PARAMETRIC AND NONPARAMETRIC DYNAMICAL SYSTEM IDENTIFICATION USING LASER-MEASURED VELOCITIES

A Thesis Presented to the Faculty of the Graduate School
University of Missouri

In Partial Fulfillment
Of the Requirements for the Degree
Master of Science in
Mechanical & Aerospace Engineering

by
XUEWEI RUAN

Dr. P. Frank Pai, Thesis Supervisor

MAY 2014
The undersigned, appointed by the Dean of the Graduate School, have examined the thesis entitled

PARAMETRIC AND NONPARAMETRIC DYNAMICAL SYSTEM IDENTIFICATION USING LASER-MEASURED VELOCITIES

Presented by Xuewei Ruan

A candidate for the degree of Master of Science in Mechanical & Aerospace Engineering

And hereby certify that in their opinion it is worthy of acceptance.

______________________________________
Professor P. Frank Pai

______________________________________
Professor Yuyi Lin

______________________________________
Professor Stephen Montgomery-Smith
ACKNOWLEDGEMENTS

I would like to express my sincere appreciation and thanks to my advisor, Dr. P. Frank Pai. He has been a tremendous mentor for me. I would like to thank him for encouraging my research and for allowing me to grow as a research scientist. His advice on my thesis work as well as my career has been priceless. I would also like to thank my committee members, Dr. Yuyi Lin and Dr. Stephen Montgomery-Smith for serving on my thesis committee and providing me valuable comments and suggestions.

A special thanks goes to my family. Words cannot express how grateful I am to my mother and father for all of their support and sacrifices for my study and life. I would also like to thank all of my friends who supported my study and encouraged me to strive towards my goal. Hao Peng, Haoguang Deng, Shuyi Jiang, Yiqing Wang and Tiancheng Xu have been giving me their personal and professional unwavering support during my stay at the University of Missouri.
LIST OF FIGURES

Fig. 2.1 Direct time-domain parametric identification of a linear system without data noise: (a) velocity, displacement and acceleration, (b) identified parameters and relative percentage errors using \( n=3 \) in Eq. (2.4), and (c) identified parameters and relative percentage errors using \( n=8 \). ................................................................. 14

Fig. 2.2 Direct time-domain parametric identification of a linear system with a sudden stiffness change: (a) velocity, displacement and acceleration, (b) identified parameters using \( n=3 \) in Eq. (2.4), and (c) identified parameters using \( n=8 \). ................................................................. 16

Fig. 2.3 Direct time-domain parametric identification of a linear system with and without noise in velocities: (a) estimation errors without noise in velocities, (b) velocity with noise, and displacement and acceleration computed from velocity, and (c) identified parameters and relative percentage errors using \( n=8 \) in Eq. (2.4). ................................................................. 17

Fig. 2.4 First step of EMD-recognize extrema from data and connect as two cubic splines ............................................................................................................................................................. 21

Fig. 2.5 The EMD decomposition of a free vibration signal of a cantilevered beam: (a) the measured velocity of a point on the beam, (b) the extracted first IMF, (c) the extracted second IMF, (d) the extracted third IMF, and (e) the residual. ................................................................. 23

Fig. 3.1 Shifting displacement by a nonzero central displacement: (a) displacement before shifting, (b) displacement after shifting, (c) the spring force function estimated using the displacement before shifting, and (d) the spring force function estimated using the displacement after shifting. ............................................................................................................................. 37

Fig. 3.2 Shifting displacement by using EMD: (a) displacement before shifting, (b) displacement after shifting, (c) the spring force function estimated using the displacement before shifting, and (d) the spring force function estimated using the displacement after shifting for the integrated signal shown in Figs. 3.1a and 3.2a. ............................................................................................................................. 39

Fig. 3.3 Shifting displacement of the nonlinear system in Eq. (3.6): (a) displacement before shifting, (b) displacement shifted by a constant value, (c) displacement shifted by using EMD.................................................................................................................................................. 40

Fig. 3.4 Spring force functions estimated from Fig. 3.3: (a) using the displacement in Fig. 3.3a, (b) using the displacement in Fig. 3.3b, and (c) using the displacement in Fig. 3.3c.
Fig. 3.5 The actual spring and damping force functions in Eq. (3.6): (a) spring force function, and (b) damping force function. ................................................................. 43

Fig. 3.6 The spring force function of Eq. (3.6): (a) from nonparametric identification, (b) from polynomial fitting of Fig. 3.6a, and (c) from piecewise linear fitting of Fig. 3.6a. 46

Fig. 3.7 Demarcation points and grouping of points on the spring force function curve: (a) determination of demarcation points, and (b) grouping data points into three segments. 48

Fig. 3.8 The damping force function of Eq. (3.6): (a) from nonparametric identification, and (b) from curve fittings of Fig. 3.8a........................................................................................................... 49

Fig. 3.9 The estimated damping force of Eq. (2.1): (a) using the simulated displacement, velocity and acceleration with noise, and (b) using the simulated velocity with noise and the displacement and acceleration computed from the velocity. .......................... 51

Fig. 3.10 Noise filtering by using EMD: (a) original signal with noise, (b) noise extracted as the first IMF, (c) the filtered signal extracted as the second IMF, and (d) the left residual. ............................................................................................................................................................................. 55

Fig. 4.1 Setup of a horizontal cantilevered steel beam for transient vibration test........ 58

Fig. 4.2 Setup of a laser vibrometer and a cantilevered beam for steady-state vibration test. ........................................................................................................................................ 59

Fig. 4.3 A plastic car with an adjustable magnetic damper........................................... 60

Fig. 4.4 EMD analysis of the beam’s transient vibration: (a) integrated displacement, (b) extracted first IMF, (c) extracted second IMF, (d) extracted third IMF, and (e) the residual. ........................................................................................................................................................................... 62

Fig. 4.5 The second IMF: (a) displacement, (b) velocity, and (c) acceleration. .......... 64

Fig. 4.6 Time-varying amplitudes and frequencies from HHT and CPD analyses: (a,b) HHT analysis, and (c,d) CPD analysis. ............................................................................................................................................................................. 65

Fig. 4.7 Nonparametric identification using the second IMF: (a) spring force function from
−\mu \delta, (b) curve fitting of (a), (c) damping force function from \( F_d(u) \), and (d) curve fitting of (c).

Fig. 4.8 Velocity responses of the plastic car with different damping values: (a,d) low damping, (b,e) medium damping, and (c,f) low damping.

Fig. 4.9 Nonparametric identification of the spring and damping forces of the plastic car: (a) spring forces under low (red) and medium (blue) dampings, and (b) damping forces under low (red) and medium (blue) dampings.

Fig. 4.10 Time-frequency analysis of the low-damping displacement response: (a) displacement, (b,c) amplitude and frequency from HHT analysis, and (d,e) amplitude and frequency from CPD analysis.

Fig. 4.11 Time-frequency analysis of the medium-damping displacement response: (a) displacement, (b,c) amplitude and frequency from HHT analysis, and (d,e) amplitude and frequency from CPD analysis.

Fig. 4.12 Time-frequency analysis of the 19th point’s displacement response of the beam: (a) displacement, (b,c) amplitude and frequency from HHT analysis, and (d,e) amplitude and frequency from CPD analysis.

Fig. 4.13 Time-frequency analysis of the plastic car’s steady-state displacement response: (a) displacement, (b,c) amplitude and frequency from HHT analysis, and (d,e) amplitude and frequency from CPD analysis.
ABSTRACT

Accurate damage inspection and reliable health monitoring of dynamical systems relies on accurate system identification techniques, where signal procession for accurate extraction of modal parameters from measured dynamical data plays the key role. In theory, if all three variables (i.e., displacement, velocity and acceleration) of each point on a dynamical system are available from measurement, parametric or even nonparametric system identification can be easily and accurately performed. In experiment, however, it is often only one variable is measured because collocating three different sensors at a point is too difficult even if the sensors are small enough not to affect the system’s dynamic characteristics. Numerical investigations reveal that velocity is the best choice because the corresponding acceleration and displacement can be estimated by numerical differentiation and integration, and because today’s laser vibrometers can provide very accurate measurements of velocities.

Real-world dynamical systems often behave nonlinearly especially when they are damaged or aged. Because dynamic characteristics (modal frequencies, damping ratios, mode shapes, etc.) of a nonlinear system change with time, system identification methods for nonlinear systems need to be capable of extracting such time-varying system characteristics and hence time-frequency analysis is essentially needed. Numerical investigation shows that direct time-domain methods based on processing of measured
time-domain data can provide accurate identification results only for linear systems. Unfortunately, damping of a linear dynamical system cannot be accurately estimated by direct time-domain methods because its value is too small as compared to the values of stiffness and mass. Because frequency-domain methods are based on the use of frequency response functions from Fourier transform and conventional, linear modal testing techniques, they are also only valid for linear systems. On the other hand, indirect time-domain methods are based on the use of the maximum displacement and velocity states or the time-varying amplitudes and frequencies of responses to perform system identification. A nonparametric system identification method based on the use a spring force function of displacement and a damping force function of velocity or a combined restoring force function of displacement and velocity is developed and shown to work for any nonlinear systems.

All these direct time-domain methods, frequency-domain methods, and indirect time-domain methods are presented in this thesis, and advantages and shortcomings of each method are demonstrated and discussed through numerical examples. This thesis concentrates on the nonparametric system identification and presents several numerical techniques for noise filtering. Except numerical examples, experimental vibration data of a beam and a plastic car measured using a PSV-200 scanning laser vibrometer are also used to verify the accuracy of these methods.
# TABLE OF CONTENTS

**ACKNOWLEDGEMENTS** .................................................................................................................. ii

**LIST OF FIGURES** ........................................................................................................................ iii

**ABSTRACT** ....................................................................................................................................... vi

**TABLE OF CONTENTS** .................................................................................................................... viii

**Chapter 1**  
**INTRODUCTION** .................................................................................................................. 1

1.1 Background .................................................................................................................................. 1

1.2 Literature Review and Discussions ......................................................................................... 3

1.3 Motivation and Goals .............................................................................................................. 7

1.4 Thesis Overview ...................................................................................................................... 8

**Chapter 2**  
**IDENTIFICATION OF LINEAR SYSTEMS** ........................................................................ 10

2.1 Introduction ............................................................................................................................... 10

2.2 Direct Time-Domain Methods ............................................................................................... 11

2.3 Indirect Time-Domain Methods ............................................................................................. 18

2.3.1 Hilbert-Huang Transform ................................................................................................. 20

2.3.2 Conjugate-Pair Decomposition ....................................................................................... 26

2.4 Frequency-Domain Methods .................................................................................................. 30

**Chapter 3**  
**NONPARAMETRIC IDENTIFICATION** ............................................................................. 34

3.1 Introduction ............................................................................................................................... 34
3.2 Initial displacement

3.3 Nonparametric Identification and Functional Forms

3.4 Noise

3.5 Filtering

3.5.1 Frequency-domain filtering

3.5.2 Time-domain filtering using EMD

3.5.3 Conventional digital filtering

Chapter 4 EXPERIMENTAL VALIDATION

4.1 Introduction

4.2 Experimental Setup

4.3 Transient Data Analysis

4.3.1 Steel Beam Experiment

4.3.2 Plastic Car Experiment

4.4 Analysis of Steady-State Vibration

4.4.1 Steel Beam Experiment

4.4.2 Plastic Car Experiment Analysis

Chapter 5 CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

5.2 Future work

References
Chapter 1 INTRODUCTION

1.1 Background

Health monitoring of aging and damaged structures is a major concern of the engineering community [1]. Aging structures can be fragile and damages may be caused by various factors such as excessive response, accumulative crack growth, and impact by a foreign object. Future intelligent structures demand high system performance, structural safety and integrity, and low maintenance cost. To meet the challenge, structural health monitoring (SHM) has emerged as a reliable, efficient, and economical approach to monitor system performance, detect damages if they occur, assess/diagnose the structural health condition, and make corresponding maintenance decisions [2]. Damage like internal stressing, rubbing, cracking and plastic deformation can cause a vibrating structure to dissipate energy, and the larger the energy dissipation the smaller the vibration amplitude [3]. Moreover, a minor crack in a structure changes the structure’s stiffness and causes stress concentration, which can lead to a large crack later. Thus time-varying modal parameters like stiffness, damping and vibration amplitude and frequency can be used to monitor the health of a structure.

