PHOTONIC ABLATION VIA QUANTUM TUNNELING

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Master of Science

by

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MAY 2015
Declaration of Approval

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Dr. Heather K. Hunt
Dedicated to my grandfather, Alfred Charles Whiteside, and his brother Bill – two of the greatest men I have ever known.
“No man is an island”

*Meditation XVII* (1624)
– John Donne

“Don’t Panic”

*The Hitchhiker’s Guide to the Galaxy* (1979)
– Douglas Adams
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## Abbreviations

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<td>AFM</td>
<td>Atomic Force Microscopy</td>
</tr>
<tr>
<td>ATR</td>
<td>Attenuated Total Reflection</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous Wave</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data Acquisition</td>
</tr>
<tr>
<td>EMC</td>
<td>Electron Microscopy Core</td>
</tr>
<tr>
<td>Er:YAG</td>
<td>Erbrium doped Yttrium Aluminum Garnet</td>
</tr>
<tr>
<td>FTIR</td>
<td>Frustrated Total Internal Reflection</td>
</tr>
<tr>
<td>IR</td>
<td>Infrared</td>
</tr>
<tr>
<td>LASER</td>
<td>Light Amplification by Stimulated Emission of Radiation</td>
</tr>
<tr>
<td>LASIK</td>
<td>Laser-Assisted In Situ Keratomileusis</td>
</tr>
<tr>
<td>MRI</td>
<td>Magnetic Resonance Imaging</td>
</tr>
<tr>
<td>MTZ</td>
<td>Microthermal Zone</td>
</tr>
<tr>
<td>N-BK7</td>
<td>Borosilicate Crown Glass</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>Neodymium doped Yttrium Aluminum Garnet</td>
</tr>
<tr>
<td>NOA 74</td>
<td>Norland Optical Adhesive # 74</td>
</tr>
<tr>
<td>PAQT</td>
<td>Photonic Ablation via Quantum Tunneling</td>
</tr>
<tr>
<td>PAS</td>
<td>PhotoAcoustic Spectroscopy</td>
</tr>
<tr>
<td>PXI</td>
<td>PCI extensions for Instrumentation</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscopy</td>
</tr>
<tr>
<td>TIR</td>
<td>Total Internal Reflection</td>
</tr>
<tr>
<td>TIRPAS</td>
<td>Total Internal Reflection PhotoAcoustic Spectroscopy</td>
</tr>
<tr>
<td>UV</td>
<td>Ultraviolet</td>
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</tbody>
</table>
Biological Terminology

Collagen | Structural protein giving skin its elasticity
Dermis | Lower layer of skin containing blood vessels and hair follicles
Epidermis | Outer layer of skin containing melanin
Fibroblasts | Collagen producing cells
Fibroplasia | Formation of fibrous tissue to create the extracellular matrix
Hyperpigmentation | Dramatic relative increase in local melanin content
Hypertermia | Elevated body temperature due to failed thermoregulation
Hypopigmentation | Dramatic relative decrease in local melanin content
Hypertrophic scar | Raised scars that protrude and distort tissue topography
Inflammation | The body’s response to harmful stimuli and damage
Lymphatic system | Vessels that transport constituents of the inflammatory response
Melanin | Broadly optically absorbing pigment in skin and hair
Melasma | Brown or grey-brown patches of discoloration in skin
Necrosis | Cell death that releases cell contents into extracellular matrix
Oedema | Area of fluid retention causing tissue swelling
Phagocytosis | Process by which a cell engulfs a solid particle
Port wine stain | Reddish area of capillary malformation in the skin
Purpura | Red or purple discolorations in the skin
Retinal melanosome | Cell that produces light-absorbing pigments within the eye
Thermal coagulation | Denaturation of blood cells through intense heat
Vascular system | System of vessels and capillaries through which blood flows
Vascularization | Formation of blood vessels and capillaries in living tissue
White blood cell | Cells responsible for engulfing and removing foreign elements
# Physical Constants

<table>
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<tr>
<th>Physical Constant</th>
<th>Value</th>
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<tbody>
<tr>
<td>Speed of Light</td>
<td>$c = 2.9979 \times 10^8$ m s$^{-1}$</td>
</tr>
<tr>
<td>Speed of Sound in water</td>
<td>$v = 1.497 \times 10^3$ m s$^{-1}$</td>
</tr>
<tr>
<td>Refractive index of Silver (532 nm)</td>
<td>$n_{Ag} = 0.1429$</td>
</tr>
<tr>
<td>Extinction coefficient of Silver (532 nm)</td>
<td>$k_{Ag} = 3.0518$</td>
</tr>
<tr>
<td>Refractive index of Titanium (532 nm)</td>
<td>$n_{Ti} = 2.479$</td>
</tr>
<tr>
<td>Extinction coefficient of Titanium (532 nm)</td>
<td>$k_{Ti} = 3.3511$</td>
</tr>
<tr>
<td>Permittivity of free space</td>
<td>$\epsilon_0 = 8.8541 \times 10^{-12}$ m$^{-3}$ kg$^{-1}$ s$^4$ A$^2$</td>
</tr>
<tr>
<td>Permeability of free space</td>
<td>$\mu_0 = 1.2566 \times 10^{-6}$ m kg s$^{-2}$ A$^{-2}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>N</td>
<td>complex refractive index</td>
</tr>
<tr>
<td>n</td>
<td>real component of N</td>
</tr>
<tr>
<td>k</td>
<td>complex component of N</td>
</tr>
<tr>
<td>t</td>
<td>thickness</td>
</tr>
<tr>
<td>λ</td>
<td>wavelength</td>
</tr>
<tr>
<td>P</td>
<td>power</td>
</tr>
<tr>
<td>I</td>
<td>intensity</td>
</tr>
<tr>
<td>H</td>
<td>Magnetic field strength</td>
</tr>
<tr>
<td>B</td>
<td>Magnetic flux density</td>
</tr>
<tr>
<td>E</td>
<td>Electric field strength</td>
</tr>
<tr>
<td>D</td>
<td>Electric displacement</td>
</tr>
<tr>
<td>j</td>
<td>Electric current density</td>
</tr>
<tr>
<td>ρ</td>
<td>Electric charge density</td>
</tr>
<tr>
<td>σ</td>
<td>Electric conductivity</td>
</tr>
<tr>
<td>μ</td>
<td>permeability</td>
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<tr>
<td>ε</td>
<td>permittivity</td>
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<tr>
<td>µ$_r$</td>
<td>relative permeability</td>
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<tr>
<td>ε$_r$</td>
<td>relative permeability</td>
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<tr>
<td>T</td>
<td>Relative transmission coefficient</td>
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<tr>
<td>R</td>
<td>Relative reflection coefficient</td>
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<tr>
<td>Γ$_g$</td>
<td>Gruneissen coefficient</td>
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<tr>
<td>H$_B$</td>
<td>Radiant beam exposure</td>
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Abstract

Due to the often extreme energies employed, contemporary methods of laser delivery utilized in clinical dermatology allow for a dangerous amount of high-intensity laser light to reflect off a multitude of surfaces, including the patient’s own skin. Such techniques consistently represent a threat to both patients and practitioners alike. The intention of this work was therefore to develop a technique that mitigates this problem by coupling the light directly into the tissue via physical contact with an optical waveguide. In this manner, planar waveguides cladded in silver with thin-film active areas were used to illuminate agar tissue phantoms with laser light operating with a 5 ns pulse-width at a wavelength of 532 nm. The light then either refracted or optically tunneled through the active area, photoacoustically generating ultrasonic waves within the phantom, whose peak-to-peak intensity directly correlated to the internal reflection angle of the beam. Consequently, angular spectra for energy delivery were recorded for sub-wavelength silver and titanium films of variable thickness. Optimal energy delivery was achieved for internal reflection angles ranging from 43 to 50 degrees, depending on the active area and thin film geometries. Silver thin-films demonstrated decreased energy fluence with increasing film thickness, whereas titanium films delivered greater energy across the angular spectrum due to their relatively high refractive index with respect to the waveguide substrate. The technique demonstrated herein therefore represents a viable method of energy delivery for biological tissue while minimizing the possibility for stray light, allowing for more control over energy delivery through choice of film thickness.
Chapter 1

Introduction

1.1 Applications of Lasers in Dermatology

Although initially not intended for such applications, lasers have been incorporated into a wide variety of medical procedures, demonstrating strong appeal as a highly controllable, non-invasive instrument. In particular, dermatology has been among the most predominant fields with applications of lasers in medicine, including tattoo removal, port wine stain treatment, skin resurfacing, treatment of acne vulgaris, and ablation of vascular lesions and cancerous tumors [1–3]. However, while many of the procedures are similar in nature, each of these applications requires the use of a different variety of laser based primarily on the intended biological or chemical target to be illuminated [4].

That being said, laser procedures in general can be broken down into five distinct classifications, depending on the characteristics of the laser being utilized and the duration of the laser pulse: photodisruption, plasma-induced ablation, photoablation, thermal interaction, and photochemical interaction [5, 6]. The first three methods utilize pulse durations in the range of femto- to nano-seconds, as shown in Figure 1.1, whereas the latter two employ durations in the millisecond to continuous wave (CW) range. It is worth noting that
Figure 1.1 A depiction of Laser-Tissue interactions as a function of pulse duration and laser power, with the two lines indicating the 1 and 1000 J/cm² irradiance limits [5].

although the shorter pulse durations do not individually exhibit thermal effects identical to those produced using longer pulses, similar results have been observed when the repetition rate of the laser and number of pulses have been sufficiently high [7]. Consequently, in certain cases, short-pulse duration approaches may be used for long-pulse duration applications; e.g., utilizing short-pulse fractionated CO₂ laser ablation for skin resurfacing.

However, while there are a plethora of applications for lasers in clinical dermatology, the method of choice for any given procedure generally reduces to one of two principally desired effects – either the localized ablation of a target chromophore, or the incitement of large area photothermal effects within the tissue. Principal examples of the former effect include the treatment of port wine stains, solar lentigines (lesions), and pigmented
tumors, along with the removal of percutaneous tattoos and other foreign pigments [8–10]. By contrast, the latter effect typically encompasses broad deep-tissue applications, such as acne scar treatment, skin resurfacing, and hair removal [11–14].

1.1.1 Selective Photothermolysis: Tattoo Removal

A primary example of the first effect, and one of the more predominant applications of lasers in dermatology, is laser tattoo removal. The technique operates on the principle of “selective photothermolysis,” in which specific wavelengths of light are chosen to photoablate molecules of tattoo ink, depending upon their relative optical absorption [10]. For example, red and yellow inks tend to clear best when ablated by laser light with a wavelength of 532 nm; whereas green, blue, and black inks tend to clear best using 755 nm or 1064 nm [7, 15].

Unfortunately, there are currently no restrictions as to the chemical composition of any tattoo inks, their pharmacological consistency, or their purity, the result of which is that each pigment may necessitate a different wavelength based on the optical absorption spectrum of the target ink, which may vary from patient to patient [16]. Additionally, the photoablation may also have adverse medical effects post-operation due to the release of potentially harmful chemicals that previously comprised the ink. For example, certain red inks tend to contain mercury or cadmium that may be released into the body when the ink is ablated, which can result in a substantial immunological response at the site of the tattoo or throughout the body [7].

Figure 1.2 demonstrates a comparison of the results of a series of four laser tattoo removal treatments using a Q-switched Nd:YAG operating at 1064 nm. Although individual treatments tend not to completely clear the tattoos, they are often preferable to more invasive surgical procedures, which necessitate complex wound closing procedures, poten-
tially resulting in wound healing complications, hypertrophic scarring, keloid formation, or anatomic distortion; consequently, laser tattoo removal procedures generally consist of a lengthy series of treatments (between 4 - 20 treatments depending on the complexity and size of the tattoo) [18–20]. Nevertheless, laser tattoo removal has shown wider appeal and greater success than other treatment procedures that tend to scar or injure the surrounding tissue, with methods being refined and improved through medical laser research every year [21].

1.1.2 Deep-tissue thermal effects: Skin Resurfacing

Another major application of lasers in dermatology is in the field of skin resurfacing. Whereas tattoo removal procedures operate through photothermolysis, skin resurfacing techniques capitalize on broader, deep-tissue photothermal effects in order to achieve
results. Laser skin resurfacing has fairly broad applications from acne scar removal to skin tightening and wrinkle removal, but the general effect is the same regardless of the structure to be resurfaced [22].

By utilizing relatively long pulse-widths (millisecond regime), the technique focuses on the thermal interactions that occur in the area surrounding the illuminated portion [14]. The high-energy laser light carbonizes and photocoagulates surface tissue, causing necrosis in the top-most layer of the epidermis and an inflammatory response in the deeper dermal tissue. The consequence of this photothermal damage is that the scarred surface tissue is destroyed and subsequently removed, whereas the deeper dermal tissue experiences an increase in collagen production to promote the generation of well-structured, healthy tissue [23].

Unlike techniques that target specific pigmented chromophores, skin resurfacing techniques target dissolved water molecules within the tissue; consequently, the lasers used tend to operate in the IR or near-IR wavelength spectra, since water is strongly optically absorbing in that range. The lasers typically used in skin resurfacing tend to be CO$_2$, Er:YAG, or Nd:YAG lasers with pulse durations from microseconds to continuous wave [12, 24, 25].

Additionally, the techniques for laser skin resurfacing can be divided into three categories – ablative resurfacing, nonablative dermal remodeling, and fractional photothermolysis – with each method focusing on a different aspect of the photothermal response [26, 27]. Ablative resurfacing is primarily intended to evenly destroy surface tissue, as shown in Figure 1.3 [left], and is useful for treating hypertrophic scarring [23, 28]. Nonablative dermal remodeling, by contrast, focuses on homogeneously encouraging the increased production of collagen without causing excess surface tissue damage, as shown in Figure 1.3 [center], and is useful for wrinkle removal procedures [29].
Chapter 1. Introduction

Fractional photothermolysis, however, relies on the heterogeneous ablation of tissue, leaving portions of healthy tissue interspersed between treated regions, as shown in Figure 1.3 [right], and is used in the treatment of pigmented lesions, melasma, and acne scarring [26, 30]. The intention of this approach is to cause partial photothermolysis in the treated regions, referred to as “microthermal zones” (MTZs), so as to achieve similar, albeit lessened, resurfacing effects as in ablative resurfacing, while leaving the surrounding tissue to experience similar inflammatory wound healing responses to those observed in nonablative dermal remodeling [23, 31]. Research has shown that Fractionated methods may result in faster recovery times by reserving healthy tissue to more readily repair the laser-damaged tissue due to keratinocyte diffusion from the untreated into the treated zones [3]. Additionally, the non-ablative thermal damage to the surrounding tissue tightens and organizes collagen fibers while simultaneously promoting neocollagen secretion by fibroblasts [32, 33]. However, non-Fractionated methods are still widely used in treatments based on manufacturers’ preferences and the relative youth of the Fractionated methods in clinical applications.
Chapter 1. *Introduction*

Figure 1.4 The severity of the thermal effects observed following illumination of biological soft tissue with incident high-energy laser light decreases with penetration depth into the tissue [34].

### 1.2 Laser-Tissue Interactions

When examining the effects of laser light on biological tissue, there are a myriad of considerations that may dramatically influence the final approach from an engineering perspective. Specific examples of such factors include inhomogenous scattering and absorption within the tissue, the tissue’s density and water content, and the degree of reflection at the surface of the tissue [34]. However, for dermatological applications it is convenient to consider only the case of soft epithelial tissues like skin. Whereas the specific effects will vary from person to person based on aspects like melanin, fat, and water content, the resultant thermal effects within the tissue remain largely consistent from person to person.
1.2.1 Photothermal Effects

Regardless of the application, illuminating soft tissues with a high-powered laser will result in a layering of distinct photothermal effects, resultant from the tissue’s poor heat conductivity into the atmosphere coupled with its inherent thermal relaxation time [34, 35]. These effects differ between applications based on the laser’s pulsewidth, energy density, and wavelength, along with the tissue’s own optical properties; however, as shown in Figure 1.4, the order of their occurrence by depth remains largely consistent: vaporization, carbonization, coagulation, oedema, and hyperthermia.

The topmost phenomenon involves the vaporization of intracellular water molecules. At temperatures in excess of 100°C, water molecules within the surface tissue vaporize, which induces ruptures and thermal decomposition resulting in localized tissue cavitation [5]. This cavitation is then reinforced by the subsequent layer of carbonization, which is a consequence of insufficient thermal conduction of excess heat away from the tissue surface.

When utilizing the laser as an optical scalpel, the carbonization is useful in sustaining the cut by preventing the tissue from reforming; however, for most dermatology applications it is considered an undesirable consequence of laser illumination.

The remaining layers of thermal coagulation, oedema, and hyperthermia are considered to be more reversible than previous two phenomena, since the body has mechanisms for their repair. These layers elicit both immunological and wound-healing responses within the tissue, which have been shown to result in increased collagen production and reduced vascularization, the principal goals of skin resurfacing and port-wine stain treatments respectively [6, 36].
1.2.2 Photothermolysis, Photoplasmolysis, and Photoacoustics

However, when considering the photoablation of individual pigmented chromophores – like tattoo inks, contrast agents, or pigmented lesions – the aforementioned photothermal effects are typically considered to be concomitant localized tissue reactions, rather than intentional biological responses. In such cases, the primary goal of the procedure is the photo-induced destruction of the target chromophore through a combination of either photothermolysis or photoplasmolysis along with inelastic photoacoustic expansion (photomechanical action), based on the pulse width and energy density of the laser [5].

Photothermolysis, as discussed in Section 1.1.1, is intended to break apart the target chromophore into much smaller molecules by thermally compromising their structural integrity. The effect is achieved by using ultra-short pulse durations in the nanosecond regime to induce an acoustic shockwave, whose intensity exceeds the fracture thresholds of the molecules [37]. The acoustic pressure wave is produced through the rapid thermal expansion of the chromophore immediately following optical absorption, in a process referred to as “photoacoustics.” Due to the extreme energies of the incident beam, the photoacoustic expansion is inelastic, fragmenting the target into smaller particles, which are more easily phagocytosed – engulfed by white blood cells – for removal by the vascular or lymphatic systems [38]. Additionally, the fragmentation of the chromophore may generate particles whose dimensions are smaller than the wavelength of visible light, and would therefore be effectively “cleared” regardless of their presence or absence in the tissue.

Photoplasmolysis acts in a similar fashion and produces a photoacoustic pressure wave through the same inelastic expansion; however, the principal difference between photoplasmolysis and photothermolysis is that the former effect results in the optical breakdown and subsequent plasma formation by the target chromophore formed by the ionizing effects of
the incident beam [39–41]. Consequently, the ensuing explosive inelastic expansion is much more extreme and results in a more thorough destruction of the pigment [34]. The two effects are distinguished by the pulse width of the incident beam, with photoplasmolysis exclusively occurring with pulse widths shorter than 1000 ps and photothermolysis occupying the longer pulse width regime up to a few hundred nanoseconds.

1.3 Contemporary Techniques

Regardless of the application, however, the means of tissue illumination in clinical practice is largely consistent, with only slight differences based on the intended application or the manufacturer’s proprietary designs. This delivery method, which has been systematically accepted throughout the clinical laser community, characteristically proceeds in the following manner: the beam of light is first redirected from a laser source into a tethered hand-piece by means of either a fiber-optic bundle or an articulated mirror system [42]. The light is then shone upon the target area of the skin through free-space propagation from a pre-determined distance away from the surface. An adjustable lens within the hand-piece focuses the light onto the tissue surface while the operating distance is typically set by an adjustable or interchangeable stand-off or plastic guard, such as the thin metal stand-off attached to the end of the hand-piece shown in Figure 1.5.

In addition to directing the laser light toward the tissue, many medical laser manufacturers utilize interchangeable hand-pieces to enhance the capabilities of their system. For example, a company called Cynosure currently produces the MedLite C6 laser tattoo removal system, which is comprised of a powerful Q-Switched Nd:YAG laser that is directed toward a hand-piece via an articulated mirror system [44]. The Nd:YAG’s principal harmonic output is a beam of 1064 nm IR light; the beam can also be frequency doubled
to output the secondary harmonic beam at 532 nm in the visible spectrum. The MedLite C6 system, also incorporates interchangeable hand-pieces, as shown in Figure 1.5, which contain dye-doped modified polymers. These polymers are pumped by the frequency doubled Nd:YAG in order to output lower frequency beams. For example, a polymer within the hand-piece doped with Pyrromethene650 (PM-650) dye will output a beam at 650 nm, whereas Pyrromethene597 (PM-597) will output a beam at 585 nm [43]. Consequently, such devices are primarily utilized in the selective photothermolysis of multi-colored tattoos for applications wherein the Nd:YAG’s harmonic outputs are insufficient (i.e. for the ablation of chromophores that are not strongly absorbing at 532 or 1064 nm).

Other companies produce similar units, each of which illuminates the tissue through free-space delivery of light out of a hand-piece. The principal difference between any two systems typically simplifies down to the lasing material (Nd:YAG, Er:YAG, CO2, etc.), the pulse width, and the manufacturer’s proprietary technologies, like the aforementioned dye-doped polymers. However, certain manufacturers also incorporate a means of fractionating the beam out of the hand-piece for fractional photothermolysis applications, as discussed in
Section 1.1.2. For example, the Fraxel Laser System, produced by Solta Medical, employs a CO$_2$ laser that is directed into a hand-piece wherein the beam is split up into an array of smaller beams to create thousands of microthermal zones (MTZs) [45].

1.4 Alternative Approaches

Despite the dangers presented by modern laser techniques, there are surprisingly few viable alternatives for achieving the same degree of effectiveness that laser procedures offer in the field of dermatology. Moreover, many of the existing alternatives may result in seriously adverse side effects including subdermal distortions and severe scarring. For example, surgical excision and dermabrasion were the primary techniques employed in tattoo removal prior to the implementation of lasers; however, considering that the former technique necessitated complex wound healing procedures and the latter utilized abrasive surfaces to scrape layers of skin off until the tattoo was removed, both techniques resulted in unacceptably high rates of scarring [8]. An additional technique offered through some private tattoo parlors, but distinctly absent in clinical settings, is the electrosurgical removal of tattoos. The process is colloquially referred to as “branding,” but is largely ineffective for removal of the ink and may result in electrosurgical burns, chronic pain, nerve damage, and moderate to severe hypertrophic scarring [46].

By contrast, skin resurfacing procedures present a much wider range of options; however, few can boast the same level of effectiveness as fractionated photothermolysis. In scar removal, for instance, microdermabrasion and chemical peels remove layers of the epidermis to exfoliate the skin, often also incorporating nutrient-rich creams or balms following the procedure [26, 47]. Deeper dermabrasion and chemical peel methods may also encourage collagen production and organization due to the relatively large fibroplasia zone [48]. How-
ever, in the event the scar tissue is anchored in the dermis, the efficacy of these techniques
is severely limited in that such techniques also produce a de-epithelialized wound, which is
prone to infection during the wound-healing process and can result in permanent scarring
[49].

1.5 Problems in Medical Photonics

Although dermatological laser procedures have progressed substantially since the tech-
nology was first put into practice by Leon Goldman in 1963, modern techniques still exhibit
many of the same problems as the initial methods [8, 50, 51]. Two of the more predominant
problems involve the high risk of ocular injury to the practitioner and the high potential for
negative tissue effects in the patient [52–55]. Both of these concerns are directly resultant
from the operational design of the light delivery systems used in dermatology settings; addi-
tionally, both of these concerns are systematically accepted in clinical applications, because
there simply are no alternative designs.

Specifically, modern dermatologic lasers operate through open-air, free-space propa-
gation of the laser light between the control hand-piece and the patient’s skin, as discussed
in Section 1.3. That being the case, there is a distinct and consistent possibility for the
un-confined light to reflect off the patient’s skin toward the practitioner or any other person
present. Considering that the vast majority of techniques utilize laser energies between 1
and 1000 $J/cm^2$, a diffuse reflection of even just 1% of the incident energy may be enough to
cause permanent eye damage to anyone in the operating room [52]. This is particularly the
case for ultra-short pulsed laser systems in the picosecond to microsecond regimes, as the
pulse can result in microcavitation or photoacoustic destruction within retinal melanosomes,
along with intra-retinal damage produced through the ionizing effects of picosecond pulse
lasers [56]. Moreover, it has been reported that the vast majority of laser related injuries involve serious damage to the retina, which consideration is only made worse by the extremely limited medical or surgical treatments available to treat ocular laser injuries [52].

Consequently, in all dermatological laser applications, the practitioner and the patient are required to wear protective eyewear with a high optical density for the wavelength being used [57]. The high optical density limits the degree to which those wavelengths can pass through the lenses of the eyewear, although most eyewear occludes a broad spectrum of wavelengths rather than a single wavelength [58]. However, to complicate matters further, the high optical density of the protective eyewear intended to make the procedures safer, in fact dramatically hinders the vision of the laser practitioner across a broad spectrum of wavelengths, which increases the difficulty of the procedure and the likelihood of unwanted overexposure.

