MU-SYNTHESIS CONTROLLER DESIGN FOR TEMPERATURE CONTROL OF A SOLAR STEAM GASIFIER WITH UNCERTAINTY MODELING AND ROBUST ANALYSIS

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By
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The undersigned, appointed by the dean of the Graduate School, have examined the thesis entitled

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NOMENCLATURE

$A =$ low frequency error specification

$A.K_f =$ state matrix for canonical form of $K_f$

$B =$ numerator coefficients of $K_f$

$d =$ disturbance input due to solar irradiation

$D =$ general matrix which commutes with $\Delta$

$E =$ denominator coefficients of $K_f$

$G =$ plant transfer function

$G_d =$ solar irradiation disturbance transfer function

$G_p =$ perturbed plant transfer function

$h =$ complex frequency response of full order controller

$I_{DN} =$ direct-normal irradiance

$j =$ imaginary unit

$k_{PID} =$ PID controller tuning gain

$K =$ $\mu$-synthesis with D-K iteration controller

$K_f =$ lower order controller fit to $K$

$K_r =$ further reduced order controller used in high fidelity simulations

$K_{PID} =$ PID controller

$LHV =$ lower heating value

$M =$ high frequency error specification

$n =$ order of output weighted transfer function

$\dot{n} =$ molar flow rate

$N =$ LFT of $P$ and $K$
\( p = \) Padé approximation

\( P = \) connectivity transfer function matrix

\( r = \) temperature reference point

\( R = \) cold gas ratio

\( R_G = \) steepest slope of step input to plant transfer function

\( s = \) Laplace operator

\( S = \) sensitivity transfer function matrix

\( T = \) complementary transfer function matrix

\( T_{\text{reac}} = \) reaction zone temperature

\( u = \) control effort

\( u_\Delta = \) uncertainty input

\( v = \) error between reference temperature and actual temperature

\( W_I = \) upper bound on multiplicative uncertainty

\( W_P = \) output error weight transfer function

\( W_u = \) control effort error weight transfer function

\( y = \) output

\( y_{\text{TF}} = \) step output from fit TFs to the high fidelity model

\( y_\Delta = \) uncertainty output

\( z = \) weighted efforts

\( \Delta = \) uncertainty

\( \Delta_p = \) input-output uncertainty

\( \hat{\Delta} = \) structured uncertainty

\( \mu = \) structured singular value
$\Pi = \text{class of plants including } G \text{ and } G_p$

$\tau = \text{time (transport) delay}$

$\omega = \text{frequency}$

$\omega_b = \text{bandwidth frequency}$
Gasifiers operate by inputting a carbon rich material, water, and heat into a reactor core to cause a chemical reaction to produce a desired chemical composition output. For this work, a unique hybrid allothermal/autothermal solar steam gasification process was utilized to produce synthesis gas by inputting lignite, water, and solar irradiation. Oxygen is also fed into the reactor to burn off a portion of the lignite if the solar irradiation is not sufficient to fuel the reactor. The gasifier is set to operate at a specific temperature to keep the total amount of syngas produced at a precise value. Since solar irradiation fluctuations would cause temperature fluctuations, temperature regulation via controlling the amount of lignite and oxygen fed into the reactor core was implemented. Two different controller synthesis methods were used: μ-synthesis with D-K iteration and Ziegler-Nichols to tune a PID controller. Both controllers maintain reactor temperature at or above a specified set point by equally varying the amount of lignite and oxygen fed into the reactor core. A high fidelity model developed in previous work was utilized to create a linear, time-invariant model for controller synthesis. Multiplicative uncertainty was used to create an uncertainty model to quantify and account for the dissimilarities between the high fidelity model and the linear model.
Performance specifications on temperature error and maximum control effort were chosen to guarantee fast response of the controller as well as reliable temperature control. Five day simulations using meteorological data to estimate solar irradiation were ran for both controllers and then compared. Simulations gave more insight into the range of operation for the high fidelity model. The linear and uncertainty model was refined to better represent the high fidelity model. The controllers were compared on three different criteria: temperature control, total syngas production, and cold gas ratio. The μ-synthesis controller performed better than the PID controller for temperature control. For total syngas production and cold gas ratio, the controllers performed equally. The μ-synthesis controller was also found to have better robust properties than the PID controller.
1.1. Introduction to Research and Motivation

Gasifiers operate by feeding carbon rich material, water, and a catalyst into a reactor to drive a reaction at a temperature, resulting in a chemical reaction output is composed of primarily hydrogen and carbon monoxide, or synthetic gas (syngas), shown in Fig. 1.

![Fig. 1. Basic gasifier operation](image)

Reactor temperature is mainly affected by change in solar irradiation or change in input feedstock. An increase in solar irradiation will increase the reactor temperature, and vice versa. Adding more water will lower the temperature of the reactor. However, for this work’s consideration, water has been kept constant. An equal increase in carbon rich materials and oxygen will increase the temperature, and vice versa. Controller design will use this effect since input feedstock is controllable, while solar irradiation cannot be. Solar steam gasification reactors, or gasifiers, are an effective means of creating synthetic natural gas (SNG), by creating the intermediary product, syngas. Syngas is comprised of primarily hydrogen and carbon monoxide. SNG can be integrated directly with natural gas usage, making it an extremely valuable resource [1, 2].
Gasifiers have conventionally been fueled by two modes: burning a portion of the feedstock within the reactor via an exothermal chemical reaction (autothermal) [3-6], or using external heat sources (allothermal) [4, 7, 8]. Autothermal can be extremely reliable for temperature control because input feedstock to change the temperature is directly controlled. Allothermal reactions can be more difficult since they commonly utilize solar irradiation [9-12], which can be inconsistent during the day, and not present during the night. However, using solar irradiation to drive the reaction can be advantageous because a larger percentage of the feedstock can be converted to syngas since part of it is being burned off as it is in autothermal operation. Previous works have investigated combining autothermal and allothermal modes [13-16], but little has been done to attempt to regulate reactor temperature by controlling input feedstock rates.

Combining the two methods of operation for temperature control is complex. Solar irradiation’s unpredictable nature can be problematic for dependable temperature control. A few have attempted to predict future solar irradiation, and will be covered in the next section, but is not used in this author’s work. Despite the complexity of combining the two methods, being able to use solar energy to drive the reaction when sufficient, and supplementing the reaction with oxygen when the solar energy is not sufficient, overcomes some of the disadvantages of operating solely autothermal or allothermal.

Controller design is based on increasing or decreasing the amount of carbon rich material and oxygen fed into the reactor, given whether the reactor temperature is below or above a set reactor temperature. Performance weights on error and control effort, as well as an uncertainty model, will be used to design one controller. A second controller,
based only on the linear model, will be used as a basis of comparison. The purpose of the uncertainty model is to quantify operation of the nonlinear model, covered in the next chapter, not represented by the linear model.

1.2. Literature Background

Several other published works have demonstrated control design for gasifiers. While no other work has shown temperature control by varying input feedstock to supplement solar irradiation, there are some similar approaches.

One of the earliest publications on gasifier control came from Petrasch et al. [17], who used LQR/LTR for a linear model of a solar thermochemical reactor. Their objective was to keep the production rate of CO at some nominal value by varying the amount of water input to the system. In order to test the performance of their controller, solar energy was stepped down 15% from the nominal value of 500 kW. While the controller maintained the production rate of CO, it was a very limiting test. As will be shown along with the simulation results later, solar irradiation is much more dynamic. Their work also did not incorporate an uncertainty model, so nothing can be said about the controller’s performance any simulation that would differ from the one presented. However, there is a similarity between their work and the thesis presented here: using a linear model. Linear control design methods are better developed since linear models are typically easier to work with than nonlinear models.