An ideal structural health monitoring system typically consists of two major components: a built-in network of sensors for collecting responses, and a data analysis algorithm/software for interpretation of measurements in terms of the physical conditions
of the structure [2]. Health monitoring refers to the use of in-situ, nondestructive sensing and analysis of structural characteristics for the purpose of detecting changes that may indicate damage or degradation [3]. For accurate measurement of a structure’s dynamic response, the sensors need to be able to work without direct contact with the structure and be nondestructive. Once damage occurs to a system or the system’s performance becomes unsatisfactory, it should be detected and reported by the health monitoring system. In other words, the structural health detection/monitoring system needs to be capable of online tracking of system characteristics. Online tracking requires a method to predict the structure’s future behavior based only on past dynamical responses. On the other hand, damage inspection does not require the method to be capable of online tracking.

To test and verify damage inspection and health monitoring algorithms before applying them to actual structures, numerical experiments using high-fidelity mathematical models are useful and important for understanding the capabilities of algorithms without expensive and time-consuming physical experiments [4]. Numerical simulation using a mathematical structural model takes the role of sensors in a structural health monitoring system, and the system model is often presented in terms of stiffness, mass and damping matrices, which can also be presented in terms of modal frequencies, mode shapes and damping ratios [4]. Extraction of such modal parameters from measured dynamical responses is the main task in system identification.
1.2 Literature Review and Discussions

There are many different ways to extract modal parameters out of dynamical data. A system can be represented by an assumed governing equation or by a response function. Many methods work by assuming a governing equation with undetermined parameters and then fitting measured dynamical data to the governing equation to determine the parameter values. These are the so-called parametric identification methods. The use of an assumed governing equation may limit the accuracy and capability for system identification, and fitting data into an unmatched assumption may result in misinterpretation. Nonparametric identification methods are to avoid such limits by not assuming the forms of unknown spring and damping functions. To obtain better estimations, the unknown functions often need to be adjusted case by case. Based on the domain of the data being processed for identification, parametric and nonparametric methods can be divided into frequency-domain methods, time-domain methods, and indirect time-domain methods.

A direct time-domain method uses an assumed governing equation. It computes the assumed unknown parameters (e.g., spring, damping and mass parameters) by fitting the measured time-domain responses (displacements, velocities and accelerations) to the assumed equation. Because all parameters in the governing equation are obtained at the same time, the so-obtained damping value often has an error much larger than those of other parameters because its value is relatively smaller. However, direct time-domain
methods are eligible of online tracking, and they also work for nonlinear systems.

Most frequency-domain methods are based on the use of Fourier transform. They transform a time series into a sum of regular harmonic signals with different frequencies and amplitudes [5]. Since Fourier transform treats the input time series as a periodic function, its starting and ending points are better to have the same value. The discontinuity between them would result in the edge effect. Because a nonlinear system’s response to a single-frequency harmonic excitation can be a quasi-harmonic and a quasi-harmonic’s spectrum contains many regular harmonics, a nonlinear response’s Fourier spectrum cannot guarantee the physical meaning of each harmonic. Moreover, a nonlinear system’s general dynamic response may contain several coupled nonlinear modal vibrations, and these modal vibrations cannot be exactly separated [6]. Even if they are numerically separated, each separated modal vibration contains many regular harmonics. Hence, frequency-domain methods are essentially not appropriate for identification of nonlinear systems.

Wavelet transform is a windowed Fourier transform using a predetermined, localized wavelet $\psi(t)$ to extract localized components similar to the wavelet [7, 8]. It decomposes a time signal into its constituent parts [9]. Wavelet transform can provide time-varying characteristics of the processed signal, but the use of an envelope for $\psi(t)$ in wavelet transform always distorts the extracted components. Because even pure harmonics of different frequencies are not exactly orthogonal to each other within an assumed window,
continuous wavelet suffers severely from the edge effect [5]. Moreover, a frequency-domain method needs to process the whole data length to obtain Fourier spectra, which makes it incapable of online tracking. Furthermore, because Fourier transform is a linear theory, it is only appropriate for treating linear systems. Hence, Fourier transform becomes less popular after Hilbert-Huang transform (HHT) and other time-frequency analysis methods are proposed in the engineering community for treating nonlinear systems.

To reveal the mathematical implications and characteristics of wavelet and Fourier transforms [5], several sliding-window fitting (SWF) methods for time-frequency analysis have been proposed by researchers [10]. SWF cannot accurately decompose and compute the time-varying frequencies and amplitudes [6], but it can be used to extract as many harmonics as needed by including major harmonics identified from the signal’s Fourier spectrum in the sliding-window fitting process [5].

HHT is more appropriate than Fourier transform[11] and wavelet transform[7] for signal decomposition and time-frequency-energy presentation of nonlinear or non-stationary signals [12, 13]. It is derived from the principles of empirical mode decomposition (EMD) and Hilbert Transform [14]. EMD can sequentially decompose an acquired signal into basic components, called intrinsic mode functions (IMFs) [15]. The intrinsic mode functions are usually physically meaningful because their characteristic scales are physical [16]. Several major shortcomings of HHT have been reported. For example, the first IMF may cover a very wide frequency range, HHT may fail to produce
an accurate frequency pattern for the inspected signal, HHT may generate pseudo components resulting in misinterpretation, and HHT may miss low-energy components [14]. In addition, HHT requires a long set of data and suffers from the edge effect, which make it incapable of online tracking.

The conjugate-pair decomposition (CPD) method only requires a few recent data points sampled at a low-frequency for sliding-window point-by-point adaptive time-frequency analysis and can be used for online frequency tracking [17]. CPD is believed to be very useful for many engineering applications that require real-time online frequency tracking [18]. However, CPD is usually applied on monocomponent. For a signal consisting of several regular/distorted harmonics, the recommended approach is to use EMD to decompose it into IMFs and then use CPD to perform time–frequency analysis on each IMF [19].

If a system’s order and/or type of nonlinearity is unknown, non-parametric identification is needed [19]. For non-parametric identification, one can use the maximum displacement states to determine the displacement-stiffness curve and the maximum velocity states to determine the velocity-damping curve [17]. Instead of tracking time-varying modal parameters, it describes the stiffness as a function of displacement and the damping as a function of velocity or velocity and displacement, which make it more flexible for nonlinear systems.
1.3 Motivation and Goals

To simulate the process of damage inspection and health monitoring of a structure by numerical experiments, a mathematical structural model is needed and the sensor needs to be modeled. To obtain more information about a structure, it is better to use more sensors at more locations and even more than one sensor at each location. But collocating several sensors at one point is too difficult even if the sensors are very tiny. Hence, only one sensor is often used at each location in real experiments. Moreover, the accuracy and sensitivity of the chosen sensor plays an important role for accurate damage inspection and monitoring of structures. The scanning laser vibrometer is probably today’s most sensitive and accurate instrument for measuring velocities of structures. Hence, the sensor assumed in our numerical simulations and actual experiments will be a velocity sensor, and the dynamical signal assumed to be experimentally obtained is velocity. Moreover, noise contained in the measured velocity will be simulated by adding normally distributed random numbers to the simulated velocity.

Signal processing plays the key role in dynamics-based damage inspection and health monitoring of structures and mechanical systems, and the focus is always on accurate extraction/tracking of dynamic characteristics in order to identify system parameters (e.g. natural frequencies, mode shapes, mass, damping, stiffness and external loading) by reverse engineering [20]. And accurate frequency tracking is the key for online
identification and health monitoring of dynamical systems [19]. Aging structures and especially damaged ones are often nonlinear. Frequency-domain methods are less popular because of incapable of dealing with nonlinear systems, and direct time-domain methods are less accurate because parameters of large and small values are simultaneously extracted. Indirect time-domain methods and nonparametric methods will be the emphasis in this thesis. Moreover, experimental data will be used to verify the feasibility and accuracy of each method.

1.4 Thesis Overview

Chapter 2 starts with an introduction to linear systems. In the whole thesis, numerical and experimental data all represent velocity data, and displacements and accelerations used in the signal processing are obtained from velocities by integration and differential, respectively. Sections 2.2-2.4 introduce direct time-domain methods, indirect time-domain methods, and frequency-domain methods. In addition to the shortcomings and advantages of each method, we also discuss the capability of these methods for online tracking and the possibility for nonlinear system identification.

Chapter 3 introduces a nonparametric identification method that is different from all those methods in Chapter 2. We perform nonparametric identification of several linear and nonlinear single-degree-of-freedom systems, and challenges in nonparametric identification are discussed.
Chapter 4 presents two experiments to verify the capability and accuracy of the methods introduced in Chapters 2 and 3. We also compare all the results from these different methods for system identification.

Chapter 5 summarizes all the numerical and experimental results and provides some concluding remarks. Moreover, some related research topics are proposed for future study.
Chapter 2 IDENTIFICATION OF LINEAR SYSTEMS

2.1 Introduction

When damage occurs to a system, stiffness and/or damping values of the system will be changed. For a single-degree-of-freedom (single-DOF) linear dynamic system, its governing equation can be presented as

\[ m \ddot{u} + c \dot{u} + ku = f(t) \]  \hspace{1cm} (2.1)

where \( k \) and \( c \) stand for the system’s spring and damping coefficients, respectively. The mass \( m \) of the system and the applied force \( f(t) \) usually are known, and the time-varying displacement, velocity and acceleration can be obtained by measurement or by integration/differentiation if only one of them is measured. That leaves \( k \) and \( c \) to be unknown. If \( k \) and \( c \) can be identified from measured dynamic response, they can be used for health diagnosis. The major problem of estimating \( k \) and \( c \) is that the value of \( c \) is much smaller than that of \( k \); their values are in different orders. With small noise contained in the measured dynamic response, one may still get a good estimation of \( k \), but accurate estimation of \( c \) is very challenging. Accurate estimation of damping and stiffness is always the emphasis of every method for system identification.

The \( u, \dot{u}, \ddot{u} \) can be directly measured using different displacement, velocity, and acceleration sensors, respectively. Unfortunately, simultaneously measuring more than one of them at the same location is practically challenging because accelerations and
displacements often require contact sensors. Hence, a practical approach is to measure one of \( u, \dot{u}, \ddot{u} \) and then use differentiation and/or integration to obtain the other two.

Data obtained from measurement always contain noise, and the influence of noise is accumulated by numerical integration and aggravated by numerical differentiation. If \( u \) is measured, \( \dot{u} \) and \( \ddot{u} \) can be obtained by one and two times of numerical differentiation, respectively. If \( \ddot{u} \) is measured, \( \dot{u} \) and \( u \) can be obtained by one and two times of numerical integration, respectively, but it involves two unknown integration constants. Hence, it is better to measure \( \dot{u} \), and then obtains \( u \) and \( \ddot{u} \) by numerical integration and differentiation, respectively. Moreover, measured velocities often contain less noise than measured accelerations. Also, today’s scanning laser vibrometers can provide very accurate measurement of velocities. Hence, velocity rather than displacement and acceleration of a system is considered to be the best choice of measurement.

