PRESERVICE TEACHER LEARNING FOR SUPPORTING
ENGLISH LANGUAGE LEARNERS TO MAKE SENSE OF MATHEMATICS

A Dissertation presented to the Faculty of the Graduate School
University of Missouri

In Partial Fulfillment of the Requirements for the Degree
Doctor of Philosophy

by

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JULY 2015
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ACKNOWLEDGEMENTS

There was a moment that I was afraid my dissertation might not come out into the world. I am deeply grateful about this piece of work and would like to express my sincere gratitude to all who have helped me go through such a challenging time. First and foremost, I thank God, whose many blessings have encouraged me to not give up on anything. There are also several people whom I am indebted to because of their crucial contribution in this dissertation research.

I am forever grateful to Dr. Barbara Dougherty for serving as the chair of my dissertation and for being my academic advisor these past four-years. Her warm encouragement, guidance, understanding, patience, and support were essential to the completion of this dissertation and to my formation as a researcher. I am also grateful to Dr. Kathryn Chval for her time, guidance, and expertise in the topic of English language learners (ELLs). Without her, my conceptual framework and codebook would not have been completed. In addition, this dissertation research was based on my term paper for the Research Seminar led by Dr. James Tarr. I would like to thank him because his feedback and guidance were useful and fundamental to the birth of this document. I am also thankful to Dr. Zandra de Araujo, whose research project, Supporting Teachers of ELLs to Achieve Reform, provided crucial insights when I designed my dissertation study. Her support, inspiration, and friendship are what I will always appreciate. Moreover, I would like to thank Dr. Carlo Morpurgo in the Mathematics Department for serving as one of my dissertation committee members and for bringing understanding, patience, and hospitality.

I would also like to extend special gratitude to three preservice teachers who
voluntarily participated in this study: Becky, Lucy, and Kate. I deeply appreciate their sincere preparation and punctuality. Without their time, support, and effort, it would not have been possible to even begin this research.

It has been a great experience to study with such intelligent and warm people at MU, especially doctoral students at office 119, and the incredible faculty members in mathematics education. A special thank you goes to Chris Bowling, our dear staff at the office of mathematics education. I am very thankful for all of the tedious but important work he has done for me. Another special thank you goes to Erin Smith, a doctoral student in mathematics education, for coding my data to help me ensure the reliability of this research.

Finally, and most importantly, I would like to thank my family with all of my heart who have sacrificed many things for me to complete my doctoral studies. My endless appreciation goes to my husband, Jinyoul, for being my shelter and home. He always held my hand whenever I was discouraged by disease, disbelief, or the unstable future. I also would like to thank to my dear son, Iluda, for always bringing so much fun, laughter, and happiness, as his name means “Dreams come true.” Additionally, I am deeply thankful to my parents in South Korea who once said girls do not have to go to college, but who then always supported me to be the first generation to go to graduate school and earn a doctoral degree in my entire family. I would like to finally extend and show my appreciation to the people of Columbia Korean Baptist Church who fed and prayed for my family and me while I was ill.

Without the love and support of all the people mentioned above, none of my dreams for the completion of my doctoral degree would have ever come true.
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ABSTRACT

This case study seeks to investigate how preservice teachers (PSTs), who have never received a training related to ELL education, support English language learners (ELLs) when making sense of highly cognitive demanding mathematical problems with complicated language use. In this study, three middle school mathematics PSTs worked with ELLs in one-on-one settings while receiving individual interventions that focused on research-based ELL teaching strategies. I examined the strategies they implemented in order to help ELLs understand mathematics problems and solutions. The purpose of this study was twofold: to investigate the strategies the PSTs applied when helping ELLs understand mathematical problems, and to determine if these teachers showed a shift in using strategies influenced by their experiences with ELLs and what they were learning from the interventions. Their written lesson plans, videotaped teaching practices, and written reflections were coded using the following five categories: mathematical content, culture/life experience, mathematical/cognitive process, mathematical/contextual language, and positioning. As time evolved, all of the PSTs began to apply life-connection strategies and to integrate various visuals closely connected to mathematical situations into their lessons, which they learned during the interventions. They indicated a positive influence from the interventions, although the levels and contents of the impact vary among participants. The results also reveal PSTs used limited strategies associated with all the categories. The findings of this study suggest that PSTs require significant preparation infused with practical experiences and examples in order to design a linguistically and conceptually rich lesson while simultaneously making meaningful connections between mathematics and ELLs’ experiences.
CHAPTER 1. INTRODUCTION

As a former adult English language learner (ELL), I still remember how scared and uncomfortable I was in my first class in the United States (U.S.). I was in a panic when I soon realized I could not understand any English sentence the teacher spoke. After painfully struggling but pretending to not struggle for one month, I finally convinced myself to tell the teacher the truth. He was shocked and then asked, “Then why didn’t you ask me a question?” I did not answer anything at the moment because constructing a correct English sentence on the right spot was not a simple task for me. I needed to find the correct words in my head such as a correct subject and a correct verb, to figure out the correct order and grammar in the sentence structure, and to check the pronunciation of each word. When I finally arrived at one complete sentence, the chance to say it out loud was already gone. However, I wanted to say something like this, “Why didn’t you ask me a question? Why didn’t you check if I understood? Why didn’t you help me out before I expressed my difficulty in front of everyone?”

Based on the K–12 teacher survey conducted by Walker, Shafer, and Liams (2004), my case illustrates my situation may be encountered by large numbers of ELLs. The results from their survey indicates that almost half (45%) of teachers believe it is ELLs’ responsibility to adapt to American culture and school life, and one of five teachers refuse to modify their instruction for ELLs. The authors argue that these unproductive beliefs may be the result of insufficient professional development or administrative supports. As statistics show, while 41% of teachers have ELLs in their classroom, only 13% are prepared to adequately teach ELLs (National Center for
Furthermore, secondary teachers in content areas such as mathematics seem to have misconceptions regarding ELLs. Harper and de Jong (2004) found that content teachers believe that “just good teaching” is also appropriate for ELLs, so there is no need to provide linguistic supports or particular scaffoldings for ELLs. Their attitude towards ELLs is described in the following vignette.

With the exception of key vocabulary, they [secondary mainstream teachers] had trouble conceptualizing the lesson in terms of language demands and developing language objectives for ELLs at different proficiency levels. They appeared uncomfortable and expressed doubts about the relevance of this exercise [developing specific language objectives for ELLs] for mainstream content area teachers (Harper & de Jong, 2004, p.156).

Insufficient preparation and professional development for content teachers related to teaching ELLs is a significant concern as the number of ELLs in mainstream classrooms is rapidly growing. Approximately 25.3 millions of both foreign-born and U.S.-born ELLs reside in the U.S. in 2011 (Whatley & Batalova, 2014) and the number of ELLs in the U.S. public schools increased by 0.6 million between 2002–03 and 2010–11 (Aud et al., 2013). Moreover, ELL-related policies such as those in California, Arizona, and Massachusetts that have banned bilingual education and negated ELL instruction (Krashen, 2004) might result in placing more ELLs in mainstream classrooms. However, current teacher education programs in many states do not require all teacher candidates to complete ELL education courses (such as second language acquisition and effective teaching practices). For instance, only 20 states require that all preservice teachers (PSTs) have some ELL preparation and less than one-sixth of college-based teacher preparation programs include content related to teaching ELLs (Ballantyne, Sanderman, & Levy, 2008). The lack of ELL education in teacher preparation programs might be one of the reasons that preserves the achievement gap between ELLs and non-ELLs: only 14% of
fourth-grade ELLs scored at or above the proficient level in mathematics while 42% of non-ELLs in the same grade reached proficiency. In eighth grade, only 5% of ELLs scored at or above the proficient level in mathematics while 35% of non-ELL students attained the same level (National Center for Education Statistics, 2011).

Although mathematics teacher educators and researchers (e.g., Artiles & McClafferty, 1998; Durgunoglu & Hughes, 2010) have contended teacher preparation programs should include rigorous ELL or multicultural education to help narrow the achievement gap, there has been little attention to teaching ELLs in content areas such as mathematics or science (Janzen, 2008). This is perhaps guided by the misconception that mathematics is less difficult for ELLs because it is based on a universal language of numbers. Contrasting to that belief, Echevarria, Vogt, and Short (2010) clarified the necessity of bringing ELL perspectives in mathematics classes as “One contributing factor to the difficulty ELs [ELLs] experience is that mathematics is more than just numbers; math education involves terminology and its associated concepts, oral or written instructions on how to complete problems, and the basic language used in a teacher’s explanation of a process or concept” (p. 1). As this statement emphasizes, language has a crucial role in constructing new knowledge (von Glasersfeld, 1995; Vygotsky, 1978). However, content teachers may not be able to see the need for using language explicitly because the role of language as a medium in teaching and learning is usually hidden (Harper & de Jong, 2004).

The linguistic demands of the instructional context are not the only aspect teachers need to consider in planning mathematics activities for ELLs. They also have to accommodate the cultural, socioeconomic, and linguistic factors associated with
cognitive activities (Campbell, Adams, & Davis, 2007). While it seems clear that ELLs have unique obstacles to overcome, high stakes mathematics tests often include a significant amount of word problems, which aggravate these obstacles. The contexts in mathematical word problems contain not only unknown words but also unfamiliar cultural or societal factors. For example, if the word problem is situated in a football game, ELLs who are from other countries in which that particular sport is not very popular might not know how to play the game and therefore not know how to count the score. In this case, the context situated in the mathematics problem denies students access to the problem. Therefore, teachers of ELLs must be aware that ELLs encounter complex challenges such as mathematical content, problem solving, linguistic demands, and cultural differences when working on word problems in mathematics. Without identifying these factors that raise cognitive demands, teachers cannot accurately assess students and provide appropriate supports.

The recent release of the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) raises an expectation of improving mathematics education, but this document does not include guidelines for teaching ELLs mathematics. Instead, the Standards for Mathematical Practice in CCSSM require enhancing students’ mathematical reasoning and sense making through mathematical discussion. The basic ideas in the mathematical practices appear in earlier documents such as the Curriculum and Evaluation Standards for School Mathematics (NCTM Commission on Standards for School Mathematics, 1989), the Principles and Standards for School Mathematics (NCTM, 2000), and mathematics proficiency in Adding It Up (Kilpatrick, Swafford, &
Findell, 2001). But given the sophisticated use of language required by the mathematical practices (e.g. Mathematical Practice 3: Construct viable arguments, and critique the reasoning of others), ELLs confront multiple challenges to manage the high level of language as well as subject knowledge simultaneously.

Recognizing the need for guidance in this area, there have been efforts to connect the mathematical practices with language use. Understanding Language Project (TESOL International Association, 2013) is one example. The mission of this project is to heighten educator awareness of the critical role that language plays in the CCSSM and the Next Generation Science Standards (NGSS Lead States, 2013). While the traditional paradigm of teaching content and language to ELLs focused heavily on vocabulary and grammar, the project emphasizes the new paradigm of the CCSSM that requires teachers focusing on such language constructs as discourse, complex text, explanation, argumentation, purpose, typical structure of text, sentence structures, and vocabulary practices. Another effort to relate CCSSM to the educational needs of ELLs is on academic language or mathematics literacy. Gottlieb and Ernst-Slavit (2013) offer ideas for teachers on how they can implement each standard of the mathematical practices in order to promote students’ academic language development. For example, to meet the first mathematical practice (MP1), “Make sense of problem, and persevere in solving them,” they suggest allowing students to use their home language, providing enough time for students to understand the language and content, using small group activities, and offering descriptive and timely feedback.

In sum, mathematics instruction aligned with CCSSM should create opportunities for students to actively engage in mathematical language and make sense mathematical
situations. Moschkovich (2012) recommends five guidelines for implementing the mathematical practices into mathematics instruction for ELLs: (1) focus on mathematical reasoning, not accuracy in using language, (2) shift to a focus on mathematical discourse practices, move away from simplified views of language, (3) recognize and support students to engage with the complexity of language in mathematics classrooms, (4) treat everyday language and experiences as resources, not as obstacles, and (5) uncover the mathematics in what students say and do.

Responding to current challenges in the field as well as research recommendations, I designed a study to investigate how PSTs learn to help ELLs understand mathematical problems. Building experience with ELLs helps PSTs alter their perspectives because PSTs could see the significance of incorporating language and scaffolding into mathematics instruction (Fernandes, 2011). Although transforming teachers’ perspective is important, teaching practices specifically designed for ELLs also need to be studied because transformed perspectives eventually lead to transforming practices. In this study, I provided PSTs with an opportunity to work with ELLs in one-on-one settings and examine what strategies they used to support ELLs as they solved mathematical word problems. The primary research focus of this study was PST learning for the ultimate purpose of designing an effective model of ELL education within mathematics teacher preparation programs.

**Research Questions**

This study seeks to investigate PSTs’ thinking, planning, and use of strategies to help ELLs make sense of highly cognitive demanding mathematical problems, which embed sophisticated language use. Hence, it is critical for teachers of ELLs to implement
MP1, “Make sense of problem, and persevere in solving them,” by providing appropriate scaffoldings in both mathematics and language. The research questions for this study were:

1. How do middle school mathematics PSTs help ELLs make sense of high cognitive demanding mathematical problems in one-on-one setting with consistent intervention?
   a) What factors do PSTs consider in order to implement high cognitive demanding mathematical problems for ELLs?
   b) What supports do PSTs provide for ELLs to analyze the meaning of a problem and look for entry points to its solution?
   c) How do PSTs guide ELLs to explain their reasoning process and solution pathways?
2. In what ways, if any, do PSTs demonstrate a shift in planning and implementing mathematical problems as they work with ELLs and receive structured interventions on their practices?

**Conceptual Framework**

The lens I use for this study is the framework for analyzing mathematics instructional contexts developed by Campbell, Adams and Davis (2007). The framework was designed for ELLs and extended from the constructivist model of mathematics teaching (Simon, 1995). As Simon reconstructed mathematical instruction based on his reflection, this proposed framework also forms a cycle of reflection for planning and implementing instruction. However, Campbell et al. (2007) found Simon’s model did not include some components such as explicit considerations of language, culture, and prior
experiences. Hence, they filled out the missing part of Simon’s model by adopting theories of language and culture (Cuevas, 1984; Echevarria, Vogt, & Short, 2004a) and employed various theories such as symbolic interaction theory (Blumer, 1969), compatible aspects of phenomenology (Berger & Luckman, 1973), socio-cultural approach to mediated action (Wertsch, 1991), and cognitive load theory (CLT; van Merriënboer & Sweller, 2005). The developed framework describes the problem solving process for ELLs in four components: (1) mathematical content, (2) cultural and life experiences, (3) mathematical and cognitive processes, and (4) mathematical and contextual language.

In the development process, Campbell et al. considered three critical components of mathematical pedagogy: classroom as a socially constructed learning environment, teacher’s perceptions, and student’s perceptions. The three components interact with each other and they are mediated by language, culture, and mathematics. In constructivist view, the teacher’s perceptions of student learning and classroom environment are mediated by the teacher’s prior experiences. When a teacher and students live in the same social or cultural community and share common experiences, the teacher’s perceptions of the students’ cognitive models of mathematics can be reasonably effective. However, when they do not share language, previous experiences, or culturally based assumptions in teaching and learning mathematics, it is difficult for the teacher to predict how students build their cognitive models for learning mathematics. Therefore, it is necessary for teachers and students to establish goals, meanings, and perspectives together in the context of mathematical tasks (Moschkovich, 2004).
Campbell et al. (2007) view solving mathematical problem as a complex practice, which involves a learner’s life experience, language, cognitive processes, and knowledge of and the ability to apply mathematical content (Figure 1). In their view, the problem-solving process begins with establishing a problem space in which the cognitive activity of the problem interacts with other components. The problem space is created when a problem solver starts to read a problem and analyze the mathematical situation in the problem based on prior experiences and knowledge. Through this initial analysis, he or she needs to identify the cognitive processes and mathematical language structures in order to make a decision to plan a solution pathway. In the problem space model (Figure

1), Campbell et al. (2007) note that the problem-solving process begins in the upper right box (mathematical and cognitive processes) and interacts with the lower right box (mathematical and contextual language) and the upper left box (culture and life experiences). Then, strategies emerge in interactions with the solver’s experiences related to the context of the mathematics problem. Understanding the context and finding proper strategies eventually yield the decision of how to use related mathematical content.

Therefore, mathematics teachers need to consider that prior knowledge and life experience, which is implied in the language and situations used in the context of mathematical problems, may not correspond with student’s experience, especially with ELLs. If mathematics teachers are aware of this fact, they can see it is necessary to identify the interactions among four elements as we see in Figure 1 and to reflect the elements on implementing mathematical problems for ELLs. It is noteworthy to point out that teachers who are able to use strategies for helping ELLs benefit other students as well (Adler, 1999) because culture, language, and previous experience also influence non-ELLs. By considering these components, teachers can examine their own instructional practices and monitor students’ thinking process of solving mathematical problems.

However, while I collected data, some strategies that did not fit into the four existing components of this framework emerged and seemed to significantly impact both teaching practice and learners’ participation. Thus, I added one additional component, *positioning*, to the framework. I will explain the details of the strategies later, but it is clear that those strategies are related to positioning theory. Harré and Lagenhove (1999) defined positioning theory as a study of “local moral orders as ever-shifting patterns of
mutual and contestable rights and obligations of speaking and acting” (p. 1). For example, people can consciously or unconsciously position themselves and others as powerful or powerless, authorized or unauthorized, or dominant or submissive in their conversations. How teachers position themselves and their students could influence learning environments, especially students’ participation in learning (Yamakawa, 2014).

![Diagram](image)

*Figure 2. Modified problem space model*

Based on positioning theory and the findings from data, I added the fifth component, positioning, to Campbell et al.’s framework as shown in Figure 2. Moreover, although Campbell et al. (2007) proposed the model to explain student actions in solving problems, I use it as a framework of teacher actions and strategies employed to help students understand and solve mathematical problems because it is important for teachers to anticipate students’ learning process to make various decisions for instructions (Simon, 1995). In order to provide effective support in problem solving, teachers need to consider what students think and use during the process of problem solving. Moreover, when
teaching, the problem-solving process could start at any component and proceed with interactions among multiple components. The positioning component is placed outside of the problem space, but interacts with the other four components in the framework model, because positioning itself does not fit in any of the existing four components. The ways teachers position themselves or students could encourage or restrict students’ participation in the problem-solving process or mathematical discussion.

Campbell et al. (2007) provide specific questions to guide teachers to plan and implement instruction for ELLs in each component. The guiding questions and explanations, adopted and extended by the researcher, are following. These questions help develop a specific guideline for analyzing data, especially for developing the code manual.

**Mathematical Content**

**How experienced are students with mathematics concepts and procedures?**

When students do not have sufficient content knowledge required to understand a problem, the cognitive level of the problem becomes higher than the teacher planned. Hence, it is crucial for teachers to provide an appropriate level of problems to maximize the learning opportunity. More specifically, teachers need to identify what mathematical concepts are required to understand and solve a problem and check students’ prior content knowledge before they begin to work on the problem. For students who do not have solid understandings of prerequisites, teachers should be ready to provide opportunities for ELLs to build meaning for all required mathematical concepts.

**How do teachers challenge or reinforce students’ understanding in mathematics?** After assessing students’ prior knowledge, teachers have a clearer picture
of how to design instruction to meet students’ needs. A teacher may want to include challenging questions that require higher cognitive demanding work or complex procedures. If students need to develop foundational understandings of a certain concept, teachers may need to develop additional activities or questions. It is also possible to modify a problem to adjust its cognitive demand according to students’ prior knowledge and capability. For example, a teacher would break down a complex problem into micro steps by adding sub-questions to make it easier. However, this kind of adjustment could reduce the cognitive demand of the original problem unnecessarily because the modified problem suggests a certain solution pathway rather than allowing students have a chance to explore their own solutions (Stein, Smith, Henningsen, & Silver, 2009).

Cultural/life Experiences

What knowledge of cultural or life experiences is needed to understand the problem statement? Curriculum materials often use cultural contexts that are popular or common in the U.S. such as famous people, sports, or historical events. ELLs, however, may not have common cultural experiences such as using a coin-operated laundromat, playing football, or cooking Thanksgiving turkey. One common approach to culture in mathematics classroom is to use ethnical tools or materials such as creating a model of Mayan base-20 system to help Latino/a students appreciate their heritage (Furner, 2009). However, the meaning of culture includes more than ethnicity. For example, Carter (2000) defines culture as “learned patterns of thought and behavior that are passed from one generation to another and are experienced as distinct to a particular group” (p. 865). Hollins (2008) argues that the definitions may provide insights into human behavior and cognition, but not be sufficient for school learning experiences in a culturally diverse
environment. She recommended PSTs should be “encouraged to systematically construct a working definition of culture that will guide decision making in planning instruction for different populations of students” (p. 35). Being aware of students’ culture including their first language helps teachers find appropriate mathematical curriculum and pedagogical strategies.

What connections need to be made between the mathematical problems and student experience? Krashen (2004) claims that bilingual education is helpful for ELLs to learn English in their grade-level curricula because students’ first language serves as a tool to learn new content knowledge and develop English proficiency. Language is not the only medium for teaching ELLs. Their prior experiences should be connected to mathematics instruction in order to engage them in mathematical learning and make mathematics accessible to them. Some examples of teacher actions for cultural connections are using familiar contexts and examples as well as greeting students with their first language.

Mathematical and Cognitive Processes

What mathematical processes are needed and how experienced are the students at using them? The Principles and Standards for School Mathematics (NCTM, 2000) identify five mathematical process standards: problem solving, reasoning and proof, communication, connections, and representations. Although problem solving is the first standard, all other standards are also involved in the problem-solving process. The teacher’s role is to monitor and guide how students develop their cognitive process through reasoning, communicating with others and self, and connecting to other subjects and real life. Similarly, the Principle to Actions (NCTM, 2014) included “implement
tasks that promote reasoning and problem solving” as one of eight Effective Teaching Practices. Moreover, as students engage in building their cognitive model, they are encouraged to think as mathematicians do (Cuoco, Goldenberg, & Mark, 1996; Lesh & Doerr, 2003). For example, they visualize a problem situation, organize a plausible conjecture, and analyze their hypothesis. Thus, teachers need to identify how to facilitate students’ development of these processes.

**What cognitive processing skills are needed?** Campbell et al., (2007) assert that students do not naturally develop cognitive processing skills. Instead, a rigorous instruction such as well-designed examples and instructional guidance is required to foster the skills (van Merriënboer & Sweller, 2005). One major tool for helping student’s cognitive process is scaffolding, which is “a form of support provided by the teacher (or another student) to help students bridge the gap between their current abilities and the intended goal” (Rosenshine & Meister, 1992, p. 26). In this sense, the basic idea of scaffolding shares its root with *the zone of proximal development* (Vygotsky, 1978). Van Merriënboer, Kirschner, and Kester (2003) explain how scaffolding works through a combination of performance support and fading. The fading process means that the amount of supports should be gradually decreased as a learner achieves each small step of solving a problem or understanding a concept. For teachers, it is necessary to consider a right amount of scaffoldings because excessive or insufficient support can take student’s learning opportunity away. Moreover, they need to choose a right type of scaffoldings to support students to develop cognitive processing skills such as making conjectures, analyzing givens, or creating coherent representations.
Mathematical and Contextual Language

Do the students’ prior experiences include the development of mathematical language and the development of the natural language in the learning of mathematics? Some researchers argue that using students’ informal language helps ELLs learn mathematics (Echevarria et al., 2004a; Herbel-Eisenmann, Cirillo, & Skowronski, 2009; Moschkovich, 2000). The natural language in students’ first or second language that ELLs bring to classrooms does not represent their deficiency. Rather, it can be considered as a resource (Moschkovich, 2007b) because they can use the informal language to build cognitive model before they are introduced to academic vocabulary or asked to use academic language to express their understanding. Cummins (2000) explains the distinction between academic language and everyday language, and discusses how they interact with context. Although the two types of languages are not completely independent, it is important for teachers to identify differences because they need to understand what language ELLs have not developed yet and how to use informal language for learning academic language. According to Cummins (2000), language learners usually take much longer to develop academic language than everyday language. If teachers are not aware of this fact, they might assume the ELLs who are fluent in everyday language do not need any support in classrooms.

Moreover, teachers need to alter the belief that mathematics is the universal language. This belief may be rooted in another misconception that mathematics is all about numbers, which is only true in a part of mathematics. In fact, mathematical tasks include many linguistic factors. For example, students need to read mathematical problems to determine what they have to find and what processes they have to apply to
find the solution as well as to communicate their thinking through written or oral explanations. All of these activities require a certain level of language skills. The linguistic factors in mathematics have been addressed under various names such as mathematical discourse (Cobb, Stephan, McClain, & Gravemeijer, 2001; Moschkovich, 2007c), mathematical literacy (Adams, 2010; Beal, Adams, & Cohen, 2009), or mathematical discussion (Moschkovich, 1999).

**Does the language used in the problem or instruction correspond to the level of English language development of ELLs?** The language development of ELLs does not correspond with the language level in their grade-level curriculum. Furthermore, they are sometimes not allowed to enroll in advanced classes such as algebra for 7th- or 8th-grade students because of their ELL status (Campbell et al., 2007). It is crucial for mathematics teachers to find an appropriate level of problems for ELLs. If the language difficulty is too high, ELLs are unable to understand the problem, and if a teacher simplifies the problem statement too much, ELLs are not going to have an opportunity to develop their language skills. With selecting an appropriate level of language in mathematical problems, mathematics teachers also need to identify unfamiliar words such as academic words or cultural words and help ELLs develop meaning for them.

**Are there words that have specialized meanings in mathematics that have different meanings in natural language?** Some words have multiple meanings in a mathematical context, which might include a different meaning from the general definition in students’ ordinary routine. For example, when the problem involves the concept of rounding numbers, some students, especially ELLs, may have confusion because what they might picture from the word “round” is a circular shape like a round
table, not estimating numbers to the nearest specified place value. Therefore, the meaning of word only becomes clear in the given context shared by all participants in the mathematical discussion, and the context involves not only academic knowledge but also social and cultural aspects (Moschkovich, 2004). For this reason, teachers need to check carefully if ELLs understand the problem correctly even if they say there is no unknown word in the problem.

**Positioning**

How do teachers position themselves and students? When people position themselves or others, they usually do not intend such positioning (Harré & van Langenhove, 1999). Rather, it takes place unconsciously or simply follows a common cultural norm. For teachers, the traditional position is described as authority; in the authority, teachers give instructions to students and expect them to follow the given rules. Wagner and Herbel-Eisenmann (2014) suggested four categories of teacher discourse patterns in mathematics classroom according to the use of teacher authority: Personal authority, discourse as authority, discursive inevitability, and personal latitude. According to the authors, teachers who used the last pattern, personal latitude, allow students, not only teachers, to participate into decision-making process. Eventually, this authority pattern leads a student-centered approach by allowing students to have authority as well as the teacher.

How does positioning influence ELLs’ participation in mathematical discussion? Positioning relates to identity. Howie (1999) states that “positive feedback to the self-advocate in terms of acceptance of the self-advocate’s self-positioning and viewpoint, and positive action in response to expression of preferences and needs, will
encourage further positive self-positioning and self-advocacy.” According to him, teacher’s positive feedback encourage students’ effort to establish positive identity, or a strong self-esteem. This kind of positioning may encourage students to actively participate in classroom discussion because it increases their confidence in academic learning. This seems particularly important in case of ELLs because they are typically positioned by others or themselves inequitably in peer-to-peer or whole-class discussions due to their lack of English proficiency (Pinnow & Chval, 2014). In this sense, Yoon (2008) argued that the ELLs’ levels of participation differ according to the teachers’ pedagogical approaches and their interactions with the ELLs, which were based on their positioning of themselves as teachers.

**Significance**

This study contributed to the field of mathematics education with authentic information on how to better support PST learning in teaching ELLs. This topic, how to teach mathematics for ELLs, was relatively new in mathematics education research. There were few research articles related to ELLs in mathematics education (Estapa, 2012). Moreover, many researchers focused on transformation of PST belief (Artiles & McClafferty, 1998; Downey & Cobbs, 2007; Durgunoglu & Hughes, 2010; Pappamihiel, 2007) rather than their pedagogical learning in mathematical instruction. Another deficiency in ELL study stems from the ambiguity between culturally diverse learners and ELLs. Although these two subgroups of students have different characteristics and different needs, they are often used interchangeably because many culturally diverse learners are also ELLs, and a large portion of ELLs in the U.S. falls into one cultural group, Latinos (Fry, 2007). In her review, Janzen (2008) found that there are a number of
articles (e.g. Rodríguez & Kitchen, 2005; Secada, Fennema, & Byrd Adajian, 1995) in the field of mathematics education that address the issue of preparing PSTs to work with culturally diverse students. However, these volumes usually do not include discussions related to working with ELLs.

Designing a linguistically and conceptually rich lesson in order to make it accessible to ELLs is an important skill for mathematics teachers of ELLs. Hence, PSTs have to develop this skill in their preparation programs and understand what they should consider as they design mathematics lessons for ELLs. The teaching practices examined in this study may illuminate what teachers need to learn to help ELLs make sense of mathematical problems with linguistic scaffoldings and also fill the gap in the literature with regard to the implementation of mathematics tasks with ELLs. Mainstream classroom teachers who have never learned any structured ELL pedagogy may find this study useful to prepare themselves for culturally and linguistically diverse students. Moreover, one benefit of this study is to support mathematics teacher educators because this study will provide a unique glimpse into how PSTs design lessons and interact with ELLs and also provide a foundation that will help mathematics teacher educators see what PSTs need and how to support their development in the area.

**Definitions**

**English Language Learner**

Ballantyne et al. (2008) define ELLs as “who are not yet proficient in English and who require instructional support in order to fully access academic content in their classes” (p. 2).

**Cognitive Demand**
Stein, Smith, Henningsen, and Silver (2009) define cognitive demand as “the kind and level of thinking required of students in order to successfully engage with and solve the task” (p. 1). This is similar to cognitive load in the cognitive load theory (CLT) (van Merriënboer & Sweller, 2005). Although Campbell et al. (2007) follow CLT closely, I would like to use the definition and the criteria of cognitive demand level provided by Stein, Smith, Henningsen, and Silver (2009) because they define it in the mathematical context.

**Scaffolding**

Grounded on the idea of the zone of proximal development (Vygotsky, 1978), scaffolding means an instructional process in which strategic control of learning is transferred from experts to novices (Campbell et al., 2007; van Merriënboer et al., 2003) and includes all materials and strategies provided by a teacher or a more experienced peer to support students to achieve their learning goals.

**Sense Making or Making Sense**

In general, sense making or making sense of problems can be interpreted as understanding. More specifically, the NCTM (NCTM, 2009) defines sense making as “developing understanding of a situation, context, or concept by connecting it with existing knowledge” (p. 4). With these definitions, the description of MP1 is used as a working definition of making sense of mathematical problems.

**Problem Solving**

NCTM (2000) defines problem solving as “engaging in a task for which the solution method is not known in advance (p. 52).” Similarly, the Program for International Student Assessment (PSIA) project provides the definition of problem
solving as follows, “Problem solving is cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver” (Bell & Burkhardt, 2002, p. 2).

**Summary**

Although a population of ELL students rapidly increases, many mathematics teachers are not adequately prepared to teach ELLs. Responding to the current needs from both research and field, this study aims to provide useful recommendations to prepare PSTs to teach highly cognitive demand mathematical problems for ELLs. Hence, the modified problem space model of Campbell et al. (2007), which was developed to help PSTs teach ELLs problem solving, was used as a conceptual framework for this study. This study investigated how three middle school PSTs helped ELLs make sense of cognitively demanding mathematical problems and how their use of ELL strategies evolved as they received structured interventions and had experience with ELLs.

In the next chapters, I provide a review of literature relevant to ELLs, PSTs, and problem solving in mathematics education. The chapter 3 is organized to provide a sufficient description of the research design, data collection, and analysis methods. The findings are provided in chapter 4, and lastly, I discuss the results and implication as well as limitations and suggestions of future research.
CHAPTER 2. LITERATURE REVIEW

In this chapter, I discuss previous studies and findings to describe how PST learning and making sense of mathematical problems has been discussed related to ELLs. I first briefly overview recent ELL studies, and then describe two groups of literatures: PST learning and problem solving. Within the second and third sections, I revisit the topic of ELLs to further discuss how each topic is elaborated in a view of ELL research. There might not exist perfect one-to-one matches between general studies and ELL specific studies, but I believe it is meaningful to compare and contrast them and seek to find related points so that they connect to each other.

**English Language Learners in Mathematics Education**

The body of ELL studies has been growing, but it seems who ELLs really are and what we need to study about them vary across researchers. In this section, I seek to elucidate the definition of ELLs in the U.S. and introduce previous studies of ELLs in mathematics education.

**Defining ELLs**

The criteria for identifying ELLs differ according to researchers and states (Young & King, 2008). It is not uncommon that students categorized as ELLs in one state are not identified as ELLs in another state. Some researchers follow the regulation of the state or the districts where their participants reside or use English language proficiency (ELP) assessments including California English Language Development Test (CELDT). ELLs are often categorized with other minority groups such as bilinguals, Latinos, culturally diverse learners, or immigrant students. Limited English proficient (LEP) students and English second language (ESL) learners are also the names used for ELLs. Since the majority of ELLs in the U.S. are Latinos, this particular group sometimes
represents ELLs (Garrison & Mora, 2005; Ortiz-Franco, Hernandez, & De la Cruz, 1999; Téllez, Moschkovich, & Civil, 2011). In addition, the statistical fact that approximately 59% of middle school and high school ELLs qualify for free or reduced price lunch (U.S. Department of Agriculture, 2007) leads the research of ELLs to naturally involving students with low socio-economic status (SES). Therefore, to avoid any possible confusion on the research subjects, it is important to clarify the meaning of ELLs.

Culturally diverse learners are the largest group among all of those ELL related groups (Figure 3) because there are some culturally diverse learners who are neither bilinguals nor immigrants such as African-American or Native American students. Bilinguals and ELLs also fall into the set of culturally diverse learners because they obviously have different cultures from English-only students and language is an essential part of culture. However, not all bilingual or multilingual students are ELLs because some of them already became fluent in English or English might be one of their first languages. I do not include the set of Latinos in the diagram (Figure 3) because no other particular ethnic groups are included in it. However, it is evident that not all Latinos are ELLs and vice versa (Gutierrez, 2002). Similarly, immigrant students are not always ELLs because they could emigrate from English speaking countries such as England, Canada, or Australia. Another essential distinction should be made between ELLs and immigrant students in the sense that ELLs are not a subset of immigrants. In the U.S., more than 75% of elementary ELLs are second-generation or even third-generation American (Grantmakers for Education, 2014) who were born in this country. They may remain in ELL classrooms because they have lived in only their ethnical communities and have not had sufficient opportunities to learn English academic language from adults. Finally, LEP students—in the smallest subset—are those who have not yet passed ELP assessment according to Ballantyne, Sanderman, and Levy (2008). It should be noted that it does not imply students who are released from LEP status by achieving ELP
assessment requirement do not need any supports for understanding academic language in content area classrooms (Francis, Rivera, Lesaux, Kieffer, & Rivera, 2006).

As described, defining ELLs is complicated because multiple terms are used and they intersect each other or are subsets of other groups. For example, although the studies of culturally diverse learners do not particularly concentrate on language issues, they cannot be completely excluded from the ELL theme because culturally diverse leaners include immigrant students, bilingual students, and ELLs.

![Diagram](image)

*Figure 3. Categories and relationships of ELL related student groups*

There exist many different definitions of ELLs. Moreover, as Echevarria, Powers, and Short (2006) indicate, it is hard to define ELLs as one particular group because they demonstrate a wide range of language proficiencies and of subject-matter knowledge as well as diverse cultures, Socioeconomic status, age of arrival in the U.S. and personal experiences living in the U.S. The Census Bureau data (U.S. Department of Education, 2004) define linguistic minority students as those who speak a language other than English at home. Furthermore, the federal No Child Left Behind (NCLB) legislation
suggests detailed criteria of ELLs in K-12 schools as follows: “[1] who were not born in the U.S. or whose native language is a language other than English, [2] who come from an environment where a language other than English has had a significant impact on the individual’s level of English language proficiency, and [3] whose difficulties in speaking, reading, writing, or understanding in English may be sufficient to deny the individual the ability to meet the state’s proficiency level of achievement or the ability to successfully achieve in the classroom where the language of instruction is English” (Young & King, 2008, p. 1). The third one may include native speakers with low English ability or with reading/writing disorders. However, this study does not include those students, so ELLs in this study have sufficient academic ability but lack English proficiency due to their different cultural and linguistic backgrounds. Thus, I will follow Ballantyne et al. (2008) who define ELLs as those “who are not yet proficient in English and who require instructional support in order to fully access academic content in their classes” (p. 2). This definition is simple and conveys the basis of other definitions, and more importantly, it focuses on instructional support.

**Previous Studies of ELLs**

Moschkovich (2007a) describes three views of ELL studies in mathematics education. According to her, early studies of ELLs in mathematics education tend to pertain to the belief that students need to have basic language skills prior to learning mathematics. These studies (e.g., Dale & Cuevas 1987) focused on low-cognitive demanding tasks (Stein et al., 2009) such as simple computation or memorization. Recent trends in mathematics education (National Council of Teachers of Mathematics (NCTM), 2014; NGA Center & CCSSO, 2010) that emphasizes critical thinking, reasoning, and problem solving do not support the effectiveness of this approach. Furthermore, focusing
on ELLs’ basic language skills and providing only low-cognitive demanding tasks has been criticized for stereotyping ELLs as low performers. Another group of ELL research highlights different kinds of language such as the conversational/academic language (Cummins, 1979, 2000) or communicative/analytic competence (Bruner, 1975). Although this approach has provided useful guidance, Moschkovich (2007) warned the separation of academic language and everyday language since the distinction should be seen in each situative context. Finally, there are sociocultural studies, which consider student learning occurs in social and cultural contexts. In this view, mathematics is defined as discourse activity, which includes using multiple material, linguistic, and social resources (Greeno, 1998), and learning is constructed through interactions with others and social activities (Forman, 1996; Lave & Wenger, 1991; Nasir, 2002; Razfar, Khisty, & Chval, 2011).

Understanding discourse in mathematics learning requires more than knowing the dictionary meaning of vocabulary. Gee (1996) defines Discourses as follows:

A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and ‘artifacts’ of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or ‘social network,’ or to signal (that one is playing) a socially meaningful role (p. 131).

This extended perspective of discourse helps us understand how ELLs make sense of mathematical concepts since they learn mathematics through processing language and other forms of discourse such as diagrams, pictures, or symbols. Cobb, Stephan, McClain, and Gravemeijer (2001) also provide a specific meaning of discourse in mathematics as “taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas” (p. 126).
Moschkovich (2007b) deepens and elaborates the discussion of discourse in mathematics. She uses the term *Discourse practices* to emphasize that the extended meaning of discourse– which Gee suggested above–includes not only languages but also perspectives and conceptual knowledge. The following statement explains her perspective about the cognitive and cultural aspects of *Discourse practice*.

Discourse is not disembodied talk; it is embedded in practices. Words, utterances, or texts have different meanings, functions, and goals depending on the practices in which they are embedded. Discourses occur in the context of practices and practices are tied to communities. I view Discourse practices as dialectically cognitive and social (Moschkovich, 2007b, p. 25).

In other words, *Discourse practices* in mathematical discussion embrace previous experiences and perspectives of learners and teachers. From this view, we assume that misunderstandings may occur in mathematical discussions when the teacher and his/her students do not have shared experiences. More importantly, ELLs are not deficient learners in this perspective because they are able to communicate mathematically through gestures, pictures, and graphs, which facilitate mathematics learning.

Moschkovich (2012) also recommends that mathematics teachers of ELLs should follow general guidelines for high-quality mathematics instructions. Moreover, it is crucial to use and maintain highly cognitive demanding mathematical tasks, for instance, by encouraging students to explain details of problem solving and the reasoning process they applied (American Educational Research Association, 2006; Stein & Henningsen, 1996). This is a different perspective from the common belief that ELLs should not be given higher-order thinking tasks due to their lack of language comprehension (Reeves, 2006). The case study of de Araujo (2012) also reveals that secondary mathematics teachers tend to select low cognitive demand mathematical tasks for ELLs. In this sense, effective instruction for ELLs uses language as a resource and emphasizes academic
achievement rather than just improving language skill (Gándara & Contreras, 2009).

Based on this belief, Echevarria, Vogt, and Short (2004) assert that teachers of ELLs need to plan and state language objectives as well as content objectives in their lesson plans. While Echevarria et al. (2004) seem to weigh both kinds of objectives evenly, Moschkovich (1999) places mathematical learning above low-level linguistic learning. For example, she asserts correcting grammar or memorizing definitions of mathematical terms should be pursued less than mathematical learning such as mathematical reasoning or mathematical discussions. However, both agree that, in order for ELLs to achieve academic success in mathematics class, it is important to provide linguistic supports aligned with general learning strategies in mathematics instructions (Richard-Amato & Snow, 2005).

Furthermore, various experiments and empirical studies (Garrison and Mora 2005; Gutstein et al. 1997; McLaughlin and Talbert 2001; Pearn 2007) have been conducted. They tend to avoid teacher-centered lecture and attempt to employ alternative teaching strategies such as physical activities (Ahn, I, & Wilson, 2011; Moses, 2002), gestures (Fernandes, 2012a; Roth, 2001), manipulatives (Boakes, 2009; Bustamante & Travis, 2005; Pearn, 2007), or students’ cultural connections (Furner, 2009). For instance, Help with English Language Proficiency (HELP) Math was created to apply the Sheltered Instruction Observation Protocol (SIOP) principles in an online environment (Freeman & Crawford 2008). The program incorporated the SIOP strategies such as visual aids, scaffoldings, first language service, and building background. The initial results show that ELLs responded effectively with HELP Math software in developing both mathematical knowledge and academic language. In sum, Chval and Chavez (2011) suggest seven strategies that support ELLs’ mathematical proficiency based on existing ELL research such as connecting students’ life experiences, creating classroom
environments rich in both language and mathematics, and use visual supports. Similarly, Celedon-Pattichis and Ramirez (2012) provide five guiding principles of mathematical instruction for ELLs. The principles include providing challenging tasks, building linguistically sensitive social environment, and considering cultural and linguistic differences as intellectual resources.

Lastly, there have been other approaches that relate ELL studies to culturally responsive pedagogy (Villegas & Lucas, 2002) and positioning theory (Harré & van Langenhove, 1999) although these features are interrelated to each other. Culturally responsive teachers “not only know their students well, they use what they know about their students to give them access to learning” (Villegas & Lucas, 2002, p. 27) and “help students make connections between their local, national, racial, cultural, and global identities” (Ladson-Billings, 2009, p. 28). Ahn et al. (2015) examined the culturally responsive teaching practices in mathematics classrooms of culturally and linguistically diverse groups in the U.S. and Japan. Their findings reveal that the teaching practices in both places share similar pedagogical methods, such as using everyday language prior to academic language, integrating literacy throughout instruction, and emphasizing critical thinking through mathematical discussions. In addition, Aguirre and del Rosario Zavala (2013) argue that culturally responsive mathematics teaching (CRMT) is essential to improve all student learning in mathematics and introduce a tool to help teachers evaluate their mathematics lessons under six categories: (1) cognitive demand, (2) depth of knowledge and student understanding, (3) mathematical discourse, (4) power and participation, (5) academic language support for ELL, and (6) cultural/community-based funds of knowledge.
Positioning theory is closely related to social justice or power relationship because positioning is placed in social interactions and structures. However, it provides another theoretical and psychological background of social phenomena. Positioning is understood as the “discursive construction of personal stories” (Harré & Van Langenhove, 1999, p. 16) that determines each person’s locations within the social interactions or conversations. In mathematics education, the impact of positioning is significant as Wagner and Herbel-Eisenmann (2009) claim that “changing the way mathematics is talked about and changing the stories (or myths) told about mathematics is necessary for changing the way mathematics is done and the way it is taught” (p. 2). Moreover, positioning is strongly influential when teaching minority groups such as ELLs because inequitable positioning may result less learning opportunities given for ELLs (Pinnow & Chval, 2014; Yoon, 2008). In this vein, when a teacher positions ELLs as successful contributors of the learning community the participation of ELLs increases. As an example, Pinnow and Chval (2015) examined the discourse occurring in a mathematics classroom focusing on a Latino ELL to analyze how he positioned himself and was positioned by peers and his teacher in their social interactions. In the episode, the authors describe how the teacher positioned the ELL as a competent problem solver through her verbal expressions and physical actions.

Preservice Teacher Learning

In the view of constructivism, teachers are active thinkers and decision makers who craft knowledge about subject matter, pedagogy, and their students (Cochran, DeRuiter, & King, 1993). Teachers construct, transform, and develop their pedagogical content knowledge (PCK; Shulman, 1986) based on their own prior and current
perspectives and the social context in which they live (Artilès & McClafferty, 1998).

Similarly, Ball (1988) states that teachers are also learners who possess diverse pre-existing knowledge and experiences and the lack of attention to what PSTs bring with them to learning to teach mathematics may explain why teacher education is often “such a weak intervention” (p. 41). Since this study aims to investigate PST learning to teach ELLs mathematics, I include literature with regard to teacher education and PST learning in this section.

**Learning to Become a Mathematics Teacher**

University-based teacher preparation programs typically consist of coursework and fieldwork requirements. PSTs are supposed to develop their pedagogical knowledge and PCK through the coursework and practical experiences. However, there exists criticism that current practices in teacher education do not satisfy the needs of the field (Brown & Borko, 2007; Walsh, 2013). Walsh (2013) argues “Teacher education has long struggled both to professionalize and to fully integrate itself into mainstream academia. At the core of this struggle was a perception that there was no body of specialized knowledge for teaching that justified specialized training” (p. 19). Despite the existing criticism on teacher preparation programs, there is evidence that the programs have an important role in educating future teachers. Alderman and Beyeler (2008) found that PSTs transfer what they learn from teacher preparation program to their teaching practice in the field, and Richardson and Liang (2008) address the impact of an inquiry-based course on elementary PSTs’ efficacy in mathematics and science. Some researchers seek to examine the effectiveness of field experiences (Charalambous, Philippou, & Kyriakides, 2008; Clark, 2005). Charalambous, Philippou, and Kyriakids (2008)
emphasized the importance of fieldwork as, “fieldwork comprises one of the most crucial parts of teachers’ preservice education, for it helps them integrate theory and practice, especially through critical observation and analysis of lessons taught by experienced teachers (p. 128).” In addition, Simon, Tzur, Heinz, Kinzel, and Smith (2000) applied constructivist perspectives to suggest a conception-based framework for teacher learning. Using the framework, they conducted the Mathematics Teacher Development Project including both PSTs and inservice teachers. Since “mathematical learning is a process of transformation of one’s knowing and ways of acting” (p. 584), they claim that teacher educators have to clarify goals for teacher development, to understand teachers’ current conceptions, and to generate useful hypotheses of how to proceed this development.

Overall, the goal of mathematics teacher education is to foster effective teachers, who perform the Mathematics Teaching Practices recommended by the Principles to Actions (NCTM, 2014):

1. Establish mathematics goals to focus learning;
2. Implement tasks that promote reasoning and problem solving;
3. Use and connect mathematical representation;
4. Facilitate meaningful mathematical discourse;
5. Pose purposeful questions;
6. Build procedural fluency from conceptual understanding;
7. Support productive struggle in learning mathematics;
8. Elicit and use evidence of student thinking. (p. 10).

Although all teachers are recommended to follow these eight standards to improve their teaching practices, the following section discuss what additional and extended consideration should be applied for PSTs to teach mathematics for ELLs.

**Preservice Teacher Learning for ELLs**

The five standards of the Council for the Accreditation of Educator Preparation (CAEP) are (1) content and pedagogical knowledge, (2) clinical partnerships and
practice, (3) candidate quality, recruitment, and selectivity, (4) program impact, and (5) provider quality assurance and continuous improvement (CAEP; 2013). However, Vomvoridi-Ivanovic and Chval (2014) argue that PSTs must learn the linguistic and cultural demands and needs in teaching and learning mathematics as follows,

“… conversations revolving around the development of PCK for PSTs must involve the critical aspects of language and culture. In other words, teachers must develop a deep knowledge of the linguistic and cultural demands that are unique to the teaching and learning of mathematics. This becomes more important when students speak (or are learning) more than one language (Valdés, Bunch, Snow, Lee, & Matos, 2005). Unfortunately, discussions in mathematics education have not given sufficient attention to developing teacher knowledge related to teaching ELLs, and this has direct implications for how mathematics teacher preparation is traditionally structured. As a result, in most U.S. teacher preparation programs, future teachers are not prepared to teach mathematics to ELLs. PSTs enter the profession with little knowledge about the needs, resources, and supports required to effectively teach mathematics to ELLs (Chval & Pinnow, 2010) (Vomvoridi-Ivanovic & Chval, 2014, p. 118).

This excerpt emphasizes the necessity of preparing PSTs to teach mathematics to ELLs and the importance of developing PSTs’ PCK related to the aspects of language and cultural demands of ELLs.

Findings from previous studies support the significance of developing PST’s PCK as well as content knowledge of mathematics. First, Ball (1990) reveals that PSTs lack explicit understanding of concepts when they perform computational procedures. The lack of conceptual understanding in mathematics for teaching indicates that PSTs had learned mathematics from their schooling through training rather than teaching (Glaserfeld, 1995) since the former yields only reproduction of a behavioral response while the latter seeks to generating autonomous conceptual understanding (Cochran et al., 1993). Based on the distinction, Cochran et al. (1993) define PCK as “a teacher’s
integrated understanding of four components of pedagogy, subject matter content, student characteristics, and the environmental context of learning” (p. 266).

In mathematics education, the idea of PCK is further developed and elaborated. Hill et al. (2008b) introduced mathematical knowledge for teaching (MKT) and its two subdomains: PCK and subject matter knowledge. PCK includes knowledge of content and students (KCS) and knowledge of content and teaching (KCT). They found a significant positive association between MKT and mathematical quality of instruction, and factors that mediate the association such as teacher beliefs about the ways of learning mathematics. According to the categories of MKT, knowledge to teach ELLs falls into KCS because teachers need to know about a particular student group. Hill et al. (2008a) propose the definition of KCS as “content knowledge intertwined with knowledge of how students think about, know, or learn this particular content” (p. 375). In ELL education, it is crucial to recognize the specific needs of ELLs. However, as I mentioned earlier, only less than half of all states require ELL related education for all teachers. De Jong and Harper (2005) claim that bilingual or ELL courses are not included as an integral part of teacher education for the assumption that teaching ELLs is a matter of pedagogical adaptations that can easily be incorporated into a content teachers’ existing instructional strategies.

Some argue that one of the crucial sociocultural factors that may impact students’ academic success is teacher beliefs and instructional approaches for ELLs (Durgunoglu & Hughes, 2010; Gutierrez, 2002). Artiles and McClafferty (1998) discussed how elementary PSTs change their perspectives on culturally diverse learners through a multicultural education course. The survey and observation showed participants had
positive attitudes regarding ELLs and their parents, but they did not feel well prepared to articulate the specific needs of ELLs. The researchers conclude that teachers who had lower self-efficacy have more stress and less stability, and PSTs clearly felt their unpreparedness to educate ELLs.

Downey and Cobbs (2007) and Pappamihiel (2007) performed empirical studies that offered opportunities to PSTs to teach mathematics for ELLs. Moreover, both studies focus on fieldwork in teacher preparation program. Downey and Cobbs (2007) noticed that there exists a lack of understanding on cultural practices and values in teacher education, so a single course in multicultural education is not enough to transform teacher beliefs. Their research was situated in a university-based teacher education program that requires elementary PSTs to complete one-on-one tutoring fieldwork including an interview with a culturally diverse student. The participants deepened their understanding in teaching mathematics and insight of the relationship between cultural diversity and mathematics learning. Pappamihiel (2007) argues that PSTs in content areas are situated in complexity because they need to teach language besides their subject curriculum for ELLs. The content-area PSTs spent 10 hours with an ELLs and the data were analyzed by the situative perspective (Lave & Wenger, 1991) and intercultural sensitivity (Bennett, 1993). Although many participants initially had misconceptions about ELLs, they altered their conceptions and finally obtained more acceptance and adaptation views towards culturally diverse students. Similarly, Fernandes (2011, 2012) observed how middle school PSTs noticed ELLs’ understanding of mathematics through task-based interviews. The results imply that the PSTs started adopting ELL strategies they learned from the intervention integrated into content courses taught by the author as well as became aware
of ELLs’ needs and challenges. Aligned with the results of the recent studies, McLeman, Fernandes, and McNulty (2012) found strong evidence from their large-scale survey that exposure to and learning about ELLs influence the conceptions of PSTs in mathematics education. They assert that providing field experiences with ELLs and reading ELL literature within teacher preparation program could be valuable for PSTs to learn instructional strategies that will support ELLs’ mathematical learning or to understand the linguistic complexity such as academic language or mathematical discourse. However, according to de Araujo, I, Smith, and Sakow (2015), merely providing fieldwork opportunity with ELLs does not make a significant difference when PSTs teach mathematics for ELLs. The result of this study implies PSTs need to receive a structured coaching to integrate ELL strategies in mathematical instruction. Teachers Radically Enhancing Education (T.R.E.E.) Project (Ahn et al., 2011; I, Ahn, & Catbagan, 2012) had a similar structure, but they provided 10 week training to PSTs using Five-Step Approach, an experiential teaching framework, benchmarked Algebra Project (Moses, 2002). In the T.R.E.E. project, ten PSTs were paired up to create and lead mathematical activities to work with a small group of Latino/a ELL students. The results revealed that multisensory activities helped ELLs understand abstract mathematical concepts.

Making Sense of Mathematical Problems

Defining Sense Making

Reasoning and sense making are at the heart of learning mathematics conceptually. It is hard to articulate a difference between reasoning and sense making. In practice, reasoning and sense making are intertwined across the spectrum from informal observations to formal deductions. Then, what is sense making? According to NCTM
(2009), the definition of sense making is “developing understanding of a situation, context, or concept by connecting it with existing knowledge” (p. 4). In this sense, the process of sense making naturally involves assimilation and accommodation in the view of radical constructivism (von Glasersfeld, 1995). Assimilation is a term referring to another part of the adaptation process, which taking new information or experiences and incorporating them into our existing ideas. The process of accommodation includes altering one's existing schemas or ideas, as a result of new information or new experiences. When students face a new mathematical concept or a new term, they need to incorporate it into their pre-existing mathematical knowledge or modify their prior knowledge to accept the new information. For problem solving, ELLs may confront an unfamiliar mathematical situation and language use; then, they may start the process of assimilation and accommodation to make sense of the new materials in light of their pre-existing schemas.

The CCSSM describe making sense of problems in MP1 as follows:

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. (NGA Center & CCSSO, 2010, p. 6)

According to this description, making sense of mathematical problems has an extended meaning beyond understanding what the problem is asking for. Students should “make conjectures about the form and meaning of the solution,” to “plan a solution pathway,” and even to “monitor and evaluate their progress.”
Making sense of mathematics is sometimes interpreted as “understanding.” The definition of understanding is also complex and ambiguous. Many psychologists and educators including Carpenter and Lehrer (1996), Davis (1992), and Putnam, Lampert, and Peterson (1990) have described what understanding means. The commonality of these definitions is that we understand something if we see how it is related or connected to other things we know (Brownell, 1935; J Hiebert & Carpenter, 1992). Carpenter et al. (1997) accepted the basic idea of this definition, but they argued that it does not reveal properly how people make connections between concepts. Thus, it is necessary to consider two other important processes in making of connections: reflection and communication. Based on this view, participating in social interaction, communicating thoughts with others and listening to others are crucial parts of understanding or making sense of mathematics.

Problem Solving

In mathematics education, the terms, problem and problem solving, have been circulated with multiple and often contradictory meanings through the years (Schoenfeld, 1992). The Programme for International Student Assessment (PISA) project uses the definition, “Problem solving is cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver” (Bell & Burkhardt, 2002, p. 2). Bell and Burkhardt (2002) also proposed their own definition: “Problem solving is the activity called into play when there is a demand to apply knowledge, skill and experience to unfamiliar situation” (p. 2). Problem-solving tasks are often problems that are situated in a real-life context, but Hiebert, Carpenter, and Fennema (1996) warn that this view may
Problem solving has been a core of reform mathematics education which began since *An Agenda for Action* (NCTM, 1980). The *Principles and Standards for School Mathematics* (NCTM, 2000) provide a definition of problem solving as “engaging in a task for which the solution method is not known in advance” (p. 52). The newly released *Principles to Actions* (NCTM, 2014) include eight mathematics teaching practices and one of them is to “implement tasks that promote reasoning and problem solving.” This teaching practice is associated with MP1 as it says “Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies” (NCTM, 2014, p. 10). All of these efforts denote that problem solving has a crucial role in mathematics education and all students are recommended learning about problem solving in their mathematics classrooms.

As one of the most seminal works in problem solving, Pólya (2004) proposed four phases of problem solving: Understanding the problem, devising a plan, carrying out the plan, and looking back. The first phase, understanding the problem, corresponds relatively to making sense of problems in MP1. Some useful questions suggested by Pólya, which teachers can apply in this crucial step are: What is the unknown? What are the data? What is the condition? Is the condition sufficient to determine the unknown? In addition, they can support students by asking to draw a figure or separate the various parts of the condition. Rather than phases of problem solving, Orton and Frobisher (1996) identified five processes within problem solving: Operational processes of collecting and
ordering data, mathematical processes of searching for patterns, reasoning processes of analysis and reflection, recording processes such as drawing or writing and communication processes in describing methods. Among these, the operational processes of collecting and ordering data, is similar to the first phase, understanding the problem, that Pólya proposed.

Studies on students’ mathematical thinking and problem solving have provided evidence that teachers’ knowledge of students’ concepts and misconceptions could significantly influence their instruction. For example, Carpenter, Fennema, Peterson, and Carey (1988) measured elementary teachers’ PCK for assessing students’ problem solving procedures under three categories: knowledge of problems, general knowledge of strategies, and teachers’ knowledge of their own students. These subsets correspond with subject matter content knowledge, knowledge of curriculum, and KCS respectively although the boundaries among them are unclear. They further sought the correlation between teachers’ knowledge and students’ achievement, but no clear correlation was found between them.

**ELLs and Sense Making of Problems**

In MP1, “explaining to themselves the meaning of a problem,” “make conjectures about the form and meaning of the solution,” and “understand the approaches of others to solving complex problems” require students’ linguistic skills in both comprehension and communication. If students are not able to understand the context of mathematical problems, how can they “look for entry points to its solution” although they have sufficient prerequisite content knowledge? If teachers do not carefully introduce mathematical problems to ELLs, mathematically capable ELLs may not have an
opportunity to participate in the problem-solving process. Therefore, Echevarria, Vogt, and Short (2004b) argue that instructional materials should be tied to students’ background and experiences, and strategies in order to scaffold students’ acquisition of knowledge and skills. In this sense, it is not surprising that building background is one of the beginning components in Sheltered Instruction.

There are various recommendations and suggestions about instructional strategies to teach mathematics for ELLs through literature (Celedon-Pattichis & Ramirez, 2012; Colombo, 2009; Echevarria et al., 2010; Vogt & Echevarria, 2008). More specifically, some researchers focus on ELLs’ achievement on mathematical problem-solving activities. For example, Secada et al. (1995) investigated relations between ELLs’ strategies when solving arithmetic problems and their language proficiency. Similarly, Chamot, Dale, O’Malley, and Spanos (1992) identified ELLs’ problem-solving strategies at different achievement levels and at the degree of teacher implementation. Further, Enright (2009) notes that the language of mathematical problems is often unnecessarily complex regardless of learning mathematics. For this reason, she provided guidelines for teachers to identify the language demands of word problems and reduce the language demands so that mathematical instruction is more accessible to ELLs. Some of her guidelines include using active voice at first, using flowcharts or graphic organizers, and incorporating cognates. Contreras (2009) developed a framework to help PSTs learn to generate mathematical problems systematically. Her framework starts from mathematical situation and designates a base problem, which is subdivided into proof problems, converse problems, special problems, general problems, and extended problems.
Many ELL studies focus on word problems because solving word problems with high linguistic demand is obviously challenging to ELLs in terms of their lack of background knowledge, unclear directions, or commonly used phrases (Gottlieb & Ernst-Slavit, 2013). Whang (1996) conducted an empirical study about ELLs with various English proficiency levels and found that students in the stage of transition toward bilinguals could have more difficulties in solving mathematics word problems than those in the earlier or later stages of transition because they are in a unstable linguistic stage. These studies are grounded on the belief that language shapes thinking and deeply involves learning process with the concept of mediation (Vygotsky, 1978). One important effort to detail the role of language in academic learning is Cummins (1984)’s basic social and interpersonal communication skills (BICS) and cognitive/academic language proficiency (CALP) although he warns that these two kinds of languages are not exclusively separated and CALP is not isolated attribute of learners (Cummins, 2000). ELLs may obtain BICS shortly, but it could take relatively longer to learn CALP. He also developed a model that emphasizes a degree of context with a degree of cognitive demand. Following this model, language learners should be challenged cognitively but provided with the contextual supports.

It is evident that teachers need to be aware of the role of language in mathematical instruction and problem solving. Lo Cicero, Fuson, and Allexaht-Snider (2005) outlined five steps of teacher-based problem solving as understanding the problem situation, problem solving, explaining the problem solving, discussing and reviewing the problem solving, and generating problems. Garrison and Mora (2005) analyze the language-concept connection in terms of unknown or known factors and the basic norm; for
example, use the known language to teach an unknown concept or use a known concept to teach an unknown language. Finally, the case study conducted by Chval, Pinnow, and Thomas (2014) describes the detailed intervention with one elementary teacher. The teacher was encouraged to use sophisticated words and forms an environment for ELLs to experience with those words in mathematical context (Khisty & Chval, 2002). Through the intervention, she developed understanding of the critical role of the contextual situations used in mathematical problems for ELLs and demonstrated the shift in practices in the sense of engaging ELLs into mathematical discussions.

Summary

This study lies on a joint point of three components: ELLs, PSTs, and problem solving in mathematics. I organized this chapter to be aligned with the three components. First, I developed the definition of ELLs in order to clearly distinguish this student group from other similar groups such as culturally diverse students. Then, previous research of ELLs, PST education, and problem solving are followed. While research addressing all three components is indeed scarce, the few studies commonly recommended the importance of PST education for teaching ELLs. For example, research related to PSTs’ fieldwork with ELLs (Ahn et al., 2011; de Araujo et al., 2015; Fernandes, 2012b; I et al., 2012) found that working with ELLs helped PSTs change their beliefs towards ELLs and made them competence in teaching mathematics for ELLs. Moreover, previous studies (Chamot, Dale, O’Malley, & Spanos, 1992; Contreras, 2009; Enright, 2009; Gottlieb & Ernst-Slavit, 2013) about ELLs’ problem solving or word problems recommends teachers prepare for linguistic difficulty ELLs may encounter while solving mathematical problems. In summation, researchers found that ELLs are able to learn problem solving
and participate in mathematical discussion (Moschkovich, 1999) if they are provided with appropriate supports. Hence, it is important that teacher preparation programs provide PSTs with rigorous ELL related courses including fieldwork with ELLs. In chapter 3, the research design and methodology I applied to this study are discussed.
CHAPTER 3. METHODS

This chapter describes the methods used in this research study. I first explain the reasons I chose the case study research method and provide a detailed rationale of each component of the research design. I also describe how I collected and analyzed various qualitative data to investigate PSTs’ teaching practices and interactions with ELLs in mathematical problem solving.

Case Study Research

Case study research is “the study of an issue explored through one or more cases within a bounded system (i.e. a setting, a context)” (Creswell, 2007, p. 73). Yin (2009) explains it in more details as “the case study method allows investigators to retain the holistic and meaningful characteristics of real-life events–such as individual life cycles, small group behavior, organizational and managerial processes, neighborhood change, school performance, international relations, and the maturation of industries” (p. 4). He also underscores case study research is distinguished from case studies that are used as a teaching method in business or medical field. Although defining case study as a research method is complex, researchers have accepted it as a strategy of inquiry, a methodology, or a type of design of qualitative research (Creswell, 2007; Merriam, 1998).

It is known that case study research is suitable for searching answers of who and why questions (Farquhar, 2012). Yin (2009) addresses that the case study method is useful if the questions seek to explain how present social phenomena work. More specifically, he clarifies the following conditions when case studies are preferred to other methods: (1) when, how, or why questions are being asked, (2) when the researcher has
Based on these criteria, I chose case study research method as the methodology of my research study because I sought to answer questions of how PSTs work with ELLs in a bounded system, one-on-one setting. Once a pair of a PST and an ELL began their session, I, the researcher did not interrupt or control the situation. In addition, my focus was obviously on a present phenomenon and there was a specified bounded system such as time (five weekly sessions) and place (classroom). The ‘case(s)’ situated in this study was a PST’s teaching or supporting an ELL in mathematical context. Thus, I determined a qualitative case study method was the best fit for this study and I designed each component of this research study following the case study method. For example, I purposively selected a small number of samples and used various data sources in order to provide an extensive description of each case.

Creswell (2007) lists the procedures of conducting a research study by using the case study method based on Stake (1995). First, researchers need to determine if a case study approach is suitable to answer the research questions and identify or select their case or cases. When collecting data in case study research, extensive multiple sources of information should be considered such as interviews, observations, surveys, and audiovisual materials. There are several ways to analyze these data. A typical way for analyzing multiple case studies is to apply a within-case analysis, which consists of a detailed description and themes in each case, and then a cross-case analysis, which is a thematic analysis across the cases, and finally an interpretation of the insights of the cases. The final phase is to report out the meaning of the case and what learned from the case.
One limitation of case study as a research method is that it is not possible to make a rigorous statement of generalization due to its small number of cases. However, it should be noted that the purpose of case study research is not to generalize the findings from a case or cases to a whole population (Farquhar, 2012). Cases should not be regarded as sampling units. Rather, they are more similar to experiments that are purposively selected by the researcher (Yin, 2009). Yin classifies the typical type of generalization as statistical generalization and defines one we can find in case study as analytic generalization. In this type of generalization, researchers use a previously developed theory as a framework to compare the empirical results of the case study.

**Context**

**Researcher’s Subjectivities**

Wertz et al. (2011) state, “… part of the rigor of qualitative research involves self-disclosure and reflexivity on the part of the investigator” (p. 84). For the reason, I here disclose my experiences as an ELL and as a mathematics teacher of ELLs so that readers understand my position and perspective, which may impact each component throughout this study. Since I believe my background and experience is uncommon in the U.S., I would provide sufficient information about my past experience.

I was born and educated in South Korea. I spoke only Korean at home and at school when I was in South Korea. During my 16 years of public education from kindergarten to college in my home country, I had hardly seen culturally or linguistically diverse people. Although I was taught English as a foreign language since 7th grade, I was not able to speak English until I came to the U.S. at age 30 because English education in South Korea was heavily test-focused rather than for real-life conversation.
In the first year in the U.S., I was studying English in an English Second Language (ESL) program for adults, and after completing the program I began to teach mathematics in a private K-12 school in California. I taught pre-algebra, algebra, and geometry with traditional textbooks. Becoming fluent in English was an extremely difficult task, which I sometimes felt almost impossible, because my first language is very different from English in many ways such as alphabets, sentence structures, and pronunciations. Despite the language barrier, I could build trusting relationships with my students, especially international students and culturally diverse students. Eventually, my interests in teaching ELLs encouraged me to obtain a Teaching English to Speakers of Other Languages (TESOL) certificate besides my teaching credentials in mathematics, physics, and chemistry.

After I earned my secondary mathematics teaching credential from a public university in California, I taught at a public high school in large urban districts. The schools I taught had a high dropout rate and a large number of culturally and linguistically diverse students including non-documented ones. Many students were not motivated to learn mathematics, but I found several successful cases in terms of both learning mathematics and changing behaviors. First, ELLs opened their minds relatively easily to me and spoke English more comfortably in my classroom because they knew I was not a native English speaker just like them. Second, I adopted various approaches from TESOL techniques such as visual aids, realia (real objects), and physical activities. In the TESOL lesson planning, how to design the beginning of each lesson, anticipatory set, is always emphasized. ESL teachers are required to use an attractive anticipatory set that makes a connection to students’ lives or interests so they can understand what the
topic is about and its context even if they do not know some words or fail to translate some sentences. I do not deny my efforts to employ these kinds of activities into mathematics instruction were not always successful, but I found these methods were useful to engage students including both ELLs and non-ELLs. My experiences and knowledge as an ELL and a teacher of ELLs, and also a mathematics teacher and a second language teacher naturally intrigued me to seek effective strategies to teach mathematics for ELLs and all students. The rest of this section describes how I designed this study in details.

**Participants**

In case studies, choosing appropriate cases is crucial. While quantitative studies usually choose samples randomly, case study researchers need to select cases that may show different perspectives on the event, which answers the research questions well. Creswell (2007) addresses this case selection process as *purposeful sampling* or *purposeful maximal sampling*. Moreover, the evidence from a multiple-case design could be more compelling, but it is also true that a theory from one case may not satisfy other cases (Yin, 1994). In this study, one PST participant was considered as a case, and also the unit of analysis, and I chose multiple-case design because I wanted to have some variation in teaching backgrounds, mathematical content knowledge, and pedagogical philosophy. However, I would involve a small number of cases because the amount of time and extensive resources required for many cases would be beyond my ability and I wanted to examine each case in-depth. Based on this rationale, I had four cases—four PSTs—in this study so that I could attend all meetings every week because each PST had
one weekly meeting with an ELL on a different day. However, I decided to drop one of the cases because the ELL student had to leave this study for her personal reason.

**Grade level.** I first considered the grade level of mathematics and made a decision to choose middle school level for several reasons. The first reason was the claim that college preparation begins in middle school. Students who take a rigorous curriculum in the middle school tend to show higher academic performance in high school and have a better chance to enter college than those who take less challenging courses because students build their academic skills, develop effective study habits, and engage in school-related activities through the rigorous courses (Wimberly & Noeth, 2005). In the U.S., high performing students in mathematics are allowed to skip one or two years of mathematics courses and to enroll in advanced classes. The local middle school where the ELL participants in this study attended allowed their students to take algebra I, geometry, and algebra II classes if their mathematics teachers recommend or they pass certain level tests. The students who are placed in an advanced mathematics class compared to their actual grade have more time to complete advanced high school mathematics courses such as Advanced Placement Calculus and have a better position to enter colleges (Atanda, 1999). Moreover, research reveals that once students are placed in different levels of mathematics classes, students often become “locked in” to these arrangements (Heubert & Hauser, 1999).

ELLs are often required to prove English proficiency as a prerequisite to enroll in advanced mathematics courses, but students in low-track courses who later acquire a higher level of English fluency are usually moved from a low-track ELL class to a low-track English-only class (Mosqueda, 2010). From the statistic analysis of scores of ELLs
and non-ELLs in mathematics, Mosqueda (2010) argues that English proficiency should not be used to limit students’ access to rigorous mathematics courses.

It is important to note that these tracking practices often begins in middle school by sorting out students into remedial mathematics courses, pre-algebra, or algebra and mathematics teachers have significant power and influence on the decision making (Wimberly & Noeth, 2005). Thus, mathematics teachers in middle schools should be aware of their power to change students’ future and be able to assess students’ real abilities of mathematics no matter what cultural or linguistic barriers they encounter.

**PST participants**. Four teacher participants were all white female middle school mathematics PSTs who were enrolled in a university-based teacher preparation program. Their participation was completely voluntary and the participants were randomly selected among nine volunteers in a middle school mathematics methodology course. I chose to recruit PSTs from this specific class because the grade level met my research design and they had not been required to take any ELL related courses by the time. Although four PSTs participated in this study, I decided to include only three cases in this paper because one PST worked with two ELLs while others interacted with only one ELL for five weeks because her first ELL student dropped in the middle of study due to her personal schedule conflict. The three remaining participants were Becky, Lucy, and Kate (all names are pseudonyms). When they joined this study, Kate was in her third year of the program and the rest of them were in their fourth and final year of the program. Their program required middle school PSTs to have a second endorsement area besides their main subject. Hence, Becky was pursuing a social science endorsement along with
mathematics teaching certification; Lucy chose English language arts; and Kate’s second subject was Spanish.

All PST participants had no official training to teach ELLs and little experience in working with ELLs. All of them had been taking some undergraduate mathematics courses, such as discrete mathematics, as a requirement from their teaching certificate program. Only Kate had experience of tutoring English as a second language to ELLs, but none of them had taught mathematics to ELLs. All participants were informed of the purpose of this research study and their responsibilities before they decided on their participation.

**ELL participants.** The primary research subject of this study was the three middle school mathematics PSTs, but I needed to recruit middle school ELLs in order to provide the PSTs with teaching experiences with ELLs. Although working on mathematical problems with a teacher could benefit the ELLs, I did not examine their learning or performance. However, it was necessary to select appropriate ELLs to maximize PSTs’ teaching experiences and learning.

Middle school ELLs in this study were chosen from a local immigrant community. All ELL participants were from one ethnic background, South Korea. Their ethnical background was not considered to be diverse because I included only three cases and it was not meaningful to pursue diversity in such a small group. Moreover, I intended to provide a similar setting for all PSTs including their students with similar backgrounds. Students in the lowest and highest level in both mathematics and English fluency were excluded in order to avoid unnecessary frustration caused by too high language demands or too easy access to mathematical problems. In order to select ELLs
who met these criteria, I gathered information, such as ELL class levels, performance on
their first language curriculum, from their parents and teachers of a local Korean School
who taught them more than one year. I used my previous knowledge to measure the
students’ mathematical and linguistic level because all of them attended a local Korean
Saturday school, where I volunteered as a mathematics teacher. In the end, I selected two
seventh-grade boys and one eighth-grade girl who were currently placed in ELL classes
at a local public middle school. There were two levels of ELL classes in the school:
beginner’s level and high level. All three students were in the high level ELL classes and
they had been in the U.S. for about fifteen months. Although I excluded very high
performers from my pool of ELL participants, all three students had competent
mathematics knowledge because two seventh-grade boys were in Challenge Math 7 class,
which was designed for good-standing students, and the eighth-grade girl was taking
Algebra 1. I purposively selected these students who had enough fundamental
mathematical knowledge, including basic operations, because I wanted to avoid the
situation that PSTs had to review elementary mathematical concepts so they could not
focus on problem solving.

Settings

Each PST was assigned to work with one ELL based on their available schedule.
Once they were assigned, the PSTs worked with the same ELL student for five weekly
meetings so they could use their previous experience with the students for the next
session. All meetings took place at a classroom on the university campus that the PSTs
attended so they could have easy access to meet with their ELLs.
Figure 4. The cycle of weekly routine of PSTs’ responsibilities

After a kickoff meeting and completing a pre-survey (see Appendix C) about their existing beliefs about ELLs and previous experience with ELLs, the PSTs had the same routine for five weeks (see Figure 4). First, I gave the PSTs a higher-order thinking mathematical problem a week ahead so they could prepare a middle school level of mathematics lesson based on the given problem. They were allowed to choose any pedagogical strategies and to modify the problem in any way, but they had to provide the rationale behind their decisions using the lesson plan template (see Appendix B). In addition, they were asked to refer to the aligned content and practice standards of the CCSSM in their lesson plans and were informed that their prospective students were ELLs. In a pre-interview, which I conducted immediately before each meeting with an
ELL, I asked about the PSTs’ lesson plans and their decisions regarding their plans. They were also interviewed after each session with the ELL to reflect on their implementation and receive intervention support with research-based ELL teaching strategies discussed in the review of the literature. All of the PST participants were given the same mathematical problem at the same week, after each intervention, but they could design and implement their own lessons. After each meeting, all PST participants submitted a written reflection via email to describe their learning from working with their students and from the interventions. After they finished all five sessions with their assigned ELL, each PST completed a post-survey with regard to the strategies they had implemented during this study and an individual exit interview about their learning and challenges to help ELLs make sense of mathematics with the researcher.

**Problem Selection**

I carefully selected higher-order thinking mathematical problems by considering their alignment with an appropriate content level and high demands in both cognitive and language levels. Stein, Smith, Henningsen, and Silver (2009) claim that not all mathematical problems create the same opportunities for learning. Smith and Stein (1998) identify mathematical problems according to levels of cognitive demands as memorization, procedures without connections, procedures with connections, and doing mathematics. Some of the main characteristics of higher-level cognitive demanding problems, which are placed in the levels of procedures with connections and doing mathematics, are listed below. Those were the criteria I used to select problems for this study. The characteristics of high-level cognitive problems:

- Are usually represented in multiple ways. Making connections among multiple representations helps develop meaning.
• Require some or considerable cognitive effort.
• Require complex and non-algorithmic thinking. A predictable, well-rehearsed approach or pathway is not explicitly suggested by the task.
• Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
• Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.

Some of these characteristics are similar to the description of MP1 such as “analyze givens, constraints, relationship, and goals” or “check their answers to problems using a different method.”

I selected mathematical problems that had potential to satisfy the criteria from various resources such as released items of the Smarter Balanced Assessment Consortium (SBAC), the Partnership for Assessment of Readiness for College and Careers (PARCC), the National Assessment of Educational Progress (NAEP) or the Program for International Student Assessment (PISA) (see Appendix A). Followed is one of the problems I adapted from sample items of SBAC.

Claire is filling bags with sand. All the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 58 pounds, another sand bag weighs 41 pounds, and another sand bag weighs 53 pounds. Explain whether Claire can pour sand between sand bags so that the weight of each bag is less than 50 pounds.

This item was designed for grade 8, but an aligned CCSSM standard is 6.SP.3, “Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.” Hence, the level of this problem is appropriate for students in seventh or eighth grades. I classified this problem as having a higher-level cognitive demand because 1) we can use multiple representations such as simple calculations, an equation, a table, or drawing to solve it, 2) the problem does not indicate a specific procedure, 3) the
embedded context is connected to students’ experience of weighing, 4) there are more than one solution pathway, and 5) the solver has to examine constraints and justify a solution.

However, the problem also has some features that might unnecessarily confuse ELLs. I analyzed the problem statement in terms of its language use and below are what I found.

1. The problem does not explicitly express the total number of sand bags.
2. “Claire” is a common name in the U.S. but might not be familiar to ELLs. They might think “C” is capitalized because it is the first word in the sentence.
3. The “another sand bag” appears twice, but it indicates a different bag at each time.
4. The first sentence uses “bag,” but the third sentence uses “sand bag” although their meanings are the same.
5. In the second sentence, “Each bag must weigh less than 50 pounds,” what truly weighs less than 50 pounds is the bag AND the sand in it.
6. The word “between” is commonly used for two objects. However, “between sand bags” in the last sentence implies “among three sand bags.”
7. There are various modal verbs and tenses used in the problem such as must, can, present progressive tense (e.g. is filling), and present tense (e.g. weighs).
8. The weight unit “pound” might not be used in the countries in which ELLs originated.

As we see from the list, when solving a word problem, ELLs have to manage complex linguistic process as well as mathematical or cognitive process. I examined if the PSTs
were able to analyze each word problem in both cognitive and linguistic aspects and made appropriate modifications and implementations for ELLs. Hence, I did not reduce linguistic demands of problems. Rather, I attempted to increase linguistic difficulty by eliminating all diagrams or pictures and providing only text so that the PSTs were not pre-occupied by the given supports. In addition, they had more freedom to implement the task in their own ways.

**Intervention**

I decided to provide interventions because de Araujo et al. (2015), which I used as a pilot study for this research, found that PSTs use limited ELL strategies after having only experience with ELLs. The results of the study indicate that PSTs need to receive structured interventions about ELL teaching strategies. Therefore, during five weeks of teaching sessions, I introduced research-based strategies of working with ELLs in mathematics. The first intervention happened before the first meeting with ELLs, and the following interventions were provided after each meeting except the last week as Figure 5 shows.

![Figure 5](image-url)  
*Figure 5. The time frame of the five intervention sessions*
The sources I employed to design the interventions include a book, *Beyond Good Teaching* (Celedon-Pattichis & Ramirez, 2012) and seven key strategies to support ELLs’ mathematical proficiency (Chval & Chavez, 2011, p. 262). Chval and Chavez summed seven research-based strategies from previous ELL research as follows:

1. Connect mathematics with students’ life experiences and existing knowledge (Barwell, 2003; Secada & De La Cruz, 1996).

2. Create classroom environments that are rich in language and mathematics content (Anstrom, 1997; Khisty & Chval, 2002).

3. Emphasize the multiple meanings of words and allow students to communicate by using gestures, drawings, or their first language while they develop command of the English language and mathematics (Moll, 1988, 1989; Morales, Khisty, & Chval, 2003; Moschkovich, 2002).

4. Use visual supports such as concrete objects, videos, illustrations, and gestures in classroom conversations (Moschkovich, 2002; Raborn, 1995).

5. Connect language with mathematical representations (e.g., pictures, tables, graphs, equations) (Khisty & Chval, 2002).

6. Write essential ideas, concepts, representations, and words on the board without erasing so that students can refer to them throughout the lesson (Stigler, Fernandez, & Yoshida, 1996).

7. Discuss examples of students’ mathematical writing and provide opportunities for students to revise their writing (Chval & Khisty, 2009).

However, I did not provide all seven strategies due to the time limit. As a result, I ended up with providing the first, the second, and the fourth strategies. This decision was made based on the results of the pilot study and other previous ELL studies, such as Ahn et al. (2011). These studies denote that connection, rich environment, and visual supports are common but core strategies for teaching ELLs as well as PSTs were able to understand these strategies in a short time. The Table 1 describes the weekly intervention organization I designed and conducted.
Table 1

*Weekly contents of the PST Interventions*

<table>
<thead>
<tr>
<th></th>
<th>Focused Topic</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Needs of ELLs</td>
<td>Read a story of a Korean ELL</td>
</tr>
<tr>
<td>2</td>
<td>Connect mathematics with life experiences and existing knowledge</td>
<td>Discuss how to begin an activity with life connections and students’ prior knowledge</td>
</tr>
<tr>
<td>3</td>
<td>Visual supports</td>
<td>Analyze examples of using visuals</td>
</tr>
<tr>
<td>4</td>
<td>Rich environments in mathematics and language</td>
<td>Use a Venn diagram to discuss how linguistic and mathematical strategies are related</td>
</tr>
<tr>
<td>5</td>
<td>Revisit the connection topic of week 2</td>
<td>Compare and analyze examples of teacher-student dialogues in terms of connection between mathematics and student’s life experiences</td>
</tr>
</tbody>
</table>

In the first meeting, the PSTs read a story in which a Korean ELL described her difficult experience in a U.S. school (Celedon-Pattichis & Ramirez, 2012, pp. 6-7) and discussed how to support her mathematical learning. The second intervention was devoted to building background in the problem-solving process by connecting mathematics with student’s prior knowledge and life/cultural experiences. Since this strategy was what I focused most in this study, I placed it at the early part of the interventions. I decided to revisit this topic at the last week because I observed that some of the PSTs had difficulty implementing the strategy. When I delivered these pedagogies, I did not directly explain them. Instead, I applied various activities as examples of ELL-friendly activities such as using a graphic organizer to visualize ideas, posing and
mobilizing concepts using post-its, or comparing conversations between a teacher and a student through role-playing. For example, in the third intervention, I presented three different versions of visual aids with a word problem written in unknown language, Korean, for the PSTs to understand the situation ELLs encounter in English-only classrooms and to conceptualize the effective use of visuals (see Appendix E). Similarly, I created two different versions of dialogues between a teacher and an ELL in the last week and had each PST discuss how those dialogues are different and in which way they are effective (see Appendix F).

**Data Collection**

In order to answer the questions of how or why in case study research, we need to collect different sources of data (Yin, 2009). Using multiple types of data strengthens the findings because the evidence is triangulated (Merriam, 1998). This is essential in case study research because having various perspectives when investigating cases provides rigorous foundations of the findings and eventually supports the arguments.

My first research question is “How do middle school mathematics PSTs help ELLs make sense of high cognitive demanding mathematical problems?” For pursuing the answer of the question, I used data from written lesson plans, observation, artifacts, and written reflections. The second question seeks PSTs’ growth in planning and implementing mathematical problems for ELLs reflecting their teaching experiences with ELLs and knowledge received from interventions. Besides observing the teaching, I used surveys to examine changes in PSTs’ perspectives about teaching ELLs and their own reflections about their growth.
Surveys

All PSTs were given a survey before their first meeting with ELLs. The purpose of the survey was to examine their initial thoughts and knowledge about teaching mathematics for ELLs. The pre-survey questions were twofold (see Appendix C). One part asked about their previous experiences with ELLs and the other part was about their pedagogical knowledge particularly in teaching ELLs mathematics. All questions were open-ended. In the post-survey, my focus was the PSTs’ learning from their experiences and interventions. I asked which lesson was the most meaningful and the most difficult to implement. Other questions were about what strategies were effective and what they found useful among what they learned and implemented, to help ELLs solve word problems in mathematics.

Lesson Plan and Pre-interview

I provided the PSTs with a linguistically and cognitively demanding mathematical problem one week before each meeting with their ELLs and asked them to compose a written lesson plan based on the given problem. They were informed they had unlimited freedom to modify the problem in a way that they think might be helpful for their ELLs. In addition, I let them use a simple lesson plan template (see Appendix B) so they did not have to spend too much time composing it. They were supposed to submit the written lesson plans at least the day before their meetings with their students so I could check its mathematical accuracy and prepare clarifying questions about their lesson plans. I also met each PST about 10 minutes before each teaching session to ask them about their rationale behind procedures in the plan. All pre-interviews were videorecorded.
Observation and Artifacts

Each PST had five teaching sessions with the same assigned ELL and they worked on different mathematical problems at each session. I observed each pair’s performance, but I minimized my interruptions. Each teaching session was videotaped to catch subtle interactions including verbal and non-verbal practices between the PST and the ELL and each video file of implementation was transcribed. In addition, the artifacts created by both PSTs and ELLs were collected as evidence of how they used multiple representations. All PSTs were encouraged to use an iPad app, Explain Everything, and some PSTs chose to use it sometimes. The Explain Everything enabled me to capture their interactive work with their talk concurrently (see Figure 6).

Figure 6. A screen capture of iPad application, Explain Everything.

Post-interview and Reflection

After each teaching session, I met individually with the PST again. This post-meeting had two goals: reflection and intervention. First, I interviewed them about what
went well and what did not go well, especially what was different from their expectations. After this, a short intervention about ELL teaching strategies was followed as I described in the previous section. After that, I provided a mathematical problem for the next meeting. All of these meetings were videotaped. Moreover, the PSTs submitted at most a one-page written reflection on their work with ELLs each week in order to provide deeper insights on their teaching experience while the post-interview provided instant perspectives after teaching.

**Data Analysis**

To analyze the data collected, I chose to use *the constant comparative analysis method* (CCA method; Fram, 2013), which is also called *the constant comparative method* (Merriam, 1998). The CCA method was developed by Glaser and Strauss (1967) as a variation of the grounded theory, but it has been used when researchers do not seek to build substantive theory. As the name indicates, this method is to constantly compare one set of data with another set.

Creswell (2007) describes general procedures for the CCA method. First, the researcher forms categories of information about the phenomenon in open coding, and then, applies axial coding as assembling the data in new ways. Subsequently, the stage of selective coding is followed. The researcher may write a story line or propositions connecting the categories and finally, develop and visually portray a conditional matrix that elucidates the conditions influencing the central phenomenon. Merriam (1998) addresses that “the development of categories, properties, and tentative hypotheses through the constant comparative method is a process whereby the data gradually evolve into a core of emerging theory” (p. 191).
Within-case Analysis

I followed this basic procedure of the CCA method, but I used NVivo (Mac version 10) in order to conduct an effective video coding. First, I watched the videotaped data of the first case and open coded the video clips, lesson plans, modified problems, and written reflections. Based on the result of open coding and the five categories of the conceptual framework, I established a draft codebook (Saldaña, 2013), which is a compilation of the codes and their brief and detailed content descriptions. This codebook was revised and refined several times through consulting with several faculty members. More specifically, I first open coded one case, and then I constructed a draft of codebook based on the open coding. Using the draft of codebook, I coded other cases with continuous comparing and contrasting and revised the codebook whenever I coded each case. After I coded all cases, I finalized the codebook. Then, I conducted axial coding and selective coding. Table 2 is a part of the codebook I used for one of the five categories, cultural/life experience.

Table 2
A sample of codes and their descriptions of the category of cultural/life experiences

<table>
<thead>
<tr>
<th>Cultural/Life experiences</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALE Assess student's life experience</td>
<td>Assess student's prior life experience relevant to a mathematical problem before or during the problem-solving process</td>
</tr>
<tr>
<td>CTC Connect to a student's culture</td>
<td>Change the context to align with the Korean culture or connect a mathematical problem with Korean culture</td>
</tr>
<tr>
<td>CTL Connect to a student's life experience</td>
<td>Change the problem context to align with a student's life experience or connect a mathematical problem with a student's life experience</td>
</tr>
<tr>
<td>ECC Explain cultural context</td>
<td>Teach or describe cultural aspects in the problem context, which might be different from student's culture</td>
</tr>
</tbody>
</table>
The codes were determined based on teaching strategies the PSTs used. In order to analyze teaching patterns and sequential changes associated with interventions, I used the query function of NVivo. First, I built a node matrix (code matrix) between each PST’s implementations and codes of each category (Figure 7).

In Figure 7, the left most column represents data sources including video files, modified problems and lesson plans from the first week to the fifth week. The top row includes the codes related to *mathematical and cognitive processes* and the numbers in each cell represent the frequency of each strategy. For example, the first code, AAP, stands for “ask a student to analyze a problem” and the number in the first column indicates how many times Lucy used the strategy during each phase of each week. In addition, the darker the blue color in the cell is, the higher frequency each code has in the data source.

![Figure 7](image_url)

*Figure 7. A sample node matrix of Lucy’s case*
Cross-case Analysis

After completing the within-case analysis, I conducted cross-case analysis. Yin (1994) addresses “In a multiple-case study, one goal is to build a general explanation that fits each of the individual cases, even though the cases will vary in their details. The objective is analogous to multiple experiments” (p. 112). As this statement, I pursued to find plausible explanations that satisfied all or multiple cases although I did not seek to generalize my findings for the whole population of mathematics PSTs.

Specifically, I searched for commonalities and distinctions across cases within the same category of the framework. For this, I used the query function again on all three cases in each category. Figure 8 shows a sample matrix of cross cases in the category, *mathematical and cognitive processes*, in the sequential order of weeks. This method enabled me to analyze what strategies the PSTs performed in similar or different patterns.

![Figure 8. A sample node matrix of cross cases](image-url)
Furthermore, in order to conveniently search answers of the research questions, I ran the query function with selected items. The first research question had three sub-questions in terms of three phases: planning, understanding problems, and justifying solutions. Hence, I made a series of node matrices of cross cases with two sets of items for all five categories. The first group of items included written lesson plans, modified problem, and pre-interview to analyze how the PSTs planned their ELL strategies. The other group consisted of the video files of teaching sessions and artifacts to search for answers both the second and the third sub-questions. From this analysis, I looked for what strategies the PSTs used in each phase of each week and how their use of strategies evolved. Whenever it was necessary, I reread transcripts and watched videos again until I could understand in what contexts the strategy was implemented and how it was conducted.

**Validity and Reliability**

Merriam (1998) suggested six basic strategies to strengthen internal validity: (1) triangulation, (2) member check, (3) long-term observation, (4) peer examination, (5) participatory or collaborative modes of research, and (6) researcher’s biases. Reliability is the ability to answer this question, “Will the study yield the same results if it is repeated?” Some researchers interpret it as dependability or consistency (Lincoln & Guba, 1985), and it means that the previous question could be changed to “whether the results are consistent with the data collect” (Merriam, 1998, p. 206). Merriam (1998) suggested several techniques to ensure the reliability of a research study such as the investigator’s position, triangulation, and audit trail.
To assess reliability and validity of this study, I employed the following methods: triangulation, reflexivity of investigator’s position, and member check. For triangulation of data, I used multiple sources of data, such as interviews, surveys, videotaped teaching sessions, and written lesson plans and reflections. The data from PST survey, observation of their teaching performance, and their interviews and written reflection were analyzed with the same codebook and triangulated. In addition, I provided my reflexivity that might cause biases. For reliability check, I used member check; another doctoral student coded one lesson of one case in the study and I compared her coding result to my coding result. We had higher than 95% agreement.

Summary

Data from PSTs’ lesson plans, videos recording pre-interview, implementation, and post-interview, reflections, and pre- and post-surveys were coded and analyzed by CCA method (Fram, 2013; Merriam, 1998). I analyzed each data source according to two research questions and five categories of the conceptual framework. The code matrices were developed by using NVivo in order to find patterns among PSTs’ frequent or consistent implementation of ELL strategies within each case and cross cases. In the next chapter, I present the results of each case and cross cases in details.
CHAPTER 4. FINDINGS

In this chapter, I present the findings from the data analysis related to the research questions. Here I restate the research questions to remind readers of the purpose of this study.

1. How do middle school mathematics PSTs help ELLs make sense of high cognitive demanding mathematical problems?
   a. What factors do PSTs consider in order to implement high cognitive demanding mathematical problems for ELLs?
   b. What supports do PSTs provide for ELLs to analyze the meaning of a problem and look for entry points to its solution?
   c. How do PSTs guide ELLs to explain their reasoning process and solution pathways?

2. In what ways, if any, do PSTs show a shift in planning and implementing mathematical problems as they work with ELLs and receive structured intervention on their practices?

I first provide the list of specific strategies I looked for in each of the five categories of the framework illustrated in Chapter 1. The results from a within-case analysis accordance with the five categories are presented next. In each case of PSTs, I begin with backgrounds of the PST and her initial perspectives about teaching mathematics to ELLs. Then, a detailed description of planning and implementation in each category follows. At the end of each case description, I summarize what strategies were mainly planned and used within the case as well as how each PST changed her use
of the strategies. After explaining the within-case analysis, I describe findings from the cross-case analysis for all three teachers in a chronological order according to three phases in each week and weekly meetings through five weeks. Finally, the emerging themes are discussed following the cross-case analysis and the integrated answers for each research question are provided in the summary of this chapter.

**Specific Strategies in Five Categories**

**Mathematical Content**

In this category, I focused on only mathematical or academic modification other than linguistic change or support such as language simplification or adding visuals because those strategies are included in the *mathematical and contextual language category*. More specifically, my focus was on what kind of problems and questions the PSTs added to the given problems and whether they changed the cognitive demand of each given problem. Because the framework provides two guiding questions in this category, “How experienced are students with mathematics concepts and procedures?” and “How do teachers challenge or reinforce students’ understanding in mathematics?” I divided the types of problems and questions into four groups: *problems to access a challenge, problems to challenge a student, problems to provide practice*, and *problems to assess student’s prior mathematical knowledge*.

**Cultural/life experience**

The guiding questions of the framework were “What knowledge of cultural or life experiences is needed to understand the problem statement?” and “What connections need to be made between the mathematical problems and student experience?” Hence, I did not include the casual conversations unrelated to the mathematical problems. I looked
for four strategies in this category: assessing student’s life experience, connecting a problem to student’s culture, connecting a problem to student’s life experience, and explaining cultural contexts embedded in problems.

Mathematical and Cognitive Processes

The framework provides two guiding questions: “What mathematical processes are needed and how experienced are the students at using them?” and “What cognitive processing skills are needed?” The specific strategies I found and looked for in this category were asking a student to analyze a problem, applying a procedural approach, modeling the process prior to student’s solving, pressing to explain a solution, seeking multiple solutions, and using multiple representations. In addition, I carefully analyzed if any of the strategies associated with reducing cognitive demands of the original problems. Two strategies, applying a procedural approach and modeling the process prior to student’s solving were considered to reduce cognitive demands by removing challenging tasks or routinizing problems (Stein et al., 2009)

Mathematical and Contextual Language

The framework had three guiding questions in this category: “Do the students’ prior experiences include the development of mathematical language and the development of the natural language in the learning of mathematics?” “Does the language used in the problem or instruction correspond to the level of English language development of ELLs?” and “Are there words that have specialized meanings in mathematics that have different meanings in natural language?” I aimed to find various strategies the PSTs applied to supporting their ELLs’ understanding in English, including assessing English proficiency, explaining mathematical terms and non-mathematical
words, using gestures, physical objects, and visuals, allowing multiple modes of communication, modifying a problem statement or presentation, and rephrasing or paraphrasing statements. Additionally, I examined if any of the strategies caused eliminating cognitive demanding tasks from the original problems.

**Positioning**

I had two questions in this category, “How do teachers position themselves and students?” and “How does positioning influence ELLs’ participation in mathematical discussion?” The strategies I found in positioning were only two: lowering teacher’s position and raising student’s position.

**The Case of Becky**

Prior to participating in this study, Becky had not had any kind of training about teaching ELLs or experience with ELLs although she had been working with a few middle school students at local public schools as a part of her fieldwork, required from her teacher preparation program. In her pre-survey, she expressed her positive perspectives towards ELLs in terms of their capability to learn high-level mathematics and the necessity of differentiated instruction for them.

I believe that anyone with enough practice and dedication can learn high levels of mathematics so yes I think it is possible [that ELLs are able to learn high level of mathematics]. There will be extra challenges with word problems for English language learners due to the extent of the reading that is expected. This can be something the teacher differentiates so that their ELL student(s) are successful in their classroom (Becky, pre-survey).

However, her suggestions and knowledge on ELL teaching strategies were limited to a vocabulary focus. For example, she suggested that giving ELLs a list of vocabulary or simplifying words should help ELLs make sense of the information in mathematical problems.
Becky worked with Se-Bum, a 7th grade boy who was identified as an ELL by his school district. He moved from a beginner’s level of ELL class to an intermediate level just before participating in this study. Se-Bum was enrolled in Challenge Math 7 class, which suggested he was competent in mathematics compared to his grade level peers. However, he was observed to have a quiet and reserved personality and hardly spoke out more than a few words.

**Mathematical Content**

This category focuses on how PSTs assessed student’s prior mathematical knowledge and what questions or problems they added to challenge or reinforce students’ understanding over the problems. Becky modified all five problems by simplifying statements, breaking them down into micro steps, or adding more questions or pictures.

Becky usually made warm-up questions and asked Se-Bum to complete them prior to the main problem. Her purpose of providing these kinds of “easier sets of problems” was to “give the student a warm up activity and allow me [Becky] to see where he is at with the content” according to her lesson plan. Hence, warm-up activities and exercises were usually coded as *problems to access a challenge* as well as *problems to assess student's prior mathematical knowledge*. For instance, her warm-up exercise in week 2 included questions of calculating time because the main problem was about speed. Although they might help Se-Bum pursue the goal of the main problem, the warm-up questions implied what approach the student had to follow, which the student should have figured out from his own reasoning.

Similarly, when she added sub-questions to the main problem (see Figure 9), she provided all crucial steps to solve the problem. In Figure 9, the three questions on the
bottom were what she included in the original problem. By doing that, she eventually removed an essential element of problem solving–searching for solver’s own solution pathways–and reduced its cognitive demand. In this setting, Se-Bum did not have an opportunity to break down the problem in his own way and to find his solution pathway.

Overall, she used significantly more problems to access a challenge and problems to provide practice than problems to challenge her student. The challenging problems were given only in weeks 4 and 5 while problems for practice or getting access to solve the main problem were consistently used multiple times in all five weeks.

### Week 2 Problem

**Anne’s family is driving to her uncle’s house. The family travels 383.5 miles between 10:15 a.m. and 4:45 p.m.**

1. Calculate the family’s average rate of travel for the day. Then fill in the blank to complete the following statement. You can enter a whole number or a decimal rounded to the nearest tenth.

   How many hours and minutes are between 10:15 to 4:45? ______________

   How many total miles did the family go? ________________

   The family’s average rate of travel for the day is _______________ miles per hour.

*Figure 9. A part of Becky’s modified problem for week 2*

**Cultural/life Experiences**

In sum, Becky used limited strategies in this category. Data show that the only strategy she consistently used was connecting a problem to student’s life experiences although she did not use the strategy every week. Moreover, it should be noted that she
had never attempted to connect problem contexts with student’s culture, for example, asking Se-Bum about his experience in his home country. Overall, the use of the strategies related to student’s culture and life experience was scarce. The numbers in the Table 3 represent the frequency of the incidences Becky implemented the strategies.

Table 3

*Frequency of the instances Becky applied the strategies in cultural/life experience*

<table>
<thead>
<tr>
<th>Week</th>
<th>Data source</th>
<th>Assess student's life experience</th>
<th>Connect to student's culture</th>
<th>Connect to student's life experience</th>
<th>Explain cultural contexts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Implementation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lesson plan</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modified problem</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Implementation</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lesson plan</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Modified problem</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Implementation</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Lesson plan</td>
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<tr>
<td></td>
<td>Modified problem</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Implementation</td>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Lesson plan</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>Modified problem</td>
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</tr>
<tr>
<td>5</td>
<td>Implementation</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lesson plan</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Another important finding was that her attempt to implement these strategies began after she received a relevant intervention. The intervention session provided after the first week’s teaching focused on how to differentiate instruction according to students’ existing knowledge and life experience based on the discussion of Garrison and Mora (2005). One emphasized scheme in the intervention was opening a lesson with life
connections instead of directly jumping into mathematics. Although this scheme was not immediately adopted in Becky’s instruction, she began to use some life connections in the following week (week 2). For instance, she used life examples when she explained calculating time such as “this is about the time you’re going to school” or “this would be like noon, this would be like a lunch time, and this would be like before bed.” However, she applied this strategy spontaneously rather than planning it in advance.

She finally planned to use this strategy in week 5 and the number of her implementing moments significantly increased in this week as well as it appears in the frequency table. In her lesson plan for week 5, she wrote, “I want to know what patterns he sees in everyday life. I will ask him about patterns he sees in songs (beats), in weather (temperature patterns), in pictures.” It is important to indicate that the topic of cultural/life connections was revisited during the intervention of the previous week with significantly more concrete examples, sample dialogues between a teacher and a student. Because she had seen the example of teacher discourse, Becky planned to use a conversation about patterns and prepared several life examples. It should be recognized that she began to apply life connections after she learned it although it did not come out successfully as she responded in her reflection,

I also learned that as much as you plan for something, it may not always work out the way you want. I had planned for dialogue about video games with the student but it did not take off as I had hoped. I should have had a backup plan in place in case this happened (Becky, reflection, week 5)

However, she recognized the importance of life connections as she wrote “Connecting the student to things he already knows is also a useful strategy. It allows the student and teacher to have open conversation and feel comfortable with each other” and “I learned to connect the content to the real life of the student. This helped to create good
conversation and set up an environment that was comfortable for both of us” in her post-
survey.

Mathematical and Cognitive Processes

Becky used various strategies under this category such as pressing for explaining
solutions or analyzing problems. However, her use of strategies in mathematical process
was rather superficial. For example, she used many why questions, but she did not go
further with follow-up questions and often ended up with providing specific ways to
solve or giving out the solutions as seen in the following vignette excerpted from week 3.

(B: Becky, S: Se-Bum)
1 B: Okay. Let’s talk about one more time. This one is...
2 S: False.
3 B: False because... that one, can you tell me why it’s false?
4 S: ......
5 B: That has to do with the money they pay. Is it equal, or is one hundred
different than the other? It was...
6 S: A hundred...
7 B: Well. You don’t have to say that. It was just three dollars and three dollars
equal to each other, so that meant they are proportional, or they’re the
same, right? Okay. Let’s move on this one.

In line 3, Becky asked a why-question, and when Se-Bum could not answer to the
question (line 4) she provided a hint and suggested a computational approach (lines 5-6).
Although Se-Bum began to answer his result of the computation (line 7), she stopped him
and uttered her solution and then moved to the next question (lines 8-10).

The strategies, modeling the process prior to student’s solving and applying a
procedural approach appeared in all five weeks. In addition, she rarely asked Se-Bum to
seek multiple solutions or multiple representations. The following dialogue from week 2
describes this type of her teaching.
B: You can use this [calculator] if you want. Awesome, so you got seven point six seven, but what do you know about these two numbers, the decimal point? They’re out of not sixty, are they? They are out of a hundred.

S: (Write 67/60)

B: So what is, um, sixty seven over a hundred equal to? In a decimal or a fraction, if you like to reduce it. So seven point six seven... (write 7.67). This would be out of a hundred because you are not using time yet, so sixty seven out of a hundred... (write 67/100 = ). And if you want to write that as a fraction, okay, you got point six seven, so now we have, we need to find point six seven of an hour (write .67), which is how many minutes? An hour is sixty minutes. So, now try that. Try to take this times sixty.

S: (Use a calculator)

B: So it would be about forty minutes, wouldn’t it?

S: Yeah.

B: Okay. So, instead of this [7.67], you’re going to use the forty minutes (write 40), right? So seven hours and forty minutes, now you know you started ten fifteen and you’ve gone seven hours and forty minutes. So, what time would that be? Instead of four forty five, it would be what? You can try to figure out with the way we did earlier, too.

Before this moment, Becky had explained how to convert a partial hour to minutes because Se-Bum kept using 100, not 60, when converting hour to minute. In line 1, she paid Se-Bum a compliment, “Awesome,” but it was followed by her indication of a wrong procedure. Pointing out the student’s error after a compliment may have confused Se-Bum. Then, Becky asked a question, “What do you know about these two numbers, the decimal point?” (line 2). After this, she immediately uttered the answer of her question, “They’re out of not sixty, are they? They are out of 100” (lines 3-4). Despite this explanation, Se-Bum did not catch his error and still wrote 67 out of 60 (line 5). Becky first ignored it and kept showing her way to get 67 out of 100. At this moment, she seized Se-Bum’s work and provided the answer of each step. There was no work left for Se-Bum. Becky’s seizing behavior repeatedly occurred in this week. Becky stated and demonstrated each step without assessing her student’s understanding or asking him to participate in the process.
Furthermore, she sometimes changed the nature of problems from reasoning to procedures. For example, when Se-Bum was asked to determine whether the statement, “If we know the areas of two apartments and the price of one of them we can calculate the price of the second [each pays proportional to the size of their apartment],” is correct or not, Becky provided him with specific numbers for the areas of two apartments and the price of the second apartment, and asked him to find the price of the first apartment. After Se-Bum found the price of the first apartment by applying cross-multiplication as she guided, she asked if the statement is true or false. Although this type of guidance helped the student find the right answer, Becky forced him to follow a particular pathway rather than let him determine his own way, and also misled him that one specific case could prove a statement for general cases.

However, there were some changes in her discourse and guidance to support her student’s mathematical process as she spent more time with Se-Bum and the researcher. The following vignette was extrapolated from her teaching in the fifth week.

1  B: So where will be the good place to start? You want to start with Xs?
2  S: Yes.
3  B: Let’s write the X pattern, so write down anything you noticed about Xs from this one (points at the picture on the paper), just like anything you noticed. You can even write a chart if you want to. (Draws a chart with 4 columns) Okay. So the first one. How many Xs does it have? Go ahead write it down what you need to get an answer.
4  S: (Writes 9, 25, 49, 81 in each column)
5  B: How did you know that?
6  S: Um.
7  B: You did them really quickly. I’m just curious how you figured it out so quick.
8  S: This one [row] is plus two, and here [column] is plus two.
9  B: Oh, so I just want to count them. Do you think that would work? So one, two, three, ... eight. You got nine. What did we do differently?
10  S: ...... I did like three time three.
In the beginning of the vignette, Becky still suggested a particular way by saying “You want to start with Xs?” and “You can even write a chart if you want to.” However, she was much more patient after that moment. When Se-Bum could not provide a response to her question, “How did you know that?” (line 9), she did not directly provide the answer. Instead, she asked again with encouragement as “You did them really quickly. I'm just curious how you figured it out so quick” (line 11). Responding to this, Se-Bum finally explained he used the pattern that the number of trees in a row and in a column increases by two each time. When Becky noticed Se-Bum made a mistake, she did not correct it, but questioned why a different method led a different result (lines 14-15). Compared to her first second week, she became much more thoughtful about her student’s reasoning and developed her questioning skills to ask her student’s thinking.

**Mathematical and Contextual Language**

Becky used various linguistic supports such as *using visuals, gestures, or modifying problem statements*. All of her attempts were to make the problems easier or more accessible to her student, but many of them resulted in reducing cognitive demand by removing challenging work or providing too much guidance. First, she removed a challenging work from the original problem when simplifying its statement. The first problem statement was originally as follows:

Claire is filling bags with sand. All the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 58 pounds, another sand bag weighs 41 pounds, and another sand bag weighs 53 pounds. Explain whether Claire can pour sand between sand bags so that the weight of each bag is less than 50 pounds.

Becky modified this problem by adding pictures and using fewer words because she thought it would allow the student to make sense of the problem. Although the problem
looked easier and less confusing, she took off “Explain” from the problem as appeared in Figure 10.

![Picture of bags](image)

**Figure 10.** The first problem Becky modified for her ELL student

However, after she realized Se-Bum was capable of solving mathematical problems in week 1, she reduced the amount of her modification and did not attempt to decrease the difficulty level of the given problems.

Although she used many different linguistic supports, her use of them was not consistent with her plans and she did not spend much time on assessing or building student’s understanding about what each problem was asking. In her lesson plan of week 2, she wrote “The original problem has a lot of words in it, however I will be using it after a series of easier questions to assess the student’s English speaking and reading” and also expressed her goal as “to see how well he can read and speak English [in] this lesson.” Although this was what she planned, she hardly assessed student’s
comprehension in English in the week. She did ask Se-Bum to read the problem statements aloud and helped him pronounce correctly when he mumbled some words, but she did not examine if he understood what he read and had her student begin to solve the problem right after reading it. However, this pattern shifted later as seen in the dialogue below, excerpted from week 4.

The conversation occurred in the beginning of the lesson. She began her instruction with asking if Se-Bum knew a word, “one-story house” (line 1). After Se-Bum answered “No,” she attempted to explain it, but stopped right away and asked another question instead, “Are there any stairs in one-story house?” (line 4). Se-Bum gave a right answer to this question probably because he actually knew what one-story house means or he had just figured it out from her guiding question. Becky added a brief explanation of one-story house before she presented the main problem. That was a significant difference from her previous teaching. At this time, she was able to give
opportunities for her student to make sense of the problem before she explained everything. Similarly, instead of going straight to solving the problem, Becky assessed Se-Bum’s understanding on the problem by using an alternative way, drawing (line 13). When Se-Bum did not understand “label the length,” she did not judge his work but rephrased her question with a clear definition of length. Becky described this moment in the post-interview as follows:

There was confusion between length and width. I didn’t know I should correct him because [anyway] he will get the same area. One of areas wasn’t right. And I was questioning him, and I was trying to act like I don’t know what’s going on, but I knew he got a wrong answer, so I tried to form a question that would like help him think about it without telling him that I actually knew the answer (Becky, post-interview, week 4).

From the analysis of the previous vignette and her statement in the post-interview, we can see that Becky became able to assess and build student’s understanding of the problem statement before working on the problem.

Another clear shift was observed from her use of visuals. In week 1, she included three sandbag pictures in her first modified problem (see Figure 10) in order to make it “easier than reading a very wordy problem that can be confusing” (Becky, lesson plan, week 1). However, it was after week 2 when she started implementing various visuals in relation with mathematical situations because she received the intervention about how to use visuals in mathematically meaningful ways during the intervention.

She simply added pictures of bags in week 1, but her week 3 and 4 lessons had charts and tables (e.g. Figure 11), which more explicitly represented the mathematical situation implied in the problem.

Types of visual aids Becky used include pictures, charts, tables, post-its, drawing, fonts, and underlines. She used manipulatives, colored cubes, in week 5 when she was
given a pattern problem. She used gestures as well, but she used only one type of
gestures, pointing. Although pointing worked well to communicate with her student, she
did not attempt to use any other gestures.

Figure 11. A table and pictures Becky included in her modified problem

Positioning

With the strategy in cultural/life connection, positioning strategies were the
evidence of the impact of interventions in Becky’s case. In the intervention of week 4, I
provided various specific strategies under the guiding principle of rich environment in
mathematics and language. One of them was pretending not to know tactic. Because Se-
Bum had been extremely voiceless and Becky had made “traditional teacher discourse”
that Wagner and Herbel-Eisenmann (2014) called teacher personal authority or discourse
as authority, such as directions, commands, or “I want you to do” type of sentences, I suggested to change the traditional teacher attitude by equalizing teacher position to student position. Specifically, the suggestion was stating “I don’t know how to solve the problem” or “I’m not sure how to do it. Why don’t we figure this out together?” types of discourse in order to set a parallel position to her student. Becky picked up this strategy seriously and aimed to implement it as a main strategy in the future weeks. The following vignette illustrates how she employed this strategy.

Becky: How about I draw one. You tell me if I did right or wrong, okay? Here we go. (Draw rooms within a big rectangle) Okay. I think I got everything. Will you look at it? Tell me if everything can work.
Student: Thirty by twenty was correct. [It] has all required rooms.
Becky: Okay. So, the living room, I made the living room really big. It works, doesn’t it? I think it does. I don’t know.
Student: (Nodding)
Becky: Okay. What about the kitchen? Does that work?
Student: Yeah.
Becky: Okay. What about the bathroom? It had to be fifty square feet. I think I did that one. Right?
Student: Yeah (nodding).
Becky: Bedrooms?
Student: Yeah.
Becky: How long did each bedroom have to be? I forgot. Like all the, I can’t remember all of them. I’m not sure I got it right or not...

Becky’s first try was not very natural as shown in this dialogue. She used “I think it does. I don’t know” (line 6) or “I forgot. I can’t remember all of them” (lines 15-16). Still, it helped the student speak out slightly more than before and her efforts led her to a student-centered activity, such as correcting teacher’s errors. These strategies were coded as lowering teacher’s position. Table 4 clearly tells Becky implemented this strategy since week 4 and as she used the strategy, lowering teacher’s position, the frequency of employing the other positioning strategy, raising student’s position, also increased. Becky explicitly expressed her excitement of her “successful” implementation of this
strategy in her reflection because she thought it was effective to boost up her student’s responses significantly.

This week I learned that acting like I don’t know the answer really allowed the student to feel more comfortable talking and discussing the problem with me. I think it put us on the same level and he felt more comfortable talking about math with me. I learned that he really is able to communicate things to me well if he relates to the subject and feels comfortable enough with me. It made the lesson feel more natural and relaxed when he was laughing (Becky, reflection, week 4).

Table 4

*Frequency of the instances Becky applied the strategies in positioning*

<table>
<thead>
<tr>
<th>Week</th>
<th>Data source</th>
<th>Lower teacher’s authority</th>
<th>Raise student’s position</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Implementation</td>
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<td>Lesson plan</td>
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<td></td>
<td>Modified problem</td>
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<td>2</td>
<td>Implementation</td>
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<td>Lesson plan</td>
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<td></td>
<td>Modified problem</td>
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<tr>
<td>3</td>
<td>Implementation</td>
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<td>Lesson plan</td>
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<td>Modified problem</td>
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<tr>
<td>4</td>
<td>Implementation</td>
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<td>Lesson plan</td>
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<td>Modified problem</td>
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<td>5</td>
<td>Implementation</td>
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<td>Modified problem</td>
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Summary of Becky

Becky performed limited use of ELL strategies in the beginning, but showed apparent shifts through five weeks. As for problem modifications, her frequently used strategies were adding warm-up activities and breaking down the problem into micro steps. Both strategies often reduced the cognitive demands of the problem because they guided the student to a specific solution pathway. She simplified the language of
problems, but she did not continue to make simplification because she realized her student was able to read most words. During her implementation, the most frequently used strategy was explanation/justification, which usually had a questioning form – asking for explaining solutions – although she did not always explicitly plan it. Visual aids were also consistently used, but she used more various types of visuals in later weeks than in early weeks such as post-it, charts, or diagrams.

Moreover, she diligently adopted the strategies she learned from interventions. Her favorite strategy was obviously lowering teacher position as she mentioned in her reflection and post-survey. Becky’s post-survey comments reveal the reason why she thought this strategy was particularly effective.

The first strategy that I found the most useful was pretending like I did not know the answer to the problem. I had to act during this lesson but I truly felt it was the most powerful strategy I used during this process. The student seemed to respond very well to this strategy and it allowed the lesson to go very well (Becky, post-survey).

She believed “acting like I didn’t know the answer” made her student comfortable and communicate with her more effectively. Among the five categories of the conceptual framework, positioning and cultural/life experience are the categories that Becky had shown distinct impacts from interventions because she did not use any of the strategies in these categories in the beginning but began to use them after the interventions.

The Case of Lucy

My second case is Lucy, another white female preservice teacher. She did not speak any other languages but English and had not had any official training or experience of teaching ELLs prior to joining this project. From her pre-survey, some of her statements drew my attention. The first statement was her perspective about mathematics,
“I believe that English language learners are not likely to experience more difficulties in learning mathematics compared to other subjects because, in my opinion, math is a universal subject. Numbers and other mathematical lingo are the same, or only slightly different, for most other countries.” This statement reveals that Lucy had a limited view about mathematics because she did not see that other countries use different mathematical conventions; mathematics also implies cultural and linguistic features; and cognitive demanding mathematical tasks are usually language demanding as well. She also had a pre-occupied perception about Asian students as follows:

Chinese, Korean, and other Asian descendants are usually proficient in math because their schools emphasize its importance. Although Korean ELLs will have to understand the meaning of American mathematical symbols, I think the transition would be easier in math than any other subject (Lucy, pre-survey).

According to this excerpt, she assumed Asian students were proficient in mathematics, and it seemed this belief is based on her perception of mathematics as “universal.”

As for instructional strategies for ELLs to understand mathematical problems, Lucy brought up the importance of understanding meanings of mathematical words as Becky did. However, I found her perspective was quite deeper than that of Becky because she extended her thought to understanding the meanings behind mathematical definitions and symbols. The following quote describes this difference.

ELLs will need to not only learn the concepts that you are teaching the entire class, but also what the symbols mean and definitions of mathematical words. As long as they understand the meaning behind these concepts and why you are doing certain procedures, they will be able to understand. These longer descriptions at the start of class could also help struggling students and act as a review for other student (Lucy, pre-survey).

The rest of this section provides detailed descriptions of Lucy’s plans and implementations of five cognitive demanding problems to work with Si-Young, a female eighth grade ELL who was taking Algebra at that time.
Mathematical Content

The most frequently used strategy in this category was assessing student’s prior mathematical knowledge. It usually appeared in forms of questions such as “Do you know what dimensions are?” or “What do you think this would be called?” Lucy did not explicitly state this strategy in her lesson plans, but she always assessed the meanings of mathematical terms and concepts. Her first step was usually asking Si-Young if she knew the word or concepts, and if Si-Young struggled to explain it, she provided scaffoldings or guiding questions.

One clear difference between Becky and Lucy was that Lucy rarely lowered cognitive demand of the problems. Reducing cognitive demand of the problem occurred only in the first week in Lucy’s case. In week 1, she planned a warm-up activity, “a couple of easy addition and subtraction problems written out (ex. 9 + 14 or 27 – 4)” (lesson plan, week 1). Then, she provided it to Si-Young in the very beginning of the week as Figure 12 shows.

Figure 12. Lucy’s first mathematical activity to assess her student’s mathematical knowledge
Although it was necessary for her to assess the student’s mathematical ability at the first meeting, the level of the activity was significantly lower than Si-Young’s grade level. Adding and subtracting within 20 is one of the first grade standards (1.OA.C.6; CCSSM, 2010) while Si-Young was in eighth grade. In her exit interview, Lucy explained the reason why she had such low expectation from her ELL student in the beginning as follows: “I expected her to be at lower grade level just because I didn’t know how long she is here to speak. I assume she knew math well, I guess her English is not good. So ... doing math in English would be a difficult part.”

However, Si-Young completed this activity with high speed and accuracy, which was apparently beyond Lucy’s expectation. After the first session, Lucy admitted her mistake, “She was a lot more advanced than I thought. I could’ve had more prepared that would challenge her than I did” (post-interview, week 1). As she claimed, she developed a completely shifted plan for the following week (week 2). While she added easy problems to assess Si-Young’s prior knowledge in the first week, she added multiple challenging works beyond the given problem in the second week such as making a line graph plotting the starting/ending distance and time, finding the slope of the graph, and creating a story using a graph and the change of distances. However, Lucy was unable to have Si-Young finish all activities she prepared within the given time. After week 2, Lucy did not plan to add too low-level activities or too high-level problems.

Cultural/life Experiences

I found that Lucy had some similarities to and differences from Becky in the use of strategy related to student’s cultural/life experiences. The similar part was that Lucy did not use any of these strategies before she received the relevant intervention from the
researcher as Table 5 shows zero frequency in week 1 and began to use all four strategies, assessing student's experience, connecting a problem to a student's culture, connecting a problem to a student’s life, and explaining cultural contexts in the problem right after the intervention.

Table 5

*Frequency of the instances Lucy applied the strategies in cultural/life experience*

<table>
<thead>
<tr>
<th>Week</th>
<th>Data source</th>
<th>Assess student’s life experience</th>
<th>Connect to student’s culture</th>
<th>Connect to student’s life experience</th>
<th>Explain cultural contexts</th>
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<td>Implementation</td>
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However, unlike Becky, Lucy often visited student’s culture such as the experiences in the student’s home country during week 2 to 5. The following dialogue was excerpted from the beginning of the second week meeting.

(L: Lucy, S: Si-Young)
1    L: We are going to be working speed and transportation and time. Do you
2    have a certain way of transportation you like? Like riding a car or...
S: Um... riding a car.
L: Riding a car? What kind of cars do you like?
S: Um... Van?
L: Vans? Is that what your parents have? Did you ride in a different way of transportation in South Korea?
S: No.
L: Did you take buses or trains?
S: Um. I rode trains and buses and a lot of things...
L: So, a lot of things? What was your favorite?
S: Bus?
L: Bus. Would you ride that a lot?
S: (Nodding)
L: Well, could you see any similarities or differences from transportation here and South Korea?
S: In South Korea, I think it’s closer, so we rode a lot of buses, but in America, they don’t have a lot of buses.
L: They don’t? Yeah. I realize that too. They do have a lot of them on campus. So I get to back and forth from my house through that. Do you have a lot of traffic in South Korea?
S: (Nodding)
L: Yeah, so it might be easier to try a bus or a bike than just trying yourself.
S: (Nodding)
L: Cool. Well we’re going to do some speed, do you know what speed is?
S: (Nodding)
L: How do you find the speed?

In this vignette, Lucy opened a conversation by introducing the topic of the lesson and asked about student’s life experience. During the conversation, Lucy asked about Si-Young’s experience in her home country multiple times such as “Did you ride in a different way of transportation in South Korea?” (line 6) or “Could you see any similarities or differences from transportation here and South Korea?” (line 15) and Si-Young was responsive because it was a topic she knew well. This conversation also presents how Lucy used a mixture of connections between the problem and student’s culture and life experiences. At the end of the dialogue above, Lucy made a smooth transition from a casual conversation to the mathematical topic, speed.

The following quote from her post-survey evidenced that she learned this strategy from the intervention as well as showed her satisfaction on the strategy.
There were many strategies that were effective to help my student understand problems in mathematics. I found it useful to connect my student’s life experiences with mathematics. This helped her understand the relevance of the material and got her to notice mathematical happenings in her everyday life. For example, when I told her about speed she couldn’t understand it, but then once I showed her a speed limit sign she recognized what the math formula meant (Lucy, post-survey).

In her description, she explained how her student’s experience about a speed limit sign helped the student understand the concept of speed.

Mathematical and Cognitive Processes

The most frequently used strategy in this category was pressing for explaining solutions. Lucy consistently used this strategy with significant amount for all five weeks.

I provide a short dialogue from week 3 to show how she used this strategy.

1 L: So, what would you do there?
2 S: Um... the price of the building is eight hundred dollars, and they took ten percent off, so it’s eighty dollars.
3 L: How did you take ten percent off?
4 S: I don’t think this is right.
5 L: Why don’t you think it’s right?
6 S: (Write some numerical sentences, but erase them.)
7 L: What do you try to work with? Let’s talk through it. Before you had eighty times what?
8 S: Eighty times, no.
9 L: Eight hundred, sorry.
10 S: Eight hundred times ten out of a hundred.
11 L: Okay. Then why did you do that?
12 S: To take ten percent off.
13 L: What did you get? So ten over one hundred, what’s ten over a hundred, ten percent?
14 S: Yes.
15 L: And so that you got ...
16 S: (Writes 800 x 10/100 = 80) Eighty.
17 L: So we know eighty is ten percent of eight hundred, right? So that’s what we figured out, eighty is ten percent of eight hundred, so how we would get ten percent off eight hundred?
18 S: Eight hundred minus eighty?
19 L: Good. So you’re on the right track. We just had to talk through it.

This conversation began after Si-Young completed some steps of her solution. In line 1,
Lucy’s first question was asking what she was trying to do. Si-Young responded it, but her response was almost the repetition of the problem statement (lines 2-3). Hence, Lucy asked again with a more specific question in line 4, “How did you take ten percent off?” Then, Si-Young suddenly said what she found was wrong (line 5). Lucy did not evaluate her work; instead, she asked Si-Young’s reasoning in line 6, “Why don’t you think it’s right?” However, Si-Young did not answer the question and kept working on her calculation (line 7). Lucy did not give up and asked one more time, “What do you try to work with?,” with a specific following question, “Before you had eighty times what?” in line 8-9. Still, Si-Young did not explain what she was up to, but indicated Lucy’s mistake (line 10). Then, she finally explained what she was trying to do (line 12). Lucy kept asking about Si-Young’s reasoning behind the process she just explained (turn 13). Like this, Lucy’s pressing for explaining a solution continued until Si-Young explained all of what she did and reasoned.

As I just described, Lucy pressed Si-Young to explain her solution and reasoning whenever Si-Young progressed each step. Even when Si-Young made an error, Lucy asked Si-Young why she took such step prior to providing another support or explanation. Her efforts seemed appropriate because Si-Young was not able to explain everything at once. Her responses were mostly short and conveyed a small piece of information, so it was needed for her teacher to keep pursuing until she could obtain enough explanation from her.

More importantly, Lucy provided significant effort for her student to analyze the problem statement. First of all, she did not have Si-Young read the whole statement at once. She had her student read only a part of the problem statement and after she made
Sure Si-Young understood the part completely, she moved the next part. In the following conversation, excerpt from week 3 again, Lucy asked many clarifying questions such as “Is there anything else that it tells us? Does everyone have the same size of apartment?” (lines 13-14) or “Then what if their apartment is bigger?” (line 25). After she was convinced Si-Young understood the first part of the problem, she moved the next part (line 33).

1  S: (Read the problem) “People living in an apartment building decide to buy the building. They will put their money together in such a way that each will pay an amount that is proportional to the size of their apartment.”
2  L: Is there anything in there you don’t understand?
3  S: (Shake her head)
4  L: Okay. So what did it basically say?
5  S: Um...
6  L: In your own words.
7  S: They will put money with the same amount? That is...
8  L: Do you want to try reading again?
9  S: Yeah (read silently). They will put their money together to the apartment?
10 L: To buy an apartment? Okay. Is there anything else that it tells us? Does everyone have the same size of apartment?
11 S: No.
12 L: No. So maybe they will give the same amount.
13 S: No.
14 L: No?
15 S: Yes.
16 L: Yes? They will? Okay, so I got (read the next statement) “They will put their money together in such a way that each will pay an amount that is proportional to the size of their apartment.” Proportional to the size of their apartment (underline). What do you think that means?
17 S: Um... they will pay less money if their apartment is small?
18 L: Good. Then what if their apartment is bigger?
19 S: Pay more money.
20 L: Good. Does that give you any clue what proportional means in this setting?
21 S: ... ... ... No
22 L: No? Basically what you said if the apartment is small they pay less, if big, pay more. Does that make sense?
23 S: Yeah.
24 L: Okay. So let's go to the next part. It will help a little bit. Do you want to read that aloud?
Unlike Becky, Lucy did not reduce cognitive demands of the mathematical problem while supporting her student to understand the problem. She usually maintained the cognitive demands and did not suggest or force her student to follow a particular method.

**Mathematical and Contextual Language**

Similar to Becky, Lucy used the strategy, *using visuals*, most frequently and consistently in various forms such as drawing, post-its, and pictures. Beside, Lucy employed manipulative, objects, gestures, and defining words. In the previous conversation, she began with *assessing student’s English proficiency* (line 4). Although Si-Young expressed she knew all words included in the statement (line 5), Lucy did not let her move to the next step and asked to explain what the problem meant in her own words (lines 6 & 8). Si-Young could not answer this question well (lines 7 & 9). It was not clear whether Si-Young could not explain it in English although she knew the answer or she did not understand the problem although she knew the meanings of all words. Nonetheless, she got another chance to read and think about the problem carefully through multiple interactions with Lucy.

As for *using gestures*, Lucy used pointing to indicate what she was talking about as Becky did, but she also employed various gestures. For example, she nodded to show her agreement or used hand gestures to describe a long distance or to mimic two hands of a clock. Her use of gesture appeared consistently through five weeks, but she did not explicitly state gestures as a strategy in her lesson plans.

The following figures (Figure 13 & 14) are examples of her application of visual supports. She sometimes drew mathematical representations using the iPad app, *Explain*
Everything (Figure 13) or used the grid paper (Figure 14) to draw floor plans with color coding. Lucy had her ELL use different colors for each room and its dimension.

![Figure 13. Lucy’s drawing to present a warm-up problem using Explain Everything](image)

It was not obvious how the intervention about visual aids influenced Lucy’s performance because Lucy used various visuals with connections to the mathematical situations in the problems before she received the particular intervention. Moreover, unlike other PSTs, she did not use a chart or a table after she saw during the intervention.

![Figure 14. A part of Si-Young’s work in week 4](image)
Positioning

Lucy’s use of positioning strategy was neither explicit nor significant (see Table 6). Although she was told one way of encouraging ELLs to speak out is pretending not to know the answer, she had never attempted to use the strategy as the third column of Table 6 is empty. Rather than lowering teacher position, raising student position was observed most frequently from Lucy’s discourse such as “I really like the way you did that because I didn’t think of that. You’re ahead of me” (week 2) and “That’s a good idea” (week 3).

Table 6

*Frequency of the instances Lucy applied the strategies in positioning*

<table>
<thead>
<tr>
<th>Week</th>
<th>Data source</th>
<th>Lower teacher’s authority</th>
<th>Raise student’s position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Implementation</td>
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<td></td>
<td>Lesson plan</td>
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<td></td>
<td>Modified problem</td>
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<tr>
<td>2</td>
<td>Implementation</td>
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<td>9</td>
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<td></td>
<td>Lesson plan</td>
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<td></td>
<td>Modified problem</td>
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<tr>
<td>3</td>
<td>Implementation</td>
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<td></td>
<td>Lesson plan</td>
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<td></td>
<td>Modified problem</td>
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<tr>
<td>4</td>
<td>Implementation</td>
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<td>3</td>
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<td></td>
<td>Lesson plan</td>
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<td></td>
<td>Modified problem</td>
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<tr>
<td>5</td>
<td>Implementation</td>
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<td>1</td>
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<td></td>
<td>Lesson plan</td>
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<tr>
<td></td>
<td>Modified problem</td>
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</tbody>
</table>

For example, she allowed Si-Young to take part in making problems during the speed activity in week 2. Whenever she proposed a new problem, she asked Si-Young to designate the next place she wanted to go and to calculate the speed given a distance and a duration. By allowing her student to make a problem, which is typically teacher’s work,
Lucy raised her student’s position and successfully engaged her in the mathematical activity. However, none of her actions related to positioning was explicitly planned in advance. It was not obvious whether she intentionally used the strategy or not because she did not include these strategies in her lesson plans.

**Summary of Lucy**

Initially, Lucy failed to set an appropriate level of the mathematical content of the first task she created. She assumed ELLs would not be able to perform high level of mathematics due to their needs in using English. She described it as the most difficult moment in her post-survey.

The first lesson was the most difficult to implement because I did not know what academic level my student would be on. I tried to start the lesson with a warm up of different arithmetic problems, but they were all too easy for her. I think that she was getting bored because it was too easy. Also, once we got to the actual sandbag problem she was able to solve it pretty quickly (Lucy, post-survey).

However, she had learned from her experience with Si-Young that ELLs are capable of doing cognitively demanding problems if they have appropriate supports. Eventually, she was able to provide an appropriate level of challenging work to her ELL.

Most importantly, she was the only one who used rich cultural connections. In her post-survey, she clearly stated that connecting student’s cultural/life experience was the most effective strategy. Other than connections, the strategies she used consistently include pressing for explaining solutions, using visuals, analyzing problems, and assessing student’s knowledge. Similar to Becky, she showed a clear shift in adapting strategies related to cultural/life experience due to the influence of interventions. However, I did not find strong association between the intervention and her use of strategies in other categories.
The Case of Kate

Because her minor was Spanish, Kate could speak both English and Spanish although she said she had not had many opportunities to talk with native Spanish speakers. Kate was a white female preservice teacher who had not learned about how to teach mathematics for ELLs or taught ELLs mathematics prior to participating in this study. Similar to Lucy, Kate had the belief that mathematics is a universal language so it would be less difficult for language learners to learn mathematics because all countries use the same mathematics. The following quote from her pre-survey expresses this belief.

I don’t believe they will experience more difficulties because math is the same in all languages. One plus one is always two no matter if it’s taught in Korean or English. I think what is difficult is teaching students to have math confidence in another language (Kate, pre-survey).

From the example Kate used in this brief statement, “one plus one,” I consider her view of mathematics could be procedure-centered. As for instructional strategies, Kate suggested various strategies such as teaching through visual aids and audio, partnering up, and one-on-one teaching. Kate’s approaches contrasted against other PSTs who initially proposed only vocabulary-related strategies.

Kate worked with Ye-Jun, a male ELL in seventh grade who was enrolled in Challenge Math 7, which was a regular mathematics course but for relatively high performers. Ye-Jun was born in South Korea, but he had lived in France for a couple of years when he was young. Yet, he said he forgot how to speak French. When he joined this study, he had been living in the U.S. for a little more than one year.

Mathematical Content

Kate did make some modifications on the given problems, but she rarely included additional problems or questions. During implementations, she repeated or rephrased the
problem statements rather than providing guiding or challenging questions. Overall, she did not add any challenging tasks and little practice questions to the problems. Even when she provided an extra question, it was unplanned. More specifically, she did not attempt to prepare any pre-assessment set in the beginning of the first meeting unlike other PSTs. In week 1, she presented the original problem right after she had an ice breaking activity. She did give one practice problem, which was very similar to the original problem, but it was not what she planned. She made it up at that moment because her student finished the given problem too quickly.

<table>
<thead>
<tr>
<th>Given</th>
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<tbody>
<tr>
<td>Claire is filling bags with sand. All the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 58 pounds, another sand bag weighs 41 pounds, and another sand bag weighs 53 pounds. Explain whether Claire can pour sand between sand bags so that the weight of each bag is less than 50 pounds.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Modified</th>
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<tbody>
<tr>
<td>Claire is filling bags with sand. All the bags are the same size. <strong>Each bag must weigh less than 50 pounds.</strong> One sand bag weighs <strong>58 pounds</strong>, another sand bag weighs <strong>41 pounds</strong>, and another sand bag weighs <strong>53 pounds</strong>. Explain whether Claire can pour sand between sand bags so that the weight of each bag is less than 50 pounds.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1st Review</th>
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<tbody>
<tr>
<td>Kyle has three bags of sand. One weighs 45 pounds, another weighs 56 pounds, and the third weighs 47 pounds. Is it possible for Kyle to pour sand from each bag into each other in order for each bag to weigh less than 50 pounds? Explain why or why not using only pictures and numbers. (without actual words)</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>2nd Review</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kyle has three bags of sand. One weighs 45 pounds, another weighs 56 pounds, and the third weighs 47 pounds. Is it possible for Kyle to pour sand from each bag into each other in order for each bag to weigh less than 50 pounds? Explain why or why not using only pictures and numbers. (without actual words) (If possible, [your]answer can be written in decimal form)</td>
</tr>
</tbody>
</table>

*Figure 15. Kate’s different versions of modification of the first problem*

However, she was the only one who planned and provided review problems. For
example, in the second week, she asked Ye-Jun to solve a similar structured problem to the first week problem before she presented the second week’s problem. The box above showed how the first problem was adapted by Kate through several versions. Kate did not change any words in the given problem in week 1, but made some important words bold (see the modified problem in Figure 15). Ye-Jun used two different methods to solve the modified version of problem. One of them was to try to make the weight of each sandbag 49 pounds or less and see if it was possible to make the last bag less than or equal to 49 pounds. To perform this method, he subtracted some pounds from the heaviest sandbag to make it 49 pounds and added the amount to another sandbag lighter than 50 pounds. The mathematical skills he used in this method were only additions and subtractions. It was the same method as what Kate (and other PSTs) expected from her student and she guided him in this way.

However, Ye-Jun demonstrated another method, which implied a concept of average, at the end of the session; he added all three numbers and checked if the sum was less than 147 pounds (it should be less than 150 pounds, but he used 147 because his goal was to make each bag equal to or less than 49 pounds). Although Kate said she was impressed by this method, which she had not known about, Ye-Jun did not employ the average method when he had the review problem (see the first Review problem in Figure 15) and went back to the addition and subtraction method. Unfortunately, the strategy was not successful this time because the sum of the numbers Kate made up for the review problem was between 147 and 150. Thus, when Ye-Jun tried to make all bags under 49 pounds, his answer was “impossible”. However, the correct answer was “possible” because the sum of all bags was 148 pounds, which was less than 150, so he could make
all three bags under 50 pounds if he used non-whole numbers. Kate did not notice this fact, and after she learned about her mistake during the post-interview, she wanted to provide another review problem to fix her previous mistake. That was the reason why her second version of review problem included “If possible, [your] answer can be written in decimal form” at the end (see the second Review problem in Figure 15).

**Cultural/life Experiences**

None of the strategies in this category occurred consistently, but the overall use of the strategies has a similar pattern to that of Becky. The most frequently used strategy was connecting a problem with student’s life experiences and Kate did not use connecting a problem with student’s culture (see Table 7).

### Table 7

*Frequency of the instances Kate applied the strategies in cultural/life experience*

<table>
<thead>
<tr>
<th>Week</th>
<th>Data source</th>
<th>Assess student's life experience</th>
<th>Connect to student's culture</th>
<th>Connect to student's life experience</th>
<th>Explain cultural contexts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Implementation</td>
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<td>Lesson plan</td>
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<td></td>
<td>Modified problem</td>
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<tr>
<td>2</td>
<td>Implementation</td>
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<td></td>
<td>Lesson plan</td>
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<td></td>
<td>Modified problem</td>
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<tr>
<td>3</td>
<td>Implementation</td>
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<tr>
<td>4</td>
<td>Implementation</td>
<td>1</td>
<td>7</td>
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<td></td>
<td>Lesson plan</td>
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<td>Modified problem</td>
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<tr>
<td>5</td>
<td>Implementation</td>
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<td>1</td>
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<td></td>
<td>Lesson plan</td>
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<td>Modified problem</td>
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</table>
In addition, similar to Becky and Lucy, her use of these strategies did not appear in the first week. The application of these strategies mostly appeared in weeks 4 and 5 when Kate actively adapted them through contextual modifications of the problems. The week 4 problem was about designing a one-story house floor plan, but Kate changed the context to building an arcade in the basement because she heard that Ye-Jun liked video games. The fifth problem was about farming: how many conifer trees should be planted around a specific number of apple trees. Kate modified the context because she thought farming was not a familiar context to a middle school student. Instead of the farmer’s story, she employed the story of an online game, *Plants vs. Zombie*, and twisted it planting trees to protect houses from zombies. Because Kate needed to present the modified problems, she threw questions about Ye-Jun’s life experiences related to the context and explained unfamiliar contextual words such as an “arcade” or “zombie apocalypse.” Therefore, her use of the strategies in this category was closely related to her contextual modifications of the problems.

**Mathematical and Cognitive Processes**

As other PSTs did, one of the main strategies Kate used in this category was *pressing for explaining solutions*. She used various questions to ask what Ye-Jun did and why he took a certain way to solve the problem, such as “What did you do from this proportion to get it down to that?” and “Can you tell why just by looking at it, why he pays less than over here?”

However, the most frequently used strategy was *modeling the process prior to student’s solving*. In other words, she directly told him how to solve the problems or taught the exact mathematical concept prior to solving the problem. With this guidance,
Ye-Jun did not have an opportunity to explore the problem and to establish his own argument. In addition, her guidance usually focused on procedural aspects rather than conceptual aspects. Her week 3 implementation showed this pattern as appeared in the following dialogue taken from the beginning part of week 3 session.

(K: Kate, Y: Ye-Jun)
1   K: So, um... What do you know about proportions?
2   Y: ... ... ... ...
3   K: It’s okay. So, a proportion is showing a relationship between two things, so and then two fractions are setting equal to each other. So, for instance, um, here, if you have three feet for ... 60 seconds, and you want to know how far you can go for 120 seconds, so down here I put the second, up here, I put the feet, I want to know how many feet I can go. For 120 seconds, and this is what we don’t know (put \(x\)), so we’re trying to figure out what \(x\) is here. What do you think we’re going to do to figure out what \(x\) is?
4   Y: We need to multiply by 2 to get 120 seconds, so multiply by 2 to 3.
5   K: Why don’t you write that? You solve on that side, and I will solve on this side. (They wrote their solutions on the board) That’s right. See, we approached differently but we still did right. So we can solve a proportion in multiple ways. This was a little easy because you were able to see you know, 60 times 2 is 120, and 3 times 2 is 6, but in the future instances, you might have tricky ones, but I did here, and cross multiplied and when you have an equals sign here, and they are two fractions, that’s the way proportion is set up, and you can simply multiply whatever’s up here and whatever’s down here and then vice verse over here, so 60 times \(x\), 60\(x\), 3 times 120, 360, divided by 60. For times when you have proportions that don’t have numbers as fine and friendly as these, um, you may think about cross multiply to find, but the way you did also works too. So, now that we kind of went over this.
6   K: Let’s jump into this first problem is asking. Do you want to read that?

In this vignette, Kate explained how to solve a typical proportional equation. She talked much longer than other PSTs and there was little interaction with her student. She first asked if Ye-Jun knew about proportions (line 1). Because he could not answer, Kate briefly explained a general meaning of proportions, “a relationship between two things” (line 3), and then moved directly to how it looks like and how to use the cross-multiplication method. When she presented the example of proportion between time and
distance, her focus was where each value should be placed rather than the relationship between two ratios.

Figure 16. The proportion example Kate created in week 3.

In line 10, Ye-Jun found the value of $x$ by using the concept of equivalent ratios, but Kate used the cross-multiplication method and pushed him to follow her way although there was no reason to use the method to solve the problem. They wrote each of their methods on the board (line 12, Figure 17).

Figure 17. (Left) Kate’s cross-multiplication method. She first applied cross multiplication to build an equation, and then divided both sides of the equation by 60. (Right) Ye-Jun’s equivalent ratio method. He found the relationship between denominators, and applied the same relationship to the numerators to find the value of $x$. 

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Although Kate agreed both methods were correct (lines 12-13), she did not provide an explanation about how they were related. Furthermore, she did not or was not able to explain the conceptual background of cross multiplication. What she described was each step of the procedures of cross-multiplication method without connecting concepts behind them (lines 16-19).

More importantly, she taught proportions before presenting the problem so that her student could assume that he had to use the same method to solve the problem. In addition, Kate pressed Ye-Jun to set up a proportion for each question in the problem and to solve it using cross multiplication. This action significantly limited Ye-Jun’s opportunity to do problem solving as well as reduced the cognitive demands of the problem by suggesting a specific method.

**Mathematical and Contextual Language**

Unlike other PSTs, Kate had never attempted to explicitly assess student’s English proficiency. Specifically, she did not ask Ye-Jun if there was any unknown word in the problem statements or if he understood the problem. Kate used *gestures*, but the only type of gestures she used was pointing as did Becky.

The most frequently used strategy to overcome the language barrier was the same as others, *visuals*. In her first meeting with Ye-Jun, she found a white board in the classroom and spontaneously decided to use it. Because she thought the lesson using the board went well in week 1, she continued to use the board in her next lessons. For example, she drew a rectangle to represent a sandbag on the board and Ye-Jun followed her way to solve the first problem (see Figure 18).
While frequently employing visuals, Kate enacted them in various ways as well as made them coherent to the mathematical concept embedded in the problem as time evolved. For example, in weeks 1 and 2, she simply added pictures, only related to the context, to the given problem. For week 1 problem, she added three sandbag photos with captions and for week 2 problem, she included one picture of a traveler’s car.

Figure 19. The visual supports Kate included in the third problem.
However, after receiving the intervention of visual supports, she developed her application of visuals more closely to the mathematical situation rather than giving a hint of the context. For instance, Kate decided to include pictures of a floor plan connected with information of the owner and the areas (see Figure 19). She purposefully chose the pictures in order to convey information that the building consisted of different sizes of unit and also personalized each owner by creating names of the owners.

Besides visuals, Kate allowed her student to communicate by using alternative methods such as writing, gesturing, or drawing. The following conversation describes this strategy, *multiple modes of communication* in week 1.

```
1  K: So one bag is going to weigh 42, one bag is going to weigh 56, and one bag is going to weigh 52.
2  Y: (Draw three rectangles and enter 42, 56, and 52 each as Figure 18) This must be less than 50?
3  K: Yes.
4  Y: (Write 7, 7, and 3 under each rectangle) Add this one here (draw an arrow from 7 under the box of 56 to 7 under the box of 42, and then write 49 and 49 under both boxes and 52 under the last box) it makes 49, 49, and this is 52. So it’s not possible.
5  K: You’re right. It is impossible. Is there any other way we can possibly solve this problem? Besides getting the fourth bag.
6  Y: Um... we add all this, (gesture including all three numbers) and it should be less than one hundred fifty, or [the] same.
7  K: You want to write that out for me? You want to write what you just thought of in your head?
8  Y: Write words?
9  K: You can write it in numbers, you can write in words, just whatever you just did in your head. Can you show me?
10 Y: (Draw 3 boxes and a big box) Add these together (draw arrows from small boxes to the big box as the right one of Figure 19 shows)
11 K: So you added those three numbers?
12 Y: Yes.
```

First, Ye-Jun used drawing to demonstrate his solution; he drew rectangles and arrows (line 3, Figure 18). When Kate asked for a different method (line 10), he used a hand gesture to express his thinking (line 12), which was adding all three numbers and
watched if the sum exceeded 150. However, Kate did not fully understand what he meant, so she asked him to write it out (line 14) in numbers or in words (line 17). Ye-Jun responded by drawing his idea as Figure 20 (line 19). In this short instance, Kate allowed Ye-Jun to use four different modes of communication: drawing, gesturing, writing numbers, and writing words.

Figure 20. Ye-Jun was explaining his alternative method by drawing.

Kate was the only one who modified the problem contexts, but what she changed was not only context. Three different versions of modification in Figure 15 illustrate how Kate considered task modifications in terms of language. For reader’s convenience, I duplicate the Figure 15 on the next page. The first modification neither changed its context nor the language. The only modification she made was to bold important words. She explained, “I bolded the important things because I don’t know where he is with his English and how he does extract the vital information from the problem” in her lesson plan. However, she made significant change in language in review problems to make the statement explicit. Rather than saying one, another, and another sand bag, she used one, another, and the third to distinguish each bag from others. However, she changed the final question from “Explain whether Claire can pour sand between sand bags so that the
weight of each bag is less than 50 pounds” to “Explain why or why not using only pictures and numbers (without actual words).” By changing the question, Kate eliminated a chance of practicing language, writing or speaking, from her student. This might be due to her belief that less use of language could help ELLs do and understand mathematics.

**Given**
Claire is filling bags with sand. All the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 58 pounds, another sand bag weighs 41 pounds, and another sand bag weighs 53 pounds. Explain whether Claire can pour sand between sand bags so that the weight of each bag is less than 50 pounds.

**Modified**
Claire is filling bags with sand. All the bags are the same size. Each bag must weigh **less than 50 pounds**. One sand bag weighs **58 pounds**, another sand bag weighs **41 pounds**, and another sand bag weighs **53 pounds**. Explain whether Claire can pour sand between sand bags so that the weight of **each bag is less than 50 pounds**.

**1st Review**
Kyle has three bags of sand. One weighs 45 pounds, another weighs 56 pounds, and the third weighs 47 pounds. Is it possible for Kyle to pour sand from each bag into each other in order for each bag to weigh less than 50 pounds? Explain why or why not using only pictures and numbers. (without actual words)

**2nd Review**
Kyle has three bags of sand. One weighs 45 pounds, another weighs 56 pounds, and the third weighs 47 pounds. Is it possible for Kyle to pour sand from each bag into each other in order for each bag to weigh less than 50 pounds? Explain why or why not using only pictures and numbers. (without actual words) (If possible, [your] answer can be written in decimal form)

*Figure 15.* Kate’s modifications of the first problem (duplication)

**Positioning**
Kate did not plan to use any *positioning* strategies. When she used *raising student position* or *lowering teacher position*, it was unintentionally used.
Table 8

*Frequency of the instances Kate applied the strategies in positioning*

<table>
<thead>
<tr>
<th>Week</th>
<th>Data source</th>
<th>Lower teacher’s authority</th>
<th>Raise student's position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Implementation</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Lesson plan</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modified problem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Implementation</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Lesson plan</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modified problem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Implementation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lesson plan</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modified problem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Implementation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lesson plan</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modified problem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Implementation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lesson plan</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modified problem</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For example, she uttered “I’m really impressed. You did a great job” or “I even didn’t think about doing it in that way, so you taught the teacher, so that’s really a good way to do it” when Ye-Jun suggested an alternative solution that Kate did not consider. She mentioned this in her post-survey, “I also really connected with him in moments where he was the teacher. Almost every lesson he would solve the problem in a completely different way than I did, and it would show me the problem in a whole new perspective.” She might use this type of expressions to cheer and motivate her student, but she did not plan it as a strategy to stimulate a communication or a discussion with him even after she was told the strategy during the intervention.
Summary of Kate

In the beginning, she expressed her confidence in understanding ELLs because she had been in a similar situation of learning a foreign language. She restated this in her post-survey as follows:

I feel I was able to connect to my student because I have been in a similar situation like him. I am still learning Spanish and even after eight years of studying, there is still a lot for me to learn. So, I would think about myself as a SLL [Spanish language learner] student and compare what helped my learning process and tried to match it with what I did while working with my student (Kate, post-survey).

However, her session included little interaction between her and her student. She did not put much effort to encourage her student to speak out. On the contrary, she provided a long explanation and allowed the student to use alternative ways to communicate. She might have tried reducing language demand to make her ELL comfortable as she eliminated verbal responses in her modified review problems in Figure 15.

The strategies she consistently used include pressing for explaining solutions, using visuals, and rephrasing, but with significantly less frequency compared with other PSTs. In terms of shifts in strategy use or influence from interventions, I did not find strong evidence except strategies in the category of cultural/life experience. Kate had a similar pattern to Becky in this category because she was influenced by the intervention of connecting mathematical problems with student’s life and cultural experience. There was no clear pattern in other categories. However, she stated that she thought the intervention about visual supports was effective, but her application of the intervention was not consistent because she did not bring various types of visuals in weeks 4 and 5 although she tried some new ways of visuals in week 3, which was the following week from the relevant intervention.
The strategy of using visual aids was by far the most effective. I can never forget when you brought in those three papers asking the same questions, all in Korean, with different number of pictures on each. I was so lost in the one with no pictures, and at that point, I just gave up immediately. Helpful pictures are the key with ELL students and I do not think that can be stressed enough! (Kate, post-survey)

Overall, she used less numbers of strategies and less interactions with her ELL comparing with other PSTs.

**Cross-Case Analysis**

In the previous sections, I presented findings from each case in terms of five components of the conceptual framework: *mathematical content, cultural and life experience, mathematical and cognitive process, mathematical and contextual language,* and *positioning*. In this section, I present findings from a cross-case analysis of all three units of analysis, PSTs. Unlike the previous sections, I follow a chronological order rather than categories in order to illustrate how they responded to the same mathematical problem and how their teaching evolved as they had more experiences. First, I present my findings across three phases: *planning, understanding problems,* and *justifying solutions*. These phases are based on the three sub-questions of the first research question. The analysis of all PSTs’ weekly performance follows in order to provide sufficient information for finding answers of the second question, regarding PSTs’ growth. Finally, I describe the emerging themes from all analyses with brief descriptions.

**Analysis of Phases**

Artzt and Armour-Thomas (1999) used three different stages of teaching in order to understand the role of cognition in teacher instructional practices. Each stage was named as preactive, interactive, and postactive. The preactive phase refers to lesson planning; the interactive phase denotes teacher monitoring and regulating; and the
postactive phase covers teachers’ evaluation and revising. I adopted this structure of phases, but I slightly modified the meaning of postactive. Instead of teacher’s reflection after lesson, I used it to refer the phase of justification after students found their solutions or a part of solution in order to align them with my research questions.

**Preactive phase.** The PSTs were asked to submit written lesson plans with modified problems and any additional materials. These materials convey crucial information such as what strategies the PSTs implemented intentionally and what strategies they believed were useful for their ELLs to understand and solve the problems. The lesson plan template required to fill out not only procedures but also their rationale for each procedure. However, some PSTs did not provide sufficient details and the rationale behind it, especially in the early weeks.

Table 9 summarizes the strategies they included in their lesson plans and modified problems. All PSTs commonly used *visuals*, but Table 9 reveals that each PST has distinct patterns. In Becky’s case, *problems to access a challenge* and *modifying problem language* besides *visuals*. She disclosed her concerns about vocabulary knowledge of ELLs in her pre-survey. This concern continued to appear her lesson plans because *modifying problem language* by simplifying sentences stemmed from her belief that ELLs have difficulty understanding the problem statements due to their lack of reading skills in English. In her lesson plans, she stated the original problem was inappropriate for ELLs because “it has a lot of words in it” (weeks 1 & 2) or “there were too many words and no visuals to help the student out” (weeks 3, 4, & 5). It should be noted that she started mentioning visuals after she received the intervention of visuals (between week 2 & 3).
Table 9

Strategies included in lesson plans and modified problems developed by each PST

<table>
<thead>
<tr>
<th>Week</th>
<th>Becky</th>
<th>Lucy</th>
<th>Kate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sandbag problem Modifying language Problems to assess Visuals</td>
<td>Problems to access Visuals</td>
<td>Modifying presentation Visuals</td>
</tr>
<tr>
<td>2</td>
<td>Average rate of distance Problems to access</td>
<td>Life connection Objects Problems to access Problems to challenge Visuals</td>
<td>Modifying presentation Visuals</td>
</tr>
<tr>
<td>3</td>
<td>Proportional apartment Modifying language Problems to access Visuals</td>
<td>Life connection Visuals</td>
<td>Modifying presentation Visuals</td>
</tr>
<tr>
<td>4</td>
<td>A floor plan drawing Modifying language Problems to access Visuals</td>
<td>Life connection Problems to access Pressing to explain Visuals</td>
<td>Life connection Modifying presentation Visuals</td>
</tr>
<tr>
<td>5</td>
<td>Tree patterns Life connection Problems to access Visuals</td>
<td>Life connection Objects Problems to access Pressing to explain Visuals</td>
<td>Life connection Modifying presentation Visuals</td>
</tr>
</tbody>
</table>

Lucy’s column included the most diverse strategies. Unlike others, she used life connections thoroughly after she learned about it between weeks 1 and 2. In her lesson plans, she explicitly addressed her rationale as “Have her connect these problems to her everyday life” (week 2). Providing problems to access a challenge was her other favorite strategy. She usually prepared a set of pre-activities that bridge the given problem and asked her student to complete it before she presented the high cognitive demand problem. However, she did not change problem statements for five weeks although she added pictures and designed it differently. In addition, she was the only one who clearly wrote
that she would have her student talk through all steps of solving the problem in her lesson plans.

Kate showed a rather simple pattern of strategy use compared with others. Besides visuals, the only strategy she used thoroughly was modifying problem presentation. In her lesson plans of weeks 1 through 4, she stated the only modification she made was bolded fonts. For example, in week 1, she wrote, “I bolded the important things because I don’t know where he is with his English and how he does extracting the vital information from the problem” and in week 4, “I bolded the important things because I know he struggles with reading word problems and this particular problem has a lot of stipulations that are important to remember while working through this problem.” From her explanation, she seemed to believe that her student would struggle finding important clues from the problem statement; thus, she bolded the key words so that her student could pick up those clues although he might be unable to understand the entire situation of the problem.

Overall, the difference by each PST was much distinguishable than that of given task. In other words, each PST had consistency on applying strategies whatever problems were given. In other words, they applied different strategies although they were given the same problem in the same week. Becky and Kate considered reducing language demand as their main foci. Lucy concentrated on connecting mathematical problems with student’s experiences rather than making language easier, especially after she learned about cultural/life connection from the researcher. Eventually, all three PSTs included life connection in their lesson plans in the last week, which was just after the second intervention of life connection was given. Related to the conceptual framework, what
PSTs considered in their lesson planning was mostly related to the category of mathematical and contextual language because their first goals were reducing language demand. In Lucy’s case, the category of cultural/life connection played a central role.

Interactive phase. In this phase, I included only the time the PSTs helped ELLs understand the problems before ELLs found their solutions. During this phase, many unplanned strategies occurred intentionally or unintentionally according to the PSTs’ perception about the needs of their ELLs.

Table 10 on the next page describes the strategies appeared clearly in each session. This table was made from analyzing frequency tables I constructed using NVivo (see Figures 7 & 8). Unlike the strategies in the preactive phase, the strategies in this phase appeared complex. However, the vertical patterns were stronger than horizontal patterns in the table although the patterns were not as clear as Table 9. For example, Lucy usually applied connecting a problem with student’s life experience and problems to access a challenge as she planned. However, Becky and Kate did not show consistency. In addition, their implemented strategies were different from what they planned. Their preactive strategies were mostly about how to modify or present problems rather than how to support their students to understand the problem and find entry points to solve it.

Again, using visuals was the most popular strategy among all PSTs, but the strategies in this phase was implemented differently from the visuals in the preactive phase. For instance, Becky did not really utilize the pictures she included in her modified problem in week 1 to help her student understand the problem; thus, I did not include visuals in the cell of Becky’s week 1 column.
Table 10

*Strategies implemented by each PST during interactive phase*

<table>
<thead>
<tr>
<th>Week</th>
<th>Becky</th>
<th>Lucy</th>
<th>Kate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Sandbag problem</td>
<td>Problems to assess</td>
<td>Problems to access</td>
<td>Rephrasing</td>
</tr>
<tr>
<td></td>
<td>Rephrasing</td>
<td>Visuals</td>
<td>Visuals</td>
</tr>
<tr>
<td>2 Average rate of distance</td>
<td>Modeling process</td>
<td>Cultural connection</td>
<td>Analyzing problem</td>
</tr>
<tr>
<td></td>
<td>Multiple representations</td>
<td>Gestures</td>
<td>Rephrasing</td>
</tr>
<tr>
<td></td>
<td>Visuals</td>
<td>Life connection</td>
<td>Visuals</td>
</tr>
<tr>
<td>3 Proportional apartment</td>
<td>Modeling process</td>
<td>Life connection</td>
<td>Analyzing problem</td>
</tr>
<tr>
<td></td>
<td>Rephrasing</td>
<td>Visuals</td>
<td>Modeling process</td>
</tr>
<tr>
<td></td>
<td>Visuals</td>
<td></td>
<td>Visuals</td>
</tr>
<tr>
<td>4 A floor plan drawing</td>
<td>Analyzing problems</td>
<td>Analyzing problem</td>
<td>Life connection</td>
</tr>
<tr>
<td></td>
<td>Gestures</td>
<td>Cultural connection</td>
<td>Visuals</td>
</tr>
<tr>
<td></td>
<td>Visuals</td>
<td>Life connection</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Problems to access</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Problems to assess</td>
<td></td>
</tr>
<tr>
<td>5 Tree planting patterns</td>
<td>Gestures</td>
<td>Problems to assess</td>
<td>Life connection</td>
</tr>
<tr>
<td></td>
<td>Life connection</td>
<td></td>
<td>Objects</td>
</tr>
<tr>
<td></td>
<td>Objects</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rephrasing</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Visuals</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Other strategies used in this phase were *asking them to analyze the problem*, *rephrasing*, *problems to assess prior mathematical knowledge*, and *gestures*. When the ELLs struggled understanding the meaning of problem or finding entry points, the PSTs usually asked them to find clues or constraints in the problem by asking “what was the condition?” or “What did the problem tell you about this?” They also rephrased or paraphrased the problem statements when students had hard time to grasp the meaning.
Sometimes, they needed to ask if they learned the mathematical concepts included in the problem statement like, “Have you heard of the word, proportion?” or “Have you learned about variables?” As for gestures, Lucy was the only one who applied various gestures and other PSTs used only pointing.

I mentioned in the earlier section that some strategies they planned sometimes ended up reducing the cognitive demand of the problem. Similar situations were observed during the interactive phases. In Table 10, *modeling the process prior to student’s solving* appeared several times in the cases of Becky and Kate. Week 3 was when they used this method most because the PSTs guided ELLs to use a particular procedure, the cross-multiplication, to solve the problem rather than let them use their own reasoning or more efficient strategies. Similarly, week 5 was the only week all of them applied the same method, *objects*. All the PSTs intended to use manipulatives in order to work on the pattern problem. Overall, all categories but *positioning* in the conceptual framework were implemented in this phase although each case had distinct patterns.

**Postactive phase.** The process of making sense of mathematics was not finished after students found their solutions. According to MP1 of CCSSM (NGA Center & CCSSO, 2010), students have to “monitor and evaluate their progress and change course if necessary,” “check their answers to problems using a different method,” and “understand the approaches of others to solving complex problems and identify correspondences between different approaches” (p. 6). The postactive phase began after the ELLs found an entry point and made some progresses on the way their solutions. The following table shows what strategies the PSTs employed to support their ELLs to explain their reasoning process and solution pathways.
Table 11

Strategies implemented by each PST during postactive phase

<table>
<thead>
<tr>
<th>Week</th>
<th>Becky</th>
<th>Lucy</th>
<th>Kate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Sandbag problem</td>
<td>Sandbag problem</td>
<td>Multiple modes of communication Pressing to explain Visuals</td>
<td>Multiple modes of communication Visuals</td>
</tr>
<tr>
<td>2 Average rate of distance</td>
<td>Multiple modes of communication Pressing to explain</td>
<td>Multiple modes of communication Pressing to explain</td>
<td>Multiple modes of communication Pressing to explain</td>
</tr>
<tr>
<td>3 Proportional apartment</td>
<td>Multiple modes of communication</td>
<td>Multiple modes of communication Pressing to explain</td>
<td>Pressing to explain</td>
</tr>
<tr>
<td>4 A floor plan drawing</td>
<td>Multiple modes of communication Pressing to explain Positioning Visuals</td>
<td>Multiple modes of communication Pressing to explain Visuals</td>
<td>Visuals</td>
</tr>
<tr>
<td>5 Tree planting patterns</td>
<td>Multiple modes of communication Pressing to explain Positioning Visuals</td>
<td>Multiple modes of communication Pressing to explain Multiple representations Objects</td>
<td>Objects</td>
</tr>
</tbody>
</table>

Compared with the previous tables, Table 11 contains a much less number of strategies. It reflects the PSTs did not have various strategies to help ELLs explain their thinking. At times, they were not able to guide the ELLs to explain how they found their answers although they could check their answers were correct. When their students could not find a correct answer, communication became a huge barrier because the ELLs had difficulty understanding the PSTs’ verbal instruction and the PSTs could not clearly understand what the ELLs tried to explain. It was often found that the PSTs misunderstood their students’ responses. For example, when the student said an incorrect answer, the PST assumed it correct and moved on the next step.
The most common strategy and almost the only strategy used regularly was *pressing to explain a solution*. The PSTs usually asked their ELLs to explain their processes or justify their solutions or a part of procedures using questions such as “Why would you do that?” “How did you get eight?” or “What made you start a different way than you did before?” All three PSTs used these questions through all five weeks. However, they struggled communicating with their ELLs because the ELLs did not speak out much. The ELLs usually had a long pause before they responded and their responses were usually fragmented sentences or disconnected words with incorrect grammar. Therefore, it was necessary for the PSTs to provide linguistic support to help ELLs explain their thinking and improve their language skills. However, I did not find various linguistic supports, such as using a sentence frame, ELLs’ first language, or graphic organizers. They focused on how to help the ELLs understand English in mathematical problems, not how to support and encourage their participation in mathematical discussion.

However, there were two distinguishable strategies other than *pressing for explaining solutions*. One of them was *multiple modes of communication*. They allowed the ELLs to use alternative methods to express their thinking such as writing, drawing, pointing or using pictures or objects. For five weeks, the multiple ways of communication were prevalent in all cases. Another strategy was *positioning* in Becky’s case. She intentionally lowered her teacher position to encourage her ELL to speak out and communicate with confidence. Although this strategy did not directly support student’s cognition, it had a positive effect on communicating mathematically. In the conceptual framework, the category of *mathematics and cognitive processes* mostly
occurred in the postactive phase although other categories, *positioning* and *mathematical and contextual language*, appeared in some cases.

**Analysis of Weekly Implementations**

**Week 1.** Before the first meeting with their ELL students, the group of all PSTs received a short intervention about understanding ELLs. They read a story of a Korean ELL and learned about the general needs of ELLs (Celedon-Pattichis & Ramirez, 2012). Therefore, the focus of all PSTs in week 1 was to learn about their students. However, the beginning was already challenging for everyone. As Kate stated in her post-survey, the “greatest challenge would be not knowing where they [ELLs] stand at that moment with both mathematics and English.” In order to figure out student’s prior knowledge, Becky and Lucy prepared a kind of pre-assessment set. However, they failed to predict the appropriate level for their ELLs and ended up with first grade level assessment sets, such as adding and subtracting whole numbers. Becky explained why she prepared such a low level pre-assessment before she taught as follows:

> I wrote a few little things because I was thinking about it and what if he comes here and doesn’t know how to [do] this at all, I need to slow him down and back to the basic. It tests both math and language. If he trouble reading this, then I will be able to tell this is this, but also ask adding or subtracting. I didn’t think if I have him read or not (Becky, pre-interview, week 1).

However, she confessed “I wasn’t expecting him to know how to do it so easily, so I tried to think about what would be beneficial for him after he figured out the problem, so then we did some of the stuff he’s doing in class because I wasn’t sure what else we could do” after her lesson. Similarly, Lucy and Kate expressed their surprise by ELLs’ unexpected high performance and they felt they needed to prepare more challenging problems in the following sessions.
The first mathematical problem was adapted from the released item of SBAC, designed for eighth grade students. The suggested solution was using the concept of average, which means if the sum of three bags is greater than or equal to 150 pounds, it is not possible to make all bags less than 50 pounds each. However, none of PSTs used this method, but was sticking to only additions and subtractions to manually make all weights equal to or less than 49 pounds. In addition, all of them did not think of the possibility of non-whole numbers until they were told. It was one of the reasons why they developed subtraction problems for their pre-assessment sets; because subtracting numbers was their method, they assessed if their students were able to do subtractions. As a result, they did not pursue multiple solutions or multiple mathematical representations that might be helpful to maintain the cognitive demand of the problem.

Nevertheless, they built a better picture of their student’s ability in both mathematics and English from the first meeting, so they could set the goal for next meetings. Becky’s goal was having her student explain his work and reasoning as she stated below.

Having him explain was struggle, so that’s what I’m going to make a focus, having him tell me why he did something instead of super fast doing it, kind of explain to me to see... That’s math teachers need to check. You can do procedure, but you have conceptual knowledge. I think he’s good at the procedure, but I don’t know where he is at other stuff, so that’s what I will try to check for next time (Becky, post-interview, week 1).

Lucy wanted to prepare more challenging problems as well as to ask questions during the lesson “such as how did you get that or could you use a different approach to this problem” (Lucy, reflection, week 1) and Kate also said she would give more challenges with mathematical calculation. The strategies they integrated in this week’s lessons were adding pictures to the problem, simplifying problem statement, drawing, and adding
easier problems that access to how to solve the given problem. The intervention provided at the end of the first meeting was about cultural/life connections. Therefore, all of them stated that one of their goals for the next session would be trying to connect the lesson to the student’s life and culture on their written reflections.

Week 2. The second problem was rather traditional because finding speed from given time and distance is a common item in general mathematics textbooks. However, I chose this problem because it has potential to be adapted with rich life connections and surely the PSTs and ELLs were familiar with this context. When I saw their lesson plans, it was positive because Lucy and Kate planned to integrate student’s travel experiences into their lessons. However, Kate did not implement the strategy during her lesson and Becky did not even mention about connections in her lesson plan, which was different from what she set her goal at the previous meeting. In sum, Becky and Kate did not use life connections as their strategy although Becky happened to use it a few times when her student struggled calculating time. In contrast, Lucy applied rich connections between the problem and student’s cultural and life experience as she planned. Lucy also planned to give out many challenging problems, but she realized she overestimated the time period and her student’s ability. It was not only Lucy who planned to challenge their students more at the end of the previous week, but other two PSTs did not add any challenging problems to this week’s lesson.

Furthermore, all of them began to pay attention to how to communicate with their students because after the first meeting they learned although ELLs were able to solve problems, it was difficult to have them explain what they did. Becky revealed her concerns about this issue in her pre-interview.
See I don’t know what to do on a situation like that. I don’t know how to help him explain more. I want him to be able to vocalize it, but if he can’t, then I don’t know how to, I guess I can ask him, so... I guess I can ask him yes or no questions, it would help prior to the answer I guess, you can say, but I don’t know, I don’t really know what else I can with a situation like that. I don’t know. I guess I can just try and error, like ask him what did you do, and have him try to explain, I don’t know (Becky, pre-interview, week 2)

Unfortunately, her concerns were not resolved this week and she continued to struggle with communicating with her student.

This week’s intervention focused on visual supports. I provided a Korean written mathematics problem with three different versions of visual aids (see Appendix E). The first version was only text, and the second one included one picture of students. Until here, they did not have a clue what the problem was about. Finally, when they saw the final version, which included pictures, captions, and a table containing pictorial symbols, numbers, and variables. They were able to figure out what the problem was asking for from the third version. From this learning experience, they planned to use visuals connected to not only its context, but also the mathematical situations of the problem in the following lessons.

**Week 3.** The third problem was about purchasing an apartment building. Although the context might not be familiar with middle-school age students and the numbers were unnecessarily complex, none of the PSTs attempted to modify the context or numbers. As a result, they had difficulty explaining the situation to students. In addition, they struggled explaining the definition of proportions. Apparently, they did not prepare to teach the concept of proportions and did not expect that their students did not know about proportions. Lucy confessed the difficulty of explaining a mathematical term that she had been using without difficulty in the following statement.
The greatest challenge in teaching mathematics to English language learners was explaining terms and words that they weren’t familiar with. Some of the words that I use in everyday language are difficult to explain to people who do not know what they mean. For example, in a lesson my student did not know what “proportion” meant. I have been using this term for ten years and have never thought about the definition; I just knew what it stood for. Her questioning key words like this made me take a step back from the problem and really think about which words I need to be prepared to explain. Many times, my student did not know what a term meant but continued on with the problem anyway. I need to be able to clear up any misconceptions from the very beginning of the lessons so that the confusion of my student does not continue (Lucy, post-survey).

More importantly, all of them used computation approaches to solve the problem that was solvable by using logical thinking or proportional reasoning. However, Lucy’s approach was slightly different. She let her student choose her own method and when her student wanted to use numbers she allowed the student to pick numbers. In contrast, Kate began her lesson with teaching how to set a proportion and each step of cross-multiplication and Becky guided her student to use the concept of equivalent ratios. Overall, their guidance heavily focused procedures without connections with life experience or mathematical concepts behind the procedure.

As for visual applications, all of them applied various visuals such as charts, tables, specific pictures with captions, and post-its. Unlike their previous use of visuals—using only context related pictures—the visuals applied this week conveyed mathematical information and the situation of the problem. Their visual applications were considered fairly successful as Becky described, “I learned that visuals truly do help people understand problems. When I broke the problem down and used tables and pictures it really enhanced my lesson” (Becky, reflection, week 3).

The topic of intervention in this week was to build a rich environment in mathematics and English. Because this is a broad strategy, I provided various specific
strategies that might be helpful to establish the rich environment in both mathematics and language such as multiple solutions and representations, asking about student’s thinking, using how and why questions, and focusing on mathematical discourse rather than correcting grammatical errors. I had them place each strategy on Venn diagram to help them think how mathematical and linguistic supports are related. All of them put all or most strategies in the intersection of the set of mathematical support and the set of linguistic support. Then, from the result, they inferred those strategies for developing mathematical understanding essentially develop or require language development. This activity specially impacted Lucy as she stated below.

Also, doing the Venn diagram of situations that help with math, English, and both helped me understand what important it is to integrate my lessons with both subjects for the ELL student. That diagram showed me how easy it is to teach English in a math classroom, because I had never thought about it before (Lucy, post-survey).

Among those strategies, Becky wanted to concentrate on lowering teacher position. Both Lucy and Kate liked to try why questions and asking students about their thinking before giving out the answer.

**Week 4.** The fourth problem was drawing a floor plan under some required conditions. Because this problem does not have the fixed answer, it was a chance for the PSTs to have their students seek multiple solutions. Lucy and Kate allowed their students to explore other solutions while Kate did not have that opportunity because her student took a long time to understand the problem. In addition, a house was a familiar context for students, so they performed many life connections by talking about student’s houses or dream houses. Overall, I observed rich applications of diverse strategies in this week,
including *visuals*, such as post-its, tables, pictures, drawing on a grid paper or a chart paper and color coding.

Kate and Becky selected this week’s lesson as the most beneficial and meaningful. As she planned, Becky applied the *positioning* strategy and she was satisfied by the result. That was one of the reasons she thought this lesson was the most beneficial.

I liked the house/floor plan lesson the most in the five week period. I liked how he was able to come up with his own solution, think it out, and explain what he did to me. He seemed very excited about this problem and willing to show me why some dimensions worked and others didn’t. I felt for the first time since we started that he had fun during this lesson. He was able to communicate with me effectively and he did not seem as shy. I liked acting like I didn’t know the answer so that he felt more comfortable sharing with me what he felt (Becky, post-survey).

Kate was the only one who changed the context of this problem aligned with her student’s interest. However, she still provided a long explanation rather than asked her student to explain his thinking. She previously stated she would be focusing on why questions, but the number of her questions rather decreased in this week.

During the intervention, I revisited *cultural/life connection* because it seemed this strategy had not been visible except Lucy’s case. At this time, I created two different versions of dialogues between a teacher and a student (See Appendix F). One involved superficial life connections with only the context, and the other contained deeper connections between life experiences and mathematical concepts. After reading them, they were asked to compare and contrast the dialogues. Their interpretation was similar to what I intended to hear as Lucy addressed, “They set up the talking for the same topic, but the second dialogue goes deeper into actually concrete mathematical problems related to their lives. The other one didn’t go into math at all” (Lucy, post-interview, week 4).
**Week 5.** The fifth problem was the only one that had diagrams, which were included to indicate a repeated pattern. Everyone used colored cubes for this lesson and everyone began the lesson with life connections. Lucy and Becky brought up patterns in our ordinary lives and Kate talked about student’s favorite games because she adapted it in the context of the modified problem. Becky addressed how she integrated what she learned from the previous intervention as “I think that for anyone it would be a good idea to start discussing patterns before you just go straight into the problem, especially for ELLs because the connections will help you communicate with each other” (Becky, pre-interview, week 5).

All of their students found the rule of each pattern easily but had difficulty building equations of the pattern with variables and comparing the rate of change in two patterns. Hence, the PSTs employed various methods to help students generalize the patterns. Lucy used post-its and graphs, Becky used tables and graphs, and Kate used number examples. Overall, they used visuals and *multiple representations*. Until the end of Kate’s lesson, her student could not answer the last question. Then, Kate just told him the answer without helping him make sense of the solution. It seemed she did not notice what she did because she wrote “This lesson was quite shorter than I anticipated, but he did leave today having successfully completed what I had asked of him” in her reflection although he did not complete the problem.

After their lesson, I conducted the exit interview with each PST. One of the interview questions was what advice they would provide to other teachers of ELLs. Their advice included using effective visuals, modifying problems with connections, making
sure students actually understand the problem, drawing, getting to know student prior to designing lessons, getting students to explain answers, and having many conversations.

**Themes Related to Research Question 1**

The first research question was “How do middle school mathematics PSTs help ELLs make sense of high cognitive demanding mathematical problems?” I had three sub-questions respectively related to planning, strategies to analyze the meaning of a problem, and supports to explain their reasoning process and solution pathways. I found four themes relevant to these questions, which were generally focused on teacher strategies, while looking at the patterns of frequency and categorizing codes.

**Theme 1: PSTs had difficulty measuring student’s ability in mathematics and English proficiency.** In the beginning, the PSTs had very low expectations from ELLs because they predicted the lack of English proficiency might restrict performance in mathematics. In other words, they considered English proficiency to designate mathematical difficulty level. As Moll argues (1992), they had a limited vision of ELLs that severely constrains what ELLs can accomplish and this deficient view led her to provide a remedial instruction for her ELL. Becky expressed her difficulty figuring out what her student needed was English or mathematics in her post-interview of the first week.

> It could’ve been English. I should’ve had him read this out loud, I don’t know. Or maybe try to explain the problem to me, so that I could understand what he was thinking about what he sees because that was the hardest part, was it English or math. I don't know. If I would’ve asked him to try to explain to me, then I could definitely try to figure that out better, but I didn’t have him try to explain it, so I guess I don’t even know it was math related or problem solving related. (Becky, post-interview, week 1)
When Kate had difficulty helping her student understand about proportions, she said the reason why he could not grasp the concept was because of his mathematical knowledge rather than his second language proficiency. However, Becky had different perception when a similar situation happened to her.

It was miscommunication I think. Also I don’t think it is necessarily his language skills because I think a lot of kids probably make that mistake rather they’re ELLs or not. But it’s hard for me to communicate it to him, so that was linguistic. It was hard for me to think a way that I can say to him that it would make sense, but... so it was not anything about his knowledge of math because he was really good at math, but it was hard for me to communicate when I was trying to get across to him. (Becky, post-survey)

According to her, the reason why he made an error on his mathematical work probably came from his mathematical knowledge, but when she tried to help him make sense of the problem and how to solve it, he could not follow her explanation because he did not fully understand the language Becky used. As Becky described, the miscommunication often misled the PSTs about where student’s errors stemmed from.

**Theme 2: Teacher support may reduce cognitive demand of problems.** As we have seen in previous sections, PSTs’ attempts to help students actually reduced cognitive demand of the problems. Breaking down the problem, simplifying language, or guiding to a particular solution pathway were common examples. When they simplified a problem statement to help the ELLs read it easily, they sometimes removed a challenging task or an opportunity to practice language from the ELLs. Another evidence was found when using procedural approaches to solve problems, such as using a formula without connecting it with life experience or mathematical meaning.

**Theme 3: PSTs commonly implemented pressing for explanation and visuals to support ELLs to solve mathematical problems.** The PSTs applied numerous
strategies for five weeks. Some strategies were applied by their intuition or prior knowledge, and others were learned from the researcher. Among the strategies, justifying questions and using visuals were consistently observed from all PSTs every week. However, the way of implementing the strategies evolved as they learned more. The following vignette describes how the PSTs think explaining processes and solutions is important in problem solving.

Because last week when we were working he knew procedural stuff, but he wasn’t necessarily as good as he could’ve been in problem solving. So, I think that one way to figure out how to solve a problem is by thinking about it logically and talking about it. Then, also having him explain also helps me figure out what he did do get to the answer, instead of just writing down a few things and I want him to be able to tell me exactly how he got his answer. I just think it is important to logically think stuff if you’re going to be a better problem-solver. (Becky, pre-interview, week 2)

All PSTs deliberately employed visuals since the first week. Without any intervention, they knew visuals would be helpful for the ELLs, but it was evident that they refined their skills to use visuals after they received the intervention.

**Theme 4: Cultural connection was not readily adopted.** With life connections, cultural connection was one of strongly encouraged strategies. However, the PSTs did not show their efforts to integrate their students’ cultural experience overall. While Lucy made some questions about her student’s experience in her home country, she did not explicitly bring cultural aspects in her lesson. One expected simple application of this strategy was to use pictures of cultural items, such as South Korean architectures, food, or traditional patterns. Although they were recommended using the Internet to search for their students’ home country, none of them tried to use cultural items in their lessons or modified problems.
Themes Related to Research Question 2

The second research question focuses on changes in PSTs’ planning and implementing mathematical problems when working with ELLs and receiving structured intervention. I found two themes that are related to PSTs’ shift in their using ELL strategies.

**Theme 5: Some ELL strategies need to be learned, not intuitively obtained.**

For developing mathematical lessons for ELLs, the first strategies the PSTs implemented were using pictures and simplifying language. It seems an immediate and intuitive response to the needs of ELLs because visual aids and simple language obviously reduce language demand. However, some other strategies did not appear until they explicitly learned from the researcher. For example, connecting a problem with cultural and life experience was not used a single time from all cases before they received the relative intervention. Moreover, it seemed that one time intervention was not sufficient for some of them. The second intervention was more effective, but still they had some difficulty and they thought using this strategy was challenging despite variation among participants.

**Theme 6: Specific examples of ELL strategies need to be provided to help PSTs implement them.** The PSTs knew pictures might be helpful for ELLs to make sense of problems, but their visual application was not effective until they saw concrete examples. Similarly, they did not grasp how to make connections between mathematical problems and life experience when they first heard about this strategy. They had a clear understanding when they read a sample dialogue of teacher discourse. The following quotes from exit interviews indicate how PSTs thought specific strategies or examples were important.
When I got the specific strategies, I really like the sticky note you had, it’s really, I can see what I can do for each one, and think about them individually, and I like those strategies a lot, they’re more specific like acting like I don’t know, I still think that was the most, he talks since like this whole five weeks. Today he did a lot better about explaining himself which I was really glad. He was able to tell me stuff when I was acting I don’t know the answer, I thought it really worked well. So I think the specific examples were good. (Becky, exit interview)

I thought the specific examples helped just because it showed me. what makes sense. I think I can picture it [general strategies] before [seeing specific examples], but after seeing examples, it became more clear of what I specifically I need to do. So just helped to clear I guess. I thought both were useful, but I like to know those specific examples (Lucy, exit interview).

I feel like, once we talked about the visual aids, I knew that visual aids are important for ELLs, but just putting a picture on paper is not enough, so making sure they are very good images and good visuals, so they can actually help students, kind of lead into the problem before actually reading it, and so I think it was the most helpful thing that we talked about (Kate, exit interview).

Summary

In this chapter, I reported what I found from the data and what themes emerged from the various data related to research questions. This section devoted to the final summary of the entire chapter. Although I have provided responses of my research questions through the whole chapter, the explicit answers of research questions were inscribed in this final summary section.

First, the table on the next page, Table 12, summarizes how each PST implemented ELL strategies and how they interpreted the effectiveness. I focused on consistency of their strategy use rather than frequency of it because frequency might not be meaningful if they simply repeated the same strategy without adjustment or if the given problem had a different nature. For example, the fourth task itself involved drawing. Then they could not help but use the visual strategy. Therefore, I think watching consistency may imply more meaningful information than frequency. According to Table
12, the strategies all PSTs consistently used were pressing to explain a solution and visuals. While they had similar patterns in the consistently used strategies, they had mixed patterns in the strategies used scarcely.

Table 12

<table>
<thead>
<tr>
<th>Strategies used consistently</th>
<th>Becky</th>
<th>Lucy</th>
<th>Kate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessing math knowledge</td>
<td>Analyzing problems</td>
<td>Pressing to explain</td>
<td></td>
</tr>
<tr>
<td>Modifying language</td>
<td>Assessing math knowledge</td>
<td>Rephrasing</td>
<td></td>
</tr>
<tr>
<td>Pressing to explain</td>
<td>Pressing to explain</td>
<td>Visuals</td>
<td></td>
</tr>
<tr>
<td>Visuals</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strategies used none/little</td>
<td>Cultural connection</td>
<td>Lowering teacher position</td>
<td></td>
</tr>
<tr>
<td>Cultural connection</td>
<td>Multiple solutions</td>
<td>Assessing English</td>
<td></td>
</tr>
<tr>
<td>Multiple solutions</td>
<td></td>
<td>Sentence frame</td>
<td></td>
</tr>
<tr>
<td>Lowering teacher position</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modifying language</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Sentence frame</td>
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</table>

*Patterns of the strategies PSTs applied to help ELLs make sense of the problems.*
As for intervention influences, Becky was the one who had the greatest impact from interventions. She evidently employed what she learned during interventions in her use of *life connection, visuals, and lowering teacher position*. She thought all of them were fairly effective to help her student understand problems and chose *lowering teacher position* strategy as her most successful one. Similarly, Lucy planned and applied *cultural/life connection* and *visuals* as she learned in the interventions, and she found that *cultural/life connection* was most effective in her case. Kate showed relatively less impact of interventions. She did not explicitly integrate those strategies introduced in interventions although her lessons usually implicitly contain mixed strategies that she heard from interventions.

The first research question was “How do middle school mathematics PSTs help ELLs make sense of high cognitive demanding mathematical problems?” I had three sub-questions to answer this question more specifically. The first question, “What factors do PSTs consider in order to implement high cognitive demanding mathematical problems for ELLs?,” focuses on their planning and problem modification. In short, there were mixed results. Each PST preferred to use different strategies overall. Main strategies they used to modify problems for ELLs include *modifying problem language, visuals, life connections, and problems to access a challenge*. Some of these strategies reduced the cognitive demands of the problems by removing challenging work or suggesting
particular solutions. Other than modifying problems, they planned to use assessing students’ prior knowledge, asking students to analyze problems, or connecting student’s life experience with problems.

The second sub-question seeks to find teacher-provided strategies during instructions that aimed to help ELLs make sense of mathematical problems. The results of three cases indicated that the PSTs provided various strategies to help ELLs understand the problems and find entry points to solve them although their use of strategies were not always consistent. For example, they used life connections, analyzing a problem, visuals and gestures to help ELLs understand what the problems were about and what they were supposed to do to solve them. It was true that whatever supports they used, it was necessary to spend sufficient time to have ELLs understand what the problem actually meant. In addition, helping them find entry points was a challenging part for all participants. They usually asked them directly, “How do you think you can solve it?” or “Do you have any idea how to start that?” However, these questions usually did not result in effective responses. Then, the PSTs provided more specific guidance, but this kind of support often reduced cognitive demand of the problem because they tended to guide their students to a certain solution pathway.

The third question asks how PSTs support ELLs to explain their reasoning process and solution pathways. In order to answer this question, I looked for what the PSTs did after their students found their solutions. The most common strategy they implemented was pressing to explain a solution, mostly with why questions. All PSTs planned and applied this strategy with significant amount for all five weeks. However, the ELLs usually struggled explaining their thinking. Consequently, it was a very slow
process until they obtained enough explanation from their ELLs. It was common that they had to ask multiple questions with variation and adjustment. During this process, the PSTs allowed or encouraged the ELLs to use *multiple mods of communication* such as drawing, writing, or pointing. However, they sometimes gave up on pressing their ELLs for decent explanation and quickly judged whether the students understood or not. It was more than few times that they discouraged ELLs from persevering with articulating their reasoning.

The second research question looks for PSTs’ shifts in planning and implementing strategies as they work with ELLs and receive structured intervention on their practices. This question concentrates on PSTs’ growth in using ELL strategies and influences from the interventions over that change. I found some shifts in their use of strategies. The strategies in *cultural/life connection* showed an obvious shifting pattern. All PSTs began to employ this strategy after the relevant intervention although they used it with different frequencies and qualities. Another strategy I found evolved significantly was *visuals*. Although they planned and used visual supports since the first week, their application became more diverse and coherent to the mathematical situations embedded in the problems and that was initiated by the relevant intervention.
CHAPTER 5. DISCUSSION, CONCLUSIONS, and IMPLICATIONS

In the previous chapter, I described my findings from the data analysis and answered my research questions. In this last chapter, I discuss the findings based on the framework and research questions in greater depth, but while considering “connections” as a hierarchical theme that accounts for other strategies based on the actions participants took to help their ELLs make sense of mathematics. After this discussion, the conclusions, implications, and a statement about future research will complete the chapter.

Discussion

The purpose of this qualitative case study was to examine what supports middle school PSTs provide to help ELLs make sense of highly cognitive demanding mathematical problems. As I addressed in Chapter 2, making sense is defined as “developing understanding of a situation, context, or concept by connecting it with existing knowledge” (NCTM, 2009). According to this definition, connections between new information and existing knowledge are key elements involved in making sense of mathematics and mathematical problems. Connections are essential when working with ELLs because they have different cultures and languages from non-ELL students in the U.S. The dissimilarity in culture and language, or previous experiences, leads ELLs to interpret a situation, context, or concept surrounded in a mathematical problem differently from non-ELL students (von Glasersfeld, 1995). Moreover, as Campbell et al. (2007) claimed, it is difficult for a teacher to expect what strategies are effective for the students who have different cultural backgrounds from him/her. However, it does not
mean that teachers are required to have the same cultural background as their students to
teach effectively. Instead, both teachers and students need to learn each other’s culture
and understand how others perceive and perform mathematics through sufficient
communication (Carpenter et al., 1997). In other words, teachers of ELLs have to put
effort into learning the various cultures of their ELLs (Ladson-Billings, 1995, 2009), as
well as into explaining specific cultural contexts of mathematical problems. In this study,
all PSTs were white American females from a university-based teacher preparation
program located in the Midwest of the U.S. who had never lived in other countries, while
all of the ELLs came from mid-income families from South Korea who had entered the
U.S. within the last two years. Hence, it is obvious that the two groups, the PSTs and the
ELLs, have different ways of understanding and performing mathematics because of their
different cultural backgrounds. They may use different mathematical discourses because,
as Gee (1996) defines, a discourse is “a socially accepted association” that “can be used
to identify oneself as a member of a socially meaningful group” (p 126). Therefore, my
primary focus when analyzing PST-conducted strategies was on how they created
connections between mathematical problems and student’s experiences. Many
researchers and educators have suggested this strategy, making connections, as one of the
key principles for teaching mathematics to ELLs. Two of the seven research-based
strategies that Chval and Chavez (2011) recommend are related to cultural and linguistic
connection, which are “Connect mathematics with students’ life experiences and existing
knowledge” and “Connect language with mathematical representations” (p. 262). One of
the Mathematics Teaching Practices in Principles to Actions is also “Use and connect
mathematical representations” (NCTM, 2014, p. 10). Similarly, Celedon-Pattichis and
Ramirez (2012) include “Linguistically sensitive social environment” and “Cultural and linguistic differences as intellectual resources” (p. 21) in their five guiding principles for teaching mathematics to ELLs. This was also the reason why I devoted two of the five intervention sessions to this topic. Responding to my effort, the results showed that the PST participants made the most vivid growth in the strategies in the category, cultural/life experience.

Furthermore, this category is not the only one that is related to connections. In a broad view, all of the other categories of the conceptual framework could be seen in view of connections with or for ELLs. As for the mathematical content, the PSTs assessed ELLs’ prior mathematical knowledge and connected their previous learning with the working problems. Related to the mathematical and cognitive processes, they asked the ELLs to analyze the problem and to explain about their solutions. This involved establishing connections between their mathematical work and the problem situation because students had to deduce an appropriate procedure from the mathematical situations of a problem and explain their procedures with their rationales in terms of the problem’s context. The mathematical and contextual language could include connections to the ELLs’ first language or connections between English academic language and everyday language or alternative methods of communication (Gee, 1996; Moll, 1988, 1989; Morales et al., 2003; Moschkovich, 2002). The positioning category suggests a different type of connection, grounded in the social interactions between the teacher and the student (Harré & van Langenhove, 1999; Pinnow & Chval, 2015; Wagner & Herbel-Eisenmann, 2009). I further discuss about connections that appeared prevalently among all PSTs’ practices in the rest of this section.


Modify Problem Language and Context

The first strategy the PSTs implemented to make connections between mathematical problems and their ELLs was modifying problem in order to make it comprehensible to the ELLs because they thought the language of the mathematical problems were unnecessarily complicated as Enright (2009) denotes. According to the survey conducted by Seifi, Haghverdi, and Azizmohamadi (2012), teachers perceive that the biggest difficulty for students when solving mathematical word problems lies in representing and understanding problems and the two most common causes of these difficulties were text difficulties and the unfamiliar contexts included in the problems. Therefore, it was an intuitive attempt for the PSTs to modify a problem text or context to increase its comprehensibility for ELLs. Although rewording word problems by using students’ everyday language may yield higher achievement (Mahajne & Amit, 2009), they occasionally resulted in the reduction of cognitive demands of the problem because their rewriting and reworking the language removed challenging aspects from the problem. This fits into one of the six factors associated with the decline of high-level cognitive demands that Stein, Smith, Henningsen, and Silver (2009) warned about: “problematic aspects of the task become routinized” (p. 16).

With language simplification, context was changed to help the ELLs have better access to the problems. One of the PSTs, Kate, changed context of a given problem so that it reflected a familiar context to her student in weeks 4 and 5. It seems appropriate because learning in context engages students through an enriched curriculum and enhances “transfer of learning through a demonstration of the links between school
mathematics and real world problems” (Boaler, 1993, p. 14). In this sense, Kate
connected the problems with her student’s life experience.

However, her attempt did not turn out successfully. Kate first thought drawing a
floor plan of a house was uninteresting to a middle school boy, so she changed it to
drawing a floor plan of an arcade because she knew her student liked video games.
However, her ELL did not know what an arcade was and building an arcade in the
basement is not common in his home country. In this instance, there was no additional
support to fully deliver her strategy—using life-related context—and as a result, the
context was not helpful to the student. She should have prepared how to explain the
word, such as showing pictures, or playing arcade games. This is evidence that modifying
context is not sufficient enough to engage a student’s learning unless the teacher provides
additional supports. Wilburne, Marinak and Strickland (2011) noted that it is crucial to
understand “the real-world context might not be the real world of some students in the
class” (p. 461). Therefore, only when students understand the context of a problem, the
role of contexts is central and students generally choose their mathematical procedures
and performance based on context (Boaler, 1993).

Connect with Life Experiences

All given problems were connected to real-life situations. However, real situations
in word problems are often extracted from the adult world or are artificially made. For
example, the sandbag problem in week 1 does not have a reason to make all bags less
than 50 pounds and the apartment problem in week 3 has an unfamiliar situation for
middle school students. Boaler (1993) argued that when students perceive these kinds of
word problems just as difficult mathematical problems rather than a real-life problem, the
real-world aspects of word problems became distractors rather than contextual clues. In this sense, teachers need to bring student-friendly real situations into instruction before the formal mathematical problems are presented because once students see the written word problems, they could notice they are related to school mathematics, and such problems are immediately disconnected from their real lives.

However, the PSTs struggled to connect the real-life situations to their students. The life-connection strategy was briefly explained in the second intervention session; but, in the following week, the PSTs merely showed the problem sheet and said “This is the problem we are going to do today.” It was not until they received another intervention with concrete examples and took part in a deeper discussion that they began their instruction with talking about relevant life experiences.

Research also emphasizes the connection between mathematics and student’s life experience. In Gutstein’s study (2003), his students who learned mathematics through real-life projects developed mathematical skills, such as inventing solution methods, seeking multiple solutions, and communicating mathematical findings. Similarly, Ahn, I, and Wilson (2011) adopted Moses’ Five-Step Approach (Moses, 2002) to teach Latino/a ELLs abstract mathematical concepts by building physical and visual experiences. Eventually, the PSTs intended to include various life connections in their instructions. However, most of the attempts were simply connecting the main context of the problem with the student’s life. For example, Kate asked, “Do you know what zombies are?” when she began her fifth session because her modified problem included a zombie story. Although understanding the meaning of zombies might help the student grasp the story of the problem, the knowledge did not convey any mathematical meaning related to the
mathematical concept implied in the problem, finding patterns of linear and quadratic relationships. On the contrary, Becky and Lucy began their lessons with questions about patterns around us, such as a rainbow and a wallpaper pattern. It was obvious that the ELL obtained sufficient information without being directed to a specific procedure before the mathematical problem was presented.

**Connect with Cultural Experiences**

In this study, I use a narrow definition of culture, which includes only ethnic culture. Then, the participants of this study brought only two cultures from the U.S. and South Korea. Therefore, my focus of analysis was twofold: how the PSTs integrated the culture of South Korea in their mathematical instructions and how they provide plausible explanations of the culture in the U.S. More importantly, I was interested in how they connected mathematics with cultural experience, not just by talking about their students’ home country during informal conversations, but by providing contextual clues related to the mathematical concepts. The results were relatively negative compared with the outcome of life connections. None of the PSTs used cultural aspects as main instruments of their lessons although using cultural connections were introduced multiple times during the intervention sessions, such as employing cultural contexts or using cultural visual aids. Because they have knowledge about their ethnic culture, the ELLs could contribute to the lesson. This could create an entirely different environment. On the other hand, the PSTs did not provide sufficient information about their own cultures either. Some problems contained cultural biases, but they did not fully address them to help ELLs avoid the cultural biases.
Wilburne et al. (2011) wrote about how teachers of ELLs should deal with the cultural biases in their mathematics textbooks and assessment sets. They suggested teachers should identify culturally biased problems within their curricula and consider adapting them to “enable all students to comprehend the mathematics rather than grapple with the cultural nuances of difficult vocabulary” (p. 465). As Wilburne et al. recommended, the PSTs modified the problem context to remove potential cultural biases later on. However, it is not possible for teachers to find all cultural biases in their curriculum before their students express their misunderstanding. In this case, teachers should be sensitive to ELLs’ reactions and deliberately assess their understanding of the situation, context, and concept embedded in the curriculum. If they notice their ELLs’ misunderstanding, it is necessary to explain the cultural aspect until ELLs grasp the correct meaning. Yet, the results indicate that the PSTs in this study were not ready to explain the cultural aspects. For instance, Kate failed to deliver the meaning of an arcade to her student and she was unprepared to teach the word. In addition, none of the PSTs recognized that the context of purchasing an apartment building was culturally biased.

There have been multiple studies about students’ achievement in mathematics with culturally-related activities. Brenner (1998) found that the children who received culturally-related mathematical activities showed larger cognitive gains. Similarly, Kitchen and Lear (2000) argued that mathematical activities strongly connected to students’ home culture can encourage students’ critical thinking about profound issues in their lives as well as develop their understanding of mathematical concepts. These studies are in line with culturally relevant pedagogy (CRP), which is described as “a theoretical model that not only addresses student achievement but also helps students to accept and
affirm their cultural identity while developing critical perspectives that challenge inequities that schools (and other institutions) perpetuate” (Ladson-Billing, 1995, p. 469).

Ladson-Billing provided three criteria of CRP: (1) students must experience academic success; (2) students must develop and/or maintain cultural competence; (3) students must develop a critical consciousness through which they challenge the status quo of the current social order (Ladson-Billings, 1995a). When considering the criteria, it is evident that the PSTs did not implement successful CRP. Although their students might have demonstrated academic success by finding correct answers, their ELLs were not encouraged to develop or maintain their cultural competence or critical thinking.

**Connect Mathematical Concepts with Existing Knowledge**

The connection between mathematical concepts and ELLs’ existing knowledge is crucial because it helps ELLs find an entry point to solve a problem. A common strategy the PSTs implemented to make this connection was assessing a student’s prior knowledge. For example, when doing the sandbag problem, some of them assessed their ELLs if they were able to subtract whole numbers. When they worked on the apartment problem, they began with asking if their students had learned proportions. When their students had knowledge of the concept and were able to use it, the PSTs did not have much difficulty helping the ELLs although they often guided their students to a specific approach or struggled communicating with their students. However, they sometimes failed to help their ELLs when they did not have existing knowledge regarding the concept. Using familiar contexts or simple language only helped ELLs notice what mathematical concept their teachers were talking about; but, when they encountered unknown concept, the ELLs were unable to connect the problem to the mathematics they
knew. In this case, their teachers needed to teach the new concept based on the student’s prior knowledge and previous experience. However, they often assumed that ELLs already knew all of the required mathematical concepts, and thus, came unprepared to support their ELLs to understand the concept based on their prior knowledge.

Garrison and Mora (2005) introduced the following basic principle for bilingual instruction: “To teach an unknown concept, use the known language; to teach an unknown language, use a known concept” (p. 37). If a teacher is teaching an unknown concept using an unknown language, it must be modified. ELLs do not fully understand verbal explanations in English, unless they notice that their teacher was talking about something they already know. Unfortunately, the PSTs in this study could not use the student’s known language, Korean, to teach the unknown concept. Hence, when they discovered that their students did not know the concept, they could not help but use English to explain the unknown concept. The greatest challenge came when they were explaining about the concept of proportion. Because their ELLs knew neither the language nor the concept, the PSTs should have used something the ELLs knew in order to construct the beginnings of new knowledge, such as real-life examples of cooking or shopping. However, they grappled with building a new knowledge without connecting it with something known and as a result, it only confused the ELLs further.

These connections are beneficial to all students, but they are essential to ELLs because ELLs are linguistically and culturally disconnected from the mainstream in their school culture. Therefore, this study aimed to investigate how PSTs make meaningful connections between mathematical problems and a student’s existing knowledge, which we call “making sense of mathematics.” Although I found some positive influence from
the interventions present in the PSTs’ teaching performance in terms of life connections and using visuals, their connections to the research-based ELL strategies were rather superficial than mathematically meaningful.

**Conclusions**

The unit of analysis in this case study was middle school mathematics PSTs, who had not received any official training of teaching mathematics to ELLs prior to this study. After teaching ELLs for five weeks and learning research-based ELL teaching strategies, these PSTs began to use more diverse strategies that fostered deeper connections. Various data resources were compared and analyzed using the modified *problem space framework* (Campbell et al., 2007; Harré & van Langenhove, 1999) and the CCA method (Fram, 2013; Merriam, 1998) through multiple phases and dimensions: phases within a session (planning, understanding, and justifying), weekly meetings (from week 1 to week 5), five components of the framework, and three cases. This section summarizes the final thoughts and conclusions of the findings.

**Strategies Used for Making Sense of Problems**

**Considerations in planning.** All three PSTs submitted lesson plans using the template (see Appendix B), but their complete lesson plans did not contain sufficient information that indicated what factors they would use to help ELLs to understand problems because the PSTs briefly wrote only general procedures. It could be inferred that the brief lesson plans disclosed these PSTs’ lack of consideration when teaching ELLs. Hence, pre-interviews and their modifications of the given problems were also analyzed. In sum, their main concerns were on how to modify the problems, such as simplifying the language, adding visuals, or adding extra questions, because they
assumed that wordy problems were difficult for ELLs in both language and mathematics. However, each PST had a different preference thus lacked consistency: Becky always simplified problem statements; Lucy generally added many analogous problems; and Kate changed the context of the problem. Whatever modifications they made, their main goals were to make the problem easier. As a result, they sometimes lessened the cognitive demands of the problems by removing problematic aspects or they emphasized procedures rather than the reasoning or understanding of the concepts (Stein et al., 2009). Life and cultural connections were usually not considered in the lesson preparation, especially in early weeks. However, all of the PSTs had a tendency to implement life connections in later weeks through their modified problems or guiding questions.

**Supports for understanding.** Visuals were commonly used because the PSTs believed visuals effectively deliver information although students do not comprehend the information through language. Hence, visuals were utilized to explain the meaning of a problem or as a communication tool (Chval & Chavez, 2011; Moschkovich, 2002; Raborn, 1995). However, it was not always successful because they sometimes chose an inappropriate image that did not convey sufficient information or confused their students. As an alternative communication tool, their use of visuals also had limitations because the ELLs barely initiated a way to express their thinking. Rather, they tended to imitate what their teachers had shown through the visuals.

Besides visual aids, the PSTs employed various strategies, such as reading aloud, asking students to explain the meaning of the problem in their own way, asking students to analyze conditions of the problem, or using life examples. Sometimes they needed to explain the meaning of words in the problem statement that had both general and
mathematical meanings (Chval & Chavez, 2011; Moll, 1988, 1989). Interestingly, this process seemed to be an obstacle to them because the PSTs were not prepared to explain such meanings. It was probably because they assumed their ELLs would have already known the terms or they believed they could easily explain these terms without preparation as Lucy confessed in her post-survey:

The greatest challenge in teaching mathematics to English language learners was explaining terms and words that they weren’t familiar with. Some of the words that I use in everyday language are difficult to explain to people who do not know what they mean. For example, in a lesson my student did not know what “proportion” meant. I have been using this term for ten years and have never thought about the definition; I just knew what it stood for. Her questioning key words like this made me take a step back from the problem and really think about which words I need to be prepared to explain. Many times, my student did not know what a term meant but continued on with the problem anyway. I need to be able to clear up any misconceptions from the very beginning of the lessons so that the confusion of my student does not continue (Lucy, post-survey).

She admitted explaining terms and words was much more challenging than she expected. As a native English speaker, it was difficult for her to identify what words might be confusing to ELLs who do not share a similar culture or life experience with her. She also said, “Many times, my student did not know what a term meant but continued on with the problem anyway.” However, there were times when the unknown terms or concepts blocked the ELLs from continuing on with the problems. How the strategies applied by the PSTs affected student’s learning is beyond the scope of this study, but it is true that how well their students could follow their guidance significantly influenced their teaching performance. Specifically, when the ELLs struggled understanding the problems or their teachers’ explanation, the PSTs had to grapple with employing unplanned strategies spontaneously. As a result, many of their unplanned attempts were not successful and more often than not, they ended up telling their students what procedures
they should take. As Lobato, Clarke, and Ellis (2005) address, these telling actions are not desirable because it “focuses only on the procedural aspects of mathematics” and “communicates to students that there is only one solution path” (p. 103).

**Supports for justifying.** After the ELLs found their solutions, the PSTs usually asked them to explain their procedures and reasoning processes. In this phase, the PSTs used many clarifying and justifying questions. However, the challenge came after this. Even when the ELLs found a correct answer, they struggled with explaining their reasoning and the PSTs did not clearly know how to help them. At times, they simply moved to the next task assuming their students had the appropriate reasoning processes because their answers were correct. This kind of teacher reaction did not encourage the students to develop their language skills or to participate in a rigorous discussion. Moreover, this does not follow the suggestion that mathematics instruction for ELLs should focus on mathematical reasoning and practices and ELLs’ participation in mathematics discussion even before they acquire English proficiency (Moschkovich 2000, 2002, 2007a). In my view, using follow-up questions is important because ELLs usually cannot provide a full answer at once. They generally use several words or a simple sentence, although they have already performed complicated cognitive work in their heads. If teachers are aware of this, they should be patient and keep asking follow-up questions until they get a full response from their ELLs. Eventually, I observed an ample amount of the PSTs’ follow-up questions and often these actions associated with *lowering teacher position* or *raising student position* and some linguistic supports. As for the strategies related to *positioning*, all four categories of teacher discourse patterns in mathematics classroom (Wagner & Herbel-Eisenmann, 2014) were coded, but the last
pattern, *personal latitude*, was mostly found as planned strategies. Another essential strategy observed in this phase was multiple methods of communication. When the ELLs could not speak in regards to their thinking immediately, the PSTs allowed them to write down numerical sentences or to draw something out to express their cognitive processes.

**Changes in Supports for ELLs**

It was evident that there were many individual differences among the three PSTs in terms of adopting research-based strategies. To sum up, Lucy was a fast adopter, Becky learned only specific methods, and Kate did not actively change her way of teaching. One positive result present across the cases was that they began to apply life-connection strategies after they learned it from the researcher. Another significant influence from interventions occurred in the way of using visuals. They integrated more diverse types of visuals that possessed deeper relations with mathematical situations into their lessons after they learned visuals should be used in mathematically meaningful ways. In Becky’s case, positioning strategy also became a main shift because she intended to actively apply the strategy after she learned it during one of the interventions and she found it useful to encourage her student to talk more actively.

Moreover, it should be noted that none of the PSTs clearly identified strategies for ELLs in the beginning of the study. They sometimes also did not recognize the ELL strategies that they included in their lesson plans. After the weeks of interventions, however, they were able to recognize more specific ELL strategies and provide more detailed explanations in their lesson plans than they did in the beginning. In addition, they were able to ask more open-ended questions and maintained the high-level cognitive demands of the problems.
Limitations

Although I had only three cases, the results of each case exposed many differences. The three PST participants had similar demographics, but their teaching patterns and adaptability were significantly distinct from each other. Thus, it was hard to find a consistent pattern that was present in all of three cases. This does not mean that I pursued generalization in the whole population of PSTs in the U.S. Although the small size sample could be a genuine limitation of case studies from the traditional view of research design, qualitative researchers strengthen external validity by using standard sampling procedures or using specific procedures of coding and analysis (Merriam, 1998). Therefore, the rigorous research design and coding procedures of this study might reduce the possibility of this limitation. Thus, the limitation I found at this point is the generalization of the three cases included in this study.

The fact that each PST worked with a different ELL could have influenced the results. Although all three ELLs were from the same country and had been living in the U.S. for similar lengths of time, the gender and grade differences might have affected their teacher’s performance. For example, the female ELL was the only 8th grade student with slightly more fluent English skills as well as more vocal personality. These characteristics could have given more availability of choosing instructional strategies to her teacher, Lucy. Moreover, the fact that the ELLs came from the same country as the researcher might have had an impact on the PSTs’ responses in the interviews and on the surveys because the PSTs might feel uncomfortable to make negative comments about Korean students or Korean culture.
The difficulty level and nature of the given problems were other factors that might have affected the results. It was true that the PSTs performed better overall when they had more open-ended tasks involving visual activities. The given problems in weeks 4 and 5 were more open-ended and included a drawing activity or a diagram in the problem itself. Hence, one of the reasons why the PSTs used more varied strategies with deeper connections during these weeks might be due to the fact that they dealt with this kind of problems. In addition, they had to work on a different mathematical concept every week because the given problems embedded all different concepts. Thus, it was not possible for the ELLs to use the mathematical knowledge constructed in the previous lesson for future lessons. If they were given the same concept-based problems for all five weeks, the results might have produced different themes as the PSTs could have had ELLs use what they learned from the previous lesson when they were working on the following problem.

The fact that the research was also the intervention provider is another limitation. PSTs might have pressure that they had to use the strategies introduced during interventions because the research was watching their teaching practices. Another limitation may lie on the lesson plan template, which might lead them to plan their instruction in a certain way. Finally, the short period of both data collection and intervention is a clear limitation of this study. Five weeks with only one meeting per week is limited time to examine general patterns or significant changes in teaching performance.

Implications

Moschkovich (2012) recommended that teachers of ELLs should follow general guidelines for ensuring high-quality mathematics instruction and maintaining high-level
cognitive demands. Following the general guidelines is already challenging to teachers, but it is not yet sufficient. Teachers have to provide appropriate linguistic supports, which should also be used to maintain a high-level of mathematical discussion (Richard-Amato & Snow, 2005). This case study is aligned with these views and provides practical information and recommendations. Furthermore, this research can impact teachers’ practice in a way that they encouraged to search for appropriate strategies and apply them in a meaningful way for their ELLs. Moreover, the results of this study will be useful for mathematics teacher educators who prepare and equip teachers to teach mathematics for ELLs because teacher educators can find ample evidence of what difficulties ELLs have when working on mathematical word problems and then have the knowledge as to what strategies should be applied from this research. Similar to prior research (Chval & Chavez, 2011; de Araujo, 2012; Ladson-Billings, 2009; Smith & Stein, 1998), this study recommends that teachers need to (1) maintain cognitive demands when modifying word problems for ELLs, (2) make life and cultural connections before they present mathematical problems or concepts, (3) realize simply showing pictures might not be effective, so they have to design visual aids with close connections to the mathematical situations of the problem, and (4) provide various methods to communicate and be patient until ELLs fully express their thinking in their own words.

More importantly, this study has a crucial link to PST education and introduces a model for PST preparation of how to teach mathematics for ELLs. Working one-on-one with ELLs prior to their student teaching helps PSTs have tangible and specific knowledge about teaching ELLs, as well as heighten their competence in working with diverse students. This type of preparation can be accomplished by concurrently providing
interventions that enable PSTs to apply the ELL strategies, introduced during the interventions, to teach a specific mathematical concept to a population of ELLs. In order to follow this model, it is necessary for teacher education programs to build a partnership with local schools, or as demonstrated in this study, local community organizations to access ELLs outside of formal school settings.

The results of this study also strengthen the findings of previous research (Downey & Cobbs, 2007; Fernandes, 2012b; Pappamihiel, 2007) that learning about ELLs and working with them has an impact on PSTs’ learning and understanding about ELL education. In addition, the results of this study indicate that having practical experience with ELLs is essential for PSTs to develop effective pedagogy for ELLs in mathematics education. As Kate stated, “In the College of Education, we don’t actually get an opportunity to take a class on how to teach these students [ELLs]. So, I think this opportunity put us a step above those who haven’t done something like this” (Kate, post-survey). In the following testimony, Lucy underscores how valuable her experience with a real ELL was even further:

Yes, I definitely feel more confident about how to teach ELLs in mathematics. Before these lessons I had no experience working with ELLs at all. Now, I will feel confident if I have an ELL student in my math class. I will know how to approach them and what strategies to use to help them not only feel comfortable in my classroom, but be able to succeed. Hopefully, I will be able to use these strategies to help ELLs not only learn the lessons that I am teaching, but understand them and be able to connect them to their everyday lives (Lucy, post-interview).

It should be noted that this statement expresses her overconfidence as well. Lucy believed she could teach any ELL in her future class because she taught one self-motivated, mathematically capable ELL from mid-income family. However, Lucy also addressed the most beneficial aspects was that she could implement the ELL strategies she learned
during interventions with actual ELL students. She said, “It would have been difficult to learn about them [ELLs] without actually having experience with them.” As she described, this study suggests a useful model for teacher preparation programs. General teacher preparation programs include fieldwork requirement, however, most fieldworks do not focus on a specific group of students such as ELLs. Working with one ELL or a small group of ELLs will expand PSTs’ capabilities to teach diverse learners because this type of experience will engender motivation to learn about diverse learners and to develop specific strategies for a specific group of students. Understanding and connecting to diverse learners help PSTs consider the needs of all students and create alternative methods to support culturally and linguistically diverse learners.

Additionally, teacher educators should be aware that effective ELL strategies are not easy to adopt without significant preparation and professional development. The ELL strategies the PSTs adopted before they learned any research-based strategies were only simplifying language and adding contextual pictures, which significantly reduced the high-level cognitive demands from the problem or did not provide much effective support. Meaningful applications came after the PSTs learned, especially when they saw specific examples. For example, the PSTs knew that visuals help ELLs, but they did not have a clear idea about what kinds of visuals were appropriate for such problems and how to present them in their instructions until they saw examples. Teacher educators should provide PSTs with concrete and specific examples with the general guidelines of ELL education. Specifically, they could show videos of exemplary lessons, examples of effective modifications of word problems, or engaging and effective mathematical activities for ELLs.
Finally, teachers and teacher educators should be aware of the power life and cultural connections possess within learning. Such connections support all students, but we should recognize they are necessary for ELLs because when they have to learn an unknown concept with an unknown language, life and cultural connections were often the only resources teachers could use to stimulate understanding. However, the results of this study also denote that understanding how to apply life connection strategies is not easy to develop in a short time because PSTs need to fully understand the mathematical concept and their students’ previous experiences to make meaningful connections between the two.

**Future Research**

This study could be a base for many other studies because the framework covers numerous ELL strategies. Seeking details in each component of the framework could lead to a set of research studies. For example, positioning theory could be one main framework and be introduced as a main strategy in a study to investigate discourses of PSTs or teachers related to *positioning* themselves and ELLs in mathematics classrooms.

Researchers might want to compare teacher beliefs regarding mathematics and their teaching practices for ELLs. It might be insightful to examine what relationships between those two exist, if any do. For instance, in this study, two participants disclosed their views of mathematics and how it is a universal language, so ELLs may have less difficulties with mathematics compared to other subjects. On the other hand, one of the participants assumed that their ELLs would perform at a very low level in mathematics and thus provided an extremely low-grade level of pre-assessment set. Her practice did not match with her view.
The unit of analysis of this case study was each PST, not the ELLs. Hence, future research might include or focus on ELLs’ mathematical achievement, especially when working on mathematical word problems written in English. It should be examined how differently they perform when they receive different kinds of scaffolding in order to measure the effectiveness of each research-based ELL strategy. This research provides a better understanding of which strategies make a difference in ELLs’ performance when solving word problems, and thus can be used to determine which ones to study further or test further.

Although all of these studies are interesting and worthy of exploration, the future research that draws my attention the most is the issue of PSTs’ mathematical content knowledge, especially conceptual understanding. It has been reported that teacher candidates lack explicit understanding of concepts when they perform computing procedures (Ball, 1990). There were multiple pieces of evidence that the PSTs in this study did not have an adequate level of conceptual knowledge. First, none of them thought of the concept of average to solve the sandbag problem in week 1. They were all concerned with finding a correct answer, so they did not seek any other method after they found the answer from simple subtractions despite the fact that their students were in 7th and 8th grades. In addition, they did not consider non-whole numbers when dealing with “less than 50.” In week 2, they did not explain what “average change of travel” (or speed) is. They simply used the speed formula, \( d = rt \), to find the answers. The third week was the worst because their lack of understanding regarding the concept of proportions significantly influenced ELLs’ performance in a negative way. As Lucy stated, “I have been using this term [proportion] for ten years and have never thought about the
definition” in the previous quote, this group of PSTs did not know how to explain proportions. It was evident that their effort to apply the procedure of solving proportions was not successful because it did not make clear sense to ELLs since it was presented without any working definitions or real-life examples. Without proper conceptual understanding of mathematical problems, it was not possible for these PSTs to provide a plausible explanation or a meaningful connection with students’ experiences. Hence, it would be important to investigate how PSTs’ conceptual knowledge influences their application of effective strategies for ELLs. Researchers could make practical recommendations for teacher educators and course developers from the results of the study.

Reflections

This study addressed an existing void of the research literature on middle school PST learning to implement instructional strategies to help ELLs solve high cognitive demand mathematical problems. Specifically, this study sought to document the strategies PSTs use to support ELLs to understand a problem by making meaningful connections between students’ prior knowledge and mathematical concepts embedded in the problems, between the problem context and mathematical concepts, and between the problem context and student’s life/cultural experience.

While conducting this study, perhaps the most eye-opening moment was when some PSTs provided extremely low-grade level of assessment sets to their middle school ELLs. This is a clear evidence that teachers assume student’s English proficiency influences their mathematical performance. This study revealed working with ELLs, even if they met ELLs only five times, significantly changed the PSTs’ misconception on
ELLs’ mathematical ability. After the PSTs noticed their ELLs were capable to perform fundamental mathematical skills, they stopped reducing the cognitive demand of a problem. For example, they turned into paying more attention to making problems accessible to the ELLs and to helping their students express their thinking in various ways of communication.

Furthermore, the most significant result is the influence of intervention about research-based ELL teaching strategies. Although the individual intervention sessions were short, the impact of the intervention of some strategies was vivid in some cases. For example, the use of strategies related to life connection significantly increased after the relevant intervention session in all cases. The intervention model in this study suggests that when mathematics teacher educators design fieldwork, they should consider a small group setting such as pairs of one PST and one ELL, introduce PSTs with concrete examples of ELL strategies, and provide concurrent intervention while working with ELLs.

As mentioned earlier, a population of ELLs increases rapidly in the U.S. and mathematics teachers are not adequately prepared to teach ELLs. Responding to the demand for effective teachers of ELLs, this research found that it is not impossible for mathematics teachers to help ELLs perform high cognitive mathematical activity if mathematics teachers learn how to teach ELLs effectively. Finally, this research encourages researchers and teacher educators to continue to search effective pedagogies not only for ELLs but also for teachers of ELLs because teachers take a crucial role to build student’s belief, identity, and learning.
REFERENCES


APPENDIX A

Original Mathematical Problems

Week 1

Claire is filling bags with sand. All the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 58 pounds, another sand bag weighs 41 pounds, and another sand bag weighs 53 pounds. Explain whether Claire can pour sand between sand bags so that the weight of each bag is less than 50 pounds.


Week 2

Anne’s family is driving to her uncle’s house. The family travels 383.5 miles between 10:15 a.m. and 4:45 p.m.

1. Calculate the family’s average rate of travel for the day. Then fill in the blank to complete the following statement. You can enter a whole number or a decimal rounded to the nearest tenth.

   The family’s average rate of travel for the day is ______________ miles per hour.

2. Anne tells her family, “It’s a good thing we traveled as fast as we did. If our rate had been 50 miles per hour, we wouldn’t have gotten to his house until about …” Fill in the blank to complete the following statement.

   If their average rate had been 50 miles per hour, Anne’s family would have arrived at her uncle’s house at ______: ________ p.m.

Week 3

People living in an apartment building decide to buy the building. They will put their money together in such a way that each will pay an amount that is proportional to the size of their apartment. For example, a man living in an apartment that occupies one fifth of the floor area of all apartments will pay one fifth of the total price of the building. Read the following statements.

1. A person living in the largest apartment will pay more money for each square meter of his apartment than the person living in the smallest apartment.
2. If we know the areas of two apartments and the price of one of them we can calculate the price of the second.
3. If we know the price of the building and how much each owner will pay, then the total area of all apartments can be calculated.
4. If the total price of the building were reduced by 10%, each of the owners would pay 10% less.

Explain if each statement is correct or incorrect. Write your reasoning.


Week 4

The Morrisons are going to build a new one-story house. The floor of the house will be rectangular with a length of 30 feet and a width of 20 feet. The house will have a living room, a kitchen, two bedrooms, and a bathroom.

Create a floor plan that shows these five rooms by dividing the rectangle into rooms. Your floor plan should meet the following conditions.

1) Each one of the five rooms must share at least one side with the rectangle, that is, each room must have at least one outside wall.
2) The floor area of the bathroom should be 50 square feet.
3) Each of the other four rooms (not the bathroom) should have a length of at least 10 feet and a width of at least 10 feet.

Be sure to label each room by name (living room, kitchen, bedroom, etc.) and include its length and width, in feet. Draw your floor plan. Remember to label your rooms by name and include the length and width, in feet, for each room.

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard. Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number \( (n) \) of rows of apple trees:

Find patterns of the number of apple trees and the number of conifer trees and express them in two equations in terms of \( n \).

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifer trees? Explain how you found your answer.

## APPENDIX B

### Lesson Plan Template

<table>
<thead>
<tr>
<th>Overview of Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Explain if the original problem is appropriate for ELLs.</td>
</tr>
<tr>
<td>2. Explain any difficulties ELLs may have for solving the problem.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Learning objectives</th>
<th>Common Core Content Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical objectives:</td>
<td>Content Standards:</td>
</tr>
<tr>
<td>Linguistic objectives:</td>
<td>Practice Standards:</td>
</tr>
<tr>
<td>Problem title:</td>
<td>Anticipated Duration:</td>
</tr>
<tr>
<td>Materials Needed, if any:</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Procedures</th>
<th>Rationale (or formative assessment if any)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
</tr>
<tr>
<td>Strategies for ELLs, if any</td>
<td>Reasons why you chose the strategies</td>
</tr>
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<td>-----------------------------</td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Modifications, if any</td>
<td>Reasons why you modify in each certain way</td>
</tr>
<tr>
<td>1.</td>
<td>1.</td>
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**Assessment (How will you know the student has met the learning goal?)**
APPENDIX C

Pre-Survey

Directions: Respond to each question below.

1. Do you speak any languages other than English? If so, please state the language(s) and your fluency level.

2. Have you learned about how to teach mathematics for English language learners? If so, please describe what information you obtained and what the resource was.

3. Have you taught English language learners before? If so, please describe the student’s (or students’) language proficiency and briefly explain how you helped them in mathematics.

4. Have you taught or heard English language learners who come from South Korea? If so, please describe your experience and beliefs about them.

5. In Columbia Public School District, do you think certain ELL groups have more difficulties than other ELL groups? Particularly, what do you think about Korean ELLs?

6. Do you believe English language learners are likely to experience more difficulties in learning mathematics compared to other subjects? Why or why not?

7. Do you believe English language learners are able to learn high level of mathematics such as abstract mathematical concepts or word problems? Why or why not?

8. Do you believe you should differentiate your instruction to teach mathematics for English language learners? Why or why not?

9. What instructional strategies do you think may be effective to help English language learners understand mathematical problems? Please provide your reason to choose each strategy.
APPENDIX D

Post-Survey

Directions: Respond to each question below.

1. Do you feel more confident about knowing how to teach English language learners in mathematics?

2. Do you feel you were able to connect to your student? If so, please describe the moments you made a connection to your student. If not, please explain the reason.

3. Which lesson (or problem) was the most meaningful and beneficial? Explain the reasoning for your choice.

4. Which lesson (or problem) was the most difficult to implement? Please describe the difficulties you experienced.

5. What strategies were effective and useful when helping your student understand problems in mathematics? Please describe each strategy you found useful and explain why.

6. What advice or intervention from the researcher was beneficial for you when implementing mathematical problems to your student? Please describe each intervention and explain how it helped or what you learned from it.

7. What was the greatest challenge in teaching mathematics to English language learners?

8. Describe your opinion regarding the most beneficial and the least beneficial aspects when you look back on your entire experience in this study.
APPENDIX E

A Sample Intervention: How to Use Visuals (week 3)

Directions: Read the following word problem and try to solve it. If you cannot solve it, explain why you are not able to solve it.

동수네 반 남학생 수는 반 전체 학생 수의 \( \frac{2}{3} \) 이다. 학급 회의 시간에

학급 문고 설치에 대한 찬반 투표를 실시하였더니 남학생의 \( \frac{8}{9} \).

여학생의 \( \frac{2}{3} \) 가 찬성하였고, 남학생 중 반대한 남학생 수는 2명이라고

한다. 기권한 학생이 한 명도 없을 때, 반대한 여학생 수를 구하여야.
동수네 반 남학생 수는 반 전체 학생 수의 \( \frac{2}{3} \)이다. 학급 회의 시간에 학급 문고 설치에 대한 찬반 투표를 실시하였더니 남학생의 \( \frac{8}{9} \), 여학생의 \( \frac{2}{3} \)가 찬성하였고, 남학생 중 반대한 남학생 수는 2명이라고 한다. 기권한 학생이 한 명도 없을 때, 반대한 여학생 수를 구하여라.
Directions: The following word problem is the same as the previous ones but includes visuals. Try to solve it again and explain how the visual aids help you understand the problem.

동수네 반 남학생 수는 반 전체 학생 수의 \( \frac{2}{3} \)이다. 학급 회의 시간에 학급 문고 설치에 대한 찬반 투표를 실시하였더니 남학생의 \( \frac{8}{9} \), 여학생의 \( \frac{2}{3} \) 가 찬성하였고, 남학생 중 반대한 남학생 수는 2 명이라고 한다. 기권한 학생이 한 명도 없을 때, 반대한 여학생 수를 구하여라.

1)

\[
\begin{align*}
\text{남학생} & : \frac{2}{3} \\
\text{여학생} & : ?
\end{align*}
\]

2)

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<th>여학생</th>
</tr>
</thead>
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<td>( \frac{2}{3} = ? ) 명</td>
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<tr>
<td>반대</td>
<td>( = 2 ) 명</td>
<td>( = x ) 명</td>
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APPENDIX F

A Sample Intervention: Comparing Dialogues (week 4)

Directions: Read the following dialogues between a teacher and an ELL and compare them in terms of how the teacher makes connections between mathematics and student’s experience.

<table>
<thead>
<tr>
<th>Dialogue 1</th>
<th>Dialogue 2</th>
</tr>
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</table>
| T: Have you traveled anywhere in the U.S.?  
S: Yes, my family went to Chicago.  
T: Cool. How did you travel to get there? Did you drive or fly?  
S: We used our car.  
T: Did you like it? Traveling by car?  
S: Yeah!  
T: Sounds good. You know what? Here is our math problem today. It is about car travel. | T: Have you traveled anywhere in the U.S.?  
S: Yes. My family went to Chicago.  
T: Chicago. It’s Cool. I have been there before. There was a lot to see. How did you travel? Did you drive or fly?  
S: We used our car.  
T: You used your car. I see. You drove. How long did it take?  
S: Hmmm… about 8 hours?  
T: 8 hours. That’s a long time.  
S: Yes.  
T: When I went to Chicago, it took about 6 hours I think. Did I drive faster than you? No, no, I drove slower than you. Am I right?  
S: No, you were faster.  
T: I was faster? How much faster?  
S: … I don’t know.  
T: You don’t know. Actually I don’t know either. Let’s figure that out. This Google map said the distance from here to Chicago is about… 400 miles (write 400 miles). You said it took 8 hours from here to Chicago (write 8 hrs). Then, what was your speed?  
S: … speed? I don’t know.  
T: Speed means the distance you go in unit time like one hour or one second. Here, the speed is mile per hour. If you move 400 miles for 8 hours, how many miles do you move for one hour?  
S: Hmmm… 50?  
T: 50? 50 what? 50 minutes?  
S: No. 50… miles.  
T: 50 miles for one hour, so that is 50 miles per hour. The speed is 50 miles per hour. You also call speed the average rate of change. |
VITA

Ji Yeong (Joann) I is currently an Assistant Professor of Mathematics Education in the School of Education at Iowa State University in Ames, Iowa. She earned a Bachelor of Science degree in Astronomy from Seoul National University in Seoul, South Korea (1999), a Master of Science for Teachers in Mathematics (2014), and a Ph.D. in Learning, Teaching, and Curriculum with an emphasis in Mathematics Education (2015) from the University of Missouri, Columbia, Missouri. Her research interests include but not limited to effective mathematics pedagogies for culturally and linguistically diverse learners, preservice teacher education in STEM subjects, and problem solving in mathematics.

Prior to her doctoral studies, Joann taught middle and high school mathematics and science in California as well as working as a member of the research team of Teachers Radically Enhancing Education (T.R.E.E.) Project, which was a mathematics intervention program for Latino/a ELLs in an urban public middle school. As a graduate research assistant during doctoral studies, she was involved with several research projects such as Measurement Approach to Rational Number (MARN), funded by the National Science Foundation, and Supporting Teachers of ELLs to Achieve Reform (STELLAR), which was the pilot study of her dissertation study. Moreover, she initiated a collaborative research study about culturally and linguistically diverse students in South Korea. The results of these studies have been published in several research and practitioner journals such as *ZDM – The International Journal on Mathematics Education* and *Teachers and Teaching: Theory and Practice.*