards each other, they are called compressive forces and they are opposite of tension forces. The compression mainly occurs on when the aircraft lands and while on the ground.

Bending force is a combination of tension and compression. The wing spars undergo a lot of bending load. The spar is the main structural component of the wing running through span wise at right angles to the fuselage. The spar carries flight loads and weight of the wings while stationery on the ground.

2.1.2 Causes of uneven force distribution

The aircraft as a structure (skin) is made up of aluminum metal and like any other metal; they contract and expand due to drastic variation in temperature and cabin pressure. It is found that the aircraft structure expands or balloons up to 20-35 centimeters during its flight path. The entire aircraft structure is made up of a large number of beams and skin is made up of basic aluminum metal and the sub-assemblies are held together by thousands of strong rivets. Rivets are permanent mechanical fasteners and because there is a head on each end of an installed rivet, it can support tension loads. However rivets are much more capable of supporting shear loads. Due to the unpredictable expansion the original symmetry is broken and the aircraft experiences a much stronger and penetrating force especially during landing. This is because the aircraft has already undergone a lot of stress due to take-off, change in temperature, pressure and the various maneuvers during the flight path. So the portion just above the rear landing struts undergoes tremendous stress and is one of the most vulnerable as well as most repaired components during flight maintenance.

This can impact the original force reaction scenario and if not addressed properly, it can create adverse effects on the integrity of the flight structure. The beam structures present just above the rear landing gear struts are under maintenance quite often and the below figure shows one of the maintenance sessions in progress.

Figure2. 1 Aircraft maintenance

2.1.3 Process to avoid uneven force distribution

Every aircraft in operation has some kind of defect especially on the skin but the aircraft manufacturing companies set a certain permissible or flyable limit for structural damage depending on its extent. The beams and the aircraft skin which constitute a major portion of the aircraft structure are held together by strong rivets. Riveting is one the most proven fastening process in airline industries from the past 50 years. Rivets are aerodynamically compatible and inexpensive in price.

For beams above the landing struts, the rivets which hold them together along the longitudinal direction are soft rivets instead of the conventional rivets. The soft rivets are identical in appearance to the conventional ones but they incorporate a body and mandrel of special soft aluminum alloy. Though it has a lower tensile limit, it helps in the uniform expansion of the beam structure along the longitudinal direction and this conversely helps in the expansion of beam structures and helps prevent asymmetric expansion as well as asymmetric forces on it. This counter strategy implemented by Southwest Airlines has seen comparatively larger life span of beams above rear landing struts.

2.2 Study of truss structures

A truss is typically a structure comprising of one or more triangular units formed with straight structures whose ends are connected at joints we refer as nodes. A truss is a slender member, i.e. the length is much larger than the cross section and hence for many of the experimental analysis the weight of the truss member is ignored. One of the very important property of trusses are that it cannot support bending load since each node in a truss element is hinged or joined and hence they can be treated as a one degree of freedom kinematic pair. Also, the cross sectional dimensions and the elastic properties of the truss element remains constant. The solutions for trusses are exact if nonlinear strain-displacement relations are used. Moreover trusses are more likely to have load buckling and bifurcated solutions than continuous structures. Since the truss consists of discrete bar elements and external loads are only applied at joints, there is no spatial discretization required in the modeling of a truss. The dimensional uniqueness of truss structures, the way they can be symmetrically modelled makes them good candidates for showing buckling, bifurcation and other non-linear phenomena without much concerns about the accuracy of the results. Moreover truss structures are one of the most widely used structural components and it has applications in buildings, aircraft, automotive and various other sectors. So it becomes vital to understand certain nuances affecting the integrity of truss structures. Hence trusses are good candidates for showing buckling, bifurcation and other nonlinear phenomena of highly flexible structures without concern about accuracy of modeling non uniform stresses in structures. For a simple analysis of how trusses react to forces, we take an example of truss structure with constraints and parameters as shown below.

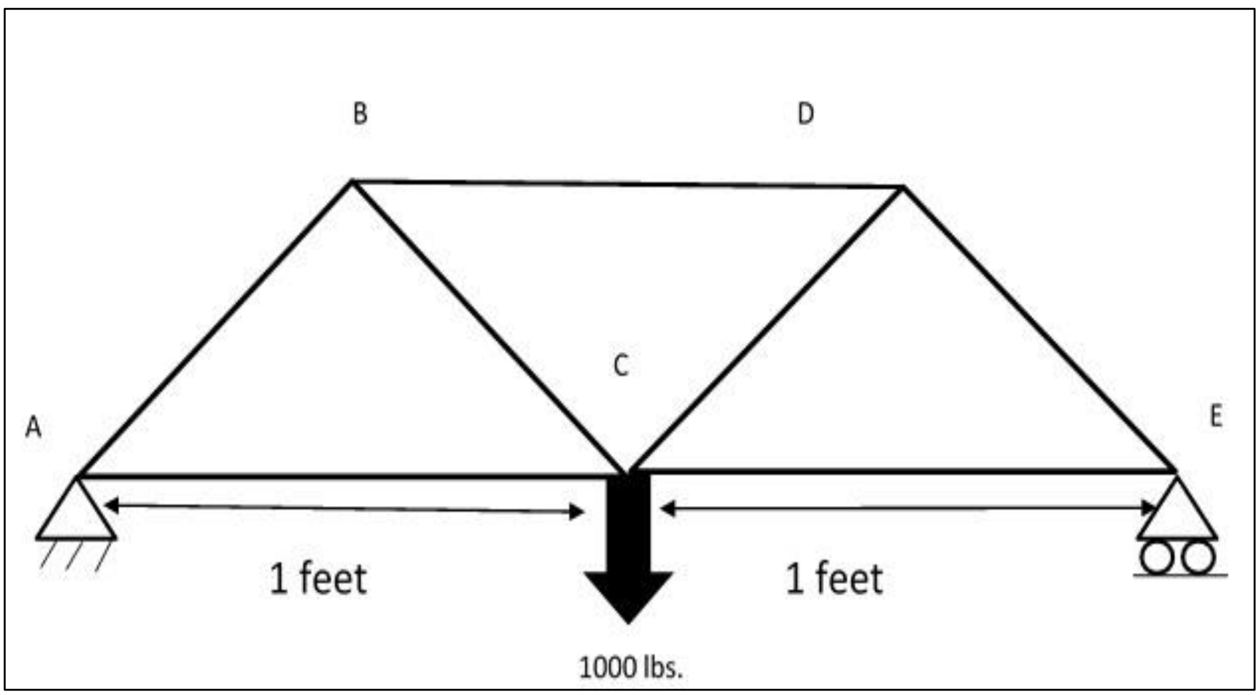

Figure2. 2 Simple seven bar truss with load at middle

The geometric and material properties of the structure are as shown below in table 2.1

Table2. 1 Geometrical & Material properties

Poisson's ratio	0.33
Mass density	15.28 slug/ft^3 or 8050 kg/m^3
Area of each structure	2.50 e-6 square feet
Young's modulus	200 GPa (4.17e9 lbs./ft^2)
Length of each structure	1 feet
Angle to the horizontal	60 degree
Number of bars	7

The structure has two constraints. On the left hand side it has a hinged joint which means it is free to rotate and on the right hand side there is a roller which enables the truss to have movement along the X-axis. A force of 1 kilo pounds is acting in the middle (Node C). With such set of constraints and parameters, the truss may represent a small foot bridge with a heavy person standing in the middle.

2.2.1 Reaction forces in truss structures

Now the reaction forces can be calculated by using the method of joints, we begin by drawing the free body diagram of the whole truss to find the reaction forces and then we segregate it joint by joint to calculate the tension in each of the members. First there are the primary reaction forces which can be represented as follows.

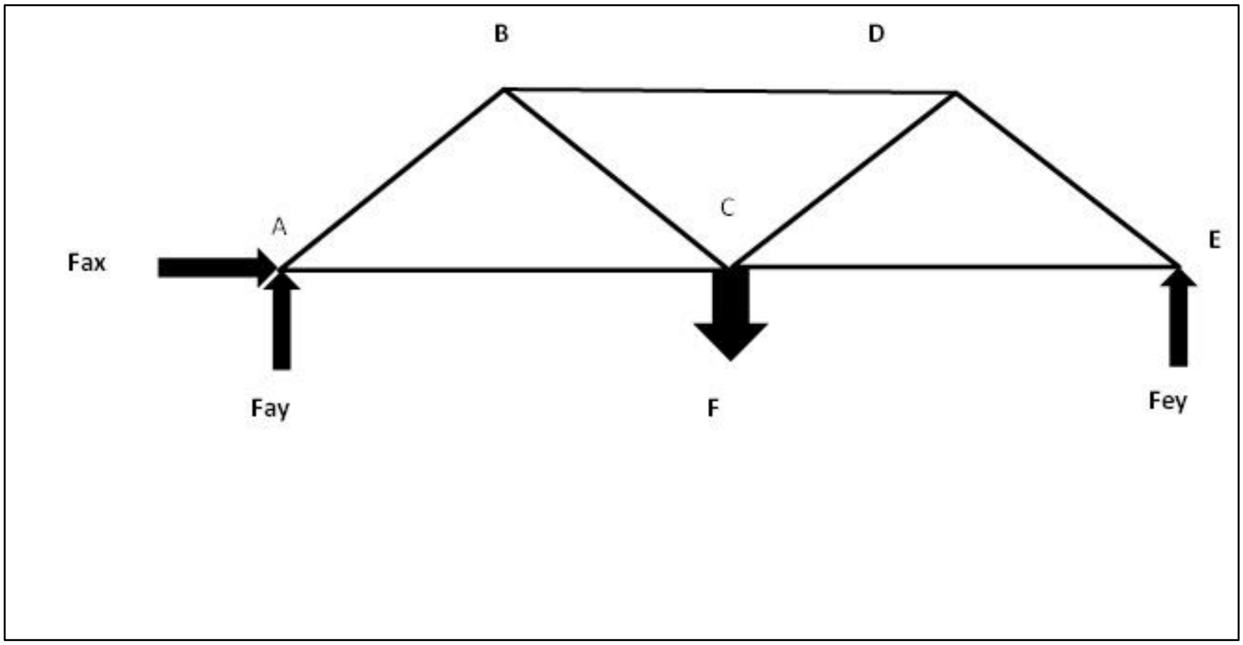

Figure2. 3 Reaction forces

If we consider the reaction forces along the X-direction there is just one force along the horizontal, Σ Fx$= 0$, hence Fax$=0$.

If we consider node A about which we compute the moment then we have two forces to consider and they are at node C and node E and hence we have moment about node A and it can be calculated as:

$$\Sigma \text{ MA}= -1000 \text{ lbs.}*1 \text{ feet} + \text{Fey}*2 \text{ feet} = 0; \tag{2.1}$$

Solving for Fey in the above equation we get Fey=500 pound force (lbs)

Similarly we have summation of forces along vertical direction is zero, Σ Fy = 0, hence:

$$\text{Fay } -1000 \text{ lbs.} + \text{Fey}=0; \tag{2.2}$$

$$\text{Fay}-1000 \text{ lbs.} +500=0;$$

Solving the above equation, we get Fay$= 500$ lbs.

2.2.2 Tension and compression in members

We can go through the truss structure joint by joint to calculate the tension in each of the members. We assume that the joints are held together by pins and hence members are free to rotate, which means that forces can be only applied in the direction of the orientation of the structure. We also assume that the weight of the truss is negligible compared to the force applied and hence weight of the members are neglected in our computations.

The first step in stress calculations is to draw the free body diagram for each node; we can calculate the stress in each member. The free body diagram for node A is shown below.

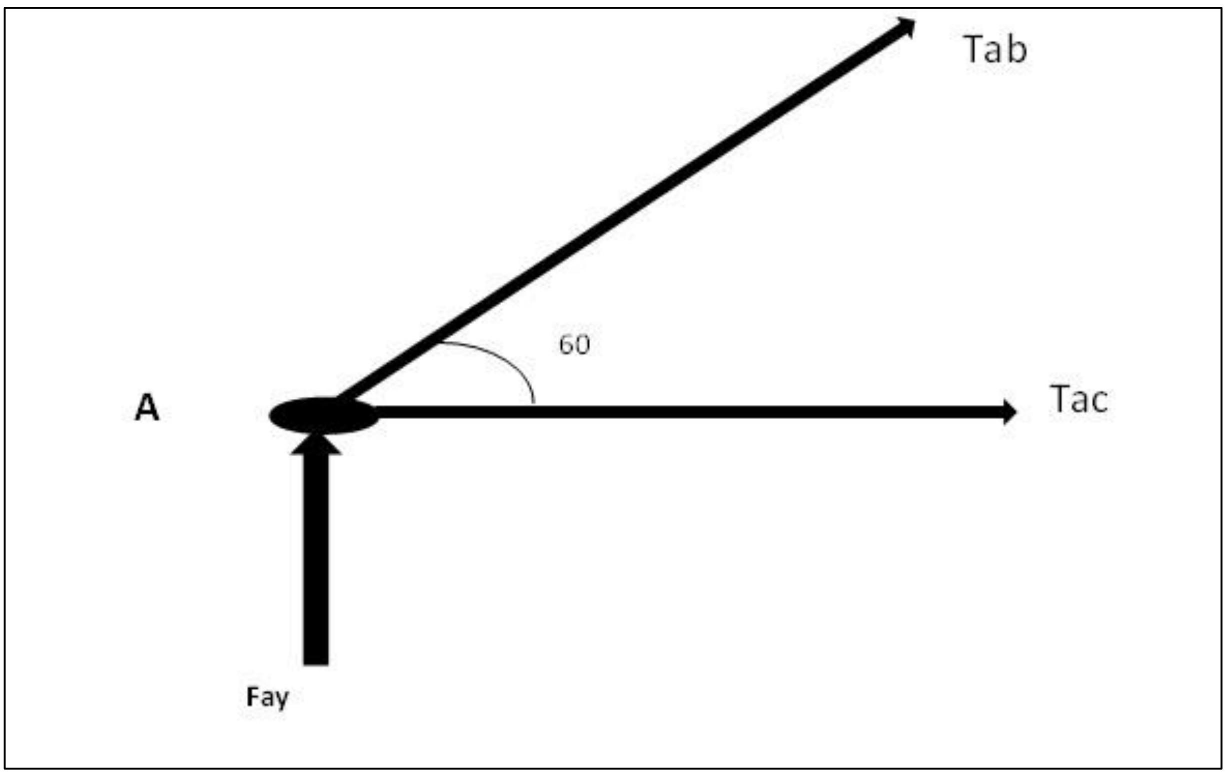

Figure2. 4 Free body diagram for Node A

The tensile forces acting can be further segregated along its axial components and then by applying the principle that sum of the forces along the horizontal and vertical directions is equal to zero, we get: Σ Fx= Tab*Cos 60 +Tac =0; *(2.3)*

Σ Fy= Tab* Sin 60 + Fay=0; *(2.4)*

Solving the above equation, we get Tab= (-500/ Sin 60) = -577.4 lbs.

Now substituting the above value in Equation (2.3), we get Tac= 288.7 lbs.

Now if we consider the node B, the free body diagram will be as shown below in figure 2.5

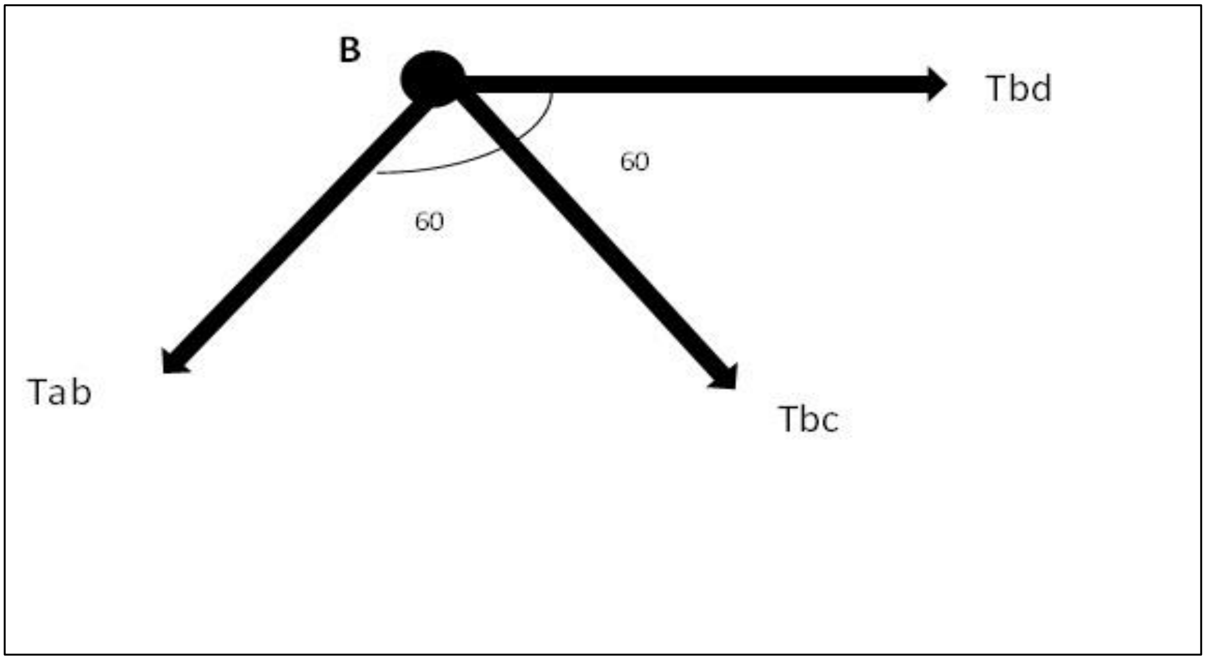

Figure2. 5 Free body diagram for node B

Applying the sum of forces, we get:

Σ Fx= -Tab*Cos 60 +Tbc Cos 60+ Tbd =0; *(2.5)*

Σ Fy= -Tab*Sin 60 –Tbc*Sin 60 =0; *(2.6)*

So Tbc= -Tab= 577.4 lbs. substituting this back in Equation (2.5), we get:

Tbd = -577.4 lbs.

Now if we consider the node C, the free body diagram will be as shown below.

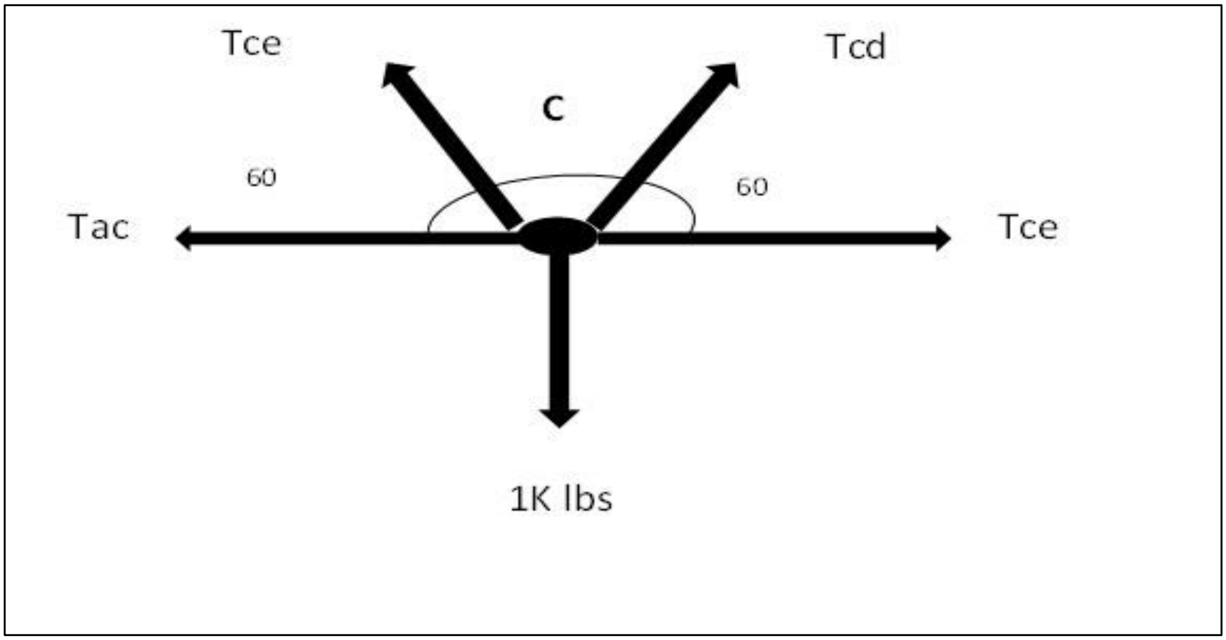

Figure2. 6 Free body diagram for Node C

Applying the sum of forces at at Node C, we get

Σ Fx= -Tac - Tbc Cos 60+ Tcd Cos 60+ Tce =0; *(2.7)*

Σ Fy= Tbc Sin 60 + Tcd Sin 60 1000 lbs.=0; *(2.8)*

Solving the above equations, we get the following values.

Tcd= 577.4 lbs, Tce= 288.7 lbs

Applying the same rules again for Node D, the free body diagram is as shown below.

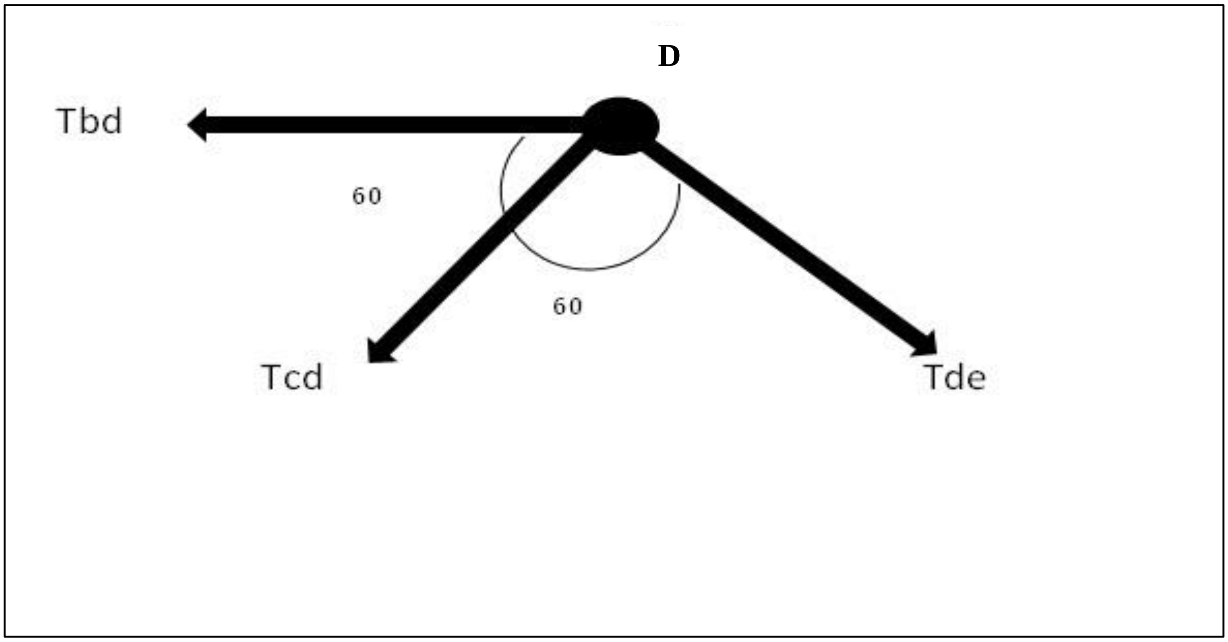

Figure2. 7 Free body diagram for Node D

Applying summation of forces, we get;

$$\Sigma Fy= -Tde * Sin 60 - Tcd *Sin 60=0; \qquad (2.9)$$

Tde= -577.4 lbs.

If all the tensile and compressive forces are assembled, the schematic representation is as shown below.

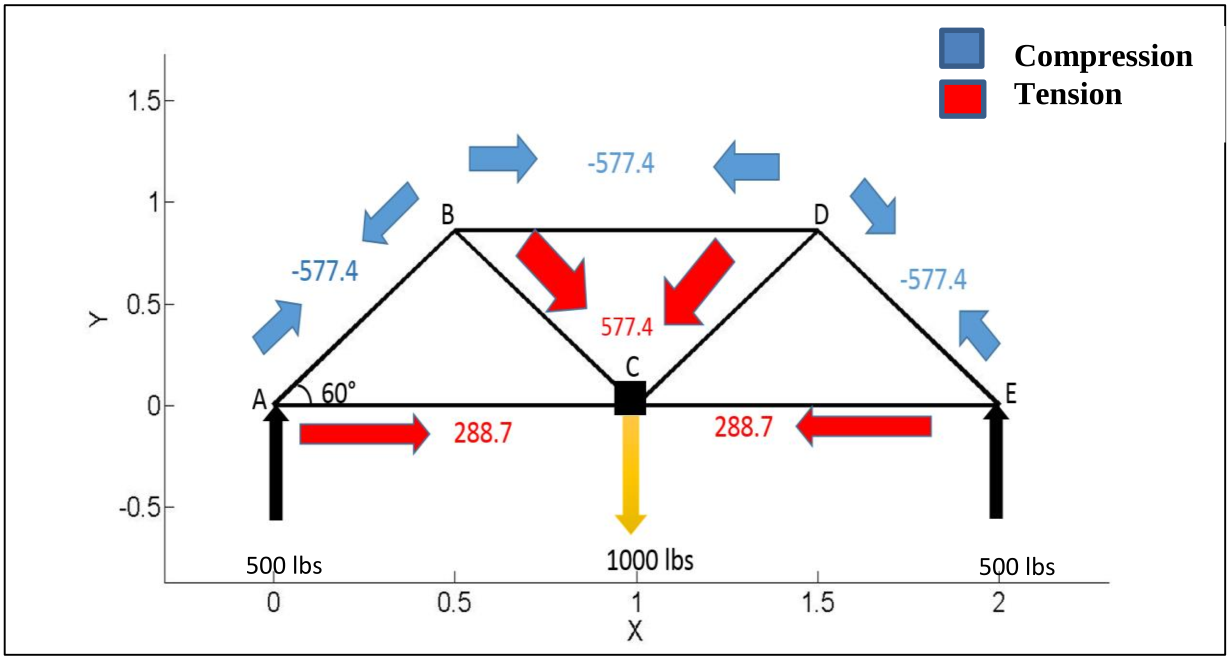

Figure2. 8 Forces acting on each member

2.2.3 Numerical verification

A similar 7 bar truss member with the same geometric and material properties as the above structure is simulated using Finite element GESA codes using MATLAB, we get a deformed geometry as shown below.