Based on the approach used for processing dynamical data from experiment or computation, methods used to extract modal parameters can be distinguished as direct time-domain methods, indirect time-domain methods, and frequency-domain methods.

### 2.2 Direct Time-Domain Methods

The main characteristic of direct time-domain methods is to estimate system parameters without transform dynamical data to the frequency domain. Direct time-domain methods use a governing equation with undetermined system parameters as the
mathematical model of a system. Eq. (2.1) is the governing equation for a single-DOF linear system, and the system parameters are $k$ and $c$. Minimizing the error of the governing equation by using least-square fitting can determine the unknowns using only time-domain data. The error is defined as:

$$\text{Error} = \sum_{i=j-n}^{i=n} \left[ m\ddot{u}_i + c\dot{u}_i + ku_i - f(t_i) \right]^2$$  

(2.2)

where it is assumed that the data point at $t_j$ is under estimation, $n$ data points from each side of $t_j$ are used for least-squares fitting, and a total of $2n+1$ points are used for estimation.

As mentioned before, velocities will be measured by using a laser vibrometer, and accelerations and displacements are obtained by numerical differential and integration. For each different case in computing displacements, the initial displacement may be unknown. If the initial displacement $u^0$ is known, accurate displacements $u$ can be obtained by integration. If the initial displacement is unknown, relative displacements $u-u^0$ can be obtained from integration. This would add one more unknown to Eq. (2.1), and the number of unknowns increases to three. The integrated relative displacement $u-u^0$ physically means the displacement with respect to the initial displacement, and accelerations can be computed by using finite difference as

$$\dot{u}(t) \equiv \int_0^t \dot{u} dt = u(t) - u^0, u^0 = u(0)$$

$$\ddot{u}(t) = \frac{\dot{u}(t + \Delta t) - \dot{u}(t - \Delta t)}{2\Delta t}$$

$$m\ddot{u}(t) + c\dot{u}(t) + k\ddot{u}(t) + ku^0 = f(t)$$  

(2.3)

To obtain more accurate accelerations, velocities of a local data segment around a
point can be fitted into a low-order polynomial function, and then the corresponding acceleration can be obtained by taking differentiation on the obtained polynomial. This data processing algorithm can be dramatically accelerated by using the Savitzky-golay (SG) filter [21].

After velocities are measured and displacements and accelerations are obtained by numerical integration and differentiation, the unknowns \( k, c \) and \( u^0 \) can be estimated by the following least-squares fitting process:

\[
\begin{align*}
\text{Error} & \equiv \sum_{i=j-n}^{j+n} (\ddot{m} \dot{u}_i + c \dot{u}_i + k \ddot{u}_i + ku^0 - f_i)^2 \\
\frac{\partial \text{Error}}{\partial c} & = 0 = \sum_{i=j-n}^{j+n} (\ddot{m} \dot{u}_i + c \dot{u}_i + k \ddot{u}_i + ku^0 - f_i) \dot{u}_i \\
\frac{\partial \text{Error}}{\partial k} & = 0 = \sum_{i=j-n}^{j+n} (\ddot{m} \dot{u}_i + c \dot{u}_i + k \ddot{u}_i + ku^0 - f_i) \ddot{u}_i \\
\frac{\partial \text{Error}}{\partial (ku^0 - f_i)} & = 0 = \sum_{i=j-n}^{j+n} (\ddot{m} \dot{u}_i + c \dot{u}_i + k \ddot{u}_i + ku^0 - f_i)
\end{align*}
\]

(2.4)

If all the \( N \) measured data points are used in Eq. (2.4), we have \( N = 2n + 1 \) and only one set of values for \( k, c \) and \( u^0 \) is obtained. However, it involves intensive computation, and any time-localized transient events cannot be revealed. On the other hand, if a localized data segment around point \( j \) is processed, the process is faster and it can reveal any time-localized transient events. In Eq. (2.4), there are three unknowns need to be determined, and hence the minimum number of points needed is three. Using a small number of points
to do least-square fitting can reveal accurate time instants of transient events, but it cannot provide accurate estimations of system parameters.

Fig. 2.1 Direct time-domain parametric identification of a linear system without data noise: (a) velocity, displacement and acceleration, (b) identified parameters and relative percentage errors
using \( n=3 \) in Eq. (2.4), and (c) identified parameters and relative percentage errors using \( n=8 \).

Fig. 2.1 shows the results from using the direct time-domain method to determine the unknowns for a linear system with \( k = 78.96, c = 1, m = 2, f = 50, u(0) = 2 \), and \( \dot{u}(0) = 0 \). A sampling time interval \( \Delta t = 0.0667 \) is used in the numerical integration using the Runge-Kutta method to obtain \( \dot{u}(t) \). Fig. 2.1a shows the displacement, velocity and acceleration. The velocity is generated by the mathematical model; the displacement and acceleration are computed by integration and differentiation. Fig. 2.1b is obtained using seven points for one section (\( n=3 \) in Eq. (2.4)), and Fig. 2.1c is obtained using seventeen points for one section (\( n=8 \) in Eq. (2.4)). Clearly, the results in Fig. 2.1c are better, indicating that the use of more data points results in a more accurate estimation. However, the use of many points increases the sliding window length and it will not be able to clearly pinpoint a sudden change in response, and the lengths of the beginning and ending segments of inaccurate estimations increase.
Fig. 2.2 Direct time-domain parametric identification of a linear system with a sudden stiffness change: (a) velocity, displacement and acceleration, (b) identified parameters using n=3 in Eq. (2.4), and (c) identified parameters using n=8.

If the stiffness changes from $k = 78.96$ to $k' = 75.80$ ($= 0.96k$) at $t=2.5s$, Fig. 2.2a shows the dynamical data from numerical integration without noise being added to $\dot{u}(t)$.

Fig. 2.2b shows the results using seven points ($n=3$ in Eq. (2.4)) to estimate the time-
varying parameters $k, c$ and $ku^0 - f$, and Fig. 2.2c shows the results using seventeen points ($n=8$ in Eq. (2.4)). It is obvious that the use of less data points provides more accurate information about the time instant of sudden change.

As a shortcoming of this direct time-domain method, all unknowns are estimated at the same time. Because the damping value is much smaller than other parameter values, the damping value is expected to be less accurate.

![Graph](image)

Fig. 2.3 Direct time-domain parametric identification of a linear system with and without noise in
velocities: (a) estimation errors without noise in velocities, (b) velocity with noise, and displacement and acceleration computed from velocity, and (c) identified parameters and relative percentage errors using n=8 in Eq. (2.4).

Fig. 2.3a is an expanded view of the errors in Fig. 2.1c, and it reveals that the damping value has a bigger relative error. The dynamical data used in Fig. 2.1 are based on numerical velocities without any noise. The error is only caused by the numerical computation of accelerations and displacements from velocities. If experimental data are used, the errors of the three estimated parameters are expected to be bigger because of the noise contained in the experimentally measured velocities. The results shown in Fig. 2.3c are obtained from the numerical velocity data with added noise (as shown in Fig. 2.3b). The noise is a vector of normally distributed pseudorandom numbers with a standard deviation $\sigma = 0.01$. Although the noise is considered to be small, the accuracy of the estimated parameters is obviously affected, as shown in Fig. 2.3c. And again, the accuracy of damping value is obviously more sensitive than other two parameters to noise. Because of the inaccuracy of direct time-domain methods for estimation of small system parameters, it is not really appropriate for identification of nonlinear systems because nonlinear systems often have small nonlinearity parameters as well as a small damping.

2.3 Indirect Time-Domain Methods

Indirect time-domain methods process time-domain data (e.g., time-varying frequencies and amplitudes) derived from the displacements measured or computed from
the measured velocities to extract a parametric or nonparametric system model. The governing equation of the single-DOF system shown in Eq. (2.1) can be rewritten as:

$$\ddot{u} + 2\zeta \omega_0 \dot{u} + \omega_0^2 u = \frac{f}{m}, \quad \omega \equiv \sqrt{\frac{k}{m}}, \quad \zeta \equiv \frac{c}{2\omega m}, \quad \omega_d \equiv \sqrt{\omega^2 - (\omega_0 \zeta)^2}$$

(2.5)

If $f = F_0 \cos \Omega t$, the response $u(t)$ consists a transient part and a steady-state part as:

$$u(t) = Ae^{-\zeta \omega t} \cos(\omega_d t - \theta) + A_0 \cos(\Omega t - \phi)$$

(2.6)

Eq. (2.6) shows that the decaying rate of the transient amplitude $Ae^{-\zeta \omega t}$ is directly related to the damping ratio $\zeta$ and the undamped natural frequency $\omega$, and the actual, damped natural frequency $\omega_d$ of $\cos(\omega_d t - \theta)$ is directly related to $\omega$. If the time-varying values of $\zeta \omega$ and $\omega$ are separately extracted from $Ae^{-\zeta \omega t}$ and $\cos(\omega_d t - \theta)$ and $m$ is known, $k$ and $c$ can be separately and accurately estimated using Eq. (2.5). If $m$ is unknown, it can be determined from the known $F_0$ and the extracted amplitude $A_0$ of the steady-state part using

$$m = \frac{F_0}{A_0 \omega^2 \sqrt{(1 - \Omega^2 / \omega^2)^2 + (2 \zeta \Omega / \omega)^2}}$$

(2.7)

Hence, how to extract the time-varying frequency and amplitude of a displacement response is the key for an indirect time-domain method for system identification. Moreover, it is necessary to have a transient part and a steady-state part in order to identify all system parameter values. For a linear system subjected to a harmonic excitation, it is possible and easy to separate the response into transient and steady-state parts. For a nonlinear system subjected to a harmonic excitation, on the other hand, it is impossible to separate the
response into a transient part and a steady-state part with a constant amplitude [22]. Even if the transient part is extracted by advanced signal processing, it is different from a pure transient response without any harmonic excitation because of the nonlinear coupling between the transient and steady parts. Hence, it is necessary to separately obtain transient and steady parts in experiment.

Because a displacement signal’s time-varying frequency and amplitude can only be obtained by time-frequency analysis, techniques for time-frequency analysis are presented next. We present a conjugate-pair decomposition (CPD) method and compare it with the Hilbert-Huang transform (HHT), which was recently patented by NASA for time-frequency analysis of nonlinear nonstationary signals.

2.3.1 Hilbert-Huang Transform

Hilbert-Huang transform (HHT) combines empirical mode decomposition (EMD) and Hilbert spectral analysis; it can be used for the extraction of sectional instantaneous frequency (sIF) and sectional instantaneous amplitude (sIA) of nonlinear and nonstationary signals [22]. EMD is a data-driven signal decomposition technique that sequentially extracts zero-mean regular/distorted harmonics from a signal, starting from high- to low-frequency components, and it is the first step of HHT [22]. During the sifting process, the signal is decomposed into $n$ intrinsic mode functions (IMFs) $c_i(t)$ and a residual $r_n$ as [22]
To extract the first IMF $c_1$ out of the original signal, extrema of $u(t)$ are first recognized and then all local minima are connected using a cubic spline and all local maxima are connected using another cubic spline, as shown in Fig. 2.4, where green stars are the maxima points and blue stars are minima points. After the two cubic splines are obtained, their average $m_{11}$ is subtracted from the signal and the residual is treated as the first estimation of $c_1$. Treat the residual signal as a new signal and repeat the same process for K times until $m_{1K} \approx 0$. Then, the residual is taken as $c_1$, i.e.,

$$c_1 = u - m_{11} \cdots - m_{1K}$$

(2.9)

Note that $c_1$ is the signal’s component that has the shortest characteristic time scale and the highest frequency. Because $m_{iK}$ actually represents the signal’s low-frequency moving
average, the process of identifying extrema explains why IMFs are extracted from high- to low-frequency components. To obtain a meaningful instantaneous frequency, the extracted component must be symmetric with respect to the local zero mean and has the same number of zero crossings and extrema [14]. So the sifting process should eliminate all low-frequency components and make the amplitude envelopes symmetric, that is, \( m_{1K} \approx 0 \).

