# CHASE CROSBY

*On Midpoints in the Middle Ages*

While the centuries surrounding the turn of the first millennium in Europe are typically associated with ignorance, superstition, and the dismissal of scientific thought in the name of religion, many brilliant, forward-thinking minds of the Middle Ages— including those of the devoutly religious—often go overlooked. Among them, is that of a 14th-century man named Nicole Oresme, a French scholar who, according to Marshall Clagett (the author responsible for the English translation and biographical information found in this paper), appears for the first time in the records of the College of Navarre in Paris, France as a student of theology in 1348 [1, p. 4]. He would later be appointed Grand Master of his aforementioned alma mater, and eventually employed by King Charles V to translate various works of Aristotle into French. His time spent at the College would yield his most interesting and revolutionary contributions to mathematics, not the least of which was a cleverly detailed attack on astrology, a pseudoscience held to be true by many during his lifetime, and in fact, many still today [1, pp. 6-7].

However, in perhaps his most remarkable and influential work, *Tractus de configurationibus qualitatum et motuum*, or *Treatise on the configuration of qualities and motions*, Oresme develops what might be considered a basic method of mapping mathematical relationships—most notably those concerning motion—which bears some resemblance to the Cartesian method of graphing relationships on a two-dimensional plane invented nearly 300 years later. In explaining the usefulness of his drawings, he also provides the first geometric proof of what is today referred to as the "mean speed theorem," meanwhile expressing an apparent knowledge of some basic tenets of Calculus, another discipline that he predates by more than two centuries.

It is useful to elaborate now on some of the vocabulary found in *Tractus*—namely the words "quality," "intensity," and "subject"—with the help of Marshall Clagett's analysis in the commentary accompanying his translation of the work. As Clagett puts it, a "quality" is an entity that is "essentially permanent or enduring in time," and it may also be thought of as a characteristic which can be acquired successively, which in this context means continuously. "Intensity" would be the rate of change of the quality with respect to the "subject," sometimes called "space." In the case of motion, "intensity" would refer to velocity, and "subject" would refer to time. Oresme constructs two-dimensional figures to represent these "qualities," using a horizontal line segment to represent the "subject" on which perpendicular line segments are erected, which represent "intensities." In the words of Clagett:

> Thus the base line of such figures is the subject when we are talking about linear qualities or the time when we are talking about velocities, and the perpendiculars raised on the base line represent the intensities of the quality from point to point in the subject or represent the velocity from instant to instant in the motion. The whole figure, consisting of all the perpendiculars, represents the whole distribution of intensities in the quality, i.e., the quantity of the quality, or in the case of motion the so-called total velocity, dimensionally equivalent to the total space traversed in the given time [1, p. 15].

This whole figure thus represents the "configuration" of the quality or motion. An examination of Nicole Oresme's work will begin where Tractus begins, in the first chapter of the first of three parts in the book. When quoting Oresme, all text appearing inside of brackets is mine.

### Excerpt from *Tractus de configurationibus qualitatum et motum*, Part I, Chapter i

Every measurable thing except numbers ["multitudes of things," by the ancient Greek distinction, called today natural numbers greater than 1] is [has magnitude] imagined in the manner of continuous quantity [such as the length of a line segment]. Therefore, for the mensuration of [act of measuring] such a thing, it is necessary that points, lines, and surfaces, or their properties, be imagined. For in them (i.e., the geometrical entities), as the Philosopher [Aristotle] has it, measure or ratio is initially found, while in other things [non-geometrical entities] it [measure or ratio] is recognized by similarity as they are being referred by the intellect [in imagination] to them (i.e., to geometrical entities). Although indivisible points, or lines, are non-existent [A point with no breadth, and a line with length but no breadth cannot be constructed.], still it necessary to feign [model] them mathematically for the measures of things and for the understanding of their ratios.

Therefore, every intensity [rate of change of a quality with respect to its subject] which can be acquired successively [continuously] ought to be imagined by a straight line [segment] perpendicularly erected on some point of the space or subject [also represented by a line segment] of the intensible thing, e.g., a quality [characteristic which can acquire continuously more or less intensity].

