# EXPERIMENTAL AND NUMERICAL INVESTIGATION OF SUBCRITICAL BIFURCATIONS IN MILLING 

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## DEDICATION

To my family

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#### Abstract

The focus of the current thesis is an experimental and numerical investigation of subcritical bifurcations in milling. More specifically, several researchers have reported results indicating that the secondary Hopf bifurcations of turning processes are subcritical. However, fewer results are available for milling - especially results that provide any substantial experimental evidence. Therefore, experimental cutting tests are performed on a relatively long aluminum workpiece. The atypical length of the work piece will be used so that the depth of cut may be slowly increased or decreased during the cutting process. This provides visual evidence of hysteresis in the bifurcation diagram and the existence of multiple stable periodic solutions. The importance of this exploratory experimental effort is that multiple attractors may exist (stable periodic solutions) for the same cutting speed and depth of cut, but only one of these solutions would be the chatter free case. An important outcome from these results is that a small perturbation in the desirable stable solution near the borders of the stability diagram could result in a jump to the unstable cutting condition.


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## Chapter 1

## Introduction

Milling is a traditional machining operation which is widely used in industry to manufacture mechanical components. In general, it is classified into end milling, face milling, and peripheral milling operations $[1,2]$. In peripheral milling, the milled surface is generated by teeth located on the periphery of the cutter body and the axis of cutter rotation is generally in a plane parallel to the workpiece surface being machined. In face milling, the milled surface results from the action of cutting edges located on the periphery and face of the cutter and the cutter is mounted on a spindle having an axis of rotation perpendicular to the workpiece surface. In end milling, the milling surface is generated by the teeth located on both the periphery and the tip of the cutter body and the cutter rotates on an axis perpendicular to the workpiece [2].

End milling is the most versatile form of milling and it is used to machine slots, contours, profiles etc. Due to the interrupted nature of end milling, these teeth are usually made helical to reduce the impact that occurs when each tooth engages the workpiece. Depending on the nature of the feature to be machined, the axis of rotation of the end mill may be either perpendicular or parallel to the finished surface. It can
also be said that an end milling operation is a combination of peripheral milling and face milling operations. Milling operations can also be classified into up-milling and down-milling according the direction of the rotation with respect to the feed direction (see Fig. 1.1).


Figure 1.1: Dynamic model of an interrupted milling process. Fig. (a) shows the up-milling process, and (b) shows a down-milling process.

### 1.1 Motivation and Current Knowledge

Material removal processes, such as metal cutting are used to achieve desired surface finishes, tolerances or shapes that are difficult to obtain otherwise. Increased economic importance and competition in the machining industry has lead to the need to optimize manufacturing processes. This provides the ability to improve the dimensional accuracy, process efficiency and manufacturing quality. Dynamic models provide the ability to predict surface accuracy and regions of stable cutting for a large combination of process parameters. Hence, the typical industrial trial and error results can be minimized and the selection of the optimum machining parameters can be implemented for best manufacturability and cost reduction.

The stability of the interrupted machining processes has become an incredibly significant research area in the early 20th century [3]. However, significant advances only started in the 1960s, when it was realized that machining can only be modeled accurately by accounting for the so-called regenerative effect [4]. Specifically, the tool cuts an already machined surface produced in the previous cutting period, and hence the chip thickness depends on the previous positions of the tool tip as well as on the current one. In other words, it can be said that the forces are coupled to the tool motion.

The focus of many recent works has been the occurrence of new bifurcation phenomena in interrupted cutting processes [5]. In addition to Neimark-Sacker or secondary Hopf bifurcations, period-doubling bifurcations have been analytically predicted in references [5-11] and confirmed experimentally in references [6-8, 12-17]. Other investigations have shown additional physical mechanisms which may influence the relative oscillations between the tool and the workpiece [18]. For instance, the thermoplastic behavior of chip formation has been reported in reference [19] and frictional effects at the tool chip interface have been examined in the reference [20].

The focus of the current research stems from an apparent void in the literature base with regards to the subcritical nature of milling bifurcations. More specifically, several researchers have reported results indicating that the secondary Hopf bifurcations of turning processes are subcritical [21,22]. However, fewer results are available for milling - especially results that provide any substantial experimental evidence. The goal of the current research project is to provide a novel series of experimental results that explore the subcritical nature of milling bifurcations.

## Subcritical bifurcation



Figure 1.2: Representative subcritical bifurcation diagram for a milling process plotted as a function of the cutting depth, $b$. Here, $x(n \tau)$ represents the amplitude of periodic displacement samples found for stable equilibria (solid red line). These stable equilibria are experimentally observable and represent both stable (chatter-free) and stable (chatter-vibration) equilibria. The dotted blue line represents equilibria that may be obtained with numerical continuation, but would not appear in an experimental setting.

The importance of understanding whether the bifurcations are sub critical is explained by the bifurcation diagram of Fig. 1.2. This graph shows the vibration amplitudes (or equilibria solutions) plotted as a function of the axial cutting depth, $b$. Starting from a location of a small cutting depth and slowly increasing, this parameter will result in a single stable equilibria solution which will also be free of chatter vibrations. As the bifurcation parameter, the depth of cut, is further increased, the chatter free equilibria solution will enter region where multiple stable equilibria coexist - both the chatter-free and a chatter-vibration solution. Towards the end of this region, the tool will reach the bifurcation point which will cause a jump in the re-
sponse of the tool to the stable chatter-vibration equilibria solution. The amplitude of the chatter vibrations will then continue to grow with an increase in the axial depth. Reversing this process by starting at a relatively larger cutting depth and progressing towards a smaller depth will result in hysteresis near the region of the multiple stable equilibria. In essence, the stable equilibria representing chatter vibration are a limit cycle that collapses to a stable chatter-free solution once the bifurcation parameter has decreased beyond the region where the multiple attractors exist. While one can argue that this method of cutting may be atypical, the relevance becomes more clear when the influence of a small perturbation is considered [23]. In particular, small perturbations within the region of coexisting stable attractors can cause the cutting tool to jump back and forth between the regions (the basins of attraction) which delineate the chatter and chatter-free equilibria.

### 1.2 Thesis Organization

In Chapter 2, an introduction to modeling and machining dynamics is discussed, which leads to the estimation of the cutting forces. Since a typical machine-tool structures may have multiple structural modes, the sections of this chapter extend the cutting force estimation to form the math model for the milling process. Chapter 3 explains the experimental system that was constructed to investigate the subcritical nature of the bifurcation diagrams. In the subsequent sections of this chapter, the math model is discussed which includes the influence of the structural modes and the cutting forces. Next, the stability border trends are studied for the experimental system for several radial immersions and helix angles. Since the goal was to focus on the experimental behavior, some of the analysis details have been omitted with the appropriate references cited. However, a numerical comparison of the experimental bifurcation diagrams is formulated which describes the results of the experimental
testing along with the methods used to visualize and process the experimental time series. The final chapter summarizes the contributions from this research and outlines the potential opportunities for future work.

## Chapter 2

## Modeling Machining Dynamics

### 2.1 Introduction

The significance of high speed machining and especially high speed milling in production has increased since new machines and tools are able to apply high material removal rates. High speed machining enables the possibility to reduce process time and improves workpiece accuracy. High speed machining is mostly related to the application of high cutting speeds, which are almost three times higher than the conventional cutting process. Within this range of cutting speeds, the cutting force decrease with increasing cutting speeds [24,25]. At very high cutting speeds, however, the cutting forces increases due to inertia forces [25]. This chapter presents the methods for modeling the cutting forces of a cutting process.

### 2.2 Cutting Forces in Machining

During a cutting process, large forces are exerted between the tool and the workpiece which often results in undesirable vibrations in the process. These naturally have an impact on the operation of the tool and the workpiece. In order to understand the tool motion and the dynamics of the cutting process, it is important to study the methods to model the cutting forces in a machining process. Some of the assumptions in the cutting forces depend upon the following factors: 1) the cutting geometry, 2) the relative displacement between the tool and the workpiece, 3) workpiece material properties. Additional influences includes frictional effects, thermal effects between the tool and the workpiece.