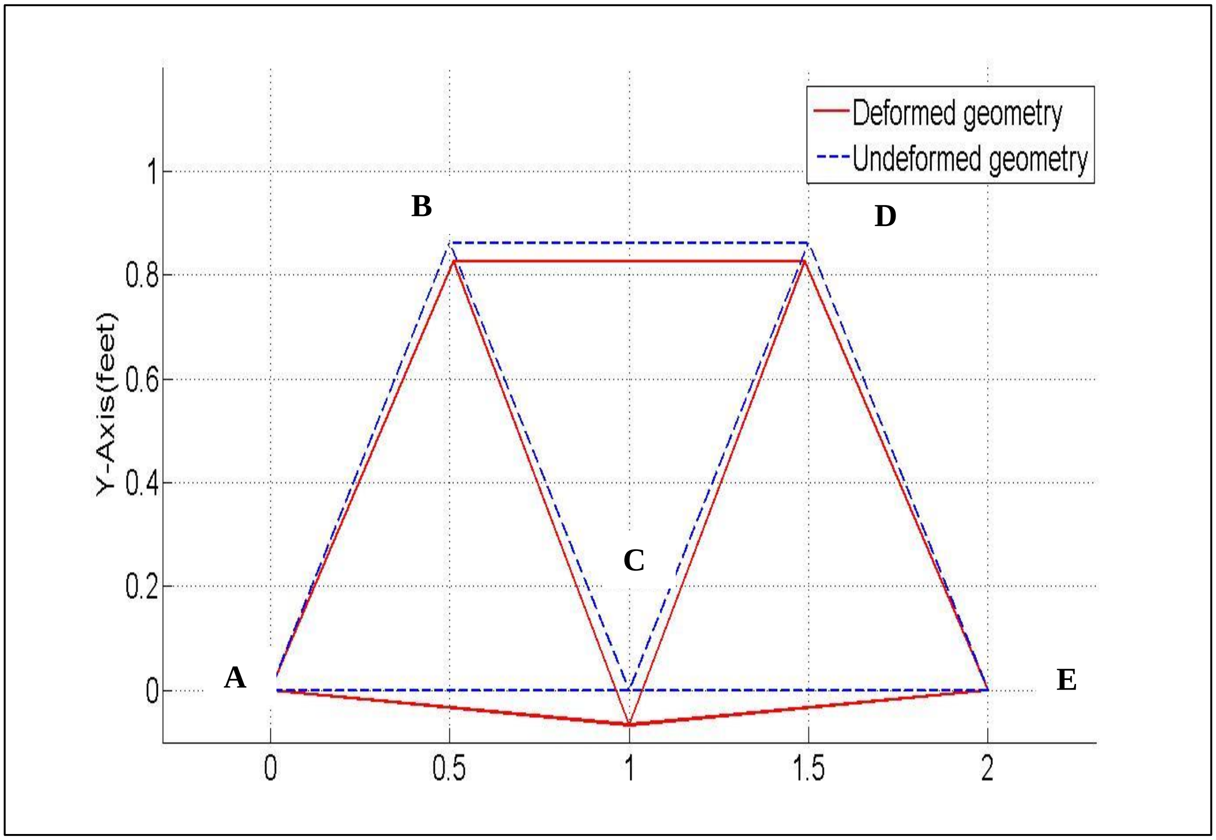

Figure2. 9 Experimental analysis

The deformed geometry conforms to the theoretical result shown in figure 2.8. Tension is evident especially in the base which induces an extension in the length whereas the compression forces act on the vertex which causes shrinkage of the vertex and this compression compensates for the extension at the base. Hence the experimental analysis conforms to the theoretical result obtained by calculating tensile and compressive forces.

2.3 Post buckling phenomena

Geometrical nonlinear static problems sometimes involve buckling or collapse behavior, where the load-displacement response shows negative stiffness and the truss structure release strain energy to remain in equilibrium. To show the post buckling phenomena, we consider a 3-bar truss structure which is one of the most commonly used structures and are more likely to have local buckling and bifurcated solution compared to the continuous structures.. A 3 bar truss member is as shown above in the figure 2.10 which is hinged at all ends. The force acting in the below truss member is at the vertex of the structure.

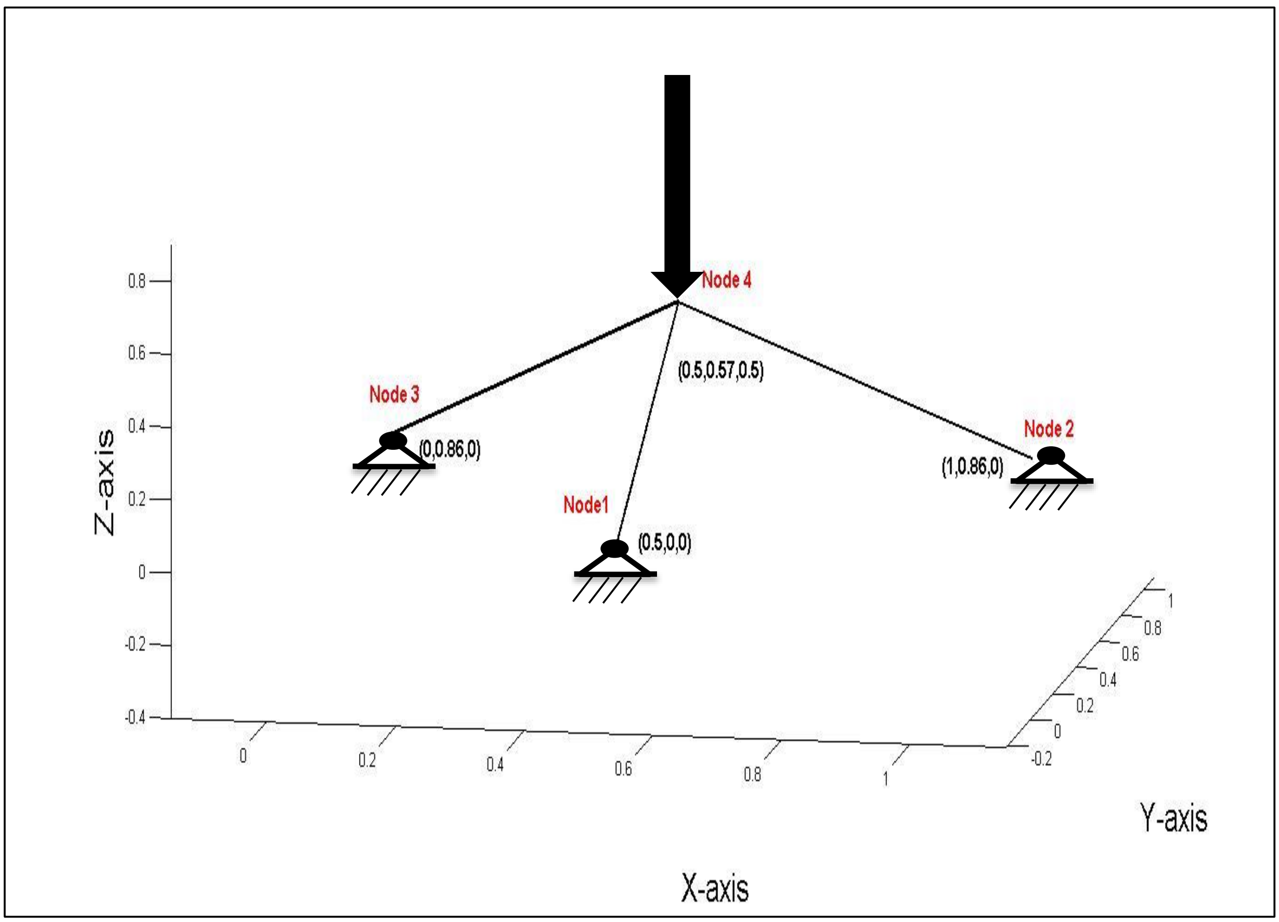

Figure2. 10 Simple 3-bar truss member

The natural and geometric and material properties of the above truss structure is given below

Table2. 2 Geometrical & Material properties

Type of material	Steel
Poisson's ratio	0.33
Mass density	15.28 feet/slug
Area of each structure	0.023 square feet
Young's modulus	72 GPa

The load deformation curve is as shown below.

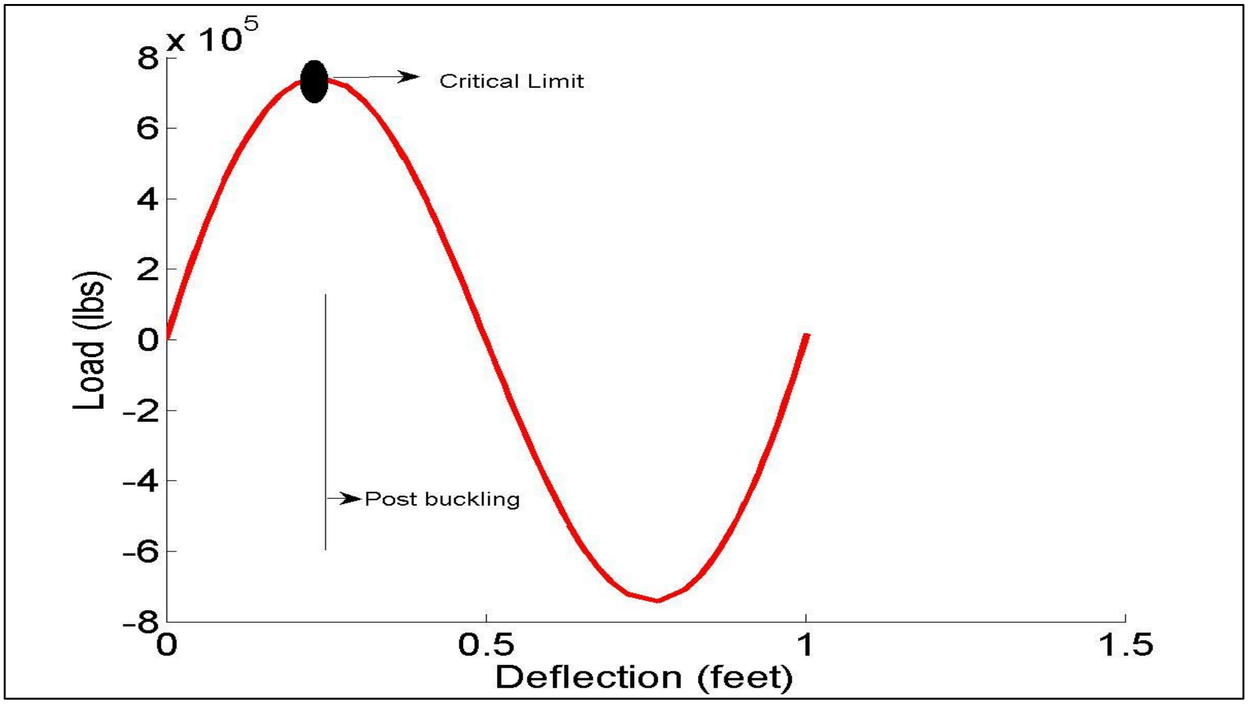

Figure2. 11 Load deformation for 3-bar truss

The critical load is the load corresponding to the situation in which a perturbation of the deformation state does not disturb the equilibrium between the external and internal forces. In case of a truss member, the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. As an applied load increases, it will slowly but steadily become large enough for the member to become unstable and buckles. In the post buckling section as shown in the figure above, if the load is further increased, there will be significant and unpredictable deformation. If the member is a part of a larger assembly, any load beyond the buckling load will cause the member to buckle and redistribute the load within the structure. For a hinged bar, Euler formula for estimation of buckling load is given by:

$$F_{cr} = \pi^2 * E * I_{min} \tag{2.10}$$

E denotes the modulus of elasticity.

I_{min} refers to the minimal moment of inertia.

L refers to the buckling length of the structure.

Nonlinear post buckling analysis with the effect of plastic deformation the yield transforms the stable post buckling behavior into unstable because after the yield point, any increase in deformation causes a decrease of corresponding load. Since the post buckling behavior can become unstable when the elasto-plastic deformations take place, it becomes extremely important to investigate the influence of imperfections on loading. These imperfections may be indifferent loading, indifferent area of cross section etc.

CHAPTER 3

METHODOLGY

3.1 Concept generation

The governing equation for any highly flexible structure can be given in Finite element notations as: $[\mathrm{M}]\{\ddot{\mathrm{q}}\} + [\mathrm{C}]\{\dot{\mathrm{q}}\} + [\mathrm{K}]\{\mathrm{q}\} = \{\mathrm{R}\}$ *(3.1)*

Where {q} is a N*1 displacement vector; {R} is a N*1 force vector caused by external loads; and [M], [C] and [K} are N*N mass, damping and stiffness matrices respectively. For any static problem, $\{\ddot{\mathrm{q}}\}$=$\{\dot{\mathrm{q}}\}$=0, and hence equation (3.1) reduces to $[\mathrm{K}]\ \{\mathrm{q}\} = \{\mathrm{R}\}$ *(3.2)*

Linear analysis is based on the assumptions that the deformed structure returns to its original form and the changes in shape is small and so is the strain. Also the material properties are considered to remain constant and hence the loading direction or magnitude does not change. Hence equation (3.2) has a linear relationship or in other words the matrix [K] largely remains unchanged and so it doesn't account for the above criteria. So all linear models assume that the displacement varies linearly with load and hence linear analysis has a major setback and isn't the most accurate of all the analysis. Hence to overcome all these setbacks Non-Linear analysis should be used.

Our study mainly deals with Non-Linear analysis of structures since highly flexible structures are designed to undergo large elastic displacements and rotations without plastic deformation under normal operations. Since large rotations change directional stiffness and mass inertias of a structure, the analysis of such structures cannot be attributed to linear analysis.

Non-Linear analysis accounts for the change in geometric properties since the stiffness matrix changes. In other words, the matrix [K] in equation (3.2) changes and the structure may not return to the original form or may be permanently deformed. Non-Linear analysis also accounts for the material properties of the material, this means that the material properties can change along the course of deformation. It also supports changes in load direction and constraint location. There are various approaches to Non-Linear analysis, the most commonly used one though would be the Newton Raphson method which illustrates the linearized incremental of equation (3.2) which leads to a different tangent stiffness matrix $[\widehat{K}]$ which is called a tangent stiffness matrix.

3.2 Newton Raphson method

The Newton Raphson method is highly effective numerical computing method for finding successively better approximations of roots of a real valued function. Like any other differential calculus methods, the Newton Raphson is based on the simple idea of linear approximation. The iterative process follows a set of guidelines to approximate one root, considering the function, its derivative and an initial value. It can be well described by a figure as shown below;

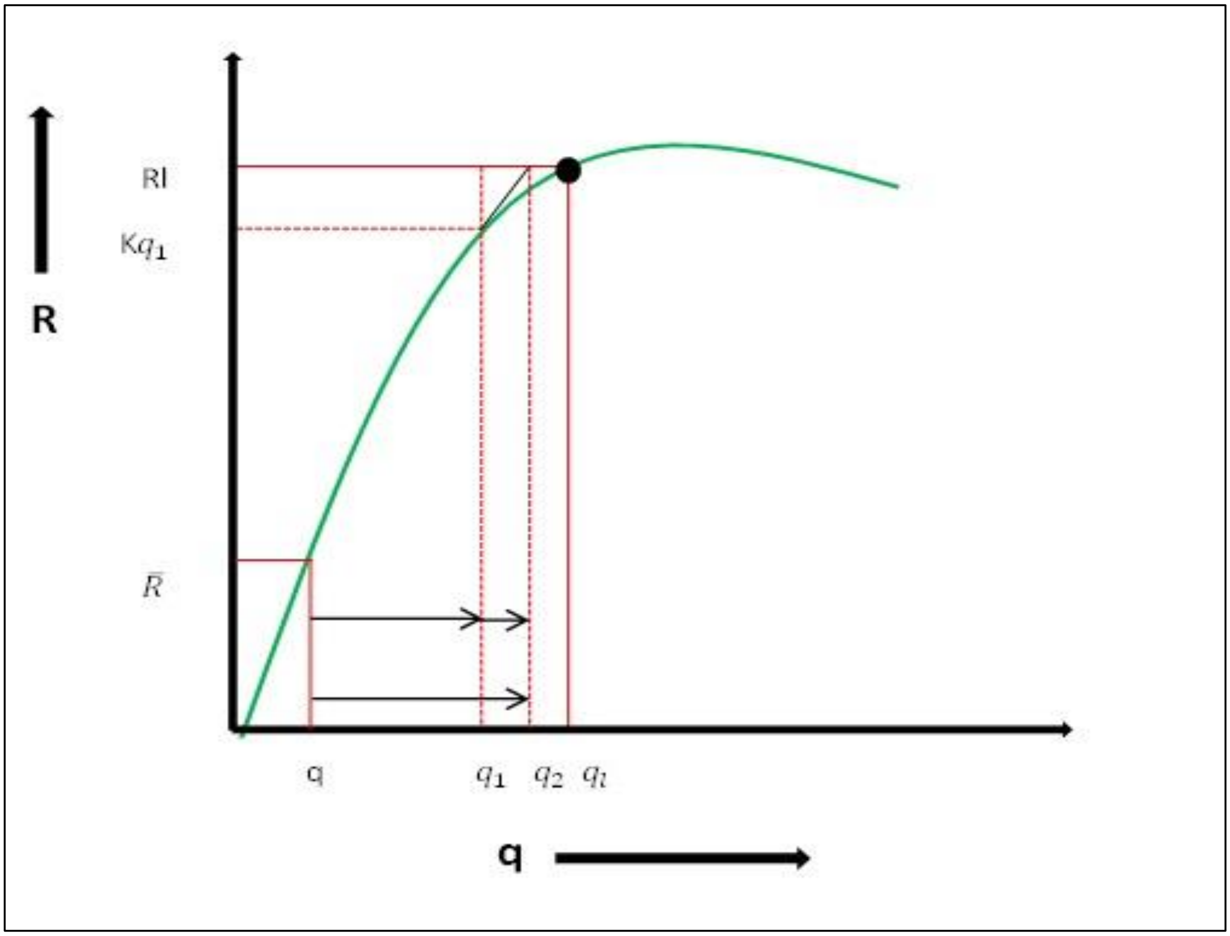

Figure3. 1 Newton Raphson Method

To derive the linearized incremental equation we consider that the displacement value

$\{q\}= \{\bar{q}\ \} + \{\Delta q\}$ *(3.3)*

In this case, the subsequent force matrix will be: $\{\{\bar{R}\} = ([K]\{q\})$ *(3.4)*

In the above equation, $\{q\} = \{\bar{q}\}$ and the term $\{\bar{q}\}$ denotes the equilibrium solution corresponding to $\{R\} = \{\bar{R}\}$ and $\{\Delta q\}$ denotes the estimated incremental displacement vector resulting from the gradually changing force increment is:

$\{\Delta R\} = (\{R1\} - \{\bar{R}\})$ *(3.5)*

When the above equation is expanded in Taylor's series and higher order terms are neglected,

we obtain $\{\mathrm{R}\} = \{\mathrm{R}\}1 = ([\mathrm{K}]\{\mathrm{q}\} + \left\{\frac{\partial \mathrm{K_{ij}q_j}}{\partial \mathrm{q_k}}\ \Delta \mathrm{q_k}\right\})$ *(3.6)*

In the above equation, $\{q\} = \{\bar{q}\}$ and hence this will simplify to:

$$([\mathrm{K}]\{\mathrm{q}\} + \left\{\frac{\partial \mathrm{K_{ij}q_j}}{\partial \mathrm{q_k}}\ \Delta \mathrm{q}\right\}) \quad \text{(3. 7)}$$

In the above equation repeated indices means summations and so the final equation can be

written as $[\hat{\mathrm{K}}]\{\Delta \mathrm{q}\} = \{\Delta \mathrm{R}\}$ *(3.8)*

In equation (3.8), $[\hat{\mathrm{K}}] = \frac{\partial \mathrm{K_{ij}q_j}}{\partial \mathrm{q_k}}$ at $\{\mathrm{q}\} =\{\bar{q}\}$ *(3.9)*

So the term $[\hat{\mathrm{K}}]$ in equation (3.9) is called the Tangent Stiffness matrix.

For linear problems, $[\hat{K}] = [\mathrm{K}]$, but for non-linear problems $[\hat{\mathrm{K}}]$ signifies the tangent stiffness along the direction of corresponding displacement when this displacement changes by an infinitesimal amount.

3.3 Finite element analysis

One of the methods used for structural analysis is the Finite element analysis or the Finite element modelling. Finite element method breaks the structure into discrete points more generally called as nodes. After this, appropriate numerical techniques can be used to approximate boundary value problems for partial differential equations. Since an arbitrary point in an element can be interpolated using the low-order polynomials and deformation of its surrounding nodes, partial differential equations are converted into ordinary differential equations. The finite element computer aided software can predict deformation, vibration, heat flow, fluid flow and many other physical effects. Finite element connects many simple element equations over small subdomains or finite domains to approximate a more larger and complex domain.

3.3.1 Finite element software packages

There are many intriguing finite element software packages in use today. Many offer open source package for numerical simulation in structural mechanics. The input to the software is the dimensions of the structure, constraints, the inherent properties of the structure and the magnitude and direction of load acting on it. The output normally would be a combination of numerical values and pertinent deformed configurations which provide a visual reference as to how the structure deforms. Abaqus, Autodesk, SolidWorks, Hyperworks products and Matlab are some of the Finite element analysis packages. For our purpose which include enormous numerical computation and calculation and for the ease of programming, Matlab was a feasible option. Its interactive environment for numerical computation, visualization and programming makes it a very sought after package for numerical computing.

3.3.2 Element discretization

The division of structure into a set of elements is called discretization or mesh generation. This is considered to be one of the most crucial phases in finite element modeling since the accuracy of the solution largely depends on the quality of meshing. The shape of the structure plays a big part to decide the quality of meshing and hence a triangle is considered to have the simplest geometric shape followed by rectangle. Generally the elements that are selected should characterize the governing equations of the problem and the accuracy is dependent on the number, shape and type of elements used. Discretization or meshing is not always a straightforward process and can get complicated and hence a sound knowledge of finite element modeling is always necessary. For example; the density of elements should be such that the regions of large gradients of the solution are adequately modeled, in other words higher order elements should be used in the regions of large gradients and mesh refinements should vary gradually from high-density regions to low-density regions.

3.3.3 Element nodes

Each element possesses a set of distinguished points called as nodal points or nodes. Nodes serve two purposes; to define the element geometry and also to define the degrees of freedom. Nodes are located at the end points of the elements or in other words, they are the connecting points between elements. The elemental nodes are always one more than the number of discretized elements. More the number of nodes, better the accuracy of the result.

3.3.4 Element geometry

The position of the geometrical nodal points defines the element geometry. Many of the elements have fairly simple geometry and are easy to define. Curved segments and straight beams like the one shown above are one-dimensional in nature whereas triangular trusses are two dimensional depending on the number of members present in it. Other members like tetrahedral are the most commonly used three dimensional structures.

3.3.5 Nodal forces

The R matrix in the universal governing equation (3.1) represents the nodal forces combined in a matrix form. There is always a set of nodal force in a one-to-one correspondence with the degrees of freedom. In mechanical elements the correspondence is established through energy arguments. The nodal displacements though are highly restrictive to the type of material used. The tangent stiffness matrix constantly keeps updating as the force is increased and the change of displacement due to increase in force.

3.4 Deformation analysis of beams

A deformation is primarily a process of change in shape and size of an object. The cause for deformation may be to an applied external force or due to change in temperature. Deformation due to forces can be attributed to tensile, compressive, shear, bending or twisting forces. When the force is applied, inter-molecular forces from the structure arise to oppose the forces. If the magnitude of the applied force is not too large, the inter-molecular forces may be sufficient to resist the applied force allowing the structure to assume a new equilibrium but if the applied force is very large, the structure may undergo a permanent deformation or the structure may even fail. In finite element analysis the structure is discretized into elements which are connected by nodes.

The appropriate way to analyze deformation numerically is by plotting a Load vs. Deformation curve. It is a curve to analyze the deformation till the limit where the material fails. We can analyze the Load vs. Deformation curve of a hinged bar element with dimension 0.02 square feet and a young's modulus of 72e9 GPa hinged beam is a beam that is attached at the boundaries and is free to rotate, like a hinge. The total length of the member is 2 feet. A schematic sketch representing a beam with force acting on the bottom is as shown below in Figure (3.2). In terms of Finite Element Modeling, the beam is discretized into 24 elements.