After the first IMF \( (c_1) \) has been obtained, one can treat the residual \( r_1(= u - c_1) \) as the new data and repeat the steps shown in Eq. (2.9) to obtain other \( c_i(i = 2,\ldots,n) \) as

\[
c_i = r_{i-1} - m_i \cdots - m_k, \quad r_{i-1} \equiv u(t) - c_1 \cdots - c_{i-1}
\] (2.10)
Fig. 2.5 The EMD decomposition of a free vibration signal of a cantilevered beam: (a) the measured velocity of a point on the beam, (b) the extracted first IMF, (c) the extracted second IMF, (d) the extracted third IMF, and (e) the residual.

In Fig. 2.5, $\dot{u}(t)$ is the velocity of one point on a vibrating beam measured by a laser vibrometer. The initial deformed beam geometry is close to the beam’s third mode shape. The $\dot{u}(t)$ contains at least the first three modes. To get a better extraction of an IMF, a signal with the frequency of the IMF and amplitude of the remaining signal should be added into the data and extract the IMF out with it. The first three natural frequency of this vibrating beam has been computed by FFT as 3.91 Hz, 24.22 Hz and 67.81 Hz. So the amplitude and frequency of the signal we added in to extract these three IMFs out is 67.81 Hz and 0.4 m/s, 24.22 Hz and 0.8 m/s and 3.91 Hz and 0.8 m/s, respectively. As the way EMD works, the extracted first IMF ($c_1$) has the highest frequency, and it contains the third mode and other minor higher-frequency modes. Moreover, $c_2$ and $c_3$ are the second and first modes, respectively, and $r_3$ is the residual.

The whole sifting process can be stopped when the residual $r_n (= r_{n-1} - c_n)$ becomes a monotonic function from which no more IMF can be extracted [22]. There is also a numerical way to decide if it should be ended. A deviation $D_v$ is defined as

$$D_v = \sqrt{\frac{\sum_{i=1}^{N} [c_{ik}(t_i) - c_{i,k-1}(t_i)]^2}{\sum_{i=1}^{N} c_{i,k-1}^2(t_i)}}$$

$$t_i = (i-1)\Delta t, \ T = N\Delta t, \ c_{ik} = u - m_{i1} \cdots - m_{ik}$$

where $T$ is the sampled period and $N$ is the total number of samples. One can set a small number to be a limit and stop the sifting process when $D_v$ is smaller than the limit. Or, a
maximum number of iterations can be set for stopping the sifting.

The second step of HHT is to perform Hilbert transform (HT) and compute the time-varying frequency $\omega_i$ and amplitude $A_i$ of each $c_i$ [22]. Because the second step is applied on each component separately, one can consider each $c_i(t)$ as a new signal $x(t)$. The HT of $x(t)$ is defined by an integral transform [23]

$$H[x(t)] = y(t) = \pi^{-1} \int_{-\infty}^{\infty} \frac{x(t)}{t-\tau} d\tau$$  \hspace{1cm} (2.12)

After $y(t)$ is obtained, $z(t)$ can be defined based on $x(t)$ and $y(t)$ as

$$z(t) \equiv x(t) + jy(t) = a(t)e^{j\varphi(t)}$$

$$a(t) = \sqrt{x^2(t) + y^2(t)}, \varphi(t) = \tan^{-1} \frac{y(t)}{x(t)}$$  \hspace{1cm} (2.13)

where $j = \sqrt{-1}$, $a(t)$ is the instantaneous amplitude of $x(t)$, which can reflect how the energy of the $x(t)$ varies with time, and the $\varphi(t)$ is the instantaneous phase of $x(t)$. The controversial instantaneous frequency $\omega(t)$ is defined as the time derivative of the instantaneous phase $\varphi(t)$ as follows [14]:

$$\omega(t) \equiv \frac{d\varphi(t)}{dt}$$  \hspace{1cm} (2.14)

Once the frequency and amplitude of each component are computed, the system’s stiffness and damping values can be estimated and the system’s health can be evaluated based on these system parameters.

After a comprehensive study of HHT, several shortcomings of HHT have been revealed. The accuracy of locating local extrema is affected by the sampling rate. A low
sampling rate means that the time step $\Delta t$ is big. When $\Delta t$ is big, each period of a high-frequency IMF is only represented by a small number of data points. Then, the located local extrema may deviate from the actual extrema. The time-velocity curve may have overshots and undershoots caused by noise, and two located adjacent extrema may even jump over several periods. Such inaccurate identification of local extrema will lead to inaccurate upper and lower envelopes, and it may end up with misinterpretation. In other words, HHT requires a high sampling rate that $\Delta t$ is shorter than one fourth of the shortest period to guarantee at least four data points in the shortest period.

The next shortcoming of EMD is about the stopping criterion ($m_{1k} \approx 0$) for sifting. A meaningful instantaneous frequency can be obtained only if the IMF’s envelopes are symmetric with respect to the local zero mean, but $m_{1k}$ can never be zero under a finite number of iterations. This leaves errors in component decomposition.

Considering the way that EMD extracts IMFs from high to low frequencies, the extracted first IMF may include other high-frequency components and noise. Hence, the first IMF may not be a monocomponent. Because the period of a low-frequency IMF is long and EMD requires several extreme points to extract an IMF, HHT needs to process a long length of data in order to extract all IMFs. This limits HHT to be an online tracking method.

HT can extract accurate frequencies and amplitudes only for monocomponent signals. For a signal consisting of multiple components, the instantaneous frequency from
HT analysis is meaningless. Unfortunately, a structure often vibrates with several modes, and an aging or damaged structure behaves nonlinearly. Hence, EMD becomes an essential tool for pre-processing a signal into IMFs before HT can be applied to extract accurate time-varying instantaneous frequencies.

The other major shortcoming of HHT is that HHT suffers from the edge effect. The edge effect, or the Gibbs phenomenon, is caused by discrete Fourier transform due to an incomplete data periodicity, i.e., the waveform has not reached a full cycle within the processed data length [23]. Because HT is often based on Fourier transform for obtaining $y(t)$, the accuracy of HHT seriously suffers from the edge effect. Extension of data on the two ends is one way of reducing the edge effect, but it will make HHT need a longer data length.

The needs of a long data length and a high sampling rate make HHT inefficient in computation and incapable of online tracking and health monitoring of structures. Next we introduce a conjugate-pair decomposition method that requires only a short data length and a low sampling rate and is capable of online tracking.

### 2.3.2 Conjugate-Pair Decomposition

Conjugate-pair decomposition (CPD) is a three-point method for tracking point wise instantaneous frequency (pIF) and point wise instantaneous amplitude (pIA) of an arbitrary signal [22]. It sets an assumed form for a signal $u(t)$ as:
\[ u(t) = C_0 + e_1 \cos(\omega t) - f_1 \sin(\omega t) \]  

(2.15)

The assumed form contains three parts. \( C_0 \) is a constant to capture the moving average, \( e_1 \cos(\omega t) \) captures the central solution without the moving average and \( f_1 \sin(\omega t) \) is the Hilbert transform of \( e_1 \cos(\omega t) \). To capture the moving average’s slope and the signal’s discontinuity, one can add an additional term \( D_0 \sinh(\omega t) \) to the function. In the assumption, \( \omega_1 \) is assumed to be known. One can get the initial guess of \( \omega_1 \) by other method, such as the Teager-Kaiser algorithm [3] or Fourier transform. Because \( C_0, e_1, f_1 \) are the three unknowns to be determined, three data points will be enough to set up three algebraic equations for their solutions. After \( C_0, e_1, f_1 \) are determined, amplitude \((a_i)\) and phase angle \((\phi_i)\) of the signal can be determined and the assumed function can be expressed as:

\[
\begin{align*}
    a_i &= \sqrt{e_1^2 + f_1^2}, \quad \phi_i = \tan^{-1} \frac{f_1}{e_1}, \quad \omega_i = \frac{d\phi_i}{dt} \\
    u(t) &= C_0 + a_i \cos(\omega i t + \phi_i)
\end{align*}
\]  

(2.16)

For a linear system, its steady-state response has constant amplitude and frequency. The use of only three data points to determine amplitude and frequency is the major merit of CPD. If the most recent three data points are used in CPD, the method can predict the following state and enable online tracking. Similar to the use of a direct time-domain method for parametric identification of a linear single-DOF system, three points is just the minimum number of data points needed to determine the unknowns. When more data points are used in such sectional estimation, the estimated parameter values will be more accurate but the
results will be less sensitive to sudden changes because they represent the values averaged over the section. Because three points only cover a short time segment, \(a_i\) and \(\alpha_i\) can be considered as the instantaneous amplitude and frequency of the central point of the three points. These instantaneous parameter values are sensitive to sudden changes and can be used to detect damage-induced transient events. For a nonlinear signal, its amplitude and frequency change with time, and this method can well capture their time-varying values.

To well capture the signal’s behavior, the time length of three points should cover at least one fourth of the signal’s period \(T\). For a signal sampled at a high sampling rate, the data length of \(3\Delta t\) may not be longer than \(T/4\). Instead of using successional points, one can use skipped data points (e.g., \(t_i, t_{i+3}\) and \(t_{i+6}\)) to increase the data length to cover at least \(T/4\). The other way is to use more successional data points to cover at least \(T/4\). When more data points are used, the results are often more accurate but the computing time will be longer and the obtained amplitude and frequency will be more of averaged values instead of instantaneous ones.

In the CPD analysis, one can pick up the central point \(t_n\) as a flag of the section, and shift other points in that section as \(\bar{t} = t - t_n\), or \(\bar{t} = \bar{t}_i = i\Delta t\), where \(i\) ranges from \(-(m-1)/2\) to \((m-1)/2\) with \(m\geq 3\) being the odd, total number of data points in the processed section. Then, the function of the sectional signal can be rewritten as:

\[
\begin{align*}
    u(t) &= C_0 + C_1 \cos(\omega t) - D_1 \sin(\omega t) \\
    C_1 &= a_i \cos(\omega t_n + \phi_i), \quad D_1 = a_i \sin(\omega t_n + \phi_i)
\end{align*}
\]

(2.17)
Same as $C_0$, $e_1$ and $f_1$, $C_0$, $C_1$ and $D_1$ can be estimated by least-squares fitting to minimize the error between the assumed function and the actual data $U(t)$ using the following equations:

\[ Error = \sum_{i=-(m-1)/2}^{(m-1)/2} [U(t_{n+i}) - u(\bar{t}_i)]^2 \]

\[ \frac{\partial Error}{\partial C_0} = 0 = \sum_{i=-(m-1)/2}^{(m-1)/2} [U(t_{n+i}) - u(\bar{t}_i)] \cdot 1 \tag{2.18} \]

\[ \frac{\partial Error}{\partial C_1} = 0 = \sum_{i=-(m-1)/2}^{(m-1)/2} [U(t_{n+i}) - u(\bar{t}_i)] \cdot \cos(\omega_1 \bar{t}_i) \]

\[ \frac{\partial Error}{\partial D_1} = 0 = \sum_{i=-(m-1)/2}^{(m-1)/2} [U(t_{n+i}) - u(\bar{t}_i)] \cdot \sin(\omega_1 \bar{t}_i) \]

The amplitude and phase can be estimated using

\[ a_i = \sqrt{C_i^2 + D_i^2}, \theta_i = \omega_i t_n + \phi_i = \tan^{-1}(D_i/C_i) \tag{2.19} \]

Except the first section, one can use the frequency of last section as an initial guess of the next section, and the frequency of next section will be updated by finite difference of the phase changes of the two successional sections. Using the first two sections as an example, we have

\[ \theta_1(t_n - p\Delta t) = \omega_1(t_n - p\Delta t) + \phi_1, \theta_2(t_n) = \omega_1 t_n + \phi_1 \]

\[ \omega_2 = \dot{\theta}_2 \approx \frac{\theta_2(t_n) - \theta_1(t_n - p\Delta t)}{p\Delta t} \tag{2.20} \]

where $t_n$ is the central point of the second section, $t_n - p\Delta t$ is the central point of the first section, and $p\Delta t$ is the time between the two central points. If the two sections have the same length, then $p\Delta t$ is also the length of the section. The backward finite difference is used to average the frequency over $p\Delta t$ in order to reduce the influence of noise in
frequency tracking [22]. The frequency updating eliminates the error introduced by the initial guess of the first section’s frequency.