In literature, several models have been proposed to model the cutting force as a function of the cutting parameters, such as the depth of cut, the uncut chip thickness, which together can be called as the uncut chip area $[1,2,25,26]$. In order to get a better understanding of the force-area relationship, we assume the cutting forces, $F_{c}(A)$ to be a function of the uncut chip area, $A_{0}$, which can be written in a Taylor series expansion as

$$
\begin{equation*}
F_{c}(A) \approx \underbrace{F_{c}(A)}_{\text {static }}+\underbrace{\frac{\partial F_{c}(A)}{\partial A}\left(A-A_{0}\right)}_{\text {dynamic }} . \tag{2.1}
\end{equation*}
$$

This is a general equation for the cutting forces with, $F_{c}$ as the resultant cutting force. Here, the cutting forces are separated into static and dynamic cutting force components. Figure 2.1, shows an interrupted turning process for orthogonal cutting [25]. Since the mathematical model for turning is much simpler than milling, one could get a better understanding orthogonal cutting mechanics involved in the cutting
process $[25,27]$. For this reason, the turning process is the best for investigation because it has a single-edged tool and thus the arrangement of the tool and the workpiece is simpler to describe (see Fig. 2.1).


Figure 2.1: A schematic of an orthogonal cutting in an interrupted turning process. Schematics (a) and (b) shows the orthogonal cutting geometry.

### 2.2.1 Cutting Force Expression in Turning

Obtaining an expression for the cutting forces involves several physical parameters. Starting with Eq.(2.1), the cutting force in turning can be expressed as follows,

$$
\begin{equation*}
F_{t}(A) \approx \underbrace{F_{t}(A)}_{\text {static }}+\underbrace{\frac{\partial F_{t}(A)}{\partial A}\left(A-A_{0}\right)}_{\text {dynamic }} . \tag{2.2}
\end{equation*}
$$

Rewriting the resultant force vector as two perpendicular force vectors, $F_{t}$ and $F_{n}$,

$$
\begin{align*}
& F_{t}(A) \approx K_{t} b(h+[y(t)-y(t-\tau)])  \tag{2.3}\\
& F_{t}(A) \approx \underbrace{K_{t} b h}_{\text {static }}+\underbrace{K_{t} b[y(t)-y(t-\tau)]}_{\text {dynamic }} . \tag{2.4}
\end{align*}
$$

where $A_{0}$ is the uncut chip area, $A$ is the instantaneous chip area, $b$ is the depth of cut, and, $h$ is the instantaneous chip thickness. From Eq.(2.4), it is apparent that the cutting force is proportional to the cutting pressure, $K_{t}$ and the area, $A_{0}$ and it is assumed to have "no vibration" in the static force component. In the dynamic part of Eq.(2.4), the cutting force is related to the tool displacement at the current time and the tool displacement from the previous tooth passage. Here, we have assumed that the tool is only compliant in the $y$-direction.

The force distribution of an orthogonal cutting process is shown in Fig. 2.2. The resultant cutting force, $F_{c}$, is at an angle $\beta_{a}$, which is sometimes called as the average friction angle between the tool's rake face and the moving chip [26]. The chip thickness ratio, $r_{c}$, and the shear plane angle, $\phi_{s}$, can be obtained from the following relationships

$$
\begin{align*}
r_{c} & =\frac{h}{h_{c}},  \tag{2.5a}\\
\phi_{s} & =\arctan \frac{r_{c} \cos \left(\alpha_{r}\right)}{1-r_{c} \sin \left(\alpha_{r}\right)} . \tag{2.5b}
\end{align*}
$$

where $\alpha_{r}$ is the rake angle.


Figure 2.2: Cutting force diagram of an interrupted turning process.

It is also assumed that the cutting edge of the tool is sharp and the deformation takes place at infinitely thin shear plane. Further assumptions include constant shear stress, $\tau_{s}$, resultant cutting force, $F_{c}$, and shear angle, $\phi_{s}$.

Referring to Fig. 2.2, the resultant force is formed from the tangential and normal forces as follows

$$
\begin{equation*}
F_{c}=\sqrt{F_{t}^{2}+F_{n}^{2}} \tag{2.6}
\end{equation*}
$$

Also, the shear force, $F_{s}$, can be correlated with the resultant cutting force as follows,

$$
\begin{equation*}
F_{c}=\frac{F_{s}}{\cos \left(\phi_{s}+\beta_{a}-\alpha_{r}\right)} . \tag{2.7}
\end{equation*}
$$

But, the shear force can be further related to the shear stress and the uncut chip area which is expressed as,

$$
\begin{equation*}
F_{s}=\tau_{s} \frac{b h}{\sin \left(\phi_{s}\right)} \tag{2.8}
\end{equation*}
$$

where

$$
\begin{align*}
& \tau_{s}=\frac{F_{s}}{A_{s}}  \tag{2.9}\\
& A_{s}=b \frac{h}{\sin \left(\phi_{s}\right)} . \tag{2.10}
\end{align*}
$$

Therefore, substituting Eq.(2.8)- Eq.(2.10) into Eq.(2.7), we get a new equation for the resultant cutting force represented as

$$
\begin{equation*}
F_{c}=\tau_{s} b h \frac{1}{\sin \left(\phi_{s}\right) \cos \left(\phi_{s}+\beta_{a}-\alpha_{r}\right)} . \tag{2.11}
\end{equation*}
$$

Now, the tangential and normal cutting forces can be written in terms of the resultant cutting force as follows

$$
\begin{equation*}
F_{t}=F_{c} \cos \left(\beta_{a}-\alpha_{r}\right), \tag{2.12a}
\end{equation*}
$$

$$
\begin{equation*}
F_{n}=F_{c} \sin \left(\beta_{a}-\alpha_{r}\right) . \tag{2.12b}
\end{equation*}
$$

Substituting Eq.(2.11) into Eqs.(2.12a)\& (2.12b), we get

$$
\begin{align*}
& F_{t}=b h\left[\tau_{s} \frac{\cos \left(\beta_{a}-\alpha_{r}\right)}{\sin \left(\phi_{s}\right) \cos \left(\phi_{s}+\beta_{a}-\alpha_{r}\right)}\right],  \tag{2.13a}\\
& F_{n}=b h\left[\tau_{s} \frac{\sin \left(\beta_{a}-\alpha_{r}\right)}{\sin \left(\phi_{s}\right) \cos \left(\phi_{s}+\beta_{a}-\alpha_{r}\right)}\right], \tag{2.13b}
\end{align*}
$$

where $\tau_{s}$ is the material shear stress:

$$
\begin{equation*}
\tau_{s}=F_{c} \frac{\sin \left(\phi_{s}\right) \cos \left(\phi_{s}+\beta_{a}-\alpha_{r}\right)}{b h} \tag{2.13c}
\end{equation*}
$$

In metal cutting literature, the cutting forces are related to the cutting parameters, known as the cutting coefficients

$$
\begin{align*}
& K_{t}=\left[\tau_{s} \frac{\cos \left(\beta_{a}-\alpha_{r}\right)}{\sin \left(\phi_{s}\right) \cos \left(\phi_{s}+\beta_{a}-\alpha_{r}\right)}\right],  \tag{2.14a}\\
& K_{n}=\left[\tau_{s} \frac{\sin \left(\beta_{a}-\alpha_{r}\right)}{\sin \left(\phi_{s}\right) \cos \left(\phi_{s}+\beta_{a}-\alpha_{r}\right)}\right] . \tag{2.14b}
\end{align*}
$$

As mentioned earlier in the section, the resultant cutting force, shear stress and shear angle are assumed to be constant, but the equations derived above shows the
cutting coefficients as a functions of the shear stress, rake angle and, shear angle, etc. The empirical formulation of the cutting forces in the tangential and normal directions are commonly written in the literature as

$$
\begin{equation*}
F_{t}=K_{t} b h, \tag{2.15a}
\end{equation*}
$$

$$
\begin{equation*}
F_{n}=K_{n} b h . \tag{2.15b}
\end{equation*}
$$

### 2.3 Cutting Force Model for Milling

The cutting force estimation in a turning process was studied in order to gain intuition which could be potentially beneficial in modeling the cutting forces in milling. Also, the modeling of a milling process is much more difficult than turning since it is a multi-point cutting tool. Furthermore, the tangential and normal forces change direction with cutter rotation. The schematics of Fig. 3.4 show a typical up-milling and down-milling operation and provide illustrations of the directional cutting forces for a milling process. This diagram also shows that milling is an interrupted cutting process - meaning that the tool enters and exits the workpiece during cutting. Therefore, the resulting math model must account for the fact that the tool has a finite entry and exit angle. Although the authors acknowledge that the shape of the chip will be trochoidal [28], the results that follow use the common approximation of a circular tool path.