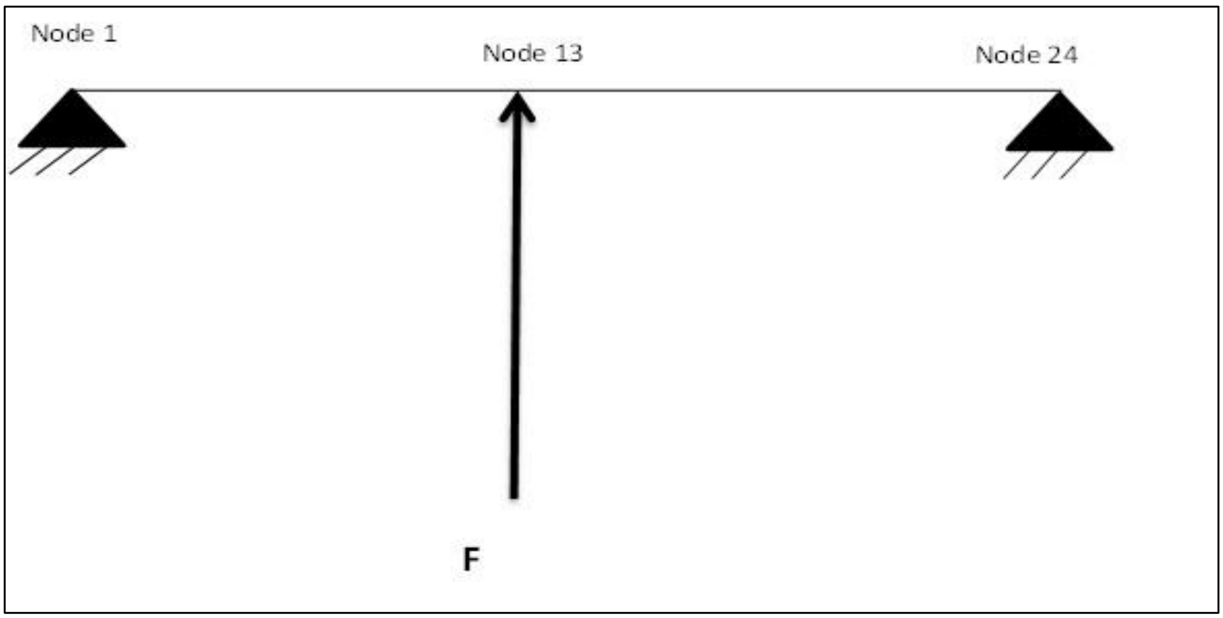

Figure3. 2 Hinged beam

Now, if a load of 30 lbs. is applied from the bottom at the center of the beam (13th Node), there will be a certain deformation that will take place and if all values of deformation are computed and plotted against the corresponding force values, a curve will be obtained as shown below.

The Load vs. Deflection curve for the beam after a force of 100 lbs. is acting on it is as shown below in figure (3.3). The nature of the graph tells a lot about the accuracy of the software and how well the software dictates the results. The curve is nonlinear in nature which means there is disproportionate relationship between the response curve and fore acting. The load deformation curve is of high significance.

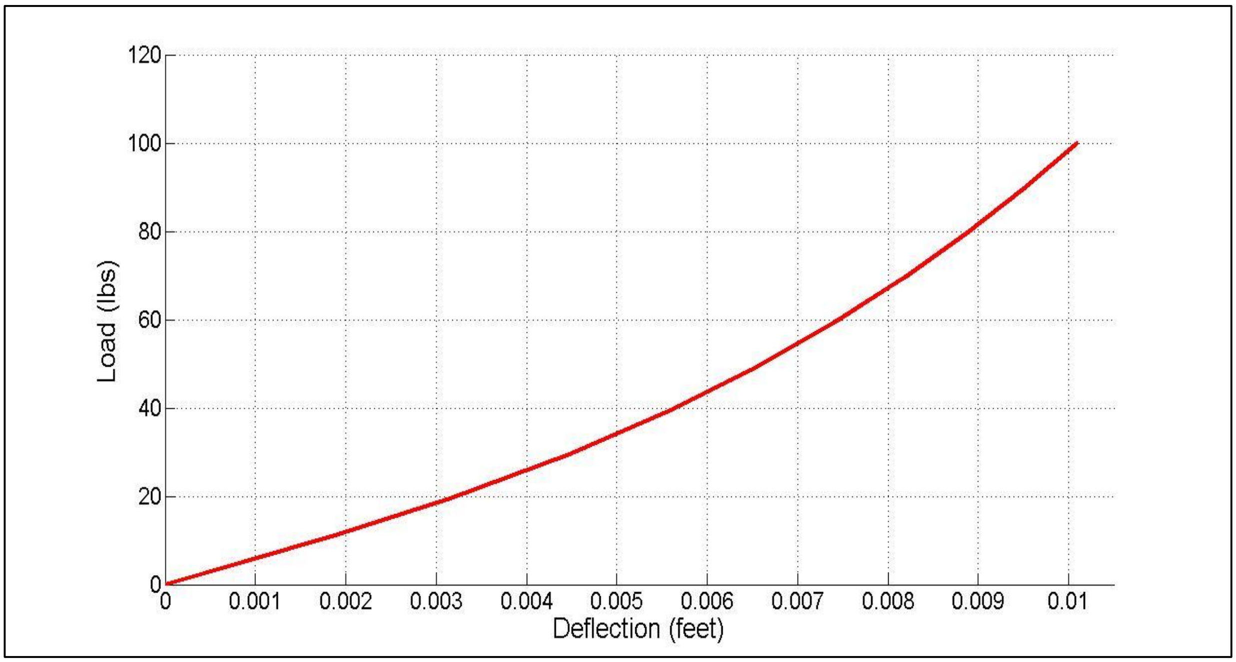

Figure3. 3 Load deformation curve for hinged beam

The deformed geometry of a straight beam is as shown below. Since the upward force is acting in the middle and the ends are constrained, the beam will deform as shown in the figure below.

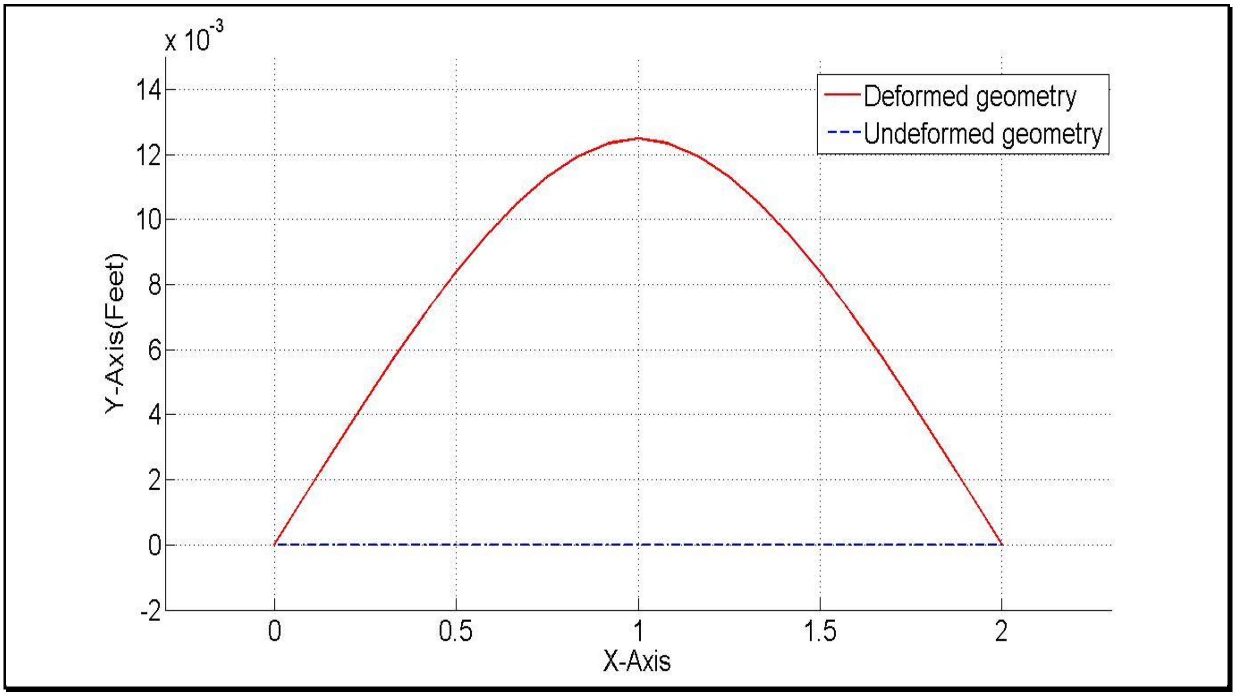

Figure3. 4 Deformed geometry of a hinged beam

3.5 GESA

GESA stands for geometrically exact structural analysis and is a total Lagrangian displacement-based finite element code for analyzing highly flexible structures. Since our study is based on nonlinear finite element theory, GESA is an ideal platform to work on since it accounts for nonlinear structural theory for structures undergoing large deformation and rotation. Cables, trusses and beam and plate element are incorporated in GESA. Isotropic as well as anisotropic materials are considered. The fact that all possible initial curvatures are included in the strain-displacement equations, governing equations of plates and shells are unified too and this strain displacement relation can be used for most of the one-dimensional and two-dimensional structure which include our study on beams and trusses. Although for two-dimensional structures, only global transitional degrees of freedom and their derivatives are used in the strain displacement relation and no independent global rotational degree of freedom are used.

For the purpose of research work, GESA for Nonlinear structural analysis is used. As like the linear analysis where there is a linear relationship between the structural response and the magnitude of the load, the nonlinear analysis deals with disproportionate relationship between the response curve and the magnitude of loading. A finite element analysis is called static if the parameters are not variably in line with change in time and hence any displacement, acceleration parameter is not accounted for.

Chapter 4

Results

4.1 Truss structure equivalence to a spring

Trusses have the unique property which in a way makes them mechanically equivalent to a spring since they have no stiffness against the applied loads except for those acting along the axis. If a spring is considered whose spring constant is denoted by k has one hinged and the other end slides down a frictionless vertical wall in response to a downward force. The symbol α denotes the angle between the spring and the horizontal direction in the absence of the downward force. The change in angle during the downward displacement of the roller is α'.

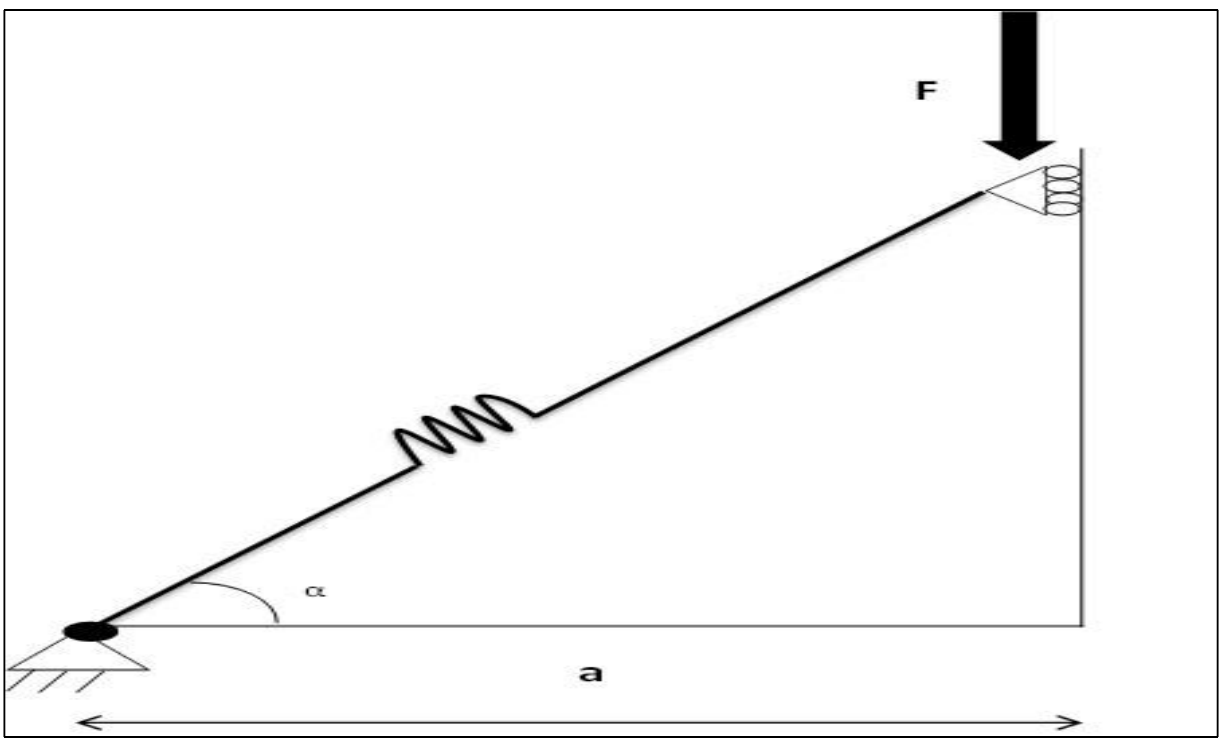

Figure4. 1 Experimental setup of a spring

The equation for the force applied can be given by $(\mathrm{K} * \nabla * \sin \alpha') - \mathrm{F} = 0$ *(4.1)*

Where ∇ refers to the change in length of the spring

The change in angle α' is proportional to the change in length of the spring and hence the change in length can be given by $\nabla = [a/\cos(\alpha) - a/\cos(\alpha')]$; which simplifies to,

$$\nabla = a*[1/\cos(\alpha) - 1/\cos(\alpha')] \quad (4.2)$$

Substituting the above equation in equation (4.1); we get force:

$\mathrm{F} = \mathrm{K}* a*[1/\cos(\alpha) - 1/\cos(\alpha')]* \sin\alpha'$, which simplifies to

$$\mathrm{F}/ (\mathrm{K}*a) = [1/\cos(\alpha) - 1/\cos(\alpha')]* \sin\alpha' \quad (4.3)$$

Function (4.03) can be plotted for various set of initial angles ranging from ∏/4 to ∏/12;

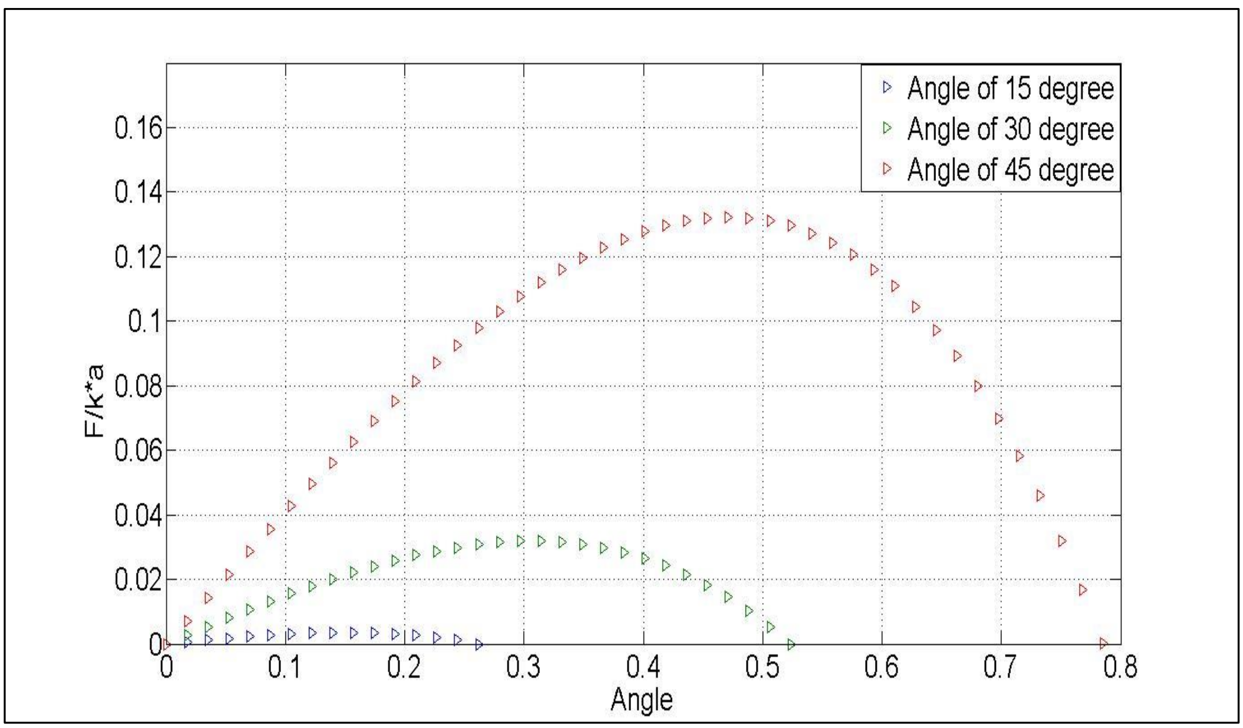

*Figure4. 2 F/k*a Vs. Angle graph for a spring*

The graph shows a similar behavior for all set of angles; initially the frictionless roller requires a force to move downwards and after a point which is referred to as critical point in truss loading, the frictionless roller moves down for itself load being decreased. Similarly the Force vs. Displacement graph for the frictionless roller can be plotted, since the reduction in angle is proportionate to the displacement of the frictionless roller. The force deformation graph will follow the same pattern as the force vs. angle graph.

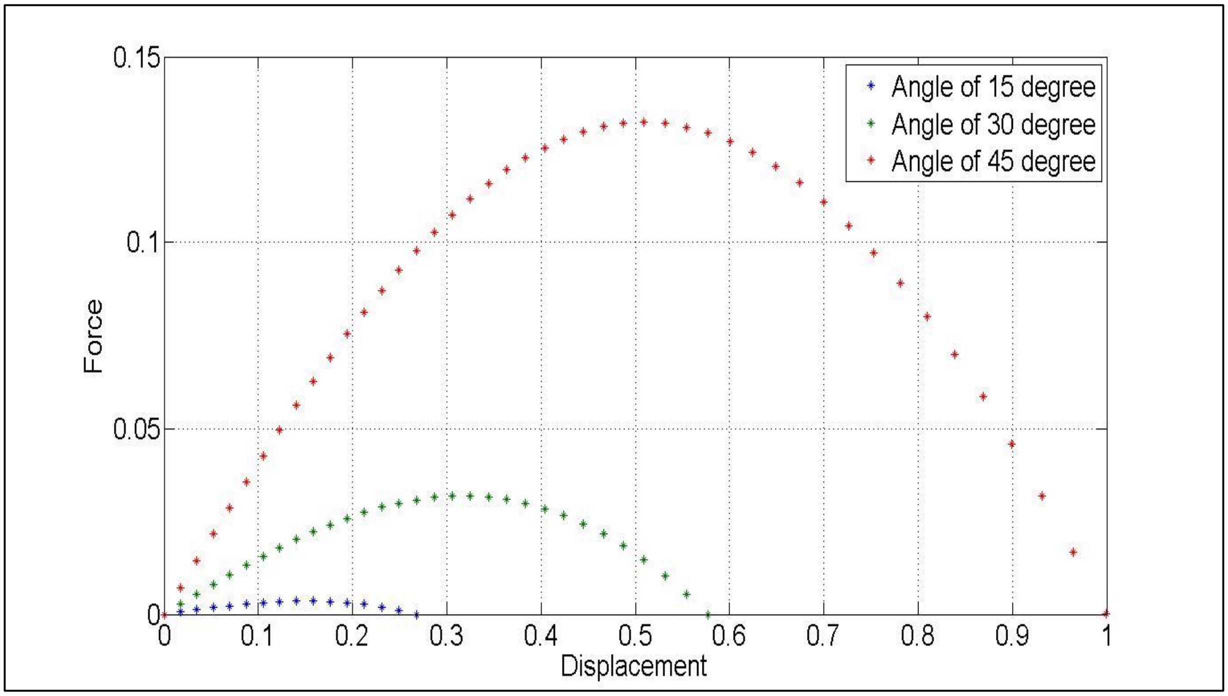

Figure4. 3 Force displacement graph for an inclined spring

The above spring structure can be congruous to a truss structure shown below. The forces acting on the vertices are equal and hence the deformation takes place in a symmetric manner without the roller being moved. Hence the force acting on the left hand side vertex can be assumed to be equal to force acting on an inclined spring as shown in the previous case. The node 1 represents the position of the frictionless roller from the previous setup and the truss structure in itself is assumed to be mechanically similar to a spring.

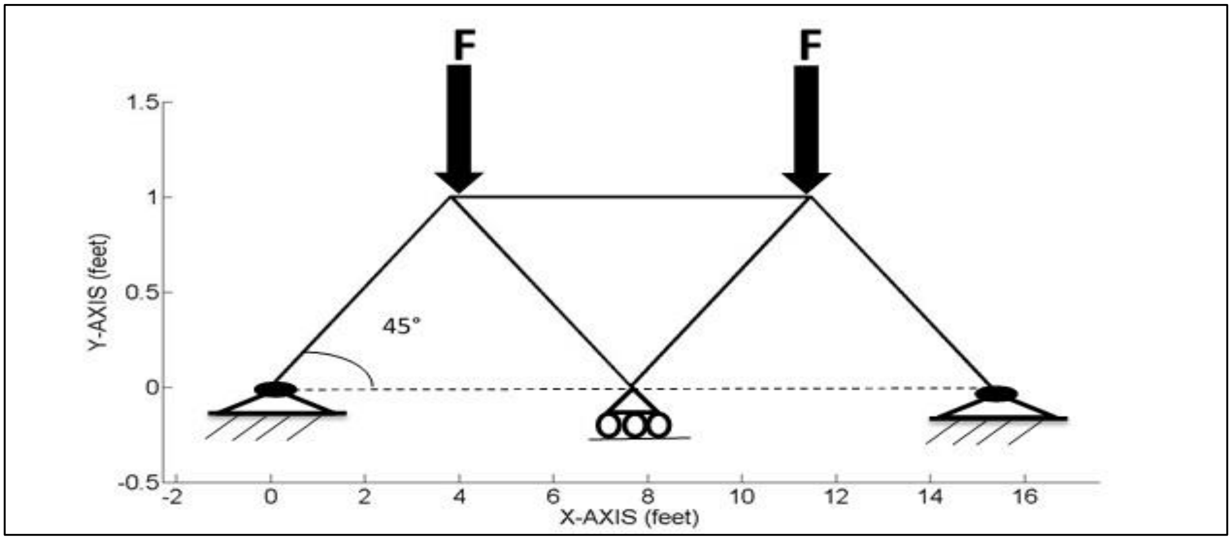

Figure4. 4 Five-bar truss structure with equal loads acting

Since the stiffness is assumed to be equal to 1 in the previous case, to replicate a similar model we assume a medium density fiberboard with a very low modulus of elasticity. This is mainly because the modulus of elasticity is a measure of stiffness. The geometrical properties are as shown below.

Table4. 1 Geometrical & Material properties

Poisson's ratio	0.33
Mass density	15.28 slug/ft^3 or 8050 kg/m^3
Area of each structure	3e-3 square feet
Young's modulus	3 GPa
Number of bars	5

The load deformation curve when a force is applied on the vertices of a medium density fireboard is as shown below in figure (4.14).

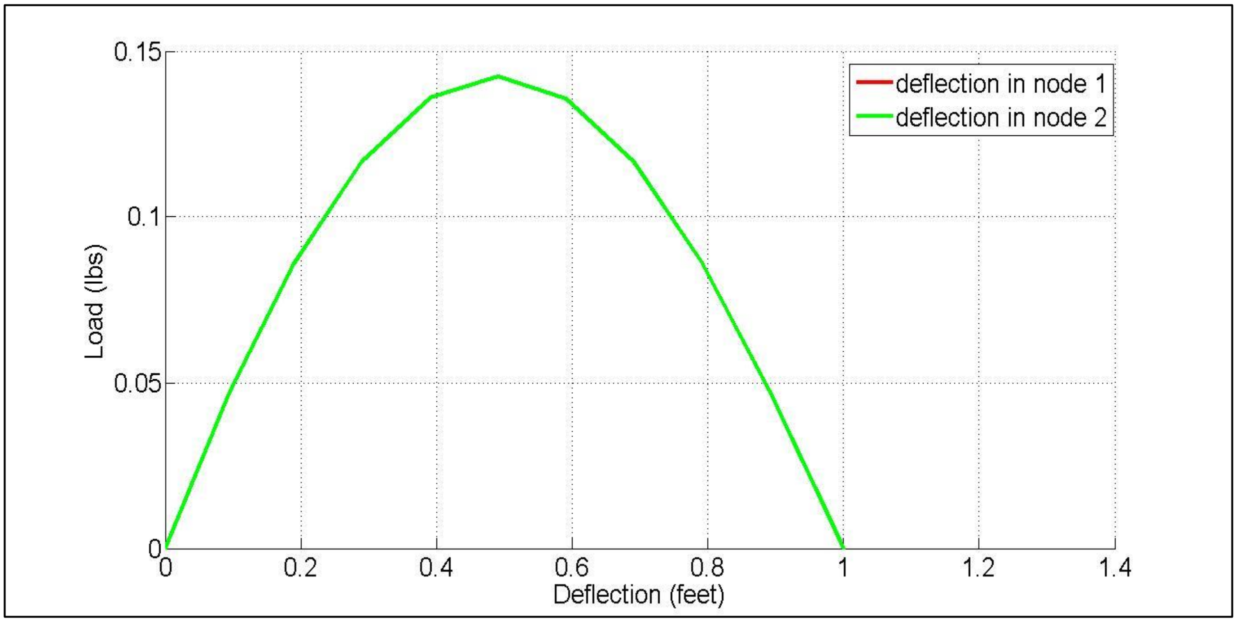

Figure4. 5 Load deformation for 5-bar spring

Since the forces applied on the vertices are symmetrical, the center point does not move and the symmetry is maintained; hence the force calculation can be carried out only on the left hand side vertex (Node 1). It can be seen that the load deformation curve for the 5 bar structure with load acting on the top follows the same pattern as the load deformation curve for frictionless roller attached to the spring and hence this analysis supports the theory that truss structure is mechanically equivalent to a spring since it has no stiffness against the load applied apart from the axial direction, although unlike a spring, a truss element can be oriented in direction in a plane and the same element will be capable of handling tension as well as compression.

4.2 Symmetry breaking in 7-bar truss structures

To understand the concept of symmetry breaking in detail, a simple example of a two-dimensional 7 bar steel truss structure can be considered as shown below.