[As seen below, given a line segment *AB* representing the "space or subject" of a quality, and a point *D* on *AB* representing a particular point in or on the subject, a perpendicular line segment *DC* is constructed to represent the quality's "intensity" (rate of change) at that particular point.]

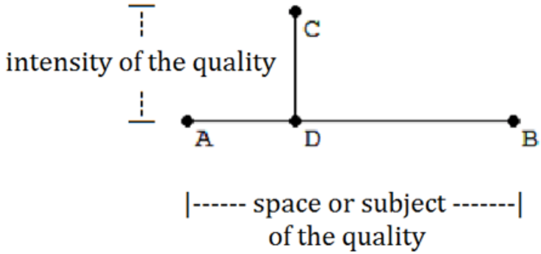


Figure 1

For whatever ratio is found to exist between intensity and intensity, in relating intensities of the same kind [as, for example, the intensity of motion—velocity—could not be usefully related in this context to the intensity of temperature within an object], a similar ratio is found to exist between [the length of] line [segment] and line [segment], and vice versa [That is, an intensity twice that of the object's at time *D* in Figure 1 would be represented by a line segment twice as long as *DC*.]. For just as one line [segment] is commensurable to another line [segment] and incommensurable to still another [Two line segments are said to be "commensurable" if the ratio of their lengths is a rational number, and "incommensurable" if it is irrational—a distinction important in Oresme's time.], so similarly in regard to intensities certain ones [line segments] are [can be] mutually commensurable and others incommensurable in any way because of their continuity. Therefore, the measure of intensities can be fittingly imagined as the measure of lines [line segments], since an intensity could be imagined as being infinitely decreased or infinitely increased in the same way as a line [segment's length].

In *Tractus*, Oresme uses his "figuration doctrine" to attempt to explain a variety of different phenomenon—from the astronomical to the psychological—falling within the broadly defined discipline of "natural philosophy" of which he was a student. For the purposes of this paper, it is most relevant, if not convenient, to focus on this doctrine as it relates to motion, a topic to which his methods—in the tone of numerous sections of his work—also apply. Oresme tends to describe his mathematics in reference to general "linear qualities," and thereafter mention that the described methods can also be used to describe motion, as one will no doubt notice in the transcribed selections below. Henceforth, one might most effectively consider his propositions as they relate to the motion of an object—with "quality" referring to motion, "intensity" to velocity, and "subject," (occasionally "extension") as referring to time—as he tends to mention this application after the fact.

Oresme sets the stage for coming mathematical insights later in Chapter i of Part I (I.i), noting that, "The consideration of these lines [line segments] naturally helps and leads to the knowledge of any intensity, as will be more fully apparent in chapter four below." In I.ii and I.iii, he establishes more language common in the rest of this work, namely that "intensities" should be referred to as "longitudes," and the space or subject in which a quality exists, or "the extension of any extended quality," should be called "latitude." He goes on:

> The aforesaid extension is designated by a line [segment] drawn in the subject [as a horizontal line segment, or base], a line [segment] on which the line [segment representative] of intensity of the same quality is erected perpendicularly. [See Figure 2.]

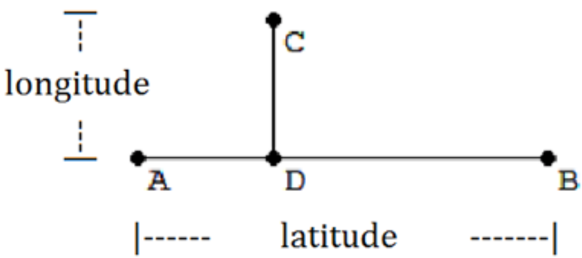


Figure 2

While the terms longitude and latitude are almost exclusively associated with cartography today, it is worth noting that in the time of Oresme, the word “longitude” actually refers to “length,” and “latitude” to “breadth.” However, either consideration of the words paints a familiar picture in the mind with regard to the mapping of relationships in modern mathematics—the line segment of latitude perhaps representing an “x-axis” and the line segments of longitude extending parallel to a “y-axis.” However, Oresme does not specifically refer to his images (drawn or imagined) using such modern terminology.