With an appropriate sampling rate, CPD uses only three points to determine a signal’s instantaneous frequency and amplitude, and it is capable of online tracking. However, CPD can only treat a monocomponent signal. For a multi-component signal, it needs to be pre-decomposed into components before the use of CPD on each component. EMD would be a great method to decompose a multi-component signal into several IMFs, and then CPD can be applied on each IMF to compute its instantaneous amplitude and frequency. Because EMD needs to process the whole data length, it makes CPD inappropriate for online tracking of multi-component signals.

2.4 Frequency-Domain Methods

The discrete Fourier transform (DFT) decomposes an arbitrary time signal \( u(t) \) into many regular harmonics with each one having a constant amplitude and a constant frequency as:

\[
u(t_k) = u_k = a_0 + 2 \sum_{i=1}^{N/2} (a_i \cos \omega_i t_k + b_i \sin \omega_i t_k) = a_0 + \text{Real} \left( 2 \sum_{i=1}^{N/2} (a_i - jb_i) e^{j\omega_i t_k} \right)
\]

\[
a_i = \frac{1}{N} \sum_{k=1}^{N} u_k \cos \omega_i t_k , \quad b_i = \frac{1}{N} \sum_{k=1}^{N} u_k \sin \omega_i t_k , \quad \omega_i = 2\pi i / T
\]

\[
U(\omega) = a_i - j b_i = \frac{1}{N} \sum_{k=1}^{N} u_k e^{-j\omega_i t_k} \quad (2.21)
\]

where \( t_k = k\Delta t \ (k = 1, 2, \ldots, N) \), \( N \) is the total number of samples (assumed to be even
here), $\Delta t$ is the sampling interval, $1/\Delta t$ is the sampling frequency $f_s$, and $T(=N\Delta t)$ is the sampled period. $U$ is the Fourier spectrum of $u(t)$, and $a_i$ and $b_i$ are the spectral coefficients and they represent amplitudes of the extracted regular harmonics [5]. To extract each set of regular harmonics ($a_i \sin \omega_i t$ and $b_i \cos \omega_i t$), the angular velocity $\omega_i$ is determined first, and then the amplitude is obtained from the extracted $a_i$ and $b_i$. The signal $u(t)$ is treated as a periodic function having a period $T$ in DFT and is expressed as the summation of $N/2$ harmonics of constant amplitudes and frequencies [5]. The highest frequency (also called the Nyquist frequency) is $0.5/\Delta t$, which corresponds to the component with $i=N/2$. This indicates that the highest-frequency component is only represented by two data points, and it satisfies Shannon’s sampling theory.

The frequency resolution $\Delta f(=1/T = f_s/N)$ of DFT is restricted by the sampled period $T$. To increase the frequency resolution, $T$ must be increased. The period $T$ needs to cover at least one fundamental period of the signal in order to identify the signal’s lowest-frequency component from the spectrum [24]. To obtain a better spectrum of a low-frequency signal, the sampled signal duration often needs to cover at least three cycles of the lowest-frequency component to be extracted. Using the length of $2\Delta t$ as the shortest period only guarantees enough points for each period of a high-frequency harmonic, but the available highest frequency is also restricted by the sampling rate. Increase of the sampling rate increases the available maximum frequency, but it decreases the frequency resolution if the same $N$ is used.
All these results are based on the assumption that the signal $u(t)$ is periodic although the original function may not be periodic [24]. In DFT, the signal is repeated to become a periodic function with a period of the data length $T$. Unfortunately, because the starting and ending points are determined by initial conditions, external excitations and sampling and are not controllable, they often have different values. Repeating the signal to become a periodic one often results in discontinuity at the connection point. At these discontinuity points, the Fourier series converges to the average of the left- and right-side values at each of these points and shows overshoots around them, which is called the edge effect in the time domain. In the frequency domain, it results in the leakage problem because many high-frequency harmonics exist in order to account for the discontinuity [24]. Moreover, the use of chosen, finite number of regular harmonics to represent the signal and the use of summation instead of integration to compute the amplitudes $a_i$ and $b_i$ in Eq. (2.21) introduce errors in DFT.

Because Fourier transform presents a dynamical signal in terms of a constant average $a_0$ and many regular harmonics with constant frequencies and amplitudes by processing a long length of data, those extracted harmonics represent the averaged harmonic contents of the signal and cannot be used to reveal short-duration transient events. Moreover, a modal vibration of a nonlinear structure is often a quasi-harmonic, whose Fourier spectrum consists of many regular harmonics. Hence, each harmonic of a nonlinear signal’s spectrum is physically meaningless. Furthermore, the spectrum of a nonlinear
multi-mode vibration signal is even more complex, and it is impossible to perform any parametric or nonparametric identify from a nonlinear signal’s Fourier spectrum. Hence, frequency-domain methods based on the use of spectra are not practical for general system identification at all.

Wavelet transform (WT) is similar to Fourier transform (FT). A wavelet having a narrow frequency band is used in WT as the basis function to extract the amplitude of each component similar to the wavelet, but a single-frequency harmonic is used in FT as the basis function to extract the amplitude of each harmonic. All the problems of FT exist in WT. Moreover, because a wavelet has a short time duration, the edge effect is more serious in WT.
3.1 Introduction

Parametric identification methods assume the forms of stiffness and damping functions with unknown parameters to be determined by processing measured signals. Their accuracy is limited by the forms and number of assumed functions. On the other hand, a nonparametric identification method is to find the forms of stiffness and damping functions. A linear/nonlinear single-DOF dynamical system can be presented using a mass $m$, a spring force $F_s(u)$ and a damping force function $F_d(\dot{u})$ as

$$m\ddot{u} + F_d(\dot{u}) + F_s(u) = f(t)$$

(3.1)

The mass $m$ of a dynamical system often remains unchanged even if the system is damaged, and it is also known in actual applications. However, a general damping force function can be a function of $u$ and $\dot{u}$, which will be examined later in Section 3.3.

Because $F_d(\dot{u})$ and $F_s(u)$ are coupled at most times, they are independent of each other only at some specific times. When $\dot{u} = 0$, $F_d(\dot{u}) = 0$, $u$ is at its local maximum and Eq. (3.1) reduces to

$$F_s(u) = f(t) - m\ddot{u}\big|_{\dot{u}=0}$$

(3.2)

Here $f(t)$ is assumed to be known by measurement, and $\ddot{u}(t)$ is obtained from the measured $\dot{u}(t)$ by finite difference or the SG filter. The identified $F_s(u)$ at discrete time instants can be fitted into a polynomial with coefficients determined by least-squares fitting:
\[ F_s(u) = c_0 + c_1u + c_2u^2 + c_3u^3 + c_4u^4 + c_5u^5 \]

Error = \[ \sum_{i=1}^{t} \left( F_s(u_i) - f(t_i) + m\dddot{u}_i \bigg|_{\dddot{u}=0} \right)^2 \]  

\[ \frac{\partial Error}{\partial c_k} = 0 = \sum_{i=1}^{t} \left( F_s(u_i) - f(t_i) + m\dddot{u}_i \bigg|_{\dddot{u}=0} \right) u_i^k, \quad k = 0, 1, 2, 3, 4, 5 \]

Here a 5th-order polynomial is assumed for \( F_s(u) \), but the assumed order can be adjusted for different cases. If the \( u - F_s(u) \) curve from the use of a 5th-order polynomial is the same as that from a 6th-order polynomial, it means a 5th-order polynomial is enough to capture the behavior of the spring force. If the results from 5th and 6th-order polynomials are very different, use the 6th-order or even higher-order one. Polynomial functions may not fit all nonlinear functions. Harmonic functions, piecewise linear or nonlinear functions, exponential functions, or hyperbolic functions can be better choices for some special dynamical systems.

After the spring force function is determined, the damping force function \( F_d(\dddot{u}) \) can be similarly computed. In this thesis, we use the same assumption that velocities will be measured using a laser vibrometer and displacements and accelerations are computed from integration and differentiation of velocities. When \( \dddot{u} = 0 \), \( |\dddot{u}| \) is at its local maximum and Eq. (3.1) reduces to

\[ F_d(\dddot{u}) = f(t) - F_s(u) \bigg|_{\dddot{u}=0} \]  

(3.4)

The identified \( F_d(\dddot{u}) \) at discrete time instants can be fitted into a polynomial with coefficients determined by least-squares fitting:
\[ F_d(\ddot{u}) = c_0 + c_1 \ddot{u} + c_2 \dot{u}^2 + c_3 \dot{u}^3 + c_4 \dot{u}^4 + c_5 \dot{u}^5 \]

\[ Error = \sum_{i=1}^{t} \left( F_d(\ddot{u}_i) - f(t_i) + F_s(u_i) \right|_{u=0}^{u=0} \right)^2 \quad (3.5) \]

\[ \frac{\partial Error}{\partial c_k} = 0 = \sum_{i=1}^{t} \left( F_d(\ddot{u}_i) - f(t_i) + F_s(u_i) \right|_{u=0}^{u=0} \right) \dddot{u}_i^k, \quad k = 0, 1, 2, 3, 4, 5 \]

A low sampling rate may result in no data points on the velocity peaks, and then the located points of maximum \( |\dddot{u}| \) may have non-zero accelerations. To obtain a more accurate damping force, one could add the term \( m\ddot{u} \) back into the \( Error \) in Eq. (3.5). However, because the error of acceleration from differentiation is always bigger than the error of displacement from integration, the acceleration here is better to be obtained from the SG filter. Similarly, the located points of maximum \( u \) may have non-zero velocities. To obtain a more accurate spring force, one may consider adding the corresponding damping force to the \( Error \) in Eq. (3.3). Unfortunately, the damping force function is unknown during the estimation of \( F_s(u) \). However, one can use a polynomial curve fitting to find accurate maximum displacement locations. Moreover, the estimated \( F_s(u) \) is often more accurate than the estimated \( F_d(\ddot{u}) \) because the magnitude of \( F_s(u) \) is much larger than that of \( F_d(\ddot{u}) \).

### 3.2 Initial displacement

The displacement signal from integration of the measured velocity signal does not account for the non-zero initial displacement. There are two ways to account for non-zero initial displacements. One easy way is to shift the integrated displacement signal by a
constant. If the system undergoes a free damped vibration, the final displacement should approach zero and the mean value of the maximum and minimum displacements when \( t \) is large is the constant for shifting.

![Diagram of shifting displacement](image)

**Fig. 3.1** Shifting displacement by a nonzero central displacement: (a) displacement before shifting, (b) displacement after shifting, (c) the spring force function estimated using the displacement before shifting, and (d) the spring force function estimated using the displacement after shifting.