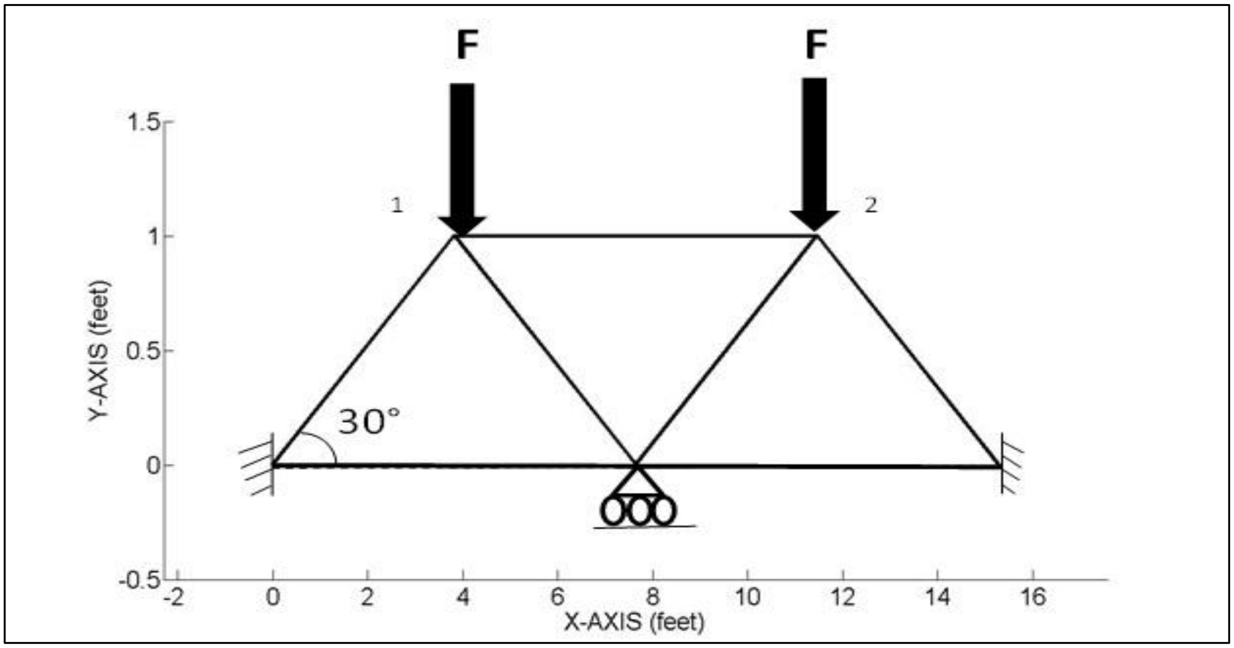

Figure4. 6 7-bar truss with symmetric load

The structure is hinged at both ends and is provided with a roller in between. This is one of the most commonly used boundary conditions in truss experimentation. Such setups can be most frequently seen in bridges and in aircraft wing spars where the truss structures lay horizontal and provide support. The hinge restricts any rotation and the roller support is provided in many big structures like bridges to aid in expansion and contraction during temperature changes. The soft rivets does a similar job as a roller in aircraft structures. The angle of inclination of the truss structure in our experimental analysis is 30 degree to the horizontal. There are two equal forces of 4000 lbs. acting on the vertices i.e. node 1 and node 2.

The following are the material properties of the structure.

Table4. 2 Geometrical & Material properties

Poisson's ratio	0.33
Mass density	15.28 slug/ft^3 or 8050 kg/m^3
Area of each structure	2.3 e-5 square feet
Young's modulus	200 GPa (4.17e9 lbs./ft^2)
Height of the structure	1 feet

The load deformation curve is as shown in figure (4.16)

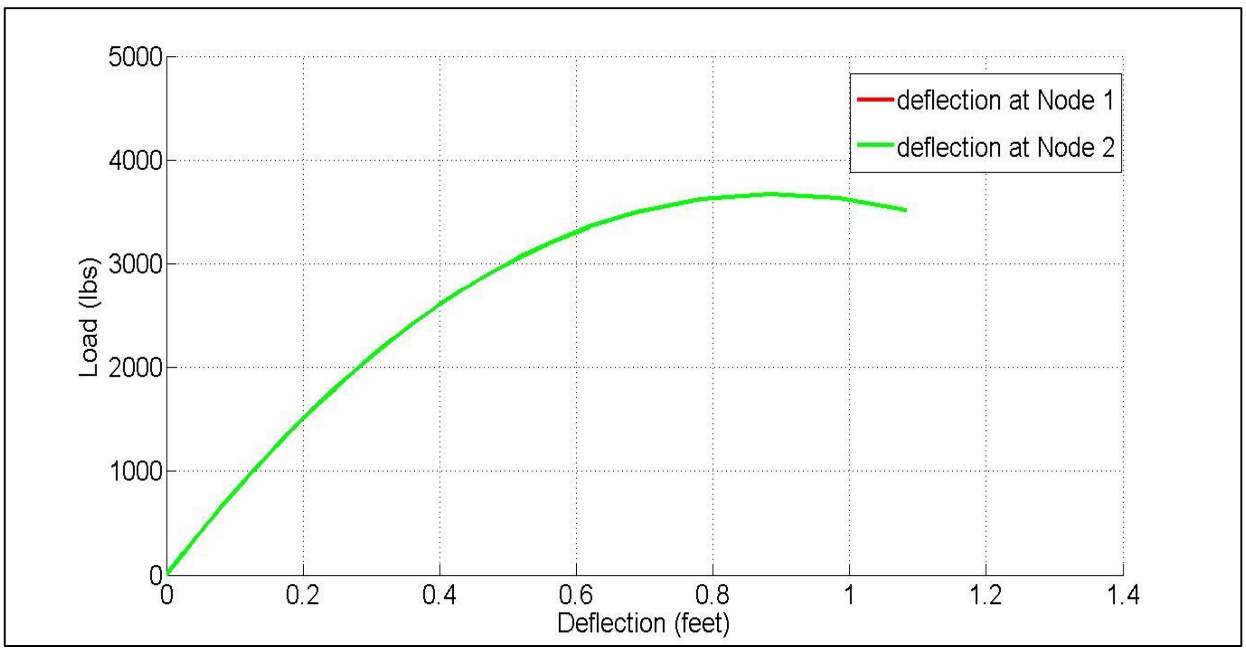

Figure4. 7 Load deformation curve for a 7-bar truss with uniform loading

The load deformation curve for the vertices does not exhibit a branching and conforms without deviating since the force applied on the vertices are the same and the roller in the middle does not move .The load deformation curve proves that the symmetry is being maintained since the two curves representing the deflection in the two nodes overlap. This means though there is a deformation in the structure, the symmetry is still very well being maintained. The buckling load is around 3750 lbs. This is can be understood by means of a deformed geometry diagram as shown below

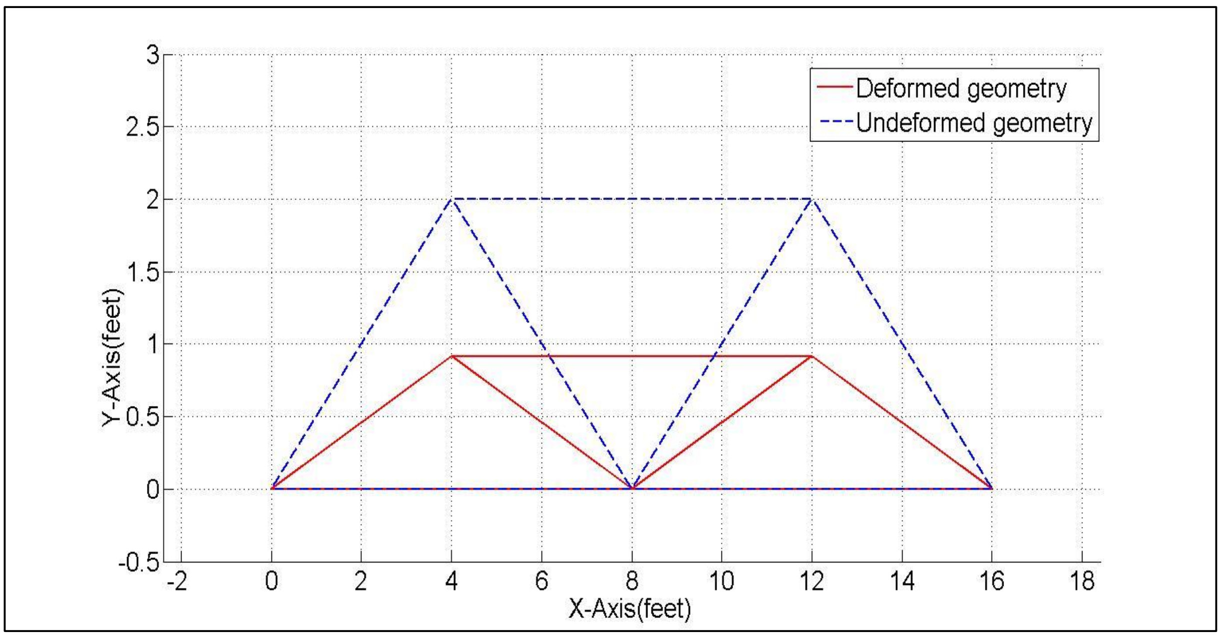

Figure4. 8 Deformed geometry for a 7-bar truss with uniform loading

The structure deforms with when two equal compressive forces act on the vertex. The compressive forces deform the structure uniformly till the buckling load is reached. Hence it is observed that there is nothing that disturbs the symmetry and the center point which consists of a roller does not move.

Now if the one of the forces acting on the vertex is altered and all the other geometrical and material properties are kept a constant, the results and effects on buckling point would be something to consider. The force on node 2 is reduced by 1% compared to node 1 as shown below in figure (4.9)

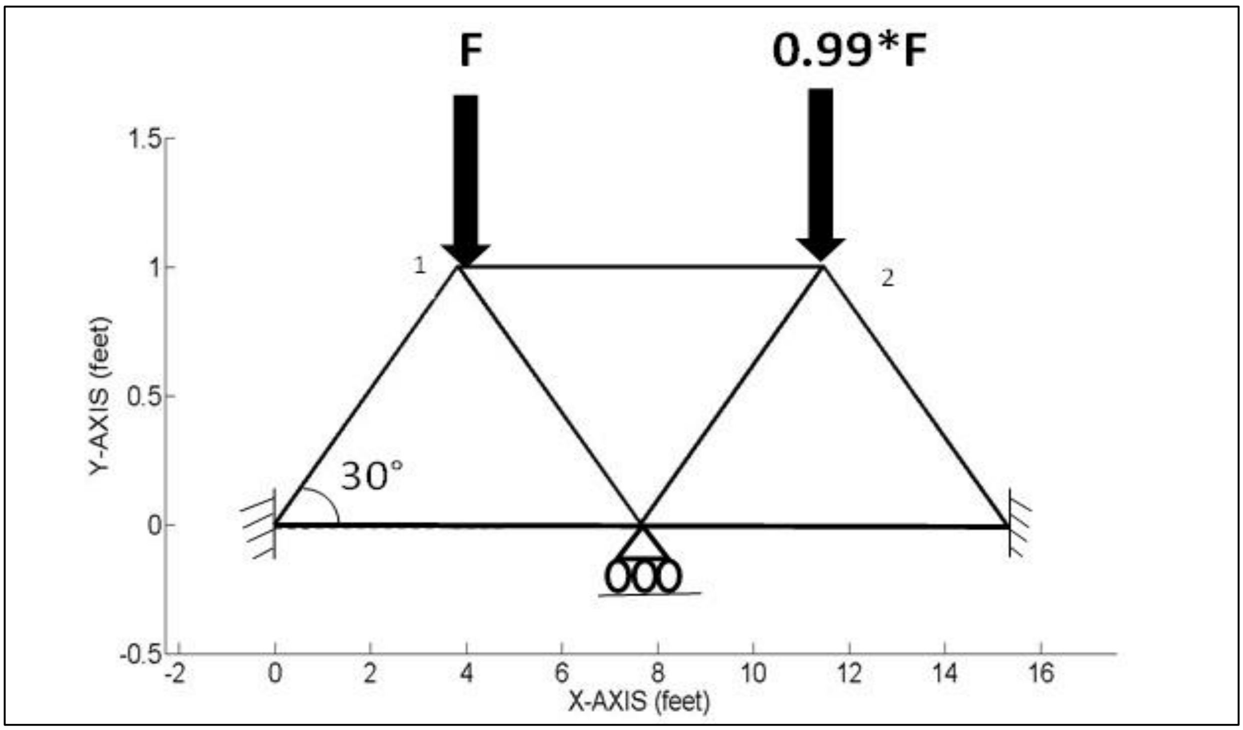

Figure4. 9 7-bar truss with non-uniform loading

The load deformation curve for the above setup is as shown below in figure (4.10)

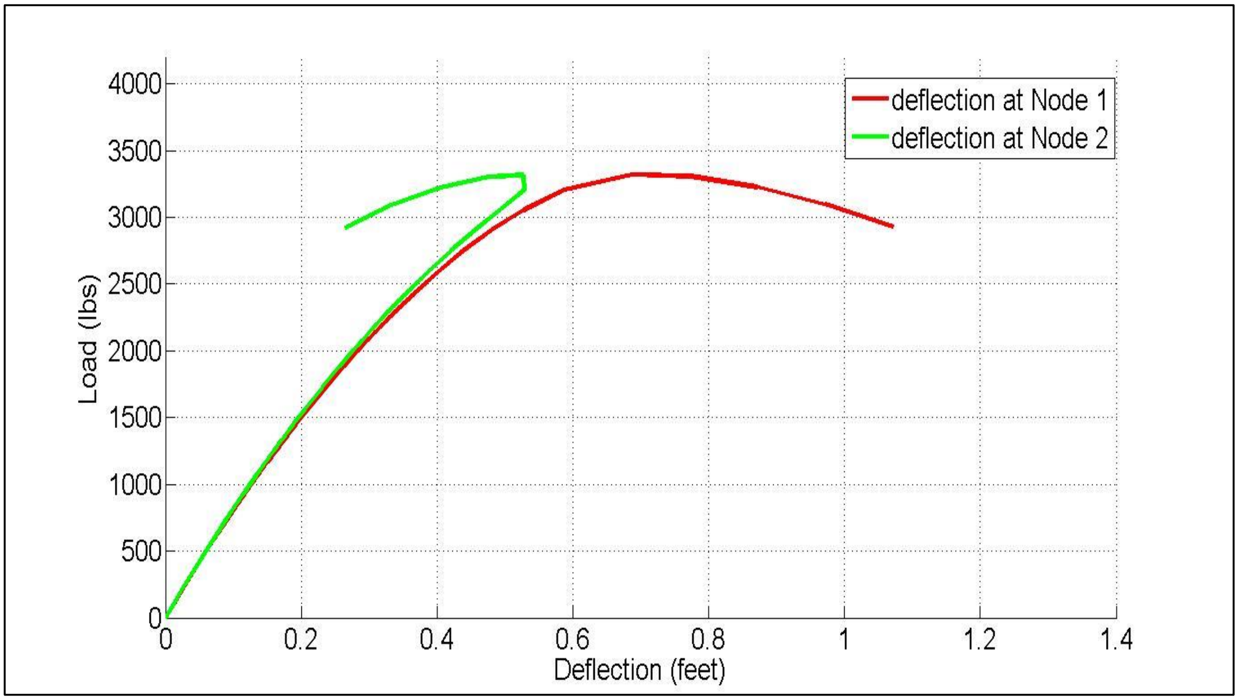

Figure4. 10 Load deformation curve for a 7-bar truss with non-uniform loading

In the above figure, there is a bifurcation that takes place, this phenomena is symmetric breaking or asymmetric response. The symmetry of the object is disturbed and this has an effect on the buckling load of the structure too, it fails at a load of 3300 lbs. which is a 12% reduction in the buckling load from the previous case where equal forces are acting on the vertex. The load deformation curve shows a bifurcation after which the two nodes are deformed in a bizarre manner since the roller placed at the center is being displaced and symmetry being disturbed. This is could be more easily understood with the deformed geometry analysis as shown in the figure (4.11)

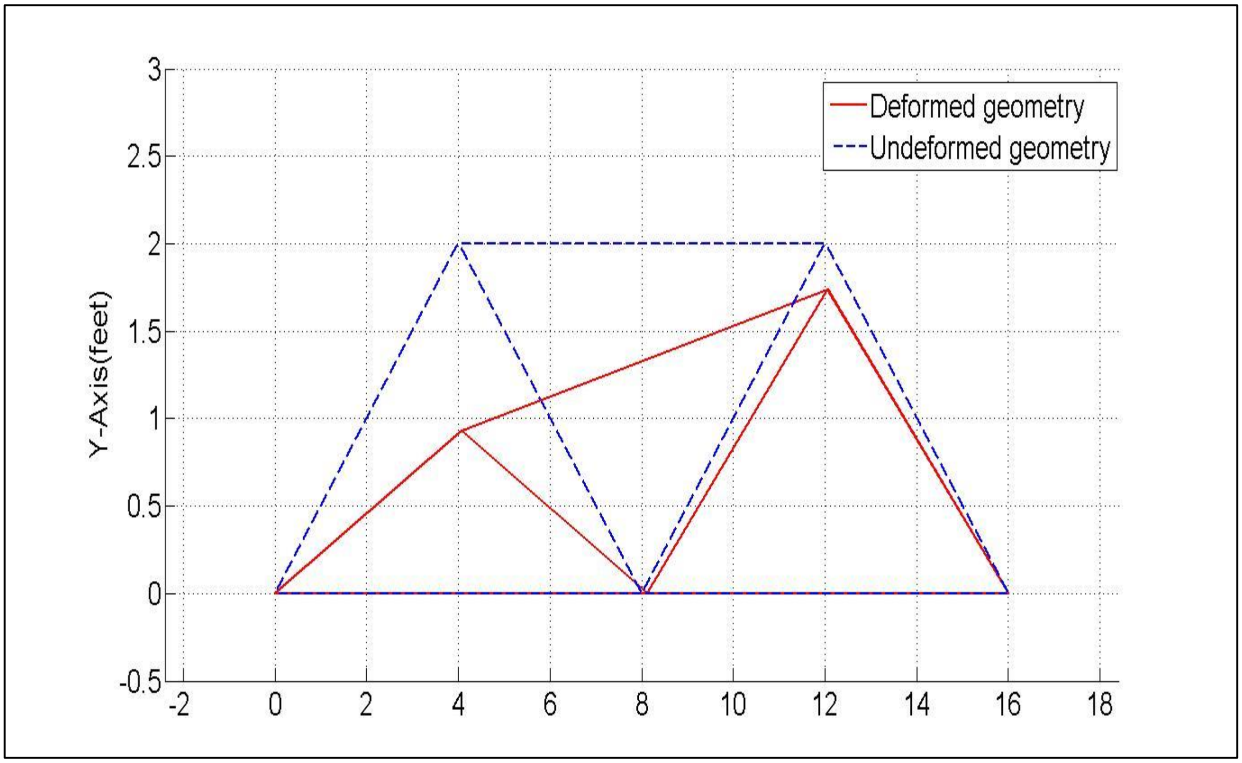

Figure4. 11 Deformed geometry for a 7-bar truss with non-uniform loading

The lack of symmetry in the response show that even miniscule difference in the forces acting can cause an asymmetric response. The node fixed to the roller gets displaced to the right due to the larger force acting on node 1. The larger force on node 1 also displaces it by a larger margin compared to node 2, but the node 2 gets displaced back after the symmetry is broken. The load deformation curve followed a symmetrical path and after the break point, the lower force node displaces in the opposite direction.

4.2.1 Effect of area of cross section on symmetry breaking

Truss structures come in various shape and size and depending on the purpose, their dimension and geometrical properties vary. In order to study the effects of cross sectional area on symmetry breaking, we take an example of the previous case but with all the material properties described in table (4.2) being the same except for the area which is reduced by 30% and has a value of 1.6e-5 square feet. The structure is as shown below in figure (4.12)

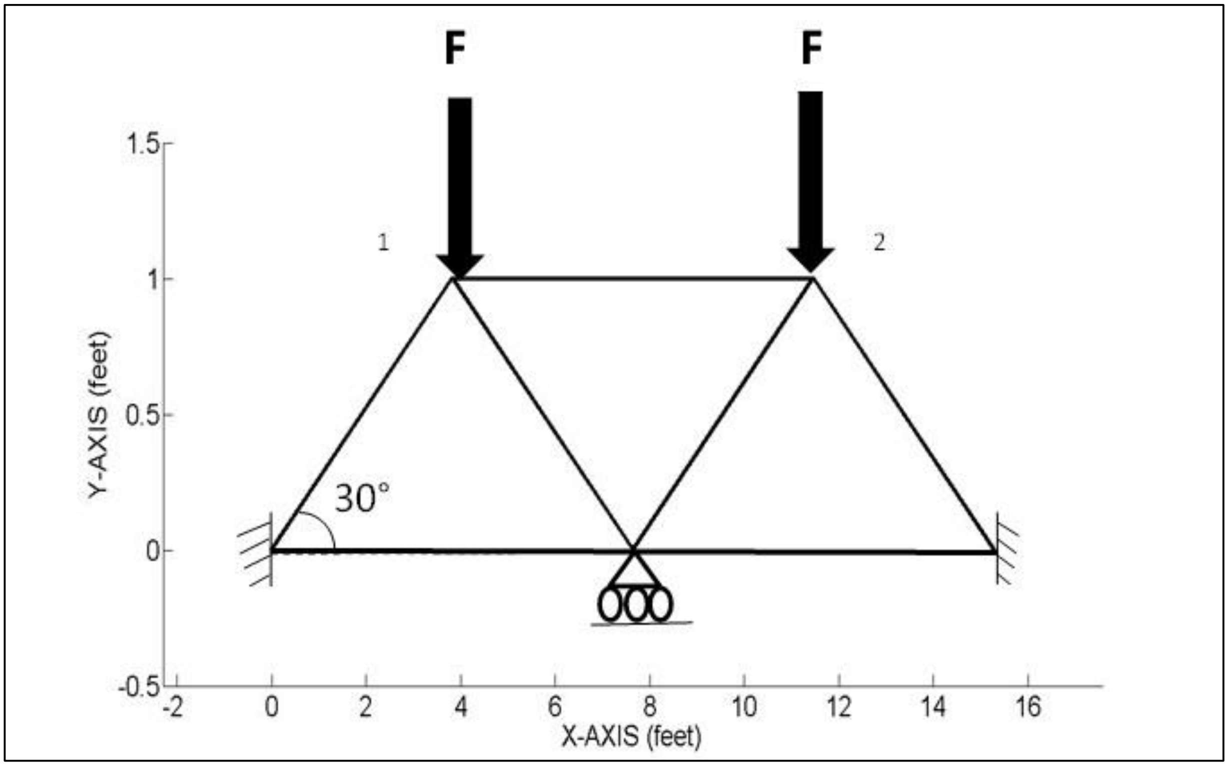

Figure4. 12 7-bar truss with uniform loading

The forces act on the vertices of the structure and are equal in magnitude with a value of 4000 lbs. For this criteria, the load deformation curve is as shown below.

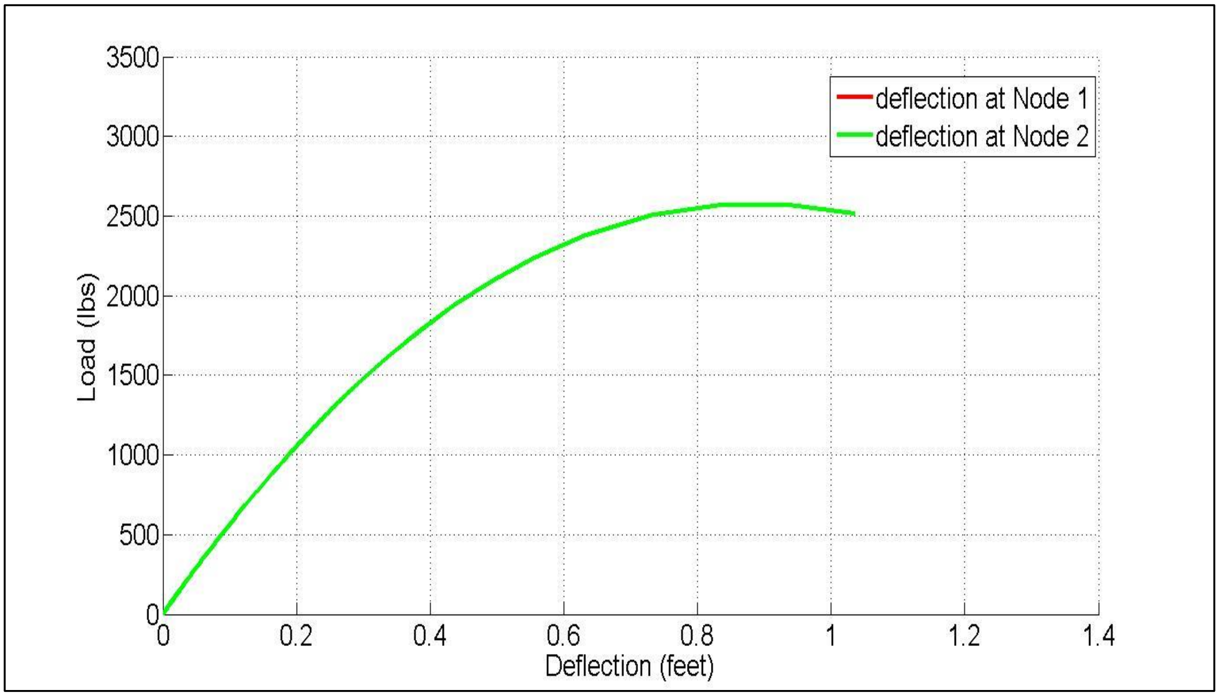

Figure4. 13 Load deformation curve for a 7-bar truss with uniform loading

Since the magnitude of forces acting on the vertices is equal in magnitude, the load deflection curves as in the previous case overlap and there is no evident disturbance in symmetry. Since the area of cross section is reduced, the buckling load will be reduced as compared to the previous case which is expected. The magnitude of buckling load is 2600 lbs. The deformed geometry is expected to behave the same as the previous case. If now the forces on the vertices are altered by a minute amount as shown below.