In addition to the ocular hazards, another consequence of the free-space propagation design is that the light is transmitted into the skin at a tissue-air interface. As discussed in Section 1.2, the poor heat conductivity at this interface prevents adequate heat dissipation, resulting in carbonization, vaporization, and a subsequent inflammatory response. Whereas most of these effects fade within a few days of treatment, more permanent hyperpigmentation, hypopigmentation, purpura, and scarring are possible side effects of excessive damage or uneven wound healing [59]. Moreover, the potential for damage increases noticeably for patients of darker skin phototypes (i.e. higher melanin content or density), so much greater care must be taken for such patients [60, 61].

There are also other factors related to the delivery design that further exacerbate these problems. For example, there is a substantial degree of operator error introduced because the hand-pieces are aimed at transdermal targets held suspended above the tissue
Figure 1.6 When placed against human skin, light within the waveguide may couple out and propagate within the tissue toward a target chromophore, such as tattoo ink or a melanotic tumor.

surface, which can result in harmful overexposure of the tissue, causing permanent scarring and increased recovery times. Moreover, due to the inconsistencies of the laser delivery methods and the rapid rate at which the oedema forms within the tissue, most procedures necessitate a lengthy series of treatments, rather than a single, more effective treatment, which only serves to compound the aforementioned concerns with every subsequent visit, particularly in pigment clearing procedures like tattoo removal [18, 20, 24].

1.6 Project Proposal

Given the consistent problems presented by modern laser dermatology equipment in addition to the distinct lack of viable alternatives, it is therefore the intention of this thesis to present a technique that either circumvents or resolves each of the more predominant problems presented by contemporary approaches. The method presented herein, entitled Photonic Ablation via Quantum Tunneling (PAQT), delivers the laser light through physical contact of an optical waveguide with the tissue, as shown in Figure 1.6. Consequently, the
Figure 1.7 An optical waveguide may be built into a simple hand-piece to couple laser light directly into the skin, while also incorporating additional technologies like contact cooling and an electronic interlock to enhance safety and efficacy.

The majority of the high-energy light is transmitted directly into the tissue, leaving no room for tissue reflections to endanger the practitioner. Not only does this render the procedure as a whole substantially safer, it may also permit the use of less optically-dense eyewear that may not impose the same degree of strain during use.

Moreover, the direct contact between the waveguide and the tissue eliminates the tissue-air interface and allows for the incorporation of contact cooling techniques. Doing so could protect the epidermis and upper-dermal layers from excessive thermal damage,
which may reduce down-time between procedures, the likelihood for scarring and other negative tissue complications, and mitigate pain by numbing nerves. Other such cooling technologies have been implemented in other clinical applications, like hair removal and port wine stain treatment [62]. However, most focus on preoperatively cooling the epidermis, rather than appropriately dissipating the photothermally generated heat, which would be achieved through contact cooling.

Additionally, by necessitating physical contact of a large flat surface with the tissue, it is much more likely that the hand-piece will not only be steadier than when using other techniques, but will also be appropriately positioned with respect to the tissue. The steadier design should reduce errors presented by vibrations and strain placed on the practitioner, while also allowing for more consistent ablation of the intended target. Figure 1.7 demonstrates a preliminary design that incorporates such a waveguide. Since the technique represents a significant paradigm shift in the approach to laser dermatology, the design is modeled after a standard computer mouse so as to be more readily approachable for practicing clinicians. However, this design also incorporates contact cooling technologies in addition to an electronic interlock, which prohibits accidental firing of the device when not in contact with the tissue.

However, there are many questions that must first be answered before PAQT can be produced as a viable medical laser modality. While we were not the first to produce planar waveguides, the majority of modern fabrication procedures are designed specifically to diminish the evanescent effects that we intend to utilize for light delivery. Consequently, our first concern was to establish a fabrication procedures designed to make use of the evanescent leaking when in contact with tissue. It has also yet to be seen whether PAQT will ablate tissue in a manner comparable to current methods. Although the laser energy
transmitted may be the same, our method of delivery is vastly different from traditional
free-space propagation and subsequently may yield different results.

1.7 Technical Objectives

Therefore, in order to initiate the development of a safer and more controllable medical
laser device, it must first be established that PAQT is a viable method for coupling laser
light into biological tissue. This thesis is intended to investigate factors primarily relating
to ease of fabrication, physical design, and efficiency of light delivery. The technical aspects
of the project are discussed in the following chapters; however, the general objectives of this
study are as follows:

• to establish cladding procedures for glass waveguides utilizing metallic coatings,

• to utilize photoacoustics to confirm significant light delivery into tissue phantoms,

• and to determine the energy delivered through the active area of the waveguides as it
relates to film thickness and angle of internal reflection.
Chapter 2

Technical Discussion

As a brief overview, the PAQT technique involves the fabrication of a planar optical waveguide through which laser light is transmitted. Upon direct physical contact of the waveguide’s active area with biological tissue, the light either refracts or optically tunnels out of the waveguide into the tissue. By tuning the composition and thickness of the active area, it is possible to control the relative portion of light that is transmitted out of the waveguide. The following sections discuss the technical aspects of the approach in greater detail before describing the research methods employed in this investigation and their subsequent results.

2.1 Planar Waveguide Mediated Light

The cardinal strength of the PAQT technique is that it is founded upon well-established optical principals, the most fundamental of which is the concept of the optical waveguide (a.k.a “lightguide”). Used to transmit electromagnetic waves in the optical spectrum, waveguides composed of optically-transparent materials function by reflecting light within their bounding surfaces, forcing the light to propagate along their length [63, 64]. Optical waveguides have been used extensively in a variety of applications, including optoelectronic
integrated circuitry, attenuated total reflection spectroscopy (ATR), and laser beam generation within planar waveguides; however, our application is intended for use in the controlled delivery of light into tissue and for that endeavor we must consider the types of interactions light can undergo along the length of the waveguides in question [65–67].

2.1.1 The Fresnel Equations

Depending on the optical properties of the waveguide and the external medium, along with the angle of the incident light, a beam incident on one of these bounds may either refract into the external medium, travel along the interface, or totally internally reflect back into the waveguide. The mathematical equations that govern each of these conditions are collectively known as the Fresnel Equations, and can be directly derived from Maxwell’s Equations in isotropic media, shown below [68].

\[
\nabla \times \mathbf{H} = j + \frac{\partial D}{\partial t} \quad (2.1) \\
\n\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t} \quad (2.2) \\
\n\nabla \cdot D = \rho \quad (2.3) \\
\n\n\nabla \cdot B = 0 \quad (2.4) \\
\n\n\n\mathbf{j} = \sigma \mathbf{E} \quad (2.5) \\
\n\n\nD = \varepsilon \mathbf{E} \quad (2.6) \\
\n\n\nB = \mu \mathbf{H} \quad (2.7)
\n\]

Expressing the Wave Equation in terms of only the Electric Field, \( \mathbf{E} \), necessitates that the magnetic components of Maxwell’s Equations (\( \mathbf{H} \) and \( B \)) be replaced with their electric equivalents. Combining and reducing Maxwell’s Equations to only their electric components
Chapter 2. Technical Discussion

yields the Laplacian of \( E \), shown below in Eq. (2.8).

\[
\nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu \sigma \frac{\partial^2 E}{\partial t^2} \tag{2.8}
\]

One solution of this equation is in the complex form of a plane-polarized harmonic plane wave:

\[
E = \mathcal{E} \exp \left[ i \omega \left( t - \frac{x}{v} \right) \right] \tag{2.9}
\]

with the condition that:

\[
\frac{\omega^2}{v^2} = \omega^2 \varepsilon \mu - i \omega \mu \sigma \tag{2.10}
\]

In this representation, \( \mathcal{E} \) represents the vector amplitude for the Electric wave. The complex representation of the refractive index (\( \tilde{n} = n - i \kappa \)) can then be readily derived from Eq. (2.10) by defining \( \tilde{n}^2 = \frac{\omega^2}{v^2} \). This representation utilizes \( n \) and \( \kappa \) to represent the real and complex components of the refractive index respectively. By accounting for the speed of light (\( c \)), relative permeability (\( \mu_r \)), relative permittivity (\( \varepsilon_r \)), and the complex refractive index, Equation (2.9) can be rewritten to incorporate some of the more useful optical properties. The equation can then be generalized for any plane-polarized wave propagating in the direction given by the unit vector \( \hat{s} = (\alpha i + \beta j + \gamma k) \), to yield the Wave Equation in terms of \( E \) in Eq. (2.11). Additionally, a more rigorous derivation is included in Appendix A.

\[
E = \mathcal{E} \exp \left[ i \left( \omega t - \frac{2\pi \tilde{n}}{\lambda} (\alpha x + \beta j + \gamma z) \right) \right] \tag{2.11}
\]

By a similar process, it is also possible to derive the Wave Equation in terms of only its magnetic components, yielding Eq. (2.12) below.

\[
H = \mathcal{H} \exp \left[ i \left( \omega t - \frac{2\pi \tilde{n}}{\lambda} (\alpha x + \beta j + \gamma z) \right) \right] \tag{2.12}
\]
Figure 2.1 The sign conventions defining the positive directions for the Electric and Magnetic components of an obliquely incident beam upon a secondary medium for [left] Transverse Magnetic (TM, p-) polarized light and [right] Transverse Electric (TE, s-) polarized light.

Whereas polarization does not have any implications for a beam that is normal to the interface, since the magnetic and electric components would both be parallel to the interface, in cases of oblique incidence in non-absorbing media, as is observed in optical waveguides, the polarization state of the beam has distinct implications with regard to the relative reflection and transmission coefficients. For the purposes of this study, we will consider the case of a beam that is linearly polarized with a transversely electric component. This polarization state is referred to as TE or s-polarized, wherein “s” stands for “senkrecht” (German for “perpendicular”). The alternative polarization state is that of the transverse magnetic beam, referred to as TM or p-polarized, wherein “p” stands for “parallel” (the derivation for the transverse magnetic component is included in Appendix A).

The positive sign conventions for both polarizations of an obliquely incident beam are shown in Figure 2.1, with the conventions for TM polarized light on the [left] whereas those for TE polarized light are on the [right]. Based on these conventions, the boundary
Chapter 2. Technical Discussion

Conditions for an obliquely incident s-polarized beam are mathematically represented below:

\[ E_i + E_r = E_t \quad (2.13) \]
\[ H_i \cos \vartheta_0 - H_r \cos \vartheta_0 = H_t \cos \vartheta_1 \quad (2.14) \]

Again, we will derive these conditions such that they deal with only the electric field. It is important to note that the \( \cos \vartheta \) terms arise from translating \( \hat{s} \) over to perpendicular incidence, so that \( E \) and \( H \) are tangential (parallel) to the surface. Furthermore, it is convenient to define new variables to represent components of the translated boundary conditions, as shown below:

\[ E_i = E_i \quad H_i = H_i \cos \vartheta_0 = Y_0 \cos \vartheta_0 E_i \]
\[ E_r = E_r \quad H_r = Y_0 \cos \vartheta_0 E_r \]
\[ E_t = E_t \quad H_t = Y_1 \cos \vartheta_1 E_t \]

Rewriting the boundary conditions in the terms shown above yields Eq. (2.15) and (2.16), from which we can obtain expressions for the amplitude reflection and transmission coefficients for p-polarized light, \( \rho_s \) and \( \tau_s \) in Equations (2.17) and (2.18) respectively, by defining \( \rho_s = \frac{E_r}{E_i} \) and \( \tau_s = \frac{E_t}{E_i} \). In this form, \( Y \) represents the optical admittance in non-absorbing media equivalent to \( Y = n \mathcal{Y} = n \sqrt{\frac{\varepsilon_0}{\mu_0}} \), where \( \mathcal{Y} \) is the characteristic optical admittance of free space.
Chapter 2. Technical Discussion

\[ H_i - H_r = H_t \]  \hspace{1cm} (2.15)

\[ y_0 \cos \theta_0 E_i - y_0 \cos \theta_0 E_r = y_1 \cos \theta_1 E_t \]  \hspace{1cm} (2.16)

\[ = y_1 \cos \theta_1 (E_i + E_r) \] (Sub. in Eq. 2.13)

\[ E_i \left( y_0 \cos \theta_0 - y_1 \cos \theta_1 \right) = E_r \left( y_0 \cos \theta_0 + y_1 \cos \theta_1 \right) \] (Distribute and factor)

\[ \frac{E_r}{E_i} = \frac{y_0 \cos \theta_0 - y_1 \cos \theta_1}{y_0 \cos \theta_0 + y_1 \cos \theta_1} \] (Rearrange fractions)

\[ \rho_s = \frac{E_r}{E_i} \] (Definition of \( \rho \))

\[ \therefore \rho_s = \frac{y_0 \cos \theta_0 - y_1 \cos \theta_1}{y_0 \cos \theta_0 + y_1 \cos \theta_1} \] (2.17)

\[ y_0 \cos \theta_0 E_i - y_0 \cos \theta_0 E_r = y_1 \cos \theta_1 E_t \] (Continued from above)

\[ y_0 \cos \theta_0 E_i - y_0 \cos \theta_0 (E_t - E_i) = y_1 \cos \theta_1 E_t \] (Sub in Eq. 2.13)

\[ E_i (2y_0 \cos \theta_0) = E_t (y_0 \cos \theta_0 + y_1 \cos \theta_1) \] (Factor)

\[ \frac{E_t}{E_i} = \frac{2y_0 \cos \theta_0}{y_0 \cos \theta_0 + y_1 \cos \theta_1} \] (Divide by \( E_i \))

\[ \tau_s = \frac{E_t}{E_i} \] (Definition of \( \tau \))

\[ \therefore \tau_s = \frac{2y_0 \cos \theta_0}{y_0 \cos \theta_0 + y_1 \cos \theta_1} \] (2.18)

Next we define the expressions for the Irradiance of each of the three beams of interest,

wherein \( I_i \) represents the Irradiance of the incident beam derived from the general expression for Irradiance, as shown below.
\[ I = \frac{1}{2} \mathbb{R} \{ E \times H^* \} \] (2.19)

\[ I = \mathbb{R} \left\{ \frac{1}{2} E (\gamma \cos \vartheta E)^* \right\} \] (Scalar Irradiance)

\[ \therefore I_i = \frac{1}{2} \gamma_0 \cos \vartheta_0 E_i E_i^* \] (2.20)

Following along the same approach, we will now define \( I_r \) and \( I_t \) in terms of \( I_i \) in order to derive the relative reflection and transmission coefficients for s-polarized light, \( R_s \) and \( T_s \) respectively.

\[ I_r = \mathbb{R} \left\{ \frac{1}{2} E_r (\gamma_0 \cos \vartheta_0 E_r)^* \right\} \] (Scalar Reflected Irradiance)

\[ = \frac{1}{2} \rho_s E_i (\gamma_0 \cos \vartheta_0 \rho_s E_i)^* \] \( (E_r = \rho_s E_i) \)

\[ = \frac{1}{2} \rho_s^2 \gamma_0 \cos \vartheta_0 E_i E_i^* \] \( \text{(Reflect the format of } I_i) \)

\[ \therefore I_r = \rho_s^2 I_i \] (2.21)

\[ I_t = \mathbb{R} \left\{ \frac{1}{2} E_t (\gamma_1 \cos \vartheta_1 E_t)^* \right\} \] (Scalar Transmitted Irradiance)

\[ = \frac{1}{2} \tau_s E_i (\gamma_1 \cos \vartheta_1 \tau_s E_i)^* \] \( (E_t = \tau_s E_i) \)

\[ = \frac{1}{2} \tau_s^2 \gamma_1 \cos \vartheta_1 \gamma_0 \cos \vartheta_0 E_i E_i^* \] \( \text{(Reflect the format of } I_i) \)

\[ \therefore I_t = \frac{\gamma_1 \cos \vartheta_1}{\gamma_0 \cos \vartheta_0} \tau_s^2 I_i \] (2.22)

From here, considering the simple energy balance equation below allows for the direct derivation of \( R_s \) and \( T_s \). It is also worth noting that by translating \( \hat{s} \) to perpendicular incidence and incorporating the \( \cos \vartheta \) components, the traditional condition that \( 1 - R = T \) still holds true.
\[ I_0 = I_1 \] (Conservation of energy)

\[
\mathbb{R} \left\{ \frac{1}{2} E_0 \times H_0^* \right\} = \mathbb{R} \left\{ \frac{1}{2} E_1 \times H_1^* \right\} \quad \text{(Sub in. Eq. 2.19)}
\]

\[
\mathbb{R} \left\{ \frac{1}{2} (E_i + E_r) (H_i - H_r)^* \right\} = \mathbb{R} \left\{ \frac{1}{2} E_t H_t^* \right\} \quad \text{(Scalar components)}
\]

\[
\frac{1}{2} (E_i + E_r) (\gamma_0 \cos \vartheta_0 E_i - \gamma_0 \cos \vartheta_0 E_r)^* = \frac{1}{2} E_t \gamma_1 \cos \vartheta_1 E_t^* \quad (H = \gamma \cos \vartheta E)
\]

\[
\frac{1}{2} \gamma_0 \cos \vartheta_0 (E_i + \rho_s E_i) (E_i - \rho_s E_i)^* = \frac{1}{2} \gamma_1 \cos \vartheta_1 \tau_s^2 E_i E_i^* \quad (E_r = \rho_s E_i)
\]

\[
\frac{1}{2} \gamma_0 \cos \vartheta_0 E_i E_i^* (1 - \rho_s^2) = \frac{1}{2} \gamma_1 \cos \vartheta_1 \tau_s^2 E_i E_i^* \quad \text{(Factor)}
\]

\[
I_i (1 - \rho_s^2) = \frac{1}{2} \gamma_1 \cos \vartheta_1 I_i \quad \text{(Sub. in Eq. 2.20)}
\]

\[
I_i - \rho_s^2 I_i = \frac{1}{2} \gamma_1 \cos \vartheta_1 I_i \quad \text{(Distribute)}
\]

\[
I_i - I_r = I_t \quad \text{(Sub in Eq. 2.21, 2.22)}
\]

\[
1 - \frac{I_r}{I_i} = \frac{I_i}{I_t} \quad \text{(Divide by } I_i)
\]

\[ \therefore 1 - R = T \quad (2.23) \]

Following from this, we find the expressions for \( R_s \) and \( T_s \) shown below in Equations (2.24) and (2.25), respectively.

\[ R_s = \rho_s^2 = \left( \frac{\gamma_0 \cos \vartheta_0 - \gamma_1 \cos \vartheta_1}{\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1} \right)^2 \quad (2.24) \]

\[ T_s = \tau_s^2 \left( \frac{\gamma_1 \cos \vartheta_1}{\gamma_0 \cos \vartheta_0} \right) = \frac{4 \gamma_0 \gamma_1 \cos \vartheta_0 \cos \vartheta_1}{(\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1)^2} \quad (2.25) \]

By substituting \( \gamma = n \gamma \), the traditional form of the Fresnel equations for s-polarized light are produced.
Chapter 2. Technical Discussion

\[ R_s = \left( \frac{n_0 \cos \vartheta_0 - n_1 \cos \vartheta_1}{n_0 \cos \vartheta_0 + n_1 \cos \vartheta_1} \right)^2 \]  \hspace{1cm} (2.26)

\[ T_s = \frac{4n_0n_1 \cos \vartheta_0 \cos \vartheta_1}{(n_0 \cos \vartheta_0 + n_1 \cos \vartheta_1)^2} \]  \hspace{1cm} (2.27)

These equations describe the degree of transmission and reflection observed at the intersections of two materials of differing optical properties, namely the waveguide and any surrounding material. When a beam of light is incident upon a planar interface, such as those shown in Figure 2.1, a portion of the light may reflect whereas the remainder may refract into the secondary medium. For each of the preceding equations, \( T_s \) represents the percent of light incident upon the intersection that is transmitted into the second medium, whereas \( R_s \) represents the percent of light reflected back within the original material. A similar derivation yields the percent reflection (\( R_p \)) and transmission (\( T_p \)) for p-polarized light, shown below in Equations (2.28) and (2.29) respectively.

\[ R_p = \left[ \frac{n_0 \cos \vartheta_0 - n_1 \cos \vartheta_1}{n_0 \cos \vartheta_0 + n_1 \cos \vartheta_1} \right]^2 \]  \hspace{1cm} (2.28)

\[ T_p = \frac{4n_0n_1 \cos \vartheta_0 \cos \vartheta_1}{(n_0 \cos \vartheta_0 + n_1 \cos \vartheta_1)^2} \]  \hspace{1cm} (2.29)

### 2.1.2 Snell’s Law

In cases of refraction, the Fresnel Equations govern the degree of reflection and refraction at the interface, leaving a portion of the incident beam to propagate within each material. However, there are two additional possible outcomes, wherein there is no observable refraction and \( T = 0 \), as shown in Figure 2.2 (b) and (c). In order to determine the conditions at which these outcomes occur, we must first derive another equation to directly
Figure 2.2 The conditions for the three possible outcomes of the Fresnel equations and Snell’s Law are: (a) when \( n_2 > n_1 \) light is refracted into the second material, (b) when \( n_2 < n_1 \) and \( \theta = \theta_c \) the light travels along the intersection, or (c) when \( n_2 < n_1 \) and \( \theta > \theta_c \) the light undergoes TIR.

relate the refractive indices and propagation angles of each the incident, transmitted, and reflected beams.

We start by considering the Irradiance of each of the three beams individually. The general expression for Irradiance can be concisely represented in the following manner:

\[
I = \frac{1}{2} n Y |E|^2 \exp \left[ -\frac{4\pi k}{\lambda} (ax + by + cz) \right]
\] (2.30)

Following that, we apply a few simple boundary conditions:

- the tangential components of \( \mathbf{E} \) and \( \mathbf{H} \) are continuous across the boundary
- \( \hat{s}_{\text{reflected}} = \langle \alpha_r, \beta_r, \gamma_r \rangle \)
- \( \hat{s}_{\text{transmitted}} = \langle \alpha_t, \beta_t, \gamma_t \rangle \)

From these conditions, we can represent each of the three wavefronts of interest – incident, reflected, and transmitted – consistently for both Electric and Magnetic components in the following forms:
Incident : \( \exp \left[ i \left( \omega_i t - \left( \frac{2\pi n_1}{\lambda_i} \right) \left( x \sin \vartheta_i + z \cos \vartheta_i \right) \right) \right] \)

Reflected : \( \exp \left[ i \left( \omega_r t - \left( \frac{2\pi n_1}{\lambda_r} \right) \left( \alpha_r x + \beta_r y + \gamma_r z \right) \right) \right] \)

Transmitted : \( \exp \left[ i \left( \omega_t t - \left( \frac{2\pi n_2}{\lambda_t} \right) \left( \alpha_t x + \beta_t y + \gamma_t z \right) \right) \right] \)

Then, in order to satisfy continuity for all \( x, y, \) and \( t \) at \( z = 0 \), it can be determined that \( \omega_i \equiv \omega_r \equiv \omega_t \). In other words, there is no change in frequency for reflected or transmitted waves as compared to the incident wave. That being the case, we can extend the identical equality to the wavelength, such that \( \lambda_i \equiv \lambda_r \equiv \lambda_t \). Additionally, since the representation of the incident beam is oriented such that it is independent of \( y \), the conclusion can be drawn that the directions of the reflected and transmitted waves are confined to the plane of incidence. The mathematical implications of these observations are as follows:

\[
0 \equiv n_1 \beta_r \equiv n_2 \beta_i \\
\left( \frac{2\pi n_1}{\lambda} \right) \left( x \sin \vartheta_i + z \cos \vartheta_i \right) \equiv \left( \frac{2\pi n_1}{\lambda} \right) \left( \alpha_r x + \gamma_r z \right) \\
\equiv \left( \frac{2\pi n_2}{\lambda} \right) \left( \alpha_t x + \gamma_t z \right) \\
\therefore \sin \vartheta = \alpha \\
\therefore \cos \vartheta = \gamma \\
n_1 \sin \vartheta_i \equiv n_1 \alpha_r \equiv n_2 \alpha_t 
\] (2.31)

Following from Eq. 2.31 and defining the angles of reflection and transmission to be \( \vartheta_r \) and \( \vartheta_t \) respectively, we find the following relation to be true.

\[
n_1 \sin \vartheta_i \equiv n_2 \alpha_t 
\] (From Eq. 2.31)

\[
= n_2 \sin \vartheta_t 
\] (2.32)

29
Redefining $\vartheta_i$ and $\vartheta_t$ to be $\vartheta_1$ and $\vartheta_2$ respectively, results in the traditional expression for Snell’s Law shown below:

$$n_1 \sin \vartheta_1 = n_2 \sin \vartheta_2$$  \hspace{1cm} (2.33)

### 2.1.3 Waveguide Operation

In order for a material to operate as a waveguide, the light must be totally internally reflected within the bounds of the material, such that all of the light incident upon the intersection of the substrate and external medium reflects back within the original medium, as shown in Figure 2.2 (c). Optical waveguides operate exclusively under this third condition, such that the refractive index of the external medium $(n_2)$ is less than that of the waveguide $(n_1)$, forcing the light to internally reflect between its surfaces as it propagates along the length of the waveguide. However, in order to achieve total internal reflection (TIR), the incident angle of the light upon the boundary must also be greater than the critical angle, $\theta_c$, which can be mathematically determined from Snell’s Law (eq. 2.33) [69].