A more recent controller design by Saade et al. [18] utilized model predictive control (MPC) to maintain the ratio of CO:CO$_2$ at some nominal value by controlling gas and steam flows. Solar irradiance was considered a disturbance input. The work here
maintains the ratio $H_2$:CO instead because the Fischer-Tropsch process to synthesize liquid fuels runs most efficiently at a specific, maintained ratio for a given catalyst [19]. However, a water-gas shift reactor downstream from the gasifier can be used to change the composition of syngas. Therefore, maintaining a total amount of syngas is more important than a ratio of syngas outputs. As was with Petrasch, Saade also utilized a linear model based on their nonlinear model. Their linear model considered nominal conditions as well as lower and upper limits considered for inputs and disturbances. MPC has its advantages: it naturally introduces a feedforward control and can prevent violations of the inputs and outputs. However, the upper and lower limitations reduce the robustness of the controller. This was noted by Saade. The minimum allowable solar irradiation is 700 W/m$^2$. This would severely limit the times when the gasifier could be operated. The meteorological data used for this work contains an entire year of data, and only 23.1% of the time is solar irradiation greater than 700 W/m$^2$. The proposed design in this work will allow for nonstop operation, including nighttime when there is no solar energy available to drive the reaction within the gasifier. In another published work [20], Saade et al. built upon their previous work to incorporate forecasting of solar irradiation using a total sky imager (TSI). They demonstrated accurate predictions of incoming solar irradiation up to one minute in advance. While one minute may not be a sufficient amount of time to noticeably increase the accuracy of temperature control for this work, the idea is very promising. As will be shown later, the main reason for temperature deviation from the reactor temperature set point is due to solar disturbance.

In previous work that used the same nonlinear model used for this thesis, controller synthesis began with a PI controller, and then improved upon via coprime
factor uncertainty to improve robustness. Temperature control for the 1100K set point was excellent (within a few degrees), and comparable to the best results presented in this work. However, as the set point changed, temperature control got worse. For 1250K set point, temperature deviation was at least an order of magnitude higher. These results are what led to this author’s work. An uncertainty model is necessary to do the design work that can result in reliable temperature control over a wide range of temperature set points.

1.3. Chapter Overview

Chapter 2 will detail the nonlinear model and its linearization, and the comparison between the two. The uncertainty model is developed in a similar manner to the linear model, so it will also be covered here. Chapter 3 details the performance requirements and performance weights, the method for designing two different controllers, and their performance with regard to the linear model and the uncertainty model. Chapter 4 explains how the controller was implemented within the high fidelity model, as well as the simulation results to show how the two different controllers performed. Simulations were also ran for a different type of feedstock, and are shown in this chapter. Chapter 5 will detail some of the limitations of the original uncertainty model. The work in Chapters 2 through 4 will be reworked, showing the improvements resulting from use of the new uncertainty model. Chapter 6 will be a summary of the results from the two different uncertainty models and the controllers associated with them. Chapter 7 concludes by summarizing how the objectives of the paper have been achieved and the significance of the results.
CHAPTER 2: DEVELOPMENT OF LINEAR AND UNCERTAINTY MODEL

2.1. Nonlinear, High Fidelity Model

Figure 2 shows the schematic for the reactor on which is the basis of a high fidelity model that is used for the simulations.

Solar irradiation, $I_{DN}$, hits (assumed to directly hit) an emitter plate made of SiC after passing through a quartz window. SiC is chosen for its material properties, namely a high absorbance rate over the solar spectrum, a high thermal conductivity, and a low coefficient of thermal expansion [21]. The window was assumed to be uniform temperature with an associated thermal capacitance. The walls are lined with $\text{Al}_2\text{O}_3$ since it acts as a ceramic insulation absorbing, emitting, and reflecting radiation [22, 23]. Specific heats and thermal conductivities were determined to be functions of temperature[24-26]. Compiled radiative property information determined spectral optical properties [27, 28]. The wall, emitter, and quartz window were modeled assuming one-
dimensional, transient heat conduction. A 20s time step was selected for iterating the model. Within the cavity, convective heat transfer was neglected and only radiative heat transfer was considered. Convective heat transfer for other components were calculated using correlations for cylinders and flat plates. Radiosity method for enclosures was employed, analyzing the radiative change in the evacuated cavity. Assumptions made within the reaction zone volume were negligible thermal capacitance, uniform absorption coefficient, gray absorption/emission, and a uniform temperature. These assumptions were made because of the reaction zone’s fast radiative exchange. The particles of feedstock inputted was assumed to have no thermal capacitance due to their actual capacitance being magnitudes smaller than the surrounding walls as well as their constant flow into and out of the reactor zone. An in-depth analysis of the high fidelity model is covered in previous work [15].

Carbon rich materials, water, and O\textsubscript{2} if the solar irradiance is not sufficient to heat the reaction, are brought to the reaction zone, resulting in a reaction to produce syngas. The carbon rich material chosen was lignite due to its abundance and relatively high reactivity [29, 30]. Sorghum is another carbon rich material that has been used in gasifiers [31-33]. Simulations with the high fidelity model and temperature control will be done using sorghum, but all modeling and controller design is based off lignite. As the syngas exits the reactor, chemical equilibrium is assumed. If O\textsubscript{2} is necessary, part of the carbon rich material is burned off to heat the reaction instead of being converted to syngas. Typical meteorological data (TMY3) was used to obtain hourly measurements of solar irradiance and ambient temperatures for an entire year [34].
The gasifier can operate on three different modes. At high enough $I_{DN}$ and a low enough temperature set point, no $O_2$ is needed and the reactor operates fully in allothermal mode. If $I_{DN}$ is not sufficient to drive the reaction at the desired temperature, $O_2$ is added to burn some of the lignite to heat the reaction – a combination of autothermal and allothermal operation. During the night, the reaction is fully driven by autothermal. For the last two modes of operation, excess lignite is added to ensure production of syngas does not drop significantly due to some of it being used to heat the reactor core. By controlling the input feedstock amount to maintain temperature, the gasifier can operate nonstop. Reactor temperature, output syngas components, and input feedstock are tracked.

2.2. Approach to Linearization

It was decided to utilize linear control design due to its simpler nature compared to nonlinear control, even though the model is nonlinear. A standard linearization technique for control design is to do a first order Taylor series approximation for the governing ODEs with respect to each state variable. However, the governing equations for the model employed are both extremely complex and non-continuous. Instead of doing a first order Taylor series, the nonlinear model was simulated with constant feedstock input and solar irradiance. The constant values are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{DN}$</td>
<td>279.22 W/m²</td>
</tr>
<tr>
<td>$\dot{n}_{O2}$</td>
<td>0.459 mol/s</td>
</tr>
<tr>
<td>$\dot{n}_C$</td>
<td>1.968 mol/s</td>
</tr>
<tr>
<td>$\dot{n}_{H2O}$</td>
<td>1.509 mol/s</td>
</tr>
</tbody>
</table>
Once the startup transients had settled, lignite and oxygen inputs were increased by 1%. The resulting dynamic response was used to fit a transfer function by minimizing the sum of the squared error between the simulation result and a step response to the transfer function, or

$$\sum_{i=1}^{m} (T_{\text{reac},i} - y_{TF,i})^2,$$

where $m$ is the number of data points from the high fidelity simulations, $T_{\text{reac}}$ is the reactor temperature, and $y_{TF}$ is the step response from the transfer function.