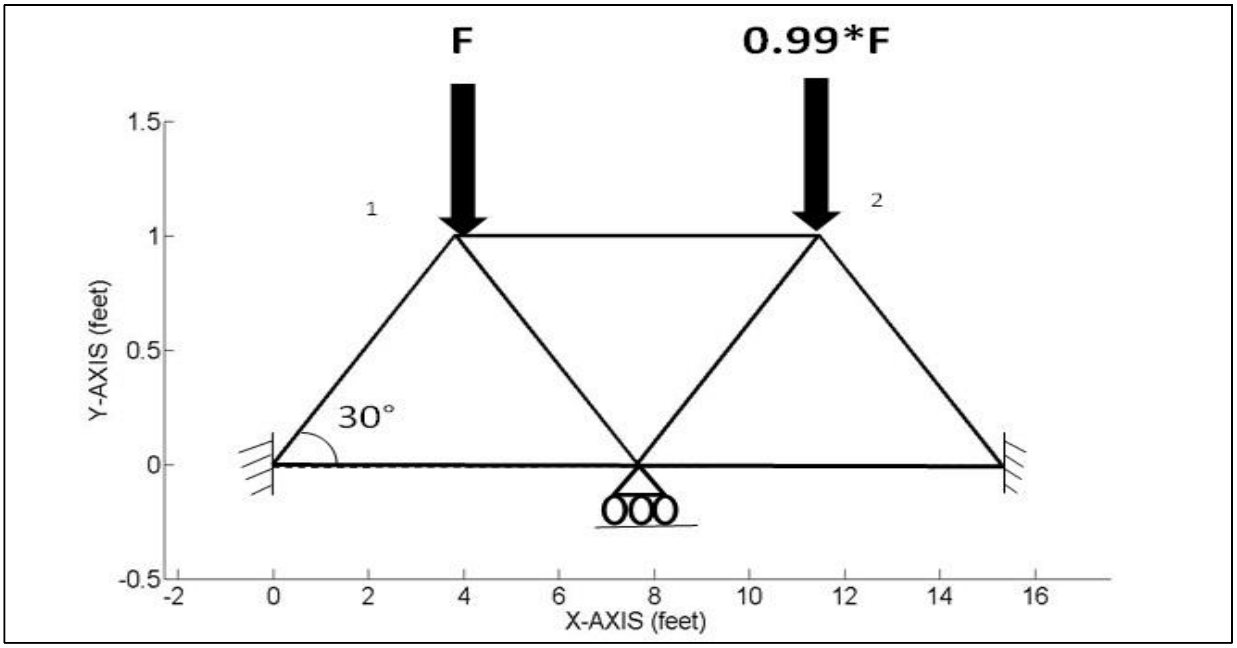

Figure4. 14 7-bar truss with non-uniform loading

When the load on the 2nd node is altered by 1% and all the other parameters are kept constant like the previous case, the load deformation curve is as shown below.

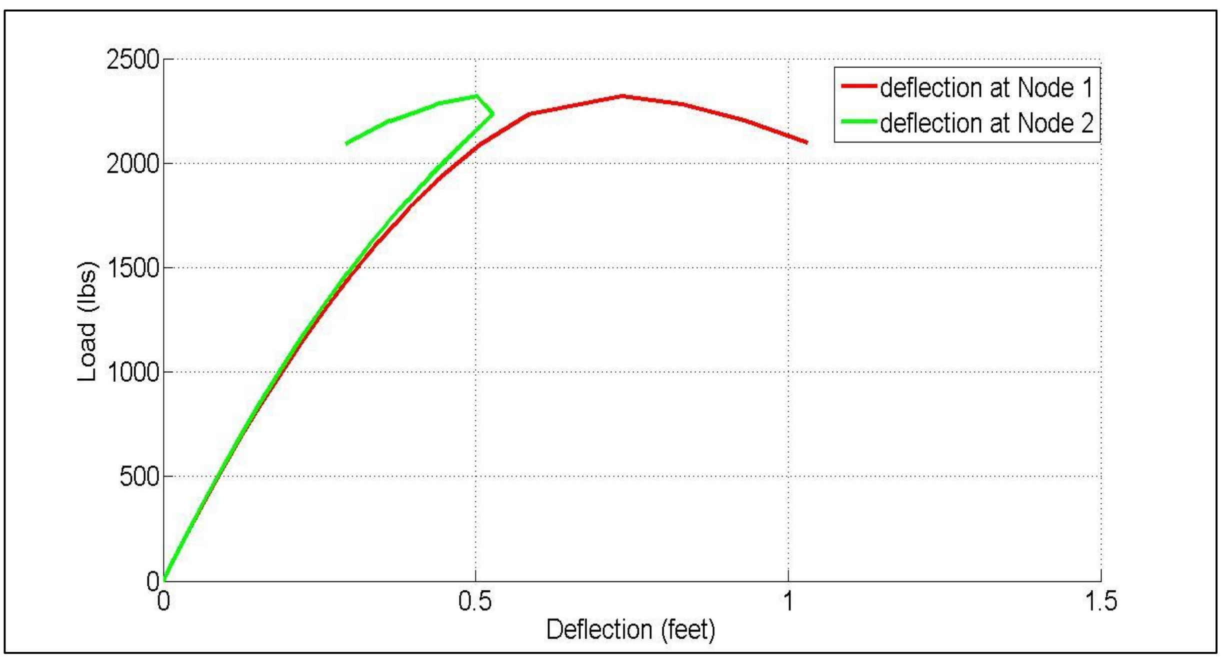

Figure4. 15 Load deformation curve for a 7-bar truss with non-uniform loading

The structure seems to fail at around 2300 lbs. which results in an approximately 12% reduction in buckling load, hence the reduction in area doesn't seem to effect the buckling load but it can't be concluded since other truss structures with different geometrical properties need to be analyzed.

4.2.2 Effect of height of the structure on symmetry breaking

The height of the structure is a crucial aspect of any analysis. The intrinsic properties of the structure will definitely play a crucial role in the buckling and asymmetric response. It is necessary to see how symmetry breaking plays a part in short structures and how quickly does the indifferent forces react according to the height of the structure. We take another structure which has the same material properties shown in table (4.2) except for the height which is 0.5 feet and so the angle of the structure to the horizontal is reduced to half as shown below.

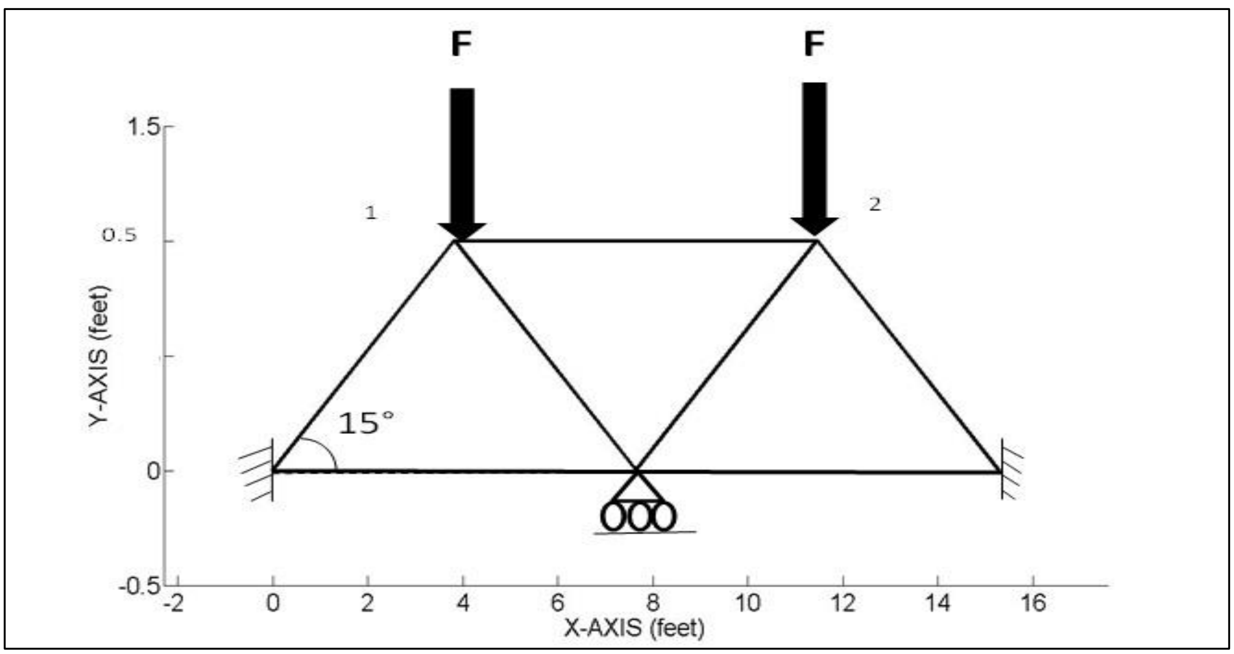

Figure4. 16 7-bar truss with uniform loading

When a load of 1000 lbs. is applied, the load deformation curve obtained is as shown below.

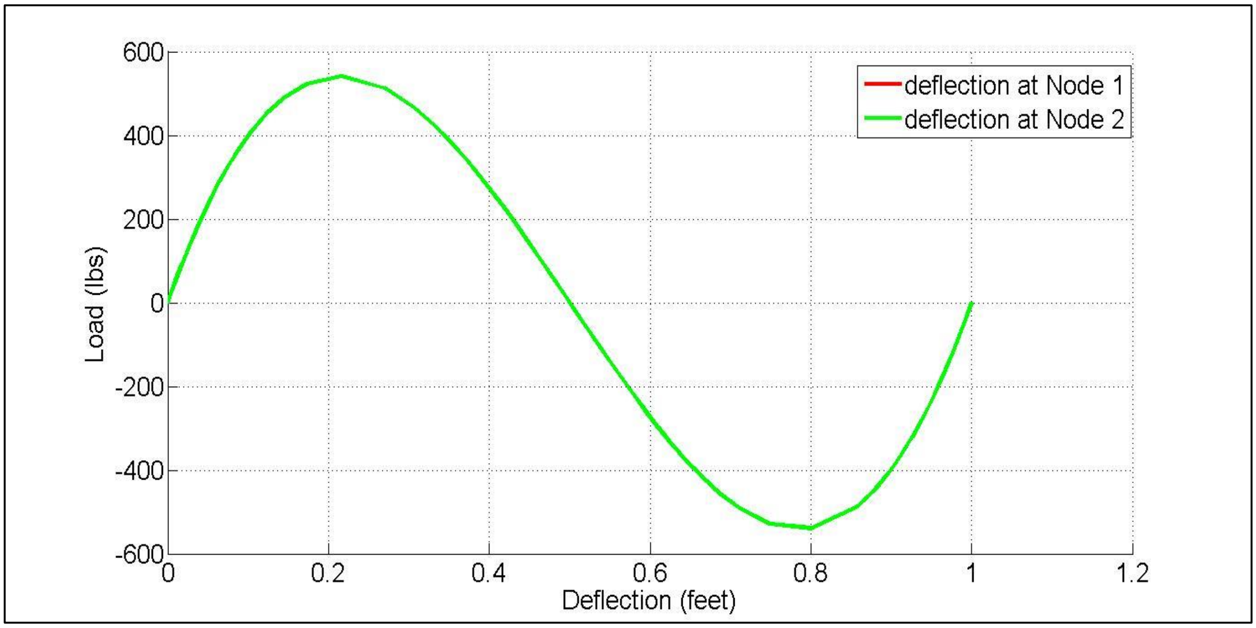

Figure4. 17Load deformation curve for a 7-bar truss with uniform loading

It can be seen that the structure fails at around 580 lbs. but the load deformation curve goes beyond buckling for both the nodes which overlap since the magnitude of forces being applied on the vertices are equal. Under compressive forces which even significantly exceed critical value, additional stresses reach quite large values and directly threaten the structural integrity. Therefore the critical case which immediately precedes rupture is not acceptable in in real life conditions. After the critical point the structure begins to deform with reduced load and this phenomenon can be observed in the deformed configuration as shown below in figure (4.18).

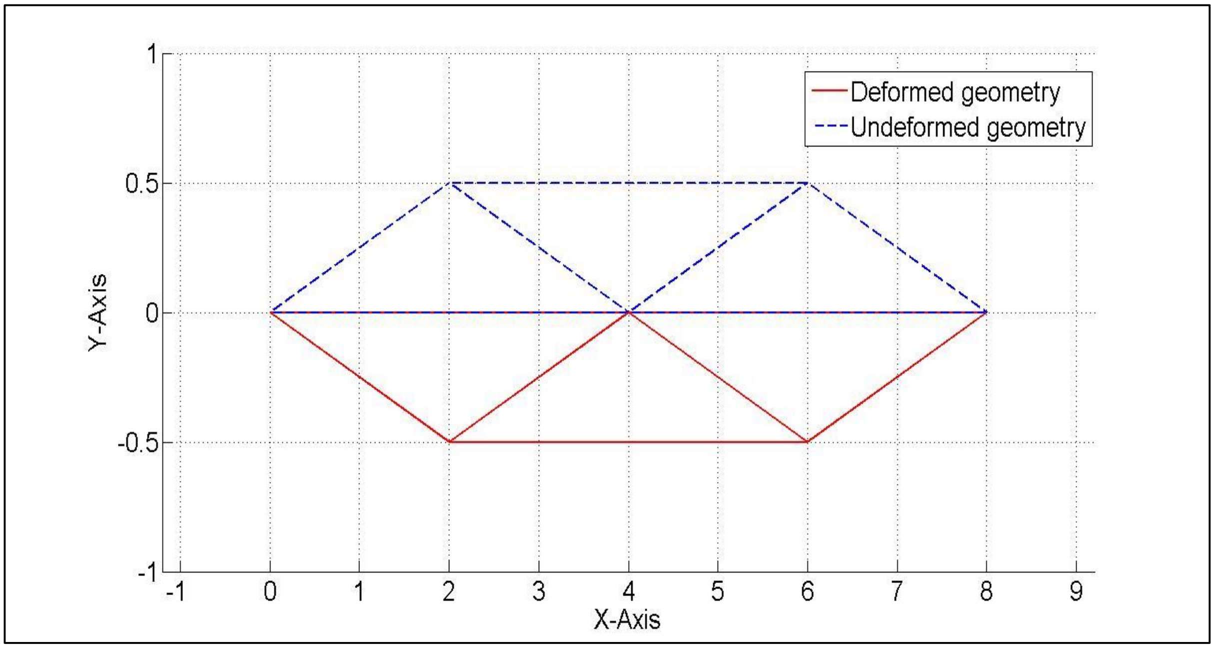

Figure4. 18 Deformed geometry for a 7-bar truss with uniform loading

If the load on the right hand side vertex is reduced by 1% to monitor the effect of asymmetry and its effects in a seven bar structure with geometrical and material properties same as in the previous case, the load deformation curve is as shown below.

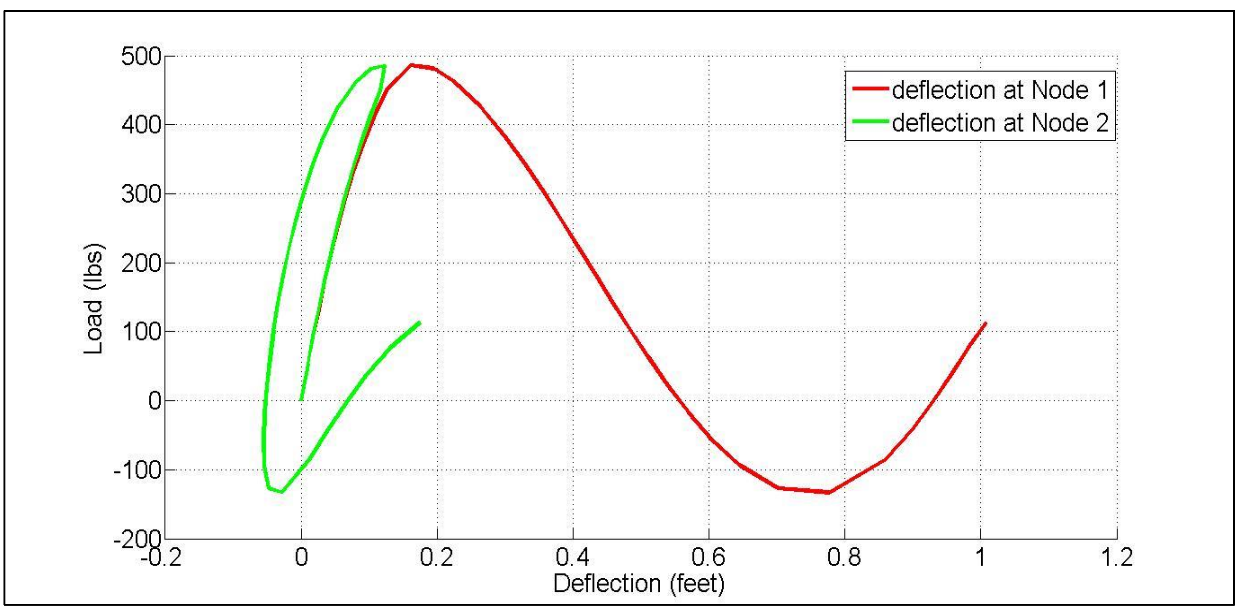

Figure4. 19 Load deformation curve for a 7-bar truss with non-uniform loading

The load deformation curve for the shortened bar shows a more pronounced symmetry breaking with respect to the previous case. The buckling point compared to the previous case is reduced by approximately 10-15 % and hence it can be concluded that buckling point gets affected in structures of the same geometrical properties regardless of the height of the structure. It is hence evident that eccentric loads causes a decrease in buckling load and the post buckling structure becomes unstable. The curves conform from for a certain set of time and then deviates in opposite direction and even during the post buckling process and hence bigger imperfections causes smaller buckling load. The deformed geometry will give us a clearer picture of how exactly the deformation occurs, it is as shown below in figure (4.20)

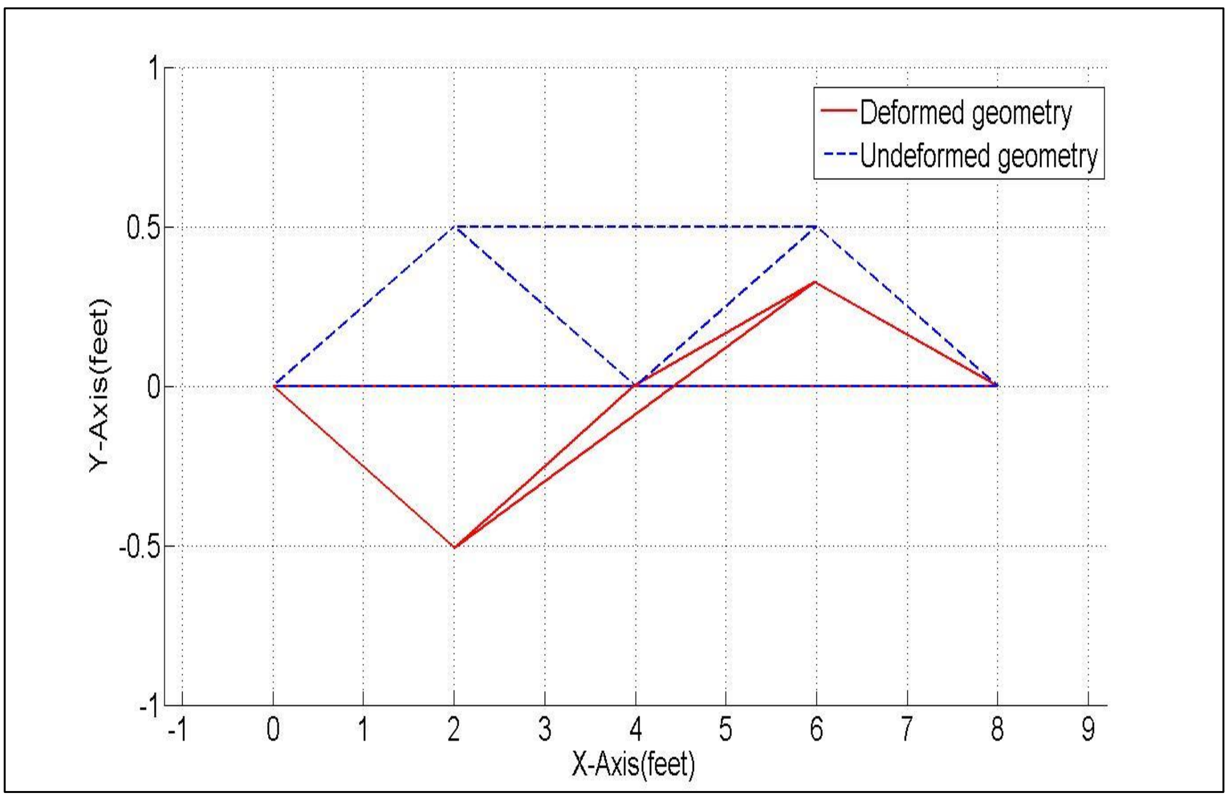

Figure4. 20 Deformed geometry for 7-bar truss with uniform loading

The deformation proceeds symmetrically initially and after the symmetry is broken, the node on the greater force area deforms to a greater extent and after the buckling point is reached the structure deforms with decrease in load. The curves conform from for a certain set of time and then deviates in opposite direction and even during the post buckling process and hence bigger imperfections causes smaller buckling load.

4.3 Symmetry breaking in 5-bar truss structure

5-BAR truss structures are widely used in modern mega structures. They will be provided with some sought of a base but in more complex structures bases mayn't be present. It would be pretty essential to get into the behavior of 5-bar truss structures under various conditions and geometrical properties and their reaction to various set of forces. It would also be a good base point to compare the reaction forces between a 7 bar structure with a base and a 5 bar structure without a base. The effects of baseless structures can be analyzed. We first consider a 5-bar truss structure with a roller in between and hinged at the ends. There are also two equal forces acting on the vertices. The setup is as shown below in figure (4.21)

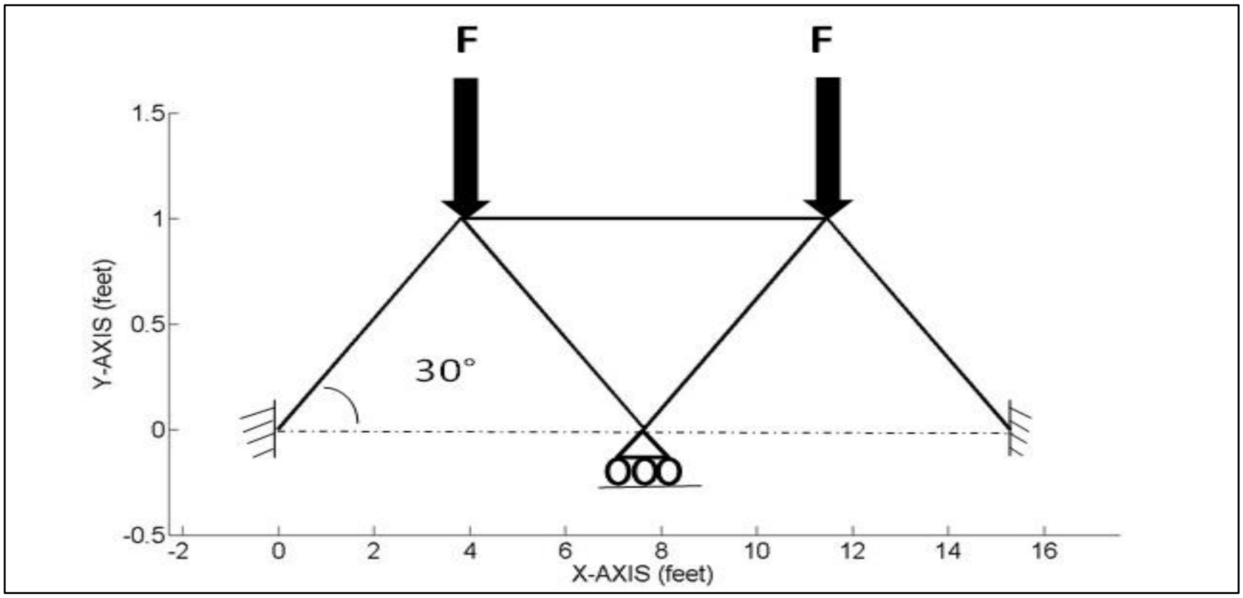

Figure4. 21 5-bar truss with uniform loading

The geometrical properties are as given below

Table4. 3 Geometrical & Material properties

Poisson's ratio	0.33
Mass density	15.28 slug/ft^3 or 8050 kg/m^3
Area of each structure	1.04 e-5 square feet
Young's modulus	200 GPa (4.17e9 lbs./ft^2)
Height of the structure	1 feet

The angle of inclination of a truss element to the horizontal is 30 degrees. The load deformation curve when a load of 2000 lbs. is acting on the vertices is as shown below in figure (4.22)

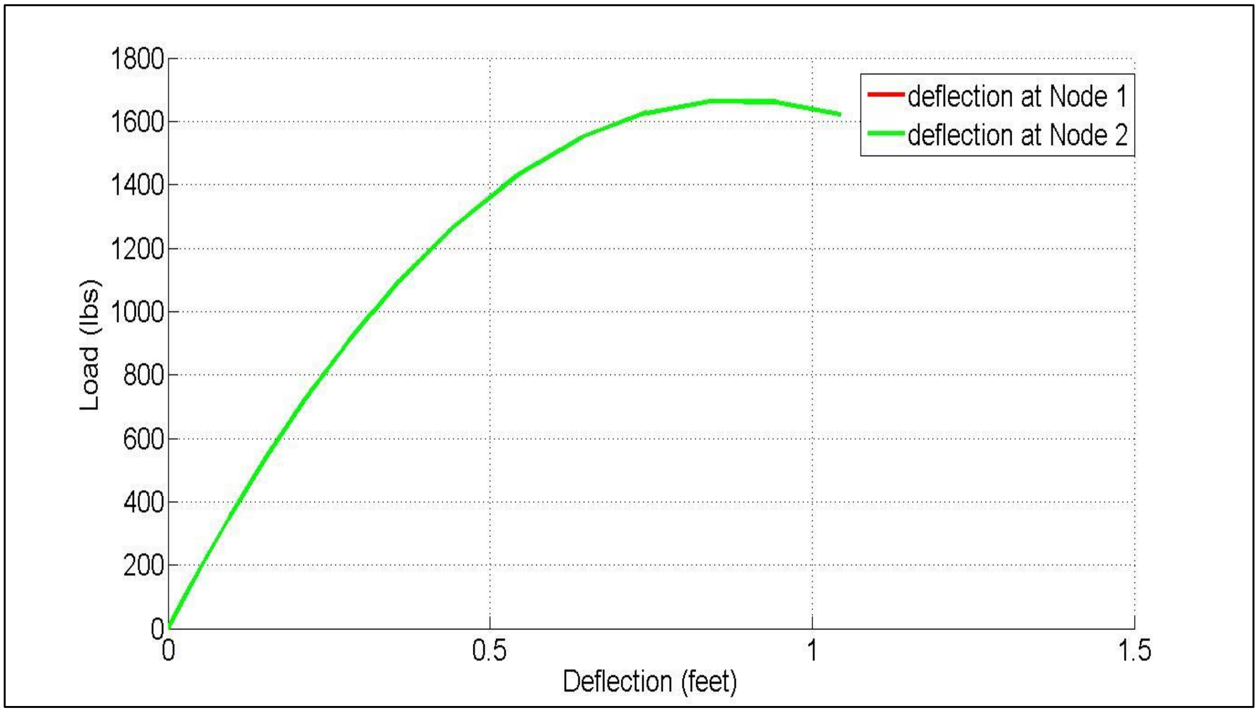

Figure4. 22 Load deformation curve for 5-bar truss with uniform loading

The curve shows a nonlinear behavior with a buckling of 3400 pound force. The deformation curve shows a very generic behavior. In terms of the structural deformation this means that there is gradual deformation along the Y-axis. The magnitude of the deformation can be seen in the figure (along the x-axis), to support this the deformed configuration curve can be analyzed which replicates the deformed geometry according to the coordinates displaced. It should also be noted that the magnitude of displacement can vary according to the type of material used, structural properties, geometric as well as the natural boundary conditions.