Recalling Eq. (2.1) from section 2.2.1, the force-area relationship for milling can be expressed as,


Figure 2.3: Cutting force diagram of an interrupted milling process. Fig.(a) shows the top view of an up-milling process, and Fig.(b) shows the top view of a down-milling process.

$$
\begin{align*}
& F_{t}(A) \approx \underbrace{F_{t}(A)}_{\text {static }}+\underbrace{\frac{\partial F_{t}(A)}{\partial A}\left(A-A_{0}\right)}_{\text {dynamic }}  \tag{2.16a}\\
& F_{t}(A) \approx K_{t} b w_{p}(t) \tag{2.16b}
\end{align*}
$$

where $A$ is the instantaneous chip area, $A_{0}$ is the uncut chip area, $K_{t}$ is the cutting coefficient in the tangential direction.

The instantaneous chip area, $w_{p}$, when the $p^{t h}$ tooth is in contact, is given by:

$$
\begin{equation*}
w_{p}(t)=\underbrace{h \sin \theta(t)}_{\text {static }}+\underbrace{[x(t)-x(t-\tau)] \sin \theta(t)+[y(t)-y(t-\tau)] \cos \theta(t)}_{\text {dynamic }} . \tag{2.17}
\end{equation*}
$$

where $h$ is the feed/ tooth, $\tau=\frac{60}{N \Omega}$, is the tooth pass period with $\Omega$, as the spindle speed and $N$, the number of cutting teeth, and $x(t)$ and $y(t)$ are the tool motions along the respective directions.

The instantaneous chip width mainly depends on the following factors: 1) the feed per tooth, $h 2$ ) the cutter rotation angle, $\theta$, and, 3 ) regeneration in the compliant structure directions. The tool motion in the $x$ and $y$ directions can be better illustrated using Fig. 2.4.

## Radial chip thickness ( $\mathrm{w}_{\mathrm{p}}$ )



Figure 2.4: Detailed schematic of the motion of the tool for a radial chip thickness, $w_{p}(t)$ in the $x \& y$ direction components.

With reference to Fig. 2.4, the radial chip thickness can be obtained by projecting the difference between the current tool displacement, described by $x(t)$ and $y(t)$,
and the displacements of the cutting tool on the previous tooth passage, $x(t-\tau)$ or $y(t-\tau)$, onto the radial direction. This represents the dynamic part of the cutting force model expressed in Eq. (3.1).

In order to obtain the math model for milling which is discussed here, one would need to resolve the forces. These forces should be translated from the rotational frame onto the fixed co-ordinate frame.


Figure 2.5: Detailed schematic of the force distribution for radial chip thickness, $w_{p}(t)$ with respect to the tool motions in the $x$ - and $y$ - direction components.

From Fig. 2.5, we could resolve the tangential and normal forces and translate them into $F_{x}$ and $F_{y}$ in the fixed frame co-ordinate as:

$$
\left[\begin{array}{c}
F_{x}  \tag{2.18}\\
F_{y}
\end{array}\right]=\left[\begin{array}{cc}
-\cos \theta & -\sin \theta \\
\sin \theta & -\cos \theta
\end{array}\right]\left[\begin{array}{c}
F_{t} \\
F_{n}
\end{array}\right] .
$$

It has been evident from the discussions earlier in this section that the cutting are not active throughout the process. For a better understanding of the cutting process, we introduce a switching function, which is also called the Heaviside step function, to Eq. (3.2). Therefore, we rewrite the equations for the cutting force components in the $x \& y$ directions on the $p^{t h}$ tooth as follows,

$$
\begin{equation*}
F_{x p}=-g_{p}(t)\left[F_{t p} \cos \theta+F_{n p} \sin \theta\right] \tag{2.19a}
\end{equation*}
$$

$$
\begin{equation*}
F_{y p}=g_{p}(t)\left[F_{t p} \sin \theta-F_{n p} \cos \theta\right] \tag{2.19b}
\end{equation*}
$$

where, $g_{p}$ is the switching function and the subscript $p$ denotes the active teeth; i.e.

$$
g_{p}(t)=\left\{\begin{array}{l}
1, \text { if the } p^{t h} \text { tooth is active }  \tag{2.20}\\
0, \text { if the } p^{t h} \text { tooth is not cutting }
\end{array}\right.
$$

The tangential and normal cutting force components for the $p^{t h}$ tooth, $F_{t p}$ and $F_{n p}$ are considered to be the product of the linearized cutting coefficients, $K_{t}$, and $K_{n}$, the axial depth of cut, $b$, and the instantaneous chip thickness, $w_{p}$. Their expressions are given by

$$
\begin{align*}
F_{t p} & =K_{t} b w_{p}(t)^{\gamma}  \tag{2.21a}\\
F_{n p} & =K_{n} b w_{p}(t)^{\gamma} \tag{2.21b}
\end{align*}
$$

where the superscript, $\gamma$, represents the nonlinear relationship between the cutting force and the feed rate. Within this thesis, the linear relationship is only considered (i.e. $\quad \gamma=1$ ). Thus, the linear model for regeneration is used for the analytic prediction of stability [29].

In the above, a linear cutting force model has been studied which depends on the tool motion at the entry and exit of the cut. However, the cutting forces shown in Eq. (3.4) are considered to be a function of the cutting coefficients, the axial depth of cut and the chip thickness. If more than one cutting tooth is engaged in the cut, the total cutting force along each direction can be written as the summation over the total number of cutting teeth

$$
\left[\begin{array}{l}
F_{x}(x, y, \theta(t), \tau)  \tag{2.22}\\
F_{y}(x, y, \theta(t), \tau)
\end{array}\right]=\sum_{p=1}^{N} g_{p}(t) b\left(h\left[\begin{array}{l}
-F_{t p}(x, y, \theta(t), \tau) \\
-F_{n p}(x, y, \theta(t), \tau)
\end{array}\right]\right) .
$$

Since these expressions couple the cutting forces to the tool motion, they pave way
for stability prediction.

### 2.4 Structural Dynamics of the tool

The experimental study of structural dynamics has provided major contributions to understand many vibration phenomena. Modal analysis is a process whereby a structure is described in terms of its natural characteristics which are frequency, damping and mode shapes, that can be often referred to as the dynamic properties. The natural frequencies and mode shapes occur in all the structures that are designed. Basically, there are characteristics that depend on the mass and stiffness of the structure which set the natural frequencies and the shape distribution of the mode shapes. Understanding the mode shapes and how the system behaves under dynamically loading conditions aids the design engineer in the development of better structures.

Modal analysis is the study of the natural characteristics of structures. It is used to help design all types of structures including automotive and aircraft structures, computers, tennis rackets, to name a few. However, there is no single test or analysis procedure which is best for all cases and so it is very important that a clear objective is defined before any test is undertaken. It is also important that a clear distinction is made between free vibration and forced vibration analysis. Typically, with vibration studies, there are two types of systems associated, they are single degree-of-freedom (SDOF) systems and multiple degree-of-freedom (MDOF) systems. For a SDOF system, a free vibration analysis yields its natural frequency and damping factor, whereas a forced excitation leads to the definition of the frequency response function (FRF) [30]. The FRF is simply the ratio of the output frequency response of
a structure to the positive frequency response of the applied force. The applied force and the response of the structure due to the applied force are measured simultaneously. The response measured can be displacement, velocity or acceleration. Now, the measured time data is transferred from the time domain to the frequency domain using Fast Fourier Transform (FFT) algorithm. Due to this transformation, the function will become complex, i.e:, functions containing real and imaginary components or magnitude and phase components to describe the function.

### 2.4.1 Experimental Modal Testing

This section describes a series of experimental modal tests that were used to obtain math model parameters that describe the experimental system that is discussed in the future sections. A schematic diagram of the experimental setup is shown in Fig. 2.6. Modal testing of the cutting tool was done using a instrumented hammer to impact, and hence excite the structure.

Impact testing is a fast, convenient, and a low cost way of finding the modes of structures and machines. The following equipment is needed to perform an impact test: 1) an impact hammer with a load cell attached to its head to measure the input force; 2) an accelerometer to measure the response acceleration at a fixed point and direction; and 3) a multi channel data acquisition system to compute FRFs.

A PCB model 086C02 impact force hammer with a calibration constant of 83.19 $\mathrm{N} /$ Volt was used to excite the structure. For response measurement, a PCB model 352B10 accelerometer and NIDAQ 6036E national instruments data acquisition system were used for this experiment to measure the FRF. Different sized hammers are required to provide the appropriate impact force, depending on the size of the


Figure 2.6: Schematic diagram of experimental modal testing.
structure; small hammers for small structures, large hammers for large structures. The selection of the hammer tip can have a significant effect on the measurement acquired. The input excitation frequency range is controlled mainly by the hardness of the tip selected $[30,31]$. The tip needs to be selected such that all the modes of interest are excited by the impact force over the frequency range to be considered.