### 2.3. LTI Model Comparison to High Fidelity Model

The TF, or plant, was found to fit best if it included a direct feedthrough to account for a near instantaneous change in temperature, as well as two first order transfer function components, each with one pole and no zeros. The first order TFs were chosen by trial and error to see what worked best. Combined to a single TF, the plant is

$$G = \frac{40.207(s^2 + 1.803 \times 10^{-4} + 1.436 \times 10^{-8})}{(s + 5.565 \times 10^{-5})(s + 4.329 \times 10^{-5})},$$

with a mean absolute error of 0.018K between the step response of the TF and the high fidelity model simulation results. The nonlinear model simulation time step is 20 seconds, so a 2$^{nd}$ order Padé approximation of a 20 second delay was appended to $G$,

$$p = \frac{s^2 - 0.3s + 0.03}{s^2 + 0.3s + 0.03}.\quad (3)$$

Any mention of $G$ forward can be assumed to be appended with the Padé approximation. Figure 3 compares the linear and nonlinear model. The plant matches the nonlinear model for at least 120,000 seconds, or over 33 hours. While the gasifier can operate longer than 33 hours continuously, the dynamics of interest for the gasifier occur on far shorter
timescales. Therefore, the linear fit is more than sufficient and a comparison of the linear and nonlinear model for a longer time period is not required.

Fig. 3. Comparison of Plant and High Fidelity Model Simulation

2.4. Uncertainty Model

A controller designed based solely on the plant presented above is potentially extremely limiting. Even if the controller achieves the performance criteria, which will be discussed in the following section, its performance cannot be determined for simulations or real world implementations that differ from the nominal conditions used for the linearization of the plant. A standard method for ensuring the controller will meet the performance criteria for a range of nominal conditions is to develop an uncertainty model to be used in the controller design.

Multiplicative Uncertainty was chosen for the method of developing the uncertainty model because the bulk of the work creating it is extremely similar to
developing the plant $G$. Multiplicative Uncertainty quantifies the uncertainty by comparing $G$ to a class of perturbed plants, $G_p$ [35, 36], defined as

$$\Pi_I: G_p(s) = G(s)\left(1 + W_I(s)\Delta(s)\right),$$  \hspace{1cm} (4)$$

where $\Pi_I$ is the class of perturbed plants and $\Delta$ is any stable TF such that $\|\Delta\|_\infty \leq 1$. $W_I$ is the bound on the multiplicative uncertainty.

The perturbed plants were created by varying the nominal values of the feedstock of the linearization process previously in Table 1. The nominal values were increased by 80% to 180% in 20% increments. Once the transient dynamics had stopped, lignite and oxygen inputs were increased by 1%. Figure 4 shows the dynamic temperature response of $G$ and each $G_p$.

![Fig. 4. Perturbed Plant Temperature Response from Nominal](image)

The perturbed plants were fit to a direct feedthrough and two first order TFs each with one pole and no zeros as well (similar to the plant model discussed earlier). To obtain the weight, $W_I$, rearrange Eq. (4) to solve for $W_I$ so that it is upper bound on the magnitude frequency response of the error between $G$ and each $G_p$ normalized by $G$:
\[ |W_I(j\omega)| \geq \left| \frac{G_p(j\omega) - G(j\omega)}{G(j\omega)} \right| \quad \forall \omega. \]  

(5)

Figure 5 shows the result of Eq. (5). It is common to choose a simple \( W_I \) first, and only use a higher order TF if the initial selection of \( W_I \) is not sufficient. Since the dip for the frequency responses around \( 10^{-4} \) rad/s may be considered as minor, it has been neglected in selecting \( W_I \), which was chosen to be constant: -4.5 dB, or approximately 0.6 absolute value.
CHAPTER 3: CONTROLLER SYNTHESIS

3.1. Performance Requirements

Controller performance was defined by two metrics: reference error and maximum control effort. Reference error is the difference between the specified set temperature and the actual temperature of the reactor core. The specified set temperature, \( r \), and actual temperature, \( y \), are shown in Fig 6. This figure will be referred back to several times.

![Block Diagram of Plant with Controller, Weighting TFs, and Uncertainty](image)

**Fig. 6. Block Diagram of Plant with Controller, Weighting TFs, and Uncertainty**

While it would be simple to single specify a small reference error, it typically isn’t realistic. The disturbance, \( d \), shown in Fig. 6, can have a significant effect on \( y \), causing the control effort to be extremely large, which can lead to instabilities. Therefore, during low frequency disturbance, the controller was designed to have a small reference error and a large error during high frequency. The frequency defining the transition from low to high frequency, or bandwidth frequency, was determined from the TMY3 data obtained from the National Solar Radiation Data Base. Since data was provided every
hour, the low frequency should at least include a frequency of every hour (or 0.0008 rad/s). Maximum control effort, $u$, shown in Fig. 6 is the maximum change the controller output undergoes compared to the plant’s maximum output (temperature). The specifics for defining the weights to bound the reference error and control effort are shown below.

Reference error, $W_p$, is defined as the ratio of error to reference set point, and is a TF represented as

$$W_p = \left( \frac{s^{\frac{A}{\sqrt{M}}} + \omega_b^n}{s + \omega_b^n \sqrt{A}} \right)^n,$$

where $A$ is the low frequency error, $M$ is the high frequency error, $\omega_b$ is the bandwidth at which error transitions from low frequency to high frequency, and $n$ is the order of the TF. The low frequency error parameter was chosen such that temperature would vary by less than approximately 5K. This value was chosen somewhat arbitrarily. During the start of this work, it still had not been determined how precise temperature control needed to be. Therefore, a precise temperature control was selected so that performance could be relaxed if allowed, instead of having to tighten the performance specifications – a harder task. $A$ was chosen to be 0.0025, or 0.25% deviation from the set point. High frequency error is going to be chosen such that there can be a lot of deviation from the set point, since low frequency allows little deviation. $M$ was chosen to be 3, or 300% deviation. The bandwidth frequency was chosen to be 0.001 rad/s. It was chosen by trial and error, but close to the 0.008 rad/s listed above. The higher the order of $W_p$, the more sharp the transition is between the low and high frequency error. A higher value can improve performance, so $n$ was chosen to be 3.
The control effort is a ratio of max change in output to input. Figure 3 showed that for a 0.05 mol/s increase in feedstock, there was a 5K increase over one time step. The time step was 20 seconds, so the ratio of output to input was 5. Max control effort was chosen to be twice this, 10, to be conservative since unconsidered conditions might cause the increase to be higher.

3.2. μ-Synthesis Controller

A controller design method, μ-synthesis with D-K iteration [35-38], was chosen since it is possible to account for all performance requirements and plant uncertainty described by weights, $W_p$, $W_u$, and $W_f$. This is advantageous over controller designs such as LQR/LTR mentioned in the background section, since those methods don’t utilize an uncertainty model during the controller synthesis. The μ-synthesis procedure minimizes the structured singular value (SSV) [39, 40] of $\mu(N(K))$, over all frequencies by finding a stabilizing controller using D-K iteration.

$$\min_K \left( \min_D \|DN(K)D^{-1}\|_\infty \right), \tag{7}$$

where $K$ is the controller, $D$ is any matrix to commute with $\Delta$, the uncertainty matrix, and $N$ is the LFT of $P$ and $K$. $P$ is the interconnectivity matrix between the outputs $[y_\Delta, z_1, z_2, v]^T$ and inputs $[u_\Delta, r, u]^T$ of the block diagram above, and is shown in Fig. 7.
Fig. 7. P-Δ-K Configuration

$P$ is calculated as

$$\begin{bmatrix} y_{\Delta} \\ z_1 \\ z_2 \\ v \end{bmatrix} = P \begin{bmatrix} u_{\Delta} \\ r \\ u \end{bmatrix}, \quad P = \begin{bmatrix} 0 & 0 & W_I \\ -GW_p & W_p & -GW_p \\ 0 & 0 & W_u \\ -G & 1 & -G \end{bmatrix}.$$ \quad (8)

$$N = \begin{bmatrix} -W_I T & W_I KS \\ -W_p GS & W_p S \\ -W_u T & W_u KS \end{bmatrix} \text{ where } S = \frac{1}{1 + GK} \text{ and } T = SGK.$$ \quad (9)

A disturbance plant, $G_d$, is shown in Fig. 6, but was not used for controller design. It is considered in analysis only later.