The 5-bar structures without a base would usually convey a part of the exerted load to the ground or the presumed base. Although a difference in the reduction the buckling load can be seen for negligence of the base as compared to the previous case. The deformed geometry is as shown below in figure (4.23).

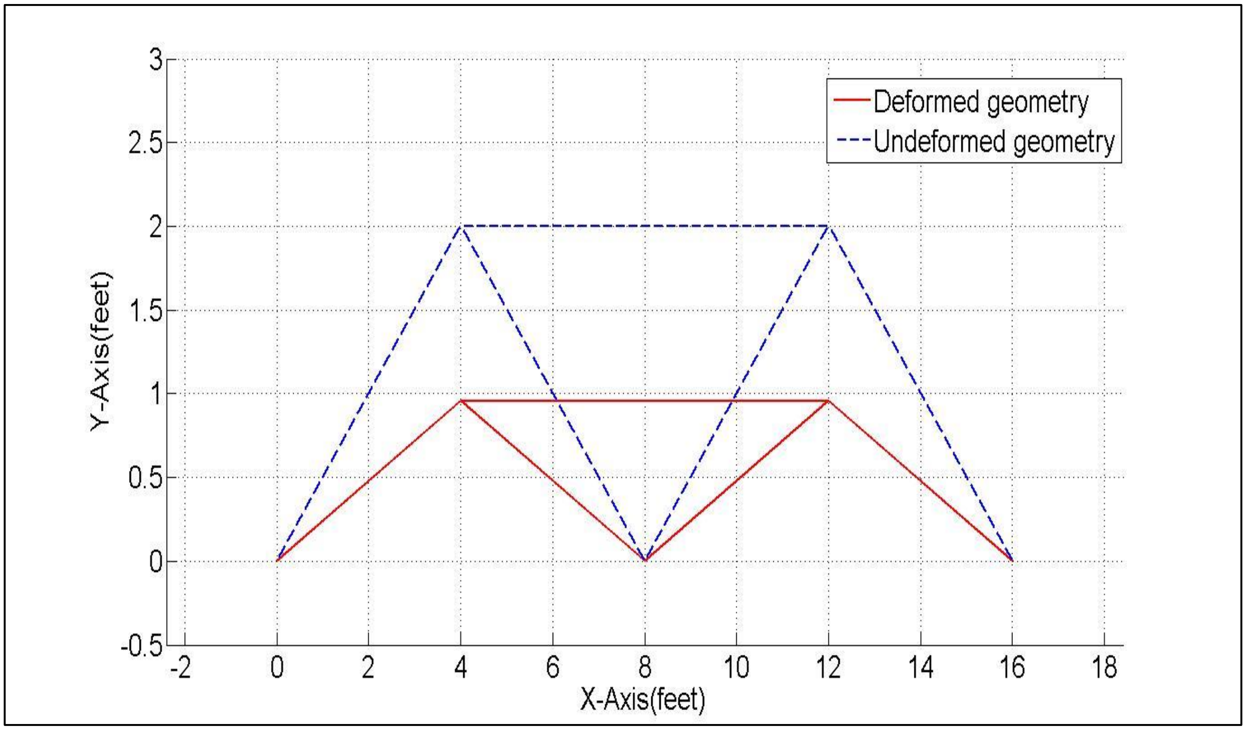

Figure4. 23 Deformed geometry for a 5-bar truss with uniform loading

Even though the base is absent, the symmetrical forces acting allows the deformed geometry to maintain symmetry as shown in the figure above. Hence asymmetrical forces acting would result in intriguing results of the above setup.

If the same setup as above is taken into consideration but with the forces acting on the vertices being altered by 1%. The setup is as shown below in figure (4.24) and has material properties same as the previous case as described in table (4.3).

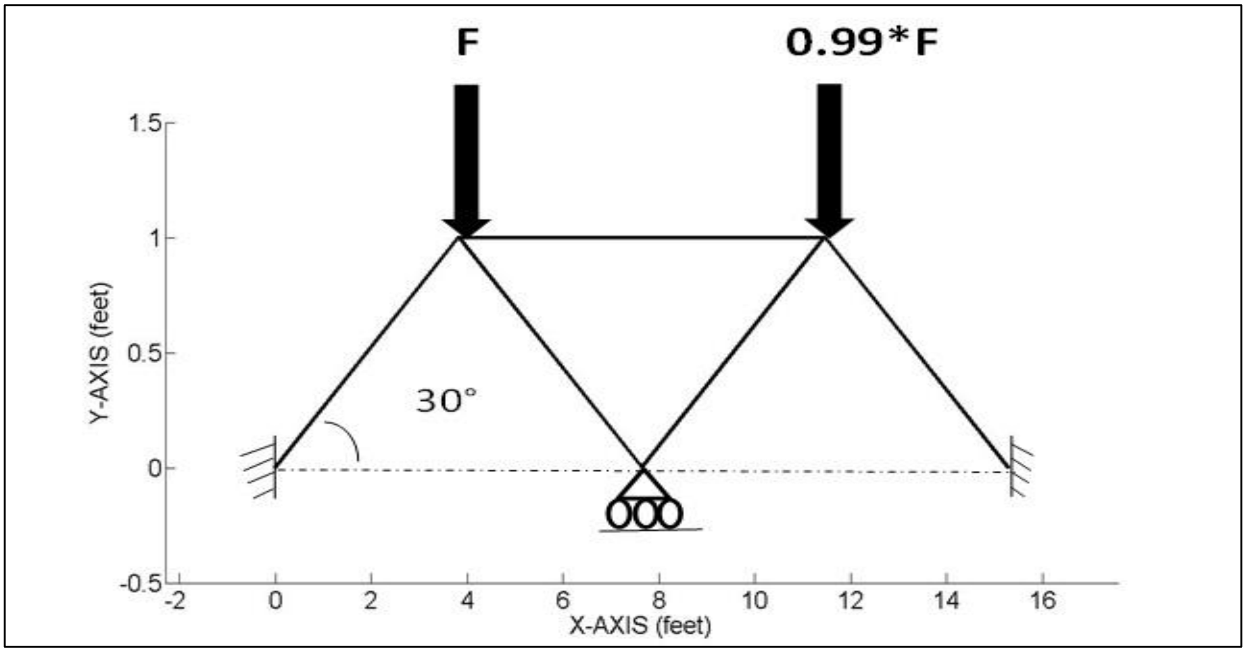

Figure4. 24 5-bar truss with non-uniform loading

The same geometric and material properties are used as in the previous case. The load deformation curve for a constant force applied is given below in figure (4.25)

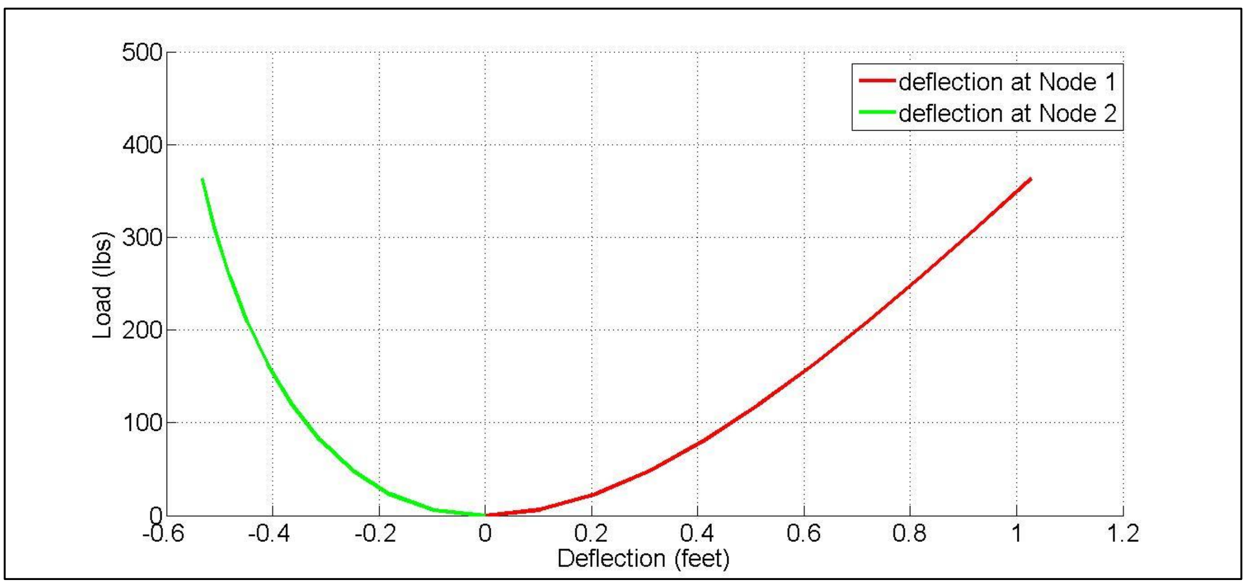

Figure4. 25 Load deformation curve for 5-bar truss with non-uniform loading

It can be observed that there is a significant dip in the buckling load of the structure as compared to the previous case. The reduction is around 70%. The non-uniformity of the forces acting tends to expedite the failure of the structure and this can be a cause of failure of many structural components across the globe. When the compressive axial load enters the elasto plastic region of deformation, the corresponding loss of stiffness causes a significant reduction in critical buckling load as evident in the figure above. Structural anomalies, external forces etc. can have a huge impact on the performance and integrity of the structure. If these are not accounted for, the structure might have premature failure points.

The deformed geometry is as shown below.

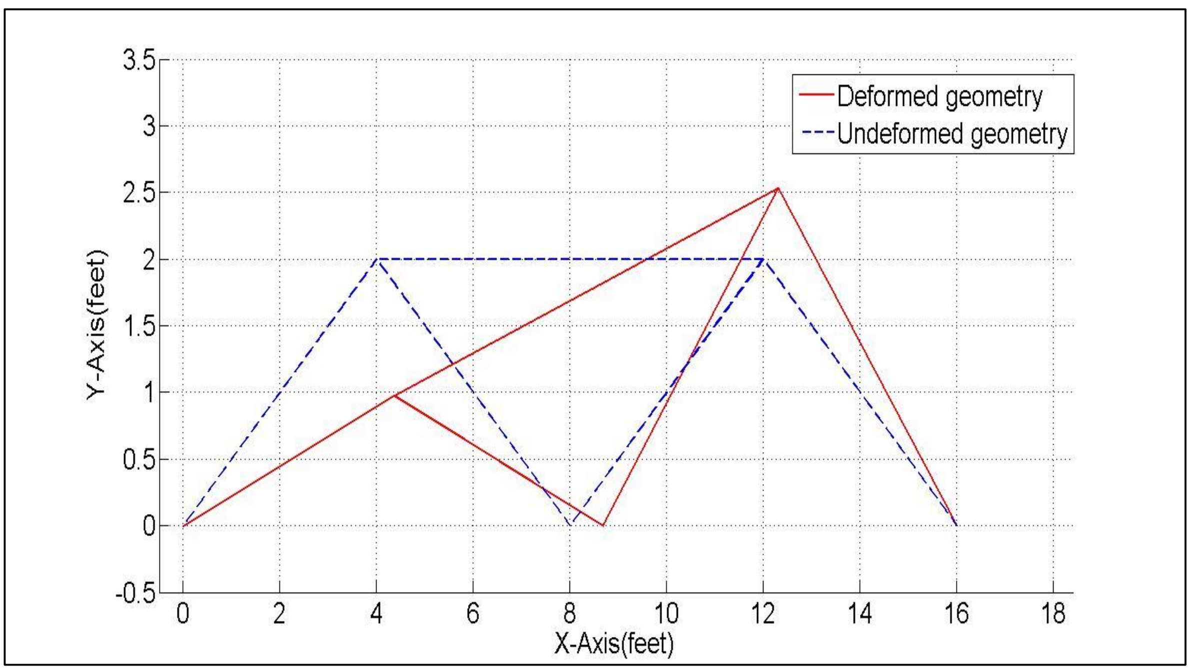

Figure4. 26 Deformed geometry for a 5-bar truss with non-uniform loading

The miniscule difference in force plus the roller presence causes the vertex on the higher force side to deform downwards and this in turn causes the vertex on the lower force side to deform in the opposite direction.

4.3.1 Effect of cross sectional area on symmetry breaking

Though reduction in cross sectional area didn't make a big difference to the reduction in buckling load for a 7-bar structure, it can't be concluded so because it may be specific to the structure and its properties. It is already shown above that the buckling load for a 5-bar structure reduces y around 70% as compared to the 15% of the 6-bar structure. Hence we take an example of 5-bar truss structure with reduced area of cross section as compared to the previous case. The schematic representation is as shown below in figure (4.27)

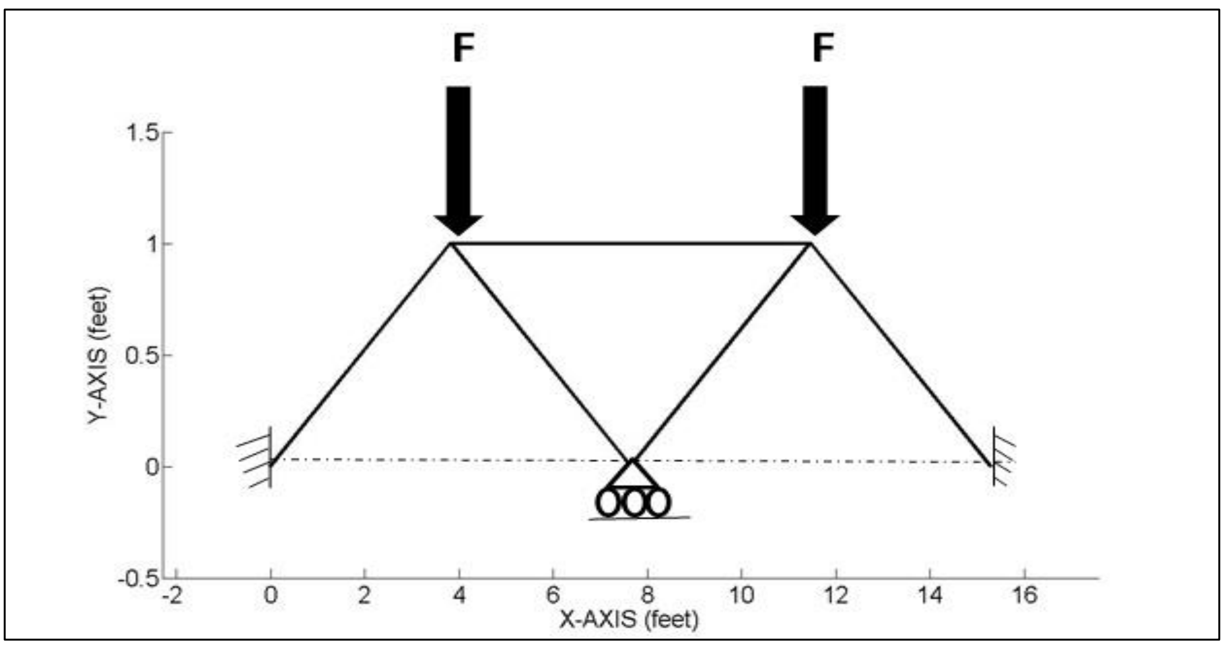

Figure4. 27 5-bar truss with uniform loading

The setup is same like the previous case with roller in the middle and hinged at the ends without a base and the geometric and material properties are same as described in table (4.3) but the area is reduced by 70%. The load deformation curve is as shown below in figure (4.28)

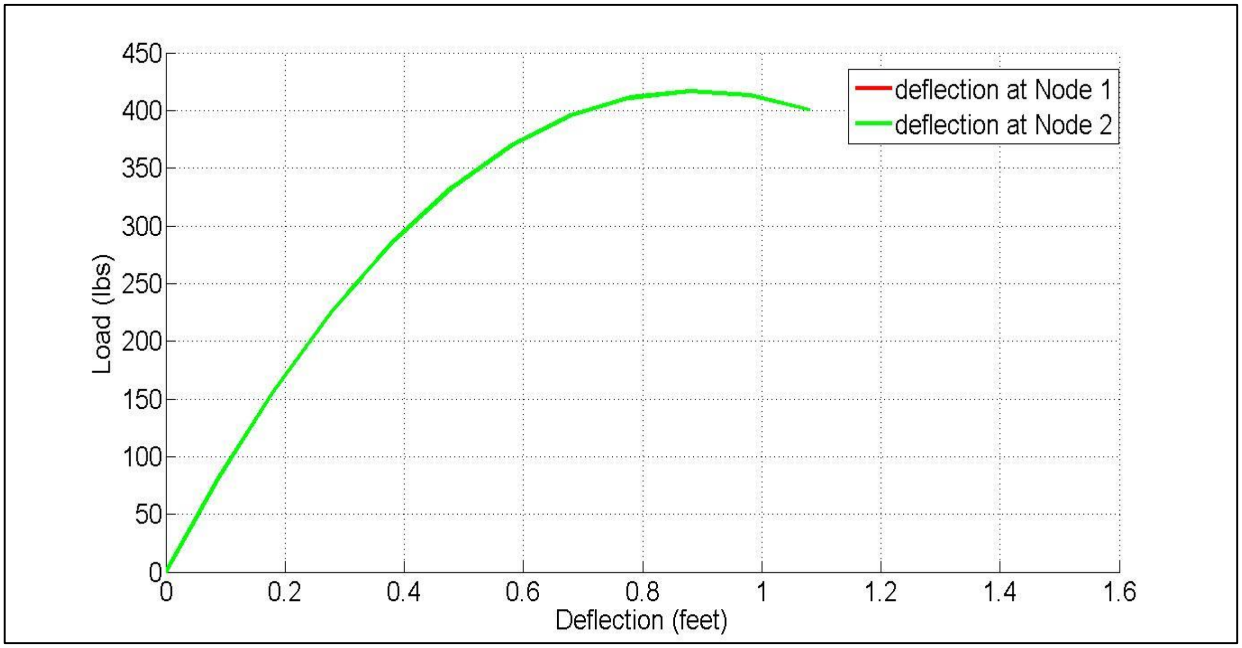

Figure4. 28 Load deformation curve for 5-bar truss with uniform loading

The buckling load is around 430 lbs. And it can be seen that due to the symmetrical forces acting on the vertices, the deformation curve for both the vertex points overlap. The magnitude of the deformation can be seen in the figure (along the x-axis), to support this the deformed configuration curve can be analyzed which replicates the deformed geometry according to the coordinates displaced. It should also be noted that the magnitude of displacement can vary according to the type of material used, structural properties, geometric as well as the natural boundary conditions. The deformed configuration is shown below in figure (4.29).

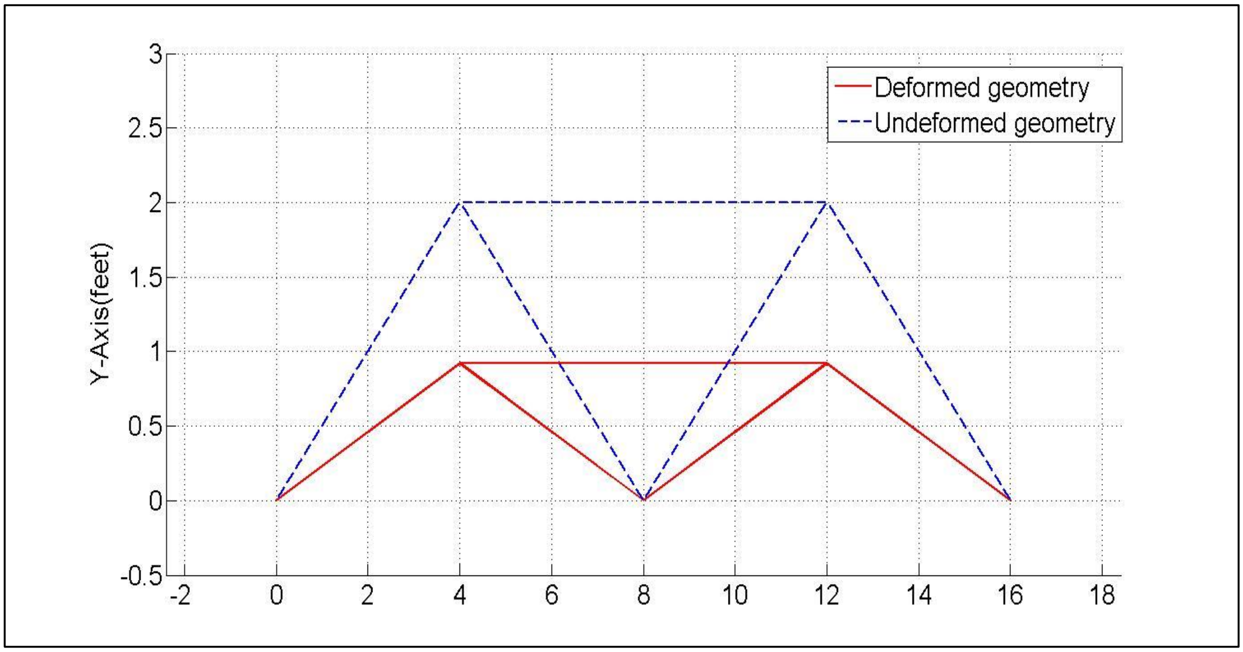

Figure4. 29 Deformed geometry for a 5-bar truss with uniform loading

The deformation in the deformed geometry corresponds with the results in the load deformation curve which means the deformation value in the load deformation graph matches the value of deformation in the deformed geometry. Since the forces are symmetric, the roller in between doesn't move and symmetry is maintained with a buckling load of around 430 lbs.

Now if the same setup as above is taken into consideration with forces at the vertices being altered by 1%, the results would be something to reckon. The material and geometrical properties are again the same as the previous case and indifferent forces are applied at the vertices of the 5 bar truss structure. The load deformation curve for this setup is as shown below in figure (4.31).

