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ECONOMIC ANALYSIS OF POTATO, CORN AND WHEAT RESPONSE TO NITROGEN AND PHOSPHORUS APPLICATION IN THE HIGHLANDS OF ECUADOR

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CONTENTS

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CHAPTER I

INTRODUCTION AND OBJECTIVES

Geographical Characteristics of Ecuador

Ecuador, a country roughly the size of the state of Colorado, lies on the northwestern coast of South America, between Colombia and Peru. Its total land area is estimated at about 271,950 square kilometers. Its territory includes Continental Ecuador and the Archipelago of Colon (the Galapagos Islands). Continental Ecuador, as its name indicates, extends north and south of the equator and has three types of climates—tropical, subtropical, and temperate.

Ecuador is divided into three distinct regions by the Andes which span the country from north to south. The fertile Pacific coastal plain, or the costa, produces the principal export crops (bananas, coffee, cocoa, beans, sugar and rice) as well as other products mostly for domestic consumption (tropical fruits, livestock products, cotton and tobacco). The highland or Sierra, consists of the eastern and western cordillera (chains) of the Andes, and the valleys between. In this temperate zone the main products are potatoes, corn, wheat, vegetables, fruit and livestock products. The area known as the Oriente lies east of the Andes and slopes gently downward towards the Amazon River Basin. This region is largely underdeveloped with cattle raising being of some importance. In the early 70's petroleum was discovered in this region which is expected to help in its development. The Galapagos Islands are relatively unimportant for agricultural purposes.

Rainfall in Ecuador varies widely by region and season. In the costa region the rainy season is from December through May, with annual precipitation varying from 12 inches in Ancon to 130 inches in Bucay, with some locations in the northeast receiving in excess of 150 inches. In the Sierra, rainfall occurs mostly between October and May. It varies from 15 to 50 inches, but is more evenly distributed than in the costa. In the Oriente, annual rainfall of more than 120 inches is distributed evenly over the entire year. Soil fertility also varies greatly, ranging from the rich soils in the Costa and some valleys of the Sierra, to the eroded hillsides in the Andes (6).

The Ecuadorian agricultural sector contributes substantially to the country's export earnings, food requirements and employment. Until the early 70's and before petroleum was discovered, agriculture accounted for 90 percent of Ecuador's export earnings. These export crops were principally bananas, cocoa, coffee and sugar. Currently domestic agricultural output supplies about 85 percent of the country's food needs and many of the agricultural products required for industry. In addition, crop and livestock production, forestry and fishing provide the livelihood for over 50 percent of the economically-active population (8).

Problem Statement

Recently there has been increased interest among developing countries, such as Ecuador, in increasing fertilizer usage as a means to increase output. In the past, these countries have been characterized as relatively low users of agriculture fertilizer. However, the recent discovery of petroleum in Ecuador, along with the construction of a government-controlled refinery, and a well-defined policy with regard to fertilizer use, the level of fertilizer usage is expected to increase rapidly in the future. As a result, research is needed so that recommendations on the most profitable use of this factor of production can be made.

Ecuadorian farmers are in need of information showing how much and what kinds of fertilizer to use to maximize profits. Agricultural extension workers and others alike, who provide production planning advice to farmers operating under widely varying conditions, recognize the need for information on the economic aspects of fertilizer use. This kind of information is also needed by government officials when making estimates on the total amount of fertilizer needed under varying agricultural policies. Researchers are thus being called upon to conduct the research necessary to answer agro-economic questions basic to development of practical and economically efficient fertilizer recommendations.

This study deals with the highlands natural region; a region where potatoes, corn and wheat are the principal crops planted. Data on the total amount planted and production for each respective crop for the last ten years are presented in Table 1. It is estimated that potatoes, corn and wheat generate approximately 64 per cent of the total agricultural employment in the highlands (3). These crops are also the principal components of the daily diet of the Ecuadorian people. In addition, a considerable number of peasant families depend upon the income forthcoming from the annual sale of these crops.

Objectives

1. To estimate a production function relating the effects of topdress nitrogen and phosphorus on potato yield.
2. Estimate the resulting potato yields per hectare for specified nitrogen and phosphorus applications.

TABLE 1.

Estimation of Planted Area and Production For Potatoes, Corn and Wheat, Ecuador, 1965-75

Year	Potatoes		Corn			
	Area Planted HECTARES	Production TONS	Area Planted HECTARES	Production TONS	Area Planted HECTARES	Production TONS
1965	44480	390842	250420	142227	68900	65088
1966	44344	347040	217465	122172	65004	62727
1967	48212	398586	303700	130736	79585	78546
1968	49159	510873	225200	102575	79399	82910
1969	41420	456686	232110	140527	100231	94099
1970	47220	541794	236980	200460	76230	81000
1971	53452	680740	241305	140385	75560	64893
1972	37729	437348	249990	170642	56054	50640
1973	43579	539198	123770	100342	46504	45189
1974	39138	503340	109615	76252	56261	54989
1975	39499	499371	108763	90247	70233	64647

SOURCE: Agricultural Ministry of Ecuador

3. Calculate the combinations of nitrogen and phosphorus required to produce specified yields (iso - quants) of potatoes and corresponding marginal rates of substitution.