The displacement shown in Fig. 3.1a is obtained by integrating the velocity of a
system with cubic nonlinearity and \( u(0) = 1 \) and \( \dot{u}(0) = 0 \). The nonzero central displacement at the end can be determined as the mean value of the local maximum and minimum (blue stars in Fig. 3.1a). The red points in Fig. 3.1c is the estimated spring force function based on the displacement data without shifting and the blue curve is the correct spring force function. It gives the right shape, but it needs to be shifted. Shifting the displacement in Fig. 3.1a by the nonzero central displacement results in the displacement in Fig. 3.1b and the spring force function in Fig. 3.1d.

Displacement response obtained from numerical integration of the measured velocity response may contain errors caused by measurement noise. Another way to account for the non-zero initial displacement and to eliminate the accumulated noise-induced errors in displacements is to use the empirical mode decomposition (EMD) to extract the moving average out. Of course, this approach is valid only if the system has symmetric response under no loading or symmetric loading.
Fig. 3.2 Shifting displacement by using EMD: (a) displacement before shifting, (b) displacement after shifting, (c) the spring force function estimated using the displacement before shifting, and (d) the spring force function estimated using the displacement after shifting for the integrated signal shown in Figs. 3.1a and 3.2a.

Fig. 3.2b & 3.2d show the displacement and the spring force function shifted by using EMD. Note that EMD makes the displacement vibrating around zero at the end of time and the spring force function overlaps with the correct one. For a response consisting of multiple modes, one needs to make sure all modes have been removed before subtracting.
the residual from the integrated displacement response

Fig. 3.3 Shifting displacement of the nonlinear system in Eq. (3.6): (a) displacement before shifting, (b) displacement shifted by a constant value, (c) displacement shifted by using EMD.

Fig. 3.3a shows the displacement of a nonlinear system response to a continuous input force, the governing equation of the system is shown in Eq.(3.6).

\[
\ddot{u} + 0.4\dot{u} \left| \dot{u} \right| + k(u) = 0.15\cos(0.01t^2),
\]

\[
k(u) = \begin{cases} 
  u - 0.06 \text{sgn}(u - 0.06), & |u| > 0.06 \\
  0, & |u| \leq 0.06 
\end{cases}
\]

\[
\Delta t = 0.1571, u(0) = 0.4, \dot{u}(0) = 0, \text{noise} = 0.01 \times \text{randn}(N,1) \times \left| \frac{\dot{u}(t)}{\left| \dot{u} \right|_{\text{max}}} \right|
\]

There are two components in the displacement response. To apply the first method by
treating the moving average as a constant value, the mean value of successional local extreme (blue stars in Fig. 3.3a) here cannot represent the vibration range of the system. Instead, the local maximum and minimum of the lower frequency signal (green stars in Fig. 3.3a) can give a better approximation of central displacement. Shifting the displacement response by the mean value of green points results in the displacement shown in Fig. 3.3b. After the two components are extracted, subtracting the residual from Fig. 3.3a yields the displacement response shown in Fig. 3.3c. Both methods give appropriate shipments and make the displacement around the zero value at the end of time. In Fig. 3.4, the estimated spring force functions are shown as red dots and the correct ones are shown as blue lines. They confirm both methods of accounting for initial displacements are accurate. Because the system here is a nonlinear one, the frequency and amplitude of each component change with time. Hence, EMD introduces small errors in the displacement response and then the estimated spring force function, as shown in Fig. 3.4c.
Fig. 3.4 Spring force functions estimated from Fig. 3.3: (a) using the displacement in Fig. 3.3a, (b) using the displacement in Fig. 3.3b, and (c) using the displacement in Fig. 3.3c.

As shown in Fig. 3.4, the spring force function here is actually a piecewise function. Therefore the assumed spring force function needs to be three piecewise linear functions.

### 3.3 Nonparametric Identification and Functional Forms

Because a non-parametric identification method does not assume the forms of spring and damping force functions, it can identify a wide range of nonlinear systems. After the spring and damping force functions are identified, high-order polynomials are often used to derive their functional forms. However, different nonlinear systems may require different nonlinear functions. How to adjust the order of polynomials is already mentioned in Section 3.1. Next we present other types of adjustment.

Eq. (3.6) represents a second-order system having backlash stiffness and quadratic damping undergoing a transient vibration with $F_s(u) = k(u)$ and $F_d(\dot{u}) = 0.4\ddot{u}|\dot{u}|$. Fig. 3.5 shows the actual spring and damping functions.
Fig. 3.5 The actual spring and damping force functions in Eq. (3.6): (a) spring force function, and (b) damping force function.
Although data from experiment include noise, and integration of velocity accumulate the noise, the characteristic of piecewise in the relationship between displacement and spring force still would be shown clear.
Fig. 3.6 The spring force function of Eq. (3.6): (a) from nonparametric identification, (b) from polynomial fitting of Fig. 3.6a, and (c) from piecewise linear fitting of Fig. 3.6a.

The velocity response from Eq. (3.6) by Runge-Kutta integration is added with 1% noise, and then displacements and accelerations are obtained from the velocity response by numerical integration and the SG filtering. Fig. 3.6a shows the $u - F_s$ curve, where $F_s(u) = f(t) - m\ddot{u}|_{t=0}$ and $f(t) = 0.15\cos(0.01t^2).$ These data points will be used to derive the form of the spring force function. The x axis in Fig. 3.6 represents the displacement. If one polynomial is used to fit the spring force function, Fig. 3.6b shows the fitted spring force function, which is obviously inaccurate. Fig. 3.6c shows the accurate result by using three separate piecewise linear functions for curve fitting, which reveals the piecewise linearity of backlash stiffness.

For the piecewise linear fitting shown in Fig. 3.6c, the first step is to determine the demarcation points and then perform the curve fitting for each of the three segments. With noise from experiment and errors from integration, the spring force function in the second segment may vary around zero, and it creates some local maximum points. With the assumption that the noise from experiment and integration does not change the sign of slope, the points with maximum/minimum displacements of those local maximum points can be used as demarcation points. However, errors may happen, as shown in Fig. 3.7a. The first and second demarcation points are marked as blue stars in Fig. 3.7a. A local maximum has been picked as the second demarcation point because noise and errors make
the displacement there unusually smaller. After that kind of points have been eliminated, and demarcation points can be more accurate, as shown by green stars in Fig. 3.7a. For all the data points used for computing the spring force function, if a point displacement is less than the first demarcation point, it would be grouped into the first segment. For a point displacement between the two demarcation points or larger than second demarcation point, it is grouped into the second or the third segment. Fig. 3.7b shows the result of correct grouping, and the derived spring force function was shown in Fig. 3.6c.
Fig. 3.7 Demarcation points and grouping of points on the spring force function curve: (a) determination of demarcation points, and (b) grouping data points into three segments.

Although the damping force function is not a piecewise function, data points still need to be grouped into three segments because the spring force function is represented by three piecewise linear functions. The grouping of each data point is determined by its displacement value. If the displacement is less than the first demarcation point in computing the spring force, it is grouped into the first segment, and the spring force function for the first segment is used in the governing equation. Similarly, if displacement is larger than the second demarcation point or between the two demarcation points, the spring force function for the third or the second segment is used in the governing equation.

Fig. 3.8a shows the relationship between the velocity and the damping force function. According to Eq. (3.4), the damping force function $F_d(\dot{u})$ is equal to $f(t) - m\ddot{u} - F_s(u)$, which can be calculated from known data. Data from three segments are marked with three different colors, where the black curve is the actual damping force function. Data points have been grouped into three different segments should give the same estimated damping force function. With noise accumulated in the earlier computation, the damping force function estimated by these data may not be accurate, as shown by the red line in Fig. 3.8b.
The van der Pol oscillator has inseparable spring and damping forces:

$$\ddot{u} - 0.065(1 - u^2) \dot{u} + u = 0, u(0) = 0.01$$  \hspace{1cm} (3.7)

In this case the damping force is a function of both velocity and displacement; so the assumed damping force should be adjusted to
\[ F_d = \sum_{i=0}^{J} \sum_{j=1}^{J} c_{ij} u^i \dot{u}^j \] (3.8)

For non-parametric identification of the damping force function, one can still use the data points when \( \ddot{u} = 0 \). However, because \( F_d(u, \dot{u}) \) is a function of \( u \) and \( \dot{u} \), derivation of the form of damping force function needs to be done by surface fitting as:

\[
\begin{align*}
    Error &= \sum_{k=1}^{K} \left( F_d(u_k, \dot{u}_k) - f(t_k) + F_s(u_k) \right|^{|\ddot{u}_k=0})^2 \\
    \frac{\partial Error}{\partial c_{ij}} &= 0 = \sum_{k=1}^{K} \left( \sum_{i=0}^{J} \sum_{j=1}^{J} c_{ij} u^i \dot{u}^j - f(t_k) + F_s(u_k) \right)^{|\ddot{u}_k=0}) u_i \dot{u}_j \\
    i &= 0, 1, 2, 3, \ldots I, j = 1, 2, \ldots, J \\
\end{align*}
\] (3.9)

### 3.4 Noise

Noise is a major problem from experiment. It affects the accuracy of parametric and nonparametric identifications of spring and damping forces. It is necessary to include noise in numerical simulations. The common way to include noise in simulation is adding to the signal an \( N \times 1 \) vector of pseudorandom numbers drawn from the standard normal distribution like:

\[
noise = 0.01 \times \text{randn}(N,1) 
\] (3.10)

where a standard deviation \( \sigma = 0.01 \) is assumed. However, it often happens in real cases that the noise value increases with the signal value. In order to obtain more realistic simulations we add noise to the simulated velocity as

\[
\dot{u}(t) + 0.01 \times \text{randn}(N,1) \times \frac{\dot{u}(t)}{|\dot{u}_{\text{max}}|} 
\] (3.11)
Although it is almost impossible to simultaneously obtain $u, \dot{u}, \ddot{u}$ for a measurement point from an experiment, it is possible in the simulation. For the linear system shown in Eq. (2.1) and Fig. 2.1, Fig. 3.9a shows the damping force estimated by using the simulated displacement, velocity and acceleration with a $\sigma = 0.02$ noise. Fig. 3.9b shows the damping force estimated using the simulated velocity with a $\sigma = 0.02$ noise and the displacement and acceleration computed by integration and the SG filtering. Because the influence of noise is accumulated in the displacement and acceleration through integration and differentiation, the result in Fig. 3.9b is not as accurate as that in Fig.3.9a.

![Fig. 3.9 The estimated damping force of Eq. (2.1): (a) using the simulated displacement, velocity and acceleration with noise, and (b) using the simulated velocity with noise and the displacement and acceleration computed from the velocity.](image-url)
3.5 Filtering

As shown in Section 3.4, accurate estimations of spring and damping forces can be obtained only if the noise in velocities can be eliminated or reduced. Hence, noise filtering on the measured velocities before computation of displacements and accelerations is important and necessary. In general, noise often appears as high-frequency components in a signal’s Fourier spectrum. Hence, we will concentrate on filtering high-frequency noise.

3.5.1 Frequency-domain filtering

As discussed in Section 2.4, any time function $u(t)$ can be sampled and presented through the discrete Fourier transform (DFT) as

$$u(t) = a_0 + 2 \sum_{i=1}^{N/2} (a_i \cos \omega t + b_i \sin \omega t) = a_0 + \sum_{i=1}^{N/2} A_i \cos(\omega t - \phi_i)$$

$$A_i = 2 \sqrt{a_i^2 + b_i^2}, \quad \phi_i = \tan^{-1} \frac{b_i}{a_i}$$

where the component amplitudes $A_i$ are functions of the frequency $\omega$. In other words, DFT transforms a time-domain signal into a signal in the frequency domain described by a set of component amplitudes $A_i$ and phase angles $\phi_i$ at different frequencies. If the high-frequency components are due to noise, the signal without noise can be obtained by eliminating the high-frequency components by setting their $A_i$ to zero and then computing the filtered signal using Eq. (3.12), which is called the inverse discrete Fourier transform (IDFT). However, how to determine an appropriate cutoff frequency for eliminating high-
frequency components from the spectrum is challenging. Using too many components in Eq. (3.12) may result in a still noisy signal, and using too less components may distort the signal. If the signal’s spectrum without noise is available for reference, it is easy to determine the cutoff frequency by comparison. Or, one can smooth the signal using another method (e.g., the SG filter) and then take the spectrum of the smoothed signal as the noise-free spectrum for comparison.