The controller design is considered successful if $\mu(N) < 1$. The checks to ensure this criteria are nominal stability (NS), nominal performance (NP), robust stability (RS), and robust performance (RP). These four checks will be explained in detail in sections 4.4 and 4.5.

The algorithm for $\mu$-synthesis with D-K iteration is quite complex because minimization of the SSV is desired at all frequencies. Therefore, the Robust Control Toolbox within MATLAB [41] was utilized which has built in functions to do $\mu$-synthesis with D-K iteration, given $P$, $W_p$, $W_u$, and $W_I$.

The resulting controller, referenced as the $\mu$-syn controller from this point forward, was a 23rd order TF with some high frequency dynamics that likely are not necessary, since high frequency error is allowed to be high by design. The controller was
reduced to a much lower order TF similar to how $G$ was obtained: by minimizing the error of the frequency response between the full order controller and a lower order TF. MATLAB has a built in tool specifically for doing frequency response least squares fitting, \textit{invfreqs}() [37], which minimizes

$$
\min_{b, e} \sum_{k=1}^{n} \left| h(k) - \frac{B(\omega(k))}{E(\omega(k))} \right|^2
$$

where $h(k)$ is the frequency response of the full order TF and $B$ and $E$ are

$$
\frac{B(s)}{E(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \cdots + b(n+1)}{e(1)s^n + e(2)s^{n-1} + \cdots + e(n+1)}.
$$

The reduced order controller was designed to have the frequency response to match the full order frequency up to approximately the bandwidth chosen for $W_p$. By trial and error, it was determined that two zeros and four poles gave the best fitting magnitude and phase for frequencies below $\omega_b$. Equation (12) shows the reduced order controller, $K_f$,

$$
\frac{6.418 \times 10^{16} (s + 0.001211)(s + 3.368 \times 10^{-5})}{(s + 1.383 \times 10^{20})(s + 1.074 \times 10^{-3})(s^2 + 1.456 \times 10^{-3}s + 2.935 \times 10^{-8})}.
$$

Equation 12 shows that there is still an extremely fast pole: $-1.383 \times 10^{-20}$. To remove this, the controller was put into state space canonical form [40], using MATLAB’s \textit{canon}(). As can be seen from the state matrix,

$$
A.K_f = \begin{bmatrix}
-1.42 \times 10^{22} & 0 & 0 & 0 \\
0 & -7.31 \times 10^{-5} & 1.55 \times 10^{-4} & 0 \\
0 & -1.55 \times 10^{-4} & -7.31 \times 10^{-5} & 0 \\
0 & 0 & 0 & 1.07 \times 10^{-4}
\end{bmatrix},
$$

canonical form does a coordinate transformation to create a new set of coordinates that are ordered by scaling. Model order reduction was used to remove the first state, using
MATLAB’s `modred()` function will match the DC gain of the reduced system to the original as close as possible. The reduced controller,

\[
K_r = \frac{4.64 \times 10^{-4}(s + 1.21 \times 10^{-3})(s + 3.36 \times 10^{-5})}{(s + 1.07 \times 10^{-4})(s^2 + 1.46 \times 10^{-4}s + 2.93 \times 10^{-8})},
\]

has the same DC gain (5.9911) as \(K_f\), but no longer has the extremely fast pole. Putting the original, 23\(^{rd}\) order controller into canonical form to remove the fast poles was attempted, but it introduced instabilities within the high fidelity simulations that could not be remedied.

Figure 8 compares the frequency response of the full order controller to the reduced controller. It can be seen that at approximately \(10^{-3}\) rad/s, the two controllers’ frequency responses begin to deviate. High frequency error was designed to be extremely large, so high frequency magnitude frequency response can be allowed since the controller was not meant to have reliable control during these periods.
3.3. PID Controller Design

$K_r$ was compared to a controller design that takes neither of the performance weights nor the uncertainty model into consideration. PID controller design is a well-established method. The particular process utilized here was the Ziegler-Nichols method [42], which constructs the controller based on a time delay, $\tau$, the maximum slope of the step response of the plant, $R_G$, and a tuning parameter gain, $k_{PID}$. It is formed by

$$K_{PID} = \frac{1.2}{R_G \tau} \left(1 + \frac{1}{2 \tau \times s} + 0.5 \tau \times s\right) k_{PID}. \quad (15)$$
\( \tau \) was chosen to be 20 seconds to match the time delay of the nonlinear model. \( R_G \) was found to be \( 1.4706 \times 10^{-3} \) K/sec by doing a step input to the plant \( G \) and calculating the maximum slope of the response. Figure 9 shows the root locus of the open loop TF.

![Root Locus](image)

**Fig. 9. Root Locus of the Open Loop TF with \( K_{PID} \)**

Depending on the gain, the open loop TF can be stable or unstable – all left half plane poles or any right hand poles, respectively. The maximum gain for the open loop TF to be stable is \( 1.87 \times 10^{-4} \), shown in Fig. 9. Though this gain leads to extremely low damping of oscillations in the linear model, simulations in the nonlinear model did not appear to have any noticeable oscillation. Therefore, to improve response time of the controller, \( k_{PID} \) was chosen to be the maximum stable gain, leading to,

\[
K_{PID} = \frac{0.07630(s + 0.05)^2}{s},
\]  

for \( K_{PID} \).
3.4. Nominal Stability

Nominal stability defines whether the nominal plant and controller is stable in an open loop TF. NS is achieved if the closed loop transfer function of the nominal plant with the controller has all left half plane poles. The PID controller was shown to be NS above. NS for $K_r$ was checked by looking at the poles of the CLTF, shown in Table 2.

<table>
<thead>
<tr>
<th>Poles</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3.36 \times 10^{-5}$</td>
<td>$-2.18 \times 10^{-1}$</td>
</tr>
<tr>
<td>$-4.31 \times 10^{-5}$</td>
<td>$-9.02 \times 10^{-5} + 7.90 \times 10^{-5}i$</td>
</tr>
<tr>
<td>$-5.60 \times 10^{-5}$</td>
<td>$-9.02 \times 10^{-5} - 7.90 \times 10^{-5}i$</td>
</tr>
<tr>
<td>$-1.07 \times 10^{-4}$</td>
<td>$-7.28 \times 10^{-5} + 1.55 \times 10^{-4}i$</td>
</tr>
<tr>
<td>$-1.29 \times 10^{-3}$</td>
<td>$-7.28 \times 10^{-5} - 1.55 \times 10^{-4}i$</td>
</tr>
<tr>
<td>$-4.28 \times 10^{-2}$</td>
<td>$-1.50 \times 10^{-1} + 8.66 \times 10^{-2}i$</td>
</tr>
<tr>
<td>$-5.63 \times 10^{-2}$</td>
<td>$-1.50 \times 10^{-1} - 8.66 \times 10^{-2}i$</td>
</tr>
</tbody>
</table>

All of the poles are within the left half plane. Therefore, NS has been achieved for both controllers.