In the next few chapters of Part I, Oresme elaborates on the function of his drawn latitudes and longitudes, demonstrating how the “plane figures” they form can be used to represent qualities and motions. While he covers a wide variety of cases, an examination of an excerpt from I.vi suffices to acquaint one with his methods, the implications of which—though described further and in greater detail in *Tractus*—are easily understood and will afterwards be briefly discussed:

### Excerpt from Tractus I.vi *On the clarification of the figures*

> For example, let line [segment] *AB* [representative of the subject, say, time] be divided in [at] point *C* in any way such that the intensity in [velocity at] point *C* is double that in [at] point *A*; and in point *B* let it be triple that in point *C*. Therefore, by the first chapter [I.i] the line [segment] imagined as rising perpendicularly above point *C* and denoting the intensity [velocity] at that point is double [the length of] the line [segment] imagined as rising [perpendicularly] above point *A*, and the line [segment] imagined as rising [perpendicularly] above point *B* is three times [the length of] the line [segment] imagined as rising [perpendicularly] above point *C*. [See Figure 3 below.]

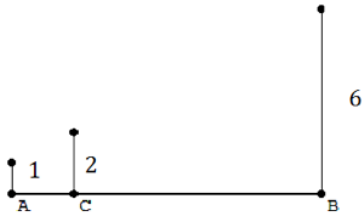


Figure 3

Therefore, this quality [mσtion] can be imagined only by the figure which at point *C* is twice as high as at point *A* or whose summit at point *C* is double [the "height" of] that at point A, and whose summit at point *B* is triple [the "height" of] that at point *C* [A figure can be formed by connecting the "summits" of the longitudinal lines at points *A, B*, and *C,* as seen in Figure 4 below. In accordance with Oresme's drawing, point *C* has been placed such that the resulting figure is a quadrangle, though, as he shortly hereafter nσtes, the three given ratios are insufficient to determine a precise figure.]

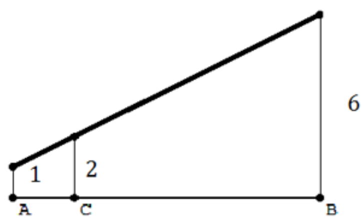


Figure 4

– with further stipulation however that the figure of this sort could be varied in altitude according to the ratio of intensities in the σther points of [on] line [segment] *AB* [That is, as we do nσt know the ratio of velocities (and therefore longitudes) of σther points in the span of time represented by *AB*, it is possible for the representative figure to take σther forms.]. But from this, it is apparent that a quality of this sort cannσt be designated [represented] by a rectangle or by a semicircle; and similarly concerning an infinite number of figures.

It follows intuitively that a quality of equal intensity at all points on the line segment AB could be represented by a rectangle, as the ratio of the longitudes at any two points on AB would always equal 1. Such a figure is seen in Figure 5.


Figure 5

Oresme refers to this type of quality—that which exhibits a constant intensity—as "uniform," and he deems those qualities that exhibit an intensity varying on their subject line "difform." A quality represented by Figure 4 would be called "uniformly difform," as its intensity varies over the subject line *AB*, but the rate of change of its intensity from point *A* to point *B* is constant, as the line segment formed by the "summits" of every longitude on *AB* forms a straight-line segment. In reference to an object in motion, Figure 4 represents one that exhibits a constant acceleration. Qualities which exhibit neither of the above – for example, a quality represented by a semicircle with *AB* as a diameter [See Figure 6.]—are called "difformly difform."

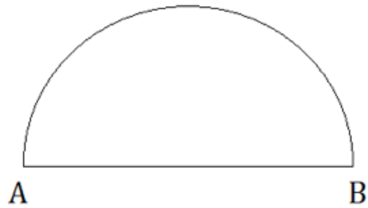


Figure 6

For the purpose of continuing our examination of Oresme's methods as they relate to mapping and measuring motion, we will skip ahead to Part III of *Tractus*, entitled *On the Acquisition and Measure of Qualities and Velocities*, in which many of his more memorable and lasting insights are recorded.