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$  \hspace{1cm} (2.34)

$$\theta_c = \arcsin \left( \frac{n_2}{n_1} \right)$$  \hspace{1cm} (2.35)

Assuming that the external material is of lower refractive index than the waveguide, it can be determined that $\sin \theta_2 \rightarrow 1$, which reduces Snell’s Law to equation (2.35), allowing for direct calculation of the critical angle when $n_1$ and $n_2$ are known. Using this value in conjunction with the Fresnel equations for reflectivity for TE and TM polarized light (Eq. (2.26) and 2.28 respectively) allows for the generation of a plot depicting polarization-dependent reflectivity versus internal reflection angle. For example, the reflectivity for a waveguide composed of BK7 glass ($n_1 = 1.519$) considering the external material to be air
Figure 2.3 The reflectivity for p-polarized light is consistently lower than for s-polarized light, until the critical angle at $\theta_c = 41.17^\circ$, after which the light undergoes TIR. (\(n_2 = 1.0\)) is shown in Figure 2.3 [70]. The graph demonstrates that the reflectivity for TE polarized light is consistently greater than for TM polarized light, until the critical angle at $\theta_c = 41.17^\circ$, after which the light undergoes total internal reflection. Additionally, the TM polarization reflectivity drops to 0 at the Brewster Angle, which is given by Equation (A.79), and is derived in Appendix (A.2.2).

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$  \hspace{1cm} (2.36)

However, in most cases it can not be assumed that only air will come in contact with the waveguide. Consequently, for most waveguides TIR is ensured by design through the incorporation of a cladding layer of known refractive index to encapsulate the waveguide, as shown in Figure 2.4 [71, 72]. Typically, optical waveguide substrates are composed of trans-
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Figure 2.4 Light in a waveguide undergoes TIR when $n_2 < n_1$ and $\theta$ exceeds the critical angle. This can be ensured by cladding the waveguide in a polymer or reflective metal layer of lower refractive index than the substrate.

Parent polymers, like Poly-methyl methacrylate (PMMA, $n = 1.49$), ceramics, or glasses, like Sapphire ($n = 1.77$) or Borosilicate BK7 glass ($n = 1.519$) [70, 73]. By selecting one such substrate with known refractive index, $\theta_c$ can subsequently be scaled by the appropriate selection of the cladding material. For example, using polymer claddings with refractive indices around $n = 1.32$ on a BK7 glass substrate typically results in a relatively large $\theta_c = 60.34^\circ$; however, choosing such a material limits the angles that support TIR to only those between $60.34^\circ$ and $90^\circ$.

On the other hand, certain metal claddings allow for a much broader range of total internal reflection angles, since they may have substantially lower refractive indices. For example, the polarization-dependent reflectivity plot for a BK7 glass waveguide clad in Silver ($n = 0.142$) is shown in Figure 2.5. The figure shows that the metal ensures a broad range of angles that support TIR after the critical angle of $5.36^\circ$, although the p-polarized light experiences a substantial decrease in reflectivity until the Brewster Angle at $5.34^\circ$. Such metal claddings demonstrate high reflectivity in the visible spectrum and support TIR throughout a much broader spectrum of angles; however, they also tend to introduce
Using Silver as a cladding allows for a wide range of acceptable angles that support TIR, from around $5.4^\circ < \theta_1 < 90^\circ$. However, the reflectivity of TM polarized light is severely limited at angles less than $\theta_c$. A degree of optical absorption, which may result in energy loss over long distances [74].

### 2.2 Evanescent Leaking and Optical Tunneling

The following section provides a discussion of the nano-scale effects observed at the bounding interfaces of an optical waveguide. Whereas the previous explanation of TIR and waveguide operation provides a conceptual overview of the phenomena involved in light transmission, the wave nature of light causes a variety of problems when describing TIR, due to common assumptions resultant from classical mechanics.

In classical mechanics, particularly on the macroscale, light is traditionally treated as a particle propagating in distinct energy packets referred to as photons. As such, diagrams like that shown in Figure 2.4 depict the paths of these photons bouncing off the surfaces
of the waveguide like particles off a barrier. However, research has demonstrated, even as far back as Isaac Newton, that the energy of the photons does not merely reflect off the surface, but rather exhibits a degree of penetration into the surrounding material [75].

2.2.1 Evanescent Fields

By contrast, treating the light as an electromagnetic wave instead of a discrete particle reveals that upon interacting with a secondary material of lower refractive index, the energy of the incident wave slightly crosses the interface of the two materials, penetrating the external medium in the form of an exponentially decaying field of energy, referred to as the evanescent field. In mathematically deriving the existence of the evanescent field, it is necessary to account for the non-propagating nature of the field first by defining the “propagation” vector \( \hat{s}_2 = \langle \sin \theta_2 i + 0 j + \cos \theta_2 k \rangle \), and second by recognizing that \( \theta_2 \) technically does not exist, since there is no refracting portion of the beam. Consequently, in the course of the derivation, all terms containing \( \theta_2 \) will need to be substituted out for terms that still apply to the situation of total internal reflection. We will also define \( E_{0,2} \) to be the electric field in the second medium and \( \mathcal{E}_{0,2} \) to be the amplitude of the field of interest. It is also convenient to temporarily define the wave vector \( k_2 = \frac{2\pi \hat{n}}{\lambda_0} \) for simpler expressions during the derivation.
\[ E_{0,2} = \mathcal{E}_{0,2} \exp \left[ i \left( \omega t - \left( \frac{2\pi n}{\lambda_0} \right) \hat{s}_2 \cdot \vec{r} \right) \right] \]  

(From Eq. A.15)

\[ = \mathcal{E}_{0,2} \exp \left[ i (\omega t - k_2 \hat{s}_2 \cdot \vec{r}) \right] \]  

(Sub. in \( k_2 \))

\[ = \mathcal{E}_{0,2} \exp \left[ i (\omega t - k_2 x \sin \theta_2 - k_2 z \cos \theta_2) \right] \]  

(Dot product)

\[ = \mathcal{E}_{0,2} \exp \left[ i \left( \omega t - k_2 x \sin \theta_2 \pm k_2 z \left( 1 - \sin^2 \theta_2 \right)^{\frac{1}{2}} \right) \right] \]  

(Trig. substitution)

\[ = \mathcal{E}_{0,2} \exp \left[ i \left( \omega t - k_2 x \frac{n_1}{n_2} \sin \theta_1 \pm k_2 z \left( 1 - \frac{n_1^2}{n_2^2} \sin^2 \theta_1 \right)^{\frac{1}{2}} \right) \right] \]  

(Snell’s law)

\[ = \mathcal{E}_{0,2} \exp \left[ i \left( \omega t - k_2 x \frac{n_1}{n_2} \sin \theta_1 \pm k_2 z \left( n_2^2 - n_1^2 \sin^2 \theta_1 \right)^{\frac{1}{2}} \right) \right] \]  

(Factor \( n_2 \))

\[ = \mathcal{E}_{0,2} e^{i \left( \omega t - k_2 x \frac{n_1}{n_2} \sin \theta_1 \right)} e^{-\frac{k_2}{n_2} z \left( n_1^2 \sin^2 \theta_1 - n_2^2 \right)^{\frac{1}{2}}} \]  

(Factor \( i \))

\[ = \mathcal{E}_{0,2} e^{i \left( \omega t - \frac{2\pi \tilde{n}_1}{n_0 n_2} x \sin \theta_1 \right)} e^{-\frac{2\pi \tilde{n}_2}{\lambda_0 n_0} z \left( n_1^2 \sin^2 \theta_1 - n_2^2 \right)^{\frac{1}{2}}} \]  

(Sub. out \( k_2 \))

\[ = \mathcal{E}_{0,2} e^{i \left( \omega t - \frac{2\pi n_1}{\lambda_0} x \sin \theta_1 \right)} e^{-\frac{2\pi}{\lambda_0} z \left( n_1^2 \sin^2 \theta_1 - n_2^2 \right)^{\frac{1}{2}}} \]  

(Simplify)

By defining \( \delta \) to be the penetration depth of the evanescent field into the secondary (rarer) medium, and representing it as shown in Equation (2.37), the final expression of the Electric field in the secondary medium can be represented as shown in Equation (2.38). Additionally, \( \delta \) is the depth at which the amplitude of the field reduces to \( 1/e \) of its maximum value

\[ \delta = \frac{\lambda_0}{2\pi \left( n_1^2 \sin^2 \theta_1 - n_2^2 \right)^{\frac{1}{2}}} \]  

(2.37)

\[ \therefore \ E_{0,2} = \mathcal{E}_{0,2} e^{i \left( \omega t - \frac{2\pi n_1}{\lambda_0} \sin \theta_1 \right) x} e^{-\frac{z}{\delta}} \]  

(2.38)

Many sources find it convenient to represent the Evanescent field in terms of irradiance, rather than Electric field intensity [76, 77]. The relationship between Irradiance and the Electric field is derived from the Poynting vector, as shown below.
\[ I = \frac{1}{2} n \mathcal{Y} |EE^*| \]  
(From Eq. A.31)

\[ = \frac{1}{2} n \sqrt{\frac{\varepsilon_0}{\mu_0}} |E|^2 \]  
(Sub. for \( \mathcal{Y} \))

\[ = \frac{1}{2} n \sqrt{\varepsilon_0 c^2} |E|^2 \]  
(\( \mu_0 = \frac{1}{\varepsilon_0 c^2} \))

\[ \therefore I = \frac{1}{2} n \varepsilon_0 c |E|^2 \]  
(2.39)

From here, it is possible to substitute in \( E_0 \), \( 2 \), and simplify the expression by defining \( I_0 \) to be the incident irradiance, represented as shown in Equation (2.41). Doing so yields Equation (2.40), which is in the same form as a simplified version of Beer’s law.

\[ I = \frac{1}{2} n \varepsilon_0 c [E_{0,2}]^2 \]  
(Sub in. Eq. 2.38)

\[ = \frac{1}{2} n \varepsilon_0 c \left[ E_{0,2}^2 e^{2i(\omega t - \frac{2\pi n_1}{\lambda_0} \sin \theta_1 x)} e^{-\frac{2z}{\delta}} \right] \]  
(Distribute)

\[ \therefore I = I_0 e^{-\frac{2z}{\delta}} \]  
(2.40)

where: \( I_0 = \frac{1}{2} n \varepsilon_0 c \left[ E_{0,2}^2 e^{2i(\omega t - \frac{2\pi n_1}{\lambda_0} \sin \theta_1 x)} \right] \)  
(2.41)

It is important to note that both \( \delta \) and \( I \) are strongly dependent upon the incident angle. Figure 2.6 demonstrates the penetration depth for a beam of 532 nm light incident upon a BK7 glass (\( n = 1.519 \)) and air (\( n = 1.0 \)) interface. The penetration depth decays quickly from a mathematically infinite value at the critical angle \( \theta_c = 41.17^\circ \). Near the critical angle, the penetration depth is on the order of the wavelength of the incident beam; however, at greater incident angles, the penetration depth reaches a horizontal asymptote around 74.05 nm. Figure 2.7 depicts the relative penetration depths for each of three interfaces: glass–air, glass–polymer (\( n = 1.32 \)), and glass–silver. Although TIR occurs throughout a
larger portion of the angular spectrum for the glass–silver interface than for the other two interfaces, the evanescent field penetration depth decays to a shorter horizontal asymptote. Consequently, a relatively thin layer of silver around 60 nm might make for a sufficiently thick cladding layer as the \( \frac{1}{e} \) depth would be shallower than the thickness for most of the angular spectrum.

\[
E_{0,2} = E_{0,2} e^{i\left(\frac{\omega t}{\lambda_0} - \frac{2\pi n_1}{\lambda_0} z \sin \theta_1\right)} e^{-\frac{2\pi}{\lambda_0} z \left(n_1^2 \sin^2 \theta_1 - n_2^2 + \kappa_2^2 - 2in_2\kappa_2\right)}^{\frac{1}{2}} \tag{2.42}
\]

It is also worth considering that \( \delta \), in this case, is calculated assuming that the secondary medium is non-absorbing. However, for metal clad waveguides, that may not necessary be applicable. Equation (2.42) represents an alternative expression for secondary materials that have a degree of optical absorption, denoted by the \( \kappa \) term from the complex
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**Figure 2.7** The penetration depth of the evanescent field created by totally internally reflecting 532 nm light decays slower for the Glass–Silver interface, but reaches a shallower horizontal asymptote than the Glass–Air and Glass–Polymer interfaces.

The optical absorption decreases the penetration depth of the evanescent field and forces the field to propagate at a slight angle to the interface, rather than along it as in the case of a non-absorbing rarer medium, which is not reflected in the graph shown in Figure 2.7.

For the mathematically inclined, this description of the evanescent field may be sufficient; however, an alternative conceptual approach would be to simply consider the photon to be a packet of energy propagating toward the interface of the two media. As the photon undergoes TIR, the momentum of the packet causes a portion of the energy to penetrate into the second medium, whereas the majority of the packet remains within the initial material, as shown in Figure 2.8. The penetrating portion of the packet reaches into the medium in the form of an exponentially decaying field, which is only present while the photon is
When a photon incident upon a boundary between two materials of differing refractive index reflects away from the boundary, a portion of the energy extends as an exponentially decaying field into the external medium.

undergoing TIR. The use of the term “evanescent”, which means “fleeting” or “temporary”, is specifically because the non-propagating field is only present during a TIR event.

Then, provided the external medium is of lower refractive index than the originating medium, the bulk of the packet reflects back away from the interface. However, the evanescent portion during its brief presence in the rarer medium, has the chance to interact with the external medium, typically in the form of optical absorption. This interaction is referred to as “evanescent leaking” and is usually a consideration to be minimized in waveguide design due to the potentially significant loss of energy, except in cases where transmission efficiency is not a limiting design factor. It is also worth noting that in a two-medium system, such as those considered above, the evanescent field does not transfer energy into the external medium, except in the case of optical absorption.

2.2.2 Frustrated Total Internal Reflection and Quantum Tunneling

Whereas most waveguides are designed to limit evanescent effects due to the potential loss of energy into the surrounding materials, our waveguides utilize that loss as a means by
if the boundary material is sufficiently thin, a portion of the evanescent field may penetrate into the external medium and revert into a propagating photon. In this process, the thickness of the secondary medium is less than the penetration depth of the evanescent field. Consequently, the field extends into a third medium, where a percentage of the light is able to escape TIR and revert into a propagating photon within the third medium, as shown in Figure 2.9. This technique has been utilized in a variety of applications including prism couplers, optical filters and switches, biosensors, and in optically characterizing thin films [77, 79–84]. Moreover, it is also the basis upon which Photon Tunneling Microscopy functions [85].

Figure 2.9 shows that as the wave of light approaches the waveguide–film interface, a portion of the wave extends past the interface, forming the evanescent field as a decaying exponential field. When the film thickness, \( d \), is sufficiently small, the peak of the evanescent wave extends beyond the thin film and into the tissue. Then, provided \( n_3 > n_1 \), the evanescent peak reverts back into a propagating wave within the tissue, having "optically tunneled" through the thin film since the light was never propagating within the film;
Figure 2.10 The diagram depicting the boundary conditions for FTIR is similar to a simple case of double refraction through an intermediary medium; however, under TIR, the beam technically does not propagate within the secondary medium $n_2$.

although it is also called quantum tunneling, since the effect is analogous to that observed by electrons under similar situations [86].

In order to mathematically demonstrate this effect, we must derive an expression for the Transmission into the third external medium. Consequently, it is necessary to consider the reflectivity at the interfaces between each subsequent layer. It is convenient, therefore, to start by considering Försterling’s solution for the amplitude reflection coefficient of a thin film between two semi-infinite isotropic dielectric media, shown in Equation (2.43) [87]. Figure 2.10 depicts the tri-layer boundary conditions considered in the derivation of the optical tunneling effect.

\[
re^{i\delta} = \frac{r_{12} + r_{23}e^{-ix}}{1 + r_{12}r_{23}e^{-ix}} \tag{2.43}
\]

wherein: $x = \frac{4\pi n_2 d}{\lambda_0} \cos \theta_2 \tag{2.44}$
In this expression, $r_{12}$ and $r_{23}$ represent the complex reflection coefficients at each boundary (1,2) and (2,3) respectively, whereas $\delta$ represents the change in phase upon reflection, and $d$ represents the intermediate film thickness. It is important to note that the evanescent field within the second medium is not a propagating beam, and therefore $\theta_2$ has no real value. Consequently, using Snell’s Law (Eq. 2.34) and substituting $x' = ix$, it is useful to rewrite Equation (2.44) in the form shown in Equation (2.45).

$$x' = +\frac{4\pi d}{\lambda_0} \sqrt{n_1^2 \sin^2 \theta_1 - n_2^2} \quad (2.45)$$

In this form, it is necessary to specify that $x'$ adopt only the positive root, as a negative root results in an evanescent amplitude that increases to infinity. Additionally, in order to obtain an expression for the transmission through the film, we must first rewrite Eq. (2.43) by defining $r_{12} = e^{i\delta_{12}}$ and $r_{23} = -r_{32} = e^{i\delta_{32}}$, based on the fact that only the phase of the wave changes upon TIR along with Stoke’s principle of reversibility.

$$re^{i\delta} = \frac{e^{i\delta_{12}} - e^{i\delta_{32}}e^{-x'}}{1 - e^{i(\delta_{12} + \delta_{32})}e^{-x'}} \quad (2.46)$$

Consequently, following the derivation presented by Court et al., utilizing the Fresnel equation for s-polarized light (Eq. 2.26, 2.27), it is possible to determine an expression for the transmission through the intermediary film into the external medium by for TE polarized light [78]. The resulting expression is given by Equation (2.47). A similar equation may also be derived for p-polarized light. Additionally, it is important to note that this equation assumes that the three media are optically transparent and have negligible absorption; however, a similar representation may be derived by using the complex form of the refractive index, ($\tilde{n} = n + i\kappa$), instead.
\[ T_s = \frac{1}{\alpha \sinh y + \beta} \]  

(2.47)

wherein:  
\[ y = \frac{2\pi n_2 d}{\lambda_0} \sqrt{N^2 \sin^2 \theta_1 - 1} \]

\[ \alpha_s = \frac{(N^2 - 1)(\eta^2 N^2 - 1)}{4N^2 \cos \theta_1 (N^2 \sin^2 \theta_1 - 1) \sqrt{\eta^2 - \sin^2 \theta_1}} \]

\[ \beta_s = \frac{\left(\sqrt{\eta^2 - \sin^2 \theta_1 + \cos \theta_1}\right)^2}{4 \cos \theta_1 \sqrt{\eta^2 - \sin^2 \theta_1}} \]

\[ N = \frac{n_1}{n_2} \]

\[ \eta = \frac{n_3}{n_1} \]

It is important to note that the intensity of the new ray of light in the third medium is dependent upon only the portion of the evanescent wave that penetrates into the third medium. Consequently, the majority of the light will be reflected back within the waveguide. Also, for our applications, biological tissue will likely have a refractive index lower than that of the waveguide, which would otherwise not allow for the tunneling to occur. However, in order to account for that issue, real tissue trials would involve an oil or optical gel specifically chosen to have a higher relative refractive index.

### 2.3 Photoacoustic Spectroscopy and Energy Measurement

The penultimate topic involves the concept of photoacoustic spectroscopy, and although it is not a core component of the technique presented in this thesis, the photoacoustic effect was employed in this study in order to measure energy fluence through the waveguide active areas. Utilized in a wide variety of applications ranging from the detection of circulating tumor cells and malarial indicators to the evaluation and optical characterization of
surfaces and thin films, photoacoustics refers to the phenomenon wherein a laser beam of sufficiently short pulse duration is absorbed by a chromophore within a medium, causing the absorber to undergo a rapid thermoelastic expansion, which results in an ultrasonic pressure wave that propagates throughout the medium. [77, 88–91]. Provided the pulse duration of the incident light is shorter than the time required for the resultant acoustic wave to depart, a condition known as “stress confinement,” this pressure wave can be described mathematically by Equation (2.49) [92, 93].

\[ \tau \leq \frac{\delta}{c_s} \]  

(2.48)

The condition for stress confinement is shown in Equation (2.48), wherein \((c_s)\) is the speed of sound in the medium \((1.5 \text{ mm/µs in water and tissue})\) and \(\delta = 1/\mu_a\), where \(\mu_a\) is the optical absorption coefficient of the target [77]. Additionally, when firing with pulse-widths on the order of a few nanoseconds, the heat dissipation away from the absorber can be neglected and all of the incident electromagnetic energy is translated into acoustic pressure, having met a condition known as “thermal confinement” [94].

\[ p(x, t) = \frac{1}{2} \Gamma g \mu_a H_B e^{-\mu_a(x-c_s t)} + \frac{1}{2} \Gamma g \mu_a H_B e^{-\mu_a(x+c_s t)} \]  

(2.49)

wherein \(\Gamma_g\) represents the Grünésian coefficient, \(H_B\) is the radiant exposure of the beam, \((x)\) is depth, and \((t)\) is time [77]. At the surface of an ultrasonic detector, where \(x = 0\), the equation describes the recorded amplitude of the pressure wave as a function of time. As the wave approaches the detector surface, the pressure exponentially increases in amplitude followed by an immediate drop when the wave hits the detector. Then, while the wave is reflecting away from the detector, the recorded pressure follows a reciprocal exponential
return to the baseline. Peak-to-peak analysis is often used in photoacoustic experimentation, as a result of this characteristic waveform profile observed by the detector.

For the purposes of this study, artificial simulations of human skin, referred to as “tissue phantoms,” were used to demonstrate the efficacy of the PAQT technique. By using a section of optically absorbing rubber as the target, the optical properties of the phantoms can be assumed to be consistent for this application. Consequently, the amplitude of the photoacoustic pressure wave was primarily dependent upon the radiant exposure of the beam upon the optical absorber, which directly corresponded to the energy fluence through the active area. Photoacoustic spectroscopy proved to be a useful tool for energy transmission measurement, as the properties of the pressure wave directly correlated to the energy delivered to the optical absorber.

2.4 PAQT: Photonic Ablation via Quantum Tunneling

Photonic Ablation via Quantum Tunneling (PAQT) represents an amalgam of the aforementioned fundamental optical concepts, in which laser light reflects between opposing surfaces of a planar waveguide until undergoing FTIR and tunneling out through a designated “active area” into the external medium, in this case human skin or a tissue phantom. This technique utilizes the FTIR evanescent leaking effect by placing a waveguide with an ultra-thin film against the skin. The waveguide has a relatively thick cladding along most of its length, but at a specified active area, the cladding is thin enough to allow for the light to tunnel into the tissue. Additionally, since the energy of the new photon corresponds with the amount of penetration, it is possible to scale the energy delivery by using thicker or thinner active area films.

By coupling the light into a planar waveguide, as demonstrated in Figure 2.11, and
incorporating a thin film active area onto its face, it is possible to illuminate tissue through direct physical contact. Doing so effectively circumvents many of the complications presented by contemporary free-space propagation approaches, discussed in Section 1.5. Primarily, this direct coupling approach significantly minimizes the possibility for hazardous reflections within the operating room, simply by reducing the operational distance to 0 and enclosing the optical system.

This technique also eliminates the tissue-air interface and replaces it with a tissue-waveguide interface, through which photothermally generated heat can be more readily dissipated, thereby potentially mitigating excess tissue damage, including carbonization and vaporization at the tissue surface. Furthermore, although not specifically investigated herein, having the waveguide adjacent to the tissue readily allows for the incorporation of contact cooling technologies, which have already been clinically demonstrated to further reduce thermal tissue damage [18, 62].
For the purposes of this study, the waveguides were fabricated using N-BK7 glass substrates, chosen for their transparency at 532 nm, which was the wavelength of the Q-Switched Nd:YAG laser that was used. The waveguides were also clad in a thin film of Silver, which was chosen for its high reflectivity and comparatively low refractive index \(n = 0.143\) at the lasing wavelength [70]. It is also important to note that since Silver does not readily adhere to glass substrates, an ultra-thin film of Titanium was deposited first to serve as an adhesion layer for the subsequent Silver films.