3.5. Nominal Performance, Robust Stability, and Robust Performance

As mentioned previously, $G_d$ was not considered during the controller design for simplicity. It represents the effect of solar disturbances on the reactor temperature. $G_d$ is

$$G_d = \frac{0.0026235(s + 4.125 \times 10^{-5})}{(s + 2.083 \times 10^{-3})(s + 2 \times 10^{-5})},$$

(17)

and was determined by a similar process used in creating $G$, except instead of implementing a step input to feedstock, a step input was done to solar irradiation. It is typical to normalize $G_d$ using a scale such that $d$ ranges from 0 to 1. The solar
irradiation data used provided readings every one hour. The maximum change found in one hour was 700 W/m$^2$, so $G_d$ was scaled accordingly.

$P$ and $N$ are also recalculated to include $G_d$ in the interconnectivity matrix relating input and outputs:

$$
\begin{align*}
\begin{pmatrix} y_\Delta \\ z_1 \\ z_2 \\ v \end{pmatrix} &= P \begin{pmatrix} u_\Delta \\ r \\ d \\ u \end{pmatrix},
\quad P = \begin{bmatrix} 0 & 0 & 0 & W_i \\
-GW_p & W_p & -G_dW_p & -GW_p \\
0 & 0 & 0 & W_u \\
-G & l & -G_d & -G \end{bmatrix}.
\end{align*}
$$

$$
N = \begin{bmatrix} -W_iT & W_iKS & -W_iG_dKS \\
-W_pGS & W_pS & -W_pG_dS \\
-W_uT & W_uKS & -W_uG_dKS \end{bmatrix}, \text{ where } S = \frac{1}{1 + GK} \text{ and } T = SGK.
$$

Figure 10 shows the $P$ interconnectivity matrix with the labeled inputs and outputs. A new input, $d$, has been included.

![New P-\Delta-K configuration.](image)

If a controller has NP, then the controller will meet the performance requirements for the nominal plant (neglecting uncertainty at this point). NP is defined as

$$
\text{NP} \iff \|N_{22}\|_\infty < 1, \quad N_{22} = \begin{bmatrix} W_pS & -W_pG_dS \\
W_uKS & -W_uG_dKS \end{bmatrix}.
$$

A controller has RS if the controller is stable for all the modeled uncertainty. RS is defined as
\[
\text{RS} \leftrightarrow \mu(N_{11}, \Delta) < 1, \quad N_{11} = -W_i T. \quad (21)
\]

However, the SSV is less convenient to calculate because \( \Delta \) is not well defined. RS can be calculated as

\[
\|N_{11}\|_\infty < 1 \Rightarrow \text{RS} \quad (22)
\]

if \( \Delta \) is unstructured. Since the input to the controller, \( u \), is a scalar, and only one type of uncertainty is considered, then \( y_\Delta \) and \( u_\Delta \) must also be scalar. Therefore, \( \Delta \) is a 1x1 matrix. Any matrix that “full”, which is the case for 1x1, is unstructured; therefore, Eq. (22) can be used to calculate RS. A “full” matrix is one that has all nonzero elements.

RP for a feedback control system means that the controller will meet the performance criteria for all considered uncertainty, and is calculated as

\[
\mu(N, \tilde{\Delta}) < 1, \quad \tilde{\Delta} = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_p \end{bmatrix}. \quad (23)
\]

The block diagram connecting \( N \) and \( \tilde{\Delta} \) is shown in Fig.11. The simplification done for RS cannot be done for RP because \( \tilde{\Delta} \) will always have structure due to the off-diagonal elements being zeros.
The MATLAB \textit{mussv()} command can calculate the SSV, given $N$ and the structure of $\hat{\Delta}$. $\Delta$ was already determined to be $1 \times 1$. $\Delta_p$ has dimensions for its rows and columns determined by the number of inputs and outputs of the block diagram in Fig. 6 (neglecting the inputs and outputs to $K$ and $\Delta$), respectively. The inputs are $r$ and $d$. The outputs are $z_1$ and $z_2$. $\Delta_p$ therefore has dimensions $2 \times 2$.

Table 3 shows NP, RS, and RP for the full order controller, the reduced order controller, and the PID controller.

<table>
<thead>
<tr>
<th>Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>NP</td>
</tr>
<tr>
<td>RS</td>
</tr>
<tr>
<td>RP</td>
</tr>
</tbody>
</table>

Reducing the controller did change the values for NP, RS, and RP. However, they are still less than one so the controller design is considered successful. The PID controller met only one of the criteria: NS. It did not meet NP, nor RS or RP.
CHAPTER 4: MODEL SIMULATION WITH CONTROLLER

4.1. Controller Implementation

The controller was implemented into the high fidelity model. The input to the controller is desired reference temperature minus current reactor temperature. The control effort is solved using Euler’s Method with a single 20 second time step. Given a positive temperature difference, the controller outputs the amount lignite and O\(_2\) that should be increased from the base level of 31.264 mols of lignite every 20 seconds (or 1.5632 mols/s) and 0 mols of O\(_2\). The number of mols of water is always kept constant at 31.264 mols. If the temperature difference is negative, then the control effort will decrease. If the control effort reaches 0, it will saturate at 0 since the number of mols of O\(_2\) being inputted cannot go below 0. Saturation can occur when solar irradiation reaches a high level if the desired reference temperature is low enough.

A five day simulation was run for the high fidelity model. Two different reference points were chosen for the reactor temperature: 1100 K and 1250 K. The 1100 K set point was selected because during the nighttime, the total syngas was at a desired output of 3.5 mol/s. The 1250 K set point was chosen to see how a significantly higher set point would affect temperature control. The solar irradiation fluctuated between 0 W/m\(^2\) during nighttime to nearly 1000 W/m\(^2\). The amount of solar irradiance also varied from day to day. Figure 12 shows the simulation results of the controller implemented in the high fidelity model. The solar irradiation is from the first 5 days of the year in the TMY3 data.
4.2. Controller Performance

During times when the control effort was nonzero, the temperature’s maximum deviation was 3.25 K below the reference for the 1100 K case and 3.5 K below for 1250 K. This results in a deviation of 0.295% and 0.28%, respectively. This is slightly worse than the specified performance, but the choice for 0.25% was conservative since the linear model is extremely simplified compared to the nonlinear model.

Control effort performance was also met. During nonzero control effort, for both temperature set points the control effort increases by approximately 14 mols. Since the temperature deviation is less than 4 K for either set points, the ratio of temperature to control effort is below one while the maximum was specified to be 10.

The same simulations were run using the PID controller. A comparison of temperature for both set points is shown in Fig 13.
Performance of the PID controller is not nearly as good as $K_r$. During the 1100K set point simulation, temperature falls below the set point during nonzero control by 15.46K, an error of 1.41%. The temperature decreases by 15.74K during the 1250K set point, an error of 1.26%. Control effort performance was met, though. The control effort ranges are approximately the same for both controllers. The PID controller responds a little slower initially, which accounts for the increase in temperature deviation from the set point, as can be seen in Fig. 14 below. The ratio of temperature deviation to control effort is around one, far less than the maximum 10 specified. $K_{PID}$ does perform better than $K_r$ during zero solar irradiation. This is to be expected, though. PID controllers, especially with a significant integral component, are known to have excellent control for type zero systems with zero disturbance. Appendix A has simulations for other solar irradiation readings as well as another set point, 1500K.
4.3. Sorghum Simulations

Sorghum is another carbon rich material that can be used in gasifiers. The chemical reaction is different, though. For lignite, the chemical equilibrium is

$$\text{CH}_{0.81}\text{O}_{0.23} + 0.77\text{H}_2\text{O} \rightarrow \text{syngas}.$$  \hspace{1cm} (24)

The baselines for water and lignite were chosen based on Eq. (24). By making the baseline the same for water and lignite, during nighttime when the control effort is nonzero, the ratio of water to lignite is approximately 0.71, close to the 0.77 given in Eq. (24). This was determined by trial and error. Sorghum’s chemical equilibrium can vary depending on the type of sorghum used. For this work, the desired ratio for water to sorghum is 0.33. Through trial and error, it was determined that, to maintain the ratio during nighttime operation, the baseline for water should be 13.853 mols and sorghum should be 28.50 mols. The same four simulations (1100 K and 1250 K for both the $\mu$-syn
and PID controller) were done as before, except with sorghum. The two set points using $K_r$ are shown in Fig. 15.

![Fig. 15. Reactor Temperature during 5 day simulation (a) and control effort (b) for sorghum using $K_r$.](image)

The controller performed extremely similar to how it did with the lignite model. For the 1100 K set point, the maximum temperature deviation was 3.46 K below the set point and 3.83 K for the 1250 K case. Figure 16 shows the temperature control using the PID controller.
Fig. 16. Reactor Temperature during 5 day simulation for sorghum using $K_{PID}$

For the 1100 K set point, the temperature deviation was 16.54 K and 17.09 K for the 1250 K case. As was shown in Fig. 14 in the previous section, $K_{PID}$ responds slightly slower than $K_r$, but the control effort is essentially the same given the same set point. This is true for the sorghum results as well. However, the maximum control effort varies from lignite to sorghum. Figure 17 compares the control effort for lignite and sorghum for the 1100 K set point, using $K_r$. 
Fig. 17. Control effort for lignite and sorghum for 1100 K set point using $K_r$

The control effort for the two carbon rich materials are similar, with the main difference being that sorghum is about 0.1 mol/s higher during nighttime operation. The maximum control effort is 15 mols. Therefore, the ratio of maximum temperature change to control effort is less than one.

These are extremely encouraging results. All that was changed was the baseline for the carbon rich material and water, and the controller was able to have reliable temperature control for a different carbon rich material and chemical equilibrium.
CHAPTER 5: REFINED UNCERTAINTY MODEL

5.1. Motivation for Refining Uncertainty Model

Although the $\mu$-synthesis with D-K iteration procedure resulted in an adequate controller design, the robust analysis found that $K_{PID}$ was not robustly stable based on analysis using the linear model and uncertainty model. Dozens of different simulations were run with the high fidelity model, and yet none were unstable. It could be that a different simulation would introduce instabilities, but another possibility is that the uncertainty model included too much uncertainty.

Looking through the results in Appendix A, the highest lignite (lignite baseline plus control effort) ever gets to is 50 mols – a 27% increase from the nominal control effort. However, the uncertainty model included an 80% increase of 39.36 mols to 70.85 mols for lignite. Essentially, perturbed plants assuming high control were likely included that were not realistic based on the results. Another issue is that in the original uncertainty model, lignite and oxygen were increased by 1% to produce a dynamic response (if lignite was 1.5 mol/s and oxygen was 0.3 mol/s, then lignite increased by 0.015 mol/s and oxygen by 0.003 mol/s). This is a problem because the controller operates differently. In the high fidelity model simulations with the controller, lignite and oxygen are increased by the same scalar value (if the control effort is 0.7 mol/s, then both lignite and oxygen need to be increased by that amount).

It was also found, upon a closer look at the original simulations for the linear model, that solar irradiation can have subtle effects on the dynamics of the temperature. Essentially, it is not obvious that the transient temperature from startup has settled unless
it is looked upon a timescale of days instead of hours. It’s unclear if this was the case for the original simulations, but it was something to address for the new $G$ and $G_p$’s. With a better understanding of how to more accurately model the dynamics of the nonlinear model, a new $G$ and uncertainty model was developed.

5.2. New Perturbed Plants

For the new uncertainty model, the new baseline for input feedstock is shown in Table 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{DN}$</td>
<td>700 W/m²</td>
</tr>
<tr>
<td>$\dot{n}_{O_2}$</td>
<td>0.25 mol/s</td>
</tr>
<tr>
<td>$\dot{n}_C$</td>
<td>1.81 mol/s</td>
</tr>
<tr>
<td>$\dot{n}_{H_2O}$</td>
<td>1.56 mol/s</td>
</tr>
</tbody>
</table>

A higher value for solar irradiation was selected to ensure there was enough energy to heat the reactor to a sufficient temperature, since the baseline for oxygen is lower than the previous uncertainty model. Nine different simulations with the high fidelity model were considered by increasing the baseline for C and O₂ which were increased in increments of 0.1 mol/s up to 0.8 mol/s (2.61 mol/s total for C and 1.05 mol/s for O₂). For each of the simulations, once the startup transients had stopped (2.5 days were simulated before the step was done to ensure the temperature had settled), C and O₂ were increased by 0.05 mol/s to cause a dynamic response in the reactor temperature. Figure 18 shows the dynamic response.
Fig. 18. Plant responses used for the new Uncertainty Model

The top curve is the baseline; the next is 0.1 mol/s increase from the baseline, and so on. The equal increase in C and O$_2$ cause the direct temperature jump as before, but instead of continuing to increase the temperature, there is a decrease. This is quite different from before. As mentioned before, the dynamic responses for the previous uncertainty model were created by doing a 1% increase to both lignite and oxygen. Since the amount of lignite being fed into the reactor was greater than oxygen, the change in lignite was higher than the change in oxygen. It’s possible that the larger increase in lignite, compared to the increase in oxygen, caused a greater part of the lignite to be burnt off, causing the temperature to increase.

The old nominal plant is not going to be used for the new uncertainty model. $G$ is going to be chosen such that $W_f$ can be minimized. Any of the nine dynamic responses can be chosen to represent the dynamics of the nominal plant. Once the dynamic responses are fit to TFs, each one will be considered as $G$ initially while the rest are the
$G_p$’s. Whichever one results in the smallest $W_I$ will be selected as $G$. Essentially the dynamic response that is most similar to the rest will be chosen to be $G$.

As before, the TFs fit to the high fidelity simulation are comprised of a direct feedthrough and two first order TFs with one pole and no zeros. Figure 19 shows the new TFs, which fit very well again. The largest mean absolute error of any of the nine is 0.0748K.

![Figure 19. Fit TFs to new Plants](image)

Using Eq. (5), multiplicative uncertainty was calculated nine different times, each time corresponding to a different plant being considered $G$. The 0.4 mol/s increase from the baseline created the lowest multiplicative uncertainty. This makes sense intuitively. The dynamic responses differ primarily only in magnitude of the response. Since 0.4 mol/s is halfway in between 0 mol/s and 0.8 mol/s, the magnitude of the response is also halfway between the extremes shown in Fig. 18. This specific response is most similar to
the rest. The selected $G$ is shown below, such that the multiplicative uncertainty is minimized,

$$G = \frac{74.852(s + 4.50 \times 10^{-4})(s + 3.22 \times 10^{-5})}{(s + 4.87 \times 10^{-4})(s + 3.39 \times 10^{-5})}.$$  \hspace{1cm} (25)

As before, a Padé approximation has been appended for use in the controller design and robust analysis. Figure 20 shows the multiplicative uncertainty, as well as $W_I$.

\[\text{Fig. 20. New Multiplicative Uncertainty with } W_I\]

$W_I$ was chosen to be constant for the reason given before (Section 3.1): -11.78 dB or 0.26 absolute. The same $W_p$ and $W_u$ was chosen in order to compare to the previous controller designs.