As a major defect of using IDFT for noise filtering, the edge effect reduces the accuracy in identification. The two data ends of the filtered signal is distorted by the edge effect. However, one can cut the beginning and ending parts of the filtered signal and use the left data to estimate the time-varying spring and damping forces.

Although noise is often of high frequencies, low- and mid-frequency noises also exist in some mechanical systems. Smoothing a dynamic signal using local polynomial fitting (e.g., the SG filter) can only eliminated high-frequency noise. On the other hand, the empirical mode decomposition (EMD) can be modified for filtering low- and high-frequency noises, as shown next.

### 3.5.2 Time-domain filtering using EMD

As introduced in Section 2.3.1, the empirical mode decomposition (EMD) can sequentially extract intrinsic mode functions (IMFs) from high- to low-frequency ones from a signal. Because the extracted first IMF always has the highest frequency, if a
chosen function with a frequency higher than that of the first IMF is added to the signal, the first extracted IMF will be the chosen function and the high-frequency noise and discontinuities contained in the original signal [17]. Then, the extracted first IMF minus the chosen function will be the high-frequency noise and discontinuities in the original signal. As shown in Fig. 3.10, \( c_1 \) is the noise extracted by EMD as the first IMF, and \( c_2 \) is the filtered signal can be used to compute force and damping functions. If the signal contains a low-frequency noise, the residual will represent it. Hence, EMD can also be used to filter low-frequency noise. Fig. 3.10a shows a nonlinear system’s response to a nonzero initial displacement, the governing equation of the system is shown in Eq.(3.13).

\[
\ddot{u} + 0.05\dot{u} + u + 20u^3 = 0
\]

\[\Delta t = 0.1571, u(0) = 1, \dot{u}(0) = 0, \text{noise} = 0.01 \times randn(N, 1) \times \frac{\dot{u}(t)}{|\dot{u}|_{\text{max}}}\] (3.13)
Fig. 3.10 Noise filtering by using EMD: (a) original signal with noise, (b) noise extracted as the first IMF, (c) the filtered signal extracted as the second IMF, and (d) the left residual.

The quality of this EMD-based noise filtering depends on the frequency and amplitude of the added high-frequency function. The frequency should be higher than those of all expected IMFs to avoid extracting any high-frequency IMFs before noise filtering. Because noise is usually small and random, the noise cannot be extracted by EMD as a regular IMF. Hence, the amplitude of the added function needs to be big enough to carry the noise to be extracted with the added function as one IMF. The amplitude should be a
little larger than the signal’s amplitude. With some attempts, one can determine appropriate
frequency and amplitude. One can do the same filtering several times to get better results.
However, we note that filtering a signal with small noise by EMD may mess up the original
signal, especially at the two data ends.

3.5.3 Conventional digital filtering

Filtering noise is a very common subject in different engineering fields, and there
are many ready-to-use codes for noise filtering. Those codes usually are very user-friendly
and easy to operate. For example, an open source code for noise filtering is available from
Download the code ‘ifilter.m’ from the website and open it by MATLAB. With the
instruction from the website, this filter is very easy to operate. Loading the data into the
code, one can adjust the pass band by keyboard to make sure the major signal will be kept
and noise will be eliminated. The adjustment can be seen right on the figure.

Most ready-to-use filterers are based on Fourier transform. Noise filters based on
Fourier transform can be used to easily perform band-pass filtering by choosing a wanted
frequency band. However, extracting harmonic components from a signal’s spectrum
always introduces errors into the filtered signal. Hence, the filtered signals always suffer
from the edge effect and often have shifted phases and distorted amplitudes.
Chapter 4  EXPERIMENTAL VALIDATION

4.1  Introduction

Signals from numerical simulations using assumed mathematical models cannot represent all actual experimental physical phenomena. Hence, experimental verifications are needed in order to validate the parametric and nonparametric system identification methods introduced in Chapters 2 and 3. Because the system identification methods are to present the spring and damping force functions of a system as $F_s(u)$ with $\dot{u} = 0$ and $F_d(\ddot{u})$ with $\ddot{u} = 0$, it is necessary that the measured signal contains a wide range of amplitude values. In other words, a transient experiment is needed. Moreover, a steady-state response’s amplitude is needed in order to determine the system’s mass, as shown in Eq. (2.7). Hence, there are two types of experiments needed for collecting dynamical data for performing system identification. One is transient experimentation by giving a mechanical system a non-zero initial condition or an impact and measuring its free-vibration velocity response(s). The second one is steady-state experimentation by giving a mechanical system a single-frequency harmonic excitation or a periodic multi-component excitation and measuring its steady-state velocity response(s) after the start-up transient vibration dies out.
4.2 Experimental Setup

First we consider the horizontal cantilevered beam shown in Fig. 4.1. For transient experiments, the beam tip was giving an initial displacement by hand.

Fig. 4.1 Setup of a horizontal cantilevered steel beam for transient vibration test.

The velocity along the $z$ axis is measured using a PSV-200 laser vibrometer. The focus of the laser beam can be adjusted to locate and focus on a specific point on the steel beam, as shown in Fig. 4.2. To have strong back scattered laser light and obtain more accurate velocity values, a retro-reflective tape is put on the beam.
Fig. 4.2 Setup of a laser vibrometer and a cantilevered beam for steady-state vibration test.

The experimental setup is summarized as follows. To generate a steady-state vibration, the steel beam is excited by an integrated QuickPack QP10N PZT patch with a harmonic voltage $180\sin \omega t$ volts that comes from the PCB-790 power amplifier with an input voltage $9\sin \omega t$ volts from the HP-33120A function generator. The excitation frequency $\omega$ is close or at one of the beam’s natural frequencies predetermined by FFT analysis of the beam’s response to a chirp excitation.

Because a mechanical system’s damping force is usually smaller than its spring force, it is more challenging to have accurate identification of the damping force. To concentrate more on the identification of damping forces, we consider another
Fig. 4.3 A plastic car with an adjustable magnetic damper.

The red plastic car is connected by two springs to the two ends of the track. An adjustable magnetic damper is integrated with the plastic car. Hence, this mechanical system consists of a known mass and unknown spring and damping forces. When the plastic car begins to move back and forth, the magnetic field of the magnetic damper induces an Eddy current in the aluminum rail and another magnetic field. The interaction of these two magnetic fields creates a damping force on the car, which is about proportional to the car’s velocity. The laser beam is focused on a point on the right end of the car.

A DC motor is connected to the left spring through a crank shaft and an inextensible string. The spinning speed of the DC motor can be controlled by changing the input current. Even if the DC motor undergoes a constant-speed rotation, the excitation to the car may not be harmonic because of the crack shaft and the string. Except to understand the damping
characteristics we also want to know whether the motion is harmonic.

4.3 Transient Data Analysis

Since frequency-domain methods are valid only for linear systems and direct time-domain methods are inaccurate for nonlinear systems, here we concentrate on the use of indirect time-domain methods for nonparametric identification of the experiment data.

4.3.1 Steel Beam Experiment

The size of the steel beam in Fig. 4.1 is $784.23\,mm \times 31.75\,mm \times 3.175\,mm$. Using the FFT mode of the laser vibrometer, the first three natural frequencies of the steel beam was experimentally determined to be 3.91Hz, 24.22Hz, and 67.81Hz. With a static initial deformed geometry similar to the third mode shape, the beam’s free transient vibration mainly consists of the first three modes. Velocity of the point with a distance of 476.63mm from the fixed point has been measured. Displacements and accelerations are computed from the measured velocity response. Because the conjugate-pair decomposition (CPD) method and nonparametric identification can only treat a monocomponent signal, the original velocity signal needs to be decomposed first. Empirical mode decomposition (EMD) provides very accurate and meaningful decomposed intrinsic mode functions (IMFs), and it is the first step of Hilbert-Huang transform (HHT). Displacements obtained by integrating the measured velocities would contain a linearly changing component.
caused by the unknown initial displacement, as shown in Fig. 4.4a. This moving average would be left over after three IMFs has been extracted out. Figs. 4.4b-4.4d show the extracted IMFs $c_1$ (the third mode), $c_2$ (the second mode) and $c_3$ (the first mode). Each of these three IMFs is extracted by adding a high frequency harmonic with amplitude and frequency being 0.01m and 67.81Hz, 0.02m and 24.22Hz, and 0.02m and 3.91Hz, respectively.

Fig. 4.4 EMD analysis of the beam’s transient vibration: (a) integrated displacement, (b) extracted first IMF, (c) extracted second IMF, (d) extracted third IMF, and (e) the residual.
With the first three modes being coupled in the velocity response, the integrated displacement response would have big errors. Hence, it is better to decompose the velocity response first, and then get the displacement and acceleration responses for each modal vibration. First we add to the original velocity signal a harmonic function with a frequency at the third natural frequency (67.81Hz) and an amplitude (0.4m/s) slightly larger than the original signal, and then EMD extracts the added function and the third mode as the first IMF. Subtracting the added function from the extracted IMF yields the true, first IMF. The second and third IMFs can be sequentially extracted in a similar way, and the corresponding frequency and amplitude of the added harmonic functions are 24.22Hz and 0.8m/s and 3.91Hz and 0.8m/s, respectively.

Since CPD and nonparametric identification can treat only monocomponent signals, the second IMF $c_2$ in Fig. 4.4c is analyzed using CPD and HHT in order to demonstrate the proposed identification method. Fig. 4.5b shows the same data from Fig. 4.4c, Fig. 4.5a and c are the displacements and accelerations computed by the velocity. When applying CPD, here we use 15 points on each side ($m=3I$ in Eq. (2.18)) to do the least squares fitting.
Fig. 4.5 The second IMF: (a) displacement, (b) velocity, and (c) acceleration.
Fig. 4.6 Time-varying amplitudes and frequencies from HHT and CPD analyses: (a,b) HHT analysis, and (c,d) CPD analysis.

The blue lines in Fig. 4.6b&4.6d represent the correct second natural frequency.

The frequency error from HHT is about 1%, and that from CPD is about 0.7%. The big errors around t=0 in Fig. 4.6c&4.6d are caused by an inaccurate initial guess of the frequency using the Teager-Kaiser algorithm in CPD. On the other hand, the big errors in Fig. 4.6a&4.6b are caused by the edge effect introduced through Hilbert transform used in HHT. In general, CPD is more accurate than HHT, at least for this case here.
To perform the proposed nonparametric identification, accelerations also need to be computed from the measured velocities. Instead of using finite difference, we use the SG filter to obtain more accurate accelerations. The SG filter uses a polynomial function to fit a local segment of velocity data, and then the acceleration of the center point is obtained by differentiation on the polynomial. However, instead of using the common least-squares fitting, the SG filter uses a set of weighting coefficients on the local velocities to efficiently compute the smoothed velocity and acceleration of the data segment’s center point. In 5, the acceleration is computed by the SG filter, the velocity is from , and the displacement is from numerical integration.