**Excerpt from III.i *How the acquisition of a quality is to be imagined***

> Succession in the acquisition of a quality can take place in two ways: (1) according to extension [in the subject], [and] (2) according to intensity, as was stated in the fourth chapter of the second part [It seems this is actually meant to be a reference to II.iii, where a nearly identical statement is made.]. And so extensive acquisition of a linear [continuously acquired] quality ought to be imagined by the motion of a point flowing over the subject line [segment] in such a way that the part traversed has received the quality and the part not yet traversed has not received the quality. An example of this occurs if point c were moved over line [segment] *AB* so that any part traversed by it would be white and any part not yet traversed would not yet be white [See Figure 7 below.].

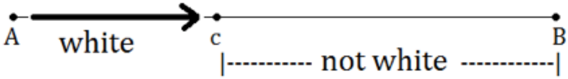


Figure 7

Thus, one of the two ways a quality can be acquired is by "extension" in the subject, represented above by the line segment *AB*. The further along *AB* point c moves, the more "whiteness" is acquired. The second means of acquiring a quality—through intensity, the vertical dimension in Oresme's drawings—is described slightly later on in the same chapter:

**Excerpt from III.i *How the acquisition of a quality is to be imagined***

> The intensive acquisition [acquisition by intensity] of punctual quality [quality at an indivisible *point* on the subject, or instant in time if referring to motion] is to be imagined by the motion of a point continually ascending [perpendicularly] over a subject point and by its motion describing [drawing] a perpendicular line [segment] imagined [as erected perpendicularly] on the same subject point. But the intensive acquisition of a linear quality [as it is acquired on a *divisible* subject, or *over* time] is to be imagined by the motion of a line [segment] perpendicularly ascending over the subject line [segment] and in its flux or ascent [across the subject line segment] leaving behind a surface by which the acquired quality is designated [represented]. For example, let *AB* be the subject line [segment]. I say, therefore, that the intension of [intensity at] point A is imagined by the motion, or by the perpendicular ascent, of point *C* [the line segment *AC*], [See Figure 8.] and the intension of line *AB* [the whole subject], or the [total] acquisition of the intensity, is imagined by the [surface created by the] ascent of line [segment] *CD* [across *AB*]. [If the quality in question were uniform, the imagined "motion" of *AC* across *AB* would yield the rectangular "surface" in Figure 9.]


Figure 8

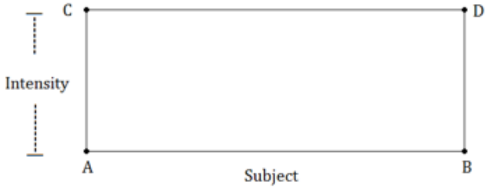


Figure 9

**Excerpt from III.vi** ***Further consideration of the same subject***

And just as we have spoken of the measure of linear quality, so we ought to speak of the measure of velocity, except that instead of extension [in the subject] the time of the duration [represented by horizontal *AB*] of the velocity is taken, and intensity according to degree [represented by vertical *AC*] is taken, and similarly for other successive entities [aside the measure of velocity]. For example, a uniform velocity taken for three days is equal to a velocity that is three times as intense which lasts for one day and similarly for pain, pleasure, and also for light, if it is imagined to be a successive entity.

Oresme's final statement in the selection above—while perhaps more intuitive when it comes to motion than to pain, pleasure, and light—is not groundbreaking in and of itself. However, an application of his figuration doctrine to a particular scenario regarding motion in the following chapter is. In his usual manner, Oresme describes the following proposition in terms of a general "linear quality," and afterwards declares that "one should speak of velocity in completely the same fashion as linear quality." Knowing this, the following selection will be explicated as it relates to the motion of an object over time.

**Excerpt from III.vii** ***On the measure of difform qualities and velocities***

Every quality [motion], if it is uniformly difform [Recall that this would describe motion with constant acceleration.], is of the same quantity [say, "total velocity," or distance traveled] as would be the quality of the same [type as] the previous [that is, motion] of equal subject [over an equal amount of time] that is uniform [demonstrates constant velocity] according to [equal to] the degree [measure of velocity] of [at] the middle point of the same subject [stretch of time]. I understand this to hold if the quality is linear.