The number of samples per second is called the sampling rate (or sampling frequency), referred to here as SR in units of Hz . The sampling rate directly affects the highest frequency that is analyzed in the computer, conventionally called the Nyquist limit frequency $\left(f_{\text {nyquist }}\right)$, which is exactly equal to one-half the sampling rate. For
most measurements, the frequency band ranges from approximately 20 Hz to 20 KHz and for this experiment, a sample rate of 5 KHz was chosen. The Nyquist frequency can be expressed as

$$
\begin{equation*}
f_{\text {nyquist }}=S R / 2 . \tag{2.23}
\end{equation*}
$$



Figure 2.7: Overlay of the theoretical frequency response (solid line) onto the experimental frequency response (denoted with dots). Graphs are labeled to agree with the math model directions of Fig. 3.4. Estimated modal parameters are given in Table 3.1.

The Frequency Response Function (FRF) is a fundamental measurement that isolates the inherent dynamic properties of a mechanical structure [31,32]. Experimental modal parameters such as natural frequency, damping and mode shapes are obtained from a set of FRF measurements. Fig. 2.7 shows the spectral amplitude as a function of frequency for $x$ - and $y$-directions of the experimental system. Modal tests were performed at the cutting tool tip using a PCB model 086C02 impact
force hammer to excite the structure. For the response measurement, a PCB model 352B10 accelerometer was used. A gradient based optimization method was used in combination with a modal parameter fitting scheme to obtain the modal parameters that are listed in Table 3.1.

| Parameters | $m_{x, y}(K g)$ | $c_{x, y}(N s / m)$ | $k_{x, y}(N / m)$ |
| :---: | :---: | :---: | :---: |
| X-direction | 0.0372 | 2.2841 | 8.51 e 5 |
| Y-direction | 0.0398 | 0.9948 | 8.94 e 5 |

Table 2.1: Modal parameters for asymmetry study.

### 2.5 Summary and Conclusions

In this chapter, cutting force model for a two degree of freedom system was presented. The cutting force expressions are correlated to the math model obtained from the experimental modal parameters. In addition, the structural dynamics of the tool are also studied which contributes to the understanding of many vibration phenomena.

## Chapter 3

## Study of subcritical bifurcations in milling

### 3.1 Introduction

Large-amplitude vibrations known as chatter are a primary impediment to improving the productivity of machining processes [2]. The recognized mechanism for chatter vibrations is known to be the relative motions between a cutting tool and workpiece system. More specifically, the relative motions can cause chatter due to self-excitation of the system's structural modes [29]. Unless avoided, these regions of large-amplitude chatter vibration may cause excessive dynamic loads on the machine spindle and table structure, damage to the cutting tool, and a poor surface finish.

The most common approach for modeling the cutting forces for machining processes is to write the forces as being functions of the uncut chip area. When these expression(s) include the tool motions, which couples the tool motions to the
cutting forces, the model is said to become a non-ideal energy source. Models using delay equations to describe the relative motion between consecutive passages of the cutting tool first appeared in the works of Koenigsberger and Tlusty [4] and Tobias [33]. These authors pioneered the stability analysis of regenerative chatter in the early 1960s. A particularly important result from these analyses was the ability to identify stable cutting regions in which relatively large material removal rates could be obtained by cutting at higher spindle speeds. Following these works, a key strategy has been to apply analytical and numerical methods to predict parameter domains where instabilities exist.

Stability predictions from many early analysis are only approximate for the case of milling since they rely on the fundamental assumption of continuous cutting. In milling, the cutting forces change direction with tool rotation and cutting is interrupted as each tooth enters and exits the workpiece [34,35]. This requires the cutting force expressions for milling to contain an additional term, such as a Heaviside step function, that will turn on the cutting forces as the tool enters into the cut and then turn them off as the tool exits the workpiece.

The focus of many recent works has been the occurrence of new bifurcation phenomena in interrupted cutting processes [5]. In addition to Neimark-Sacker or secondary Hopf bifurcations, period-doubling bifurcations have been analytically predicted in references [5-11] and confirmed experimentally in references [6-8, 12-17]. Other investigations have shown additional physical mechanisms which may influence the relative oscillations between the tool and the workpiece [18]. For instance, the thermoplastic behavior of chip formation has been reported in reference [19] and frictional effects at the tool chip interface have been examined in the reference [20].

The focus of the current thesis stems from an apparent void in the literature base with regards milling bifurcations. More specifically, several researchers have reported results indicating that the secondary Hopf bifurcations of turning processes are subcritical [21,22]. However, fewer results are available for milling - especially results that provide any substantial experimental evidence. The goal of the current paper is to provide a novel series of experimental results that explore the subcritical nature of milling bifurcations.


Figure 3.1: Schematic shows the stroboscopic map of the chosen states, $X\left(\tau_{n}\right)$ and $X\left(\tau_{n}+\Delta t\right)$. This shows the mapping of the periodic displacement samples to the torus and sectioned to provide a Poincaré map.

The schematic of Fig. 3.1 shows a transverse sectioning of the phase plane that has been mapped to the torus. Here, the sectioning of the continuous flow generates a Poincaré mapping by plotting $X\left(\tau_{n}\right)$ versus $X\left(\tau_{n}+\Delta t\right)$. This corresponds to the stroboscopic sampling of the velocity and displacement. With reference to Fig. 3.1, the stroboscopic map of milling time series can often be used to qualitatively distinguish between chatter-free and a chatter-vibration behavior [35].

The importance of understanding whether the bifurcations are subcritical is explained by the bifurcation diagram of Fig. 3.2. This graph shows the vibration am-
plitudes (or stroboscopic samples) plotted as a function of the axial cutting depth, b. Starting from a location of a small cutting depth and slowly increasing, this parameter will result in periodic motions which will also be free of chatter vibrations. As the bifurcation parameter, the depth of cut, is further increased, the chatter free vibration solution will enter region where multiple stable equilibria coexist - both the chatter-free and a chatter-vibration solution. Towards the end of this region, the tool will reach the bifurcation point which will cause a jump in the response of the tool to the stable chatter-vibration equilibria solution. The amplitude of the chatter vibrations will then continue to grow with an increase in the axial depth. Reversing this process by starting at a relatively larger cutting depth and progressing towards a smaller depth will result in hysteresis near the region of multiple periodic attractors. In essence, the periodic attractor representing chatter vibration is a limit cycle that collapses to a stable chatter-free solution once the bifurcation parameter has decreased beyond the region of multiple attractors. While one can argue that this method of cutting may be atypical, the relevance becomes more clear when the influence of a small perturbation is considered [23]. In particular, small perturbations within the region of coexisting stable attractors can cause the cutting tool to jump back and forth between the regions (the basins of attraction) which delineate the chatter and chatter-free attractors. This very phenomenon is what we explore in the pages that follow.

### 3.2 Description of the experimental apparatus

The experimental system described within this section was developed to capture the desired bifurcation behavior. Milling tests were performed on a three-axis Cincinnati CFV 1050 SI CNC machine (see Fig. 3.3). The procedure for the experiment consisted of up-milling a 1016 mm long aluminum 6061-T6511 workpiece at a radial

## Subcritical bifurcation



Figure 3.2: Representative subcritical bifurcation diagram for a milling process plotted as a function of the cutting depth, $b$. Here, $x(n \tau)$ represents the amplitude of periodic displacement samples found for stable equilibria (solid red line). These stable equilibria are experimentally observable and represent both stable (chatter-free) and stable (chatter-vibration) equilibria. The dotted blue line represents equilibria that may be obtained with numerical continuation, but would not appear in an experimental setting.
immersion of $3 \%$ and feed rate of $\mathrm{h}=0.2032(\mathrm{~mm} /$ tooth $)$. Since the goal was to obtain experimental bifurcation diagrams, which require the quasi-static variation of a control parameter, the axial depth of cut was slowly increased by cutting along an incline as shown in Fig. 3.3. Here, both an incline and decline cutting operation were used to investigate the potential role of hysteresis.