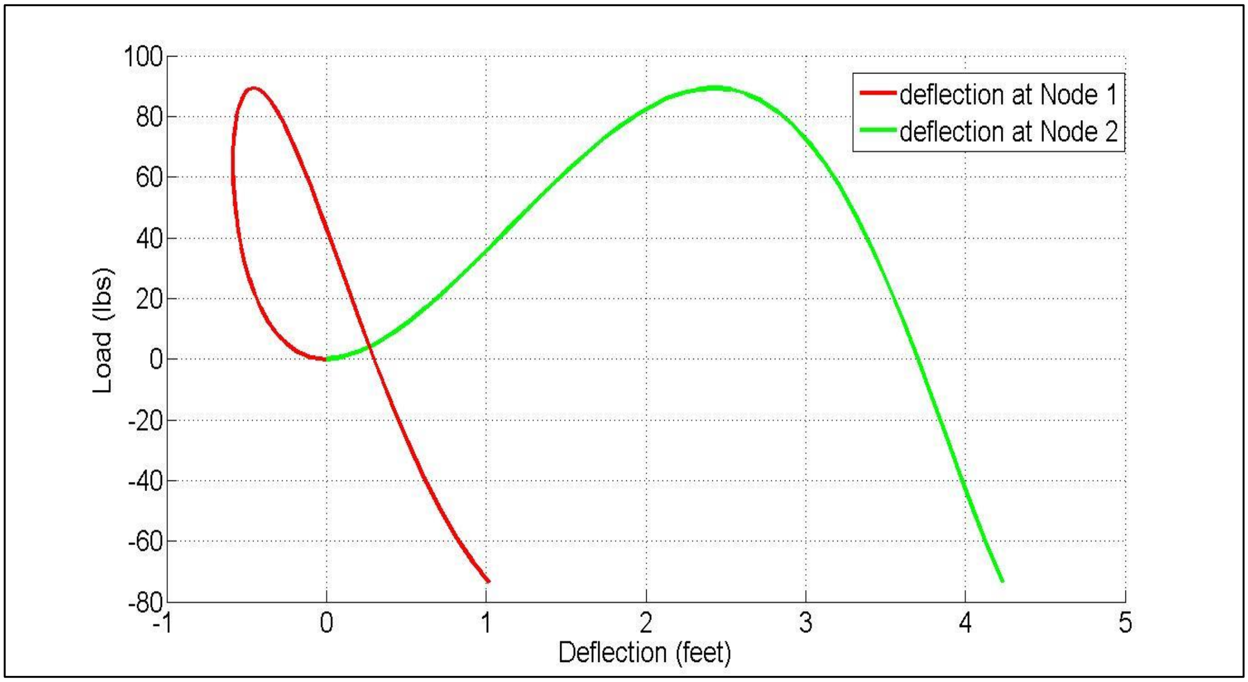

Figure4. 30 Load deformation curve 5-bar truss with non-uniform loading

Even though the curve shows a non-linear behavior there is a weird behavior that the structure executes which can be seen in the graph above. The structure tends to fail at around 90 lbs. So the buckling load for this structure is reduced by around 80% which is a significant margin. The deformation of the two vertices is in the opposite direction unlike the previous case. It can be observed that there is a large difference in the buckling point of a 5 bar structure as compared to a 7 bar structure and since the critical load is calculated relative to the base of the structure, the base becomes a very influencing factor in symmetry breaking. More information on the exact deformation analysis could be understood by studying the deformed geometry curve as shown below in figure (4.32)

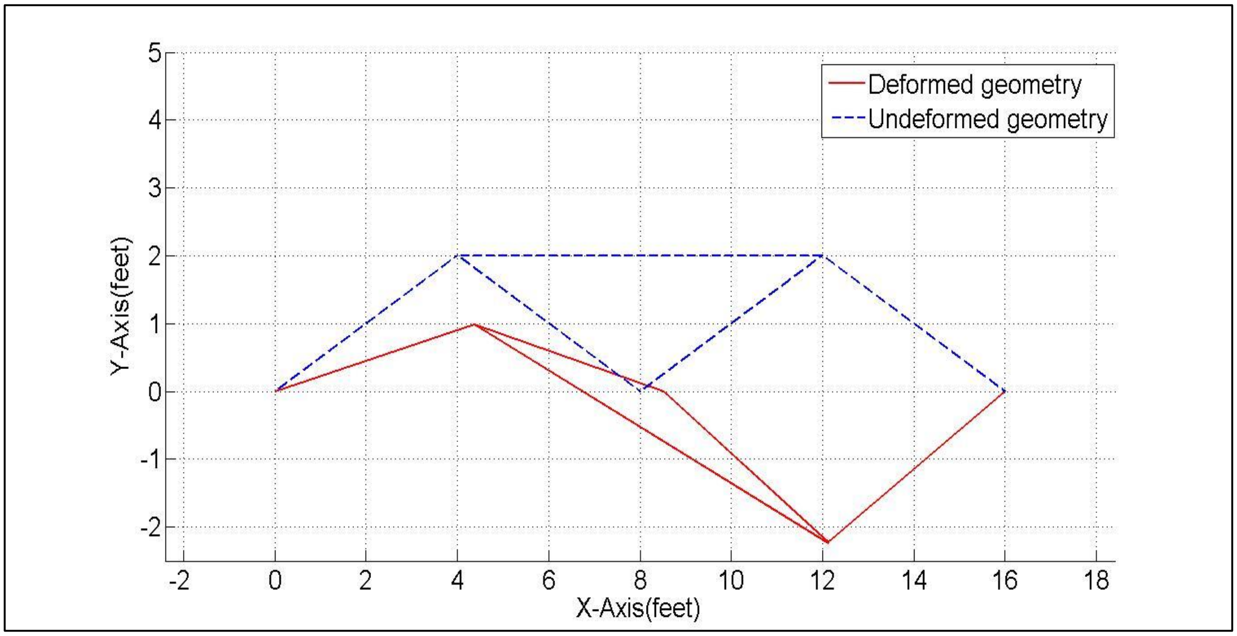

Figure4. 31 Deformed geometry for 5-bar truss with non-uniform loading

The miniscule difference in force plus the roller presence causes the vertex on the higher force side to deform downwards and this in turn causes the vertex on the lower force side to deform in the opposite direction.

4.3.2 Effect of height of a structure on a symmetry breaking

Considering all the parameters to be the same and reducing the height of the truss bar from the previous case, the phenomena of symmetry breaking can be analyzed for a 5 bar truss structure. The setup is as shown below in figure (4.33).

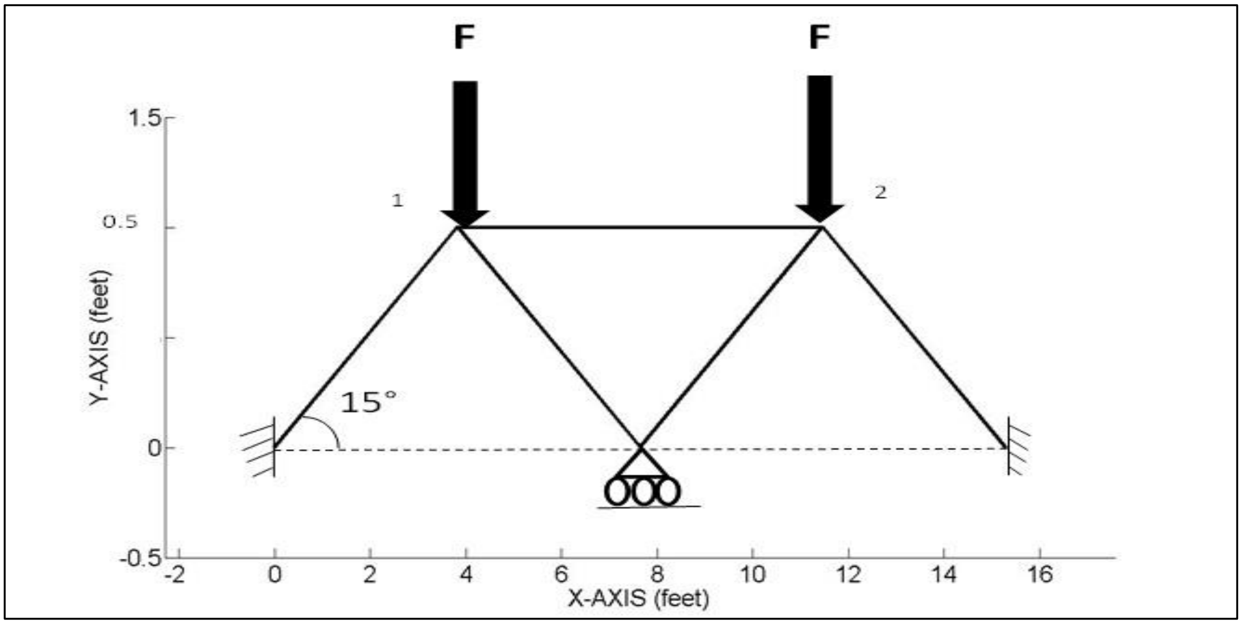

Figure4. 32 5-bar truss with uniform loading

The load deformation curve obtained is as shown below in figure (4.34).

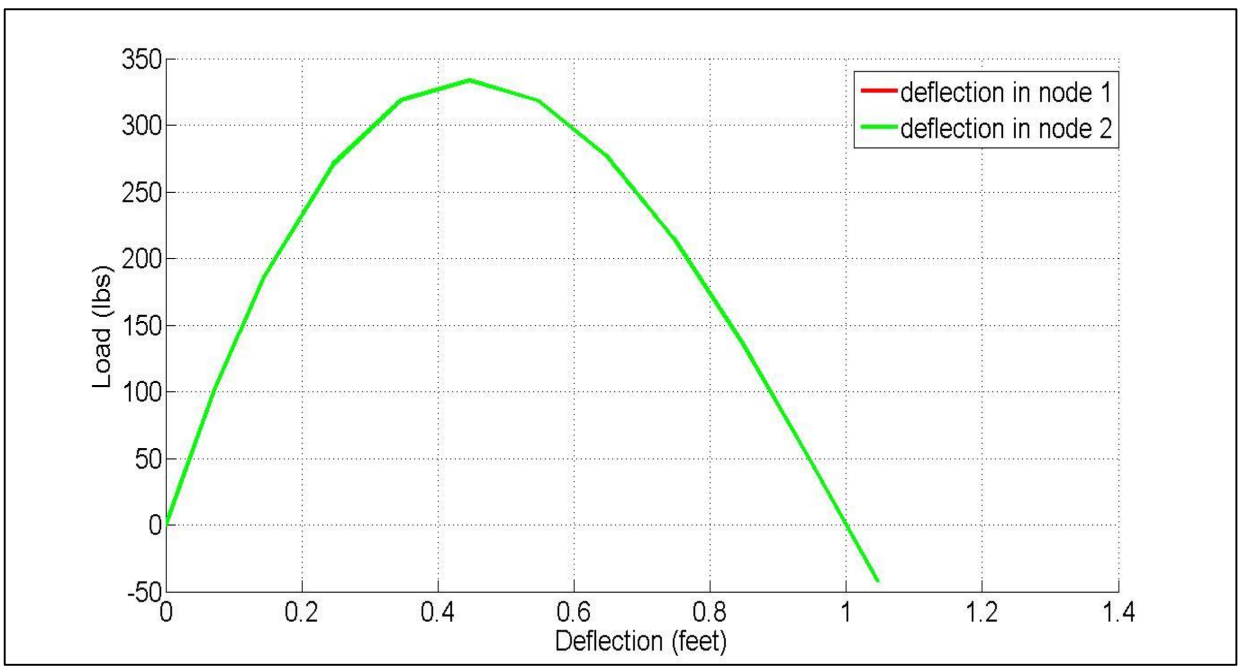

Figure4. 33 Load deformation curve for a 5-bar truss with uniform loading

From the Load deflection curve, it can be infered that the structure fails at 330 lbs. and there is a period of postbuckling. The deformed geometry is as shown below. Under compressing forces which even significantly exceed critical value, additional stresses reach quite large values and directly threaten the structural integrity. Therefore the critical case which immediately precedes rupture is not acceptable in in real life conditions. After the critical point the structure begins to deform with reduced load and this phenomenon can be observed in the deformed configuration as shown below in figure

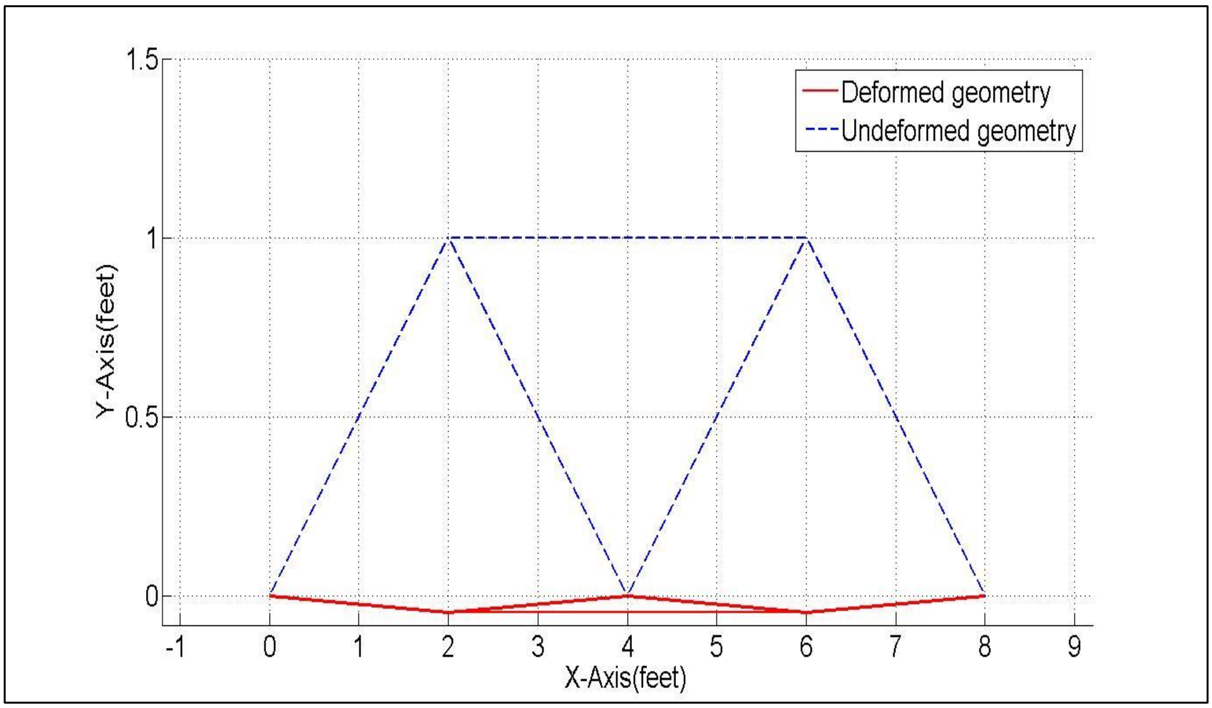

Figure4. 34 Deformed geometry of a 5-bar truss with uniform loading

The postbuckling phenomenon is evident in the deformed configuration. The strucutre deforms till the critical limit and then the decrease in load results in increase in deflection.

Conidering all the parametrs to be the same again, if the forces on the vertices are altered by 1% to monitor the effects of symmetry breaking with geometrical and material properties shown in table 4.3; the load defrmation curve for the above setup is as shown below.

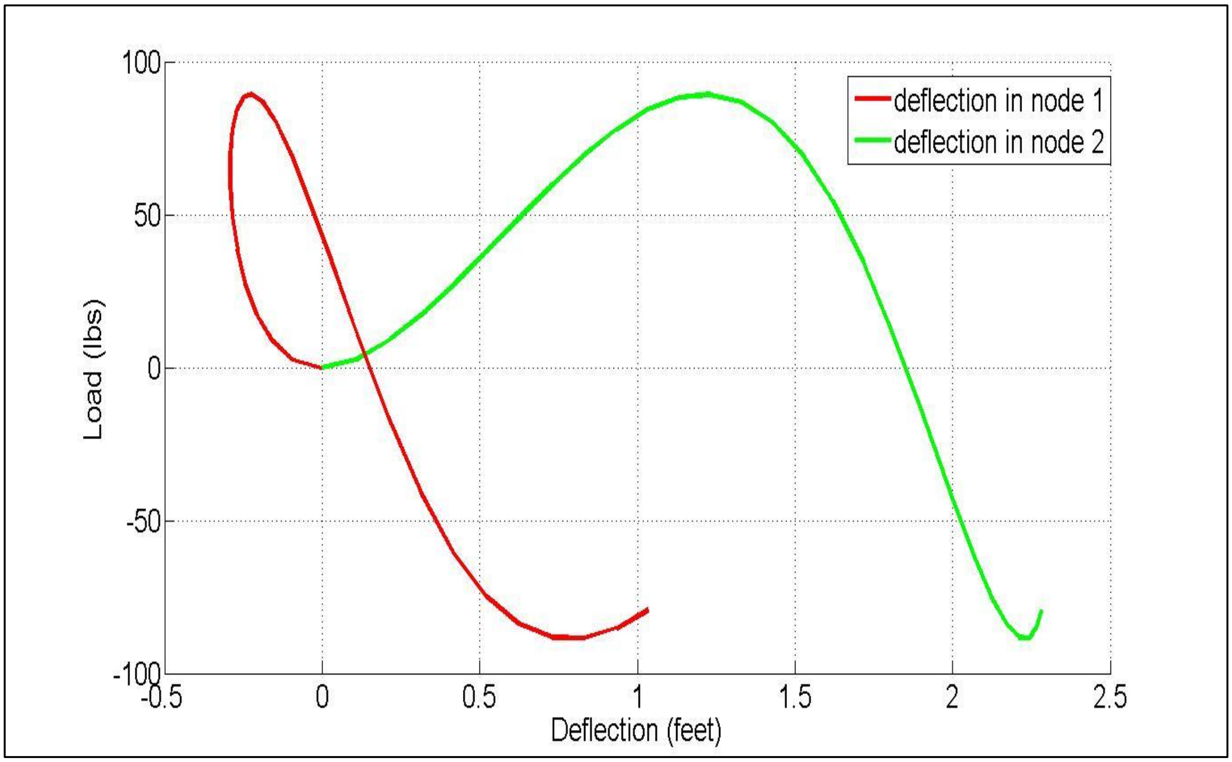

Figure4. 35 Load deformation curve for a 5-bar truss with non-uniform loading

The load deformation curve for the shortened bar shows a more pronounced symmetry breaking with respect to the previous case. The buckling point compared to the previous case is reduced by approximately 70 %. It is hence evident that eccentric loads causes a decrease in buckling load and the post buckling structure becomes unstable. The curves do not conform and deviate right at the origin from for a certain set of time and then deviates in opposite direction and even during the post buckling process and hence bigger imperfections causes smaller buckling load. The deformed geometry will give us a clearer picture of how exactly the deformation occurs, it is as shown below in figure (4.20)

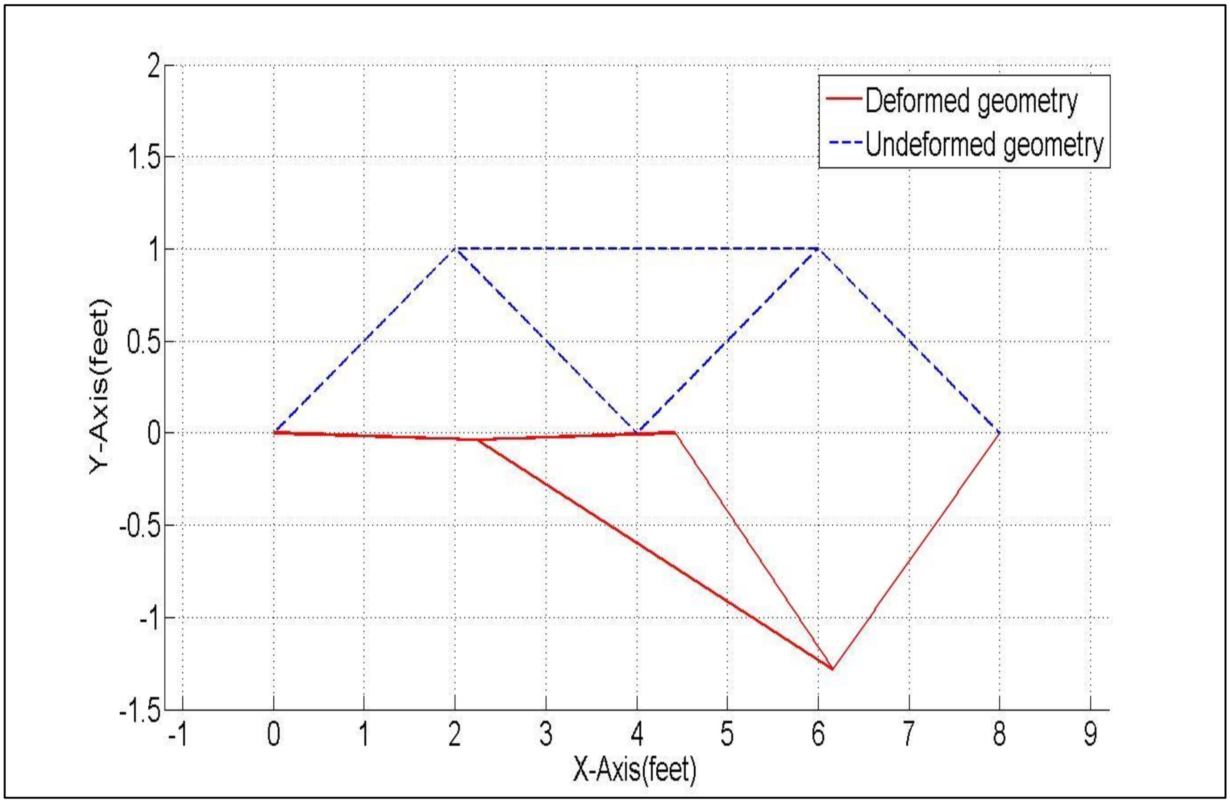

Figure4. 36 Deformed geometry for a 5-bar truss with non-uniform loading

4.4 Bifurcation without conforming and influence of base

It can be seen that the load deformation curve for the asymmetrically loaded 5 bar truss strucutre bifurcates without conforming even once unlike the 7 bar strucutre. The cause may be attributed to lack of base for the support strucutre and it influences a premature failure of the strucutre. This phenomenon creates a drastic reduction in the buckling load of the strucutre from the original value. For the asymmetrically loaded 7-bar truss structure, the reduction in buckling load comapred to to the symmetrically loaded strucutre was around 10% but this increases to around 70% in the case of a 5-bar strucutre. This phenomenon can be further verified by having a half base to the above 5 bar strucutre as shown below.

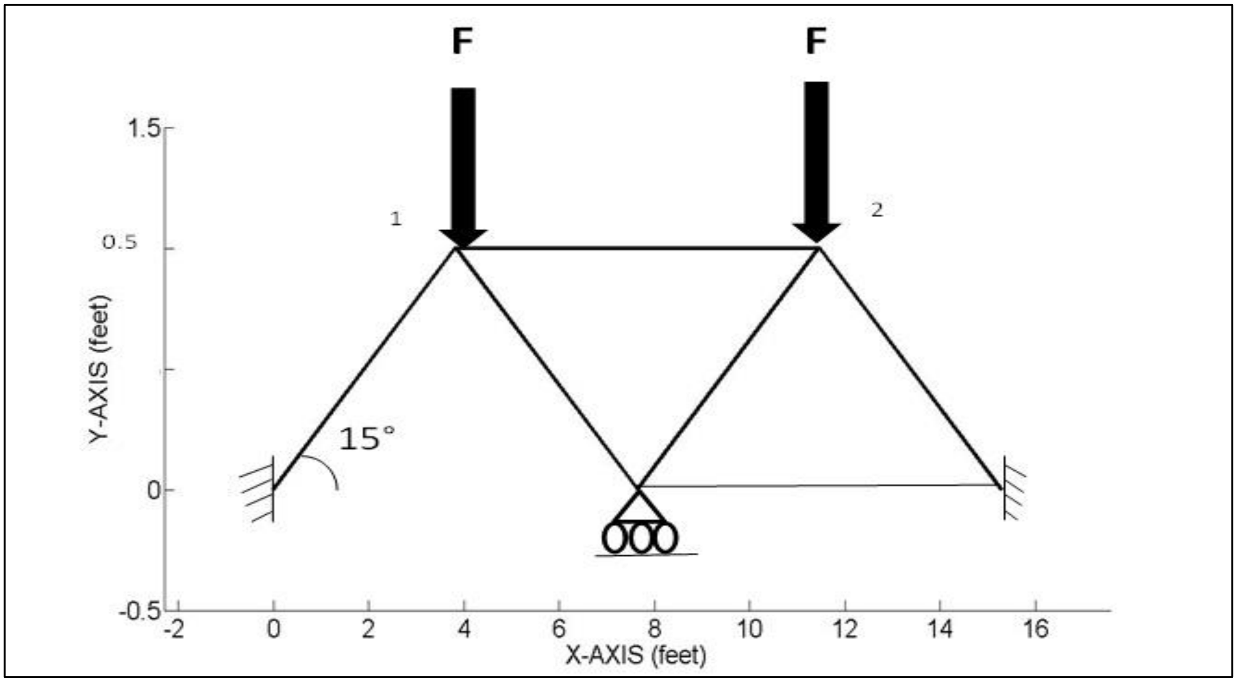

Figure4. 37 Truss structure with half base

The geometric and material properties are as shown below.

Table4. 4 Geometrical & Material properties

Poisson's ratio	0.33
Mass density	15.28 slug/ft^3 or 8050 kg/m^3
Area of each structure	1.38 e-5 square feet
Young's modulus	200 GPa (4.17e9 lbs./ft^2)
Height of the structure	0.5 feet
Number of bars	6

The load deformation curve for the above set up is as shown below.

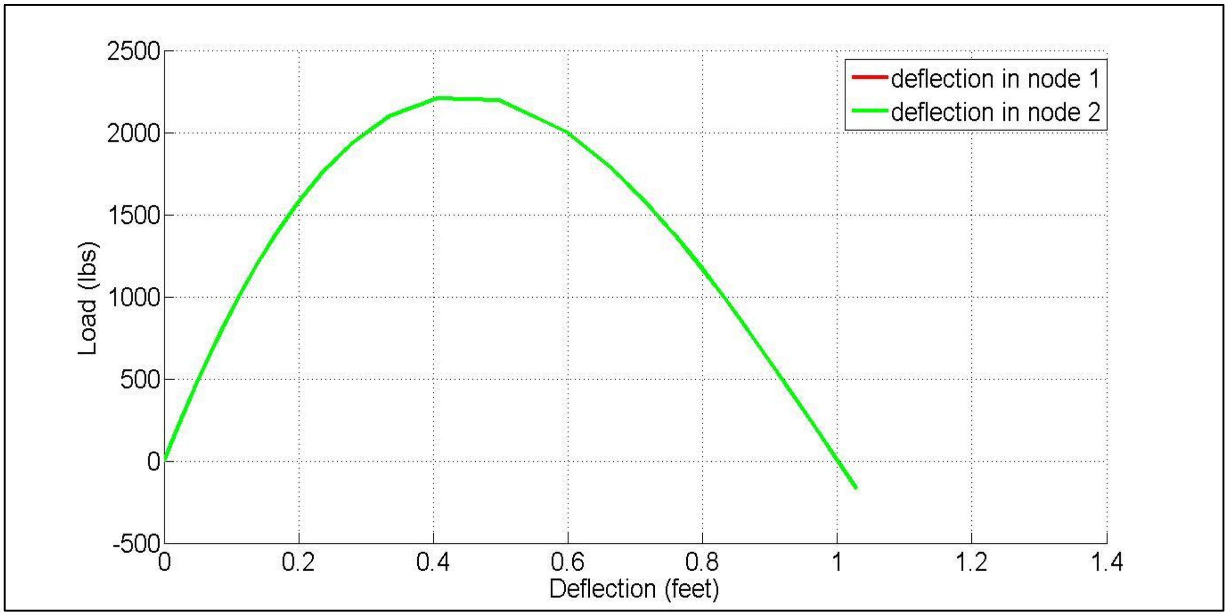

Figure4. 38 Load deformation curve for truss structure with half base

The strucutre seems to have critical load of around 2200 lbs., if now the load on the right hand side is altered by 1% and all the geomterical and material properties are kept the same as the previous case, the load deformation curve obtained is as shown below.