Using the simplified form of Snell’s Law (eq. 2.35) along with the refractive index of N-BK7 glass \(n = 1.519\), it can be determined that the critical angle at the glass-Silver interface is \(\theta_c = 4.87^\circ\). Therefore, this design allows the waveguides to support TIR throughout a broad range of internal reflection angles within approximately \(5^\circ < \theta < 90^\circ\). However, the transmission efficiency with which the light escapes the waveguide into the tissue, whether by refraction or FTIR, will vary with respect to the internal reflection angle. Consequently, the waveguides were designed to accept a broad range of entrance angles so that the optimum internal reflection angle may be experimentally determined [69].

Figure 2.4 depicts the pair of hemicylindrical lenses that were used to couple the incident beam into the waveguide at any angle ranging from \(0^\circ\) to \(75^\circ\). The lens pair acts as a 1:1 Keplerian beam expander so that the beam remains collimated within the waveguide, and are refractive-index-matched to the glass substrates. Other techniques for coupling light into a planar waveguide have been demonstrated in prior works, including prism coupling and nano-particle coupling; however, neither case would have readily allowed for the variation of the internal reflection angle [95, 96]. Figure 2.4 also demonstrates that the beam is forced to propagate along the length of the waveguide, escaping only through the designated active area. For the purposes of this study, the active areas were formed
by simply masking off a rectangular area of the substrate during the deposition process, leaving a bare glass surface. In that geometry, the beam simply refracts into the tissue at angles corresponding to the Fresnel conditions shown in Figure 2.1 (a). However, in order to control the degree of energy delivered, the active areas were then coated in a number of consecutive thin layers of Titanium and Silver to encourage FTIR and allow for a modicum of control over the degree of energy delivered into the tissue phantoms.
Chapter 3

Research Methodology

This chapter will discuss the specific aspects of the experimental procedures, the electronic and optical equipment utilized, and the fabrication protocols for the waveguides and tissue phantoms. In addition to the thorough discussion of the hardware involved, this chapter will also discuss the automated control software that was designed in-house to control the pulsed laser, record useful information from the data acquisition unit, and control the motorized components of the optical apparatus. Additional information regarding the custom designed LabView control software may be found in Appendix C.

3.1 Waveguide Design and Fabrication

Under ideal situations, the waveguides designed for these experiments were intended to match the geometry shown in Figure 3.1. The rectangular slab incorporates a semi-cylindrical prism into one end of the waveguide in order to couple the light into the waveguide while maintaining a coherent beam within the material. However, since slabs in such geometries could not be custom ordered from a manufacturer, waveguides with a functionally similar design were fabricated instead. These waveguides consisted of an N-BK7
Chapter 3. Research Methodology

**Figure 3.1** The ideal design for an optical waveguide incorporates an optical quarter-round into the substrate slab in order to allow for the incident light to be coupled into the waveguide using a separate cylindrical lens in Keplerian beam expander formation.

Glass slab substrate clad with thin layers of Titanium and Silver, using an approximately cylindrical lens to couple the light into the waveguide.

Waveguide fabrication proceeded in four steps: substrate preparation, Titanium deposition, Silver deposition, and coupling prism adhesion. Substrate preparation involved the use of a strong cleaning agent in order to remove any oils or dust from the substrate surface that may have been deposited during their fabrication. This step was essential since the slabs used were not fabricated for the purpose of being used as optical waveguides, but rather for use as microscope slides. Originally, the cleaning procedure involved using a 10% (w/v) NaOH solution to clean the substrates. While this solution did a wonderful job removing oil deposits, it also slightly etched the substrates and imparted slight structural weaknesses that resulted in their frequent fracture during testing. As such, the cleaning procedure was changed to a Nochromix H₂SO₄ bath followed by soaking in high-purity Methanol. The final step was to use a cotton-tipped applicator to apply high-purity Isopropanol to clean the surface of any excess deposits from the cleaning procedure and remove streaking. The Isopropanol was wiped off using Thorlabs lens tissue and blown dry using compressed air.
With the slabs cleaned, the entrance aperture and active area were masked off using a thin electrical tape that leaves a minimal amount of residue when removed. The entrance aperture was set to be 6 mm in width, measured from one end of the slab. The active area was set to start at approximately 21.10 mm, measured from the entrance aperture end of the slab, and end at approximately 45.0 mm. This active area geometry was chosen to match the dimensions of the tissue phantoms, which will be discussed in the next section.

Prior research has shown that Silver does not readily adhere to glass substrates, so an ultra-thin intermediary Titanium layer was deposited first to serve as an adhesion layer. The Titanium thin film oxidizes in open air and the resulting TiO$_2$ covalently bonds with the glass substrate. The subsequent Silver layer, when deposited over the surface of the Titanium, forms a Silver-Titanium alloy that bonds with all subsequent Silver deposition layers. Using an EMS575X cold sputtering vacuum deposition system, located in the University of Missouri Electron Microscopy Core (EMC), waveguides were coated in Titanium using the following settings:

- Titanium target from Refining Systems, Inc., 0.995% purity
- Sputtering
- Oxidizing
- Current: 30 mA
- Sputtering time: 00:01:00 (HH:MM:SS)
- Pump Hold Off
- Pump Hold Delay: 00:00:00 (HH:MM:SS)

When this step is omitted and the Silver is deposited on its own, the thin film can be easily rubbed off the surface of the glass substrate with minimal pressure; however, the Silver-Titanium alloy that forms at the interaction surface significantly enhances bulk film adhesion such that even rigorous cleaning with Acetone or Isopropanol does little to remove the metal layers. From this point, with the Titanium film deposited on both sides of the waveguide, the Silver layer is deposited using the following settings:
Figure 3.2 A planar BK7 glass substrate with silver cladding has $\theta_c = 4.87^\circ$, which allows the waveguide to support TIR under a wide range of internal reflection angles from $5^\circ < \theta < 90^\circ$

- Silver target from Denton Vacuum, 0.995% purity
- Sputtering
- Oxidizing
- Current: 90 mA
- Sputtering time: 00:03:00 (HH:MM:SS)
- Pump Hold Off
- Pump Hold Delay: 00:00:00 (HH:MM:SS)

Assuming that the Silver and Titanium deposit at similar rates to Platinum, whose deposition rates had been established by the EMC, the Titanium layer should be around 5 to 7 nm thick, while the Silver layer should be around 100 nm thick. Although these assumptions may not be accurate, they were functionally effective in that the Titanium film served to adhere the Silver layer, while the Silver layer was sufficiently thick so as to severely limit the transmission of 532 nm light and restrict laser propagation within the bounds of the substrate. Further development of the technique will include metal film deposition rate analysis using Scanning Electron Microscopy (SEM) or Atomic Force Microscopy (AFM), so as to better characterize the film thickness.

Figure 3.2 more accurately demonstrates the experimental design of the waveguides fabricated in this study. With the BK7 substrates prepared and coated in Silver, the final
fabrication step involved the adhesion of an approximately hemi-cylindrical coupling lens atop 6 mm entrance aperture. The lens was a Thorlabs LJ1878L2-A BK7 lens with a 5.2 mm focus, and was adhered to the glass substrate using NOA 74 (Norland Optical Adhesive #74), which is refractive index matched to BK7 glass. Since the NOA 74 is a UV curing adhesive, the waveguides were left to cure in a Leica EM UV curing oven for 20 minutes.

The entrance aperture width was chosen to optimize energy fluence for internal reflection angles between 40° and 55° following initial testing. Matlab simulations, described in Appendix B, determined the approximate coupling efficiency for an elliptical beam incident upon a waveguide at angles ranging from 0.2 to 75° with entrance aperture widths ranging from 5 to 9 mm.

### 3.2 Tissue Phantom Construction

Tissue phantoms comprised of transparent agar and an optically absorbing rubber (3M 300LSE) were utilized to simulate the structure and refractive index of human skin. The agar served to mimic the refractive index of human tissue, as both agar and tissue are principally comprised of water. The rubber was intended to represent an optically absorbing tattoo approximately 2 to 3 mm below the surface of the tissue. This had also been attempted with a variety of dyes; however, each dye that was tested would slowly and unevenly diffuse into the rest of the phantom over time. Additionally, the rubber did not vary in refractive index or optical absorption between tests, which provided another benefit over the use of optical dyes.

It is worth noting that the phantoms used do not have a layer designed to simulate the optically absorbing epidermis. This omission was an intentional component in the phantom design, as an epidermal layer at the surface could have absorbed the light evanescently and thrown doubt upon whether the light could be made to propagate down into the tissue to ablate a tattoo. This study was intended to demonstrate that the PAQT waveguide technique could be utilized to couple light into the tissue. Once the light is propagating within the tissue, it will behave in exactly the same manner as in contemporary laser dermatology techniques, so the epidermal layer was excluded since the local absorption plays no role in coupling the light into the tissue, only in obstructing its further propagation.
Chapter 3. Research Methodology

Figure 3.3 A block of transparent Acrylic was milled to have a 1 mm deep groove exactly the width of the glass substrates used in waveguide fabrication. A rectangular mold was cut into the mount with a circular port to be sealed by an ultrasonic transducer.

The phantoms were prepared by heating and stirring 10 mL of 1 X TAE buffer with 100 mg of Agarose powder until the mixture cleared and established a rolling boil. At this point, 2 mL of the solution was pipetted into a 22 mm × 24 mm × 9 mm mold that had been milled into the waveguide mount, as shown in Figure 3.3. The base of the mold had a tube cut through to the rear of the mount to be sealed by an ultrasonic transducer, which was press-fit into place such that its surface was flush with the base of the mold. The agar layer was allowed to set for approximately 4 min, after which point a 1 mm thick piece of red rubber, with external dimensions to match those of the mold, was placed over top of the semi-set agar. While the first layer was setting, the remainder of the Agar solution was kept stirring on the hotplate to prevent it from setting prematurely. Finally, 2.5 mL of Agar was pipetted on top of the rubber, making sure to avoid bubble formation under the rubber. It is important to note that the amount of Agar used was determined experimentally so that
the phantom would be consistently in contact with the waveguide for at least two hours during the testing, or approximately 20 min longer than a full sequence of 15 tests. The phantoms were made to have a distinct convex meniscus, which protruded around 0.5 mm above the rim of the mold.

Figure 3.3 also shows that a second acrylic block was used as a clamp to ensure consistent contact between the waveguide active area and the phantom; however, the figure does not show that the clamp had a small piece of rubber on the underside in the center of the phantom to ensure contact was maintained while minimizing pressure on the waveguide. The clamp was held in place with four screws that were tightened using a torque wrench to 15 ozf.in, with washers placed between the clamp and the mount to prevent the mount from shattering the waveguide with excess pressure. This torque setting was experimentally determined to ensure that the waveguides would not be fractured by applying too much pressure.

3.3 Optical Apparatus

Seeing as the appropriate alignment of the waveguide coupling lens with the beam at all angles of incidence was paramount to obtaining reliable transmission data, the majority of the optical apparatus was mounted in a 30 mm cage system. The laser source that fed the optical train was a Quantel Brilliant Nd:YAG that has a 5 ns pulse width and delivers a vertically polarized beam with a wavelength of 532 nm in a circular gaussian profile. The first of the components in the optical train was a planoconcave lens (Thorlabs LC-1582) with a $-75$ mm focal length, followed by a planoconvex lens (Thorlabs LA-1433) with a 150 mm focal length arranged in a Galilean beam expander formation.

For s-polarized testing, the expanded beam was passed through a removable zero-order half-wave plate (Thorlabs WPH05H-532) to rotate the polarization from vertical to horizontal. This component could be removed for p-polarized testing, as the vertically polarized output from the source was already p-polarized with respect to the waveguide. Regardless of the desired polarization, the beam was delivered through a polarizing beamsplitter cube (Thorlabs CM1-PBS251), which was 90° rotatable and directed light with the undesired polarization into an optically absorbing beam dump.
With the light properly polarized, the beam was passed into a 50:50 beamsplitter cube (Thorlabs BS013), which redirected half of the beam toward a rubber absorber adhered to the surface of an ultrasonic transducer (Olympus Panametrics PI15-2). A small portion of the light reflected back through the beamsplitter toward a Silicon–based photodiode (Thorlabs DET10A), which served as the trigger source for the data acquisition device. The other half of the beam was directed to a dichroic mirror (Thorlabs DMLP505), which served to overlay the 473 nm CW alignment beam onto the path of the pulsed beam.

Since the waveguide substrate was only 1 mm thick, the circular beam was contracted in the horizontal direction by a 4X mounted anamorphic prism pair (Thorlabs PS883-A), which converted the circular beam into an elliptical beam with a 1 mm beam waist, as shown in Figure 3.4. Due to the diffractive effects resultant from such an extreme contraction, the beam was passed through a planoconvex cylindrical lens (Thorlabs LJ1558L1-A) with a 300 mm focal length in order to re-collimate the beam. The light was passed through a planoconvex cylindrical lens (Thorlabs LJ1878L2-A) with a 10 mm focal length to couple the beam into the waveguide. The waveguide was fabricated using an identical lens and was positioned in a Keplerian beam expander formation so as to maintain the beam collimation within the waveguide.

Finally, the waveguide was positioned on a series of four stages to ensure proper...
alignment and ease of testing. The first stage was an X-Y translational stage that was utilized to align the central axis of the second stage, which was a motorized rotational mount (Thorlabs CR1-Z7). An optical plate was affixed on top of the rotational mount, on which an X-Y micro-translational stage was mounted. The final component, attached to the micro-translational mount, was the acrylic waveguide mount shown in Figure 3.3 that served to maintain consistent alignment between testing, consistent tissue phantom geometries, and consistent positioning of the ultrasonic transducer (Olympus Panametrics V319).

3.4 Electronic Equipment

3.4.1 Experimental Equipment

The experimental apparatus makes use of a variety of electronic components for excitation, alignment, measurement, and analysis. The principal component is the Quantel Brilliant Q-Switched Nd:YAG laser, which is incorporated into a Vibrant 355 II laser unit. The Brilliant laser source is pumped by a flashlamp and produces a 1064 nm laser beam that is horizontally polarized. The beam is then passed through a $\lambda/2$ wave plate to rotate the polarization in order to match the phase requirements of the second harmonic generator. The 532 nm component of the beam is reflected toward the experimental apparatus using a dichroic mirror that allows any remaining 1064 nm light to pass through to a beam dump. Laser operations, including flash lamp controls, laser firing, and Q-Switch delay, were controlled by a computer through an RS232 (DB-9) connector.

The optical train was aligned using a continuous wave (CW) low power laser beam that was superimposed on the 532 nm beam output from the Vibrant laser system. A Diode-pumped Solid State (DPSS) laser with a 473 nm beam output was used for this purpose (Laserglow Technologies LRS-0473-TSM-00100-10). This beam was chosen for an alignment beam for two principal reasons: first, because it is much easier to align an optical system using a CW beam as opposed to a nanosecond-pulsed beam; second, because a 473 nm beam can be readily incorporated into the path of a 532 nm beam by the use of an angled dichroic mirror with a 505 nm high-pass cut-off wavelength (Thorlabs DMLP505).

Another crucial component utilized in the system is the PicoScope 4224 data acqui-
sition unit. As opposed to standard oscilloscopes that incorporate screens and electronic controls, the PicoScope unit is a purely analog system with 2 BNC input channels operating with 20 MHz bandwidth, 32 MS memory, and 80 MS/s sample rate. The PicoScope is controlled through a USB 3.0 cable connected to a computer operating Windows 7. Although the unit was supplied with operational software in order to operate with the same capabilities as a standard oscilloscope with similar specifications, the software was insufficient for the needs of this project. As such, data acquisition was controlled through a custom LabView interface, which will be described in Section 3.5.

The data recorded comes from three different sources - two ultrasonic transducers and one photodiode. The first transducer was an Olympus Panametrics NDT V319 15 MHz ultrasonic transducer with a 0.5 inch detection area. This transducer was an unfocused model and was used to record the photoacoustic waveforms generated within the tissue phantom on Channel A of the PicoScope 4224 unit. The second transducer was an Olympus Panametrics PI15-2 15 MHz ultrasonic transducer with a 0.25 inch detection area. This transducer was a focused model with 1 inch focus that served to measure the incident energy of the laser through the detection of photoacoustic waveforms generated within a 1 mm thick piece of optically absorbing red rubber on Channel B of the PicoScope. It is important to note that this rubber was cut from the same stock as that used in the tissue phantom, so that the photoacoustic comparison between the signals from one transducer and those from the other are directly comparable with regard to frequency. It should also be noted that although the second transducer was a focused model, the rubber adhered to its surface never moved or was altered in any way, so its focus is irrelevant in this context.

Data acquisition through the PicoScope 4224 unit was triggered by use of a photodiode (Thorlabs DET10A), on Channel B. The DET10A utilizes a Silicon PIN photo detector whose responsivity at 532 nm was approximately 0.30 A/W. Seeing as there were only two channels on the PicoScope, but three signals to record, the optical trigger signal from the photodiode was multiplexed with the energy measurement signal from the secondary ultrasonic transducer. The signals were separated in the time domain due to the thickness of the rubber optical absorber attached to the surface of the transducer. However, the unaltered intensity of the signal from the photodiode over saturated the channel and rendered energy
measurement ineffective. Consequently, the signal from the photodiode was passed through a 10 dB signal attenuator along with a 50 Ω feed-through terminator. The PicoScope was therefore set to trigger at 100 mV, so that it would trigger when the photodiode exceeds the threshold, but not when it had recorded the signal from the second transducer, which did not exceed the threshold.

Additionally, the system also incorporated a Ritec Broadband Receiver BR-640A, which filtered and amplified the signal from the tissue phantom transducer. The Ritec unit was set to 28 dB gain, 100 kHz High Pass filter, no Low Pass filter, and a 50 Ω input impedance. It is important to note that the system also incorporated a 50 Ω feed-through terminator between the Ritec and the PicoScope in order to filter out electronic reflections in the BNC cables due to impedance mismatching.

With the laser sources and measurement equipment already defined, the experimental apparatus also incorporated a motorized rotational stage, in order to control the internal reflection angle of the beam within the waveguide. The incident angle of the beam was controlled and varied through the use of a Thorlabs CR1-Z7 rotational stage connected to the computer through a T-Cube DC motor controller unit (Thorlabs TDC001). The stage operates with a 360° continuous motion worm drive in order to rotate the stage. The DC-servo motor is tracked using a built-in optical encoder that records the relative motion from the initial position at startup, which is accounted for in the LabView control software. It is also important to note that in order to ensure rotation around the central axis, the stage utilizes a Thorlabs CR1 adapter plate, which has a 1/4”-20 threaded hole at its center.

3.4.2 Additional Equipment

In addition to the electronic equipment utilized during the experiments, there were three other principal components that were crucial to the success of this project. The first was the Coherent FieldMax TOP Laser Power/Energy Meter. Prior to every test, the incident laser energy was measured using this meter in terms of incident energy in mJ. The meter was set to measure and average 100 pulses with three digits of precision. Unfortunately, this system was an older model and could not be directly interfaced with an oscilloscope or computer; consequently, it could not be used to measure the energy of each
shot during testing, which would have been preferable to the photoacoustic method used.

Another component of note was the control computer that operated the LabView software discussed in Section 3.5. The system incorporated a 2.67 GHz Intel Xeon 64-bit quad-core processor with 4 Gb of onboard RAM. The computer was running Windows 7 Enterprise Edition and operated using LabView 2013 Full Development System, utilizing LabView Service Pack 1 and the Signal Processing Toolkit.

The final component was the EMS575X cold sputtering system, which was housed in the Electron Microscopy Core. The Silver deposition target was a 50 mm square target from Denton Vacuum (online part # 2975). The Titanium deposition target was also a 50 mm square target from Denton Vacuum (online part # 3006).

3.5 LabView Automated Control Program

Data collection, preliminary analysis, laser firing, and motor automation were controlled by a custom designed LabView program. The program communicated to the Nd:YAG laser system by an RS232 cable, and controlled the Q-switch delay, flashlamp operation, and laser firing by sending short string commands to the system. The communication protocols were set to 9600 baud, 8 data bits, 0 parity, no stop bit, no flow control, and CR + LF termination characters. The Q-switch delay was kept between 330 and 335 $\mu$s, which was varied to maintain the incident energy measured before the waveguide between 1.80 and 2.10 mJ in order to ensure sufficient photoacoustic waveform generation without causing any damage to the waveguide or other optical components.

The PicoScope and T-cube motor controller were both controlled through a USB connection, with data acquisition settings and rotational increments set within the program. The PicoScope was set to record 5000 post-trigger samples at the maximum sample rate of 80 MS/s. The trigger threshold was set to 80 mV with a rising trigger direction to detect the signal from the photodiode. Both Channel A, which recorded the waveguide transmission waveforms, and Channel B, which recorded the photodiode and incident energy waveforms, were set to 200 mV.

The automation portion of the software was comprised of a series of loops and sequence structures, the block diagrams for which are included in Appendix C. The system
automation loop proceeded in the following steps:

- Trigger the laser to fire a single shot
- Wait for a signal from the photodiode to break the trigger threshold
- Record 5000 samples from each input channel
- Measure peak-to-peak and absolute integrated intensities
- Check if the system has rotated to 75°
  - If it has, the loop is complete
  - If not, the system rotates by 0.2°
- Plot the intensity curves and save the data
- Repeat until the system has rotated through the full range of angles

The loop took approximately 380 seconds to complete and could also be set to repeat itself as many times as was necessary; however, it was found that after around 20 complete runs, the data sometimes become unreliable as the tissue phantom tended to dry out too much. Consequently, the energy fluence was tested for 15 consecutive tests in order to mitigate any concerns about statistical anomalies or outliers. A new tissue phantom would be constructed for each sequence of tests to avoid potential problems from over drying. The system could also be set to record the full photoacoustic trace data from Channel A for the first test in a sequence (380 files).
Chapter 4

Results and Data Analysis

The following sections will introduce both the empirical and qualitative results of the work; however, the more in-depth discussion and interpretation of their ramifications will be reserved for Chapter 5. The first section covers the computational simulations that were performed in order to optimize the waveguide fabrication procedures. Subsequent sections then deal with energy fluence data through waveguide active areas with Titanium and Silver coatings, respectfully.

4.1 MATLAB Coupling Efficiency Simulations

In the course of waveguide fabrication, certain design approximations were necessarily made in lieu of ordering custom-manufactured waveguides, as discussed in Section 3.1; consequently, these approximations in addition to the incident beam spot size resulted in the coupling efficiency of the waveguides deviating from the theoretical values. Specifically, since the waveguides made use of hemi-cylindrical lenses, rather than quarter-cylindrical lenses, it was necessary to coat a portion of the interface between the lens and the waveguide substrate with silver, in order to prevent the incoming light from escaping the waveguide.

However, since the angle of the incoming beam was to be varied throughout the course of testing, the spot size incident upon the waveguide substrate would grow in width, while the reflections would translate along the length of the waveguide. As a result, a portion of the incident beam would reflect off the mirrored surface without ever entering the waveguide substrate; whereas another portion of the beam would reflect off the rear
Figure 4.1 The coupling efficiency through the adhered cylindrical lens is directly related to the incident angle, since portions of the beam will be cut off as the geometry and position of the projected beam change throughout the angular spectrum.

face of the waveguide substrate and escape confinement entirely, as depicted in Figure 4.1. Consequently, the coupling efficiency of the waveguide was directly related to the dimensions of the entrance aperture and the geometry of the mirroring between the lens and the waveguide.

In order to determine an appropriate width for the entrance aperture, a series of computational simulations were performed using MATLAB (R. 2014a). The simulations generated a 3-dimensional elliptical gaussian profile to represent the incident beam, which was initially a circular gaussian spot that was then compressed along the horizontal axis by the anamorphic prism pair. Since the waveguide was to be rotated through a range of angles during testing, the short axis of the elliptical profile was programmed to increase accordingly, as a result of the ellipse being projected onto the surface of the waveguide substrate at increasing incident angles. Following that, the right portion of the ellipse along the short axis was cut off in a “knife-edge” fashion to represent the portion of the beam that would reflect off the mirrored surface without entering the waveguide proper. At the same time, the left portion of the ellipse was similarly cut off to represent the portion of the
Figure 4.2 The geometry chosen was that of an entrance aperture 1.00 mm from the central axis of the lens, shown in [red], as it had a local maximum that corresponded to experimentally observed maxima.

beam that entered the waveguide, but subsequently reflected away without being coupled into the main body of the slab.

The simulations were performed for incident angles from 0 to 75°, as that was the range of angles to be experimentally tested. Throughout the process, the area under the modified gaussian was recorded and taken as a fraction of the total area under the unadulterated beam profile. These fractions were represented as percentages and plotted against the internal reflection angles to generate an overall coupling efficiency graph as a function of internal reflection angle.