In order to investigate several regions of the parameter space, the spindle speed was changed for each set of incline/decline tests. In addition, for multiple cutting tests, clean up passes were performed prior to every recorded cut to create standard reference for the next pass. A 12.75 (mm) diameter, two flute, solid carbide end mill with a $30^{\circ}$ helix angle was used during all stability tests (see modal parameters in Table 3.1). Tool displacements, measured approximately $5(\mathrm{~mm})$ from the tool tip, were recorded at a sample rate of 25 KHz using a laptop data acquisition system and Lion Precision capacitance probes. A once per revolution timing signal was obtained using a laser tachometer that was created by heat shrinking an aluminum sleeve onto the tool.

To study the dynamic behavior of a particular milling process (i.e. tool, tool holder, and spindle combination), modal parameters of the specific system are required. Modal tests were performed at the cutting tool tip using a PCB model 086C02 impact force hammer to excite the structure. For the response measurement, a PCB model 352B10 accelerometer was used. A gradient based optimization method was used in combination with a modal parameter fitting scheme to obtain the modal parameters that are listed in Table 3.1.

In the section that follow, stability predictions are provided for the experimental system. Here, the governing equations are first described prior to presenting the


Figure 3.3: Schematic diagram of the experimental system that was used to generate experimental bifurcation diagrams. The 1016 (mm) long workpiece was machined along an incline/decline to create depth of cut sweeps.

| Parameters | $m_{x, y}(\mathrm{Kg})$ | $c_{x, y}(N s / m)$ | $k_{x, y}(N / m)$ |
| :---: | :---: | :---: | :---: |
| X-direction | 0.0372 | 2.2841 | 8.51 e 5 |
| Y-direction | 0.0398 | 0.9948 | 8.94 e 5 |

Table 3.1: Experimentally identified modal parameters for the cutting tool.
results of the stability analysis.

### 3.3 Math Model for Milling

During a machining process, large forces are exerted between the cutting tool and the workpiece. This can often result in undesirable vibrations that may damage the tool or the workpiece. In this section, the model for a two degree of freedom milling process is derived. In addition, this section provides a brief explanation for the dynamic forces in milling which can lead to instability. In this section, we apply a model that is in agreement with the prior works of references $[10,12,14,29,36]$. However, a brief derivation is provided for completion. Those familiar with milling process may wish to proceed to Section 3.4.

### 3.3.1 Cutting Force Expression in Milling

The schematics of Fig. 3.4 show a typical up-milling and down-milling operation and provide illustrations of the directional cutting forces for a milling process. This diagram also shows that milling is an interrupted cutting process - meaning that the tool enters and exits the workpiece during cutting. Therefore, the resulting math model must account for the fact that the tool has a finite entry and exit angle. Although the authors acknowledge that the shape of the chip will be trochoidal [28], the results that follow use the common approximation of a circular tool path.


Figure 3.4: Schematic diagrams of the math models for: (a) up-milling; and (b) down-milling processes. The directional cutting forces are defined as $F_{t}$ and $F_{n}$ in the tangential and normal directions. The term $F_{c}$ is used to describe the resultant cutting force.

The instantaneous radial chip thickness, $w_{p}$, on the $p^{\text {th }}$ tooth can be approximated as

$$
\begin{equation*}
w_{p}(t)=h \sin \theta_{p}(t)+[x(t)-x(t-\tau)] \sin \theta_{p}(t)+[y(t)-y(t-\tau)] \cos \theta_{p}(t), \tag{3.1}
\end{equation*}
$$

where $h$ is the feed per tooth, $\theta$ is the cutter rotation angle, $\tau=\frac{60}{N \Omega}$, is the tooth pass period, and $\Omega$, is the spindle speed. The number of cutting teeth is denoted by $N$, and $x(t)$ and $y(t)$ are the tool motions along the respective directions.

The radial chip thickness can be obtained by projecting the difference between the current tool displacement, described by $x(t)$ and $y(t)$, and the displacements of the cutting tool in the previous tooth passage, $x(t-\tau)$ and $y(t-\tau)$, onto the radial direction. This represents the dynamic part of the chip thickness expressed in Eq.(3.1) [26].

In order to obtain the math model for milling, the forces in the tangential and normal components must be transformed from the rotating reference frame of the tool into the fixed frame. The tangential and normal forces $F_{t}$ and $F_{n}$ can be resolved and transformed into $F_{x}$ and $F_{y}$ in the fixed frame as follows

$$
\left[\begin{array}{l}
F_{x}  \tag{3.2}\\
F_{y}
\end{array}\right]=\left[\begin{array}{cc}
-\cos \theta & -\sin \theta \\
\sin \theta & -\cos \theta
\end{array}\right]\left[\begin{array}{l}
F_{t} \\
F_{n}
\end{array}\right] .
$$

The tangential and normal cutting force components for the $p^{t h}$ tooth, $F_{t p}$ and $F_{n p}$ are considered to be the product of the linearized cutting coefficients, $K_{t}$, and $K_{n}$, the axial depth of cut, $b$, and the instantaneous chip thickness, $w_{p}$. Their expressions are given by

$$
\begin{align*}
& F_{t p}=K_{t} b w_{p}(t)^{\gamma}  \tag{3.3a}\\
& F_{n p}=K_{n} b w_{p}(t)^{\gamma} \tag{3.3b}
\end{align*}
$$

where the superscript, $\gamma$, represents the nonlinear relationship between the cutting force and the feed rate. Within this paper, the linear relationship is only considered (i.e. $\gamma=1$ ). Thus, the linear model for regeneration is used for the analytic prediction of stability [29].

In the above, a linear cutting force model has been studied which depends on the tool motion at the entry and exit of the cut. However, the cutting forces shown in Eq. 3.4 are considered to be a function of the cutting pressures, the axial depth of
cut and the chip thickness. If more than one cutting tooth is engaged in the cut, the total cutting force along each direction can be written as the summation over the total number of cutting teeth

$$
\left[\begin{array}{l}
F_{x}(x, y, \theta(t), \tau)  \tag{3.4}\\
F_{y}(x, y, \theta(t), \tau)
\end{array}\right]=\sum_{p=1}^{N} g_{p}(t) b\left(h\left[\begin{array}{l}
-F_{t p}(x, y, \theta(t), \tau) \\
-F_{n p}(x, y, \theta(t), \tau)
\end{array}\right]\right) .
$$

The switching function, $g_{p}(t)$, acts as a square wave to provide a discontinuity in the cutting forces by assuming a value of one when the $p^{t h}$ tooth is active and zero when it is not cutting. These expressions for the cutting forces pave way to the prediction of the local stability.

For the analysis of a tool with a non-zero helix angle, there exists different force expressions for each cutting situation, i.e. entry, middle and exit, and each case [26]. The cutting situation is important because the depth of cut will depend on the rotation position. Also, the different cases depend on whether or not the tool fully enters the cut before exiting. More details are provided by Edes [37] in addition to the force expression derivation.

### 3.3.2 Modal Equation of Motion

Combining the cutting forces of the previous section with the math model for the structural dynamics results in the final governing equation. Assuming either a compliant tool or a compliant structure, a summation of forces gives the following equation of motion for a two degree of freedom system

$$
\left[\begin{array}{cc}
m_{x} & 0  \tag{3.5}\\
0 & m_{y}
\end{array}\right]\left[\begin{array}{l}
\ddot{x}(t) \\
\ddot{y}(t)
\end{array}\right]+\left[\begin{array}{cc}
c_{x} & 0 \\
0 & c_{y}
\end{array}\right]\left[\begin{array}{l}
\dot{x}(t) \\
\dot{y}(t)
\end{array}\right]+\left[\begin{array}{cc}
k_{x} & 0 \\
0 & k_{y}
\end{array}\right]\left[\begin{array}{l}
x(t) \\
y(t)
\end{array}\right]=\left[\begin{array}{l}
F_{x}(x, y, \theta(t), \tau) \\
F_{y}(x, y, \theta(t), \tau)
\end{array}\right]
$$

where the terms $m_{x, y}, c_{x, y}$, and $k_{x, y}$ are the modal mass, damping, and stiffness in the respective directions of the system. The equations for the milling process can be expressed in more compact form, here the consolidated force expression becomes

$$
\begin{align*}
{\left[\begin{array}{c}
F_{x}(x, y, \theta(t), \tau) \\
F_{y}(x, y, \theta(t), \tau)
\end{array}\right] } & =\sum_{p=1}^{N} g_{p}(t) b\left(h\left[\begin{array}{cc}
-K_{t} s c & -K_{n} s^{2} \\
K_{t} s^{2} & -K_{n} s c
\end{array}\right]+\right. \\
& {\left.\left[\begin{array}{ll}
K_{t} s c-K_{n} s^{2} & -K_{t} c^{2}-K_{n} s c \\
K_{t} s^{2}-K_{n} s c & K_{t} s c-K_{n} c^{2}
\end{array}\right]\left[\begin{array}{l}
x(t)-x(t-\tau) \\
y(t)-y(t-\tau
\end{array}\right]\right), } \tag{3.6}
\end{align*}
$$