Nonparametric identification extracts the spring force function using the maximum displacement states, which correspond to zero-velocity states. Because the data here are from a free transient vibration, the input force is zero. With \( f = \ddot{u} = 0 \), Eq. (3.2) reduces to

\[
F_s(u) = -m\ddot{u} \tag{4.1}
\]

Fig. 4.7a shows all data points in Fig. 4.6a with zero velocity. The gap between the two parts is because the displacements haven’t decayed to zero as shown in a. In Fig. 4.7b, the red line is the spring force function obtained from Fig. 4.7a by curve fitting, and the blue line is the correct spring force function. Fig. 4.7c shows all data points in Fig. 4.6a with zero acceleration, which correspond to

\[
F_d(\dot{u}) = -F_s(u)|_{\ddot{u}=0} \tag{4.2}
\]

The gap here is because the accelerations haven’t decayed to zero as shown in c. In Fig.
4.7d, the red line is the damping force function obtained from Fig. 4.7c by curve fitting, and the blue line is the accurate damping force function calculated using

\[ F_d = 2\zeta \omega m A = -2\zeta m \dot{A}, \quad \zeta = \frac{-\frac{1}{\omega} \frac{d(\ln A)}{dt}}{\omega A} = -\frac{\dot{A}}{\omega A} \] (4.3)

where \( A \) and \( \omega \) are the time-varying amplitude and frequency of the velocity shown in Fig. 4.6b. Fig. 4.7 shows again that the spring force function estimated from nonparametric identification is more accurate than the damping force function. That can be explained as the errors in the estimated spring force function are transferred to and aggravate the errors in the estimated damping force computation.
Fig. 4.7 Nonparametric identification using the second IMF: (a) spring force function from $-m\ddot{u}$, (b) curve fitting of (a), (c) damping force function from $-F_s(u)$, and (d) curve fitting of (c).

### 4.3.2 Plastic Car Experiment

To focus on identification of damping, three transient experiments with low, medium and high magnetic attraction forces were performed with about the same initial displacement, and the measured velocities are shown in Fig. 4.8a-c. The flat parts at the right end indicate that the car was already stopped by the magnetic damping force. Fig.
4.8d-f show the non-zero parts of Fig. 4.8a-c within 0.70~11.90 sec, 0.74~6.50 sec, and 1.60~4.49 sec, respectively. Fig. 4.8f shows that the data with high damping only has one cycle of vibration, which is too short for obtaining accurate system identification. Hence, only Fig. 4.8d & 4.8e will be further analyzed.
Fig. 4.8 Velocity responses of the plastic car with different damping values: (a,d) low damping, (b,e) medium damping, and (c,f) low damping.

Fig. 4.9 compares the spring and damping forces estimated from Fig. 4.8d & 4.8e, where red and blue lines represent low- and medium-damping cases, respectively. Note that the spring force in Fig. 4.9a remains the same under different damping forces,
indicating no change of the two springs and accurate identification of the spring force function. Fig. 4.9b clearly reveals that the damping force of the response in Fig. 4.8d is lower than that of the response in Fig. 4.8e. However, the medium (blue) damping force curve does not pass the (0,0) point. It is because the errors due to non-zero initial displacement and integrated measurement noise are not exactly removed by the EMD analysis, and it causes inaccurate estimation of $F_s(u)|_{u=0}$ that is used in estimation of $F_d$. The estimated damping function can be improved by using a longer length of data.
Fig. 4.9 Nonparametric identification of the spring and damping forces of the plastic car: (a) spring forces under low (red) and medium (blue) dampings, and (b) damping forces under low (red) and medium (blue) dampings.

Fig. 4.10a shows the displacement response under low damping by integrating the velocity response in Fig. 4.8d and removing the moving average using EMD. The time-varying amplitude and frequency in Fig. 4.10b&c are from HHT analysis, and those in Fig. 4.10d&e are from CPD analysis. Because the displacement after $t=9$ s is decreased to zero, it is hard to identify its amplitude and frequency from the displacement data. Similarly, Fig. 4.11a shows the displacement response under medium damping by integrating the velocity response in Fig. 4.8e and removing the moving average using EMD. The time-varying amplitude and frequency in Fig. 4.10b&c are from HHT analysis, and those in Fig. 4.11d&e are from CPD analysis. Again the large errors in Figs. 4.11b-e are due to that the displacement after $t=5$ s is too small to have accurate extraction of its amplitude and frequency.
Fig. 4.10 Time-frequency analysis of the low-damping displacement response: (a) displacement, (b,c) amplitude and frequency from HHT analysis, and (d,e) amplitude and frequency from CPD analysis.
Fig. 4.11 Time-frequency analysis of the medium-damping displacement response: (a) displacement, (b,c) amplitude and frequency from HHT analysis, and (d,e) amplitude and frequency from CPD analysis.

4.4 Analysis of Steady-State Vibration

Nonparametric identification of spring and damping force functions requires a free
transient response with a decaying vibration amplitude. However, a steady-state response with a constant amplitude is also needed in order to estimate the system’s mass, and it also enables the estimation of the system’s stiffness value and damping coefficient.

4.4.1 Steel Beam Experiment

The size of the steel beam in Fig. 4.1 for steady-state experiment is $831.9 \text{mm} \times 31.75\text{mm} \times 3.175\text{mm}$. The first three natural frequencies are roughly determined using the FFT mode of the laser vibrometer as 3.906Hz, 21.88 Hz and 60.16Hz. Under an excitation voltage through the integrated PZT patch at 21.88Hz, the steel beam vibrates at its second mode. Velocities of 37 equidistant points on the beam were measured by the laser vibrometer. The first several points near the fixed end and the points near the second mode’s node have small displacements and are not good for analysis. The 19th point has an appropriate amplitude and is chosen for analysis. To prepare the data for CPD and HHT analyses, the displacement response is obtained by integrating the measured velocity response and removing its moving average using EMD and is shown in Fig. 4.12a. Fig. 4.12b-e show the amplitudes and frequencies obtained from HHT and CPD analyses. It is clear that CPD provides more accurate time-frequency analysis of amplitude and frequency than HHT. The blue lines in Fig. 4.12c&e represent the accurate second natural frequency. The frequency error from HHT is about 5%, but the one from CPD is only about 0.2%. It
shows that the accuracy of CPD analysis on steady-state signals is accurate.

Fig. 4.12 Time-frequency analysis of the 19th point’s displacement response of the beam: (a) displacement, (b,c) amplitude and frequency from HHT analysis, and (d,e) amplitude and frequency from CPD analysis.
4.4.2 Plastic Car Experiment Analysis

With a quasi-harmonic displacement from the DC motor’s crank shaft (Fig. 4.3) pulling the left spring connected to the plastic car, the car moves back and forward harmonically. Again, displacement response is obtained by integrating the velocity response and removing its moving average using EMD. As shown in Fig. 4.13a, the displacement amplitude is a little bit larger at the middle time period, and Fig. 4.13b&d also indicate that. Because CPD uses an initial guess of frequency from other method, it results in significant errors around \( t=0 \), but its accuracy recovers after a short time. Fig. 4.13c indicates that the spring has quadratic nonlinearity because the time-varying frequency modulates at a frequency equal to the signal’s vibration frequency [17].
Fig. 4.13 Time-frequency analysis of the plastic car’s steady-state displacement response: (a) displacement, (b,c) amplitude and frequency from HHT analysis, and (d,e) amplitude and frequency from CPD analysis.
Chapter 5  CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

This thesis develops/presents different methods for parametric and nonparametric identifications of linear and nonlinear dynamical systems based on the use of velocity signals measured using a laser vibrometer. With measured velocities, the SG filter is used to filter noise out of velocities and provide accurate accelerations without using finite difference, and displacements are obtained by using numerical integration and then using the empirical mode decomposition to remove the part due to non-zero initial conditions and the part due to integration of noise. The proposed methods include frequency-domain methods, direct time-domain methods, and indirect time-domain methods.

Parametric identification works by assuming a governing equation with undetermined parameters and then fitting measured dynamical data to the governing equation to determine the parameter values. Nonparametric identification uses an unknown spring force function of displacement and an unknown damping force function of velocity (and displacement) to represent the system and then uses the maximum displacement and velocity states to find the spring and damping forces, respectively.

Frequency-domain methods are revealed to be invalid for nonlinear systems because a nonlinear system’s response to a harmonic excitation is often not harmonic and hence a response spectrum’s components lose their physical meaning. Fourier and wavelet
transformations suffer from the edge effect and require a long signal length to ensure accuracy. They also require a high sampling rate in order to have a wide frequency range for analysis.

Direct time-domain methods based on processing the displacements, velocities and accelerations can provide accurate identification results only for linear systems, and the estimated damping is often inaccurate because of its small value. For a nonlinear system, because there are many small nonlinear parameters in addition to the small damping parameter, it is difficult to simultaneously identify these parameters. However, direct time-domain methods are capable of capturing small transient events.

Indirect time-domain methods use the maximum displacement and velocity states to reveal the spring and damping forces, respectively. Then, the functional forms of spring and damping forces can be derived by curve fitting, but different functional forms need to be tried for different cases. Moreover, because these methods can only treat monocomponent signals, signal decomposition is needed before a multi-component signal can be analyzed. The spring force function can also be derived from the time-varying amplitude and frequency of the displacement signal from time-frequency analysis. The damping force function can also be derived from the time-varying amplitude and frequency of the velocity signal from time-frequency analysis.

For time-frequency analysis, Hilbert-Huang transform (HHT) and conjugate-pair decomposition (CPD) can be used. HHT uses empirical mode decomposition (EMD) and
Hilbert transform (HT) for time-frequency analysis. EMD decomposes a signal into a few physically meaningful intrinsic mode functions (IMF), but it requires a high sampling rate and a long data length in order to have accurate decomposition. Moreover, because HT uses Fourier transform, it causes the edge effect and reduces the accuracy of HHT. Hence, HHT is incapable of online tracking.

Conjugate-pair decomposition (CPD) can provide accurate time-frequency analysis for a signal having a narrow frequency band. Because CPD requires only a few data points for sliding-window fitting to extract the instantaneous frequency and amplitude, it is capable of online tracking. But, because CPD can treat only monocomponent signals, a multi-component signal needs to be decomposed before CPD analysis. EMD is a good choice for signal decomposition, but it makes CPD incapable of online tracking. In addition, CPD needs an initial guess of frequency using other method. However, an inaccurate initial guess of frequency will be updated and improved during iterations, and it only introduces errors at the beginning time.

Nonparametric identification describes the stiffness as a function of displacement and the damping as a function of velocity or velocity and displacement, which makes it more general for nonlinear systems. It captures the spring and damping force functions via system responses (i.e., displacements, velocities, and accelerations). It can provide accurate estimation of stiffness from the spring force function. However, data points for computing the damping force function often have small displacement values, which results in a small
spring force. And, because the errors accumulated by integration and differentiation, the damping force function is not as accurate as the spring force function. For data containing several modes, EMD can be applied as a pre-processor, and then the nonparametric identification method can be applied on each IMF.

5.2 Future work

From all the presented results and discussions, we recommend the following tasks for future research in order to improve the proposed methods:

1. Nonparametric identification of spring and damping forces using the time-varying amplitude and frequency from time-frequency analysis needs further study.

2. Hilbert transform can be computed in the time domain by following the definition shown in Eq. (2.12), instead of using the discrete Fourier transform. Then, the edge effect in HHT can be significantly reduced.

3. For highly nonlinear systems, time-frequency analysis results from CPD analysis are less accurate than those from HHT analysis. Hence, CPD needs to be improved for highly nonlinear systems.

4. The use of the SG filter for smoothing velocities, providing accelerations, and prediction to improve the accuracy and efficiency of system identification is worth study.

5. The accuracy of nonparametric identification of the damping force function needs
to be improved.
References:


