THE CYCLICAL BEHAVIOR
OF PRICES AND INFLATION

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by
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THE CYCLICAL BEHAVIOR OF PRICES AND INFLATION

Xue Li

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ABSTRACT

This paper documents business cycle facts of prices and the inflation rate for the United States from 1959:Q1 to 2013:Q3. Prices are countercyclical and the inflation rate is procyclical. In addition, prices lead the overall cycle by two quarters and the inflation rate lags the overall cycle by three quarters. To account for the observed cyclical behavior, two models are applied and extended including a business cycle model with endogenous money supply (Freeman and Huffman 1991) and a DSGE model with sticky prices (Ireland 2003). The former model only generates countercyclical prices but not procyclical inflation or the phase shift of prices relative to the overall cycle. For the latter model, its sticky-price version captures all the observed cyclical facts; whereas its flexible-price version fails to capture the procyclical behavior of inflation and the phase shift of prices relative to output. Better performance of the sticky-price model indicates that nominal rigidity can account for the cyclical behavior of prices and inflation. Thus, a powerful empirical business cycle model should incorporate a reasonable degree of price stickiness.
CHAPTER 1
INTRODUCTION

1.1 Motivation

The primary goal of modern business cycle theories is to account for business cycle regularities using equilibrium models with optimizing agents. How do we understand business cycles? How do we measure and account for the cyclical properties of key economic variables? This paper aims to answer these questions by focusing on two business cycle facts: the cyclical behavior of aggregate price level and the inflation rate.

According to Kydland and Prescott (1990), a comprehensive documentation of business cycle facts includes three measures. The first measure is the amplitude of cyclical fluctuations. The other two measures involve the cyclical co-movements between key macr...
cycles as fluctuations in employment, output, and prices (both consumer and wholesale prices). Business cycle studies before the 1970s usually apply such definitions of business cycles.

Economic activity in market economies features sustained growth, thus most macroeconomic series share a common trend or have closely related trends. If the trend(s) dominate the raw levels of economic series, one would obtain positive correlations among most variables so long as their trends are positively correlated. This result is not wrong but it provides little insight into the underlying relationship among key variables at business cycle frequencies.

Considering the obvious upward trend in many macroeconomic series, Lucas (1977) defines business cycles as deviations of aggregate output from trend. Measurement of business cycles requires a formula to extract the “trend” from a raw time series, but Lucas does not provide such a formula or even define the “trend”. Hodrick and Prescott (1981) propose a formula to extract the trend component from a raw time series and the remainder is the cyclical component. Such a formula has been widely used in business cycle research and is now named the Hodrick-Prescott (H-P) filter. Throughout this dissertation the H-P filter is applied to identify the cyclical components of relevant economic series.

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2 A common feature of the two definitions is that each business cycle is identified as a combination of four phases that evolve from one into another: expansion, crisis, depression, and revival.
3 A raw time series consists of the natural logarithms of observed values of the corresponding variable.
4 Hodrick and Prescott (1981) have been reprinted in Hodrick and Prescott (1997).
5 A mathematical formula for the H-P filter is provided at the beginning of Chapter 4.
1.2 Significance of the Research Question

Having a profound influence on business cycle studies, the price-output relationship is important for two reasons, among others. First, reliable characterization of business cycle regularities helps identify the nature and sources of cyclical fluctuations. The belief in procyclical prices leads to the adoption of monetary disturbances as the source of cyclical fluctuations. For example, Friedman and Schwartz (1963) considered monetary innovations as the main determinant of aggregate fluctuations. Later empirical studies found it difficult to justify the crucial role of monetary shocks in output fluctuations. Thus, researchers focus instead on other factors, including technology shocks, nominal rigidity, etc.

Second, identified business cycle regularities have been used to restrict business cycles models. Before the 1970s, the conventional wisdom was that prices are procyclical. Even Lucas (1977) regards procyclical prices as one of the business cycle regularities.\(^6\) The popularity of such a belief strongly impacts business cycle studies in the 1970s and early 1980s. For instance, the positive correlation between prices and output leads to an identification restriction that supply remains constant when demand fluctuates. To accommodate such an implication, some researchers adjust their models for the procyclicality of prices, like the equilibrium models of Lucas (1972), Sargent (1976), and Barro (1979), and the nonmarket clearing models of Taylor (1979, 1980) and Gordon (1982). Such an adjustment would be unnecessary and even harmful if prices and output

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are not actually positively correlated. As shown in Section 4, the price level is indeed not procyclical and thus such an adjustment is indeed unnecessary.

The cyclical behavior of inflation is important since the traditional focus of business cycle models, like rational expectations models along the lines of Lucas (1972, 1976), has been the inflation rate instead of the price level. For example, Lucas (1976) mentions the changes in nominal prices and wages (inflation), instead of the price level itself, at the peak and at the trough of the business cycle when motivating his theoretical model. It is tempting to expect that the price level and the rate of change of prices over time, namely the inflation rate, exhibit similar cyclical properties since both of them are measures of prices. However, nothing guarantees that this is the case.

When reporting business cycle facts, it is important to distinguish between the cyclical behavior of prices and that of the inflation rate. Such a distinction is crucial in discriminating between alternative business cycle models based on stylized facts. Also, this distinction is important for the motivation of identification restrictions when constructing an empirical business cycle model. Whether the countercyclical behavior of prices results in the same countercyclical behavior of inflation? The answer is frequently “No”. As reported in Chapter 4, the contemporaneous correlation between the inflation rate and output is positive. Put differently, countercyclical prices do not necessarily indicate countercyclical inflation.

1.3 Dissertation Organization

The plan of this dissertation is as follows. Chapter 2 reviews the literature on the characterization and interpretation of the cyclical behavior of prices and inflation. Chapter 3 introduces the data and methodology. Empirical results are reported in Chapter
4. To account for the observed cyclical behavior of prices and inflation, two models are applied and augmented in Chapters 5 and 6. Chapter 5 assesses a business cycle model with endogenous money supply and its extension – a business cycle model with endogenous monetary policy rule. Chapter 6 interprets these cyclical facts using a sticky-price dynamic stochastic general equilibrium (DSGE) model. Chapter 7 summarizes the findings.
CHAPTER 2
LITERATURE REVIEW

Since the historical definitions of business cycles have been reviewed briefly in Chapter 1, this chapter reviews the characterization and explanation of the business cycle regularities for the price level and inflation. For the cyclical behavior of prices, the literature is divided into two parts. The first part features the procyclicality of prices, which used to be an accepted “fact”. And the second part features the countercyclicality of prices after World War II (WWII), which overturns the assertion that prices are procyclical. For the interpretation of the observed cyclical behavior, five categories can be found in the literature, including the AD-AS framework, the business cycle framework with a monetary sector, changes in the monetary policy, an atheoretical explanation, and unidentified models.

2.1 Evolving Views on the Cyclical Behavior of Prices

2.1.1 Procyclical Behavior of Prices

Before 1989, many researchers started with the premise that the price level is procyclical, which has profoundly influenced business cycle modeling. The earliest study on the co-movement between prices and output dates back to 1930 by Kuznets. He measured the relation between the cycles in various economic series. His definition of “cycle” is somewhat unique. Kuznets (1930) decomposed a time series into cyclical fluctuation and
“secular movements”. The secular movements were then decomposed into two components: the primary trend and the secondary movements. The secondary component was called “major cycles”, which was used to characterize the relation between cycles in different variables. Kuznets (1930) examined production and prices on disaggregated levels over the 19th and early 20th centuries and found a positive correlation between the cycles in production and prices.

Other researchers prior to Lucas (1977) generally implement the definition of business cycles by Mitchell (1913, 1927), Burns and Mitchell (1946) or Hansen (1951). An economic series – output, for example – can be visually identified as either increasing generally over time or decreasing generally over time. The latter is a contraction and the former is an expansion. Hence, they generally use raw economic series in the documentation of cyclical facts. Mills (1936), for example, analyzed wholesale price indexes (WPI) for 1929 – 1936 U.S. and for other 32 countries over 1928 – 1936. He found that crisis and recession were featured by a general reduction of prices, which reflected the procyclicality of prices for the U.S. and other countries considered. Mills (1946) examined price movements for 64 individual commodities, their prices and quantities with reference cycle over contractions and expansions. The sample period covers nine business cycles from 1904 to 1938. The procyclical pattern of prices is significant for 5/6 of all commodities studied.

Burns and Mitchell (1946) defined business cycles as a class of fluctuations in

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7 Following Cournot and Fisher (1897, p. 25), “secular movements” are defined as continuous and irreversible changes underlying cyclical movements of a time series.

8 Such a definition of “cycles” is unique since other researchers either use a raw time series or the cyclical component of the corresponding variable to measure business cycles.
general economic activity. They studied over a thousand of time series for four countries including the U.S., the Great Britain, Germany, and France. The data used are WPI and individual prices over all contractions and expansions from 1854 to 1933. Similar to previous work, their results confirmed the procyclical behavior of prices for major industrialized economies. Defining business cycles in a way similar to Mitchell (1913, 1927), Hansen (1951) analyzed monthly industrial production, factor employment, and retail prices of food to urban workers in the U.S. from 1919 to 1939. He provided figures indicating a positive correlation between prices and production. In other words, when production is generally increasing the price level is generally increasing.

Given the studies discussed above and other relevant work, the procyclical behavior of prices had played a prominent role in the literature by the 1970s. In fact, this commonly held belief had continued to impact business cycle modeling even after the 1970s during which Lucas (1977) proposes a better definition of business cycles. Specifically, he defines “the business cycle” as movements or deviations of gross national product (GNP) around trend. Lucas (1977) himself claims the procyclical behavior of prices appears to be robust across countries and time periods. Put differently, although Lucas contributes much to the definition of business cycles, he fails to re-examine the cyclical behavior of prices using his own definition.

The influence of procyclical prices continued until the end of the 1980s. Hall and Taylor (1986) document a positive correlation between the price level and output as a

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9 For example, Harrod (1936, p. 41) regarded as a fact that the price level rises in expansions and drops in recessions. Another example involves Mitchell (1951, p. 170), who stated that most prices usually increase in expansions and decrease in recessions, although not always.
business cycle fact. Bernanke (1986) and Mankiw (1989) criticize the findings reported by King and Plosser (1984) because their model cannot explain why prices are procyclical. In addition, Olson (1989) advocated the view in Lucas (1977) that “prices are generally procyclical”.

As discussed in Chapter 1, the assertion of procyclical prices is partially attributed to the specification of data used to characterize the cyclical behavior of prices. Prior to the 1980s, researches mainly used the natural logarithms of prices and output, which have closely related trends. The high correlation between the trend components of prices and output arises from the sustained growth in developed economies; combined with the upward trend in the price level, it which provides little insight into the underlying co-movement between prices or inflation and output at business cycle frequencies. Another drawback is that the literature typically focuses on the contemporaneous correlation while neglecting the lead-lag relationship between prices and output, which is another important aspect of the cyclical co-movements.

2.1.2 Countercyclical Behavior of Prices

Since the 1980s, the procyclical behavior of prices has been questioned by various researchers. It was first challenged by Friedman and Schwartz (1982). They found that changes in prices and output were generally negatively correlated between cycle phases for the U.S. and the U.K. of 1867 – 1975.10

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10 See Friedman and Schwartz (1982, p. 9). They used phase-average data on prices and output where the average cycle phase was 2.0 years for the U.S. and 2.8 years for the U.K. Thus the price-output correlation was measured “between cycle phases”.

9
In their documentation of business cycle facts, Kydland and Prescott (1990) implement Lucas’ definition of business cycles. Formally, they define business cycles by extracting deviations of aggregate output from trend. Moreover, Kydland and Prescott propose a procedure to decompose a raw time series into a trend component and a cyclical component. They find a negative correlation between de-trended prices and output using real GNP, the GNP deflator, and the Consumer Price Index (CPI) for post-Korean war U.S. (1954 – 1989). In light of their empirical evidence, Kydland and Prescott argue that the procyclical behavior of prices should no long be regarded as an important guidance or indication of macroeconomic models.

Cooley and Ohanian (1991) conduct a comprehensive study on the cyclical behavior of prices. In their study, they extend the history, using quarterly and annual real GNP, GNP deflator, and WPI for the U.S., from the 1820s to 1990. The full sample is divided into four sub-periods, including the pre-Civil war (1822 – 1859), pre-WWI (1869 – 1911), inter-war (1919 – 1941), and post-WWII (1948 – 1987) periods. The cyclical components are extracted by three methods, including first differencing, linearly detrending, and the H-P filter. They find mixed results for the pre-Civil war period, no consistent correlation for the pre-WWII period, a positive and robust correlation for the inter-war period, and a negative and robust correlation for the post-WWII period. Similar to Kydland and Prescott (1990), Cooley and Ohanian (1991) conclude that it is unnecessary to adjust business cycle models for a positive price-output correlation.

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12 Kydland and Prescott (1990) obtain a contemporaneous correlation coefficient of -0.55 between the cyclical components of implicit GNP deflator and real GNP, and a coefficient of -0.57 between the cyclical components of CPI and real GNP.
The break point of the cyclical behavior of prices has been challenged by Wolf (1991). He finds that prices become countercyclical only after 1973, not after 1953 as in Kydland and Prescott (1990). Wolf uses quarterly Industrial Production Index (IP), Producer Price Index (PPI), real GNP, and Wholesale Price Index (WPI) for the U.S. over the period 1957 – 1988. The H-P filter is applied to extract the cyclical components of economic series involved. Average contemporaneous correlations between detrended output (IP) and price level (PPI) for five-year rolling window samples reveal that prices are generally procyclical until the end of the 1960s, generally countercyclical in the 1970s, and become mildly procyclical again in the late 1980s. Wolf also examines the cyclical behavior of various commodity prices for two sub-periods: 1957 – 1972 and 1973 – 1989. Still, most price indexes are much more countercyclical since 1973 compared to the first sub-period.

Backus and Kehoe (1992) compare the cyclical properties of nominal and real variables across ten industrialized countries. The ten countries examined consist of the U.S., the U.K., Canada, Germany, Australia, Denmark, Italy, Japan, Norway, and Sweden. Backus and Kehoe (1992) use annual output and price levels over a century (1850 or 1860 or 1870 – the 1980s). The cyclical components are extracted by the H-P filter. They find that the cyclical behavior of prices varies across time periods: prices are procyclical before WWII and countercyclical afterward. They also find that inflation rates are more persistent after WWII than before.


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13 Wolf (1991) points out that the results are qualitatively robust to alternative detrending methods, including first-order differencing and the exponential smoothing filter.
except for the period in the 1950s and the 1960s. The ten countries considered are the same as those in Backus and Kehoe (1992). The data in Smith’s sample include annual and quarterly real GDP or GNP, output deflator and CPI over a century. He decomposes raw time series with first-difference, a linear trend, and the H-P filter. The results are mixed. Specifically, Smith finds that prices are generally procyclical from the late 19th century until WWII with the exception of a period around WWII, and prices are countercyclical for the post-Depression period with the possible exception of a period in the 1950s and 1960s.

Chadha and Prasad (1993) question the common adoption of the cyclical component of the price level in the characterization of its cyclical behavior. They prefer to use the rate of inflation to measure the cyclical co-movement between prices and output. The reason is that inflation might be stationary, thus a filter is unnecessary to make it stationary. Using postwar U.S. data from 1947 to 1989, Chadha and Prasad (1993) find that the inflation rate is consistently and usually strongly positively correlated with the cyclical components of output extracted by various methods.\footnote{Output is measured by quarterly real GNP (measured in 1982 dollars). Price series are measured by the implicit GNP deflator and the consumer price index (CPI).}\footnote{These detrending methods include linearly detrending, segmented (linearly) detrending, the H-P filter, the Beveridge-Nelson (BN) decomposition, and the Blanchard-Quah (BQ) (1989) decomposition.} Still, for the cyclical component of prices, they find a consistent and often strongly negative correlation between detrended prices and output.

Motivated by the distinct cyclical behavior of prices and inflation for the U.S., Chadha and Prasad (1994) detect similar distinction for the G-7 countries.\footnote{These countries include the U.S., the U.K., Germany, Japan, France, Canada, and Italy.} They use
postwar quarterly real GDP or GNP, implicit price deflators, and unemployment rate (as an alternative indicator of the cycle). Five decomposition methods are applied, including first difference, a linear trend, a segmented linear trend (Perron 1989, 1990), the H-P filter, and the Blanchard-Quah (B-Q) (1989) decomposition. Chadha and Prasad (1994) find that prices are countercyclical and inflation is procyclical for all countries considered, thus they emphasize the necessity to distinguish between the levels of inflation and the cyclical component of prices when reporting business cycle facts.

Stimulated by the focus on developed countries, Kim (1996) examines the cyclical behavior of prices and inflation in two rapidly growing developing economies: Korea and Taiwan. He uses quarterly GNP, GNP deflator, GDP, GDP deflator, and CPI for Korea from 1970 to 1995 and for Taiwan from 1959 to 1995. The cyclical components are extracted by five methods including first difference, a linear trend, a segmented trend, the H-P filter, and the Phase Average Trend (PAT). Kim (1996) finds that the cyclical components of prices and output are negatively correlated, while the levels of inflation are positively correlated with the cyclical component of output. These findings suggest that the countercyclicality of prices and the procyclicality of inflation are robust to Korea and Taiwan, both of which are developing economies.

Cooley and Ohanian (1991) discover the time-varying nature of the price-output relationship at business cycle frequencies. Cover and Hueng (2003) adopt the definition by den Haan (2000), who defines the price-output correlation as the contemporaneous correlation between forecast errors of output and the price level from bivariate VARs at
various horizons.\textsuperscript{17,18} In order to measure these time-varying correlations, Cover and Hueng estimate a sequence of the price-output correlation for the U.S. using a vector autoregressive model (VAR) with a bivariate generalized autoregressive conditional heteroskedasticity (GARCH) error process.\textsuperscript{19} Using quarterly real GDP, nominal GDP, and GDP deflator of the U.S. from 1875 to 1999, they find that estimated correlations are generally positive before 1945, zero during 1945 – 1963, and negative after 1963.\textsuperscript{20} Cover and Hueng (2003) thus argue that researchers have overstated the idea that the price-output correlation becomes negative after WWII. Moreover, they argue that researchers should not construct or look for business cycle models which limit the sign or degree of the price-output correlation.

Another study estimating the price-output correlation sequence is Lee (2006), but Lee provides different empirical evidence from Cover and Hueng (2003). In order to detect possible changes in conditional price-output correlations over time, Lee (2006) adopts Engle’s (2002) dynamic conditional correlation (DCC) – GARCH model. Estimated by the maximum likelihood (ML) method, the DCC-GARCH model suggests procyclical prices before WWII and countercyclical prices after WWII. The empirical results are proved robust to alternative data specifications by re-estimating the conditional mean equation using first-differenced and de-trended data. Lee’s (2006)

\textsuperscript{17} Page 18 briefly reviews the paper by den Haan (2000).
\textsuperscript{18} As argued by den Haan (2000), this alternative definition frees researchers from having to choose between various detrending methods to stationarize relevant economic series, which is the task of the traditional approach, i.e., the correlation between the cyclical components of different variables.
\textsuperscript{19} The multivariate GARCH model has time-varying conditional variance-covariance matrix for its residuals, allowing for quarterly estimates of the contemporaneous correlation between prices and output.
\textsuperscript{20} According to Cover and Hueng (2003), negative correlation estimates mean that the frequency of negative correlations rises dramatically after 1963, but the negative correlations are no more important than zero correlations.
empirical results and the similarity of his evidence compared with that provided by others further suggest that prices are procyclical before WWII and countercyclical afterwards.

Haslag and Hsu (2012) re-examine business cycle regularities for prices and inflation using Lucas’ (1977) definition of business cycles. The data used are quarterly real GDP and GDP deflator of the U.S. over the period 1947 – 2012. They decompose these time series using the H-P filter and focus on the contemporaneous price-output correlation and inflation-output correlation. Different from Chadha and Prasad (1993, 1994) and Kim (1996) where levels of inflation are used, Haslag and Hsu (2012) use HP-filtered inflation when characterizing the cyclical behavior of inflation. They find a negative correlation between the cyclical components of prices and output, and a positive correlation between the cyclical components of inflation and output.

2.2 Interpreting the Cyclical Behavior of Prices and Inflation

2.2.1 The AD-AS Framework

Using nearly a hundred of years of annual data (1890 – 1978), Gordon (1980) finds a contemporaneous correlation coefficient of around 0.5 between the percent change in the price level (inflation) and the percent change in detrended output (output growth). He also finds an increase in the autocorrelation of inflation after WWII. Hence, Gordon proposes an AD-AS model with increased inflation persistence and higher volatility of “supply shocks” after WWII. He attributes the dramatic rise in the persistence of inflation to a change in the economic structure after WWII, which explains the distinct cyclical behavior of prices before and after WWII.

Chadha and Prasad (1993) account for observed cyclical behavior of prices using a
sticky-price demand-driven model and the correlation between levels of inflation and detrended output. They examine a sticky-price demand-driven model and a flexible-price supply-driven model. For the former model, they find procyclical inflation and countercyclical price level even when the price level is procyclical by construction. Thus, they argue that the inflation-output correlation accurately reflects the cyclical behavior of the price level. For the latter model where the price level is countercyclical by construction, simulated price level is countercyclical. However, simulated inflation is countercyclical which contradicts observed procyclicality of inflation. They conclude that the cyclical behavior of prices is best described by the inflation-output correlation which distinguishes between demand and supply shocks. However, their arguments require identification of demand and supply shocks in order to characterize the cyclical behavior of prices.

Spencer (1996) argues that an AD-AS model with only demand shocks leads to an ambiguous sign of the price-output correlation, thus both demand and supply shocks are required to explain observed cyclical behavior of prices. He finds that for an AD-AS model with only demand shocks, simulated price-output correlation is positive sometimes and negative at other times (e.g., Chadha and Prasad 1993, Judd and Trehan 1995).

To interpret observed cyclical behavior of prices, Spencer (1996) suggests that one examines of the distinct dynamic impacts of demand and supply shocks on prices and

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21 See Footnote 14 for the measurement of output and prices in real economy.
22 In a sticky-price demand-driven model, the cyclical component of output adjusts primarily to movements in demand and thus higher demand increases output. In addition, higher demand raises the price level (slowly due to price stickiness). Hence, the price level is procyclical by the construction of this model.
23 In a sticky-price demand-driven model, the cyclical component of output adjusts primarily to movements in supply and thus higher supply increases output. In addition, higher supply lowers the price level. Hence, the price level is countercyclical by the construction of this model.
output. Using structural VARs with the Blanchard-Quah (B-Q) identification, Spencer finds that demand shocks affect prices permanently and output temporarily, while supply shocks have permanent impacts on both prices and output. Such empirical evidence is consistent with the predictions of an AD-AS model. Spencer (1996) thus concludes that conventional AD-AS models adequately account for the countercyclical behavior of prices after WWII. Moreover, demand shocks play an important role in the short run.

Similarly, den Haan (2000) denies the ability of a sticky-price model with only demand shocks (e.g., Chadha and Prasad 1993) to explain observed countercyclical behavior of postwar prices. He proposes an alternative measure of the price-output correlation which involves the correlation between forecast errors of prices and output from VAR systems at various horizons. Den Haan (2000) finds that prices are procyclical in the “short run” (at short forecast horizons) and countercyclical in the “long run” (at long forecast horizons). He accounts for such empirical results using a model with money demand shocks dominant in the “short run” and productivity shocks dominant in the “long run”.

Pakko (2000) highlights multiple shocks in the interpretation of the cyclical behavior of prices. Decomposing the price-output correlation into frequency components, Pakko finds certain common features of the correlation’s cospectrum which are robust across sample periods and de-trending methods. The only exception occurs in the interwar era, during which negative components of the price-output cospectrum appear at fairly low frequencies, while positive contributions emerge from higher frequencies. Pakko (2000) then examines a demand-driven Keynesian model of Judd and Trehan (1995). He finds that it is difficult to account for changes in the nominal-real covariance
structure by varying parameter values in a single-shock model. He suggests at least two shocks so that one shock explains a low-frequency negative co-movement and the other shock contributes to a high-frequency positive co-movement between prices and output.

Slightly different from others, Cover and Hueng (2003) find that the price-output correlation is positive (during recessions and expansions) before 1945, zero during 1945 – 1963, and negative (only during expansions) after 1963. They attribute the change in the price-output correlation after 1945 to changing behavior of prices during expansions after WWII. The reasons they suggest include a change in the relative dominance of demand and supply shocks over time, and a change in the response of monetary authorities to such shocks, or both.

Cover and Hueng (2006) provide a more concrete explanation for the change in the cyclical behavior of prices since WWII. Using a structural VAR with bivariate GARCH(1,1) errors, they find that postwar price-output correlation is significantly negative only when the demand shock is less volatile than the supply shock. With the estimated demand shock less variable than the estimated supply shock, Cover and Hueng (2006) attribute the distinct behavior of postwar prices to reduced volatility of the demand shock. They suggest that the ultimate reason might be an improving monetary policy which increases U.S. economic stability since WWII.

2.2.2 The Business Cycle Framework with a Monetary Sector

King and Plosser (1984) illustrate the cyclical behavior of prices using a real business cycle (RBC) model with an explicit banking sector. Their model includes two productive sectors. The first sector is the final goods-producing industry and final goods serve as either consumption goods or production inputs. The other sector is the financial industry
which supplies transaction services as an intermediate good. Households and firms desire transaction services which reduce transaction costs.

The response of prices to productivity shocks is determined by two factors, including the impact of output on the demand for outside money (currency) and the impact of nominal interest rate on the demand for outside money. King and Plosser find that the cyclical behavior of prices depends on the monetary policy. Prices are procyclical if the money supply is sufficiently procyclical to dominate the countercyclical effect of the demand for real balances on the price level. Conversely, prices move countercyclically if the money supply is not adequately procyclical.

Similarly, Smith (1992) suggests that the business cycle framework with a monetary sector can account for the countercyclical behavior of the price level. According to Smith (1992), higher income and consumption increase the transaction demand for money in most models of economic fluctuations, indicating procyclical behavior of the demand for real balances. Therefore, in any model where the money stock is not responsive to real activity, the price level has to be reduced to satisfy increased demand for real balances, i.e., the price level moves countercyclically. Smith (1992) argues that such a rationale applies to a variety of models, including a model driven by government spending shocks, a model with Keynesian investment spending shocks, and RBC models driven by productivity shocks (e.g., King and Plosser 1984, Cooley and Hansen 1989).

equilibrium model of economic fluctuations, i.e., a Cash-in-Advance (CIA) model with nominal wage rigidities, with an endogenous monetary policy rule. The monetary policy is endogenous since the central bank aims to minimize a weighted sum of variations in output and the inflation rate. Floden’s findings imply that small changes in the central bank’s preference dramatically alter the corresponding money supply rule and the nominal-real correlations among economic variables.

2.2.3 Changes in the Monetary Policy

Given the widely accepted view that prices are procyclical before WWII and countercyclical afterward, several researchers ascribe the change in the sign of the price-output correlation to changes in monetary policies. Cagan (1986, p. 664) argued that the central bank resists dramatic price changes after WWII but failed to do so before WWII. Accordingly, the price level moves countercyclically and inflation becomes more persistent after WWII. Similarly, Bordo et al. (1987) point out that the monetary policy is distinct after WWII compared with its counterpart under the gold standard. They illustrate their view with the exceptionally countercyclical behavior of prices in prewar Japan, which was not on the gold standard during most of the prewar period.

Backus and Kehoe (1992) detect the change in the cyclical behavior of prices for countries other than the U.S. In light of the international phenomenon that prices behave differently after WWII, Backus and Kehoe emphasize the need for an explanation that holds across countries. Furthermore, they document a change in the sign of the correlation between money growth and output growth since WWII from positive to negative. Accordingly, Backus and Kehoe account for the changing cyclical properties of
prices across WWII with a declined relation between the money supply and output after WWII.

2.2.4 An Atheoretical Explanation

Haslag and Hsu (2012) provide an atheoretical explanation for the countercyclical behavior of prices and the procyclical behavior of inflation. Motivated by the evidence that prices lead output at business cycle frequencies, they suggest a positive phase shift of HP-filtered price series relative to HP-filtered output series as an explanation of observed cyclical facts. They illustrate that an appropriate degree of phase shift guarantees the negative sign of the price-output correlation and the positive sign of the inflation-output correlation. Haslag and Hsu further interpret that the proposed phase shift has implications for Granger-causality from prices to output. An F-test confirms their interpretation that prices Granger-cause output, but output does not Granger-cause prices for the U.S. after WWII.

2.2.5 Unidentified Models

Different from other researchers eager to explain the cyclical behavior of prices, Judd and Trehan (1995) are pessimistic about the feasibility of illustrating stylized facts of business cycles as indicators of certain classes of economic models. Based on quarterly real GDP and GDP deflator of postwar U.S. (1947 – 1992), the authors argue that using the sign of the price-output correlation to evaluate business cycle theories or models is questionable. The reason is that the commonly-used countercyclical behavior of prices can stem from a variety of models, including a sticky-price demand-driven Keynesian model where the price level is procyclical by construction. Moreover, minor modifications of the model
alter the sign of the price-output correlation. Judd and Trehan (1995) conclude that it is hard to interpret observed business cycle facts without referring to certain specific economic theory.

Judd and Trehan (1995) doubt the ability of the price-output correlation to characterize important business cycle regularities in the absence of any particular economic theory. Put differently, they give theories priority over empirical evidence and argue that economic theories should guide the interpretation of empirical evidence for business cycles. In my view, business cycle models should be built on objective facts about business cycles since they cannot be altered intentionally by anyone. Therefore, careful characterization of business cycle regularities is the prerequisite for the construction of business cycle models.

In the characterization and interpretation of the cyclical behavior of prices and inflation, the literature emphasizes contemporaneous correlations which capture one important aspect of the business cycle regularities. Another important aspect involves the phase shift of prices relative to the overall cycle, which has been paid little attention. Besides the contemporaneous price-output correlation, this paper also documents the correlations at leads and lags to detect phase shift of prices relative to output. Moreover, this paper aims to illustrate such a lead-lag relationship using theoretical models.
CHAPTER 3
DATA AND METHODOLOGY

3.1 Data

For the U.S., quarterly real GDP and GDP deflator are available from 1947:Q1. This paper aims to investigate the price-output relationship in a larger system including monetary aggregates and nominal interest rates, while monetary measures are available from 1959:Q1. Therefore, the sample includes quarterly data on real GDP \( y \), real personal consumption expenditures \( C \), real gross private domestic investment \( I \), the GDP deflator \( p \), the monetary base \( MB \), the M1 money stock \( M1 \), the M2 money stock \( M2 \), the 3-month Treasury Bill (TB) rate \( i \), and the civilian noninstitutional population (age 16 and over) for the U.S. from 1959:Q1 to 2013:Q3.\textsuperscript{24,25} Other two variables involved are Money Multiplier 1 \( MM1 \) and Money Multiplier 2 \( MM2 \), which are defined as the ratio between M1 and MB, and the ratio between M2 and MB, respectively.

3.2 Methodology

To provide a roadmap for subsequent analysis, this section introduces the methodology used. The objective of this paper is to comprehensively investigate the cyclical behavior

\textsuperscript{24} As will be explained in Section 3 of Chapter 6, data on consumption and investment are used in place of data on output when estimating parameters of the sticky-price model. As will be covered in Section 7 of Chapter 6, observations of the civilian noninstitutional population are used to obtain per-capita measures in robustness check.

\textsuperscript{25} All data are obtained from the FRED database from the Federal Reserve Bank of St. Louis.
of prices and inflation and to account for observed cyclical properties with a theoretical model. Accordingly, empirical results are reported in Chapter 4 and two candidate models are examined in Chapter 5 and Chapter 6. The first candidate model is a business cycle model with endogenous money supply initiated by Freeman and Huffman (1991) and furthered developed by Freeman and Kydland (2000) and Henriksen and Kydland (2010). And the other model is a sticky-price dynamic stochastic general equilibrium (DSGE) model specified by Ireland (2003). These two models will be evaluated according to their ability to match empirical moments of key variables.

Chapter 4 consists of three sections, including business cycle regularities for key economic aggregates, autocorrelation functions (ACFs) and vector autoregressive (VAR) analysis. According to Kydland and Prescott (1990), a comprehensive documentation of business cycle facts consists of three measures. The first measure is the amplitude of cyclical fluctuations. The second measure is the degree of contemporaneous correlation between the price level and output, and that between the inflation rate and output at business cycle frequencies. And the last measure is the lead-lag relationships between prices, inflation, and output, which characterize the phase shift of prices relative to the overall cycle, as defined by output.

As an important component of time-series properties, ACFs display the persistence evident in the cyclical components of key economic aggregates. Several researchers have documented stylized facts about output dynamics in the United States by examining the time-series properties of economic aggregates. For example, using annual data, Nelson...
and Plosser (1982) and Cochrane (1988) find that the sample autocorrelations in output growth are significantly positive at the first lag and insignificantly negative at higher lags. Cogley and Nason (1995) replicate such empirical evidence using quarterly data and assess the ability of real-business-cycle (RBC) models to capture these stylized facts by comparing the model-generated ACFs with empirical ACFs. Motivated by Cogley and Nason (1995), this paper investigates the sample ACFs of the HP-filtered levels of key macroeconomic variables, especially output, the price level, and the inflation rate, to document their dynamics at business cycle frequencies. Moreover, the sample ACFs are compared to the model-generated ACFs of the corresponding variables in Chapter 5 and Chapter 6 as a way to evaluate the fit of each candidate model.

VAR models are used since they provide a theory-free method to estimate a variety of relationships among relevant variables.\(^{27}\) Sims (1980b) investigates the sources of output fluctuations using VARs including the natural logarithms of monthly money supply (the M1 money stock), output (industrial production or IP), price level (wholesale price index or WPI), and nominal interest rate (the rate on 4-6 month prime commercial paper). He finds that in a trivariate VAR including M1, IP, and WPI, M1 Granger-causes and explains a large fraction of variance in IP. Interestingly, when the nominal interest rate is added, M1 no longer Granger-causes output, while the interest rate Granger-causes both M1 and IP. Sims thus suggests that the observed money-output correlation is partially attributed to common reactions in response to shocks to the interest rate.

The most striking result reported by Sims (1980b) is that monetary shocks have not

\(^{27}\) See Sims (1980a).
driven output fluctuations, after conditioning on the nominal interest rate. The seminal work of Sims has stimulated a variety of studies on the determinants of output fluctuations. Many researchers have checked the sensitivity of Sims’ findings to alternative measures of the same variable, alternative frequencies and specifications of data, alternative time periods, and alternative lag structures. Their results are mixed. Using the natural logarithms of both monthly and quarterly data from 1948 to 1983, Litterman and Weiss (1985) corroborate Sims’ findings that the contribution of money to output fluctuations is very sensitive to the inclusion of the short-term interest rate.

On the other hand, Sims’ findings are suspected by many researchers who provide empirical evidence that minor modifications of Sims’ model dramatically alter the role of monetary shocks in the determination of output. For example, King (1983) adds three lags and a time trend to Sims’ (1980b) model and estimates the VAR on a slightly later time period. King finds that the fraction of output variation attributable to monetary innovations increases from Sims’ 4% to 24%. Other related work has been conducted by

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28 The specification of data refers to the nature of the specific values used. Some researchers (e.g., Sims 1980b, Litterman and Weiss 1985) use the natural logarithms of corresponding variables, and some researchers (e.g., Eichenbaum and Singleton 1986) use the growth rates of corresponding variables; while other researchers (e.g., King 1983, Todd 1990) use linearly detrended levels of corresponding variables.


30 Litterman and Weiss (1985) use data on output (the industrial production index), price level (the consumer price index less shelter), money (M1), and nominal interests (yield on 3-month treasury securities) for the U.S. from 1948 through 1983.

31 In this paragraph, Sims’ model refers to Sims’ (1980b) four-variable unrestricted VAR including the rate on 4-6 month price commercial paper (R), the M1 money stock (M1), wholesale price index (WPI), and industrial production (IP).
Runkle (1987), Spencer (1989), etc. By estimating hundreds of variations of Sims’ four-variable VAR, Todd (1990) reconcile two sides of the debate by showing that the role of money in output fluctuation is sensitive to alternative specifications of the model, but Sims’ evidence against the specific form of monetarism is robust.

To provide more insights into the debate on the role of the money stock (and interest rates) in the determination of output, VAR models will be built to assess the sensitivity of Sims’ (1980b) results to changes in the measure of the same variable, the specification of data, the lag structure, and time periods. This paper contributes to the literature by incorporating HP-filtered variables in VARs. Other papers either use the natural logarithms, or the growth rates, or linearly detrended levels of variables of interest. None of them has addressed the issue raised by Sims using the H-P filter.

Previous studies (e.g., King 1983, Litterman and Weiss 1985) have shown that the specification of data is critical to the estimation of and inference on VARs. Failure to exclude the trend component may commingle the behavior of the cyclical component and the trend component in VARs if the focus is the non-trending component. To address the question raised by Sims (1980b) from another perspective where the effects of the

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32 Runkle (1987) proposes two methods of computing confidence intervals for variance decompositions (and impulse response functions) of unrestricted VARs. He finds that those confidence intervals are so large that few firm conclusions can be drawn about the relationships between money, interest rates, prices, and output from unrestricted VARs. To check the robustness of the role of monetary innovations in output fluctuations, Spencer (1989) alters the specification of the four-variable VAR including the same data as in Litterman and Weiss (1985). The specifications examined by Spencer include alternative orderings of the variables, alternative lag orders of the VAR, alternative detrending methods, and alternative frequencies of the data (monthly, quarterly, semiannual, and annual). He finds that for the specifications considered, the fraction of output forecast error variance accounted for by monetary shocks varies from zero to over 50%.

33 Todd (1990) agrees with Sims (1980b) in the sense that few models examined by Todd lend support to the version of monetarism Sims assessed.
sustained growth trend are removed, this paper builds VARs using the cyclical components identified by the H-P filter. For the specification of the lag structure, researchers often use rules of thumb or arbitrarily selected lag length(s). Only a few of them have realized the influence of lag structure specification upon statistical inferences on estimated VARs. Considering such an crucial role of lag length selection, this paper applies a likelihood ratio (LR) test to identify the lag structure of each VAR considered.

Another important purpose of building VARs is to investigate the Granger-causal relationship between prices and output at business cycle frequencies. The Granger-causal relationship between two variables can be identified using Granger-causality tests. The Granger-causality test examines whether movements in one variable (Variable 1) temporally precede movements in another variable (Variable 2) in a VAR system. Put differently, it assesses whether lags of Variable 1 help predict current values of Variable 2. The Granger-causality statistic can be obtained from an F-test in which the null hypothesis is that the set of coefficients corresponding to lags of Variable 1 is zero. Variable 1 Granger-causes Variable 2 if the relevant set of coefficients is jointly significant at a predetermined significance level, say 5%. Conversely, Variable 1 does not Granger-cause Variable 2 if the coefficients involved are jointly zero at the predetermined significance level.

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34 For example, Sims (1980b) uses twelve lags for all VARs examined without an explicit explanation for such a choice.
36 The lead-lag relationship between prices and output in Section 1.3 of Chapter 4 suggests that prices lead output by about two quarters. Therefore, it is reasonable to assume that the price level Granger-causes output. See also Haslag and Hsu (2012).
To investigate the Granger-causal relationship between the price level and output, the simplest VAR will be built including HP-filtered values of output and the price level. To check the robustness of observed Granger-causality between prices and output in the bivariate VAR, each of other variables will be added to build trivariate VARs. And then four-variable VARs will be built including prices, output, and two other variables. Finally, two five-variable VARs will be built including base money, the money multiplier (MM1 or MM2), output, the 3-Month TB rate, and the price level.

To explain observed cyclical behavior of prices and inflation, two candidate theoretical models will be examined and assessed based on their ability to capture the empirical moments of key variables, especially output, prices, and inflation. The first model is Freeman and Huffman’s (1991) business cycle model with endogenous money supply. Freeman and Kydland (2000) propose a more explicit model based on Freeman and Huffman (1991). Henriksen and Kydland (2010) expand the model of Freeman and Kydland (2000) by allowing for curvature with respect to the size of goods in the consumption argument of households’ utility function. The model in Chapter 5 is thus directly attributed to Freeman and Kydland (2000) and Henriksen and Kydland (2010).

Freeman and Kydland (2000) and Henriksen and Kydland (2010) only consider two monetary policy rules, including a constant money growth rate and serially correlated money growth rates. However, in reality central banks respond to real activity. For example, Eichenbaum and Singleton (1986) provide empirical evidence that the monetary policy rule involves response to both lagged money growth and output. Gavin and Kydland (1999) consider an endogenous monetary policy rule in which the growth rate of fiat money responds to lagged output and lagged money stock. Hence, the monetary
policy in Freeman and Kydland (2000) will be endogenized by allowing money growth rate to change in response to deviations in output and the money stock. The augmented model will be evaluated by examining the cyclical properties of output, the price level, and the inflation rate.

The other candidate model is a sticky-price dynamic stochastic general equilibrium (DSGE) model specified by Ireland (2003). According to Woodford (2003), the stickiness of nominal prices plays an important role in the equilibrium allocation of resources. With the fact that commodity prices change infrequently, it is worthwhile to investigate the role of nominal price rigidity in the cyclical behavior of the price level and the inflation rate.
From Chapter 1, the H-P filter has been widely used to extract a smooth trend from the natural logarithm of a certain economic series, and the residual is defined as the cyclical component. The idea is that the trend is more sensitive to long-term fluctuations than to short-term fluctuations. The sensitivity of the trend to short-term fluctuations is controlled by a smoothing parameter $\lambda$. Let $y_i$ denote the natural logarithm of a time series (e.g., real GDP), then

$$y_i = y_i^T + y_i^c$$ for $t = 1, 2, \ldots, T$$

(4.1)

where $y_i^T$ is the trend component and $y_i^c$ is the cyclical component of the observable $y_i$. $T$ is the sample size. The trend component $y_i^T$ is the solution of the following minimization problem

$$\min_{y^T} \left\{ \sum_{t=1}^{T} (y_t - y_t^T)^2 + \lambda \sum_{t=2}^{T-1} \left[ (y_{t+1}^c - y_t^c) - (y_t^c - y_{t-1}^c) \right]^2 \right\}$$

(4.2)

Hodrick and Prescott (1981) suggest that $\lambda = 1600$ for quarterly data.

4.1 Business Cycle Facts

4.1.1 Cyclical Volatilities

The first component of business cycle regularities involves the amplitude of cyclical fluctuations of each economic series examined. For the inflation rate, HP-filtered levels of the inflation rate ($\pi$) are used to characterize its cyclical properties. I choose to follow
Haslag and Hsu (2012) rather than Chadha and Prasad (1993, 1994) and Kim (1996), who use raw levels (the natural logarithm) of the inflation rate, for two reasons. First, the levels of inflation in the sample have a unit root at 1% significance level based on the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test and are only marginally stationary at 5% significance level based on the augmented Dickey–Fuller (ADF) test. By contrast, HP-filtered inflation is stationary at 1% significance level based on both KPSS and ADF tests. Investigation of the cyclical co-movements between macroeconomic time series is meaningful only when these series are stationary, so HP-filtered levels of inflation seem to be a better measure of inflation if one wants to compute the inflation-output correlation at business cycle frequencies.

The second reason involves a comparison of the contemporaneous correlation coefficients between HP-filtered real GDP and two different measures of inflation. For the sample examined, the contemporaneous correlation coefficient between HP-filtered output and HP-filtered inflation is 0.1875, which is significant at 1% significance level. However, the contemporaneous correlation between HP-filtered output and the levels of inflation is only 0.1119, which is significant at 10% level. To document the cyclical co-movements between inflation and output, this paper chooses HP-filtered levels of inflation which is more closely related to the overall business cycle.

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37 The null hypothesis in the KPSS test is trend-stationarity and the null hypothesis in the ADF test is unit root. The associated p-value in the KPSS test is 0.010 and the associated p-value in the ADF test is 0.048.
38 The associated p-value is 0.100 in the KPSS test and 0.001 in the ADF test.
Table 4.1 Standard Deviations of Cyclical Components: 1959:Q1 – 2013:Q3

<table>
<thead>
<tr>
<th>Variable</th>
<th>(y)</th>
<th>(p)</th>
<th>(\pi)</th>
<th>MB</th>
<th>M1</th>
<th>M2</th>
<th>MM1</th>
<th>MM2</th>
<th>(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>1.53</td>
<td>0.78</td>
<td>0.27</td>
<td>5.63</td>
<td>2.53</td>
<td>1.35</td>
<td>5.19</td>
<td>5.62</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Notes: \(y\) = Real GDP; \(p\) = the GDP deflator; \(\pi\) = gross quarterly inflation rate computed from the GDP deflator; MB= the Monetary Base; M1= the M1 money stock; M2= the M2 money stock; MM1= the Money Multiplier 1 defined as the ratio between M1 and MB; MM2= the Money Multiplier 2 defined as the ratio between M2 and MB; \(i\) = the 3-Month Treasury bill (TB) rate; the cyclical components are obtained by applying the H-P filter with a smoothing parameter of 1600 to the natural logarithms of corresponding variables; all cyclical components are measured in percentage terms; the figures represent the standard deviations of the cyclical components of corresponding variables.

Table 4.1 displays the standard deviations of HP-filtered levels of real GDP \((y)\), the GDP deflator \((p)\), the inflation rate \((\pi)\), base money \((MB)\), two monetary aggregates \((M1\) and \(M2)\), two corresponding money multipliers \((MM1\) and \(MM2)\), and the 3-month TB rate \((i)\). MB, MM1, and MM2 are the most volatile, with standard deviations over 5.15%, indicating that the average heights of the cyclical components of base monetary and two money multipliers are close to or over 5.2% at business cycle frequencies. The M1 money stock is relatively less volatile, with a standard deviation of 2.53% which is less than a half of 5.2%. Hence, the average height of the cyclical component of M1 is 2.53%, which is less than a half of the average height of the cyclical component of MB at business cycle frequencies. \(^{39}\) Output and M2 are relatively stable, with standard deviations of 1.53% and 1.35%, respectively. \(^{40}\)

\(^{39}\) With M1 as the product of Monetary Multiplier 1 (MM1) and base money (MB), the evidence suggests that movements in base money and the multiplier are frequently in opposite directions.

\(^{40}\) Similar to the previous footnote, the evidence suggests that movements in base money and the multiplier
From Table 4.1, the price level is even more stable, with a standard deviation (0.78%) about only a half of the standard deviation of output (1.53%). Put differently, the average height of the cyclical component of the price level is about only a half of the average height of the cyclical component of output at business cycle frequencies. The most stable variables are the cyclical components of the inflation rate and the short-term interest rate, whose standard deviations are less than 0.30%. This table indicates that monetary measures are the most volatile in the sense of having the largest standard deviations. Interestingly, the evidence confirms that the quantity theory does not hold at business cycle frequencies since the volatile money supply does not result in volatile price level or inflation.

4.1.2 Contemporaneous Cross-correlations with Output

The second component of business cycle facts measures the cyclical co-movements of key economic aggregates with output. Table 4.2 reports contemporaneous correlation coefficient between output and each of other variables studied. Except for the M1 money stock, all other variables are significantly correlated with output at 1% level. Specifically, the price level is countercyclical, with a contemporaneous correlation coefficient of -0.48. The inflation rate is procyclical, with a contemporaneous correlation of 0.19.

The MB-output correlation is -0.29, indicating countercyclical behavior of base money. Despite the fact that base money and output move contemporaneously in opposite directions, the M2 money stock moves in the same direction as output, with a
contemporaneous M2-output correlation of 0.23. Two money multipliers are procyclical since the contemporaneous MM1-output and MM2-output correlations are both 0.34. However, the M1 money stock is acyclical because the corresponding correlation (0.07) is insignificant at any conventional significance levels.

Table 4.2 Contemporaneous Correlations with Output: 1959:Q1 – 2013:Q3

<table>
<thead>
<tr>
<th>Variable</th>
<th>$p$</th>
<th>$\pi$</th>
<th>MB</th>
<th>M1</th>
<th>M2</th>
<th>MM1</th>
<th>MM2</th>
<th>$i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr. w/ $y$</td>
<td>-0.48***</td>
<td>0.19***</td>
<td>-0.29***</td>
<td>0.07</td>
<td>0.23***</td>
<td>0.34***</td>
<td>0.34***</td>
<td>0.43***</td>
</tr>
</tbody>
</table>

*Notes: See Table 4.1 for notations; the figures show the contemporaneous correlation coefficients between the cyclical components of output and the corresponding variable; *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.*

Countercyclical base money and acyclical M1 money stock cast doubt on the assertion among some researchers that monetary shocks cause output fluctuations. On the other hand, procyclical M2 money stock lends support to the idea of a positive correlation between money supply and output. Finally, the short-term nominal interest rate ($i$) is procyclical, with a contemporaneous correlation of 0.43. In short, the price level and base money are countercyclical; inflation, the M2 money stock, the money multipliers, and the nominal interest rate are procyclical; and the M1 money stock is acyclical.

4.1.3 Lead-lag Relationships with Output

As the third measure of business cycle regularities, the lead-lag relationship between a certain variable and the overall cycle is detected by estimating the correlation sequence at leads and lags between this variable and output. Figure 4.1 displays the correlation coefficients between the cyclical components of output and other variables at various
leads and lags up to five. Put differently, this figure plots the correlations between date-\(t\) output and date-\(t+j\) values of other variables against integers \(j\), where \(j \in [-5, 5]\). These correlations present the degree of cyclical co-movements of relevant series with output. The most important function of the correlation sequences is to display the lead-lag relationships for pairs of economic series considered.

![Cyclical Properties of the Price Level](image1)

![Cyclical Properties of Inflation](image2)

![Cyclical Properties of Base Money](image3)

![Cyclical Properties of M1](image4)

![Cyclical Properties of M2](image5)

![Cyclical Properties of MM1](image6)

![Cyclical Properties of MM2](image7)

![Cyclical Properties of 3-M TB Rate](image8)

Figure 4.1 Lead-lag Relationships with Output: 1959:Q1 – 2013:Q3

The upper-left panel shows the correlations between date-\(t\) output and date-\(t+j\) price level. Setting \(j = 0\) yields the contemporaneous price-output correlation, which is close to \(-0.5\). Furthermore, the countercyclicality of the price level is the strongest when \(j\) is about \(-2\), suggesting that the degree of cyclical co-movement between date-\(t\) output
and date - t − 2 price level is the highest. Hence, the price level tends to lead the overall cycle by approximately two quarters or half a year. 41 As a rule, there exists a phase shift of a certain variable relative to output only if the peak or trough of the correlation sequence occurs somewhere other than j = 0. Moreover, the given variable leads the overall cycle if the peak or trough occurs when j < 0. By contrast, the given variable lags the overall cycle if the peak or trough occurs when j > 0.

From the upper-middle panel, the contemporaneous correlation between HP-filtered inflation and output is positive, confirming the procyclical behavior of inflation. Moreover, the correlation sequence peaks when j is around 3. Put differently, inflation lags the overall cycle by three quarters.

How do we interpret such a result? It is tempting to translate the countercyclical behavior of prices into the countercyclical behavior of inflation. However, inflation measures the rate of change in the price level over time, thus the cyclical properties of inflation depend on the relative co-movement of the cycle with date - t price level and date - t − 1 price level. Formally, since all variables are measured in logarithm, it follows that

\[
\text{cov}[\pi(t), y(t)] = \text{cov}[p(t) - p(t-1), y(t)] = \text{cov}[p(t), y(t)] - \text{cov}[p(t-1), y(t)].
\]

The sign of a correlation is the same as the sign of the corresponding covariance. Thus, the sign of the contemporaneous inflation-output correlation actually depends on the size of the covariance between output and the price level relative to the size of the covariance between output and the first lag of the price level. Since both covariances are negative,

41 The fact that the price level leads the overall business cycle by around half a year has implications for price stickiness, which will be explained in more detail in Chapter 7.

42 See Ross (2004, Lemma 4.7.1 on P. 122). See Appendix A for the content and proof of this lemma.
inflation is procyclical if the lagged price series is more countercyclical compared to the current price series.\textsuperscript{43} This point is reflected in the upper-left panel, where the degree of countercyclicality of the price level is higher when \( j = -1 \) (and even higher when \( j = -2 \)).\textsuperscript{44}

From the upper-right panel, there is no evidence that the monetary base (\( MB \)) leads the overall cycle. Base money is generally countercyclical and slightly lags the overall cycle at best, which seems counterintuitive, especially for researchers advocating a causal relation from the money supply to real activity. Such a result becomes easier to interpret if HP-filtered base money is contrasted with HP-filtered output, which is illustrated in Figure 4.2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart.png}
\caption{Cyclical Co-movement of Base Money with Output: 1959:Q1 – 2013:Q3}
\end{figure}

\textsuperscript{43} \( \text{Corr}(X,Y) = \frac{\text{cov}(X,Y)}{(\sigma_x \sigma_y)} \Rightarrow \text{cov}(X,Y) = \text{Corr}(X,Y)(\sigma_x \sigma_y) \). The standard deviations of date-\( t \) and date-\( t - 1 \) price level are almost the same, so multiplying each correlation by the product of corresponding standard deviations does not alter the relative sizes of the covariances.

\textsuperscript{44} Only \( j = -1 \) matters here because the inflation rate is the log difference of the price level and its first lag. As will be conjectured in Chapter 7, the price-output correlation is the strongest when \( j = -2 \) because the adjustment of prices to the market-clearing level takes about two quarters.
Figure 4.2 plots the cyclical components of the two economic aggregates from 1959 to 2013, along with eight recessions over this period. This figure indicates that base money and output became inversely related over some periods since 2002. Particularly, negative MB-output correlation is mainly due to the countercyclical movement of base money since 2006. Specifically, base money dropped sharply since 2006, when overall business activity was slightly above the trend. Base money then rose considerably during the financial crisis since 2008, which is a significant recession in the history of U.S. economy. In fact, there is a “structural” break in the cyclical behavior of base money at around 1979, which is confirmed by examining the MB-output correlation in two overlapping sample periods: 1959 – 1979 and 1959 – 2013. The contemporaneous MB-output correlation coefficients for the two sample periods are 0.4271 and -0.2852, respectively. Clearly, base money is procyclical before 1979 and becomes countercyclical afterwards, resulting in the countercyclical behavior of base money over the whole sample period 1959 – 2013.

The middle-left and center panels of Figure 4.1 display the lead-lag relationships between output and two monetary aggregates: the M1 money stock and the M2 money stock. Although the M1 money stock is uncorrelated with output contemporaneously, the correlations between output and lags of M1 are significantly positive. In addition, when $j = -4$ M1-output correlation sequence peaks at 0.20, which is significant at 1% level. Hence, the M1 money stock tends to lead output by four quarters. From the center panel,

45 The corresponding p-values for the contemporaneous MB-output correlation coefficients are 0.0001 and 0.0000 over the period 1959 – 1979 and the period 1959 – 2013, respectively.
the M2 money stock is positively correlated with output contemporaneously (when $j = 0$) and the degree of the M2-output correlation is the highest when $j = -2$. Thus, the M2 money stock is procyclical and leads the overall cycle by about two quarters. These observations partially confirm the idea that money stock might be a determinant of output fluctuations. The procyclical behavior of the two monetary aggregates ($M_1$ and $M_2$) seems to contradict the countercyclical behavior of base money ($MB$). To reconcile such a contradiction, I will look at the multipliers in particular.

The middle-right and lower-left panels of Figure 4.1 present the cyclical co-movements of the two money multipliers ($MM_1$ and $MM_2$) with output. Both money multipliers are procyclical and coincide with the overall cycle. These findings confirm an empirical pattern in the literature, namely monetary measures are related to output primarily in the form of the money multiplier, rather than monetary aggregates (e.g., $M_1$, $M_2$).\textsuperscript{46,47,48} The last panel suggests moderately procyclical behavior of the short-term nominal interest rate. The corresponding contemporaneous correlation is approximately 0.5. Also, the correlation sequence between the cyclical components of the 3-month TB rate and output peaks at 0.52 when $j$ is about 2, indicating that the nominal interest rate lags the overall cycle by approximately two quarters.

\textsuperscript{46} See Cagan (1965), King and Plosser (1984), and Champ, Freeman, Haslag (2011, p. 167).
\textsuperscript{47} Although contemporaneous M2-output correlation (0.23) is significant, the degree of such a correlation is lower than the degree of contemporaneous MM2-output correlation (0.34). That is, MM2 is more closely correlated with output than is the M2 money stock.
\textsuperscript{48} Such evidence motivates why a model like the one specified by Freeman and Huffman (1991) with endogenous money multiplier is a good starting point.
4.2 Autocorrelation Functions (ACFs)

As an important component of time-series properties, autocorrelation functions (ACFs) exhibit the persistence evident in relevant time series. Formally, an autocorrelation function shows the similarity between observations as a function of the time lag between them. Using annual data over 1909 – 1970, Nelson and Plosser (1982) find that the sample autocorrelations of first-differenced real GNP, nominal GNP, and real per-capita GNP are significantly positive at lag one but insignificant at higher lags. In addition, they find that the first differences of price indexes (including the GNP deflator and consumer prices) display more persistent positive autocorrelation. Nelson and Plosser (1982) also examine the sample ACFs of linearly detrended series and report that most sample autocorrelations start at around 0.9, followed by an approximately exponential decline.\footnote{The results of Nelson and Plosser (1982) illustrate that different detrending methods yield distinct sample ACFs of the same time series. This is intuitive since different detrending methods imply different cyclical filters which extract different components from the same time series (see Canova 2007, Chapter 3). This also explains the difference between the sample ACFs of HP-filtered variables I obtained and the sample ACFs of the correspondingly variables detrended by other methods in the literature.}

Cogley and Nason (1995) replicate such empirical evidence using quarterly real per-capita GNP growth from 1954:Q1 to 1988:Q4. In particular, Cogley and Nason report that output growth is positively and significantly autocorrelated at the first two lags and weakly negatively autocorrelated at higher lags.

Figure 4.3 plots sample ACFs of the cyclical components of output \( (y) \), the price level \( (p) \), the inflation rate \( (\pi) \), base money \( (MB) \), monetary aggregates \( (M1 \text{and} M2) \), the money multipliers \( (MM1 \text{and} MM2) \), and the short-term nominal interest rate \( (i) \) for the
United States from 1959:Q1 to 2013:Q3. The two solid lines parallel with the x-axis in each panel corresponds to the approximate upper and lower confidence bounds for the corresponding sample ACF. This figure suggests that most sample autocorrelations start at about 0.9, except that for the inflation rate, whose sample autocorrelations start at about 0.5.\(^{50}\)

---

\(^{50}\) The sample autocorrelation at lag zero is not considered here since its value is always one by construction.
From the upper-left panel, the sample autocorrelations in HP-filtered output are significantly positive at the first four lags, which is similar to the first difference case of Nelson and Plosser (1982) and Cogley and Nason (1995), although here the autocorrelations start at a much higher value. At higher lags, the autocorrelations are generally negative, which is different from the first difference case in Nelson and Plosser (1982) or Cogley and Nason (1995), where the autocorrelations are essentially zero at lags over one year. The sample ACF of HP-filtered output here is quite different from the linearly detrending case of Nelson and Plosser (1982), where the sample ACF of output is significantly positive at the first five lags, corresponding to five years with annual data used. As noted in Footnote 49, the differences in the sample autocorrelations of output between my results and others cited are mainly due to the fact that different detrending methods are used.

The upper-middle panel indicates that the sample autocorrelations in HP-filtered price level are significantly positive at the first five lags and negative at higher lags. Again, this is different from the sample ACFs of price indexes in Nelson and Plosser (1982), where the sample ACFs of first differenced GNP deflator and consumer prices are significantly positive for the first two lags (or two years with annual data used) and essentially zero for higher lags. The upper-right panel suggests that HP-filtered levels of inflation rate are positively autocorrelated at the first four lags and negatively autocorrelated for higher lags up to 13. Unlike other variables which are highly persistent, the inflation rate is less persistent since its autocorrelation at the first lag is only around 0.5.
The middle panel plots the sample autocorrelation functions of the cyclical components of monetary measures including base money and two monetary aggregates. From the middle-left panel, the sample autocorrelations in base money are significantly positive at the first three lags and generally negative at higher lags. The center panel suggests that the sample autocorrelations in the M1 money stock are significantly positive at the first five lags. At higher lags, the autocorrelations are moderately negative. From the middle-right panel, the M2 money stock is positively autocorrelated at the first four lags and negatively autocorrelated for higher lags up to 13. In contrast, first differenced money stock is positively autocorrelated over the first three years and essentially uncorrelated over longer horizons in Nelson and Plosser (1982). For linearly detrended money stock in Nelson and Plosser (1982), although the sample ACF starts at 0.95 (as in this paper), it decays fairly slowly, which is again different from the sample ACF of HP-filtered money stock I obtain.

The lower panel displays the sample ACFs of HP-filtered levels of two money multipliers (MM1 and MM2) and the short-term nominal interest rate (i). Both money multipliers and the nominal interest rate are positively autocorrelated at the first four lags and negatively autocorrelated at higher lags. In contrast, Nelson and Plosser (1982) report that the bond yield (as a measure of the nominal interest rate) displays more persistent autocorrelations in first difference and linearly detrended levels. As noted previously, the difference in the sample ACF of nominal interest rates between my results and Nelson and Plosser (1982) stems from the application of different detrending methods.

In short, Figure 4.3 shows that base money is positively autocorrelated at the first three lags. For all other variables examined, their sample autocorrelations are positive at
the first four or five lags. Another point is that the inflation rate is only moderately persistent but all other variables are highly persistent. The sample ACFs of the cyclical components of these macroeconomic aggregates, especially output, the price level, and inflation, will be compared to their counterparts generated by the models in Chapter 5 and Chapter 6 as a way to evaluate the fit of each candidate model.

4.3 Vector Autoregressions (VARs)

As introduced in the methodology part (Section 2 of Chapter 3), VARs will be built to provide more insights into the debate on the role of money (and interest rates) in the determination of output. Specifically, the counterparts of Sims’ (1980b) VARs will be constructed to assess the sensitivity of Sims’ results to changes in the measure of the same variable, the specification and frequency of data, the lag structure, and the time period. Another purpose of building VARs is to investigate the Granger-causal relationship between the price level and output at business cycle frequencies.

The simplest VAR includes HP-filtered levels of output and the price level. To check the robustness of the observed Granger-causality between the price level and output in the bivariate VAR, each of other variables (base money, monetary aggregates, and the nominal interest rate) will be added to build trivariate VARs. And then four-variable VARs will be built including the price level, output, and two other variables. Finally, two five-variable VARs will be built using data on base money, MM1 or MM2, output, the nominal interest rate, and the price level.

4.3.1 Robustness Check for Sims (1980b)

This subsection compares the results from two VARs using HP-filtered levels of quarterly
data from 1959:Q1 to 2013:Q3 with the results from corresponding VARs in Sims (1980b) using the growth rates of monthly data from 1948 to 1978. The aim is to check the robustness of findings reported by Sims (1980b) to alternative measures of the same variable, alternative specifications (and frequencies) of data, alternative lag structures, and alternative time periods. Table 1 in Sims (1980b) presents forecast error variance decomposition (FEVD) estimates of a trivariate VAR including the growth rates of M1, IP, and WPI. The Granger-causality between prices and output will be emphasized since this paper focuses on the cyclical behavior of prices (and inflation).

Table 4.3 48-Month/16-Quarter FEVD of Three-variable VARs

<table>
<thead>
<tr>
<th>Sims (1980b, Table 1) - monthly</th>
<th>My Results - quarterly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable Explained</td>
<td>By Innovations in</td>
</tr>
<tr>
<td></td>
<td>M1  IP  WPI</td>
</tr>
<tr>
<td>M1</td>
<td>97   2   1</td>
</tr>
<tr>
<td>IP</td>
<td>37   44  18</td>
</tr>
<tr>
<td>WPI</td>
<td>14   7   80</td>
</tr>
</tbody>
</table>

Notes: M1 = the M1 Money Stock; IP = Industrial Production; WPI = Wholesale Price Index; y = real GDP; p = the GDP deflator; output is measured by IP in Sims (1980b) and by real GDP in this paper; the price level is measured by WPI in Sims (1980b) and by the GDP deflator in this paper; Sims (1980b) uses 12 lags for all VARs considered, while this paper chooses an optimal lag length (which is 6 here) selected by the LR test; all figures are shown in percentage.

46

Table 4.3 compares Table 1 of Sims (1980b) with FEVD estimates of the

51 Forecast error variance decomposition (FEVD) indicates the percentage of the forecast error variance of each variable that can be accounted for by exogenous shocks to each of other variables in a VAR. FEVD shows the amount of information that each variable contributes to the other variables in the VAR.
corresponding trivariate VAR constructed here. The left panel copies postwar results from Table 1 of Sims (1980b) using monthly data over 1948 – 1978 and the right panel reports the FEVD estimates I obtained using quarterly data from 1959 to 2013. The forecast error variance of the M1 money stock (M1) can be explained in a similar way in both panels, where exogenous shocks to M1 account for no less than 90% of its own forecast error variance, while exogenous shocks to output or to the price level explain no more than 8% of forecast error variance of M1.

The determination of output fluctuations I obtained is partially similar to Sims (1980b), where about one half of output forecast error variance is attributed to its own innovations. However, the other half of output forecast error variance is explained differently in the two panels. In Sims (1980b, Table 1), the percentages of 48-month forecast error variance of output attributable to monetary and price shocks are 37% and 18%, respectively. However, in my trivariate VAR the fractions of 16-quarter forecast error variance of output explained by exogenous shocks to M1 and the GDP deflator (p) are 16% and 26%, respectively. That is, in Sims (1980b, Table 1) the M1 money stock (M1) is more important than the wholesale price index (WPI) in output fluctuations, whereas according to my results, the GDP deflator (p) plays a more important role than does the M1 money stock (M1) in the determination of output.

Finally, the determination of the forecast error variance of the price level is distinct

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52 Sims (1980b) considers both the interwar (1920 – 1941) and the postwar (1948 – 1978) periods. Only the results for the postwar period will be compared since the literature suggests a change in the cyclical behavior of the price level across WWII.
from the two panels of Table 4.3. In the left panel, price innovations contribute to 80% of its own variance, while exogenous shocks to the M1 money stock (M1) and output (IP) explain only 14% and 7% of the 48-month forecast error variance of WPI, respectively. By contrast, my results suggest that only 40% of the 16-quarter forecast error variance of the GDP deflator is attributed to its own innovations, while over one half of the variance can be accounted for by exogenous shocks to real GDP (y), suggesting a crucial role of output in the determination of price fluctuations. The M1 money stock plays a minor role in the determination of the price level, with a percentage of only 9%.

Table 4.4 48-Month/16-Quarter FEVD of Four-variable VARs

<table>
<thead>
<tr>
<th>Variable Explained</th>
<th>Sims (1980b, Table 2) - monthly</th>
<th>My Results - quarterly</th>
</tr>
</thead>
<tbody>
<tr>
<td>By Innovations in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>50</td>
<td>i 61</td>
</tr>
<tr>
<td>M1</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>WPI</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>IP</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>By Innovations in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>61</td>
<td>18</td>
</tr>
<tr>
<td>p</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>y</td>
<td></td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: See Table 4.3 for notations; R = the rate on 4-6 month prime commercial paper; i = the 3-month TB rate; the nominal interest rate is measure by the yield on 4-6 month price commercial paper in Sims (1980b) and by the 3-month TB rate in this paper; Sims (1980b) uses 12 lags for all VARs considered, while this paper chooses an optimal lag length (which is 13 here) by the LR test; all figures are shown in percentage.

Table 4.4 contrasts 48-month FEVD estimates of a four-variable VAR of Sims (1980b, Table 2) to 16-quarter FEVD estimates of a comparable VAR I obtained. The left panel displays Sims’ postwar results using monthly data from 1948 to 1978 and the right panel exhibits my results using monthly data from 1959 to 2013. The determination of nominal interest rate fluctuations is similar in both panels, where one half or more of the
forecast error variance of the interest rate is explained by its own innovations, followed by exogenous shocks to the money stock and output (between 17% and 28%), and then price innovations (4%). Such a result indicates that the nominal interest rate is exogenously in the sense that its own innovations account for one half or more of its own forecast error variance.

The FEVD of the M1 money stock is different in the two panels. Specifically, in Sims (1980b, Table 2) over one half of the forecast error variance in the M1 money stock is attributed to interest rate innovations, suggesting a major role of interest shocks in the movements of the money stock. Moreover, monetary shocks account for 42% of its own forecast error variance, which suggests that forecast error variance of M1 is jointly explained by exogenous shocks to the nominal interest rate and the money supply. However, my results indicate that interest shocks play a minor role in the forecast error variance of the M1 money stock, with a percentage of only 18%. By contrast, monetary innovations explain over 70% of its own forecast error variance. Hence, in my four-variable VAR the M1 money stock is exogenous in the sense that over 72% of its forecast error variance is attributable to its own innovations. In both panels, price and output innovations play a minor role in the determination of money fluctuations.

The determination of the price level is somewhat different in the two panels. Specifically, in Table 2 of Sims (1980b) 60% of the forecast error variance of the wholesale price index (WPI) is attributed to its own innovations, and the corresponding fractions for exogenous shocks to M1, output, and the interest rate are 32%, 6%, and 2%, respectively. Similarly, based on my results price shocks explain over 70% of its own forecast error variance. However, the percentages the FEVD in the price level fluctuations
attributable to M1, output, and the interest rate are 7%, 9%, and 12%, respectively. In short, in Sims (1980b, Table) monetary innovations account for the second-highest proportion (32%) of the FEVD in the price level fluctuations; while from my results the M1 money stock plays a minor role relative to the nominal interest rate in the determination of the price level.

Finally, the two panels suggest different forecast error variance decompositions of output. In the left panel over one half of the output variation is due to its own innovations, while in the right panel the corresponding percentage is only 24%. Interest rate shocks contribute to 30% of output fluctuations in Table 2 of Sims (1980b), whereas according to my results the corresponding fraction is 44%, suggesting that the nominal interest rate is a major determinant of output fluctuations. In addition, my results indicate that price shocks explain twice the fraction of output fluctuations (28%) compared to the left panel. In short, this paper corroborates Sims’ findings that the nominal interest rate plays a much more important role than does the M1 money stock in the determination of output.

**4.3.2 Granger-causal Relationship between Prices and Output**

Table 4.5 reports P-values associated with Granger-causality tests between prices and output for all VARs considered. The first column displays variables included in corresponding VARs, which are built upon a bivariate VAR including the price level and output. The second column presents the optimal lag length of each VAR selected by the likelihood ratio (LR) test. This column indicates that simple models have shorter lags in general and more complex models typically have much longer lags. The last two columns show the P-values associated with the F-test that the price level Granger-causes output, and the F-test that output Granger-causes the price level, respectively.
Table 4.5 Granger-causality Tests between Prices and Output for Various VARs

<table>
<thead>
<tr>
<th>Variables Included in VAR</th>
<th>Optimal Lag Length</th>
<th>P-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y, p$</td>
<td>5</td>
<td>0.005***</td>
</tr>
<tr>
<td>MB,$y, p$</td>
<td>5</td>
<td>0.001***</td>
</tr>
<tr>
<td>M1,$y, p$</td>
<td>6</td>
<td>0.003***</td>
</tr>
<tr>
<td>M2,$y, p$</td>
<td>4</td>
<td>0.005***</td>
</tr>
<tr>
<td>$i, y, p$</td>
<td>13</td>
<td>0.007***</td>
</tr>
<tr>
<td>MB,MM1,$y, p$</td>
<td>12</td>
<td>0.054*</td>
</tr>
<tr>
<td>MB,MM2,$y, p$</td>
<td>5</td>
<td>0.005***</td>
</tr>
<tr>
<td>$i, M1, p, y$</td>
<td>13</td>
<td>0.017**</td>
</tr>
<tr>
<td>$i, M2, p, y$</td>
<td>16</td>
<td>0.005***</td>
</tr>
<tr>
<td>MB,MM1,$y, i, p$</td>
<td>16</td>
<td>0.144</td>
</tr>
<tr>
<td>MB,MM2,$y, i, p$</td>
<td>16</td>
<td>0.026**</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; the sample period is 1959:Q1 – 2013:Q3 for the U.S.; the optimal lag length for each VAR is determined by the likelihood ratio (LR) tests where the maximum lag order is set to 20; the third VAR corresponds to the one in Sims (1980b, Table 1); the eighth VAR corresponds to the one in Sims (1980b, Table 2); *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

If the predetermined significance level is 5%, the price level Granger-causes output in 9/11 of the models considered, while output Granger-causes the price level in only one of the VARs. Setting the significance level at 10%, prices Granger-cause output in 10/11 of the models involved, whereas output Granger-causes prices in only 2/11 of the VARs. These results suggest a stronger Granger-causality from prices to output and a much weaker Granger-causality from output to prices. The Granger-causality from prices to output is robust to most VAR specifications, except for the tenth VAR including base money (MB), Money Multiplier 1 (MM1), output ($y$), the nominal interest rate ($i$), and the price level ($p$). Such a finding illustrates the findings in Section 1.3 of Chapter 4 that the
price level leads or temporally precedes the overall cycle, which is defined by output.\textsuperscript{53}

Another interesting point is that the Granger-causality between the price level and output becomes two-way at 5\% significant level in the ninth VAR including \( i, M_2, p, \) and \( y \). In addition to the ninth VAR, two-way Granger-causality between the price level and output becomes significant at 10\% level in the last VAR including \( MB, MM_2, y, i, \) and \( p \). It is worth noting that the price level could Granger-cause output even if output does not Granger-cause the price level, whereas output Granger-causes the price level only when the price level Granger-causes output at a higher significance level.\textsuperscript{54}

Specifically, in the simplest bivariate VAR, the price level \( (p) \) Granger-causes output \( (y) \) at 1\% significance level, while output does not Granger-cause the price level. Such a result remains for all trivariate VARs where base money \( (MB) \), or one of the two monetary aggregates \( (M_1, M_2) \), or the nominal interest rate \( (i) \) is added into the simplest bivariate VAR consisting of output and the price level. When both base money and Money Multiplier 1 \( (MM_1) \) is added into the bivariate VAR, the Granger-causal relationship from the price level to output becomes significant at only 10\% level, while the Granger-causal relationship from output to the price level remains insignificant.

The Granger-causality from prices to output becomes significant at 5\% level if base

\textsuperscript{53} The finding also has implications for the existence of price stickiness in real economy, where the sluggish response of nominal prices to exogenous shocks affects resource allocation. Hence nominal prices temporally precede and Granger-cause output, reflecting the impact of price stickiness on resource allocation.

\textsuperscript{54} See the ninth VAR including \( i, M_2, p, \) and \( y \) and the last VAR including \( MB, MM_2, y, i, \) and \( p \).
money (MB) and Money Multiplier 2 (MM2), or the short-term nominal interest rate (i) and one of the two monetary aggregates are included into the baseline VAR composed of output and the price level. As mentioned earlier, output also Granger-causes the price level at 5% level in the four-variable VAR including the nominal interest rate, the M2 money stock, the price level, and output. Put differently, the Granger-causality between the price level and output becomes two-way in the four-variable VAR consisting of the nominal interest rate, the M2 money stock, the price level, and output.

Finally, the Granger-causality between the price level and output vanishes in a five-variable VAR including base money, Money Multiplier 1, output, the nominal interest rate, and the price level. That is, in the five-variable VAR considered, the price level does not Granger-cause output (the corresponding P-value is 0.144), output does not Granger-cause the price level (the corresponding P-value is 0.208), either. However, the Granger-causality between prices and output becomes two-way, at least at 10% significance level, when the five-variable VAR includes Money Multiplier 2, instead of Money Multiplier 1. In short, Table 4.5 suggests a much stronger Granger-causality from the price level to output than that from output to the price level.
CHAPTER 5
A BUSINESS CYCLE MODEL WITH ENDOGENOUS MONEY SUPPLY

To account for observed cyclical behavior of the price level and the inflation rate, this chapter applies and extends Freeman and Huffman’s (1991) model with endogenous money supply as the first candidate model. Freeman and Huffman (1991) build a general equilibrium model based on the observations that observed Granger-causality from nominal money stock to output is attributed to inside, not outside, money, and that fiat money stock does not affect real activity. In their model the money supply (the M1 aggregate) consists of both inside (deposits) and outside money (currency). The model’s money multiplier is endogenously determined by the relative rates of return of deposits and currency net of transaction costs.

Freeman and Kydland (FK) (2000) propose a more explicit model based on Freeman and Huffman (1991). Henriksen and Kydland (2010) expand the model of Freeman and Kydland by allowing for curvature with respect to the size of the goods in the consumption argument of households’ utility function. Although the essence of the first candidate model in this paper is attributed to Freeman and Huffman (1991), the direct references are Freeman and Kydland (2000) and Henriksen and Kydland (2010). The model environment will be described shortly, followed by a definition of competitive equilibrium and an analysis of the impulse responses of key economic variables. And then the model will be solved and simulated, followed by a detailed analysis of the simulation results. Finally, the baseline FK (2000) model is augmented by incorporating

55 These observations are consistent with the empirical findings reported in Section 1.3 of Chapter 4.
endogenous monetary policy rule, whose effects on the simulation results are examined.

5.1 Model Environment

In this model, time is discrete and infinitely long, which can be denoted as \( t = 0, 1, 2, \ldots \). The model economy is occupied by four types of agents, including perfectly competitive firms and commercial banks, a large number of infinitely-lived homogeneous households, and a central bank. Firms produce consumption goods and commercial banks provide financial services which channel funds from savers to lenders. Households provide capital and labor to firms and purchase consumption goods from firms.\(^{56}\) Households also make deposits at commercial banks and earn interest from their deposits. The central bank issues fiat money and transfers net revenues from printing money to households as a lump-sum transfer. Hence, the commodity space consists of labor, capital good, consumption good, and fiat money. The following subsections first introduce the four types of agents in detail, and then cover the capital market and money market.

5.1.1 Firms

At date \( t \), perfectly competitive firms rent capital \( k_t \) and hire labor \( l_t \) from households to produce output \( y_t \) according to a constant-returns-to-scale Cobb-Douglas production function:

\[
y_t = z_t f(k_t, l_t) = z_t k_t^\alpha l_t^{1-\alpha}
\]

\(^{56}\) As will be covered later in Section 1.4 of this Chapter, households can purchase consumption goods using either fiat money or bank deposits.
where $z_t$ is the technology level realized at date $t$ and $\alpha$ measures the capital’s share in output. The technology level evolves according to the following stationary AR(1) process

$$z_t = \alpha z_{t-1} + \epsilon_t \sim N(\mu_x, \sigma_x^2)$$

(5.2)

Let $\delta$ denote the depreciation rate of the capital stock, and $i_t$ denote net investment in period $t$. Then next-period capital stock is the sum of undepreciated capital and current-period net investment. Accordingly the law of motion for the capital stock is

$$k_{t+1} = (1 - \delta) k_t + i_t$$

(5.3)

### 5.1.2 The Central Bank

In period $t$, the central bank issues intrinsically useless fiat money $M_t$ at a (gross) growth rate $\mu_t$. Thus, the money supply is determined by

$$M_t = \mu_t M_{t-1}$$

(5.4)

The central bank transfers net revenues from printing fiat money to households as a lump-sum transfer $X_t$ whose aggregate nominal value is

$$X_t = M_t - M_{t-1} = (\mu_t - 1) M_{t-1}$$

(5.5)

The central bank is assumed to follow two types of monetary policies, Policy I and Policy II, in the baseline model and the augmented model, respectively. Policy I involves autocorrelated money growth rates, which is Policy C considered in Freeman and
Kydland (2000). Policy II implements a Taylor-type rule, which is a more realistic assumption about the way the central bank reacts to real activity.57

5.1.3 Commercial Banks

At date \( t \), perfectly competitive commercial banks accept deposits \( n_t \) from households and make loans to firms short of funds for investments. Commercial banks pay interest at a rate \( \bar{r}_{t+1} \) on deposits received in period \( t \) and charge interests at a rate \( r_{t+1} \) on loans made in period \( t \). Perfect competition leads to zero profit for commercial banks, but the above two interest rates are different due to the fact that commercial banks are subject to a reserve requirement. The central bank requires that commercial banks must hold a fraction \( \theta \) of the deposits as reserves.

Let \( p_t \) denote the price level at data \( t \). The return on fiat money is the ratio between the value of money in period \( t + 1 (1/p_{t+1}) \) and the value of money in period \( t (1/p_t) \). Accordingly, the gross rate of return on fiat money is

\[
\frac{1/p_{t+1}}{1/p_t} = \frac{p_t}{p_{t+1}}
\]

(5.6)

Commercial banks own two types of assets: loans issued to firms and fiat money held as reserves. On the other hand, deposits are the liability for commercial banks. Zero profit condition makes the interest rate paid to depositors \( \bar{r}_{t+1} \) equal to a linear combination of the rate of return on loans or capital \( r_{t+1} \) and the rate of return on reserves or fiat money.

---

57 A more detailed description of Policy I and Policy II will be provided in Section 3 of Chapter 5.
\[ \tilde{r}_{t+1} = (1 - \theta) r_{t+1} + \theta \left( \frac{p_t}{p_{t+1}} \right) \]  

(5.7)

### 5.1.4 Households

There are a large number of infinitely lived homogeneous households, with the population size normalized to one. Each household is endowed with an initial capital stock \( k_0 \) at date 0 and one unit of time at each date \( t \geq 0 \). Capital can be rented to firms with production technologies and time can be allocated to productive activity, leisure, and transactions. There is a continuum of consumption good types ordered by size and indexed by \( j \) with \( j \in [0, 1] \). Households derive utility from both consumption and leisure.

Let \( c_t(j) \) denote the consumption of type \( j \) goods in period \( t \), and \( \alpha_t \) denote leisure in period \( t \).

The representative household aims to maximize the expected life-time utility:

\[
E \sum_{t=0}^{\infty} \beta^t u \left\{ \min \left[ \frac{c_t(j)}{(1 - \omega) j^{-\omega}} \right], \alpha_t \right\}
\]

(5.8)

In Equation (5.8), \( \beta \in (0, 1) \) is the discount factor, and \( \omega \) measures the curvature of the argument in the "min" function. The instantaneous utility \( u \left\{ \min [c_t(j)/(1 - \omega) j^{-\omega}], \alpha_t \right\} \) is a Leontief utility function. For a given desired level of total consumption at date \( t \): \( c_t^* \), the Leontief-type instantaneous utility function suggests that the representative household sets \( c_t^* = c_t(j)/(1 - \omega) j^{-\omega} \) such that
\[
\begin{align*}
    c_j(j) = (1 - \omega)j^{-\omega}c_j;
\end{align*}
\]

Equation (5.9) stipulates how much the household should consume each type of goods for \( j \in [0, 1] \) to maximize utility.

The representative household earns income from four sources. First, the household can supply labor \( l_t \) to firms in return for wages at a real rate \( w_t \). Second, the household can invest its capital in production by renting capital to firms and obtains returns at a real rate \( \bar{r} \). Third, the representative household can deposit its money at commercial banks and earns interest at a rate \( \bar{r} \). Finally, the household can receive a lump-sum subsidy with nominal value \( \chi_t \) from the central bank at the beginning of each period.

The representative household stores its wealth in three forms: non-intermediate capital \( a_t \), nominal bank deposits \( h_t \), and fiat money \( m_t \). Both bank deposits and fiat money can be used to purchase consumption goods, but using deposits incurs a real fixed cost \( \gamma \). The household chooses between fiat money and deposits for consumption based on their relative rates of return, net of transaction costs. At the beginning of each period, the household decides its real money balances and the ratio between deposits and currency. To consume the desirable level of goods, the household replenishes its money balances \( n_t \) times at date \( t \). The replenishment of money balances incurs a cost in terms of time, with \( \varphi \) units of time each trip. Accordingly, at date \( t \) the total amount of time spent on the replenishment of money balances is \( \varphi n_t \).

Due to the fixed cost of using deposits for consumption, the rate of return on using deposits net of transaction costs approaches negative infinity for fairly small sizes of
purchase (when \( j \to 0 \)). Consequently, there exists an optimal cut-off purchasing size \( j_i^* \) such that money is a desired means of payment for any purchasing sizes below \( j_i^* \), and that deposits are preferred for any purchasing sizes above \( j_i^* \). Since the representative household replenishes its money balances \( n_t \) times in period \( t \), per-period total consumption using cash equals the product of real money balances \( m_t / p_t \) and the number of trips to banks \( n_t \). The demand for real money and real deposits can be expressed using the optimal cut-off value \( j_i^* \) in the following way

\[
\begin{align*}
    n_t \left( \frac{m_t}{p_t} \right) &= \int_0^{j_i^*} c_i(j) \, dj = \int_0^{j_i^*} (1 - \omega)^{-\omega} c_i^* \, dj = \left( j_i^* \right)^{-\omega} c_i^* \quad (5.10) \\
    n_t \left( \frac{h_t}{p_t} \right) &= \int_0^{j_i^*} c_i(j) \, dj = \int_0^{j_i^*} (1 - \omega)^{-\omega} c_i^* \, dj = \left[ 1 - \left( j_i^* \right)^{-\omega} \right] c_i^* \quad (5.11)
\end{align*}
\]

Plugging Equation (5.9) into the household’s instantaneous utility function, the maximization problem for the representative household becomes

\[
\max \sum_{t=0}^{\infty} \beta^t \left[ u(c_i^*, d_t) \right] = \max \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\nu} \left[ (c_i^*)^{\mu} (d_t)^{-\zeta} \right]^{1-\nu} \right\} \quad (5.12)
\]

Equations (5.10) and (5.11) constitute two liquidity constraints for the representative household. Two other constraints involve the budget constraint and the time constraint. The budget constraint is given by

\[
w_i l_t + r_i a_{t-1} + \bar{r}_t \left( \frac{h_{t-1}}{p_{t-1}} \right) + \frac{m_{t-1}}{p_t} + \frac{X_t}{p_t} = c_i^* + a_i + \frac{h_t}{p_t} + \frac{m_t}{p_t} + \gamma(1 - j_i^*) \quad (5.13)
\]
Since the time endowment at each date has been normalized to one, the time constraint can be written as

$$1 = l_t + d_t + \phi n_t \quad (5.14)$$

### 5.1.5 Capital Market and Money Market

In the capital market, firms need capital as an input in their production function. There are two sources of capital: non-intermediated capital from households’ investments and intermediated capital from bank loans. Therefore, the size of total capital stock at date $t + 1$ is determined by the representative household’s decision at date $t$ on how much to invest in capital and how much to deposit at banks. Mathematically,

$$k_{t+1} = a_t + (1 - \theta) \left( \frac{h_t}{p_t} \right) \quad (5.15)$$

For the money market, at date $t$ the supply of fiat money $M_t$ is controlled by the central bank, and the demand for currency stems from households and commercial banks. The reason is that households hold fiat money for transaction purposes and banks are required by the central bank to hold reserves in proportion to the amounts of deposits they accept. Another straightforward way to understand this is to apply the definition of base money, which equals the sum of currency circulating in the public and the reserves held by commercial banks. Formally,

$$M_t = m_t + \theta h_t \quad (5.16)$$
where $m_t$ is the amount of fiat money held by the representative household, $\theta$ is the required reserve ratio, and $h_t$ is the amount of deposits that commercial banks accept from the household.

At date $t$ the money supply or the M1 money stock $M_{1,t}$ is composed of currency circulating in the public $m_t$ and nominal deposits $h_t$. Let $MM_1$ denote the money multiplier which is defined as the ratio between the M1 money stock and base money. Accordingly the money supply can be expressed as the product of the monetary base $M$ and the money multiplier $MM_1$:

$$\frac{M_{1,t}}{M_t} = m_t + h_t = M_t \cdot MM_1 = M_t \left[ 1 + \frac{h_t(1-\theta)}{m_t + \theta h_t} \right]$$

(5.17)

### 5.2 Equilibrium

The model economy is characterized by six state variables: the technology level $z$, the money grow rate $\mu$, lagged price level $p_{-1}$, lagged size of non-intermediated capital $a_{-1}$, lagged nominal deposits $h_{-1}$, and lagged fiat money balances $m_{-1}$. A competitive equilibrium consists of a sequence of allocations $\{e^*, a^*, h^*, m^*, j^*, \ell^*, n^*\}$ and a sequence of prices $\{p_t, w_t, r_t\}$ such that:

I. The representative household maximizes its expected lifetime utility (5.12) subject to four constraints (5.10), (5.11), (5.13), and (5.14), taking all prices as given.

II. Each firm maximizes its profits, taking all prices as given.

III. Each commercial bank maximizes its profits, taking all prices as given.
IV. The central bank’s budget constraint (5.5) is satisfied.

V. The consumption good market, the capital market (5.15), the labor market, and the fiat money market (5.16) clear.

5.3 Empirical Strategy

The general equilibrium model in Section 1 of this chapter is solved by log-linearizing the efficiency conditions around the steady state and calibrated following Henriksen and Kydland (2010). Specifically, the share of capital in output \( \alpha \) is set equal to 0.3485, and the depreciation rate \( \delta \) is calibrated to 0.025. The autocorrelation coefficient of the technology level \( \varepsilon \) is set equal to 0.95, and the technology shock is assumed to have a standard deviation of 0.0076. The reserve requirement ratio \( \theta \) is set equal to 0.10 and the discount factor \( \beta \) is calibrated such that the net annual real rate of return on capital is 4% in steady state. For households’ utility, the parameter \( \omega \) is set equal to -1.5, the risk-aversion parameter \( \gamma \) is set equal to 2, and consumption’s share in utility function \( \zeta \) is calibrated to 0.33. Finally, transaction cost parameters \( \gamma \) and \( \phi \) are calibrated such that net annual rate of inflation is 3% and the deposit-to-currency ratio is 9. Consequently, \( \gamma \) and \( \phi \) are calibrated to 0.00595 and 0.00076, respectively.\(^{58}\)

The model in Section 1 of this chapter is simulated and evaluated under two assumptions, corresponding to the two monetary policies mentioned previously. Specifically, the baseline FK (2000) model considers Policy I and the extended model incorporates Policy II. For both monetary policies, the average growth rate of fiat money

\(^{58}\) Readers are invited to see Henriksen and Kydland (2010) for more details.
is set equal to 3% annually. Policy I adopts Policy C in FK (2000), in which money growth rate follows an AR(1) process with an autocorrelation coefficient of 0.7 and random shocks with a standard deviation of 0.2. Policy II implements a Taylor-type rule, in which the growth rate of fiat money responds to lagged output and money stock. The reason for this extension is that central banks are believed to respond to real activity in reality. For example, Eichenbaum and Singleton (1986) provide empirical evidence that the monetary policy rule involves response to both lagged money growth and output. Gavin and Kydland (1999) consider an endogenous monetary policy rule in which the growth rate of fiat money responds to lagged output and lagged money stock.

5.4 Baseline FK (2000) Model with Autocorrelated Money Growth Rates

As mentioned previously, the baseline model involves Freeman and Kydland’s (2000) model and Policy I, in which fiat money growth rate follows an AR(1) process with an autocorrelation coefficient of 0.7 and random shocks with a standard deviation of 0.2. This section first presents impulse response analysis of the baseline FK (2000) model, and then examines the cyclical properties and autocorrelation functions of key economic variables simulated by this baseline model.

5.4.1 Impulse Response Analysis

Figure 5.1 plots the reaction of key economic variables in response to a positive one-percent shock to the technology level under Policy I. In each panel, the horizontal axis represents the number of years after the productivity shock, and the vertical axis represents percent deviation from the steady state level of the corresponding variable. The upper-left panel shows that output initially rises to about 1.4% above its steady state.
level, followed by a smooth decline to around 0.4% above its steady state level in the tenth year after the shock. Such a response is straightforward by examining the production function in Equation (5.1) where the technology level serves as a multiplier of output.

Figure 5.1 Responses to a Positive 1% Technology Shock under Monetary Policy I

The upper-right panel indicates that consumption initially jumps to about 0.5% above its steady state value, and then climbs slowly and peaks at approximately 0.6% above its steady state level in around the third year after the shock. From there on, consumption declines slowly to about 0.4% in the tenth year after the shock. The rise in consumption stems from raised output, real wage, and all other returns for households. The upper panel of Figure 5.1 suggests that a technology shock has long-term impacts on physical quantities, including output and consumption.
The middle-left panel suggests that the M1 money stock initially rises to approximately 0.9% above its steady state level, followed by a linear decrease to about 0.3% above its steady state level in the tenth year after the technology shock. Without a positive shock to the monetary base, the observed response of the money supply is attributed to the response of the money multiplier MM1 to the technology shock, which is illustrated by the middle-right panel.

From the middle-right panel, the response of the money multiplier to the technology shock is almost identical to that of the money supply. The change in the money supply here is endogenous in the sense that there is no change in base money $M$, which is directly controlled by the central bank. A positive shock to the technology level alters the relative returns on various assets, leading to a change in the holdings of different assets, and thus a change in the endogenously determined money multiplier MM1.

From the lower-left sub-plot, the price level reduces smoothly and reaches a trough at around -0.5% below its steady state level in around the fifth year after the technology shock. A comparison of the responses of output and the price level suggests a negative correlation between these two variables.

How do we explain such a response of the price level? This model includes two competing effects on price movements. The first effect contributes to procyclical prices in response to a positive technology shock. Higher technology level raises the capital returns (and thus deposits) relative to the return on money, leading to more deposits and less money holdings. Hence, the money multiplier increases due to higher deposit-to-currency ratio. Larger money multiplier tends to decrease real demand for currency $m/p$, and thus raises the price level.
The second effect leads to countercyclical price level in response to a positive shock to the technology level. Higher technology level stimulates output and thus increases income and consumption. Higher level of desired consumption increases the demand for both means of payment: real deposits \( h/p \) and real money balances \( m/p \), both of which can be raised by lowering the price level. The observed response of the price level suggests that the second effect dominates the first one, resulting in observed countercyclical behavior of the price level.

5.4.2 Business Cycle Properties of Simulated Economic Variables

This subsection reports the cyclical properties of key economic variables generated by the baseline business cycle model under monetary policy I. The model is simulated for 1000 times, and all the business cycle properties reported in this section are the averages of the corresponding cyclical properties across the 1000 simulations. Table 5.1 displays the model-generated cyclical volatilities of these variables, along with their empirical counterparts. A comparison of the actual (sample) and the model-generated standard deviations provides insights into the fit of the first candidate model. In this table, the first row presents the model-generated standard deviations of HP-filtered levels of output \( (y) \), the price level \( (p) \), implied inflation rate \( (\pi) \), base money \( (MB) \), the money multiplier \( (MM1) \), and the money supply \( (M1) \). And the second row displays the standard deviations of HP-filtered levels of these economic variables for the U.S. over the period

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59 The business cycle properties include standard deviations of the cyclical components, contemporaneous correlations between the cyclical components of output and other key variables, and lead-lag relationships between the cyclical components of output and other key variables.
Table 5.1 Standard Deviations of Cyclical Components: Simulated vs. Empirical

<table>
<thead>
<tr>
<th>Variable</th>
<th>( y )</th>
<th>( P )</th>
<th>( \pi )</th>
<th>MB</th>
<th>MM1</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Data: 219 Quarters</td>
<td>1.40</td>
<td>0.84</td>
<td>0.61</td>
<td>0.56</td>
<td>0.97</td>
<td>1.15</td>
</tr>
<tr>
<td>U.S. Data: 1959:Q1-2013:Q3</td>
<td>1.53</td>
<td>0.78</td>
<td>0.27</td>
<td>5.63</td>
<td>5.19</td>
<td>2.53</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; all variables in the first row are simulated using FK’s (2000) business cycle model under monetary policy I; 219 quarters corresponds to the number of quarters over the sample period 1959:Q1 – 2013:Q3; all cyclical components are extracted by the H-P filter and are measured in percentage terms; the figures represent the standard deviations of the cyclical components of corresponding variables.

Table 5.1 suggests that the baseline model captures the cyclical volatilities of output and the price level quite well, since the standard deviations of the cyclical components of simulated output and price level are quite close to their actual counterparts. However, there exists a discrepancy between the model-generated variances (or standard deviations) and their empirical counterparts for the inflation rate and all monetary measures, including base money, the money multiplier, and the M1 money stock. Specifically, implied inflation from the model-generated price level is too volatile, with a standard deviation of 0.61%, but the standard deviation of actual inflation is only 0.27%. In addition, the model-generated base money, money multiplier, and M1 stock are too stable, with standard deviations of 0.56%, 0.97%, and 1.15%, respectively. By contrast, the standard deviations of actual monetary measures are much more volatile, with standard deviations over 5.10% for base money and the money multiplier and over 2.50% for the

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60 Remember from Section 1 of Chapter 4 that inflation is measured as gross quarterly rate of inflation.
M1 money supply.

Table 5.2 Contemporaneous Correlations with Output: Simulated vs. Empirical

<table>
<thead>
<tr>
<th>Variable</th>
<th>( P )</th>
<th>( \pi )</th>
<th>MB</th>
<th>MM1</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Data: 219 Quarters</td>
<td>-0.22***</td>
<td>-0.16**</td>
<td>-0.01</td>
<td>0.92***</td>
<td>0.77***</td>
</tr>
<tr>
<td>U.S. Data: 1959:Q1-2013:Q3</td>
<td>-0.48***</td>
<td>0.19***</td>
<td>-0.29***</td>
<td>0.34***</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; all variables in the first row are simulated using FK’s (2000) business cycle model under monetary policy I; 219 quarters corresponds to the number of quarters over the sample period 1959:Q1 – 2013:Q3; all cyclical components are extracted by the H-P filter and are measured in percentage terms; the figures show the contemporaneous correlation coefficients between the cyclical components of output and the corresponding variable; *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

Table 5.2 compares the model-generated and actual contemporaneous correlations between the cyclical components of output and other key economic variables. This table suggests that the business cycle model with endogenous money supply only captures the countercyclical behavior of the price level and the money multiplier. Specifically, simulated price level is mildly countercyclical, which is similar to the cyclical behavior of its empirical counterpart. The model-generated money multiplier is procyclical which is similar to the behavior of its actual counterpart, although to a different degree.

However, the baseline FK (2000) model fails to match the actual cyclical behavior of other three variables, including the inflation rate, base money, and the M1 money stock. Specifically, implied inflation based on simulated price level is mildly countercyclical, while inflation in the real economy is mildly procyclical. The model-generated monetary base is acyclical, while its actual counterpart is moderately countercyclical. Finally, the model-generated money supply is highly procyclical, but its
actual counterpart is acyclical contemporaneously. Concentrating on prices and inflation which are the focuses of my study, Table 5.2 suggests that the baseline FK (2000) model only generates countercyclical price level but not procyclical inflation rate.

Figure 5.2 Lead-lag Relationships with Output: Simulated vs. Empirical

Figure 5.2 contrasts the model-generated to actual (sample) lead-lag relationships between output and other key economic variables at business cycle frequencies. In addition to contemporaneous correlations, this figure also plots the correlations between
output and other variables at leads and lags up to 5 quarters. The fit of the baseline business cycle model in generating appropriate lead-lag relationships is assessed by comparing the solid line with circular markers to the dotted line with triangular markers in each panel. From the upper-left panel, simulated price level is mildly countercyclical and coincides with the overall cycle. The model-generated price level matches the cyclical behavior of its empirical counterpart in the sense that they both move countercyclically, but the simulated price level does not lead the overall cycle as its actual counterpart does.

The upper-right panel illustrates graphically that the inflation rate computed from the simulated price level is mildly countercyclical since the model-generated inflation-output correlation sequence is about -0.2 when the horizontal axis takes the value 0. Moreover, the blue dotted line with triangular markers reaches a trough at 0 on the horizontal axis, suggesting that the model-generated inflation coincides with aggregate output. Such model-generated behavior of the inflation rate does not match the behavior of its empirical counterpart, which moves procyclically and lags the overall cycle. Thus, the business cycle model with endogenous money supply fails to capture the actual cyclical behavior of the inflation rate in any sense.

The middle-left panel implies that the model-generated monetary base is acyclical across various lags and leads since the simulated MB-output correlations are nearly zero at all leads and lags considered. This is different from the empirical cyclical behavior of

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61 As mentioned in Section 1.3 of Chapter 4, the lead-lag relationship between output and another variable, say the price level, is characterized by the correlations between date-\( t \) output and date-\( t + j \) values of the price level, where \( j \in [-5, 5] \).
base money, which moves countercyclically and slightly lags the overall cycle. From the middle-right panel, simulated M1 money stock is highly procyclical and coincides with simulated aggregate output since the simulated M1-output correlation sequence peaks when the horizontal axis takes the value 0. This is different from the actual cyclical properties of the money supply, which moves acyclically.

The lower-left panel suggests that the model-generated money multiplier is highly procyclical and coincides with the overall cycle, which is similar to the behavior of its actual counterpart. The only difference between the two is the degree of procyclicality: the empirical money multiplier is only moderately procyclical whereas its model-generated counterpart is highly procyclical. The middle and lower panels imply that the first candidate fails to capture the cyclical behavior of any monetary measures, except for the money multiplier.

In short, the baseline business cycle model (Freeman and Kydland 2000) only captures a few aspects of the empirical cyclical properties of key economic variables. For contemporaneous cross-correlations with output, this model only captures the countercyclical behavior of the price level and the procyclical behavior of the money multiplier. It fails to generate procyclical behavior of the inflation rate, which is one of the focuses of this paper. For the lead-lag relationships with output, this baseline business cycle model fails to capture the phase shift of any variable with the overall cycle, which is defined by output.

5.4.3 Autocorrelation Functions (ACFs) of Simulated Economic Variables

To assess the ability of the baseline business cycle model to capture the persistence evident in the cyclical components of key economic series, this subsection plots the
model-generated autocorrelation function of each variable of interest. Each model-generated ACF is estimated by averaging autocorrelations of the corresponding variable across 1000 simulations. Figure 5.3 displays the model-generated autocorrelation functions of HP-filtered output, price level, inflation, base money, M1 money stock, and money multiplier. A comparison between Figure 5.3 and Figure 4.3 in Chapter 4 evaluates the fit of the baseline FK (2000) model in generating economic series with similar persistence or dynamics as corresponding actual economic series.

Figure 5.3 The Model-generated Autocorrelation Functions (ACFs): 219 Quarters

From the upper-left panel of Figure 5.3, the model-generated autocorrelations of output are significantly positive at the first three lags and slightly negative at higher lags. This is similar to the upper-left panel of Figure 4.3, where the empirical autocorrelations of output are significantly positive at the first four lags and negative at higher lags. The upper-middle panel suggests that the model-generated autocorrelations of prices are
significantly positive at the first three lags and mildly negative at higher lags. The empirical counterpart in Figure 4.3 is somewhat different, since the sample autocorrelations of prices are positive the first five lags and generally negative at higher lags. The upper-right panel implies that the model-generated inflation rate behaves like a random walk, since it is uncorrelated with its own lags, i.e., the corresponding autocorrelation is effectively zero at any positive lags.

The lower-left panel implies that simulated base money is positively autocorrelated at the first four lags and negatively autocorrelated at higher lags. This is similar to the ACF of actual base money where the sample autocorrelations are positive at the first three lags and generally negative at higher lags, as displayed in the middle-left panel in Figure 4.3. The lower-middle panel of Figure 5.3 indicates that the model-generated autocorrelation in the M1 money stock is positive at the first three lags and mildly negative at higher lags. This differs from the ACF of actual M1 money stock since the center panel of Figure 4.3 suggests positive sample autocorrelations of M1 at the first five lags and negative sample autocorrelations at higher lags.

The last panel implies that the model-generated money multiplier is positively autocorrelated at the first three lags and slightly negatively autocorrelated at higher lags. This corresponds to the lower-left panel of Figure 4.3, where the sample autocorrelations in MM are positive at the first four lags and negative for higher lags. In short, Figure 5.3 indicates that the business cycle model with endogenous money supply captures the persistence in output, base money, and the money multiplier but not in other variables considered, especially the inflation rate.

Examination of the results in Section 4 of this chapter simulated by the baseline FK
(2000) model suggests that a business cycle model with endogenous money supply is inadequate to capture all the business cycle properties and autocorrelation functions of key variables, especially the price level and the inflation rate. Focusing on the price level and inflation, the baseline business cycle model matches the countercyclical behavior of the price level but not the procyclical behavior of the inflation rate. It fails to generate the phase shift of the price level relative to the overall cycle, either. In addition, this baseline business cycle model cannot explain the moderate persistence evident in actual inflation rate.

5.5 Extended FK (2000) Model with Endogenous Monetary Policy Rule

The most sophisticated monetary policy considered by Freeman and Kydland (2000) and Henriksen and Kydland (2010) involves serially correlated money growth rates. However, in reality central banks respond to real activity. For example, Eichenbaum and Singleton (1986) provide empirical evidence that the monetary policy rule involves response to both lagged money growth and output. The inability of the baseline FK (2000) model to generate procyclical inflation might result from the adoption of such a simple monetary policy. Motivated by Eichenbaum and Singleton (1986) and Gavin and Kydland (1999), a more realistic monetary policy (Policy II) is incorporated to see whether such an extension is able to improve the simulation results. Put differently, the growth rate of fiat money is allowed to respond to lagged output and money stock to see whether such an augmentation can generate procyclical inflation.

Remember from Equation (5.4) that the growth rate of fiat money at date $t$ is denoted by $\mu_t$, thus the endogenous monetary rule (Policy II) is described by
\[ \mu_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 M_{t-1} + \epsilon_{\mu t} \quad \text{with} \quad \epsilon_{\mu t} \sim N\left(0, \sigma_{\mu}^2\right) \]  \hspace{1cm} (5.18)

where \( \alpha_1 \) is the proportional response of money growth to lagged output and \( \alpha_2 \) is the policy coefficient on lagged money stock. In addition, \( \epsilon_{\mu t} \) denotes the monetary shock at date \( t \), which is normally distributed with zero mean and standard deviation of \( \sigma_{\mu} \). The standard deviation of money growth shock \( \sigma_{\mu} \) is set equal to 0.3\% per quarter such that the standard deviation of HP-filtered levels of simulated output matches that of actual output for properly chosen values of \( \alpha_1 \) and \( \alpha_2 \). Equation (5.18) implies that in steady state,

\[ \mu^* = \alpha_0 + \alpha_1 y^* + \alpha_2 M^* \]  \hspace{1cm} (5.19)

where the superscript \( * \) denotes steady-state values. Accordingly, the value of coefficient \( \alpha_0 \) can be determined by \( \alpha_0 = \mu^* - \alpha_1 y^* - \alpha_2 M^* \).

Following Gavin and Kydland (1999), I do not estimate or calibrate monetary policy coefficients \( \alpha_1 \) or \( \alpha_2 \) in this paper. Instead, I first find ranges of policy coefficients within which contemporaneous price-output and inflation-output correlations are sensitive to changes in the value of \( \alpha_1 \) or \( \alpha_2 \). And then I search for the combination of contemporaneous price-output correlation and inflation-output correlation that is consistent with empirical evidence, i.e., the price-output correlation is negative and the inflation-output correlation is positive. If countercyclical price level and procyclical inflation can be found in simulated data by varying the value of \( \alpha_1 \) or \( \alpha_2 \), I will examine the lead-lag relationships in search for appropriate phase shift of the price level relative to the overall cycle. If countercyclical price level and procyclical inflation cannot be found
simultaneously within chosen range of $\alpha_1$ or $\alpha_2$, FK’s (2000) business cycle model with endogenous money supply cannot capture the actual cyclical behavior of inflation.

This section reports the cyclical properties of the price level and inflation generated by the augmented FK (2000) model under monetary policy II. The value of policy coefficient on lagged output $\alpha_1$ varies between -0.1 and 0.1 in increments of 0.02. And the size of policy response to lagged money stock $\alpha_2$ ranges from -0.04 to 0.1 in increments of 0.014. When the cyclical properties of prices and inflation are examined for alternative values of $\alpha_1$, the size of $\alpha_2$ is fixed at 0.05 such that contemporaneous price-output and inflation-output correlations are sensitive to choices for values of $\alpha_1 \in [-0.1, 0.1]$. Similarly, the value of $\alpha_1$ is set equal to -0.01 when investigating the cyclical properties of prices and inflation for alternative values of $\alpha_2 \in [-0.04, 0.1]$. The model is simulated for 1000 times, and all the cyclical properties reported in this section are the averages of the corresponding properties across the 1000 simulations.

Figure 5.4 plots the cyclical properties of simulated price level and inflation for alternative values of $\alpha_1 \in [-0.1, 0.1]$ and $\alpha_2 = 0.05$. The upper panel presents the standard deviations (sd) of HP-filtered levels of simulated price level and inflation, and the lower panel shows the contemporaneous correlation coefficients between HP-filtered output and price level/inflation. From the upper panel, the standard deviations of simulated price level and inflation exhibit an asymmetric “V” shape when the value of $\alpha_1$ varies between -0.1 and 0.1. In addition, HP-filtered price level is always more volatile than HP-filtered inflation, which is consistent with empirical evidence. When the money growth is made mildly countercyclical, i.e., when $\alpha_1 = -0.02$, the standard deviations of the price level and inflation rate drop to 0.17% and 0.05%, respectively. Deviation of $\alpha_1$ from -
0.02 makes the price level and inflation more volatile, with standard deviations up to 0.77% and 0.57%, respectively.

Figure 5.4 Cyclical Properties for Alternative Policy Responses to Output

The lower panel implies that the choice of $\alpha_i$ plays an important role in the cyclical behavior of simulated price level and inflation. As $\alpha_i$ increases from -0.1 to 0.1, the price level and inflation change from procyclical to countercyclical. Specifically, the price level is procyclical for $\alpha_i \leq -0.03$ and countercyclical for $\alpha_i > -0.03$. Moreover, size of policy response ($\alpha_i = -0.02$) that minimizes the cyclical volatility of inflation also makes inflation highly countercyclical. Inflation is procyclical for $\alpha_i \leq -0.06$ and countercyclical for $\alpha_i > -0.04$. Remember from Chapter 4 that contemporaneous price-
output and inflation-output correlations are -0.48 and 0.19, respectively. However, the lower panel of Figure 5.4 suggests that inflation is countercyclical whenever the price level is countercyclical, i.e., it is impossible to find a value of $\alpha_i \in [-0.1, 0.1]$ that yields both countercyclical price level and procyclical inflation.

![Standard Deviations of Price Level and Inflation](image1)

**Figure 5.5 Cyclical Properties for Alternative Policy Responses to Money Stock**

Figure 5.5 plots the cyclical properties of simulated price level and inflation for alternative values of $\alpha_2 \in [-0.04, 0.1]$ and $\alpha_i = -0.01$. From the upper panel, the standard deviations of simulated price level and inflation exhibit an elbow shape as $\alpha_2$ ranges between -0.04 and 0.1. Specifically, when the proportional response of money growth to the money stock is -0.04, the price level and inflation are highly volatile with standard deviations of 3.69% and 2.83%, respectively. As the policy coefficient on money stock
\(\alpha\) increases to -0.026, the standard deviations of HP-filtered price level and inflation drop to 0.25\% and 0.17\%, respectively. The standard deviations of the price level and inflation remain roughly constant for any higher values of \(\alpha\).

The lower panel indicates that the size of \(\alpha\) affects the cyclical behavior of simulated price level and inflation. As \(\alpha\) increases from -0.04 to 0.1, movements in the price level and inflation change from procyclical to countercyclical. Specifically, the price level moves procyclically for \(\alpha \leq -0.012\) and countercyclically for any positive values of \(\alpha\). In addition, inflation is procyclical for \(\alpha < -0.026\) and countercyclical for \(\alpha > -0.02\). Still, the lower panel of Figure 5.5 suggests that inflation is countercyclical whenever the price level is countercyclical. That is, it is impossible to find a value of \(\alpha_t \in [-0.1, 0.1]\) that generates both countercyclical price level and procyclical inflation.

By examining the simulation results in Sections 4 and 5 for the original and augmented FK (2000) models, a business cycle model with endogenous money supply is inadequate to capture the business cycle properties all key macroeconomic variables, especially the price level and the inflation rate. Focusing on the price level and inflation, the first candidate model matches the countercyclical behavior of the price level but not the procyclical behavior of the inflation rate. The original FK (2000) model also fails to generate the phase shift of the price level relative to the overall cycle. In addition, the original FK model cannot explain the moderate persistence evident in the actual inflation rate.
CHAPTER 6
A STICKY-PRICE DSGE MODEL

The second candidate model is a sticky-price dynamic stochastic general equilibrium (DSGE) model specified by Ireland (2003). This model allows, but does not require, nominal prices to react sluggishly in response to any shocks that affect the model economy. Woodford (2003) argues that price stickiness is an important determinant of resource allocation in equilibrium. Besides the fact that commodity prices change infrequently, it is worthwhile to investigate the role of nominal rigidity in the cyclical behavior of the price level and inflation. The model environment will be described shortly, followed by a definition of symmetric competitive equilibrium. And then the empirical strategy will be introduced, followed by an interpretation of parameter estimates and conditional variance decompositions of key variables, and a detailed analysis of the simulation results. The final part involves robustness check, where per-capita data and an alternative measure of the money stock are used to check the robustness of the simulation results obtained in Section 6 of this chapter.

6.1 Model Environment

In this model, time is discrete and continues forever, which can be denoted as \( t = 0, 1, 2, \ldots \). There are four types of agents in this model economy, including a representative household, a continuum of intermediate goods-producing firms, a representative finished goods-producing firm, and a central bank. The representative household rents capital and sells labor to intermediate goods-producing firms and purchases finished goods from finished goods-producing firms. Finished goods-
producing firms purchase intermediate goods from intermediate goods-producing firms and aggregate them into finished goods. The central bank issues fiat money and transfers net revenues from printing money to households as a lump-sum transfer. Hence, the commodity space consists of labor, the capital good, intermediate and finished goods, and fiat money. The following subsections introduce the four types of agents and their relationships in detail.

6.1.1 Households

At date 0, the representative household is endowed with initial money balances \( M_0 \), initial bonds \( B_0 \), and an initial capital stock \( K_0 \). At each date \( t \geq 0 \), the household is endowed with one unit of time. At the beginning of each period \( t = 1, 2, 3, \ldots \), the household holds \( M_{t-1} \) units of cash carried over from last period, \( B_{t-1} \) units of discount bonds purchased last period, and \( K_{t-1} \) units of capital accumulated until this period. At the beginning of period \( t \), the discount bonds purchased in the previous period mature, yielding \( B_{t-1} \) units of additional money. Furthermore, the household receives nominal lump-sum subsidies \( T_t \) from the central bank.

The household supplies capital at a nominal rental rate \( Q_i \) and labor at a nominal wage rate \( W_i \) to each intermediate goods-producing firm in competitive markets. There is a continuum of intermediate goods-producing firms indexed by \( i \) with \( i \in [0, 1] \), so the amount of capital supplied to firm \( i \) is \( K_t(i) \) and the amount of labor hired by firm \( i \) is \( L_t(i) \). The household’s decision on capital and labor supply must satisfy the following two conditions
\[ K_t = \int_0^t K_t(i)di \]  \hspace{1cm} (6.1)

where \( K_t \) is the total amount of capital supplied and

\[ l_t = \int_0^t l_t(i)di \]  \hspace{1cm} (6.2)

where \( l_t \) is the total amount of time worked.

The household also receives all nominal profits \( D_t(i) \) earned by each intermediate goods-producing firm \( i \in [0, 1] \), so total nominal profits are

\[ D_t = \int_0^t D_t(i)di \]  \hspace{1cm} (6.3)

In period \( t \), the representative household derives utility from consumption \( C_t \), real money balances \( M_t/P_t \), and leisure \((1-l_t)\). As will be covered later, the consumption goods are produced by the representative finished goods-producing firm which aggregates all types of intermediate goods.

The household’s expected life-time utility is

\[ \mathbb{E} \sum_{t=0}^\infty \beta^t u\left( C_t, \frac{M_t}{P_t}, l_t \right) \]  \hspace{1cm} (6.4)

where \( \beta \in [0, 1] \) is the discount factor and the instantaneous utility function is

\[ u\left( C_t, \frac{M_t}{P_t}, l_t \right) = a_t \left( \frac{\gamma}{\gamma - 1} \right) \ln \left[ C_t^{\gamma/(\gamma - 1)} + e_t^{1/\gamma} \left( \frac{M_t}{P_t} \right)^{(\gamma - 1)/\gamma} \right] + \eta \ln (1 - l_t) \]  \hspace{1cm} (6.5)
In Equation (6.5), \( \alpha_t \) is a preference shock that enters into the Euler equation representing the household’s elasticity of intertemporal substitution, namely the responsiveness of the growth rate of consumption to the real interest rate. \( \gamma > 0 \) is a preference parameter which measures the interest elasticity of money demand \([\gamma/(\gamma - 1)]\). 

\( \zeta_t \) is a money demand shock and \( \eta > 0 \) is the weight on leisure. Both preferences shocks evolve according to stationary AR(1) processes such that

\[
\ln(\alpha_t) = \rho_\alpha \ln(\alpha_{t-1}) + \zeta_{at} \quad \text{with} \quad \zeta_{at} \sim N(0, \sigma_\alpha^2) \tag{6.6}
\]

where \( \rho_\alpha \in [0, 1) \) and

\[
\ln(e_t) = (1 - \rho_e)\ln(e) + \rho_e \ln(e_{t-1}) + \zeta_{et} \quad \text{with} \quad \zeta_{et} \sim N(0, \sigma_e^2) \tag{6.7}
\]

where \( \rho_e \in [0, 1) \) and \( e > 0 \) denotes the steady-state value of the money demand shock.

The representative household allocates funds in four ways. First, it purchases products from the representative finished goods-producing firm, which charges a nominal price \( P_t \) for each unit of product. The household allocates the purchased finished goods between consumption \( C_t \) and investment \( I_t \). The transformation of finished goods into capital goods incurs a capital adjustment cost (in real terms)

\[
\left( \frac{\phi_k}{2} \right) \left( \frac{K_{t+1}}{gK_t^2} - 1 \right)^2 K_t \tag{6.8}
\]

where \( \phi_k \) is the capital adjustment cost parameter and \( g \) is the steady-state value of capital growth rate. Also, the household purchases discount bonds \( B_t \) at the nominal price \( B_t/r_t \)
where $r_t$ is the gross nominal interest rate between dates $t$ and $t+1$. Finally, the household’s wealth can be stored in fiat money $M$. The household’s budget constraint (in real terms) is given by

$$
\frac{M_{t-1} + B_{t-1} + T_t + Q_t K_t + W_l l_t + D_{t-1}}{P_t} = C_t + I_t + \left( \frac{\phi_{k_t}}{2} \right) \left( \frac{K_{t+1}}{gK_t} - 1 \right)^2 K_t + \left( \frac{B_{t-1}/r_t + M_t}{P_t} \right) \tag{6.9}
$$

In Equation (6.9), the left-hand side lists the sources of income and the right-hand side displays the allocation of resources available for the representative household. Let $\delta$ denote the depreciation rate of capital, and $x_t$ denote shocks to the marginal efficiency of investment at date $t$. The law of motion for the capital stock is

$$
K_{t+1} = (1 - \delta)K_t + x_t I_t \tag{6.10}
$$

where the investment efficiency shock $x_t$ follows the following stationary AR(1) process

$$
\ln(x_t) = \rho_x \ln(x_{t-1}) + \epsilon_x \quad \text{with } \epsilon_x \sim \mathcal{N}(0, \sigma_x^2) \tag{6.11}
$$

The representative household chooses consumption, labor supply, money holdings, bond holdings, investment (and thus next-period capital stock) for all periods to maximize its expected life-time utility (6.4) subject to the constraints (6.9) and (6.10).

### 6.1.2 Intermediate Goods-producing Firms

There is a continuum of intermediate goods-producing firms indexed by $i \in [0,1]$. Each firm produces perishable intermediate goods of a unique type which can also be indexed by $i \in [0,1]$ and hence firm $i$ produces intermediate goods of type $i$. At date $t$, an
intermediate goods-producing firm \( i \) rents capital \( K_i(i) \) and uses labor \( l_i(i) \) to produce \( Y_i(i) \) units of intermediate goods of type \( i \) according to a constant-returns-to-scale Cobb-Douglas production function

\[
Y_i(i) = K_i(i)^{\alpha} \left[ g' z_i l_i(i) \right]^{1-\alpha}
\]  

(6.12)

In Equation (6.12), \( \alpha \) is the share of capital in output, \( g \) is the steady-state capital growth rate and \( z_i \) is the productivity or technology level at date \( t \). \( g' \) and \( z_i \) play a labor-augmenting role. The productivity level \( z_i \) evolves according to the following stationary AR(1) process

\[
\ln(z_t) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt} \text{ with } \varepsilon_{zt} \sim N(0, \sigma_z^2) \]  

(6.13)

where \( z > 0 \) is the steady-state level of productivity.

Intermediate goods of type \( i \in [0,1] \) are purchased by a representative finished goods-producing firm at the nominal price \( P_i(i) \) to produce the finished good \( Y_i \), which is then sold to the household.\[^{62}\] Intermediate goods of different types substitute imperfectly for each other in the finished goods-producing firm’s production function, as will be covered in the next subsection. Thus intermediate goods-producing firm \( i \in [0,1] \) possesses some monopoly power in the market of intermediate good \( i \), i.e., the market of each intermediated good is monopolistically competitive. Accordingly, firm \( i \) can set its own

\[^{62}\text{Activities conducted by the representative finished goods-producing firm will be introduced shortly.}\]
price of intermediated good \( i \). Since the model incorporates price stickiness, the adjustment of nominal price by intermediate goods-producing firm \( i \) incurs a price adjustment cost (in real terms)

\[
\left( \frac{\phi_p}{2} \right) \left[ \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t
\]  

(6.14)

where \( \phi_p \) is the price adjustment cost parameter and \( \pi \) is the steady-state inflation rate.

Firm \( i \) aims to maximize its total market value or the discounted sum of its shareholder’s utility derived from real dividends \( D_t(i)/P_t \). Hence intermediate goods-producing firm \( i \) chooses capital input \( K_t(i) \), labor input \( l_t(i) \), and sets price \( P_t(i) \) to maximize

\[
E \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[ \frac{D_t(i)}{P_t} \right]
\]  

(6.15)

subject to the constraint (6.12). In Equation (6.15), \( \beta \) is the discount factor and \( \Lambda_t \) is the Lagrangian multiplier on the household’s budget constraint (6.9) which measures the household’s marginal utility of consumption at date \( t \). The dividends paid to the household come from profits net of any price adjustment costs such that

\[
\frac{D_t(i)}{P_t} = \frac{P_t(i)Y_t(i) - Q_tK_t(i) - W_tL_t(i)}{P_t} \left( \frac{\phi_p}{2} \right) \left[ \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t
\]  

(6.16)

6.1.3 Finished Goods-producing Firms

At date \( t \), the representative finished goods-producing firm produces \( Y_t \) units of the
finished good using intermediate goods of type \( i \in [0, 1] \) as inputs according to a constant-
returns-to-scale production function

\[
Y_t = \left[ \int_0^1 Y_t(i)^{(\theta-1)/\theta} \, di \right]^{(\theta-1)/\theta} \tag{6.17}
\]

In Equation (6.17), \( \theta > 1 \) measures the price elasticity of demand for each type of intermediate goods. As mentioned previously, the finished goods-producing firm purchases \( Y_t(i) \) units of input at the nominal price \( P_t(i) \) from intermediate goods-producing firm \( i \) and sells the finished good to the household at the nominal price \( P_t \). The finished goods-producing firm aims to maximize profit

\[
P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, di \tag{6.18}
\]

by choosing inputs \( Y_t(i) \) for all \( i \in [0, 1] \) and hence \( Y_t \) subject to (6.17).

The finish good market is perfectly competitive, so the representative finished goods-producing firm earns zero profit in equilibrium. The zero-profit condition implies that the price of the finished good is determined as

\[
P_t = \left[ \int_0^1 P_t(i)^{1-\theta} \, di \right]^{1/(1-\theta)} \tag{6.19}
\]

6.1.4 The Central Bank

At date \( t \), the central bank issues fiat money \( M_t \) at a growth rate \( \mu_t \). Hence the money supply is determined by
The central bank transfers net revenues from printing money to the representative household as a lump-sum transfer $T_t$ with nominal value

$$T_t = M_t - M_{t-1} = (\mu_t - 1)M_{t-1}$$ (6.21)

The central bank implements a Taylor-type monetary policy rule by fully adjusting the nominal interest rate $r_t$ and partially adjusting the money growth rate $\mu_t$ in response to deviations of inflation and output from their steady-state values. The monetary policy rule is

$$\ln\left(\frac{r_t}{r}\right) - \omega_{\mu} \ln\left(\frac{\mu_t}{\mu}\right) = \omega_{\pi} \ln\left(\frac{\pi_t}{\pi}\right) + \omega_{y} \ln\left(\frac{y_t}{y}\right) + \ln(v_t)$$ (6.22)

where $r, \mu, \pi, y$ are the steady-state levels of the nominal interest rate $r_t$, the money growth rate $\mu_t$, the inflation rate $\pi_t$, and detrended output $y_t (= Y_t / g_t)$, respectively. In Equation (6.22), $\omega_{\mu}$ is the weight on money growth relative to the weight on the nominal interest rate, and $\omega_{\pi}$ and $\omega_{y}$ are the policy coefficients on log-deviations of inflation and detrended output from their steady-state values. $v_t$ is the monetary policy shock which follows a stationary AR(1) process such that

$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \varepsilon_v \text{ with } \varepsilon_v \sim N(0, \sigma_v^2)$$ (6.23)
6.2 Symmetric Equilibrium

The symmetry imposed on private agents’ tastes and firms’ technologies suggests a symmetric equilibrium in which all intermediated goods-producing firms make identical decisions. Thus, \( P_i(t) = P_i, Y_i(t) = Y_i, K_i(t) = K_i, l_i(t) = l_i, \) and \( D_i(t) = D_i \) for all \( i \in [0, 1] \) and for all \( t = 0, 1, 2, \ldots \). Moreover, the central bank’s budget constraint (6.21) holds and the bond market clears for all \( t = 0, 1, 2, \ldots \) such that

\[
B_t = B_{t-1} = 0 \quad (6.24)
\]

This model economy is characterized by seven state variables: the capital stock \( K \), lagged real money balances \( m_{t-1}(= M_{t-1}/P_{t-1}) \), and the five exogenous shocks including two preferences shocks \( a \) and \( e \), the investment efficiency shock \( x \), the productivity shock \( z \), and the monetary policy shock \( v \). The five innovations \( \varepsilon_{at}, \varepsilon_{et}, \varepsilon_{xt}, \varepsilon_{zt}, \) and \( \varepsilon_{vt} \) are assumed to be uncorrelated contemporaneously and across time.

A competitive equilibrium consists of a sequence of allocations \( \{ C_t, I_t, K_t, l_t, m_t, D_t \} \) and a sequence of prices \( \{ P_t, Q_t, W_t, r_t \} \) such that:

I. The representative household maximizes its expected life-time utility (6.4) subject to two constraints (6.9) and (6.10), taking prices as given.

II. Each intermediate goods-producing firm maximizes its expected total market value (6.15) subject to (6.12), taking prices as given.

III. The representative finished goods-producing firm maximizes its profit (6.18)

---

63 Lagged real money balance is a state variable due to price stickiness in this model economy.
subject to (6.17), taking prices as given.

IV. The central bank’s budget constraint (6.21) is satisfied.

V. All markets clear, including the intermediate good market (6.12), the finished
good market (6.17), the capital market (6.1), the labor market (6.2), the fiat
money market, and the bond market (6.24).

6.3 Empirical Strategy

This sticky-price DSGE model includes 24 parameters, among which 3 parameters
\((\eta, \delta, \theta)\) are calibrated following the literature, especially Ireland (2003). The other 21
parameters \((\beta, \gamma, \alpha, \phi_r, \phi_k, \mu, \omega_r, \omega_k, e, z, \rho_a, \rho_e, \rho_z, \rho_e, \sigma_a, \sigma_r, \sigma_k, \sigma_e)\) are
estimated using Bayesian methods. The steady-state levels of money growth \(\mu\), the
money demand shock \(e\), and the productivity shock \(z\) are treated as estimable “parameters”.

With observations available on some endogenous variables (e.g., consumption,
investment, and the inflation rate) for the U.S. from 1959:Q1 – 2013:Q3, this chapter uses
Dynare to estimate the 21 model parameters with Bayesian methods.\(^{64}\)

This model economy is driven by five exogenous shocks, so five observed series are
selected to avoid the problem of stochastic singularity. The five variables include
consumption, investment, real money balances, inflation, and the short-term nominal
interest rate. As explained by Ireland (2003), the reason for using both consumption and
investment, instead of output alone, is to better estimate the capital adjustment cost

---

\(^{64}\) Dynare is a software platform which can be used to handle a variety of economic models, especially
DSGE and overlapping generation (OLG) models.
parameter $\phi_k$. With a more accurate estimate of this parameter, the simulated economy can fit the real economy better.

In the National Income and Product Accounts (NIPA), the broadest measure of consumption is real personal consumption expenditures and the broadest measure of investment is real gross private domestic investment. Following Ireland (2003), real money balances are measured by the ratio between the M2 stock and the GDP deflator. Since the monetary authority is assumed to have perfect control over the money stock, measuring the money supply with M2 inappropriately postulates that the central bank also controls the money multiplier.\(^{65}\)

To check the sensitivity of the results to alternative measures of the money stock, this chapter also estimates and simulates the entire model using base money as a measure of the money stock, wherein real money balances are computed as base money divided by the GDP deflator. To match the measure of inflation in the model, the inflation rate is computed as quarterly changes in the GDP deflator. Finally, the short-term nominal interest rate is measured by the rate on 3-Month Treasury bills since three months correspond to one quarter which is one period in the model.

Unlike Ireland (2003) who uses consumption, investment, and real balances in per-capita terms, this paper uses gross real personal consumption expenditures, real gross private domestic investment, and real balances in order to be consistent with the empirical exercise in Chapter 4. Provided that population grows smoothly enough, such a change will not contaminate the dynamics of key variables in the model. To back up this

\(^{65}\) Remember that M2 is the product of base money and the money multiplier.
claim, this chapter also estimates and simulates the sticky-price model using per-capita measures of consumption, investment, and real balances. And then the model-generated cyclical properties of key endogenous variables, especially the price level and inflation, are compared to that based on gross data.

Following Ireland (2003), the three parameters \( \eta, \delta, \) and \( \theta \) are calibrated as follows. First, the weight on leisure in the representative household’s utility function \( \eta \) is set to 1.5, suggesting that in the steady state the household spends one third of its time working and the rest enjoying leisure. Second, the (quarterly) depreciation rate of capital is calibrated to 0.025, implying an annual depreciation rate of around 10%. Finally, the price elasticity of demand for intermediate goods \( \theta \) is set to 6, corresponding to a steady-state markup of 20%.

In the Bayesian estimation of the remaining 21 parameters, the mode is computed using a Monte-Carlo based optimization routine. With the three calibrated parameters and the Bayesian estimates of the remaining 21 parameters, the sticky-price model is solved using the first-order Taylor expansion around the steady state.

6.4 Parameter Estimates

This section reports and interprets Bayesian estimates of the 21 parameters based on real personal consumption expenditures, real gross private domestic investment, the M2 money stock, the GDP deflator, and the rate on 3-Month Treasury bills. Table 6.1 reports Bayesian estimation results of Ireland’s (2003) sticky-price DSGE model for the U.S.

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66 Readers are invited to see Dynare wiki for more details.
over the period 1959:Q1 – 2013:Q3. The first column displays the name of each parameter, whose notation is introduced in the second column. The third column shows the prior distributions imposed on the corresponding parameters, and the fourth column and the fifth column present the prior mean and prior standard deviation of each parameter. Finally, the last column reports the posterior mean of each parameter, which is then used to simulate the sticky-price model.

The parameters estimated can be roughly classified into five groups. The first group includes the first three parameters, which describe tastes and production technologies. The second group involves adjustment cost parameters, which corresponds to the fourth and fifth parameters. And the third group involves the coefficients in the monetary policy rule, including the steady-state money growth rate $\mu$ and policy coefficients $\omega_y$, $\omega_y$, and $\omega_y$. The fourth group includes the steady-state money demand shock $e$ and technology level $z$, which match the steady-state levels of real money balances and output in the model and the average values of detrended real money balances and output in the real economy. Finally, the last group of parameters describes the five shocks that hit the model economy.

The first group of parameters includes the representative household’s discount factor $\beta$, the parameter measuring the household’s interest elasticity of $M_2$ demand $\gamma$, and the capital share in the production function $\alpha$. The three parameters are all between zero and one by construction, so their prior distributions are chosen to be the Beta distribution which is defined on the interval $[0,1]$. Their prior means and prior standard deviations are determined following the literature, especially Ireland (2003).
Table 6.1 Bayesian Estimation of Ireland’s (2003) Sticky-price DSGE Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Prior dist.</th>
<th>Prior mean</th>
<th>Prior s.d.</th>
<th>Post. mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>The discount factor</td>
<td>$\beta$</td>
<td>Beta</td>
<td>0.99</td>
<td>0.01</td>
<td>0.9966</td>
</tr>
<tr>
<td>Interest elasticity of M2 demand: $\gamma/(\gamma - 1)$</td>
<td>$\gamma$</td>
<td>Beta</td>
<td>0.05</td>
<td>0.10</td>
<td>0.0253</td>
</tr>
<tr>
<td>Capital share in the production function</td>
<td>$\alpha$</td>
<td>Beta</td>
<td>0.30</td>
<td>0.10</td>
<td>0.2401</td>
</tr>
<tr>
<td>Price adjustment cost parameter</td>
<td>$\phi_p$</td>
<td>Normal</td>
<td>150</td>
<td>100</td>
<td>286.90</td>
</tr>
<tr>
<td>Capital adjustment cost parameter</td>
<td>$\phi_k$</td>
<td>Normal</td>
<td>22.0</td>
<td>50.0</td>
<td>30.940</td>
</tr>
<tr>
<td>Steady-state money growth rate</td>
<td>$\mu$</td>
<td>Gamma</td>
<td>1.05</td>
<td>0.07</td>
<td>1.0022</td>
</tr>
<tr>
<td>Monetary policy coefficient on M2 growth</td>
<td>$\omega_p$</td>
<td>Normal</td>
<td>1.00</td>
<td>1.05</td>
<td>0.0009</td>
</tr>
<tr>
<td>Monetary policy coefficient on output</td>
<td>$\omega_y$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.05</td>
<td>-0.0390</td>
</tr>
<tr>
<td>Monetary policy coefficient on inflation</td>
<td>$\omega_e$</td>
<td>Normal</td>
<td>2.00</td>
<td>1.50</td>
<td>-0.9570</td>
</tr>
<tr>
<td>Steady-state money demand shock</td>
<td>$e$</td>
<td>Gamma</td>
<td>3.00</td>
<td>2.30</td>
<td>2.0225</td>
</tr>
<tr>
<td>Steady-state productivity level</td>
<td>$z$</td>
<td>Gamma</td>
<td>7000</td>
<td>4000</td>
<td>5258.0</td>
</tr>
<tr>
<td>Autocorrelation of preference shock</td>
<td>$\rho_u$</td>
<td>Beta</td>
<td>0.85</td>
<td>0.15</td>
<td>0.4104</td>
</tr>
<tr>
<td>Autocorrelation of money demand shock</td>
<td>$\rho_e$</td>
<td>Beta</td>
<td>0.85</td>
<td>0.15</td>
<td>0.9998</td>
</tr>
<tr>
<td>Autocorrelation of investment efficiency shock</td>
<td>$\rho_s$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.20</td>
<td>0.9844</td>
</tr>
<tr>
<td>Autocorrelation of productivity shock</td>
<td>$\rho_z$</td>
<td>Beta</td>
<td>0.85</td>
<td>0.15</td>
<td>0.9966</td>
</tr>
<tr>
<td>Autocorrelation of monetary policy shock</td>
<td>$\rho_v$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9893</td>
</tr>
<tr>
<td>S.d. of innovation to preference shock</td>
<td>$\sigma_u$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0133</td>
</tr>
<tr>
<td>S.d. of innovation to money demand shock</td>
<td>$\sigma_e$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0105</td>
</tr>
<tr>
<td>S.d. of innovation to investment efficiency shock</td>
<td>$\sigma_s$</td>
<td>InvGam</td>
<td>0.10</td>
<td>Inf</td>
<td>0.2329</td>
</tr>
<tr>
<td>S.d. of innovation to productivity shock</td>
<td>$\sigma_z$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0153</td>
</tr>
<tr>
<td>S.d. of innovation to monetary policy shock</td>
<td>$\sigma_v$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0034</td>
</tr>
</tbody>
</table>

Notes: “s.d.” denotes the standard deviation; “InvGam” denotes the Inverse Gamma distribution; “Inf” means infinity; “Post.” means posterior; the model is estimated using the Bayesian method based on data on consumption, investment, real money balances ($M_2/p$), inflation, and the short-term nominal interest rate for the U.S. from 1959:Q1 to 2013:Q3.

The posterior mean of the discount factor $\beta$ is 0.9966, implying a low rate of time preference exhibited by the representative household. The posterior mean of the parameter measuring the interest elasticity of money demand is 0.0253, corresponding to an interest elasticity of $M_2$ demand of -0.026. That is, the household’s demand for fiat money decreases by about 0.03% if the nominal interest rate increases by 1%, ceteris paribus. The posterior mean of capital’s share in the production function of intermediate
goods is 0.2401, implying that capital income accounts for about 24% of national income.

The second group of parameters involves the price adjustment cost parameter $\phi_p$ and the capital adjustment cost parameter $\phi_k$. By construction their values are non-negative, thus their prior distributions are truncated normal distributions bounded above zero. The prior mean and standard deviation for each adjustment cost parameter is determined following the literature. The posterior mean of the price adjustment cost parameter is 286.90, suggesting a strong nominal rigidity in the intermediate-goods industry. The posterior mean of the capital adjustment cost parameter is 30.94, which is about only one tenth of the Bayesian estimate of the price adjustment cost parameter.

The monetary policy rule in Equation (6.22) is described by the third group of parameters. The steady-state money growth rate $\mu$ is determined by policy and thus estimated using the actual data. The posterior mean of the gross quarterly rate of the steady-state money growth is 1.0022, corresponding to a net annual rate of 0.88%. The average of the net annual growth rate of the M2 money stock is 6.81% for the U.S. over the sample period, which is different from its estimate. However, the main purpose of the estimation exercise is to “calibrate” the model economy so that simulated cyclical properties of key variables match the cyclical properties of their actual counterparts. Hence, the discrepancy between the estimated and actual values of $\mu$ is not a major concern in this paper.

The small estimate of the monetary policy coefficient on M2 growth (0.0009) implies that the major instrument of the monetary policy is the nominal interest rate, rather than the money supply. Alternatively, the Fed does not adjust the nominal interest rate very actively in response to deviations of the money growth rate from its steady-state
level. The small estimate of the policy coefficient on the output gap (-0.039) suggests that the nominal interest rate is not very responsive to changes in the output gap. Finally, the policy coefficient estimate on inflation is -0.957, indicating that Fed accommodates inflation. In short, the actual data suggest that the Fed actually targets inflation by adjusting the short-term nominal interest rate.

The fourth group of parameters consists of the steady-state levels of the money demand shock and the productivity level. Their prior distributions are the Gamma distribution because the two shocks are non-negative by construction. The Bayesian estimates of the two steady-state levels allow the steady-state levels of real money balances and output in the model economy to match the average values of detrended real money balances and output in the actual data.

Finally, the last group of parameters describes the persistence and volatility of the five shocks that affects the model economy. The posterior mean of the autocorrelation of the preference shock $\rho_a$ is 0.41, implying moderate persistence in the preference shock. The Bayesian estimates of the autocorrelations of the last four shocks $\rho_e, \rho_x, \rho_c,$ and $\rho_v$ are greater than 0.98, suggesting that these four exogenous shocks are highly persistent. For the standard deviations of the innovations to the five shocks, their Bayesian estimates indicate that the investment efficiency shock is much more volatile than other four shocks.

### 6.5 Conditional Variance Decompositions

The variances in the cyclical components of output, real money balances, inflation, and the nominal interest rate can be decomposed into orthogonal components, which are attributable to each of the five shocks driving the model. In addition to model parameters,
Dynare also computes posterior distributions of the conditional variance decomposition of endogenous variables for selected periods. For example, the conditional variance of output is given by $\text{var}(y_{t+k} | t)$, where $k$ is the number of periods. Table 6.2 presents the posterior mean of the conditional variance decomposition of four key endogenous variables in Ireland’s (2003) version of the sticky-price DSGE model. In this table, the first column lists the four endogenous variables whose conditional variances are decomposed into five orthogonal components. And these orthogonal components are attributed to each of the five shocks displayed in Table 6.2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Preference shock $\alpha_t$</th>
<th>Money demand shock $\epsilon_t$</th>
<th>Investment efficiency shock $x_t$</th>
<th>Productivity shock $z_t$</th>
<th>Monetary policy shock $v_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.1</td>
<td>0.2</td>
<td>57.5</td>
<td>42.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Real balances</td>
<td>0.1</td>
<td>67</td>
<td>16.4</td>
<td>15.5</td>
<td>1</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.8</td>
<td>0.3</td>
<td>12.2</td>
<td>11.1</td>
<td>75.6</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>1.2</td>
<td>0.7</td>
<td>12.5</td>
<td>12.4</td>
<td>73.2</td>
</tr>
</tbody>
</table>

Notes: The posterior distribution of the conditional variance decomposition are computed using the Bayesian method based on data on consumption, investment, real money balances, inflation, and the short-term nominal interest rate for the U.S. from 1959:Q1 to 2013:Q3; each figure corresponds to the posterior mean of the percentage of the conditional variance of each variable attributable to each shock.

From this table, more than 99% of the conditional variance of output is accounted for by the investment efficiency shock (57.5%) and the technology shock (42.1%). Hence, the sticky-price DSGE model suggests that movements in output are primarily driven by the productivity shock as well as the investment efficiency shock. The money demand
shock explains 67% of the conditional variance of real money balances, the rest of which are mainly ascribed to the investment efficiency shock (16.4%) and the productivity shock (15.5%). On the other hand, the preference shock and the monetary policy shock do not play any significant role in the determination of real balances.

For the inflation rate, about 76% of its conditional variance is attributed to the monetary policy shock, and the other 23% is explained by the investment efficiency shock (12.2%) and the productivity shock (11.1%). Hence, the conditional variance of inflation is jointly determined by the monetary policy shock, the investment efficiency shock, and the technology shock, instead of the preference shock or the money demand shock. Finally, 73% of the conditional variance of the nominal interest rate is accounted for by the monetary policy shock, followed by the investment efficiency shock (12.5%) and the productivity shock (12.4%). Similar to the inflation rate, the preference shock and the money demand shock are not important for cyclical movements in the nominal interest rate.

6.6 Simulation Results

6.6.1 Business Cycle Properties

This subsection reports the cyclical properties of key endogenous variables generated by Ireland’s (2003) sticky-price model based on observed data on consumption, investment, real balances, the price level, and short-term nominal interest rate. Table 6.3 compares the model-generated cyclical volatilities of these key variables with the cyclical volatilities of corresponding actual variables. A powerful model should be able to capture the empirical volatilities of key economic aggregates, especially output, the price level,
and inflation, which are the focuses of this paper. In Table 6.3, the first row presents simulated standard deviations of the cyclical components of output($y$), the price level($p$), the inflation rate($\pi$), the money supply($M2$), and the short-term nominal interest rate($i$). And the second row displays the standard deviations of HP-filtered levels of actual variables for the U.S. over the period 1959:Q1 – 2013:Q3.

Table 6.3 Standard Deviation of Cyclical Components: Simulated vs. Empirical

<table>
<thead>
<tr>
<th>Variable</th>
<th>$y$</th>
<th>$p$</th>
<th>$\pi$</th>
<th>$M2$</th>
<th>$i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Data: 219 Quarters</td>
<td>2.58</td>
<td>1.11</td>
<td>0.34</td>
<td>3.36</td>
<td>0.25</td>
</tr>
<tr>
<td>U.S. Data: 1959:Q1-2013:Q3</td>
<td>2.12</td>
<td>0.78</td>
<td>0.27</td>
<td>1.34</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; output is measured as the sum of consumption and investment in both the model and data; all variables in the first row are simulated using Ireland’s (2003) sticky-price model; 219 quarters corresponds to the number of quarters over the sample period 1959:Q1 – 2013:Q3; all cyclical components are extracted by the H-P filter and are measured in percentage terms; the figures represent the standard deviations of the cyclical components of corresponding variables.

Table 6.3 implies that the sticky-price model captures the cyclical volatilities of most variables including output, the price level, the inflation rate, and the short-term nominal interest rate. However, the model-generated money supply is more than twice volatile than the M2 money stock in the actual data. This paper concentrates on the cyclical co-movements of the price level and inflation with output, so these three variables will be focused on when evaluating the fit of the model. Focusing on the first

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67 The standard deviation of HP-filtered levels of output is different from its counterpart in Table 4.1 because data on output (GDP) is not directly used in the Bayesian estimation of the model. Data on output is recovered from data on consumption and investment instead.
three economic series, Ireland’s (2003) sticky-price DSGE model captures the cyclical volatilities of key economic variables, including output, the price level, and inflation.

Table 6.4 Contemporaneous Correlations with Output: Simulated vs. Empirical

<table>
<thead>
<tr>
<th>Variable</th>
<th>$P$</th>
<th>$\pi$</th>
<th>M2</th>
<th>$i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Data: 219 Quarters</td>
<td>-0.21***</td>
<td>0.16**</td>
<td>0.30***</td>
<td>-0.10</td>
</tr>
<tr>
<td>U.S. Data: 1959:Q1-2013:Q3</td>
<td>-0.50***</td>
<td>0.11*</td>
<td>0.25***</td>
<td>0.34***</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; output is measured as the sum of consumption and investment in both the model and data; all variables in the first row are simulated using Ireland’s (2003) sticky-price model; 219 quarters corresponds to the number of quarters over the sample period 1959:Q1 – 2013:Q3; all cyclical components are extracted by the H-P filter and are measured in percentage terms; the figures show the contemporaneous correlation coefficients between the cyclical components of output and the corresponding variable; *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

Table 6.4 presents the model-generated contemporaneous correlations between the cyclical components of output and other key variables, along with their actual counterparts. Again, the first row shows simulated contemporaneous correlation coefficients and the second row presents their empirical counterparts for the U.S. over the sample period 1959:Q1 – 2013:Q3. The model-generated contemporaneous price-output correlation is -0.21, which is significant at 1% level, suggesting countercyclical behavior of simulated price level. Since the price level in the real economy is also countercyclical, although to a higher degree, Ireland’s (2003) sticky-price DSGE model captures the empirical cyclical behavior of the price level qualitatively.

The model-generated inflation rate is mildly procyclical, which matches the cyclical behavior of actual inflation rate. In addition, this sticky-price model generates moderately procyclical money supply which behaves similarly to the M2 money stock in the actual
data. However, simulated nominal interest rate is acyclical, while its empirical counterpart is moderately procyclical. Again, focusing on the price level and the inflation rate, this sticky-price DSGE model performs well in accounting for countercyclical price level and procyclical inflation rate.

![Cyclical Properties of the Price Level](image1)

**Cyclical Properties of Inflation**

![Cyclical Properties of M2](image2)

**Cyclical Properties of Nominal Interest Rate**

Figure 6.1 Lead-lag Relationships with Output: Simulated vs. Empirical

Figure 6.1 compares the model-generated and the actual lead-lag relationships between output and other key economic variables at business cycle frequencies. Four types of lead-lag relationships are considered, including the price-output correlation sequence, the inflation-output correlation sequence, the M2-output correlation sequence, and the interest rate-output correlation sequence. All correlation sequences are plotted against integers $j \in [-5, 5]$, as shown on the horizontal axes. In this figure, the solid lines
with circular markers correspond to actual correlation sequences and the dotted lines with triangular markers correspond to the correlation sequences generated by Ireland’s (2003) sticky-price DSGE model.

The fit of this model in capturing actual lead-lag relationships is assessed by comparing two lines in each panel. From the upper-left panel, the model-generated correlation between the price level and output is the strongest when \( j = -2 \). This means that Ireland’s (2003) sticky-price model generates the same phase shift as its empirical counterpart, where the price level leads output by two quarters. The upper-right panel implies that the model-generated inflation is procyclical but it does not lag the overall cycle as inflation in the actual data. By contrast, implied inflation simulated by Ireland’s (2003) sticky-price model coincides with the overall cycle, which is defined by output.

From the lower-left panel, the model-generated M2-output correlation sequence almost coincides with its actual counterpart except for the slightly different positions where the two correlation sequences peak. Specifically, simulated M2-output correlation sequence peaks when \( j = -1 \) whereas the actual M2-output correlation sequence peaks when \( j = -2 \). This suggests that the model-generated M2 money stock is moderately procyclical and leads the overall cycle by one quarter, which roughly matches the actual cyclical behavior of M2 where the money supply is moderately procyclical and leads output by two quarters.

The lower-right panel indicates that Ireland’s (2003) sticky-price DSGE model cannot explain the actual cyclical behavior of the nominal interest rate. The actual interest rate-output correlation is about 0.4 when \( j = 0 \) while the corresponding correlation sequence generated by the model is slightly negative when \( j = 0 \). Hence, simulated
interest rate is acyclical but its empirical counterpart is moderately procyclical, which graphically illustrating the point in the last column of Table 6.4. The solid line peaks at about 0.5 when \( j = 2 \), suggesting that the nominal interest rate lags output by two quarters. However, the dotted line does not peak at any positive values on the horizontal axis, implying that simulated interest rate does not lag the overall cycle.

In short, Ireland’s sticky-price DSGE model captures the cyclical volatilities of output, the price level, the inflation rate, and the nominal interest rate. It also captures the contemporaneous correlations between output and most variables, including the price level, inflation, and the M2 money stock. In addition, this candidate model captures the empirical phase shifts of the price level and the M2 money supply relative to the overall cycle. Based on all the simulation results, Ireland’s sticky-price DSGE model outperforms Freeman and Huffman’s business cycle model with endogenous money supply in illustrating the cyclical behavior of the price level and the inflation rate.

### 6.6.2 Autocorrelation Functions (ACFs)

To assess the ability of Ireland’s (2003) sticky-price DSGE model to capture the persistence evident in the cyclical components of key economic series, this subsection plots the autocorrelation functions of key endogenous variables generated by this model. Each model-generated autocorrelation function is estimated by averaging autocorrelations of the corresponding variable across 1000 simulations. Figure 6.2 displays the model-generated autocorrelation functions of HP-filtered levels of output, the price level, inflation, the M2 money stock, and the short-term nominal interest rate. By comparing Figure 6.2 and Figure 4.3 in Chapter 4, one can evaluates the fit of Ireland’s (2003) sticky-price DSGE model in capturing actual persistence evident in each key economic
variable.

Figure 6.2 The Model-generated Autocorrelation Functions (ACFs): 219 Quarters

From the upper-left panel of Figure 6.2, the model-generated autocorrelations of output are significant positive at the first three lags and slightly negative at higher lags. This is similar to the upper-left panel of Figure 4.3, where the empirical autocorrelations of output are significant positive at the first four lags and negative at higher lags. The upper-middle panel suggests that the model-generated autocorrelations of prices are significantly positive at the first five lags and moderately negative at higher lags, which are quite similar to the empirical autocorrelation functions of the price level.

The upper-right panel implies that the model-generated autocorrelations in the inflation rate are positive at the first two lags and slightly negative at higher lags. This is different from the empirical persistence in inflation, which is significantly positive at the first four lags and moderately negative at higher lags. In addition, simulated
autocorrelation in inflation is about 0.7 at the first lag, indicating highly persistent behavior of the inflation rate in the model. However, the corresponding autocorrelation is no more than 0.5 in inflation from the actual data, implying only moderate persistence in inflation in the real economy.

From the lower-left panel, the model-generated autocorrelations of the M2 money stock is significant positive at the first four lags and mildly negative at higher lags, which are quite similar to the empirical autocorrelations of the M2 money stock. Finally, the lower-middle panel indicates that the nominal interest rate is positively autocorrelated at the first three lags and mildly negatively autocorrelated at higher lags. This is somewhat similar to the autocorrelations in the 3-Month Treasury bill rate, which are significantly positive at the first four lags and moderately negative at higher lags. In short, the sticky-price model captures the actual degree and pattern of the persistence in the cyclical components of most variables, especially output, the price level, and the money stock.

6.7 Robustness Check

6.7.1 Per-capita Measures of Consumption, Investment, and Real Balances

As mentioned in Section 3, Ireland (2003) uses data on consumption, investment, and real money balances in per-capita terms when estimating the sticky-price model. The reason is that the model economy is assumed to be occupied by one representative household, and thus consumption, investment, and money balances in this model can be attributed to one “person”. Pervious estimation and simulation of the model are based on data on consumption, investment, and real balances. To check the robustness of results to alternative measures of key endogenous variables, this subsection estimates the sticky-
price model using per-capita consumption, investment, and real balances, and then simulates the model using parameter estimates obtained based on per-capita data.\textsuperscript{68} The estimation results are presented by Table B.1 in Appendix B and the simulation results are reported here.

Table 6.5 Standard Deviation of Cyclical Components: Simulated vs. Empirical

<table>
<thead>
<tr>
<th>Variable</th>
<th>$y$</th>
<th>$P$</th>
<th>$\pi$</th>
<th>M2</th>
<th>$i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Data: Per-capita</td>
<td>2.25</td>
<td>0.88</td>
<td>0.35</td>
<td>2.12</td>
<td>0.26</td>
</tr>
<tr>
<td>U.S. Data: Per-capita</td>
<td>2.10</td>
<td>0.78</td>
<td>0.27</td>
<td>1.34</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; per-capita output is measured as the sum of per-capita consumption and per-capita investment in both the model and data; all variables in the first row are simulated using Ireland’s (2003) sticky-price model based on per-capita data; 219 quarters corresponds to the number of quarters over the sample period 1959:Q1 – 2013:Q3; all cyclical components are extracted by the H-P filter and are measured in percentage terms; the figures show the contemporaneous correlation coefficients between the cyclical components of output and the corresponding variable; *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

Table 6.5 compares the cyclical volatilities of key variables simulated by the sticky-price model based on per-capita data with the cyclical volatilities of actual variables in per-capita terms. The first row displays the model-generated standard deviations of HP-filtered output ($y$), price level ($P$), inflation ($\pi$), money stock (M2), and short-term nominal interest rate ($i$). And the second row shows the standard deviations of corresponding variables observed in real economy. This table suggests that Ireland’s (2003) sticky-price

\textsuperscript{68} Real money balances are measured as the ratio between the M2 money stock and the GDP deflator. Per-capita consumption is computed by dividing data on consumption by the civilian noninstitutional population, age 16 and over. Per-capita data on investment and real balances are obtained in a similar way.
model captures the cyclical volatilities of most series even if all variables are measured in per-capita terms. Specifically, the standard deviation of HP-filtered output, price level, inflation, and nominal interest rate are close to their actual counterparts. However, the cyclical volatility of simulated M2 money stock exceeds that of actual M2 money supply by about 58%.

As argued previously in Section 6, only output, the price level, and inflation are emphasized when assessing the fit of the model since this paper focuses on the cyclical co-movements between these three economic series. Table 6.5 indicates that previous simulation results in Table 6.3 are robust even when the sticky-price model is estimated using per-capita measures of consumption, investment, and real money balances. That is, Ireland’s (2003) sticky-price DSGE model is capable of capturing the cyclical volatilities of key macroeconomic series in the real economy, including output, the price level, and inflation.

Table 6.6 Contemporaneous Correlations with Output: Simulated vs. Empirical

<table>
<thead>
<tr>
<th>Variable</th>
<th>(P)</th>
<th>(\pi)</th>
<th>M2</th>
<th>(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Data: Per-capita</td>
<td>-0.15**</td>
<td>0.33***</td>
<td>0.61***</td>
<td>0.45***</td>
</tr>
<tr>
<td>U.S. Data: Per-capita</td>
<td>-0.51***</td>
<td>0.11*</td>
<td>0.23***</td>
<td>0.34***</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; per-capita output is measured as the sum of per-capita consumption and per-capita investment in both the model and data; all variables in the first row are simulated using Ireland’s (2003) sticky-price model based on per-capita data; 219 quarters corresponds to the number of quarters over the sample period 1959:Q1 – 2013:Q3; all cyclical components are extracted by the H-P filter and are measured in percentage terms; the figures show the contemporaneous correlation coefficients between the cyclical components of output and the corresponding variable; *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

Table 6.6 displays the model-generated and actual contemporaneous correlations.
between the cyclical components of per-capita output and other key variables, including inflation, per-capita M2 aggregates and the nominal interest rate. Again, the first row corresponds to correlations between simulated series and the second row corresponds to correlations between actual series for the U.S. from 1959:Q1 to 2013:Q3. The contemporaneous correlation between simulated price level and per-capita output is -0.15, which is significant at 5% level, indicating countercyclical movements in simulated price level. Simulated inflation is procyclical, with a contemporaneous inflation-output correlation of 0.33. Simulated M2 money stock and nominal interest rate are both procyclical, which captures the procyclical behavior of actual M2 money supply and the nominal interest rate.

Table 6.6 suggests that the model-generated contemporaneous correlations between per-capita output and other variables match the signs of corresponding correlations between actual economic series. In addition, when estimated and simulated based on per-capita data, Ireland’s (2003) sticky-price model captures the procyclical behavior of the nominal interest rate compared to previous simulated results in Table 6.4. In short, this table indicates that previous simulation results in Table 6.4 are robust even when the sticky-price model is estimated using per-capita measures of consumption, investment, and real money balances. That is, Ireland’s (2003) sticky-price DSGE model captures the contemporaneous correlations between output and other key economic series in the real economy, especially the price level and inflation.

Figure 6.3 compares the model-generated and actual lead-lag relationships between HP-filtered levels of per-capita output and other four variables, including the price level, inflation, per-capita M2 stock, and the nominal interest rate. The solid lines with circular
markers correspond to actual correlation sequences and the dotted lines with triangular markers correspond to correlation sequences generated by Ireland’s (2003) sticky-price model based on per-capita data. From the upper-left panel, the correlation between simulated price level and per-capita output is the strongest when $j = -2$, suggesting that simulated price level leads output by two quarters. From the upper-right panel, simulated inflation rate is procyclical and coincides with the overall cycle. By contrast, actual inflation lags the overall cycle by three quarters, which indicates that Ireland’s (2003) sticky-price model does not capture actual lead-lag relationship between inflation and output.

![Cyclical Properties of the Price Level](image1)

![Cyclical Properties of Inflation](image2)

![Cyclical Properties of M2](image3)

![Cyclical Properties of Nominal Interest Rate](image4)

**Figure 6.3 Lead-lag Relationships with Output: Simulated vs. Empirical**

The lower panel implies that simulated M2 stock and nominal interest rate are procyclical and coincides with per-capita output. Thus, Ireland’s (2003) sticky-price model...
model only partially captures the actual cyclical behavior of the M2 money stock and nominal interest rate. Figure 6.3 indicates that previous model-generated lead-lag relationships in Figure 6.1 are qualitatively robust, at least for the price level and inflation, even when the model is estimated and simulated based on per-capita data. This subsection confirms that the cyclical properties of the price level and inflation simulated based on gross data in Section 6.1 are qualitatively equivalent to that simulated based on per-capita data.

6.7.2 An Alternative Measure of the Money Stock: Base Money

As explained in Section 3, the monetary authority is assumed to have perfect control over the money stock while in reality the central bank only controls base money. Thus, measuring the money stock with M2 inappropriately assumes that the central bank also controls the money multiplier. To justify the use of M2 as a measure of the money stock, this subsection estimates and simulates Ireland’s (2003) sticky-price model using base money as an alternative measure of the money supply, wherein real money balances are computed as base money divided by the GDP deflator. The estimation results are displayed in Table B.2 in Appendix B and the simulation results are reported here.
Table 6.7 Standard Deviation of Cyclical Components: Simulated vs. Empirical

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\gamma$</th>
<th>$P$</th>
<th>$\pi$</th>
<th>MB</th>
<th>$i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Data: 219 Quarters</td>
<td>2.82</td>
<td>0.70</td>
<td>0.26</td>
<td>7.66</td>
<td>0.37</td>
</tr>
<tr>
<td>U.S. Data: 1959:Q1-2013:Q3</td>
<td>2.11</td>
<td>0.77</td>
<td>0.27</td>
<td>5.61</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; output is measured as the sum of consumption and investment in both the model and data; all variables in the first row are simulated using Ireland’s (2003) sticky-price model; 219 quarters corresponds to the number of quarters over the sample period 1959:Q1 – 2013:Q3; all cyclical components are extracted by the H-P filter and are measured in percentage terms; the figures represent the standard deviations of the cyclical components of corresponding variables.

Table 6.7 compares the model-generated and actual cyclical volatilities of key endogenous variables, including output, the price level, inflation, base money, and the short-term nominal interest rate. From this table, the standard deviations of HP-filtered levels of simulated output, price level, inflation, and nominal interest rate are close to that of corresponding variables in real economy. However, the model-generated base money is a little more volatile than the actual base money. Comparing Table 6.3 and Table 6.7, the model-generated cyclical volatilities of key series are qualitatively robust to alternative measures of the money stock. Focusing on the cyclical behavior of the price level and inflation, Ireland’s (2003) sticky-price model captures the cyclical volatilities of output, the price level, and inflation, even if the money stock is measured by base money.
Table 6.8 Contemporaneous Correlations with Output: Simulated vs. Empirical

<table>
<thead>
<tr>
<th>Variable</th>
<th>( P )</th>
<th>( \pi )</th>
<th>MB</th>
<th>( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated Data: 219 Quarters</td>
<td>-0.12**</td>
<td>0.14**</td>
<td>0.55***</td>
<td>-0.45***</td>
</tr>
<tr>
<td>U.S. Data: 1959:Q1-2013:Q3</td>
<td>-0.50***</td>
<td>0.11*</td>
<td>-0.31***</td>
<td>0.34***</td>
</tr>
</tbody>
</table>

Notes: See Table 4.1 for notations; output is measured as the sum of consumption and investment in both the model and data; all variables in the first row are simulated using Ireland’s (2003) sticky-price model; 219 quarters corresponds to the number of quarters over the sample period 1959:Q1 – 2013:Q3; all cyclical components are extracted by the H-P filter and are measured in percentage terms; the figures show the contemporaneous correlation coefficients between the cyclical components of output and the corresponding variable; *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

Table 6.8 presents the model-generated and actual contemporaneous correlations between the cyclical components of output and other endogenous variables, including the price level, inflation, base money, and the nominal interest rate. The correlation between simulated output and price level is -0.12, which is significant at 5% significance level, indicating countercyclical behavior of simulated price level. Simulated inflation is procyclical, with a contemporaneous correlation of 0.14. However, the model-generated MB-output correlation and interest rate-output correlation have opposite signs compared to corresponding actual correlations. Again, focusing on the price level and inflation, the sticky-price model fits the actual economy in explaining countercyclical price level and procyclical inflation.

Figure 6.4 compares the model-generated and actual correlation sequences between output and other endogenous variables, including the price level, inflation, base money, and the nominal interest rate. From the upper-left panel, the model-generated price-output correlation is the strongest when \( j = -2 \), implying that simulated price level leads the overall cycle by two quarters. The upper-right panel suggests that simulated inflation is
procyclical and coincides with simulated output. From the lower panel, Ireland’s (2003) sticky-price model does not account for the cyclical behavior of base money or the nominal interest rate. Figure 6.4 indicates that previous model-generated lead-lag relationships in Figure 6.1 are qualitatively robust, at least for the price level and inflation, to alternative measures of the money stock.

![Figure 6.4 Lead-lag Relationships with Output: Simulated vs. Empirical](image)

Figure 6.4 Lead-lag Relationships with Output: Simulated vs. Empirical

This subsection confirms that the cyclical properties of simulated price level and inflation are qualitatively robust to alternative measures of the money supply. That is, the qualitative results remain the same even if the central bank is assumed to control the M2 money stock, instead of just base money. Since Ireland’s (2003) sticky-price model does not incorporate financial intermediaries, the size of the money multiplier might not be
large enough to attenuate the central bank’s control over the part of the money supply other than base money.
CHAPTER 7
SUMMARY AND CONCLUSION

This paper investigates and interprets the cyclical behavior of the price level and the inflation rate for the United States over the period 1959:Q1 – 2013:Q3. The contemporaneous correlation between the cyclical components of the price level and output is negative, and the inflation-output correlation is positive at business cycle frequencies. Thus, the price level is countercyclical and the inflation rate is procyclical. In addition to the contemporaneous cross-correlations commonly documented in the literature, this paper also studies correlations at leads and lags to detect a phase shift of prices relative to output. The price level leads the overall cycle by two quarters and the inflation rate lags the overall cycle by three quarters.

The positive phase shift of the price series relative to the output series suggests a Granger-causal relationship from the price level to output. Using the Wald test, Haslag and Hsu (2012) confirms that in a bivariate VAR containing HP-filtered price level and output, the price level Granger-causes output but output does not Granger-cause the price level. To check the sensitivity of the Granger-causality between prices and output observed by Haslag and Hsu (2012), this paper examines eleven VARs ranging from simple bivariate VAR to five-variable VARs containing measures of base money, the money multiplier, and the nominal interest rate, in addition to the price level and output. Results from the F-test imply a stronger Granger-causality from prices to output relative to that from output to prices.

For the interpretation of the cyclical behavior of prices and inflation, the literature generally focuses on the contemporaneous correlation but neglects the lead-lag
relationship between prices and output. Another point is that few papers account for the coexistence of countercyclical prices and procyclical inflation. For example, King and Plosser (1984), Smith (1992), and Floden (2000) show that the business cycle framework with a monetary sector is capable of generating countercyclical prices. Also, Chadha and Prasad (1993) and Judd and Trehan (1995) provide evidence that a standard sticky-price model with only demand shocks explains the countercyclical behavior of prices. Yun (1996) demonstrates that sticky-price models can explain the observed co-movement of inflation with the cyclical component of output.

This paper aims to explain countercyclical prices, procyclical inflation, and the phase shift of prices relative to output simultaneously. To achieve this aim, two candidate models are applied and augmented including a business cycle model with endogenous money supply (Freeman and Huffman 1991) and a sticky-price DSGE model (Ireland 2003). Built upon Freeman and Huffman’s model, the model of Freeman and Kydland (2000) generates countercyclical prices but not procyclical inflation or the phase shift of prices relative to output. The incorporation of endogenous monetary policy rule does not resolve the problem either. This suggests that a business cycle model with endogenous money supply is inadequate to explain the procyclical behavior of inflation or the lead-lag relationship between prices and output.

Ireland’s (2003) sticky-price model allows the price level to adjust sluggishly in response to shocks that hit the economy. It has been recognized as a fact that commodity prices change infrequently, implying a possibility of price stickiness. The large estimate

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69 Haslag and Hsu (2012) is the only paper I have found so far which accounts for such a phenomenon.
of the price adjustment cost parameter confirms the existence of price stickiness in the actual economy. Ireland’s (2003) sticky-price model generates countercyclical price level, procyclical inflation, and the actual phase shift of the price level relative to output. That is, the price level simulated by the sticky-price model moves countercyclically and leads the overall cycle by two quarters. As argued by Woodford (2003), among others, in optimizing models with perfect price flexibility, monetary policy has only limited effects on the equilibrium allocation of resources. By contrast, monetary policy has profound influence on real activity in optimizing models with nominal rigidities. That is, price stickiness renders money non-neutral. It is thus possible that nominal price rigidity also affects the nominal-real correlations as generated by models with flexible prices.

In Ireland’s (2003) model, nominal prices are sticky due to the cost of adjusting prices. Depending on the size of price adjustment costs, the adjusting process can be slow. Suppose there is a one-time unexpected shock to the productivity level. Holding money constant, the positive technology shock leads to a higher output level and lower prices if prices are flexible.70 However, in a sticky-price model, prices may respond quite slowly and thus the current price level could be close to the lagged price level. Therefore, lagged prices determine or forecast current prices to some extent. If the current price level is not low enough to clear the market, demand will decrease as thus the output level will decline. Since the lagged price level affects the current price level, the lagged price series has important effects on the current output series. Put differently, the lagged price series

70 The reason is that a higher output level stimulates consumption, and thus increases the demand for real money balances. Since the money stock is held constant, the price level has to be reduced to increase real money balances.
tends to temporally precede the current output series due to the sluggish adjustment of prices and the interaction between prices and output. Empirical results in Chapter 4 suggest that it takes about six months for prices to adjust to the market-clearing level since the price level leads output by two quarters at business cycle frequencies.

Nominal price rigidity also explains the procyclical behavior of inflation. As noted in Chapter 4, the sign of the contemporaneous inflation-output correlation depends on the correlation between output and the current price series relative to the correlation between output and the lagged price series. Sluggish adjustment of prices results in a stronger negative correlation between output and the lagged prices at business cycle frequencies, and thus the positive inflation-output correlation. Hence, the nice performance of Ireland’s sticky-price DSGE model in illustrating the cyclical behavior of prices and inflation simultaneously is attributed to its incorporation of nominal price rigidity. Put differently, nominal price rigidity resulting from costly price adjustment accounts for the observed countercyclical behavior of prices, procyclical behavior of inflation, and the positive phase shift of prices relative to the overall business cycle. Therefore, a powerful empirical business cycle model should incorporate a reasonable degree of price stickiness.
APPENDIX A
MATHEMATICS APPENDIX

A.1 Definition of covariance in Ross (2004)

Let $X$ and $Y$ denote two random variables, then the covariance of $X$ and $Y$, i.e., $\text{Cov}(X, Y)$, is defined by

$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

(A.1)

where $\mu_X$ and $\mu_Y$ are the means of $X$ and $Y$, respectively.

Expanding the right hand side of Definition (A.1) yields a useful expression for $\text{Cov}(X, Y)$:

$$\text{Cov}(X, Y) = E[XY] - \mu_X E[Y] - \mu_Y E[X] + \mu_X \mu_Y$$

(A.2)

Lemma 4.7.1 in Ross (2004):

$$\text{Cov}(X + Z, Y) = \text{Cov}(X, Y) + \text{Cov}(Z, Y)$$

Proof of Lemma 4.7.1:

$$\text{Cov}(X + Z, Y) = E[(X + Z)Y] - E[(X + Z)]E[Y] \quad \text{from Equation (A.2)}$$


$$= \text{Cov}(X, Y) + \text{Cov}(Z, Y)$$
## APPENDIX B

### BAYESIAN ESTIMATION RESULTS

#### B.1 Bayesian Estimation of Ireland’s (2003) Model Based on Per-capita Data

Table B.1 Bayesian Estimation of Ireland’s (2003) Model Based on Per-capita Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Prior dist.</th>
<th>Prior mean</th>
<th>Prior s.d.</th>
<th>Post. mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>The discount factor</td>
<td>$\beta$</td>
<td>Beta</td>
<td>0.99</td>
<td>0.01</td>
<td>0.9966</td>
</tr>
<tr>
<td>Interest elasticity of M2 demand: $\gamma/(\gamma - 1)$</td>
<td>$\gamma$</td>
<td>Beta</td>
<td>0.06</td>
<td>0.10</td>
<td>0.0320</td>
</tr>
<tr>
<td>Capital share in the production function</td>
<td>$\alpha$</td>
<td>Beta</td>
<td>0.30</td>
<td>0.10</td>
<td>0.2249</td>
</tr>
<tr>
<td>Price adjustment cost parameter</td>
<td>$\phi_p$</td>
<td>Normal</td>
<td>130</td>
<td>70.0</td>
<td>77.850</td>
</tr>
<tr>
<td>Capital adjustment cost parameter</td>
<td>$\phi_k$</td>
<td>Normal</td>
<td>22.0</td>
<td>40.0</td>
<td>26.750</td>
</tr>
<tr>
<td>Steady-state money growth rate</td>
<td>$\mu$</td>
<td>Gamma</td>
<td>1.02</td>
<td>0.02</td>
<td>1.0069</td>
</tr>
<tr>
<td>Monetary policy coefficient on M2 growth</td>
<td>$\omega_p$</td>
<td>Normal</td>
<td>0.40</td>
<td>0.30</td>
<td>0.4954</td>
</tr>
<tr>
<td>Monetary policy coefficient on output</td>
<td>$\omega_y$</td>
<td>Normal</td>
<td>-0.01</td>
<td>0.02</td>
<td>0.0002</td>
</tr>
<tr>
<td>Monetary policy coefficient on inflation</td>
<td>$\omega_\pi$</td>
<td>Normal</td>
<td>0.80</td>
<td>0.60</td>
<td>1.0169</td>
</tr>
<tr>
<td>Steady-state money demand shock</td>
<td>$\epsilon$</td>
<td>Gamma</td>
<td>3.00</td>
<td>1.50</td>
<td>4.0841</td>
</tr>
<tr>
<td>Steady-state productivity level</td>
<td>$z$</td>
<td>Gamma</td>
<td>7000</td>
<td>2500</td>
<td>6028.0</td>
</tr>
<tr>
<td>Autocorrelation of preference shock</td>
<td>$\rho_a$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.15</td>
<td>0.9013</td>
</tr>
<tr>
<td>Autocorrelation of money demand shock</td>
<td>$\rho_e$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.15</td>
<td>0.9990</td>
</tr>
<tr>
<td>Autocorrelation of investment efficiency shock</td>
<td>$\rho_x$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.15</td>
<td>0.9784</td>
</tr>
<tr>
<td>Autocorrelation of productivity shock</td>
<td>$\rho_z$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.15</td>
<td>0.9990</td>
</tr>
<tr>
<td>Autocorrelation of monetary policy shock</td>
<td>$\rho_v$</td>
<td>Beta</td>
<td>0.55</td>
<td>0.25</td>
<td>0.4930</td>
</tr>
<tr>
<td>S.d. of innovation to preference shock</td>
<td>$\sigma_a$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0207</td>
</tr>
<tr>
<td>S.d. of innovation to money demand shock</td>
<td>$\sigma_e$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0095</td>
</tr>
<tr>
<td>S.d. of innovation to investment efficiency shock</td>
<td>$\sigma_x$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.1154</td>
</tr>
<tr>
<td>S.d. of innovation to productivity shock</td>
<td>$\sigma_z$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0205</td>
</tr>
<tr>
<td>S.d. of innovation to monetary policy shock</td>
<td>$\sigma_v$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0050</td>
</tr>
</tbody>
</table>

*Notes: “s.d.” denotes standard deviation; “InvGam” denotes Inverse Gamma distribution; “Inf” means infinity; “Post.” means posterior; the model is estimated with the Bayesian method using data on consumption, investment, real money balances ($M2/p$) in per-capita terms, and data on inflation and the short-term nominal interest rate for the U.S. from 1959:Q1 to 2013:Q3.*
### B.2 Bayesian Estimation of Ireland’s (2003) Model Based on Base Money

Table B.2 Bayesian Estimation of Ireland’s (2003) Model Based on Base Money

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Prior dist.</th>
<th>Prior mean</th>
<th>Prior s.d.</th>
<th>Post. mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>The discount factor</td>
<td>$\beta$</td>
<td>Beta</td>
<td>0.99</td>
<td>0.01</td>
<td>0.9922</td>
</tr>
<tr>
<td>Interest elasticity of M2 demand: $\gamma/(\gamma - 1)$</td>
<td>$\gamma$</td>
<td>Beta</td>
<td>0.06</td>
<td>0.10</td>
<td>0.1447</td>
</tr>
<tr>
<td>Capital share in the production function</td>
<td>$\alpha$</td>
<td>Beta</td>
<td>0.30</td>
<td>0.10</td>
<td>0.2859</td>
</tr>
<tr>
<td>Price adjustment cost parameter</td>
<td>$\phi_p$</td>
<td>Normal</td>
<td>130</td>
<td>70.0</td>
<td>222.24</td>
</tr>
<tr>
<td>Capital adjustment cost parameter</td>
<td>$\phi_k$</td>
<td>Normal</td>
<td>22.0</td>
<td>40.0</td>
<td>58.170</td>
</tr>
<tr>
<td>Steady-state money growth rate</td>
<td>$\mu$</td>
<td>Gamma</td>
<td>1.02</td>
<td>0.02</td>
<td>1.0088</td>
</tr>
<tr>
<td>Monetary policy coefficient on M2 growth</td>
<td>$\omega_p$</td>
<td>Normal</td>
<td>0.40</td>
<td>0.40</td>
<td>0.1186</td>
</tr>
<tr>
<td>Monetary policy coefficient on output</td>
<td>$\omega_x$</td>
<td>Normal</td>
<td>-0.01</td>
<td>0.02</td>
<td>-0.0167</td>
</tr>
<tr>
<td>Monetary policy coefficient on inflation</td>
<td>$\omega_z$</td>
<td>Normal</td>
<td>0.80</td>
<td>0.40</td>
<td>1.7982</td>
</tr>
<tr>
<td>Steady-state money demand shock</td>
<td>$e$</td>
<td>Gamma</td>
<td>3.00</td>
<td>1.00</td>
<td>1.4406</td>
</tr>
<tr>
<td>Steady-state productivity level</td>
<td>$z$</td>
<td>Gamma</td>
<td>7000</td>
<td>2500</td>
<td>2507.0</td>
</tr>
<tr>
<td>Autocorrelation of preference shock</td>
<td>$\rho_u$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.15</td>
<td>0.8955</td>
</tr>
<tr>
<td>Autocorrelation of money demand shock</td>
<td>$\rho_e$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.15</td>
<td>0.9996</td>
</tr>
<tr>
<td>Autocorrelation of investment efficiency shock</td>
<td>$\rho_s$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.15</td>
<td>0.8738</td>
</tr>
<tr>
<td>Autocorrelation of productivity shock</td>
<td>$\rho_z$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.15</td>
<td>0.9042</td>
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<tr>
<td>Autocorrelation of monetary policy shock</td>
<td>$\rho_v$</td>
<td>Beta</td>
<td>0.60</td>
<td>0.20</td>
<td>0.5374</td>
</tr>
<tr>
<td>S.d. of innovation to preference shock</td>
<td>$\sigma_u$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0238</td>
</tr>
<tr>
<td>S.d. of innovation to money demand shock</td>
<td>$\sigma_e$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0390</td>
</tr>
<tr>
<td>S.d. of innovation to investment efficiency shock</td>
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<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.1532</td>
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<tr>
<td>S.d. of innovation to productivity shock</td>
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<td>InvGam</td>
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<td>Inf</td>
<td>0.0316</td>
</tr>
<tr>
<td>S.d. of innovation to monetary policy shock</td>
<td>$\sigma_v$</td>
<td>InvGam</td>
<td>0.01</td>
<td>Inf</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

**Notes:** “s.d.” denotes standard deviation; “InvGam” denotes Inverse Gamma distribution; “Inf” means infinity; “Post.” means posterior; the model is estimated using the Bayesian method based on data on consumption, investment, real money balances (MB/p) in per-capita terms, and data on inflation and the short-term nominal interest rate for the U.S. from 1959:Q1 to 2013:Q3.
BIBLIOGRAPHY


Xue Li was born in Pengzhou, Sichuan Provence, China in 1986. After completing her schoolwork at Pengzhou Middle School in Pengzhou in 2004, Xue entered Jilin University in Changchun, Jilin Province, China. She received a Bachelor of Economics with a major in International Economics and Trade from Jilin University in May 2008. During October 2008 and May 2009, she was employed as an accounting assistant at Yuehua Real Estate & Property Development Group, China. In August 2009, she entered University of Missouri-Columbia to pursue her graduate study. She will receive a Ph.D. in economics from University of Missouri-Columbia in July 2015.