where $s=\sin \theta(t), c=\cos \theta(t), N$ is the number of teeth, and $\tau$ is the time delay between consecutive tooth passages. Eq.(3.5) can now be rewritten as

$$
\begin{equation*}
\mathbf{M} \ddot{\vec{X}}(t)+\mathbf{C} \dot{\vec{X}}(t)+\mathbf{K} \vec{X}(t)=\mathbf{K}_{c}(t) b[\vec{X}(t)-\vec{X}(t-\tau)]+\vec{f}_{0}(t) b \tag{3.7}
\end{equation*}
$$

where $\vec{X}(t)=[x(t), y(t)]^{\top}$ is the position vector and $\mathbf{M}, \mathbf{C}$, and $\mathbf{K}$ are the modal mass, damping, and stiffness matrices. The terms $\mathbf{K}_{c}(t)$ and $\vec{f}_{0}(t)$ are compact notation for

$$
\begin{align*}
& \mathbf{K}_{c}(t)=\sum_{p=1}^{N} g_{p}(t)\left[\begin{array}{cc}
-K_{t} s c-K_{n} s^{2} & -K_{t} c^{2}-K_{n} s c \\
K_{t} s^{2}-K_{n} s c & K_{t} s c-K_{n} c^{2}
\end{array}\right],  \tag{3.8a}\\
& \vec{f}_{0}(t)=\sum_{p=1}^{N} g_{p}(t) h\left[\begin{array}{r}
-K_{t} s c-K_{n} s^{2} \\
K_{t} s^{2}-K_{n} s c
\end{array}\right], \tag{3.8b}
\end{align*}
$$

as noted in [27].

To analyze the dynamics of a milling process, it is essential to study the stability behavior of the system. The section that follows will discuss the asymptotic stability borders from the cutting force models derived in this section. In an effort to focus on the experimental observations, the analytical details of solving the governing equations are omitted. Instead, the authors cite the appropriate references.

### 3.4 Stability Prediction

The local stability of the milling process can be obtained using the temporal finite element analysis (TFEA). This method can be used to analytically predict the stability of a milling process. While details of this solution technique have been omitted for the sake of brevity, the interested reader can consult references $[8,11,12,17,27,29,38]$. The stability of the dynamic map equation and the system it describes is determined from the eigenvalues of the transition matrix $\mathbf{Q}=\mathbf{A}^{-1} \mathbf{B}[11,12,14,39]$. If the magnitude of any eigenvalue is greater than one for a given spindle speed $(\Omega)$ and depth of cut (b), the milling process is considered unstable. Two distinct types of instability are illustrated by eigenvalue trajectories in the complex plane: 1) a flip bifurcation or period-doubling phenomenon occurs when a negative real eigenvalue passes through the unit circle; and 2) a Hopf bifurcation occurs when a complex eigenvalue obtains a magnitude greater than one $[11,12,14,39]$.

Figure 3.5 shows the stable and unstable cutting regions used in the experiments as a function of spindle speed and depth of cut. The dots in this figure represents the experimental cases described in Section 3.5. The parameters chosen for stability predictions are those listed in Table 3.1. The applied values of cutting coefficients, $K_{t}$


Figure 3.5: Stability chart for the studied experimental system predicted from characteristic multiplier magnitudes. Figure shows the stability predictions from the modal parameters listed in Table 3.1. The dots in this figure represents the experimental cases described in Section 3.5. The experimental cases described in the next section are for $\Omega$ values of $13500,14000,15314$, and 15350 (rpm) with depth of cut values varying from $0-3(\mathrm{~mm})$.
and $K_{n}$ were chosen as $5.36 \times 10^{8}\left(\mathrm{~N} / \mathrm{m}^{2}\right)$, and $1.87 \times 10^{8}\left(\mathrm{~N} / \mathrm{m}^{2}\right)$, respectively [40]. The type of instabilities that occur by characteristic multiplier (CM) trajectories in the complex plane are illustrated in Fig. 3.6. Two unstable behaviors such as Hopf bifurcation and flip bifurcation are shown for different experimental cases. The top two graphs (i.e., $a$, and $b$ ) showing the eigenvalues projecting out of the unit circle is plotted for a spindle speed of $\Omega=13,500(\mathrm{rpm})$, and $\Omega=14,000$ (rpm), respectively with the depth of cut varying from $0-3(\mathrm{~mm})$. Cases $c$, and $d$ are plotted for values of $\Omega=15,314$ (rpm), and $\Omega=15,350(\mathrm{rpm})$, respectively. For cases $c$, and $d$, even though it shows a flip bifurcation or period doubling bifurcation in the complex plane, the experimental results that follow could not capture these effects. The authors suspect that this could be due to the existence of either stochastic variations in the cutting forces or the chip dynamics which were not taken into account to determine stability. Another possible reason could be the effect of cutter run-out in a milling process which had appeared in the data making it difficult to distinguish from perioddoubling behavior.

### 3.5 Experimental and Numerical Observations

This section describes the results of the experimental cutting tests performed to investigate the bifurcation behavior of the milling process. Experimental bifurcation diagrams were created for different cutting conditions to investigate the presence of hysteresis and the presence of multiple periodic attractors. Measurement responses obtained from the experiments were used to create the bifurcation diagrams and Poincaré sections that provide insight into the qualitative behavior of the system.


Figure 3.6: Characteristic multiplier magnitudes plots for the experimental system discussed in Section 3.2. Different unstable parameter combinations penetrates the unit circle in the complex plane: (a and b) characteristic multiplier trajectories for a secondary Hopf bifurcation for spindle speed values of $\Omega=13,500(\mathrm{rpm})$, and $\Omega=$ $14,000(\mathrm{rpm})$, respectively; and (c and d) characteristic multiplier trajectories for a flip bifurcation for spindle speed values of $\Omega=15,314(\mathrm{rpm})$, and $\Omega=15,350(\mathrm{rpm})$, respectively with the depth of cut varying from $0-3$ (mm).

### 3.5.1 Experimental Observation and Data

Bifurcation studies involve the qualitative change of the system behavior under the quasi-static variation of a control parameter. Figure 3.7 shows the experimental vibration signals measured at a spindle speed of 13500 (rpm). To capture a once per revolution timing signal, a laser tachometer was used to sense a black-white transition on the rotating tool holder. Figure 3.7 (a) shows a zoomed view of the tachometer signal, where each square wave represents one cutter revolution. The black dot denote the once per revolution timing pulse. Cases (b) and (c) show the raw displacement signal in the $x$ - and $y$-directions. These signals were periodically sampled at the tooth passage frequency to create the Poincaré sections and bifurcation diagrams that follow (see Figs. 3.8 and 3.10).

In Fig. 3.8, a secondary Hopf or Neimark-Sacker bifurcation was observed in the experimental measurements for both forward and reverse sweep cases. This graph was generated by varying the axial depth of cut as a control parameter during the experimental investigation. Results from these experiments verify the presence of hysteresis. The two dimensional circle maps shown in Fig. 3.10 provides an insight on the criterion used to locate the bifurcation points. This schematic illustrates different snapshots in time of the bifurcation diagram of Fig. 3.8, progressing from a larger cutting depth to a smaller one. It is apparent that the once per period samples begin to slowly vary from a chatter-vibration solution to a chatter-free solution (see Fig. 3.10). Also, from visualization it could be noted that the forward sweep starts at a zero depth with the onset of chatter vibration at approximately $0.55(\mathrm{~mm})$. Whereas, in case (b) (i.e. the reverse sweep), the cutting process starts at a larger depth which is already in the unstable region and becomes stable at around 0.4 (mm). This accounts to the fact that starting from a smaller cutting depth and slowly increasing the control parameter will result in a single stable equilibria solution which will be


Figure 3.7: Experimental signals acquired during the experiment: a) Laser tachometer, b) and c) capacitance probe measurement in the $x$ - and $y$-directions. In case (a), the measured response from the tachometer was used to capture a once per revolution timing signal, where each square wave represents one cutter revolution and the black dot denotes once per revolution timing pulse. Whereas, cases (b) and (c), the raw displacement signal in the $x-$ and $y$-directions are acquired using the capacitance probes. These experimental results are obtained for an down-milling case at a spindle speed of 13500 (rpm) and varying depth of cut, recorded for a forward sweep.
a chatter-free solution (see Fig. 3.2). However, if the control parameter is increased further, the chatter-free solution (trivial solution) looses stability giving way to small limit cycles [12]. Consequently, the system response is attracted to a large-amplitude limit cycle $[12,41]$. This phenomena is usually referred to as a subcritical bifurcation. Subcritical bifurcation instabilities are one of the most difficult nonlinear phenomena to deal with because even a small perturbation can cause the system to jump from a small to large amplitude vibration [42].


Figure 3.8: Down-milling bifurcation diagram for the experimental system described in Section 3.2. The periodic samples in the $x$-direction, labeled as $X_{n}$, for a cutting speed, $\Omega=13,500(\mathrm{rpm})$. The axial depth of cut, b , which varied from $0-3(\mathrm{~mm})$ was used as the control parameter. The bifurcation points in the case of a forward sweep is approximately $0.55(\mathrm{~mm})$, whereas, in case (b), it is close to $0.4(\mathrm{~mm})$. This predicts subtle effect of hysteresis in the bifurcation diagrams when comparing the a) forward sweep; and b) reverse sweep cases.

Repeated experimental tests were conducted to ensure the consistency in the variation of the bifurcation parameter to obtain the same results for both forward and reverse sweeps. Although, the authors claim the effect of hysteresis in Fig. 3.8, one
could argue that it is not very prominent. Figure 3.9 presents another down-milling experimental result where there effect of hysteresis are very subtle. However, there are several other experimental tests which show a more dominant hysteresis effect. These experimental cases will be discussed further after presenting some Poincaré sections from the results of Fig. 3.8.


Figure 3.9: Experimental bifurcation diagram for a spindle speed of $\Omega=14,000 \mathrm{rpm}$. The axial depth of cut, $b$, was kept as the bifurcation parameter in this case as well, which varied from $0-3(\mathrm{~mm})$. The periodic samples in the $x$-direction are labeled as $X_{n}$, which shows forward and reverse sweep cases. This result was obtained for a down-milling case where there the hysteresis is effect is not very prominent.

Mappings are frequently invoked as models to analyze the qualitative behavior of Poincaré sections. Some illustrations of this are shown in Fig. 3.10, which presents a two-dimensional circle map (Poincaré section) of the bifurcation diagram shown in Fig. 3.8. Six different snapshots in time were used from the bifurcation diagram of Fig. 3.8 (a) to create the Poincaré sections of Fig. 3.10. In the first diagram (case a), the Poincaré section was plotted between $b$-values of 2.7 and 2.8 (mm). For the other unstable cases (i.e., b, c, and d), the Poincaré sections capturing the depths
between 2.3 and $2.4(\mathrm{~mm}), 1.7$ and $1.8(\mathrm{~mm})$, and 1.2 and $1.3(\mathrm{~mm})$, respectively were used. Case $f$ is a snapshot in the stable cutting region which shows a stable attractor capturing the depths between 0.2 and $0.3(\mathrm{~mm})$. These are snapshots in time of the bifurcation diagram which begin to grow as the control parameter is slowly increased and jumps to another final attractor. Case $e$ is particularly interesting because the circle maps are plotted in the transition region; i.e. from a stable region to an unstable region. Here, it is apparent that the system grows in the center and starts to spiral out to another attractor as the control parameter is slowly increased. Mapping of this diagram was captured between depth values of 0.5 and $0.6(\mathrm{~mm})$.


Figure 3.10: Poincaré sections plotted for the bifurcation digram of Fig. 3.8. Cases $a-d$ show the slices for an unstable secondary Hopf bifurcation, case $e$ shows the transition from a stable region to an unstable region, and case $f$ shows the mapping in the stable region. Six snapshots were randomly chosen with equal intervals in between to represent once per tooth passage samples. The range for different cases ( $a$ $-f$ respectively) are: (a) $2.7-2.8(\mathrm{~mm})$; (b) $2.3-2.4(\mathrm{~mm})$; (c) $1.7-1.8(\mathrm{~mm})$; (d) $1.2-1.3(\mathrm{~mm}) ;(\mathrm{e}) 0.5-0.6(\mathrm{~mm})$; and (f) $0.2-0.3(\mathrm{~mm})$.

The experimental results for two additional cases are shown in Figs. 3.11 and 3.12. The parameter combinations used to create the bifurcation diagram in Fig. 3.11 was chosen to be, $\Omega=15,350$ (rpm) where the axial depth of cut was varied from $0-$ $3(\mathrm{~mm})$. The effect of hysteresis in this figure, is much more dominant than the bifurcation diagrams of Figs. 3.8, and 3.9, even though the exact location of the bifurcation point is not obvious. In particular, the reverse sweep case (see Fig. 3.11 (b)) shows sensitivity to small perturbations resulting in a switching between the chatter-free and chatter-vibration solutions. In this case, it is quite apparent that the onset of chatter begins at a cutting depth of approximately $1(\mathrm{~mm})$, but as the control parameter increases further, it starts to move back and forth between solutions and increases above a certain value giving birth to a Hopf bifurcation and leading to a limit cycle [43].


Figure 3.11: Experimental bifurcation diagram for a different parameter combination is shown. The periodic samples in the $x$-direction, labeled as $X_{n}$, for a cutting speed, $\Omega=15,350(\mathrm{rpm})$. The axial depth of cut, b , which varied from $0-3(\mathrm{~mm})$ was used as the control parameter. This predicts dominant effects of hysteresis when comparing the a) forward sweep; and b) reverse sweep cases.

Another interesting result which shows multiple attractors is shown in the bifurcation diagram of Fig. 3.12. This diagram was obtained by varying the control parameter from $0-3(\mathrm{~mm})$ with a spindle speed, $\Omega=15,314$ (rpm). It can be seen that, in the forward sweep as the bifurcation parameter is slowly increased, the chatter-free solution enters a region where multiple solutions coexist. On the other hand, reversing this process will result in hysteresis near the region of the multiple stable equilibria. At this point, the relevance become more obvious that there exist a small domain of attraction which is very sensitive to small perturbations. These results provide experimental evidence of a sensitivity to small perturbations and the coexistence of multiple attractors for certain parameter combinations.


Figure 3.12: Experimental bifurcation diagram for a spindle speed of $\Omega=15,314$ (rpm). The axial depth of cut, b , which varied from $0-3$ (mm) was used as the control parameter. The periodic samples in the $x$-direction are labeled as $X_{n}$, which shows forward and reverse sweep cases with dominant effects of hysteresis.

### 3.5.2 Numerical Investigation

Since the motion dependent discontinuity of chatter vibrations must account for the cutting tool jumping out of the cut, numerical simulation was used to study the milling process bifurcation behavior. This sub-section embarks a brief overview of the numerical strategies used for solving time delay equations in milling. Since numerical methods for delay differential equations (DDE) are intended for problems with solutions that have several continuous derivatives, discontinuity in lower order derivatives is also a major issue that requires special attention. The analystical solutions of DDE suffer discontinuity which affect the numerical methods employed to solve them [44-46]. In Fig. 3.13 the bifurcation diagram is captured numerically for the down-milling experimental case illustrated in Fig. 3.8. This shows presence of hysteresis in the bifurcation diagrams for both forward and reverse sweep cases.


Figure 3.13: Numerical bifurcation diagrams for the experimental system for a cutting speed, $\Omega=13,500(\mathrm{rpm})$ and varying depth of cut $(b=0-3(\mathrm{~mm}))$. The model illustrates hysteresis in the bifurcation diagrams for forward and reverse sweeps.

Figure 3.14 is obtained by numerical simulation to capture the experimental bi-
furcation behavior shown in Fig. 3.9. In this numerical study, the nonlinearity in the cutting depth variation is not considered. Also, motion dependent discontinuity of the tool is another potential application to the numerical investigation which requires further study.


Figure 3.14: Numerical bifurcation diagrams for the experimental system for a cutting speed, $\Omega=14,000(\mathrm{rpm})$ and varying depth of cut $(b=0-3(\mathrm{~mm}))$. The model illustrates hysteresis in the bifurcation diagrams for forward and reverse sweeps.