The graphs for each aperture geometry are included in Figure 4.2, wherein the geometries were demarcated by their distance from the central axis of the coupling lens, around which the waveguide rotated. The final geometry chosen for waveguide fabrication was that of an entrance aperture 1.00 mm from the central axis, and therefore 6.00 mm from the end of the waveguide. This geometry was chosen since the local maximum of the curve, shown
Table 4.1 Conditions Studied

<table>
<thead>
<tr>
<th>Active Areas</th>
<th>Measurement Conditions</th>
<th>Types of Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare</td>
<td>0 – 75° by 0.2° increments</td>
<td>Mean energy fluence</td>
</tr>
<tr>
<td>Ti</td>
<td>15 tests per active area</td>
<td>Photoacoustic peak-to-peak</td>
</tr>
<tr>
<td>Ti x 2</td>
<td>1 laser pulse per increment</td>
<td>Integrated intensity</td>
</tr>
<tr>
<td>Ti x 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti x 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti + Ag</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti + Ag x 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti + Ag x 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A total of 8 different active area geometries were studied, each of which underwent the same measurement conditions and identical methods of data analysis.

in Figure 4.2 in [red], corresponded to the experimentally observed maxima in the range of 40 to 50°. Consequently, all waveguides made from that point onward were fabricated with a 6.0 mm wide entrance aperture, as described in Section 3.1. For further reference, the MATLAB code that governed these simulations is included in Appendix B.

4.2 Titanium Active Areas

With the fabrication designs finalized following the aforementioned MATLAB simulations, waveguides with bare active areas were fabricated as blank slates for further testing. Since Titanium was used as an adhesion layer for the Silver films, it was necessary to understand any effects that the Titanium layer might have on the energy fluence through the active area. Consequently, the energy fluence of waveguides with purely Titanium active areas was photoacoustically tested with regard to a spectrum of internal reflection angles. The testing was performed to determine if the Titanium film substantially affected the transmission of light, but given that the layer was ultra-thin, it was initially not expected to have any noticeable effect.

A compilation all of the conditions studied is included in Table 4.1. Regardless of active area geometry, each waveguide was tested under identical conditions and analyzed using the same methods, which are discussed in the sections that follow.
Figure 4.3 Energy transmission through a Ti film was consistently greater than through a bare active area for all internal reflection angles.

4.2.1 Energy fluence

As described in Section 3.5, the waveguides were aligned such that the central axis of the hemi-cylindrical lens was coaxial with the rotational axis of the motorized stage. The photoacoustic peak-to-peak energy was recorded at each internal angle from 0 to 75° in 0.2° increments, while the entire spectrum was recorded fifteen times for each active area composition and geometry. Figure 4.3 depicts the mean value curves for each the bare active area transmission and Titanium thin-film active area transmission. The graph demonstrates an unexpected increase in energy fluence through the Titanium thin-film, resulting in a (1.99x) higher maximum energy fluence and a 291% increase in overall energy fluence across the full spectrum, calculated by integrating the area under the energy fluence curves and dividing that of the Titanium film spectrum by that of the bare active area spectrum.

To confirm these results, the tests were repeated with another Titanium coated waveguide active area. Figure 4.4 also demonstrates a similarly substantially enhanced energy
fluence. The Titanium active area demonstrated a (3.701x) higher maximum energy fluence and a 298.3% increase in overall energy fluence across the full spectrum. These results prompted further investigation into the potentially increased energy fluence when using Titanium thin-film active areas. It is also worth noting that the energy appeared to be consistently delivered from one test to the next. Figure 4.5 shows the same mean energy fluence curve as in 4.4, but with the 95% confidence intervals included to demonstrate the consistency of the increased fluence throughout the testing, which are extremely narrow throughout the spectrum, only noticeably growing in amplitude during the peak transmission regions. After four depositions of Titanium, the energy fluence no longer demonstrated the significantly increased energy fluence, as shown in Figure 4.6. Possible reasons for this discrepancy will be discussed in Section 5.1.

4.2.2 Photoacoustic analysis

All of the energy fluence graphs shown thus far have been measured using photoacoustic peak-to-peak analysis. Typical photoacoustic signals are characterized by an exponen-
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Figure 4.5 The 95% confidence intervals, shown in grey, remain fairly narrow throughout the angular spectrum, but consistently demonstrate increased energy fluence over the bare active area.

Figure 4.6 After four Titanium depositions, the energy transmission does not appear to be significantly greater than that of the bare active area.
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Figure 4.7 Transmission through a Titanium film with a 45° internal reflection angle produces an ideal photoacoustic waveform within the tissue phantom. (11-14-2014-PAdatatrace_228.csv)

Partially increasing upstroke followed by an inversion and a logarithmically increasing signal that returns to baseline, mathematically described by Equation (2.49). Consequently, measuring the difference between the peak of the upstroke and the tip of the downstroke is a standard technique in photoacoustic analysis, referred to as “Peak-to-Peak” analysis. One such signal is demonstrated in Figure 4.7, which was recorded using a waveguide with a Titanium active area.

The photoacoustic signals were identified as being generated by the rubber absorber within the tissue phantom based on their position in the time domain trace along with an estimation of the speed of sound in Agar. Considering that Agar, like human tissue, is comprised predominantly of water, it is safe to consider the speed of sound to be approximately 1.5 mm/µs [97]. That being the case, for a signal that occurs at 3 µs, the source of the acoustic wave should be approximately 4.5 mm away from the detector, which corresponds to the position of the rubber absorber within the phantom.

Furthermore, using these same estimations, we can determine the sources of other signals that occur farther along in time, and subsequently identify them as either reflections.
Figure 4.8 As the internal reflection angle changes, multiple photoacoustic signals are produced and their signals tend to convolve and distort peak-to-peak measurements. (11-14-2014-PAdata/trace_251.csv)

or other photoacoustic sources. Figure 4.9 shows the full ultrasonic trace from which the photoacoustic waveform shown in Figure 4.8 was extracted. The graph shows that there were a multitude of signals detected by the ultrasonic transducer; however, by taking into account the approximate speed of sound in Agar, we can identify the peaks that occur at (a) and (b) as independent sources. By contrast, the signals at (c) through (f) are merely acoustic reflections within the chamber. These reflections also demonstrate a ringing, which is typically characteristic of reflected signals.

Considering that the peak at point (a) occurs around 4.5 $\mu$s, it can be identified as the rubber absorber within the phantom. However, the peak that occurs at point (b) around 6.5 $\mu$s indicates an acoustic source that is approximately 9.75 mm away from the transducer surface. Based on the thickness of the tissue phantom, the most likely source for this signal is the small piece of rubber adhered to the underside of the acrylic clamp. Given the lack of cladding material, it is likely that there is a degree of evanescent leaking into the rubber, which absorbs a portion of the energy and produces a photoacoustic signal. This can be confirmed by increasing the internal reflection angle to exacerbate the evanescent absorption.
Figure 4.9 Taking into account the speed of sound in Agar, peaks (a) and (b) can be identified as photoacoustic sources, whereas (c) through (f) are revealed to be merely reflections within the chamber. (11-14-2014-PAdata/trace_251.csv)

within the optically absorbing rubber medium. Figure 4.10 shows the photoacoustic peak generated within the phantom at point (a), while also showing the much larger evanescently produced peak at point (b).

However, in the course of the energy fluence tests, the photoacoustic signals intermittently strayed from the ideal and often represented two waveforms that overlapped in the time domain. The result is an overall decrease in the recorded energy fluence, since the measured peaks were diminished in intensity due to the convolution of the two waveforms. Figure 4.8 demonstrates such a split peak waveform trace, recorded from the same energy fluence test as Figure 4.7. The ideal photoacoustic signal from the first plot was recorded at a 45° incident angle using a waveguide that had undergone four Titanium depositions covering the active area; whereas the split peak signal was recorded at 49.5° incidence.

Throughout the angular spectrum, the photoacoustic waveforms alternate between the ideal type and the split peak variety, resulting in the undulations observed in the
energy fluence graphs presented in the previous section. These two graphs represent the behavior of the photoacoustic waveforms when creating the characteristic peaks and valleys in the energy fluence graphs shown thus far. It should also be noted that upon retrospective analysis, some of the photoacoustic waveforms appear to saturate the PicoScope at 200 mV, which may suggest that the energy fluence was indeed higher with thicker Titanium films.

Seeing as these split peaks bring the peak-to-peak energy transmission graphs into question, integrated intensity curves were generated instead. These curves represent the area under the photoacoustic waveform after taking its absolute value, corrected by the equivalent integrated incident intensity curve. Figure 4.11 depicts one such energy transmission curve, which demonstrates that there may be a more significant difference in energy fluence using thicker films; however, this measurement technique is still subject to the same flaws as the peak-to-peak fluence curves, only to a slightly lesser extent. A further explanation of these concerns along with possible solutions to resolve the energy delivery measurement issues
Figure 4.11 Integration of the photoacoustic trace suggests that there may be a more substantial difference in energy fluence using Ti thin films, although it is not an ideal measurement technique.

are discussed in greater detail in Section 5.2.

4.3 Silver Active Areas

The testing for the waveguides with Silver active areas occurred concurrently with those that had Titanium active areas. Consequently, the experiments were beset with the same concerns regarding energy fluence measurement, photoacoustic analysis, and evanescent absorption.

4.3.1 Energy fluence

Unlike the Titanium waveguides, the Silver waveguide energy transmission graphs agreed with initial expectations, in that the overall energy fluence decreased with the increasing thickness of the Silver film in the active area. However, as demonstrated in Figure 4.12, the energy fluence seemed to drop off nonlinearly with film thickness. It is worth noting that the Silver waveguides also had a single Titanium adhesion layer, as mentioned in
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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{corrected_peak_to_peak_energy_transmission_vs_angle}
\caption{Corrected peak-to-peak energy transmission vs. angle}
\end{figure}

Figure 4.12 Energy fluence decreases with Ag film thickness, but in an apparently nonlinear relationship given the stark transition between the first and second Ag film depositions.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{integrated_energy_transmission_vs_angle}
\caption{Integrated energy transmission vs. angle}
\end{figure}

Figure 4.13 Measuring transmission using the integrated photoacoustic waveform revealed a slightly more steady decrease in energy fluence, although still very much nonlinear with film thickness.
Section 3.1. Both graphs demonstrate a shift in the peak position and a change in the shape of the overall transmission curve between the bare active area and the subsequent thin-film tests. This is likely due to a slight change in alignment between tests, which prompted a second round of tests with a new waveguide.

The deposition duration time utilized for the waveguide that yielded the energy fluence graphs shown in Figures 4.12 and 4.13 was 60 sec, which proved to be long enough that the second deposition effectively reduced the transmission to nearly 0. Consequently, for the reproduced tests, the deposition rate was cut in half to 30 sec, which resulted in a much more gradual decrease in energy fluence. The peak-to-peak and integrated intensity graphs for those transmission curves are shown in Figures 4.14 and 4.15 respectively. The second graph is perhaps the most convincing in that the energy fluence appears to reduce exponentially, which would be as expected due to the exponentially decreasing penetration depth of the evanescent field through the silver film with increasing thickness.

4.3.2 Photoacoustic analysis

As was the case with Titanium waveguides, the peaks in the energy fluence curves were characterized by photoacoustic waveforms that demonstrated the characteristic exponential upstroke and subsequent return to baseline. Such a waveform is shown in Figure 4.16. By contrast, Figure 4.17 demonstrates a split peak waveform, which was characteristic of the valleys in the fluence curves. These two waveforms were both extracted from the same test, only differentiated by the internal reflection angle within the waveguide at each point, 43° and 46.4° for the ideal and split photoacoustic traces respectively.

It is likely that this split peak is the result of two photoacoustic waves slightly separated in time spatially overlapping, producing an unexpected local maximum prior to the characteristic logarithmic increase. Nevertheless, following the same measurement transition as before, the photoacoustic analysis switched from simple peak-to-peak measurement over to an integrated intensity curve. While this was not an ideal solution, it did more accurately reflect the differences between the amount of energy transmitted into the tissue phantom as related to the thickness of the active area layers. Figure 4.13 shows the integrated intensity curve for a Silver thin film active area waveguide, demonstrating that the
Figure 4.14 The decrease in energy fluence was more patently observable for the waveguide with shorter deposition rates, likely due to the limited penetration depth of the evanescent field through silver films.

Figure 4.15 Integrated intensity curves demonstrate that the decrease in energy fluence corresponding with the exponential decrease in penetration depth of the evanescent field through the silver films with increasing thickness.
decrease in energy fluence may not have been as immediate a transition as was suggested by the initial peak-to-peak analysis.

Given that the Silver waveguides differed from the Titanium waveguides with respect exclusively to the deposited active area film, the Silver waveguides demonstrated the same evanescent leaking effects as was observed in the Titanium waveguides. This was confirmed through analysis of a full photoacoustic trace, such as the one shown in Figure 4.18. Again, by estimating the speed of sound in Agar to be $1.5 \text{ mm/}/\mu\text{s}$, the peaks indicated by points (a) and (b) represent two independent photoacoustic sources; whereas the peaks indicated by points (c) through (f) are simply the reflections of those two waves within the bounds of the chamber.

Even with a 45° internal reflection angle, there is a significant photoacoustic peak observable. Although it is substantially smaller than the peak generated by the rubber within the phantom, the energy loss into the rubber clamp is enough to result in a distinct photoacoustic waveform, as shown in Figure 4.19. Due to the absence of a thick, non-absorbing cladding layer, the energy of the photons incident upon the surface of the waveguides is enough to produce a distinct photoacoustic wave within the material, even at lower internal reflection angles.
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**Figure 4.16** Transmission through a Silver film with a 43° internal reflection angle produces in an ideal photoacoustic waveform within the tissue phantom. (11-11-2014-PAdatala/trace_598.csv)

**Figure 4.17** As the internal reflection angle changes, multiple photoacoustic signals are produced and their signals tend to convolve and distort peak-to-peak measurements. (11-11-2014-PAdatala/trace_615.csv)
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Figure 4.18 Taking into account the speed of sound in Agar, peaks (a) and (b) can be identified as photoacoustic sources, whereas (c) through (f) are reflections within the chamber. (11-11-2014-PAdata/trace_608.csv)

Figure 4.19 The evanescently generated photoacoustic signal (b) is indicative that a portion of the energy is being absorbed by the rubber clamp. (11-11-2014-PAdata/trace_608.csv)
Chapter 5

Discussion, Conclusions, and Continued Investigation

The following sections discuss the observations made during the course of data analysis and the subsequent conclusions that can be drawn. The chapter also incorporates sections regarding the investigative work that is planned for the continuation of the research along with a number of potential applications of the techniques presented herein.

5.1 Energy Fluence

With regard to the energy fluence through the Titanium thin-film active areas, Figures 4.3 and 4.4 both independently demonstrated increased energy fluence through the Titanium as compared to the bare active area transmission for their respective waveguides. The data was recorded using two different waveguides and in both cases resulted in approximately a 290% increase in overall energy fluence across the angular spectrum. Given that the energy transmission was significantly greater for Titanium film layers than for both the Silver and bare active areas, it is likely that the Titanium layer had a profound effect on the relative transmission coefficient at the active area boundary.

One possible explanation for this improved transmission is due to the relative refractive indices of the Titanium films as compared to the waveguide substrates. Titanium is known to have a refractive index of 2.479 at 532 nm, whereas the refractive index of the BK7 glass waveguide substate was 1.519 [70, 98]. Additionally, the refractive index of agarose gel,
which comprised the tissue phantom, is reported by Lee et al. to be \( n = 1.34 \) \[99\]. As such, it is possible that the Titanium thin-film significantly increased the relative transmission coefficient at the interface, even with such a thin layer, which may result in greater optical energy fluence through the active area.

As for the Silver active areas, the energy transmission graphs shown in Figure 4.12 demonstrated that the transmission decreased with increasing thickness of Silver in the active area. The initial peak-to-peak transmission curves suggested that the transition from high fluence to low fluence was abrupt and non-linear, which would render control over transmission somewhat more difficult as it would be highly sensitive to the film thickness at the individual nanometer level. Investigation of the integrated intensity curves shown in Figure 4.13 revealed a slightly more gradual transition from one deposition to the next; however, the fall in energy fluence was still much more significant after the second Silver deposition than it was after the first.

Therefore, a second series of tests were performed using Silver films deposited for half the duration as the first (i.e. for 30 sec at 30 mA instead of 60 sec). These repeated tests, shown in Figures 4.14 and 4.15, further clarified and confirmed this decrease through the deposition of thinner Silver films, which demonstrated a much more gradual decrease in energy fluence.

This was to be expected, due to the relatively low refractive index of Silver at 532 nm, which is \( n = 0.1429 \), as compared to the refractive index of the waveguide substrate, is \( n = 1.519 \) \[70\]. Consequently, the substantial difference in refractive indices encourages total internal reflection to a greater degree with thicker films of Silver. Based on the modified version of Snell’s Law (Eq. 2.35), the critical angle at the glass-Silver interface was calculated to be \( \theta_c = 4.87^\circ \). That being the case, total internal reflection occurs for all internal angles greater than approximately 5°, necessitating that any light transmission through the film be the result optical tunneling due to the penetration of the evanescent field into the external medium. The PAQT technique was named for this tunneling effect, since control of the Silver film geometry provides a modicum of control over the relative transmission through the film.

According to the simulations, based on the theoretical penetration depth of the evanes-
Figure 5.1 The theoretical penetration depth of the evanescent field into a Silver film reduces to less than 100 nm for the majority of the angular spectrum that was experimentally measured.

cent field at the glass-Silver interface, shown in Figure 5.1, the penetration depth throughout the majority of the angular spectrum decays to less than 100 nm by 34° and is only 57.98 nm at 75°. As such, the layers that were deposited for 60 sec may have been substantial enough to limit the degree of penetration to only a small fraction of the evanescent field. Each subsequent deposition, therefore, would have reduced the relative transmission according to the exponential decay function that governs penetration depth (Eq. 2.37). That limitation may have quickly resulted in a relative transmission that was insufficient to generate a distinguishable photoacoustic signal within the tissue phantom. Although the absolute transmission may not have been 0, it would have otherwise not been photoacoustically discernible from the background noise. This observation is further supported by the repeated tests, in that by depositing for 30 sec, the thinner individual Silver films would have allowed for a greater portion of the evanescent field to penetrate into the external medium and thereby tunnel out into the tissue phantom.
It is also worth noting that the general shape of the energy transmission curves is similar to the coupling efficiency curves simulated in MATLAB, as discussed in Section 4.1. This would suggest that the transmission is strongly related to the coupling efficiency of the cylindrical lens, which is to be expected. The first half of the spectrum appears to be strongly governed by this coupling; however, the latter half is likely governed by the penetration of the evanescent field into the external medium, and therefore the transmission through FTIR.

5.2 Photoacoustic Analysis

Throughout the angular transmission spectra, undulating series of peaks and valleys were consistently observed regardless of thin-film composition or geometry. These perturbations led to an investigation of the reliability of the peak-to-peak measurement standard that had been used to establish the transmission curves. Observation of the photoacoustic waveforms during the course of an angular spectrum test revealed that the waveforms were often far from ideal, demonstrating split peaks and unexpected undulations, which suggested that there were two photoacoustic signals overlapping in time.

It was determined that as the CR1-Z7 stage rotated through the angular spectrum, the photoacoustic waveforms would alternate back and forth between ideal photoacoustic peaks and split peaks, demonstrated in Figures 4.16 and 4.17 respectively, resulting in local maxima and minima along the energy fluence curves. Considering that the geometry of the active area is first characterized by a straight knife-edge aperture, it is likely that when the laser beam was only partially incident upon this edge it would split into two beams, with one beam refracting into the phantom while the other would reflect back into the waveguide first, before refracting into the phantom after a secondary reflection, as depicted in Figure 5.2. This would result in the generation of two independent, spatially separated photoacoustic sources within the tissue phantom. That being the case, one of the sources would theoretically be slightly closer to the ultrasonic detector, such that by the time the second waveform arrived at the transducer, the first would be reflecting away, resulting in a negative pressure zone. The two waves would overlap and result in unexpected spikes in pressure where it should be at a minimum, resulting in measured deviations away from the
Figure 5.2 Occasionally during the course of a test, the beam may be partially incident upon the knife-edge of the active area, causing it to split into two spatially separated beams that each generate a photoacoustic signal.

Due to the discrepancies created between the ideal peaks and the overlapping peaks, a new data analysis scheme was devised. This method attempted to account for the loss in measured intensity by integrating the area under the absolute value of the waveform trace. Although this technique is recognized as still being subject to the same concerns as the peak-to-peak measurement system, it served to more clearly demonstrate that the Silver films may have more gradually decreased energy fluence.

Another notable observation that was drawn from the more in-depth analysis of the photoacoustic traces was the revelation of the substantial evanescent energy absorption by the rubber clamp that held the waveguide in place against the tissue phantom. Typically, evanescent fields do not transfer energy; however, in the case of a non-zero optical absorption coefficient in the external medium, a portion of the evanescent field is absorbed, which is the basis of the TIRPAS technique discussed in Section 5.5.2 [77]. At lower angles, the loss was not very significant; however, as the internal reflection angle became more extreme, the photoacoustic waveform resultant from the evanescent loss began to dwarf
that of the beam transmitted into the tissue phantom. These signals were identified as evanescent loss partially due to their location in the time domain trace, but also due to the multiple reflections that followed in their wake. These reflections are directly resultant from the boundary conditions of the source, in that the acoustic waves generated within the rubber clamp would have reflected between the glass waveguide and the acrylic clamp before transmitting into the tissue phantom, thus causing a series of reflections steadily decreasing in amplitude. This lossy effect served to demonstrate the need for a relatively thick, optically transparent cladding layer in future investigations.

The analysis of the photoacoustic data recorded for Silver films was similar to that of the Titanium films. Initial testing revealed that the peaks and valleys in the energy transmission curves corresponded to a periodic variation between ideal photoacoustic waveforms, such as the one shown in Figure 4.16, and split peak waveforms, such as the one shown in Figure 4.17. Additionally, the Silver waveguides exhibited the same evanescent leaking effects as the Titanium waveguides did. Since the photoacoustic peaks occurred at the same point in time regardless of waveguide or active area, we can reliably state that the secondary peak consistently occurring around $6.5\mu$s was the result of evanescent leaking into the rubber clamp. For the purposes of this application, the loss of energy into the clamp suggests the need for an optically transparent cladding layer, such as a low refractive index polymer. However, it may also be possible to make use of the energy loss in order to characterize optical or physical properties of the rubber clamp. While this may seem trivial, given that the properties of the rubber piece is already well known, replacing the rubber with a biological analyte or a film of unknown optical properties could encourage the development of PAQT into an analytical technique, rather than a strictly ablative technique, which will be discussed in further detail in Section 5.5.2.

## 5.3 Conclusions

Although a more thorough analysis of energy transmission as a function of metal film thickness may be necessary before the PAQT technique can be applied as an analytical tool, this study has demonstrated the possibility of using planar waveguides to deliver light into tissue in a somewhat unconventional manner. As opposed to standard fiber optic
illumination or free-space propagation, the technique investigated herein circumvents or resolves many of the problems concerning light delivery into tissue. In accordance with the principal goal of this project, PAQT clearly has the potential to entirely avoid problems with laser light reflecting off the tissue, while still coupling the light into the skin in the direction of an optical absorbing target. Rather than considering the bulk tissue to be the target, PAQT treats the tissue as an optical propagation medium, thereby allowing light to be refracted or evanescently leaked into the tissue.

Furthermore, the results of the Silver film active area transmission studies demonstrate that it may be possible to regulate energy transmission through the precise control of active area film thickness. The investigation revealed that the fluence decreased with increasing film thickness, albeit in a much more immediate transition than was expected. Further investigations would do well to consider a greater number of thinner layers to observe the transmission transition more in greater detail. Additionally, by recording not only the time domain data, but also that of the frequency domain, it may be possible to either deconvolve the overlapping waveforms in time or instead consider the measured frequency response at each internal reflection angle and determine energy fluence by relative frequency amplitude as a function of internal reflection angle.

Moreover, the Titanium film active area studies also demonstrated the potential to couple light into the tissue. Based on the peak-to-peak energy fluence and integrated intensity curves, it is possible that due to Titanium’s much higher refractive index, the light in the active area experiences a higher relative transmission coefficient, causing potentially increased energy fluence. However, experimental errors in both recording and analysis cast some doubt on any significant conclusions that could otherwise be drawn about the energy fluence through the film. Future investigations should again utilize frequency analysis and a greater number of thinner intermediary layers to determine the potential for increased energy fluence. Additionally, an optically transparent cladding layer would permit the use of an optical couplant, which could further increase energy transmission to the sort of levels observed in the initial waveguide data.
5.4 Future Work

In addition to the efforts that could be made to improve the reliability of the results from this study, there are a number of alterations that could be made to substantially improve the functionality of the waveguide technique as a whole. Each of these propositions adds their own level of complexity to the waveguide design, albeit some of them are more easily applied than others. Nevertheless, each of the following components enhances the PAQT technique without significantly straying from the operating principles outlined by this study.