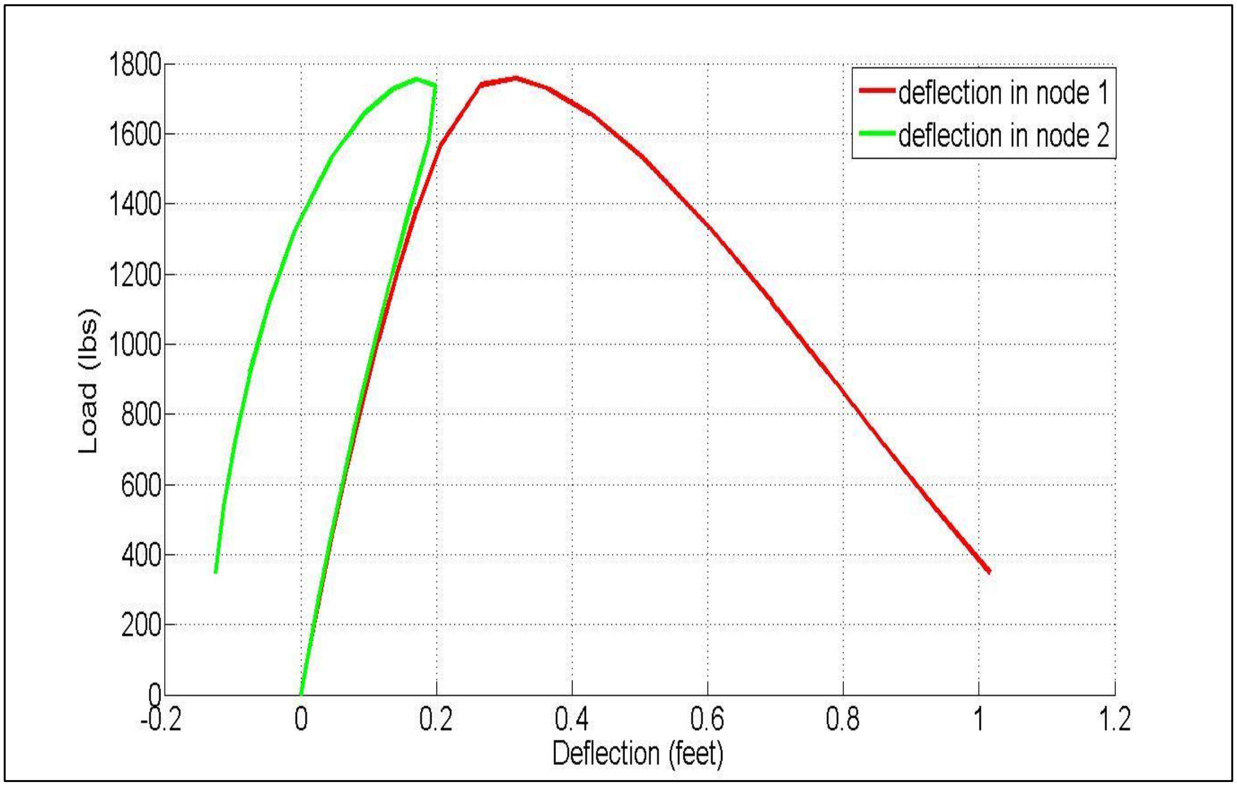

Figure4. 39 Load deformation curve for an asymmetrically loaded truss structure with half base

The buckling load is around 1750 lbs. compared to the 2300 lbs of the previous case which is a 23% reduction in buckling load and hence it can be conculded that the base of the truss strucutre influences buckling load and smaller base or base with a lower cross section results in premature buckling load and proportinaitely leads to non conforming load deformation curve.

4.5 Event investigation

Figure4. 40 Indiana roof collapse tragedy

The above image (found on http://graphics8.nytimes.com/images/2011/08/15/us/15indiana-span/15indiana-span-articleLarge.jpg) is a picture of the Indiana fair tragedy that took place in August 2011. The case investigated by Thornton Tomasetti concluded that heavy winds up to 60 mph caused the barriers to pivot from their original position and this caused the lateral load resisting system to fail. The structure can be broken down and a similar truss model can be assumed as shown below. External factors like winds and temperature variation cause remarkable impacts on the behavior of the structure, and hence the geometrical and material properties play a key role for selecting structures for areas where external forces like winds and temperature variation play a big part. An experimentation analysis can be carried out by taking an example of a simple symmetric truss member which resembles the truss structures used in the assembly of the Indiana stage roof. An assumption a truss model is as shown below in figure (4.41).

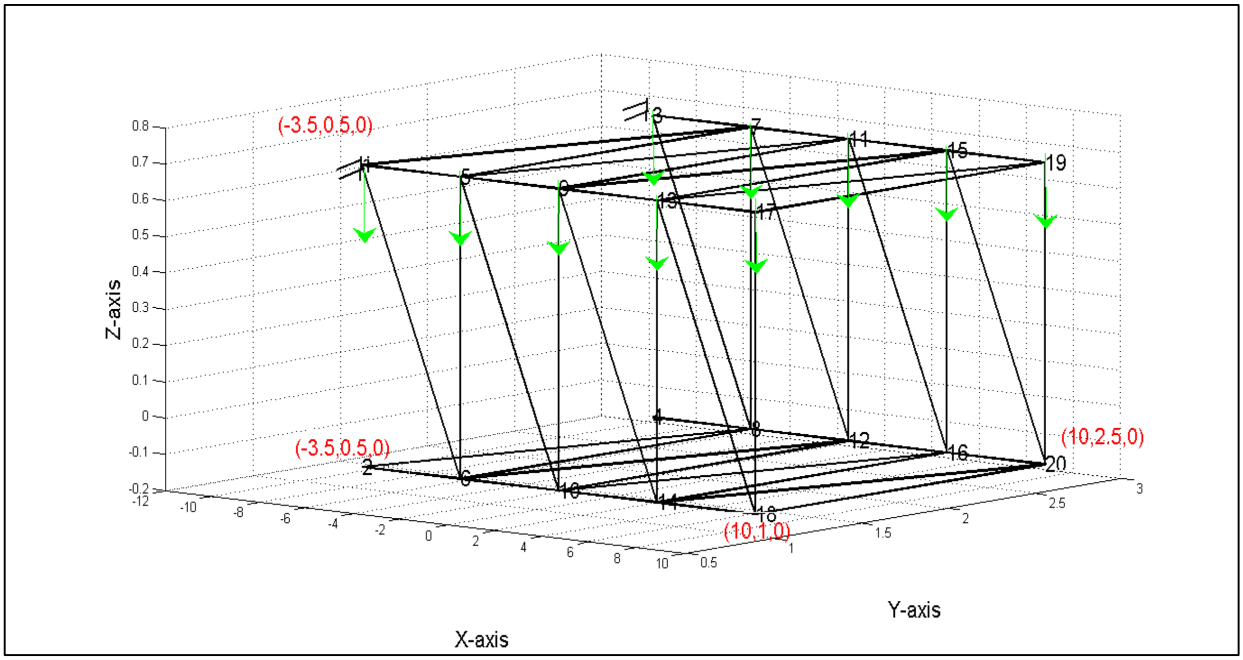

Figure4. 41 Experimental analysis of the event

The 4 bay truss structure with 20 nodes is hinged at the ends and forces of equal magnitude are assumed to be acting on the upper nodes as shown above. The geometrical properties of the above cantilevered truss structure is as shown below.

Table4. 5 Geometrical & Material properties

Poisson's ratio	0.33
Mass density	15.28 slug/ft^3 or 8050 kg/m^3
Area of each structure	2.3 e-3 square feet
Young's modulus	200 GPa (4.17e9 lbs./ft^2)

If now a force equal to 10000 lbs. is applied on the vertices, the truss structure undergoes deformation. The load deflection curve obtained is as shown below.

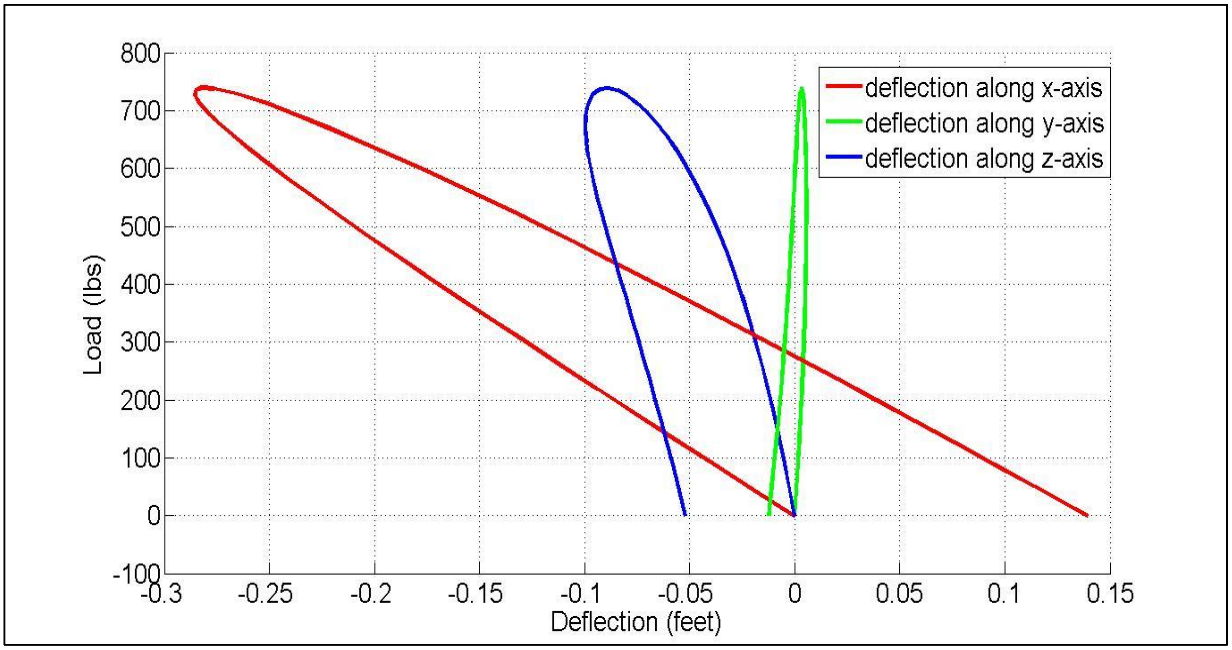

Figure4. 42 Load deformation curve for the event investigation

The above figure shows deformation along all the three axis. It can be observed that the critical load is around 750 lbs. and there is region of post buckling and from here on the structure doesn't respond to any further load. The deformation along the z-axis is the most since it is direction where the force is applied and hence this load deformation curve is consistent with the input load and the deformed geometry configuration is as shown below.

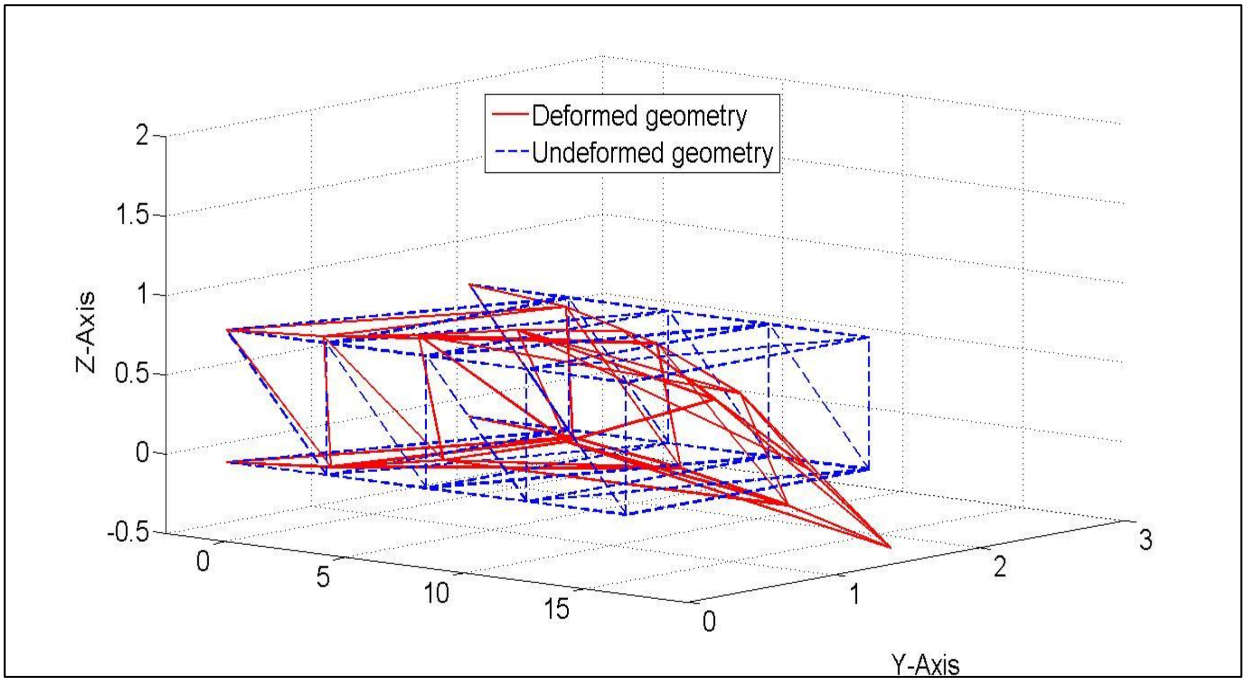

Figure4. 43 Deformed geometry

The forces acting on the vertices deform the truss structure downwards and the structure reaches a failure at around 750 lbs. There is a significant deformation at all the vertices apart from the fixed ends. Now if we assume there was a wind force acting along the x-axis, the scenario changes a little bit. The assumed model is as shown below in the figure (4.44)

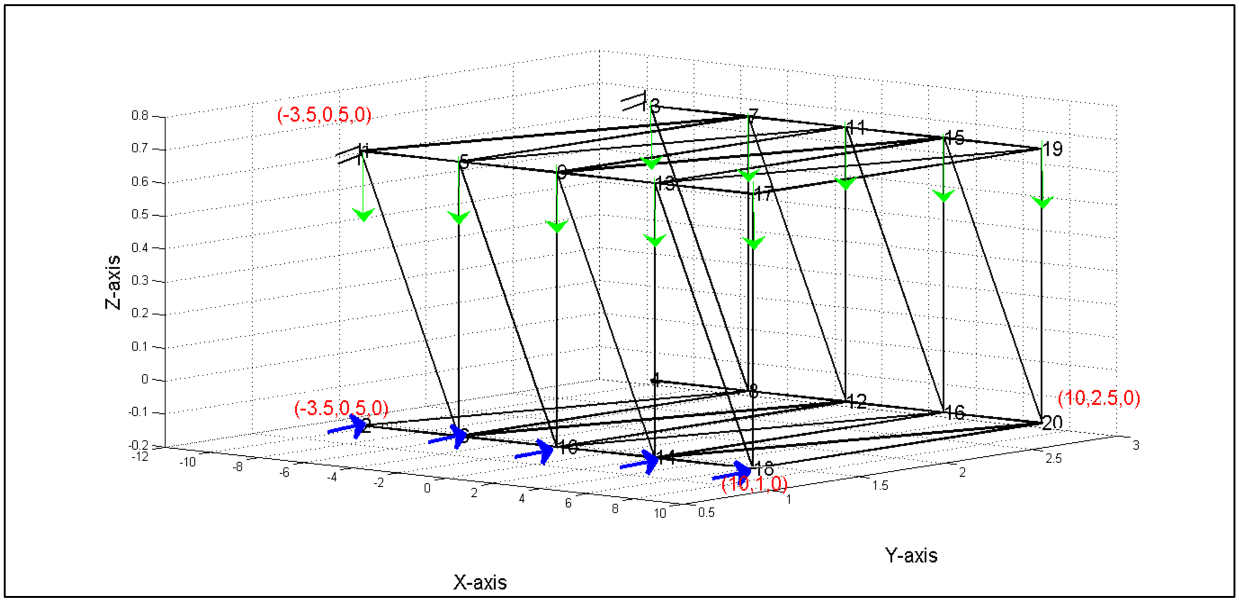

Figure4. 44 Experimental analysis with wind forces

The wind force if assumed to act on the nodes as shown above will create a different impact on the final buckling load. The velocity pressure can be given by:

$$q_z = 0.00256 * K_z * K_{zt} * K_d * V^2 * I \qquad (4.4)$$

Where K_z, K_{zt} and K_d are modifiers which are considered to be equal to unity in our case.

I is the importance factor from 0-1 which signifies the degree to hazard to human life

V is the peak velocity of the wind in mph.

Hence $q_z = 0.00256 * 1 * 1 * 1 * 60^2 * 1 = 9.216$ lbs./ft^2

By using velocity pressure, we can calculate force by the formula:

$$F = q_z * G * C_f * A_f \qquad (4.5)$$

G and C_f are modifiers and can be considered equal to 1 for the sake of simplicity.

A_f is the area exposed to wind loads and can be considered equal to 16 square feet

$F = 9.216 * 16 = \ \ 146$ lbs.

A schematic representation of the load forces of 140 lbs. acting along the horizontal direction is shown below in figure (4.45). Other properties including the material and geometrical properties, vertical forces acting are the same as the previous case.

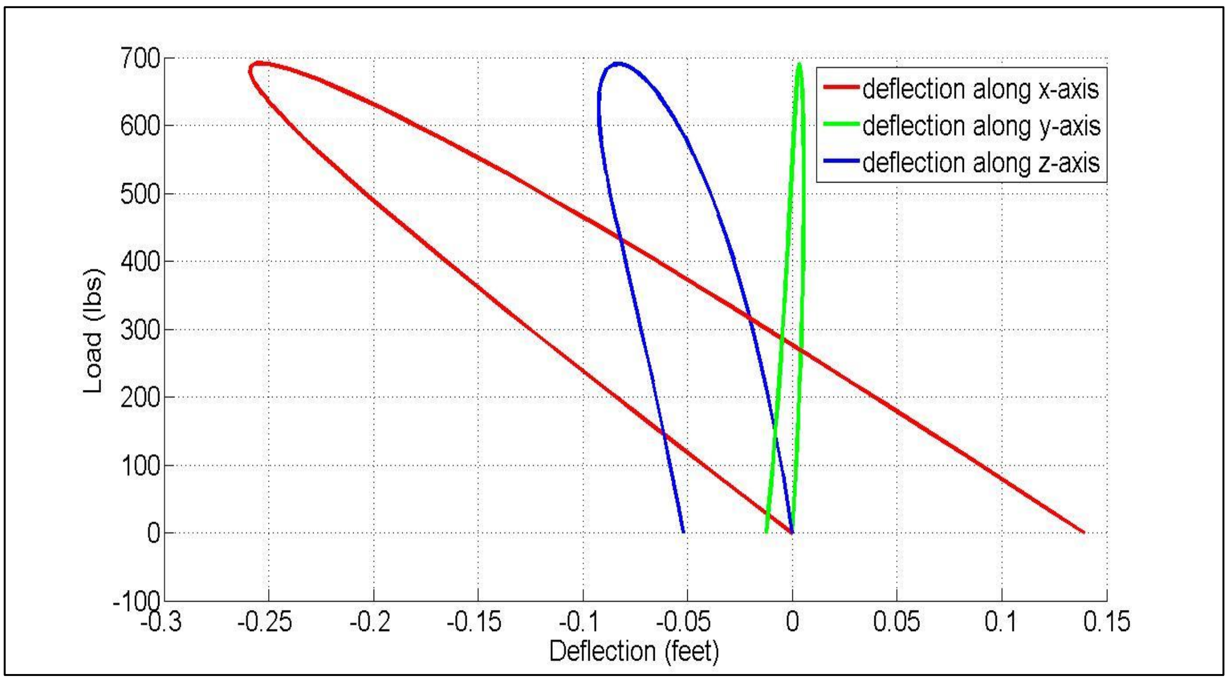

Figure4. 45 Load deformation curve for experimental analysis

The load deformation curve follows a similar pattern like in the previous case but it can be observed that the buckling load of the structure has come down to 690 lbs and that it a considerable 10% decrease in the buckling load as compared to the previous case. If simple modifications in the setup can lead to up to 10% reduction in the buckling load, then the other discrepancy can make a significant difference in the performance of the structure. The other discrepancies may include indifferent area of cross section due to temperature variation and pivot offset due to wind force.

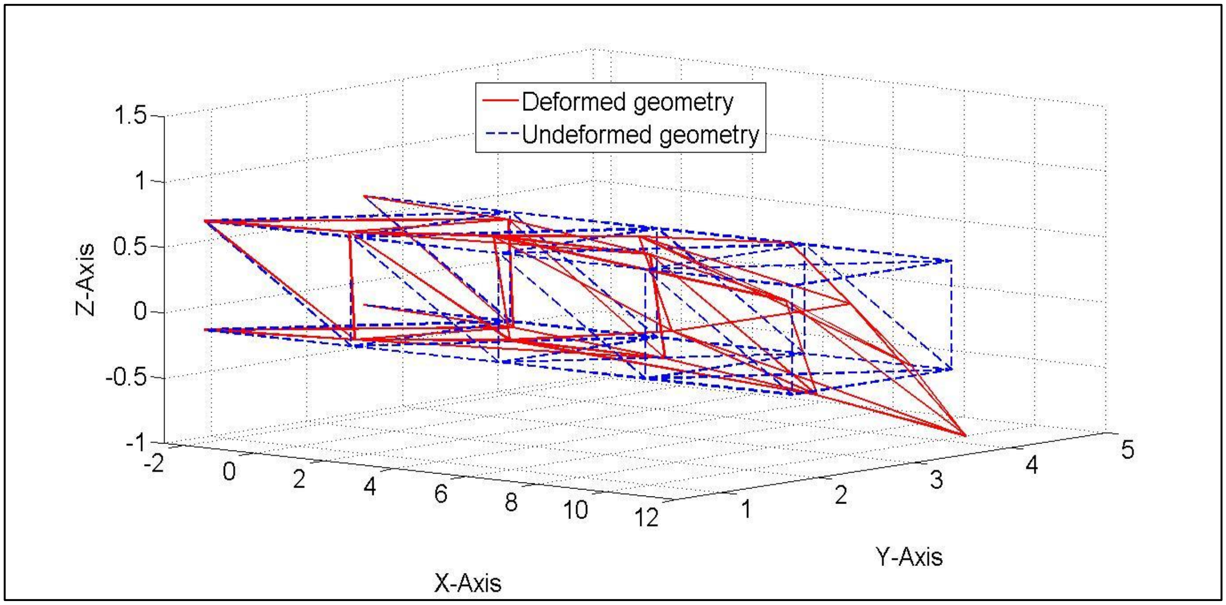

Figure4. 46 Deformed geometry after wind forces

The deformed geometry varies slightly as compared to the previous case. The presence of wind forces deforms the structure slightly along the y-axis as shown above. This eventually contributes in the premature failure of the structure and the premature failure can be attributed to the absence of symmetry in the forces acting on the structure. In real world scenarios, there can be numerous factors that can lead to premature failure of structures and wind forces are definitely one to be considered.

Chapter 5

Conclusion and future work

5.1 Conclusion

This thesis has demonstrated and outlined the effects of imperfections in structure and that asymmetric loading can have large impacts in the behavior of the structure and can lead to premature failures due to reduction in buckling load. The Newton Raphson method or a similar approximation method has to be used in the nonlinear analysis takes into account all the material and geometrical properties. Nonlinear post buckling responses show that imperfections or very small values of eccentric loading can cause a significant decrease of critical buckling load and post buckling response becomes unstable. Truss structure have been used in our study since their geometrical uniqueness make them good candidates to show buckling and the analysis of symmetry breaking. A truss structure is mechanically equivalent to a spring since it can have only tensile or compressive loads.

Symmetry breaking influences reduction in buckling load and the reduction is specific to the dimensional properties of the structure. A 7 bar structure with a base has a less reduction in buckling load for asymmetric loading whereas a 5 bar structure without a base has a significant dip in buckling load. In structures the effects if symmetry breaking is independent of the area and height of cross section. Asymmetric conditions can be attributed to the following cases:

- Asymmetric loads acting
- External factors like temperature and wind
- Manufacturing defects

In real world cases residual stresses introduced into a bar by manufacturing process are also an imperfection since material like fibers with residual compressive stress can reach the limit of proportionality before the theoretical critical limit is attained. Symmetry breaking is pronounced in shorter structures.

The load deformation curve is a good indicator of the effect of symmetry breaking. When two nodes are considered, the effect of symmetry breaking can be discerned by the nature of the graph; more the two curves conform, less is the effect of symmetry breaking.

5.2 Future work

The phenomenon of symmetry breaking can be analyzed in more different cases such as:

1. Structures having indifferent geometrical properties
2. Structures with base having a lower area of cross section compared to the other parts.
3. Materials with lower modulus of elasticity
4. Structures like circular arches which are widely used and can be good candidate to show the phenomenon of symmetry breaking.

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