### 3.6 Summary

In this chapter, the nonlinear behavior of a highly interrupted cutting process which illustrates secondary Hopf and flip bifurcations were discussed. In particular, a two degree of freedom fixed frame model for low radial immersion up-milling was experimentally verified to investigate the subcritical nature of bifurcation behavior. Stability predictions were carried out using the temporal finite element method which forms an approximate solution by dividing the time in the cut to a finite number of elements. The characteristic multipliers (CM) are used to determine the
stability of the system. The stability condition requires that the CM be less than one for given spindle speed and depth of cut.

At the time of this research, the authors believe the experiments reported within this thesis to be the first experimental bifurcation diagrams for milling data. This served as the motivation for the current study. Our conclusions are that the data shows a sensitivity to small perturbations which may result in a switching between multiple equlibria solutions. Furthermore, the experimental observations provide interesting results of hysteresis in the bifurcation diagrams (see Figs. 3.11 and 3.12). However, there are only subtle effects for some parameter combinations (see Fig. 3.8). Lastly, the authors hope that the presented experimental results will act as motivating examples for future theoretical investigations.

## Chapter 4

## Summary

### 4.1 Completed work summary

The importance of understanding whether the bifurcations are subcritical was the main focus of this thesis. As the bifurcation parameter, is further increased, the chatter-free equilibria solution will enter a region where multiple stable equilibria coexist - both the chatter-free and a chatter-vibration solution. Towards the end of this region, the tool will reach the bifurcation point which will cause a jump in the response of the tool to the stable chatter-vibration equilibria solution. The amplitude of the chatter vibrations will then continue to grow with an increase in the axial depth. Reversing this process by starting at a relatively larger cutting depth and progressing towards a smaller depth will result in hysteresis near the region of the multiple stable equilibria. In essence, the stable equilibria representing chatter vibration are a limit cycle that collapses to a stable chatter-free solution once the bifurcation parameter has decreased beyond the region where the multiple attractors exist. While one can argue that this method of cutting may be atypical, the relevance becomes more clear when the influence of a small perturbation is considered [23]. In particular, small
perturbations within the region of coexisting stable attractors can cause the cutting tool to jump back and forth between the regions (the basins of attraction) which delineate the chatter and chatter-free equilibria.

### 4.2 Primary conclusions from thesis

In this thesis, the nonlinear behavior of a highly interrupted cutting process which illustrates secondary Hopf and flip bifurcations were discussed. In particular, a two degree of freedom fixed frame model for low radial immersion up-milling was experimentally and numerically verified to investigate the subcritical nature of bifurcation behavior. Stability predictions were carried out using the temporal finite element method which forms an approximate solution by dividing the time in the cut to a finite number of elements. The characteristic multipliers (CM) are used to determine the stability of the system. The stability condition requires that the CM be less than one for given spindle speed and depth of cut.

At this time, we believe that the experiments reported within this thesis to be the first experimental bifurcation diagrams for milling data. This served as the motivation for the current study. The main conclusions are that the experimental data shows a sensitivity to small perturbations which may result in a switching between multiple equlibria solutions. Furthermore, the experimental observations provide interesting results of hysteresis in the bifurcation diagrams. However, there are only subtle effects for some parameter combinations. Lastly, the hope is that the presented experimental results will act as motivating examples for future theoretical investigations.

### 4.3 Theoretical subcritical nature predictions

From engineering viewpoint, one of the most dangerous dynamic phenomena is the presence of an unstable limit cycle in the phase space. Engineers usually design machines and systems with linear methods. If a desired motion or stationary state is linearly stable, it may still have only a small domain of attraction, that is, it may still be sensitive for perturbations if otherwise undetectable unstable oscillations which are close to the desired states. In this thesis, the experimental investigation of this phenomena is addressed with maximum efficiency. However, there are several other factors which also needs to be considered in the future studies. Apart from experimental and numerical studies, theoretical predictions also plays an important role for this type of behavior. Researchers have addressed the theory in machining processes such as turning as reported by [21, 22]. But, further theoretical background is necessary in milling to completely support the other set of studies.

### 4.4 Role of run-out and stochastic variations in cutting forces

### 4.4.1 Effects of run-out

Cutter run-out is a common phenomenon affecting the cutting performances in milling operations. It is a common problem in most interrupted machining operations. It may arise from a number of sources such as cutter axis offset, cutter axis tilt, errors in the grinding of the cutter, lack of dynamic balancing, irregularities in the cutter pockets, insert size, or the setting of the inserts. Run-out remains an important issue in machining because commercially-available cutter bodies often
exhibit significant variation in the teeth radial locations; therefore, the chip load on the individual cutting teeth varies periodically [47]. This varying chip load influences the machining process and can lead to premature failure of the cutting edges. Several previous run-out studies are available in the literature, interested reader may consult references [48].

It has previously been shown in many literatures such as references [47-49], that run-out creates a greater problem for the dimensional accuracy of parts created by a milling process. It appears that run-out also has a more significant effect on the surface quality of end milled parts. It influences the machined surface accuracy and tool life greatly [26]. The presence of cutter run-out causes chip load to vary over the rotation of a multi-tooth cutter. This varying chip load will alter the average forces, peak forces and instantaneous force profile to varying degrees depending on the cutting conditions, cut geometry and degree and nature of the run-out. This will, of course, impact the problems of cutter breakage, cutter wear, the surface error generation mechanism and the dynamic behavior of the machine tool and cutting process. The surface roughness traces reveal large peak to valley variations with a period of twice the chip load [49]. This means that one of the two cutting edges on the tool creates a deeper cut than the other. So, it is important that improving machine tool run-out will have a very significant effect on the surface quality of end-milled parts. In an effort to minimize the role of run-out in the cutting process, we hope to implement an experimental and numerical study as a part of the future research.

### 4.4.2 Some future work

The role of perturbation methods and bifurcation theory in predicting stability and complicated dynamics of machining has become an important area of research in
the recent years. Initial investigations found in literature $[43,50]$ which followed linear theory to predict stability where the bifurcation parameter, the cutting depth, which increases above a certain value giving birth to a Hopf bifurcation and leading to a limit cycle [43]. As the cutting depth is further increased, it results large-amplitude limit cycles, hysteresis, multiple attractors, and subcritical instability. These periodic solutions undergo a secondary Hopf bifurcation, leading to a two-period quasi-periodic motion (two-torus) [51]. As the bifurcation parameter increases furthermore, the torus breaks down, resulting in chaotic motion.

Manufacturing engineering needs accurate predictions of machine-tool performance to select operating parameters, such as spindle speed, tool feed, etc., that lead to the best possible part created in the most efficient manner. In order to provide that, a perturbation method may be employed to the future research to reveal the dynamics of self-excited oscillations that occur during a single-point machining of metals. This phenomenon which is commonly known as chatter-vibrations, is a dynamic instability of inherent in the cutting process. Chatter-vibrations limits the operating regions available to the machine-tool user, since combinations of spindle speeds and feeds that makes unstable cuts must be avoided to prevent damage to the machine tool, the workpiece, or the operator. Vibrations of any kind in the machine-tool system can cause poor surface finish, introducing machining errors. Chatter may also increase the dynamic loading of the machine-tool structure where the tool break or the workpiece is damaged beyond salvage.

As a part of future research, we hope to perform experimental investigations where perturbations are induced to excite the system under study. With the current knowledge and skills in this field, we hope to minimize the effect of dynamic instabilities. Sufficient theoretical and numerical investigations are also included in the future in-
vestigations to support the experimental findings.

### 4.5 Revised stability diagrams with nonlinear effects incorporated

Since the primary intend of a stability chart is to convey the regions of chatter-free cutting, the authors believe that the including the regions of multiple attractors would be a beneficial addition to the asymptotic stability charts. The stability criteria discussed in this thesis does not account for the cutter run-out and the possible stochastic variations in the cutting forces or the chip dynamics. Stable cutting tests were determined with respect to the tool position and velocity at instances separated by tooth passage periods converging to a constant value. Considering the fact that the stochastic variation in the cutting forces, a revised stability chart is needed to address the problem.

Figure 4.1 shows the stability diagram which qualitatively accounts for the effect of stochastic variations in the cutting forces. The special zone in between the lobes is were the machining operation should be done with care. While performing cutting tests in this region, it is possible that even a small perturbation could result in a jump between chatter-free and chatter-vibration solution. Future investigations will also include the chip dynamics in the stability diagrams and also would consider for the run-out effects.


Figure 4.1: Schematic diagram of the revised stability chart with nonlinear effects incorporated. This qualitative analysis provide further insight into the effects of stochastic components in the cutting forces. The zone between the two stability lobes is where is future investigation plays an important role in determining stable cutting borders.

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