First and foremost, only two metal coatings were considered for this project. The two were chosen primarily because Silver is highly reflective and allows for a greater degree of internal reflection angles, but requires an oxidizing adhesion layer that can covalently bond to the glass, for which Titanium was chosen. However, by investigating a variety of other metals, it may be possible to expand upon the designs presented herein. For example, since this study suggested that due to Titanium’s relatively high refractive index, the light within the waveguide was more readily coupled out, it would be advisable to consider the use of other high-refractive index metals. Having said that, many of the higher refractive index metals are either toxic to human skin or particularly expensive, so the cheaper metals should be considered first. One metal in particular, Molybdenum, has a much higher refractive index than Titanium while still being relatively cheap and non-toxic. If Molybdenum were tested and resulted in higher energy transmission as well, it would suggest that our initial investigations with Titanium yielded results of some merit.

Another consideration would be to utilize multilayer active areas with many consecutive layers of different metals. Using thinner depositions, it may be possible to lessen the degree to which the Silver layers inhibit energy transmission. Therefore, by using multiple layers of metal in the active area, the laser energy could be spread out across the length of the active area to result in a higher number of reflection points in contact with the tissue. Alternatively, multiple layers may allow for customized transmission geometries and scaled energy fluence. For example, if it were desirable to have a variable energy delivery waveguide, one such geometry may be to gradate the Silver layers in a step geometry to
incorporate a decrease in film thickness, and consequent increase in percent transmission, over the length of the active area. Varying the metal deposited, the layer thickness, and the deposition order may also allow for a much greater degree of control over energy delivery into the the adjacent tissue.

Along a slightly different approach, another area of investigation would be to replace the metal film active area with a zeolite or mesoporous silica thin film. It is possible that such films may exhibit fewer problems with adhesion to the waveguide substrates than some metals do, but in thin enough layers would still allow for the optical tunneling to occur, as it does for the Silver layers. Another consideration is that metal layers tend to have a relatively high extinction coefficient, which results in a non-zero degree of optical absorption in the visible wavelengths used for laser dermatology. Zeolite and mesoporous silica films can be fabricated in such a way as to remain largely transparent or opaque, and thereby circumvent concerns about energy loss or heating due to optical absorption. Their porous nature may also be of benefit to other potential applications beyond laser dermatology like cancer detection and other forms of biosensing.

With regard to the experimental observations concerning photoacoustic peak analysis, considering that in the case of the split peak traces there seems to be two photoacoustic signals convolved in time, a more appropriate analysis method may be to perform a Discrete Fourier Transform (DFT) on the data to convert it into the frequency domain, where such signals would be multiplied instead of convolved. Since both of the signals originate in the same rubber absorber, it is likely that they would have very similar frequency responses, so a multiplication of their frequency domain traces may more accurately represent the degree of energy transmission into the sample.

With regard to further laser dermatology investigations, this study has demonstrated its potential as a new method of laser delivery; however, the project did not incorporate clinically relevant energies, which may damage or otherwise compromise the efficacy of this technique. Future research should replace the optical components involved with those made of a material that is suitable for high power optical applications, such as fused silica or sapphire. With the energy ramped up to a level equivalent to that used in contemporary laser treatments, the waveguide technique should be used to ablate target chromophores.
in samples of real tissue, in order to demonstrate the true merits of the technique. A histological study of the ablated tissue would reliably confirm ablation of the chromophore, whether it be tattoo ink particles or a cancerous lesion.

5.5 Potential Applications

There are a variety of possible applications that build on the foundation set by this study. Although the project was aimed at laser tattoo removal applications, the focus was principally chosen as it was the simplest manifestation of the technique, requiring only the waveguide and optical coupling components. By incorporating additional components, the PAQT technique can be built into a plethora of different applications.

5.5.1 Photoacoustic Tomography

In medical applications, a large portion of tissue imaging is performed using ultrasonic technologies; however, when these techniques use ultrasound sources that originate outside the body, they suffer from serious problems with resolution near the surface of the tissue. Furthermore, the ultrasonic wavelength tends to be too long to properly resolve smaller anomalies like tumors and early cancerous lesions. Photoacoustic tomography is an emerging field that attempts to bridge the gap left by ultrasound techniques and image smaller elements of the tissue nearer the surface. Using a laser to incite ultrasonic waves within the tissue allows for the imaging of much smaller aspects of the tissue within the limits of the tissue’s optical penetration depth [100–104]. Photoacoustic tomography has also been utilized for a variety of microscopy applications, including label-free microscopy and erythrocyte morphology analysis [105–108].

Therefore, one of the most immediate applications of the PAQT technique requires very little additional equipment, if any at all. By placing an ultrasonic detector on the rear face of the waveguide, as shown in Figure 5.3, PAQT can be applied as a photoacoustic tomography probe. Moreover, this technique neatly resolves one of the principal problems that has plagued photoacoustic tomography – namely, that with traditional free-space illumination of tissue, the ultrasonic detector occludes the path of the laser beam. There have been a few major research labs who have produced methods that circumvent this problem.
Figure 5.3 Pulsed light that couples out of the waveguide into tissue may be absorbed by a chromophore and produce a photoacoustic pulse, which may be detected at the rear surface of the waveguide.

One lab utilizes a dual laser Fabry-Pérot cavity detector, wherein one laser is used to incite the photoacoustic signal within the tissue and the other is used to scan the cavity. Another lab uses a custom manufactured ultrasonic transducer with a disc-shaped detector that has a hole in the center for an optical fiber to illuminate the tissue. Both of these techniques have demonstrated impressive results using photoacoustics to map sub-epidermal structures, but both have their drawbacks with regard to expense and complexity. By contrast, using a PAQT waveguide to illuminate tissue may cheaply and effectively resolve the issue without the need for complex laser delivery systems.

5.5.2 TIRPAS Resonator

Another possible, albeit more complicated, application would be to capitalize on the evanescent leaking effects noticed in Sections 5.2 and 4.3.2. Total Internal Reflection Photoacoustic Spectroscopy (TIRPAS) is a parallel technique that operates on the same well-established optical principles, but utilizes the evanescent field to optically characterize thin films on the surface of a prism [91].

The TIRPAS systems typically operate using a prism that is rotated through a series of angles to determine the Brewster angle at which photoacoustic signals transition from a
Figure 5.4 By scanning through a series of internal reflection angles, laser light may escape the waveguide and directly incite photoacoustic pulses within an external medium up to the Brewster angle, after which the photoacoustic pulses are generated indirectly via the evanescent field.

direct generation by refraction into the medium over to an evanescent generation only within the first few hundred nanometers of the medium. Figure 5.4 demonstrates the difference between the PAS (photoacoustic spectroscopy) and TIRPAS regimes. The TIRPAS system utilizes the discrepancies between these two types of signals in order to draw conclusions about material properties such as refractive index and film thickness.

As a consequence of its waveguide design and multiple internal reflections, the PAQT system effectively behaves as a TIRPAS resonator along its entire length. In this manner, using a series of internal reflection points instead of just a single point may serve to enhance the sensitivity of the technology. Alternatively, utilizing an array of transducers corresponding to each internal reflection point may instead allow for a faster series of tests since each of the internal reflection points would act as its own TIRPAS signal generator.

5.5.3 Biosensing

Another application that would make use of the aforementioned evanescent leaking effect would be in the field of biosensing [109–111]. As was the case with TIRPAS, the evanescently generated photoacoustic signals can be used to characterize chromophores near the surface of the waveguide [112–114]. The diagram in Figure 5.5 demonstrates a simple representation of how the evanescent field interacts with absorbing particles near
Figure 5.5 Since the evanescent field penetrates into the external medium upon reflection, any optically absorbing biological analyzes near the interface may absorb the energy and produce a photoacoustic pulse.

the surface of the waveguide to create photoacoustic signals.

As was mentioned in Section 5.4, the PAQT technique could also incorporate mesoporous silica thin films on the surface of the waveguide. These porous films can be fabricated to capture chromophores of specific geometries and sizes. If such films were used in congruence with the evanescent photoacoustic generation, PAQT could be transformed into a clean, tag-less sensor for optically absorbing analytes. Additionally, if the tag-less approach is insufficient, the films can be functionalized with specific proteins and antibodies to specifically capture non-absorbing analytes. Upon their capture, the local refractive index should change, which theoretically may be observed in the amplitude of the photoacoustic signal since the degree of evanescent leaking is dependent upon the refractive index of the local medium.
Appendix A

Theoretical Derivations

A.1 Maxwell’s Equations

\[ \nabla \times \mathbf{H} = j + \frac{\partial \mathbf{D}}{\partial t} \quad (A.1) \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (A.2) \]
\[ \nabla \cdot \mathbf{D} = \rho \quad (A.3) \]
\[ \nabla \cdot \mathbf{B} = 0 \quad (A.4) \]
\[ j = \sigma \mathbf{E} \]
\[ \mathbf{D} = \varepsilon \mathbf{E} \]
\[ \mathbf{B} = \mu \mathbf{H} \]

\[ \varepsilon = \varepsilon_r \varepsilon_0 \]
\[ \mu = \mu_r \mu_0 \]

\[ \mathbf{H} = \text{Magnetic field strength} \]
\[ \mathbf{B} = \text{Magnetic flux density} \]
\[ \mathbf{E} = \text{Electric field strength} \]
\[ \mathbf{D} = \text{Electric displacement} \]
\[ j = \text{Electric current density} \]
\[ \rho = \text{Electric charge density} \]
\[ \sigma = \text{Electric conductivity} \]
\[ \mu = \text{permeability} \]
\[ \varepsilon = \text{permittivity} \]
\[ \varepsilon_0 = \text{permittivity of free space} \]
\[ \mu_0 = \text{permeability of free space} \]
\[ \varepsilon_r = \text{relative permittivity} \]
\[ \mu_r = \text{relative permeability} \]
\[ c = \text{speed of light in a vacuum} \]

A.1.1 Wave Equation in terms of \( \mathbf{E} \)

In order to get an expression for the wave equation in terms of the Electric Field, \( \mathbf{E} \), all of the magnetic components \( \mathbf{H} \) and \( \mathbf{B} \) must be replaced with their electric equivalents. The following rigorous derivation demonstrates this process step by step, until reaching the general expression for the wave equation in terms of \( \mathbf{E} \) (Eq. A.8). In the derivation below and those that follow, equations of some significance are accompanied by numerical tags for reference purposes; whereas intermediate steps are accompanied by a brief note describing
Appendix A. Derivations

the action required to proceed from the previous step, included on the right of each line.

\[ \nabla \times \mathbf{H} = j + \frac{\partial D}{\partial t} \]  
(From Eq. A.1)

\[ = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \]  
(Substitute \( D = \varepsilon \mathbf{E}, \ j = \sigma \mathbf{E} \))

\[ \nabla \times \left( \frac{1}{\mu} \mathbf{B} \right) = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \]  
(Substitute \( B = \mu \mathbf{H} \))

\[ \frac{1}{\mu} (\nabla \times \mathbf{B}) = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \]  
(Factor in the Curl of \( \mathbf{B} \))

\[ \therefore \nabla \times \mathbf{B} = \mu \sigma \mathbf{E} + \mu \varepsilon \frac{\partial \mathbf{E}}{\partial t} \]  
(A.5)

\[ \nabla \cdot \mathbf{D} = 0 \]  
(Charge density in a vacuum is 0)

\[ = \nabla \cdot (\varepsilon \mathbf{E}) \]  
(Substitute \( \mathbf{D} = \varepsilon \mathbf{E} \))

\[ = \varepsilon (\nabla \cdot \mathbf{E}) \]  
(Factor in the divergence of \( \mathbf{E} \))

\[ \therefore \nabla \cdot \mathbf{E} = 0 \]  
(A.6)

The expressions derived in Equations A.5 and A.6 will be utilized in the subsequent derivation, but are presented at the start of the process for ease of reference. The derivation of the wave equation will henceforth proceed unimpeded until the complete expression has been found.
\[ \nabla \times (\nabla \times E) = \nabla \times \left( -\frac{\partial B}{\partial t} \right) \]  
\[ = -\frac{\partial}{\partial t} (\nabla \times B) \]  
(Take the curl of Eq. A.2)

\[ \nabla (\nabla \cdot E) - \nabla^2 E = -\frac{\partial}{\partial t} (\nabla \times B) \]  
(Factor in the Curl of \( B \))

\[ \nabla (\nabla \cdot E) - \nabla^2 E = -\frac{\partial}{\partial t} (\nabla \times B) \]  
(Curl of the Curl identity)

\[ 0 - \nabla^2 E = -\frac{\partial}{\partial t} (\nabla \times B) \]  
(Substitute in Eq. A.6)

\[ \therefore \nabla^2 E = \frac{\partial}{\partial t} (\nabla \times B) \]  
(A.7)

\[ \nabla^2 E = \frac{\partial}{\partial t} \left( \mu \sigma E + \mu \varepsilon \frac{\partial E}{\partial t} \right) \]  
(Substitute in Eq. A.5)

\[ = \mu \sigma \frac{\partial E}{\partial t} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2} \]  
(Distribute the derivative)

\[ \therefore \nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2} \]  
(A.8)

One solution of Eq. A.8 is in the complex form of a plane-polarized harmonic plane wave:

\[ E = \mathcal{E} \exp \left[ i \omega \left( t - \frac{x}{v} \right) \right] \]  
(A.9)

with the condition that:

\[ \frac{\omega^2}{v^2} = \omega^2 \varepsilon \mu - i \omega \mu \sigma \]  
(A.10)

In this representation, \( \mathcal{E} \) represents the vector amplitude for the Electric wave. The complex representation of the refractive index \( N = n - i \kappa \), where \( n \) and \( \kappa \) are the real and complex components of the refractive index respectively, can then be readily derived from Eq. A.10, and subsequently a much more useful form of the plane wave equation.
\[
\frac{\omega^2}{c^2} = \omega^2 \varepsilon \mu - i \omega \mu \sigma \\
= \omega^2 \varepsilon \mu - 0 \\
= \omega^2 \varepsilon_0 \mu_0 \\
\therefore c^2 = \frac{1}{\varepsilon_0 \mu_0} \\
(A.11)
\]

\[
\frac{\varepsilon \mu}{\varepsilon_0 \mu_0} = \frac{\varepsilon \mu}{\varepsilon_0 \mu_0} - i \frac{\mu \sigma}{\omega \varepsilon_0 \mu_0} \\
= \varepsilon_r \mu_r - i \frac{\mu_r \sigma}{\varepsilon_0 \omega} \\
\therefore \frac{c^2}{v^2} = \varepsilon_r \mu_r - i \frac{\mu_r \sigma}{\varepsilon_0 \omega} \\
(A.12)
\]

\[
\frac{c^2}{v^2} = N^2 \\
N^2 = (n - i k)^2 \\
= n^2 - i 2 n k - k^2 \\
= (n^2 - k^2) - i 2 n k \\
\therefore (n^2 - k^2) = \varepsilon_r \mu_r - i \frac{\mu_r \sigma}{\varepsilon_0 \omega} \\
(A.13)
\]

\[
\therefore 2 n k = \frac{\mu_r \sigma}{\varepsilon_0 \omega} \\
(A.14)
\]

Equation A.8 can then be rewritten to incorporate some more useful elements like the complex refractive index term \(N\) and the wavelength term \(\lambda\). Subsequently, the equation can be generalized for any plane-polarized wave propagating in the direction given by unit vector \(\hat{s} = (\alpha i + \beta j + \gamma k)\), as shown below:
\[ E = \mathcal{E} \exp \left[ i \left( \omega t - \frac{\omega N x}{c} \right) \right] \]  
(Substitute \( v = \frac{x}{t} \) into Eq. A.8)

\[ = \mathcal{E} \exp \left[ i \left( \omega t - \left( \frac{2\pi N}{\lambda} \right) x \right) \right] \]  
(Substitute \( \lambda = \frac{2\pi c}{\omega} \))

\[ \therefore E = \mathcal{E} \exp \left[ i \left( \omega t - \left( \frac{2\pi N}{\lambda} \right) (\alpha x + \beta j + \gamma z) \right) \right] \]  
(A.15)

### A.1.2 Wave Equation in terms of \( H \)

A similar expression can be derived for the Wave Equation in terms of the Magnetic Field, \( H \). Much of the derivation is identical to that which led to Eq. A.15, but the portions that differ are shown below. Again, Equation A.16 is included for use later on in the derivation of the wave equation, but is included at the start for ease of reference.

\[ \nabla \cdot B = 0 \]  
(From Eq. A.4)

\[ = \nabla \cdot (\mu H) \]  
(Substitute \( B = \mu H \))

\[ = \mu (\nabla \cdot H) \]  
(Factor in the divergence of \( H \))

\[ \therefore \nabla \cdot H = 0 \]  
(A.16)

\[ \nabla \times (\nabla \times H) = \nabla \times \left( j + \frac{\partial D}{\partial t} \right) \]  
(Take the curl of Eq. A.1)

\[ = \sigma (\nabla \times E) + \varepsilon \frac{\partial}{\partial t} (\nabla \times E) \]  
(Substitute \( j = \sigma E, D = \varepsilon E \))

\[ = -\sigma \frac{\partial B}{\partial t} - \varepsilon \frac{\partial^2 B}{\partial t^2} \]  
(Substitute Eq. A.2 for \( \nabla \times E \))

\[ = -\varepsilon \mu \frac{\partial^2 H}{\partial t^2} - \sigma \mu \frac{\partial H}{\partial t} \]  
(Substitute \( B = \mu H \))

\[ \nabla (\nabla \cdot H) - \nabla^2 H = -\varepsilon \mu \frac{\partial^2 H}{\partial t^2} - \sigma \mu \frac{\partial H}{\partial t} \]  
(Curl of the Curl identity)

\[ 0 - \nabla^2 H = -\varepsilon \mu \frac{\partial^2 H}{\partial t^2} - \sigma \mu \frac{\partial H}{\partial t} \]  
(Substitute in Eq. A.16)

\[ \therefore \nabla^2 H = \varepsilon \mu \frac{\partial^2 H}{\partial t^2} + \sigma \mu \frac{\partial H}{\partial t} \]  
(A.17)
Appendix A. Derivations

One solution of Eq. A.17 is in the complex form of a plane-polarized harmonic plane wave, as was the case in the derivation of the Wave Equation in terms of $E$. A nearly identical equation results from the derivation of the Wave Equation in terms of $H$, where $H$ represents the vector amplitude for the magnetic wave:

$$H = H \exp \left[ i \left( \omega t - \left( \frac{2\pi N}{\lambda} \right) (\alpha x + \beta j + \gamma z) \right) \right] \quad (A.18)$$

A.1.3 Characteristic Optical Admittance, $Y$

The following derivation utilizes portions of sections A.1.1 and A.1.2 in order to gain a relation between $E$ and $H$ in terms of the characteristic optical admittance $Y$. Following from Eq. A.15, we first gain an expression for the curl of $H$ in terms of $E$, before finding an alternate expression for the curl of $H$ that suits more appropriately suits our intentions.

$$\frac{\partial}{\partial t} E = i \omega E \exp \left[ i \left( \omega t - \left( \frac{2\pi N}{\lambda} \right) (\alpha x + \beta j + \gamma z) \right) \right] \quad \text{(Derive Eq. A.15)}$$

$$= i \omega E \quad (A.19)$$
\[ \nabla \times \mathbf{H} = j + \frac{\partial D}{\partial t} \]  
(Maxwell’s Eq. A.1)

\[ = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \]  
(Substitute for \( j \) and \( D \))

\[ = \sigma \mathbf{E} + \varepsilon (i\omega \mathbf{E}) \]  
(Substitute in Eq. A.19)

\[ = (\sigma + i\omega \varepsilon) \mathbf{E} \]  
(Factor out \( \mathbf{E} \))

\[ = (\sigma + i\omega \varepsilon) \frac{N^2}{\varepsilon_r \mu_r - i\frac{\mu_r\sigma}{\omega \varepsilon_0}} \mathbf{E} \]  
(Substitute in Eq. A.12)

\[ = (\sigma + i\omega \varepsilon) \frac{N^2}{\varepsilon_0 \mu_0 - i\frac{\mu_0\sigma}{\omega \varepsilon_0}} \mathbf{E} \]  
(Replace \( \mu_r \) and \( \varepsilon_r \))

\[ = (\sigma + i\omega \varepsilon) \frac{N^2 \epsilon \mu \omega}{\varepsilon_0 \mu_0 \omega} \mathbf{E} \]  
(Common denominator)

\[ = \frac{(\sigma + i\omega \varepsilon) N^2 \omega}{c^2 (\varepsilon \mu \omega - i\mu \sigma)} \mathbf{E} \]  
(Rearrange fractions, \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \))

\[ = \frac{i (\sigma + i\omega \varepsilon) N^2 \omega}{c^2 \mu (\varepsilon \omega - i\sigma)} \mathbf{E} \]  
(Multiply top and bottom by \( i \))

\[ = \frac{i (\sigma + i\omega \varepsilon) N^2 \omega}{c^2 \mu (\sigma + i\omega \varepsilon)} \mathbf{E} \]  
(Factor in \( i \) on the bottom)

\[ \therefore \nabla \times \mathbf{H} = i \frac{N^2 \omega}{c^2 \mu} \mathbf{E} \]  
\( \text{(A.20)} \)

\[ \frac{\partial \mathbf{H}}{\partial x} = -i \frac{2\pi N}{\lambda} \alpha \mathbf{H} \exp \left[ i \left( \omega t - \frac{2\pi N}{\lambda} (\alpha x + \beta y + \gamma z) \right) \right] \]  
(Derive Eq. A.18)

\[ = -i \frac{2\pi N}{\lambda} \alpha \mathbf{H} \]  
(Substitute \( \mathbf{H} \))

\[ = -i \frac{\omega N}{c} \alpha \mathbf{H} \]  
\( \text{(A.21)} \)

\[ \frac{\partial \mathbf{H}}{\partial y} = -i \frac{\omega N}{c} \beta \mathbf{H} \]  
\( \text{(A.22)} \)

\[ \frac{\partial \mathbf{H}}{\partial z} = -i \frac{\omega N}{c} \gamma \mathbf{H} \]  
\( \text{(A.23)} \)
\[ \nabla \times \mathbf{H} = \left( \frac{\partial H}{\partial y} - \frac{\partial H}{\partial z} \right) i + \left( \frac{\partial H}{\partial z} - \frac{\partial H}{\partial x} \right) j + \left( \frac{\partial H}{\partial x} - \frac{\partial H}{\partial y} \right) k \]  

(Definition of the curl of \( \mathbf{H} \))

\[ = -i \frac{\omega N}{c} \left[ (\beta \mathbf{H} - \gamma \mathbf{H}) i + (\gamma \mathbf{H} - \alpha \mathbf{H}) j + (\alpha \mathbf{H} - \beta \mathbf{H}) k \right] \]  

(Substitute Eq. A.21, A.22, A.23)

\[ -i \frac{\omega N}{c} (\hat{s} \times \mathbf{H}) \]  

(Definition of \( \hat{s} \times \mathbf{H} \))

\[ = i \frac{N^2 \omega}{c^2 \mu} \mathbf{E} \]  

(From Eq. A.20)

\[ (\hat{s} \times \mathbf{H}) = - \frac{N}{c \mu} \mathbf{E} \]  

(Combine fractions)

\[ \therefore \mathbf{E} = \frac{c \mu}{N} (\hat{s} \times \mathbf{H}) \]  

(\( \hat{s}, \mathbf{E}, \mathbf{H} \) form a right-handed set)

\[ \therefore \mathbf{H} = \frac{N}{c \mu} (\hat{s} \times \mathbf{E}) \]  

(A.24)

The optical admittance of free space is given by \( \mathcal{Y} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \). Typically, \( \mu = \mu_r \mu_0 \), but for optical frequencies \( \mu_r = 1 \), so \( \mu \approx \mu_0 \). As such, the characteristic optical admittance of any medium can be given by the following expression:

\[ \mathcal{Y} = \frac{N}{c \mu} \]  

(Definition of optical admittance)

\[ = \frac{N}{\mu_0} \sqrt{\frac{\varepsilon_0 \mu_0}{\mu_0}} \]  

(\( \mu \approx \mu_0 \), \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \))

\[ = \frac{N}{(\sqrt{\mu_0})^2} \]  

(Rewrite the denominator)

\[ = N \sqrt{\frac{\varepsilon_0}{\mu_0}} \]  

(Cancel \( \sqrt{\mu_0} \))

\[ \therefore \mathcal{Y} = N \mathcal{Y} \]  

(A.25)

That being the case, Eq. A.24 can be rewritten in the more convenient manner shown below.