5.3. Controller Synthesis

Using $dksyn()$ in MATLAB again, a 25th order controller was synthesized. Therefore, reduction was done again to eliminate high frequency dynamics and unnecessary complexity. Using Eq. (10) to do a fit, five zeros and six poles were found
by trial and error to be the give the best results. As before, fast states still remains in the
poles (this time there are two), which can be easily seen when converted to canonical
form,

\[ A_K \]

\[
\begin{bmatrix}
-7.35 \times 10^{18} & 0 & 0 & 0 & 0 & 0 \\
0 & -12.2 & 0 & 0 & 0 & 0 \\
0 & 0 & -1.46 \times 10^{-4} & 3.94 \times 10^{-5} & 0 & 0 \\
0 & 0 & -3.94 \times 10^{-5} & -1.46 \times 10^{-4} & 0 & 0 \\
0 & 0 & 0 & 0 & -1.05 \times 10^{-4} & 0 \\
0 & 0 & 0 & 0 & 0 & -2.82 \times 10^{-5}
\end{bmatrix}
\] (26)

Removing the first two fast states with \textit{modred()}, a new \( K_f \),

\[
\frac{6.82 \times 10^{-3}(s + 0.022)(s + 3.20 \times 10^{-5})(s^2 + 1.43 \times 10^{-3}s + 6.92 \times 10^{-7})}{(s + 1.05 \times 10^{-4})(s + 2.82 \times 10^{-5})(s^2 + 2.91 \times 10^{-4}s + 2.28 \times 10^{-8})}
\] (27)

which compares well to the full order controller up until around the bandwidth of 0.001
rad/s and then deviates. Figure 21 shows this.
Since $G$ is different from before, a new PID controller was also designed. The time constant stayed 20 seconds, but $R_G$ was found to be the new value of 6.7958 K/sec. The tuning parameter was selected via the root locus of the open loop TF with $K_{PID}$ in Fig. 22. During use of the controller within the high fidelity model, oscillations were found if the gain was pushed to the imaginary axis as it was before, so a value of 40 was chosen to have the overshoot be approximately 10%.
With $k_{PID}$ selected, $K_{PID}$ is

$$K_{PID} = \frac{3.4963(s + 0.05)^2}{s}.$$  \hfill (28)

5.4. NS, NP, RS, and RP

Both new controllers have NS. The PID controller was shown to have nominal stability above by the root locus. NS for $K_r$ was determined by the poles of the CLTF, shown in Table 5.

<table>
<thead>
<tr>
<th>Table 5. Poles of the CLTF for $K_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poles</td>
</tr>
<tr>
<td>$-2.82 \times 10^{-5}$</td>
</tr>
<tr>
<td>$-3.20 \times 10^{-5}$</td>
</tr>
<tr>
<td>$-3.22 \times 10^{-5}$</td>
</tr>
<tr>
<td>$-3.39 \times 10^{-5}$</td>
</tr>
<tr>
<td>$-1.05 \times 10^{-4}$</td>
</tr>
<tr>
<td>$-4.50 \times 10^{-4}$</td>
</tr>
<tr>
<td>$-4.87 \times 10^{-4}$</td>
</tr>
<tr>
<td>$-7.59 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Since every pole is in the left half plane, NS is achieved.

Nominal performance, robust stability, and robust performance are shown in Table 6.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Old Reduced</th>
<th>New Reduced</th>
<th>Old PID</th>
<th>New PID</th>
</tr>
</thead>
<tbody>
<tr>
<td>NP</td>
<td>0.7468</td>
<td>0.7298</td>
<td>2.6489</td>
<td>4.1400</td>
</tr>
<tr>
<td>RS</td>
<td>0.6302</td>
<td>0.2747</td>
<td>3.9709</td>
<td>0.2576</td>
</tr>
<tr>
<td>RP</td>
<td>0.9869</td>
<td>0.9768</td>
<td>6.6199</td>
<td>4.3969</td>
</tr>
</tbody>
</table>

The new $K_r$ is very similar to the old one, except for the RS norm, which is smaller. $K_{PID}$ is much more reflective of what was expected originally (and reflected in the simulations). NP and RP are still not achieved, but this time the PID controller does have RS. Simulations using the high fidelity model for the original PID controller showed no signs of instability, even though it was predicted to be unstable by linear robustness analysis.

5.5. Simulation Results

The same five-day simulations that were run for the previous controllers in the high fidelity model were run for the new ones. Figure 23 shows the new results.
The performance of the new $K_r$ is similar to the previous results. During times of nonzero control for the 1100 K simulation, the temperature fell 6.46 K below the set point, or 0.59%. For the 1250 K case, the temperature fell 5.83 K below the set point – 0.47% error. This performance is worse than the previous $K_r$. While this is unfortunate, the new controller is more appropriate. The previous $G$ and uncertainty model were not reflective of how the gasifier would be operated.

The difference in control effort from the old $K_r$ to the new $K_r$ is insignificant, in terms of maximum control effort. It is still approximately 14 mols. With the temperature deviation in both cases less than 13 K, the ratio of temperature change to control effort is less than one – well below the specified of 10 for $W_d$.

The new uncertainty model and robust analysis states that the new PID controller should be stable, but will not perform as well as the new $K_r$. Figure 24 compares the two new controllers as was shown before for the old controllers in Figure 13.
For the 1100 K set point, the PID controller goes 41.24 K below the set point – an error of 3.77%. For the 1250 K case, temperature goes 41.96 K below the set point – an error of 3.36%. The maximum control effort for the new $K_{PID}$ is 14 mols (as it has been with all the controllers). Even with the temperature deviation for this controller, the ratio of maximum temperature deviation to control effort is still less than the specified value for $W_u$. This is worse than the previous PID controller, but the same reasons given for why the new $K_r$ is more appropriate apply to this controller as well. The nominal model and uncertainty model must reflect the nonlinear model, or else the conclusions drawn from the robustness analysis do not have any meaning.

The previous sorghum simulations were ran again, using the new $K_r$ and $K_{PID}$. Figure 25 shows the 1100 K simulations (a), 1250 K simulations (b), and the control effort for both set points (c).
Fig. 25. Sorghum simulations for 1100 K (a), 1250 K (b), and control effort (c) using the new $K_r$ and $K_{PID}$

The greater temperature deviations seen in the lignite simulations with the new controllers are present in the new sorghum simulations. For the 1100 K set point, the temperature deviates 6.1 K for $K_r$ and 44.26 K for $K_{PID}$. The 1250 K had temperature deviations of 6.34 K and 45.4 K for $K_r$ and $K_{PID}$, respectively. The maximum control effort for the sorghum simulations with the new controllers do not differ from the previous simulations with the old controllers: around 15 mols.
CHAPTER 6: DISCUSSION

As was mentioned in Chapter 5, the new $K_r$ appears to be worse than the previous design presented in Chapter 4. However, in at least one way, the new $K_r$ is better than the previous design. The maximum temperature deviations are relatively short, compared to the timescale the gasifier is expected to operate on. Figure 26 compares the reactor temperature of the high fidelity model with the old $K_r$ to the new one.