Here we have a written proposition of what is today known as the "mean speed theorem," written nearly two centuries before the birth of Galileo, to whom the theorem is commonly credited. Later in the same chapter, Oresme goes on to provide a geometrical proof, using the figuration doctrine he developed in the first part of *Tractus*:

Hence let there be a quality [motion] imaginable by $\triangle ABC$, the quality being uniformly difform [exhibiting constant acceleration] and terminated at no degree [reaching a velocity of zero] in [at] point *B* [the end of the stretch of time containing the motion]. And let *D* be the middle point of the subject [time] line [segment]. The degree of this point, or its intensity [velocity], is imagined [represented] by [the length of] line [segment] *DE*. [See figure 10.]

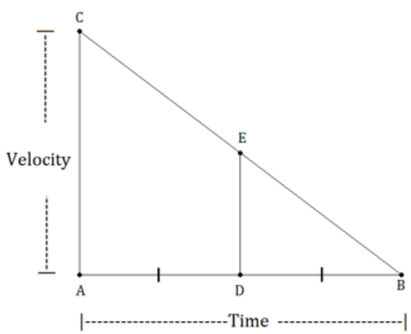


Figure 10

Therefore, the quality [motion] which would be uniform [have constant velocity] throughout the whole subject [over an equal length of time] at degree *DE* [equal to the velocity of the original motion at point *D*] is imaginable by rectangle *AFGB* [see Figure 11], as is evident by the tenth chapter of the first part [wherein Oresme shows that the motion of an object of constant velocity can be represented by a rectangle].

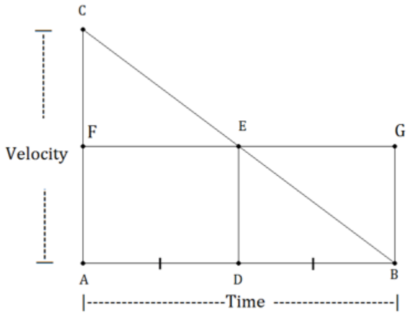


Figure 11

Therefore, it is evident by the 26th of I of Euclid [Euclid I-26, which states that two triangles which have two angles of equal measure and one side of equal length are congruent] that the two small [right] triangles *EFC* and *EGB* are equal [in area]. Therefore, the larger △*BAC*, which designates [represents] the uniformly difform quality, and the rectangle *AFGB*, which designates [represents] the quality uniform in the degree of the middle point, are equal [in area]. And this is what has been proposed. [End of proof.]

He concludes, later in III.vii:

And so it is clear to which uniform quality or velocity a quality or velocity uniformly difform is equated.

Thus, Nicole Oresme presents an elegant geometric proof of the mean speed theorem, which in modern terminology might read, "an object traveling with a constant acceleration travels the same distance as an object with a constant velocity equal to half of the initial (in our example) velocity of the object with acceleration." While other French scholars contributed to the development of this theorem, Oresme helped introduce, in the words of Clifford Truesdell, "the connection between geometry and the physical world that became a second characteristic habit of Western thought" [3, p. 35].

Oresme's perhaps unpolished demonstration of the idea that the area under a velocity curve describing an object's motion is equal to the distance traveled by the object is equally remarkable given the time period in which he resided. Unfortunately, his innovative idea of representing qualities and motions with geometric figures would suffer the same fate as those of many other mathematicians in Europe during the medieval period, and soon be lost [2, pp. 358-359]. It would be nearly 250 years before his techniques would reappear in the work of Galileo Galilei [2, p. 357]—the Italian mathematician acknowledged by many today as the "father of modern physics." However, in an apparent allusion to the future of this field of study, Nicole Oresme made a few interesting observations in the final paragraph of III.vii:

> Further, if a quality or velocity is difformly difform, and it is composed of uniform or uniformly difform parts, it can be measured by its parts, whose measure has been discussed before. Now, if the quality is difform in some other way, e.g. with the difformity designated by a curve, then it is necessary to have recourse to the mutual mensuration of the curved figures, or to these with rectilinear figures; and this is another kind of speculation. Therefore, what has been stated is sufficient.

This other "kind of speculation" might be that considered by the likes of Isaac Newton and Gottfried Leibniz some three centuries later.

## Works Cited

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