\[ \mathbf{H} = \mathcal{Y} (\hat{s} \times \mathbf{E}) \]  

(A.26)

\[ \mathbf{H} = N \mathcal{Y} (\hat{s} \times \mathbf{E}) \]  

(A.27)
A.1.4 The Poynting Vector, Irradiance, and the Optical Absorption Coefficient

In optics, the Poynting vector is defined as the instantaneous rate of flow of energy across unit area and is oriented along the same direction as \( \hat{s} \). That being the case, the mean value of the Poynting vector is defined as the optical Irradiance (i.e. the optical intensity), \( I \), shown below where \( H^* \) is the complex conjugate of \( H \), whereas \( E, H, \) and \( I \) are the respective scalar magnitudes.

\[
I = \frac{1}{2} \Re \{ E \times H^* \} \tag{A.28}
\]

\[
||I|| = I = \Re \left\{ \frac{1}{2} EH^* \right\} \tag{A.29}
\]

By incorporating in the expression derived for \( H \) in Eq. A.24, we can express the scalar irradiance as shown below.

\[
I = \frac{1}{2} \Re \{ E \times \gamma (\hat{s} \times E) \} \tag{Substitute in Eq. A.24}
\]

\[
I = \Re \left\{ \frac{1}{2} \gamma EE^* \hat{s} \right\} \tag{A.30}
\]

\[
\therefore I = \frac{1}{2} \gamma \sigma EE^* \tag{A.31}
\]

In order to gain a more useful expression for the irradiance, however, we must first determine the Real component of the Electric vector amplitude.
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\[ ||E|| = E = \mathcal{E} \exp \left[ i \left( \omega t - \frac{2\pi N}{\lambda} (\alpha x + \beta y + \gamma z) \right) \right] \]  

(From Eq. A.15)

\[ = \mathcal{E} \exp \left[ i \left( \omega t - \frac{2\pi [n - ik]}{\lambda} (\alpha x + \beta y + \gamma z) \right) \right] \quad (N = n - ik) \]

\[ = \mathcal{E} \exp \left[ \frac{-2\pi k}{\lambda} (\alpha x + \beta y + \gamma z) \right] \exp \left[ i \left( \omega t - \frac{2\pi n}{\lambda} (\alpha x + \beta y + \gamma z) \right) \right] \]

\[ \Re \{E\} = \mathcal{E} \exp \left[ \frac{-2\pi k}{\lambda} (\alpha x + \beta y + \gamma z) \right] \quad (A.32) \]

\[ \therefore EE^* = \mathcal{E}\mathcal{E}^* \exp \left[ \frac{-4\pi k}{\lambda} (\alpha x + \beta y + \gamma z) \right] \quad (A.33) \]

As a result, the Irradiance can be concisely represented in the following manner.

\[ I = \frac{1}{2} n \mathcal{Y} |\mathcal{E}|^2 \exp \left[ -\frac{4\pi k}{\lambda} (\alpha x + \beta y + \gamma z) \right] \quad (A.34) \]

Since the amplitude of the Electric field at \((x, y, z)\) is given by Eq. A.32, the expression for Irradiance can be generalized as \(I = \frac{1}{2} n \mathcal{Y} (\text{amplitude})^2\). Therefore, as opposed to the traditional \(I \propto (\text{amplitude})^2\), a more appropriate relation can be expressed as follows.

\[ I \propto n \times (\text{amplitude})^2 \quad (A.35) \]

Additionally, for expressions utilizing \(s\), \((\alpha x + \beta y + \gamma z)\) is defined as the distance along the direction of propagation. Following from that expression, it can be found that \(I\) drops to \(\frac{1}{4} I\) at a distance given by \(\frac{\lambda}{4\pi k}\). It is convenient to define the optical absorption coefficient of a medium as the inverse of this distance:

\[ \alpha = \frac{4\pi k}{\lambda} \quad (A.36) \]

A.1.5 Snell’s Law

We can readily derive Snell’s law by applying a few simple boundary conditions, namely that:
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- tangential components of \( E \) and \( H \) are continuous across the boundary

- \( \hat{s}_{\text{reflected}} = \langle \alpha_r, \beta_r, \gamma_r \rangle \)

- \( \hat{s}_{\text{transmitted}} = \langle \alpha_t, \beta_t, \gamma_t \rangle \)

From these conditions, we can represent each of the three wavefronts of interest – incident, reflected, and transmitted – consistently for both Electric and Magnetic components in the following forms:

Incident: \( \exp \left[ i \left( \omega_i t - \left( \frac{2 \pi n_0}{\lambda_i} \right) (x \sin \vartheta_0 + z \cos \vartheta_0) \right) \right] \)

Reflected: \( \exp \left[ i \left( \omega_r t - \left( \frac{2 \pi n_0}{\lambda_r} \right) (\alpha_r x + \beta_r y + \gamma_r z) \right) \right] \)

Transmitted: \( \exp \left[ i \left( \omega_t t - \left( \frac{2 \pi n_1}{\lambda_t} \right) (\alpha_t x + \beta_t y + \gamma_t z) \right) \right] \)

Then, in order to satisfy continuity for all \( x, y, \) and \( t \) at \( z = 0 \), it can be determined that \( \omega_i \equiv \omega_r \equiv \omega_t \). In other words, there is no change in frequency for reflected or transmitted waves as compared to the incident wave. That being the case, we can extend the identical equality to the wavelength, such that \( \lambda_i \equiv \lambda_r \equiv \lambda_t \). Additionally, since the representation of the incident beam is oriented such that it is independent of \( y \), the conclusion can be drawn that the directions of the reflected and transmitted waves are confined to the plane of incidence. The mathematical implications of these observations are as follows:

\[
0 \equiv n_0 \beta_r \equiv n_1 \beta_t \\
\left( \frac{2 \pi n_0}{\lambda} \right) (x \sin \vartheta_i + z \cos \vartheta_i) \equiv \left( \frac{2 \pi n_0}{\lambda} \right) (\alpha_r x + \gamma_r z) \\
\equiv \left( \frac{2 \pi n_1}{\lambda} \right) (\alpha_t x + \gamma_t z) \\
\therefore \sin \vartheta = \alpha \\
\therefore \cos \vartheta = \gamma \\
n_0 \sin \vartheta_i \equiv n_0 \alpha_r \equiv n_1 \alpha_t \quad \text{(A.37)}
\]
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Following from Eq. A.37 and defining the angles of reflection and transmission to be $\vartheta_r$ and $\vartheta_t$ respectively, we find the following relation to be true.

$$n_0 \sin \vartheta_i \equiv n_1 \alpha_t \quad \text{(From Eq. A.37)}$$

$$= n_1 \sin \vartheta_t \quad \text{(A.38)}$$

Redefining $\vartheta_i$ and $\vartheta_t$ to be $\vartheta_0$ and $\vartheta_1$ respectively, results in the traditional expression for Snell’s Law shown below. Additionally, we can represent this expression equivalently as shown in Eq. A.40 and A.41.

$$n_0 \sin \vartheta_0 = n_1 \sin \vartheta_1 \quad \text{(A.39)}$$

$$\alpha_r^2 + \gamma_r^2 = 1 \quad \text{(A.40)}$$

$$\alpha_t^2 + \gamma_t^2 = 1 \quad \text{(A.41)}$$

A.2 Boundary conditions

This section deals with the various boundary conditions encountered when dealing with borders between two adjacent media. The first sections will deal with non-absorbing media for simplicity, seeing as the absorbing nature of a medium necessitates the incorporation of complex components.

A.2.1 Normal incidence in non-absorbing media

This section deals with perhaps the simplest boundary conditions in that they do not incorporate angular components and deal only with straightforward interactions. The pair of conditions are that the electric and magnetic vectors are continuous across the boundary. They are represented mathematically as shown below:

$$\mathcal{E}_i + \mathcal{E}_r = \mathcal{E}_t \quad \text{(A.42)}$$

$$\mathcal{H}_i - \mathcal{H}_r = \mathcal{H}_t \quad \text{(A.43)}$$
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It is convenient to rearrange these conditions such that they deal with only the electric field; although the choice between $\mathbf{E}$ and $\mathbf{H}$ is an arbitrary one. Rewriting the second condition in terms of $\mathbf{E}$ results in the following derivations of the amplitude reflection coefficient, $\rho$, and the amplitude transmission coefficient, $\tau$.

\[
\mathcal{H}_i - \mathcal{H}_r = \mathcal{H}_t \quad \text{(From Eq. A.43)}
\]

\[
y_0\mathbf{E}_i - y_0\mathbf{E}_r = y_1\mathbf{E}_t \quad \text{(\mathcal{H} = y\mathbf{E})}
\]

\[
= y_1 (\mathbf{E}_i + \mathbf{E}_r) \quad \text{(Substitute in Eq. A.42)}
\]

\[
\mathbf{E}_i (y_0 - y_1) = \mathbf{E}_r (y_0 + y_1) \quad \text{(Distribute and factor)}
\]

\[
\frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{y_0 - y_1}{y_0 + y_1} \quad \text{(Rearrange fractions)}
\]

\[
= \frac{n_0\mathcal{Y} - n_1\mathcal{Y}}{n_0\mathcal{Y} + n_1\mathcal{Y}} \quad \text{($\mathcal{Y} = (n - ik)\mathcal{Y}, k = 0$)}
\]

\[
= \frac{n_0 - n_1}{n_0 + n_1} \quad \text{(Divide by $\mathcal{Y}$)}
\]

\[
\rho = \frac{\mathbf{E}_r}{\mathbf{E}_i} \quad \text{(Definition of $\rho$)}
\]

\[
: \therefore \rho = \frac{n_0 - n_1}{n_0 + n_1} \quad \text{(A.44)}
\]

\[
y_0\mathbf{E}_i - y_0\mathbf{E}_r = y_1\mathbf{E}_t \quad \text{(Continued from above)}
\]

\[
y_0\mathbf{E}_i - y_0 (\mathbf{E}_t - \mathbf{E}_i) = y_1\mathbf{E}_t \quad \text{(Substitute in Eq. A.42)}
\]

\[
2y_0\mathbf{E}_i = \mathbf{E}_t (y_0 + y_1) \quad \text{(Distribute and factor)}
\]

\[
\frac{\mathbf{E}_i}{\mathbf{E}_t} = \frac{2y_0}{y_0 + y_1} \quad \text{($\mathcal{Y} = n\mathcal{Y}$)}
\]

\[
= \frac{2n_0}{n_0 + n_1} \quad \text{(Divide by $\mathcal{Y}$)}
\]

\[
\tau = \frac{\mathbf{E}_t}{\mathbf{E}_i} \quad \text{(Definition of $\tau$)}
\]

\[
: \therefore \tau = \frac{2n_0}{n_0 + n_1} \quad \text{(A.45)}
\]

From these expressions for $\rho$ and $\tau$, we can observe that $\tau$ is always positive and real, indicating that there is no phase shift between incident and transmitted beams. We can also see that if $\rho$ is positive (i.e. if $n_0 > n_1$), then there will be no phase change; however,
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if $\rho$ is negative, there will be a phase change of $\pi$.

Having defined $\rho$ and $\tau$, we can determine expressions for the Irradiance of each of the three beams of interest – incident, reflected, and transmitted.

$$I = \frac{1}{2} \gamma \mathcal{E} \mathcal{E}^*$$  \hspace{1cm} (From Eq. A.31)

$$\therefore I_i = \frac{1}{2} \gamma_0 \mathcal{E}_i \mathcal{E}_i^*$$  \hspace{1cm} (A.46)

Here, $I_i$ above represents the Irradiance of the incident beam. While we can express $I_r$ and $I_t$ in similar terms, it is more convenient for future derivations to express them instead in terms of $I_i$, as shown below.

$$I_r = \frac{1}{2} \gamma_0 \mathcal{E}_r \mathcal{E}_r^*$$ \hspace{1cm} (Reflected irradiance)

$$= \frac{1}{2} \gamma_0 (\rho \mathcal{E}_i) (\rho \mathcal{E}_i)^*$$ \hspace{1cm} (Substitute $\mathcal{E}_r = \rho \mathcal{E}_i$)

$$= \frac{1}{2} \gamma_0 \rho^2 \mathcal{E}_i \mathcal{E}_i^*$$ \hspace{1cm} (Reflect the format of $I_i$)

$$\therefore I_r = \rho^2 I_i$$ \hspace{1cm} (A.47)

$$I_t = \frac{1}{2} \gamma_1 \mathcal{E}_t \mathcal{E}_t^*$$ \hspace{1cm} (Transmitted irradiance)

$$= \frac{1}{2} \gamma_1 (\tau \mathcal{E}_i) (\tau \mathcal{E}_i)^*$$ \hspace{1cm} (Substitute $\mathcal{E}_t = \tau \mathcal{E}_i$)

$$= \frac{1}{2} \gamma_0 \frac{\gamma_1}{\gamma_0} \tau^2 \mathcal{E}_i \mathcal{E}_i^*$$ \hspace{1cm} (Reflect the format of $I_i$)

$$\therefore I_t = \frac{\gamma_1}{\gamma_0} \tau^2 I_i$$ \hspace{1cm} (A.48)

At this point, we can utilize our expressions for $\rho$ and $\tau$ along with the expressions for Irradiance shown above in order to derive the expressions for the relative reflection and transmission coefficients $R = \frac{I_r}{I_i}$ and $T = \frac{I_t}{I_i}$ through a simple energy balance equation based on Eq. A.28.
\[ I_0 = I_1 \]  
\[ \Re \left\{ \frac{1}{2} E_0 \times H_0^* \right\} = \Re \left\{ \frac{1}{2} E_1 \times H_1^* \right\} \]  
\[ \Re \left\{ \frac{1}{2} (E_i + E_r) (H_i - H_r)^* \right\} = \Re \left\{ \frac{1}{2} E_t H_t^* \right\} \]  
\[ \frac{1}{2} (E_i + E_r) (\gamma_0 \varepsilon_i - \gamma_0 \varepsilon_r)^* = \frac{1}{2} \gamma_1 \varepsilon_t \varepsilon_t^* \]  
\[ \frac{1}{2} \gamma_0 (\varepsilon_i + \rho \varepsilon_i) (\varepsilon_i - \rho \varepsilon_i)^* = \frac{1}{2} \gamma_0 \frac{\gamma_1}{\gamma_0} \tau^2 \varepsilon_i \varepsilon_i^* \]  
\[ \frac{1}{2} \gamma_0 \varepsilon_i \varepsilon_i^* (1 - \rho^2) = \frac{1}{2} \gamma_0 \frac{\gamma_1}{\gamma_0} \tau^2 \varepsilon_i \varepsilon_i^* \]  
\[ I_i (1 - \rho^2) = \frac{\gamma_1}{\gamma_0} \tau^2 I_i \]  
\[ I_i - \rho^2 I_i = \frac{\gamma_1}{\gamma_0} \tau^2 I_i \]  
\[ I_i - I_r = I_t \]  
\[ 1 - \frac{I_r}{I_i} = \frac{I_t}{I_i} \]  
\[ \therefore 1 - R = T \]  
(A.49)

A.2.2 Oblique incidence in non-absorbing media

p-polarized light (TM polarized)

This section deals with oblique incidence, and therefore necessitates separate approaches based on the polarization state of the incident beam – either p- or s- polarized. The two approaches are similar, but require different sign conventions and result in slightly differing representations for the relative reflection and transmission coefficients. We will begin by stating the boundary conditions for p-polarized light, mathematically represented as shown below:

\[ \varepsilon_i \cos \vartheta_0 + \varepsilon_r \cos \vartheta_0 = \varepsilon_t \cos \vartheta_1 \]  
(A.50)  
\[ \mathcal{H}_i - \mathcal{H}_r = \mathcal{H}_t \]  
(\( \mathcal{H} = \gamma \varepsilon \))  
\[ \gamma_0 \varepsilon_i - \gamma_0 \varepsilon_r = \gamma_1 \varepsilon_t \]  
(A.51)
Appendix A. Derivations

Again, we will rearrange these conditions such that they deal with only the electric field. It is important to note that the \( \cos \vartheta \) terms arise from translating \( \hat{s} \) over to perpendicular incidence, so that \( \mathbf{E} \) and \( \mathbf{H} \) are tangential (parallel) to the surface. Furthermore, it is convenient to define new variables to represent components of the translated boundary conditions, as shown below:

\[
E_i = \mathcal{E}_i \cos \vartheta_0 \\
E_r = \mathcal{E}_r \cos \vartheta_0 \\
E_t = \mathcal{E}_t \cos \vartheta_1 \\
H_i = \gamma_i \mathcal{E}_i = \frac{\gamma_i}{\cos \vartheta_0} E_i \\
H_r = \frac{\gamma_r}{\cos \vartheta_0} E_r \\
H_t = \frac{\gamma_t}{\cos \vartheta_1} E_t
\]

Rewriting the boundary conditions in the terms shown above yields Eq. A.52 and A.53, from which we can obtain expressions for the amplitude reflection and transmission coefficients for p-polarized light, \( \rho_p \) and \( \tau_p \) respectively.

\[
E_i + E_r = E_t \\
\frac{\gamma_0}{\cos \vartheta_0} E_i - \frac{\gamma_0}{\cos \vartheta_0} E_r = \frac{\gamma_1}{\cos \vartheta_1} E_t \\
= \frac{\gamma_1}{\cos \vartheta_1} (E_i + E_r) \\
E_i \left( \frac{\gamma_0}{\cos \vartheta_0} - \frac{\gamma_1}{\cos \vartheta_1} \right) = E_r \left( \frac{\gamma_0}{\cos \vartheta_0} + \frac{\gamma_1}{\cos \vartheta_1} \right) \\
\frac{E_r}{E_i} = \frac{\gamma_0}{\cos \vartheta_0} - \frac{\gamma_1}{\cos \vartheta_1} \\
= \frac{\gamma_0}{\cos \vartheta_0} + \frac{\gamma_1}{\cos \vartheta_1} \\
\rho_p = \frac{E_r}{E_i} \\
\therefore \rho_p = \frac{\gamma_0}{\cos \vartheta_0} - \frac{\gamma_1}{\cos \vartheta_1} \\
= \frac{\gamma_0}{\cos \vartheta_0} + \frac{\gamma_1}{\cos \vartheta_1}
\]

(A.54)
Appendix A. Derivations

\[
\begin{align*}
\frac{\gamma_0}{\cos \vartheta_0} E_i - \frac{\gamma_0}{\cos \vartheta_0} E_r &= \frac{\gamma_1}{\cos \vartheta_1} E_t \\
\frac{2\gamma_0}{\cos \vartheta_0} E_i &= E_t \left( \frac{\gamma_0}{\cos \vartheta_0} + \frac{\gamma_1}{\cos \vartheta_1} \right) \\
E_t &= \frac{2\gamma_0}{\cos \vartheta_0} \\
\tau_p &= \frac{E_t}{E_i} \\
\therefore \tau_p &= \frac{2\gamma_0}{\cos \vartheta_0} + \frac{\gamma_1}{\cos \vartheta_1} \quad (A.55)
\end{align*}
\]

From here we define the expressions for the Irradiance of each of the three beams of interest. Considering that the oblique incidence will add some otherwise unexpected components to the Irradiance, we will begin with the original definition for the optical Irradiance, \(I\), and proceed to find expressions for \(I_i\), \(I_r\), and \(I_t\).

\[
I = \frac{1}{2} \Re \{ \mathbf{E} \times \mathbf{H}^* \} \quad (\text{From Eq. A.28})
\]

\[
I = \Re \left\{ \frac{1}{2} E \left( \frac{\gamma_0}{\cos \vartheta_0} E \right)^* \right\} \quad (\text{Scalar Irradiance})
\]

\[
\therefore \quad I_i = \frac{1}{2} \left( \frac{\gamma_0}{\cos \vartheta_0} \right) E_i E_i^* \quad (A.56)
\]

Again, \(I_i\) represents the Irradiance of the incident beam. We will also express \(I_r\) and \(I_t\) in terms of \(I_i\) as we did in the previous section in order to derive expressions for the relative reflection and transmission coefficients for p-polarized light, \(R_p\) and \(T_p\) respectively.

\[
I_r = \Re \left\{ \frac{1}{2} E_r \left( \frac{\gamma_0}{\cos \vartheta_0} E_r \right)^* \right\} \quad (\text{Scalar Reflected Irradiance})
\]

\[
= \frac{1}{2} \rho_p E_i \left( \frac{\gamma_0}{\cos \vartheta_0} \rho_p E_i \right)^* \quad (E_r = \rho_p E_i)
\]

\[
= \rho_p^2 \frac{1}{2} \left( \frac{\gamma_0}{\cos \vartheta_0} \right) E_i E_i^* \quad (\text{Reflect the format of } I_i)
\]

\[
\therefore \quad I_r = \rho_p^2 I_i \quad (A.57)
\]
Appendix A. Derivations

\[ I_t = \Re \left\{ \frac{1}{2} E_t \left( \frac{\gamma_1}{\cos \vartheta_1} E_t \right)^* \right\} \]  
(Scalar Transmitted Irradiance)

\[ = \frac{1}{2} \tau_p E_i \left( \frac{\gamma_1}{\cos \vartheta_1} \tau_p E_i \right)^* \]  
\( (E_t = \tau_p E_i) \)

\[ = \tau_p^2 \frac{1}{2} \gamma_1 \cos \vartheta_0 \left( \frac{\gamma_0}{\cos \vartheta_0} \right) E_i E_i^* \]  
(Reflect the format of \( I_i \))

\[ \therefore I_t = \frac{\tau_p^2 \gamma_1 \cos \vartheta_0}{\gamma_0 \cos \vartheta_1} I_i \]  
(A.58)

Consider the simple energy balance equation below in order to derive \( R_p \) and \( T_p \). It is important to note that by translating \( \hat{s} \) to perpendicular incidence by incorporating in the \( \cos \vartheta \) components, the traditional condition that \( 1 - R = T \) still holds true.

\[ I_0 = I_1 \]  
(Conservation of energy)

\[ \Re \left\{ \frac{1}{2} E_0 \times H_0^* \right\} = \Re \left\{ \frac{1}{2} E_1 \times H_1^* \right\} \]  
(Sub. in Eq. A.28)

\[ \Re \left\{ \frac{1}{2} (E_i + E_r) (H_i - H_r)^* \right\} = \Re \left\{ \frac{1}{2} E_i H_i^* \right\} \]  
(Scalar components)

\[ \frac{1}{2} (E_i + E_r) \left( \frac{\gamma_0}{\cos \vartheta_0} E_i - \frac{\gamma_0}{\cos \vartheta_0} E_r \right)^* = \frac{1}{2} E_t \left( \frac{\gamma_1}{\cos \vartheta_1} E_t \right)^* \]  
\( (H = \frac{\gamma_0}{\cos \vartheta_0} E) \)

\[ \frac{1}{2} \frac{\gamma_0}{\cos \vartheta_0} (E_i + \rho_p E_i) (E_i - \rho_p E_i)^* = \frac{1}{2} \frac{\gamma_1}{\cos \vartheta_1} \tau_p^2 E_i E_i^* \]  
(Sub. for \( E_r, E_t \))

\[ \frac{1}{2} \frac{\gamma_0}{\cos \vartheta_0} E_i E_i^* (1 - \rho_p^2) = \frac{1}{2} \frac{\gamma_1}{\cos \vartheta_1} \tau_p^2 E_i E_i^* \]  
(Factor)

\[ I_i (1 - \rho_p^2) = \frac{\gamma_1}{\gamma_0 \cos \vartheta_1} \tau_p^2 I_i \]  
(Sub. in Eq. A.56)

\[ I_i - \rho_p^2 I_i = \frac{\gamma_1}{\gamma_0 \cos \vartheta_0 \cos \vartheta_1} \tau_p^2 I_i \]  
(Distribute)

\[ I_i - I_r = I_t \]  
(Sub. in Eq. A.57, A.58)

\[ 1 - \frac{I_r}{I_i} = \frac{I_t}{I_i} \]  
(Divide by \( I_i \))

\[ \therefore 1 - R_p = T_p \]  
(A.59)
It may also be useful to express $R_p$ and $T_p$ in terms of $\gamma$ and $\vartheta$.

$$R_p = \frac{I_r}{I_i}$$  \hspace{1cm} \text{(Definition of $R_p$)}

$$= \frac{\rho_p^2 I_i}{I_i}$$  \hspace{1cm} \text{(Sub in Eq. A.57)}

$$= \rho_p^2$$  \hspace{1cm} \text{(Simplify)}

$$\therefore R_p = \left(\frac{\gamma_0 \cos \vartheta_0 - \gamma_1 \cos \vartheta_1}{\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1}\right)^2$$  \hspace{1cm} (A.60)

$$T_p = \frac{I_t}{I_i}$$  \hspace{1cm} \text{(Definition of $T_p$)}

$$= \frac{\gamma_1 \cos \vartheta_0}{\gamma_0 \cos \vartheta_1} \rho_p^2 \frac{I_i}{I_i}$$  \hspace{1cm} \text{(Sub in Eq. A.58)}

$$= \frac{\gamma_1 \cos \vartheta_0}{\gamma_0 \cos \vartheta_1} \rho_p^2$$  \hspace{1cm} \text{(Simplify)}

$$= \frac{\gamma_1 \cos \vartheta_0}{\gamma_0 \cos \vartheta_1} \left[\frac{4\gamma_0^2}{\cos \vartheta_0} \left(\frac{\gamma_0}{\cos \vartheta_0} + \frac{\gamma_1}{\cos \vartheta_1}\right)^2\right]$$  \hspace{1cm} \text{(Sub in Eq. A.55)}

$$\therefore T_p = \frac{4\gamma_0 \gamma_1}{\cos \vartheta_0 \cos \vartheta_1} \left(\frac{\gamma_0}{\cos \vartheta_0} + \frac{\gamma_1}{\cos \vartheta_1}\right)^2$$  \hspace{1cm} (A.61)

s-polarized light (TE polarized)

Having derived the expressions for $p$-polarized light, we must now consider the case of $s$-polarized light. The boundary conditions for the $s$-polarization state are as follows:

$$\mathcal{E}_i + \mathcal{E}_r = \mathcal{E}_t$$  \hspace{1cm} (A.62)

$$\mathcal{H}_i \cos \vartheta_0 - \mathcal{H}_r \cos \vartheta_0 = \mathcal{H}_t \cos \vartheta_1$$  \hspace{1cm} (A.63)

Again, we will rearrange the conditions such that they deal only with the electric field.