![Figure 26. Comparing temperature control with the old $K_r$ to the new $K_r$](image)

Even though the new $K_r$ has a higher maximum deviation, for the majority of the time, it is very much improved over the old design. The old controller has a mean absolute error of 1.86 K, while the new one has a mean absolute error of 0.734 K. During the nighttime with no $I_{DN}$, the new $K_r$ has a very steady temperature control at 1250 K, while the old controller has an offset error of 2.5 K that appears to be reached at approximately $36 \times 10^4$ s in Fig. 26.

The question then is: what causes the difference in performance? The excellent control during the nighttime for the new controller suggests that the new $G$ and
uncertainty model much more accurately represents the high fidelity model. It is only when solar irradiation is changing rapidly that the new $K_r$ performs worse than the previous design. Comparison of the magnitude frequency response for the old $K_r$ and the new $K_r$ is shown in Fig. 27.

![Magnitude frequency response of the two different $K_r$ controllers](image)

**Fig. 27.** Magnitude frequency response of the two different $K_r$ controllers

At low frequencies, the gain of the old $K_r$ is lower than the new one. Therefore, it should perform worse during lower frequencies compared to the new one. The high fidelity simulations results discussed in the previous chapter confirm this. As the frequency increases though, the original controller design has a higher gain, so it will perform better than the newer design at higher frequencies. This was also confirmed in the previous chapter.

The same reasoning can explain the difference between the two PID controllers designed, shown in Fig 28.
The new PID controller has worse control during times of high disturbance compared to the old PID controller, the same way the $\mu$-synthesis controller does, since the old PID controller has a higher gain at higher frequencies. The reason nighttime isn’t worse for the newer controller, even though the gain is lower at low frequencies as well, is because of how good PID controllers can work during zero and low frequency disturbance in general, as was mentioned previously (Section 4.2). Comparison of the two PID controllers’ performance in the high fidelity model is shown in Fig. 29.
Figure 30 shows temperature control for all four controllers for an 1100K set point (b) and a 1250K set point (c), given a solar disturbance (a). Ignoring the temperature spikes in the 1100K from when the control goes offline (zero control effort), the temperature deviation for a certain controller appears nearly identical regardless of the set point. A comparison of the deviation from the set point for the two new controllers is shown in Fig. 31.
These results show that the degree to which temperature is controlled is independent of the set point. For the same solar disturbance, the controller response to keep the temperature at the specified value is also the same. This is an excellent result. It suggests that neither controller will ever cause the simulation to destabilize based on a chosen realistic set point. The controller will perform the same.

Though the focus of this work was not on syngas production, it was important to ensure its production was acceptable. In previous work, it was determined that during
times of zero solar irradiation, 3.5 mol/s was the desired production rate for the 1100K set point [15]. Figure 32 plots the syngas production for the 1100K (a) and 1250K (b) set points for both the new $K_r$ and $K_{PID}$.

![Graph showing syngas production](image)

**Fig. 32. Total syngas production for the 1100K (a) and 1250K (b) set points for the new controllers**

Syngas production rate of 3.5 mol/s for the 1100K case during no solar irradiation was achieved. Production dips below this value during times of solar irradiation since less feedstock is being fed into the reactor. Syngas production rates are higher during the higher set point, which is expected – more feedstock means more syngas production. The
production rate was essentially the same for $K_r$ and $K_{PID}$. Comparing the two controllers on this metric doesn’t show any benefit for either one over the other.

Another metric to compare the two controllers by is the effect of control on the cold gas ratio, or

$$ R = \frac{\dot{n}_{CO} LHV_{CO} + \dot{n}_{H_2} LHV_{H_2}}{\dot{n}_{feedstock} LHV_{feedstock}} $$

(29)

where $R$ is the cold gas ratio and $LHV$ is lower heating value. The cold gas ratio is a measure of energy in the products to energy in the reactants. It can be viewed as a measure of efficiency of the process. $R$ can go above 100% efficiency during times of solar irradiation because the energy stored in solar energy that gets converted to the energy in the products is not considered in the equation. A plot of $R$ is shown in Fig 33.
The cold gas ratio for $K_r$ and $K_{PID}$ are virtually indistinguishable. The mean absolute error for the cold gas ratio between the simulations is $4.15 \times 10^{-4}$, and the maximum absolute error between the two is 0.03. Its shape is dictated by the shape of the solar irradiance curves. The two controllers performing the same was to be expected, though. The cold gas ratio would only differ significantly if either the input feedstock or syngas production varied between the two controllers significantly. Figure 14 shows that the control effort, which determines the input feedstock changes, only differs momentarily...
since the PID controller responds a bit slower, and Fig. 32 shows the syngas production was not significantly different.

Both syngas production rates and the cold gas ratio show that there is no noticeable difference in $K_r$ and $K_{PID}$. It calls to question how well temperature needs to be controlled. At the start of this work, it was not known how well temperature needed to be controlled. Therefore, the performance specifications were chosen to have extremely reliable temperature control. The work so far shows that a PID controller is sufficient in terms of syngas production and cold gas ratio. However, as was mentioned in Section 2.1, the high fidelity model works under some assumptions. One such assumption is the reaction kinetics, or the rate of the reactions, can be simplified. The reaction kinetics are temperature dependent, but have not been included in the model. A common equation for temperate dependency is the Arrhenius equation, or

$$ k = Ae^{-\frac{E_a}{RT}}, $$

where $k$ is the reaction rate, $A$ and $E_a$ are dependent on the reaction, $R$ is the gas constant, and $T$ the temperature. Depending on the other values, a temperature variation of only 30K (the approximate difference between max temperature deviation for $K_r$ and $K_{PID}$) could more than double, or halve, the reaction rate. Temperature control could become much more important once the model incorporates the reaction rates’ dependence on temperature. Updating the model is outside the scope of this author’s work, but is being done by others in future work.
CHAPTER 7: CONCLUSION

This thesis has presented a method for reliable temperature control for a gasifier utilizing both allothermal and autothermal modes of operation. It has taken an extremely complex model, and shown an effective way of linearizing it so that established methods of controller design could be taken advantage of. Two different controller designs have been demonstrated. Using μ-synthesis with D-K iteration, while more complicated, offers better temperature control. If less accuracy is sufficient for temperature control, a simpler controller synthesis, a PID controller, has been presented. A multiplicative uncertainty model was also developed. The uncertainty was created in such a way that it differed minimally from the nominal model, and yet it was effective. The extremely accurate temperature control during times of zero disturbance validated the method by which the uncertainty model was created. A refinement of the nominal and uncertainty model was also shown. Its improvement for $K_r$ during zero solar irradiance show that the new models better reflect the high fidelity model. The original PID controller was stable for the high fidelity simulations, but the linear robustness analysis determined it was not. The new uncertainty model gives a more realistic robust stability result for the PID controller.

Though some performance specifications were not met, namely the temperature deviation, the selection for maximum allowable temperature error was extremely conservative. This indicates that further work allow a relaxation of the maximum temperature deviation specifications if desired, instead of having to tighten the deviation requirements, which would be a more difficult task.
The designed controllers were also shown to have reliable temperature control for different types of feedstock. Lignite and sorghum had very comparable results, despite the fact that the chemical equilibrium between the two is quite different. This may allow future work to look at different feedstock options without having to redesign the controllers, which would be beneficial.
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Fig. 35. Simulation for first 5 days in April using original controllers
Fig. 36. Simulation for first 5 days in August using original controllers
Fig. 37. Simulation for first 5 days in April using new controllers
Fig. 38. Simulation for first 5 days in August using new controllers