We will also define new variables to represent the components of the translated boundary conditions, as we did for the $p$-polarization state.
Appendix A. Derivations

\[ E_i = \mathcal{E}_i \quad H_i = \mathcal{H}_i \cos \vartheta_0 = \gamma_0 \cos \vartheta_0 E_i \]
\[ E_r = \mathcal{E}_r \quad H_r = \gamma_0 \cos \vartheta_0 E_r \]
\[ E_t = \mathcal{E}_t \quad H_t = \gamma_1 \cos \vartheta_1 E_t \]

From there the derivation follows the same format as that of the p-polarized light. First we will obtain the expressions for the amplitude reflection and transmission coefficients for s-polarized light, \( \rho_s \) and \( \tau_s \) respectively.

\[ H_i - H_r = H_t \quad \text{(From Eq. A.63)} \]
\[ \gamma_0 \cos \vartheta_0 E_i - \gamma_0 \cos \vartheta_0 E_r = \gamma_1 \cos \vartheta_1 E_t \]
\[ = \gamma_1 \cos \vartheta_1 (E_i + E_r) \quad \text{(Replace } H_i, H_r, \text{ and } H_t) \]
\[ E_i (\gamma_0 \cos \vartheta_0 - \gamma_1 \cos \vartheta_1) = E_r (\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1) \quad \text{(Distribute and factor)} \]
\[ \frac{E_i}{E_i} = \frac{\gamma_0 \cos \vartheta_0 - \gamma_1 \cos \vartheta_1}{\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1} \quad \text{(Rearrange fractions)} \]
\[ \rho_s = \frac{E_r}{E_i} \quad \text{(Definition of } \rho) \]
\[ \therefore \rho_s = \frac{\gamma_0 \cos \vartheta_0 - \gamma_1 \cos \vartheta_1}{\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1} \quad \text{(A.64)} \]

\[ \gamma_0 \cos \vartheta_0 E_i - \gamma_0 \cos \vartheta_0 E_r = \gamma_1 \cos \vartheta_1 E_t \quad \text{(Continued from above)} \]
\[ \gamma_0 \cos \vartheta_0 E_i - \gamma_0 \cos \vartheta_0 (E_t - E_i) = \gamma_1 \cos \vartheta_1 E_t \quad \text{(Sub in Eq. A.62)} \]
\[ E_i (2\gamma_0 \cos \vartheta_0) = E_t (\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1) \quad \text{(Factor)} \]
\[ \frac{E_t}{E_i} = \frac{2\gamma_0 \cos \vartheta_0}{\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1} \quad \text{(Divide by } E_i) \]
\[ \tau_s = \frac{E_t}{E_i} \quad \text{(Definition of } \tau) \]
\[ \therefore \tau_s = \frac{2\gamma_0 \cos \vartheta_0}{\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1} \quad \text{(A.65)} \]

Next we define the expressions for the Irradiance of each of the three beams of interest in a similar manner as before. Again, \( I_i \) represents the Irradiance of the incident beam.
Appendix A. Derivations

derived from the general expression for Irradiance, as shown below.

\[ I = \frac{1}{2} \Re \{ \mathbf{E} \times \mathbf{H}^* \} \]  
\[ I = \Re \left\{ \frac{1}{2} \mathbf{E} \left( \gamma \cos \vartheta \mathbf{E} \right)^* \right\} \]  
\[ \therefore I_i = \frac{1}{2} \gamma_0 \cos \vartheta_0 E_i E_i^* \]  
(A.66)

Following along the same approach as before, we will now define \( I_r \) and \( I_t \) in terms of \( I_i \) in order to derive the relative reflection and transmission coefficients for s-polarized light, \( R_s \) and \( T_s \) respectively.

\[ I_r = \Re \left\{ \frac{1}{2} \mathbf{E}_r \left( \gamma_0 \cos \vartheta_0 \mathbf{E}_r \right)^* \right\} \]  
\[ = \frac{1}{2} \rho_s E_i \left( \gamma_0 \cos \vartheta_0 \rho_s E_i \right)^* \]  
\[ = \frac{1}{2} \rho_s^2 \gamma_0 \cos \vartheta_0 E_i E_i^* \]  
(Reflect the format of \( I_i \))
\[ \therefore I_r = \rho_s^2 I_i \]  
(A.67)

\[ I_t = \Re \left\{ \frac{1}{2} \mathbf{E}_t \left( \gamma_1 \cos \vartheta_1 \mathbf{E}_t \right)^* \right\} \]  
\[ = \frac{1}{2} \tau_s E_i \left( \gamma_1 \cos \vartheta_1 \tau_s E_i \right)^* \]  
\[ = \frac{1}{2} \tau_s^2 \gamma_1 \cos \vartheta_0 \cos \vartheta_0 \gamma_0 \cos \vartheta_0 E_i E_i^* \]  
(Reflect the format of \( I_i \))
\[ \therefore I_t = \frac{\gamma_1 \cos \vartheta_1 \tau_s^2}{\gamma_0 \cos \vartheta_0} I_i \]  
(A.68)

Consider the energy balance equation below in order to derive \( R_s \) and \( T_s \). As for the p-polarized condition, we will find that the traditional condition that \( 1 - R = T \) still holds true.
\[ I_0 = I_1 \] (Conservation of energy)

\[ \mathbb{R} \left\{ \frac{1}{2} \mathbf{E}_0 \times \mathbf{H}_0^* \right\} = \mathbb{R} \left\{ \frac{1}{2} \mathbf{E}_1 \times \mathbf{H}_1^* \right\} \] (Sub in. Eq. A.28)

\[ \mathbb{R} \left\{ \frac{1}{2} (\mathbf{E}_i + \mathbf{E}_r) (\mathbf{H}_i - \mathbf{H}_r)^* \right\} = \mathbb{R} \left\{ \frac{1}{2} \mathbf{E}_i \mathbf{H}_i^* \right\} \] (Scalar components)

\[ \frac{1}{2} (\mathbf{E}_i + \mathbf{E}_r) (\gamma_0 \cos \vartheta_0 \mathbf{E}_i - \gamma_0 \cos \vartheta_0 \mathbf{E}_r)^* = \frac{1}{2} \mathbf{E}_t \gamma_1 \cos \vartheta_1 \mathbf{E}_t^* \] (H = \gamma \cos \vartheta E)

\[ \frac{1}{2} \gamma_0 \cos \vartheta_0 (\mathbf{E}_i + \rho_s \mathbf{E}_i) (\mathbf{E}_i - \rho_s \mathbf{E}_i)^* = \frac{1}{2} \gamma_1 \cos \vartheta_1 \tau_s^2 \mathbf{E}_i \mathbf{E}_i^* \] (E_r = \rho_s \mathbf{E}_i)

\[ \frac{1}{2} \gamma_0 \cos \vartheta_0 \mathbf{E}_i \mathbf{E}_i^* (1 - \rho_s^2) = \frac{1}{2} \gamma_1 \cos \vartheta_1 \tau_s^2 \mathbf{E}_i \mathbf{E}_i^* \] (Factor)

\[ I_i \left(1 - \rho_s^2\right) = \frac{1}{2} \gamma_1 \cos \vartheta_1 I_i \] (Sub. in Eq. A.66)

\[ I_i - \rho_s^2 I_i = \frac{1}{2} \gamma_1 \cos \vartheta_1 I_i \] (Distribute)

\[ I_i - I_r = I_t \] (Sub in Eq. A.67, A.68)

\[ 1 - \frac{I_r}{I_i} = \frac{I_t}{I_i} \] (Divide by \( I_i \))

\[ : 1 - R = T \] (A.69)

Again, we will express \( R_s \) and \( T_s \) in terms of \( \gamma \) and \( \vartheta \).

\[ R_s = \frac{I_r}{I_i} \] (Definition of \( R_s \))

\[ = \rho_s^2 \frac{I_i}{I_i} \] (Sub in Eq. A.67)

\[ = \rho_s^2 \] (Simplify)

\[ : R_s = \left( \frac{\gamma_0 \cos \vartheta_0 - \gamma_1 \cos \vartheta_1}{\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1} \right)^2 \] (A.70)
Appendix A. Derivations

\[ T_s = \frac{I_t}{I_i} \]  
\[ = \frac{I_s}{I_i} \]  
\[ = \frac{\gamma_1 \cos \vartheta_1 I_i}{2 \gamma_0 \cos \vartheta_0 I_i} \]  
\[ = \frac{\gamma_1 \cos \vartheta_1}{2 \gamma_0 \cos \vartheta_0} \]  
\[ = \frac{\gamma_1 \cos \vartheta_1}{\gamma_0 \cos \vartheta_0} \cdot \frac{4 \gamma_0^2 \cos^2 \vartheta_0}{(\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1)^2} \]  
\[ = \frac{4 \gamma_0 \gamma_1 \cos \vartheta_0 \cos \vartheta_1}{(\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1)^2} \]  
\[ \therefore T_s = \frac{4 \gamma_0 \gamma_1 \cos \vartheta_0 \cos \vartheta_1}{(\gamma_0 \cos \vartheta_0 + \gamma_1 \cos \vartheta_1)^2} \] (A.71)

Optical admittance for oblique incidence

The optical admittance for light incident upon a surface at an oblique angle takes a slightly different form than it did for normal incidence. Since the expressions for H and E differ based on the polarization state of the beam, we must redefine the optical admittance term in such a way that it applies for any obliquely incident beam upon a non-absorbing medium, independent of its polarization. In order to distinguish this term from the optical admittance for normal incidence, we will define it as follows:

normal incidence : \( \eta = \gamma = n \gamma \)

oblique incidence : \( \eta = \frac{H}{E} \)

Proceeding from this definition for oblique incidence, we can derive new expressions for \( \rho \) and \( \tau \) that are applicable regardless of the polarization state of the incident beam. In order to demonstrate this, we will derive the expression in sequence for p- and s- polarizations.

\[ \eta_p = \frac{\gamma}{\cos \vartheta} \]  
\[ \rho_p = \frac{\gamma_0}{\cos \vartheta_0} - \frac{\gamma_1}{\cos \vartheta_1} \]  
\[ = \frac{\eta_{p,0} - \eta_{p,1}}{\eta_{p,0} + \eta_{p,1}} \]  
\[ (H \text{ for p-polarized light}) \]  
\[ (\text{Definition of } \rho_p) \]  
\[ (\text{Substitute in } \eta_p) \]  
\[ (A.72) \]
Appendix A. Derivations

\[ \eta_s = Y \cos \vartheta \]  
\[ \rho_s = \frac{Y_0 \cos \vartheta_0 - Y_1 \cos \vartheta_1}{Y_0 \cos \vartheta_0 + Y_1 \cos \vartheta_1} \]  
\[ = \frac{\eta_{s,0} - \eta_{s,1}}{\eta_{s,0} + \eta_{s,1}} \]  
\[ \rho_s = \eta_s,0 \eta_{s,0} + \eta_{s,1} \]  
\[ \therefore \rho = \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} \]  
\[ (A.73) \]

The two derivations reveal that the amplitude reflection coefficient, \( \rho \), can be represented in the same manner irrespective of the polarization state of light by writing it in terms of the optical admittance, \( \eta \).

\[ \eta_p = \frac{Y}{\cos \vartheta} \]  
\[ \tau_p = \frac{2Y_0 \cos \vartheta_0}{\cos \vartheta_0 + \cos \vartheta_1} \]  
\[ = \frac{2\eta_{p,0}}{\eta_{p,0} + \eta_{p,1}} \]  
\[ (A.74) \]

Continuing in a similar manner, we can derive an expression for the amplitude transmission coefficient, \( \tau \). The resultant equation is again independent of the polarization state, provided the appropriate optical admittance is used.

\[ \eta_s = Y \cos \vartheta \]  
\[ \tau_s = \frac{2Y_0 \cos \vartheta_0}{Y_0 \cos \vartheta_0 + Y_1 \cos \vartheta_1} \]  
\[ = \frac{2\eta_{s,0}}{\eta_{s,0} + \eta_{s,1}} \]  
\[ \therefore \tau = \frac{2\eta_0}{\eta_0 + \eta_1} \]  
\[ (A.74) \]

Based on these new definitions, we can further express the relative reflection and
transmission coefficients, $R$ and $T$ in terms of the optical admittance, $\eta$, as shown below.

$$R = \left( \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} \right)^2 \quad (A.75)$$

$$T = \frac{4\eta_0 \eta_1}{(\eta_0 + \eta_1)^2} \quad (A.76)$$

The Brewster Angle

The equations derived above can be used to determine the degree of reflectance for each polarization at a simple boundary between extended media. In cases wherein there is no absorption in the medium, it can be determined that there is a definitive angle at which the reflectance for p-polarized light goes to zero. This angle is referred to as the Brewster Angle, but is also known as the polarizing angle, since the all of the reflected light will be s-polarized. The expression for this angle can be derived using a combination of Snell’s Law (Eq. A.39) and the expression for the relative reflection coefficient for p-polarized light (Eq. A.60). Since the Brewster angle is defined as the angle at which $R_p = 0$, we will start with the conditions necessary for that outcome to occur, and derive an expression that allows us to eliminate $\vartheta_1$ from the expressions.

$$\frac{\eta_0}{\cos \vartheta_0} = \frac{\eta_1}{\cos \vartheta_1} \quad (R_p = 0)$$

$$\frac{n_0 \eta}{\cos \vartheta_0} = \frac{n_1 \eta}{\cos \vartheta_1} \quad \text{(Substitute } \eta = n \eta)$$

$$\cos \vartheta_1 = \frac{n_1 \cos \vartheta_0}{n_0} \quad (A.77)$$

$$n_0 \sin \vartheta_0 = n_1 \sin \vartheta_1 \quad \text{(Snell’s Law)}$$

$$\sin \vartheta_1 = \frac{n_0 \sin \vartheta_0}{n_1} \quad (A.78)$$

Now that we have two independent equations, we can eliminate $\vartheta_1$ using trigonometric
identities, as follows:

\[
\sin^2 \vartheta_0 + \cos^2 \vartheta_0 = 1 \quad \text{(Trig. identity)}
\]

\[
= \sin^2 \vartheta_1 + \cos^2 \vartheta_1 \quad \text{(Apply to } \vartheta_1)\]

\[
= \left( \frac{n_0 \sin \vartheta_0}{n_1} \right)^2 + \left( \frac{n_1 \cos \vartheta_0}{n_0} \right)^2 \quad \text{(Sub in Eq. A.77 and A.78)}
\]

\[
\sin^2 \vartheta_0 - \frac{n_0^2}{n_1^2} \sin^2 \vartheta_0 = \frac{n_1^2}{n_0^2} \cos^2 \vartheta_0 - \cos^2 \vartheta_0 \quad \text{(Rearrange)}
\]

\[
\left( \frac{n_1^2 - n_0^2}{n_1^2} \right) \sin^2 \vartheta_0 = \left( \frac{n_1^2 - n_0^2}{n_0^2} \right) \cos^2 \vartheta_0 \quad \text{(Isolate sin and cos)}
\]

\[
\frac{\sin^2 \vartheta_0}{\cos^2 \vartheta_0} = \frac{\left( \frac{n_1^2 - n_0^2}{n_0^2} \right) n_1^2}{n_0^2 \left( n_1^2 - n_0^2 \right)} \quad \text{(Rearrange)}
\]

\[
\left( \frac{\sin \vartheta_0}{\cos \vartheta_0} \right)^2 = \left( \frac{n_1}{n_0} \right)^2 \quad \text{(Simplify)}
\]

\[
\tan \vartheta_0 = \frac{n_1}{n_0} \quad \text{(Trig identity)}
\]

\[
\vartheta_0 = \tan^{-1} \left( \frac{n_1}{n_0} \right) \quad \text{(Solve for } \vartheta_0)\]

\[
\therefore \vartheta_B = \tan^{-1} \left( \frac{n_1}{n_0} \right) \quad \text{(A.79)}
\]
Appendix B

MATLAB Simulations

As a result of fabricating the waveguides in-house, certain approximations were made due to limited resources and time constraints that necessitated deviations away from our ideal designs. One such alteration was the use of the Thorlabs LJ1878L2-A cylindrical lens adhered to the surface of a glass slab substrate, as opposed to the use of a single piece of glass that incorporated a quarter-cylinder. As a result, the entrance aperture of the waveguide needed to be widened slightly to allow a sufficient amount of light to couple into the waveguide at the broader angles, where fluence was anticipated to be greatest. The MATLAB simulations were performed to determine the optimum thickness for this allowance, which was eventually determined to be 1.0 mm wider than the radius of the lens. Waveguides were fabricated with 6.0 mm entrance apertures as a result of the simulation curves generated by the code included in this appendix.

The code incorporates a variety of necessary aspects that contribute to the coupling efficiency of the cylindrical lens. One such aspect was the fact that as the incident angle changed from 0 to 75°, the beam spot size would grow along its narrowest dimension. The beam was assumed to be an elliptical gaussian, and was simulated using a 3-dimensional intensity distribution, which was projected onto a the waveguide surface at incident angles in 0.2° increments throughout the full range of anticipated incident angles.

Another such consideration was that portions of the light would not be coupled into the waveguide, but would instead simply reflect off the silvered rear surface of the glass. Additionally, at broader angles, portions of the projected ellipse would not be coupled
into the waveguide as they would be incident beyond the entrance aperture and would consequently reflect of the near face of the waveguide. The code incorporates loops to simulate entrance apertures from 5.0 to 9.0 mm in increments of 0.5 mm, depicting coupling efficiency resultant from the use of the cylindrical lens. The optimum entrance aperture was chosen as the first curve that demonstrated a maximum at 45° incidence.

```matlab
% GaussianActiveArea2.m
% Author: Paul J.D. Whiteside
% Date: 10/7/14

format short
clear all
clc

kappa = .96; % Coupling efficiency
I_0 = 1.5; % Measured incident energy (mJ)

r_1 = .75; % Short axis radius (mm)
r_2 = 5; % Long axis radius (mm)
internal_angle = 1:15:75; % vary internal reflection angle
height = 1; % thickness of waveguide
var_aperture = .5;

R1 = r_1./cosd(internal_angle); % Project ellipse at an angle
R2 = r_2; % Projection does not affect

sigma_x = .5*R1; % Std. dev. in x
sigma_y = .5*R2; % Std. dev. in y
```
Appendix B. MATLAB

21 \( \sigma_x^{\text{incident}} = 0.5 \times r_1; \)
22 \( A^{\text{incident}} = \kappa \times (I_0)/(2 \times \pi \times \sigma_x^{\text{incident}} \times \sigma_y); \)
23
24 \% Establish elliptical gaussian
25
26 \([X, Y] = \text{meshgrid}(-5:0.01:5,-10:0.05:10);\)
27 \(Z^{\text{incident}} = A^{\text{incident}} \times \exp(-X^2/(\sigma_x^{\text{incident}})^2+Y^2/(\sigma_y)^2);\)
28 \(A_z = \text{sum}\left(\text{sum}\left(Z^{\text{incident}}\right)\right); \quad \% \text{energy of original incident beam profile}\)
29
30
31 \% Establish elliptical gaussian
32
33 \% Transmission at Reflection Interfaces
34 \%-----------------------------------------------
35
36 \(n_0 = 1.519; \quad \% \text{Refractive index of waveguide}\)
37 \(n_1 = 1.33; \quad \% \text{Refractive index of tissue phantom}\)
38
39 \text{script\_Y} = 2.6544E-3; \quad \% \text{Free space optical admittance}\)
40 \text{script\_y\_0} = \text{script\_Y} \times n_0; \quad \% \text{Waveguide optical admittance}\)
41 \text{script\_y\_1} = \text{script\_Y} \times n_1; \quad \% \text{Coupling oil optical admittance}\)
42
43 \(\theta_c = 180/\pi \times \text{asin}(n_1/n_0);\)
Appendix B. *MATLAB*

```matlab
theta_0 = internal_angle;

L1 = 20; % active area begins 20mm from the end
L2 = 40; % active area is 20mm long
x_0 = 5; % coordinates of the center of rotation
y_0 = height;

L1_rel = L1 - x_0; % position of the active area relative to x_0
L2_rel = L2 - x_0;
Length = 60; % waveguide is 60mm long

Total_Transmission = zeros(size(internal_angle));
[aa,bb] = size(X);

for kk = 1:length(internal_angle)

    % Determine reflection and transmission coefficients as a function of
    % internal reflection angle

    if theta_0(kk) < theta_c
        theta_1 = asind(n_0*sind(internal_angle(kk))/n_1);
        T = (4*script_y_0*script_y_1*cosd(theta_0(kk))*cosd(theta_1)) / (script_y_0*cosd(theta_0(kk)) + script_y_1 *cosd(theta_1))^2;
        R = ( (script_y_0*cosd(theta_0(kk)) - script_y_1*cosd(theta_1)) / (script_y_0*cosd(theta_0(kk)) + script_y_1 *cosd(theta_1)) )^2;
    
    end
```

122
else if theta_0(kk) == theta_c
    theta_1 = 90;
    T = 0;
    R = 1;
else
    theta_1 = theta_0(kk) + 90;
    T = 0;
    R = 1;
end

% = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =
% Internal Reflection Points
% = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =

Gamma = height.*tand(internal_angle(kk));
    % distance moved in the x direction between reflection points
reflections = round(Length/(height.*tand(internal_angle(kk)))); % number of internal reflections
Transmission = zeros(length(internal_angle),reflections);
    % Transmission matrix by reflection point

transmitted_Z = zeros(size(Z_incident));
reflected_Z = Z_incident;

h = waitbar(0,'Calculating reflections...');
for nn = 1:reflections
    X_new = X + (2*nn-1)*Gamma;
    for ii = 1:aa
for jj = 1:bb
    if (L1_rel) < X_new(ii, jj) && X_new(ii, jj) < (L2_rel)
        % if the spot is within the active area, then
        % the portion
        % of reflected_Z in the active area is
        transmitted_Z(ii, jj) = reflected_Z(ii, jj)*T;
        reflected_Z(ii, jj) = reflected_Z(ii, jj)*R;
    end
end
end
waitbar(nn/reflections,h)
Transmission(kk, nn) = sum(sum(transmitted_Z));
end
Total_Transmission(kk) = sum(Transmission(kk,:));
end
plot(internal_angle, Total_Transmission)
Appendix C

LabView Control Software

This appendix is intended to present the primary block diagram for the LabView automated control program developed for control of the apparatus used in this project. The diagram shown in Figure C.1 shows the data collection and processing portion of a three-part automation frame sequence. The next frame in the sequence determined the current rotational angle and issued commands to the rotational stage; whereas the final frame updated the plots and saved the data, before returning to the starting frame.

Only the first frame is included, since it was a crucial component of the data collection and analysis scheme. The data commands flow from the base of the diagram to the top (left to right when the image is shown in landscape as opposed to portrait). Upon being triggered by the photodiode, 5000 data points from each Channel A and B of the PicoScope are recorded. The relevant photoacoustic signal is then extracted from the data so that the reflections within the chamber do not distort the analysis. Following the extraction, the user-limited data is measured in two ways: a peak-to-peak analysis, and an integrated intensity analysis. The data is then passed along to the final frame of the sequence, which handles plotting and saving data in real time as the system is proceeding through the angular spectrum.
Figure C.1 Programming loop to acquire signals. Does not include automation or file saving frames.
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