4. To estimate the optimum level of application for one nutrient when the level of the other is considered to be fixed at different levels and with specific potato and nutrient price relationships.

5. Estimate the minimum cost combinations of nitrogen and phosphorus for specified potato yield levels under varying nutrient price ratios.

6. Determine the optimum combination of nutrients (nitrogen and phosphorus) for specific potato, nitrogen and phosphorus price relationships.

7. Estimate the production function and the optimum combination of nitrogen and phosphorus under current price relationships for corn and wheat.

For purposes of this study, an in-depth economic analysis will be completed on potatoes. For corn and wheat only a partial analysis will be completed and included in the appendix. A complete analysis of corn and wheat would obviously require duplication of the procedures used for potatoes. Potatoes are a primary food crop in Ecuador and therefore were chosen for the principal analysis. In addition, the data for this crop were better than for the other crops.

CHAPTER II

METHODOLOGY

The Data

For this study cross sectional experimental data for potatoes, corn, and wheat were acquired from the Soils Department of the National Agricultural Research Institute of Ecuador. These experiments were conducted in the Santa Catalina Agricultural Research Station and in selected locations representing principal crop production areas. In total there were seven farms where experiments were conducted.

The data were collected under the randomized block experimental design. Thus, the selected treatments were allocated at random among the experimental units. The experiment included a total of 24 treatments for the seven farms in the analysis with nitrogen, phosphorus, and potassium as the nutrients being varied. The source of nitrogen was urea with superphosphate as the phosphorus source.

For purposes of this study, nitrogen and phosphorus were considered as the only variable inputs. Potassium was excluded as a variable input due to the fact that the potassium nutrient level for most soils in the highlands of Ecuador is not a limiting constraint. Thus, the two treatments with varying levels of potassium were excluded from the experiment leaving a total of 22 treatments for the seven farms. The experimental potato yield levels given in Table 2 are an average of four replications on each of the seven farms.

TABLE 2

Observed Potato Yields Resulting From Different Levels of Nitrogen and Phosphorus for Each Farm, Ecuador, 1975

Treatments Kg/Ha		Observed Yields Tons/Ha						
Nitrogen	Phosphorus	Farm 1 Chisinche	Farm 2 Aychapicho	Farm 3 Bonanza	Farm 4 Monteverde	Farm 5 Indujel	Farm 6 S'Rosa	Farm 7 LaVictoria
0	0	14.84	28.25	16.95	8.64	15.44	11.62	22.20
0	320	17.05	33.66	15.38	8.51	14.17	10.55	30.74
50	320	22.61	38.04	22.63	13.82	26.95	15.49	31.72
100	320	27.31	46.10	24.75	16.30	42.08	17.15	32.86
150	320	37.56	46.86	28.66	17.20	38.22	19.62	32.37
200	320	35.79	49.40	29.26	21.17	42.01	20.02	44.46
250	320	32.51	41.12	27.31	24.78	37.73	18.69	43.23
300	320	38.28	42.86	27.25	19.98	35.26	18.65	45.47
200	0	29.22	23.16	24.68	17.11	28.14	16.89	34.85
200	80	35.04	40.10	27.17	18.30	39.70	18.59	41.72
200	160	33.82	42.69	25.33	18.13	38.98	17.33	37.22
200	240	35.27	42.75	26.85	22.02	36.92	18.38	41.37
200	400	37.95	52.92	28.49	19.08	38.73	19.50	36.90
200	480	27.53	37.55	27.09	22.75	37.80	18.52	42.94
50	80	27.16	38.04	23.03	12.80	29.78	15.76	27.38
150	240	35.16	50.05	25.65	18.73	37.70	17.56	37.93
250	400	36.22	45.08	23.19	18.38	39.33	15.87	39.66
300	480	37.42	34.58	25.16	22.14	40.30	17.21	40.92
300	320	34.84	47.08	25.59	19.42	37.60	17.50	45.36
200	320	36.86	46.10	28.93	21.37	36.36	19.80	39.53
200	320	37.36	44.10	23.24	20.45	41.17	15.90	34.20
200	320	36.38	43.45	26.70	31.73	38.73	18.26	43.76

Estimation of the Production Function

In deriving production functions, numerous algebraic equation formulas can be used. For a detailed discussion of the different production function forms refer to Heady and Dillon (5). Hence, one of the first decisions in a study of this nature is the selection of a production function consistent with the phenomena under investigation and the theories of the sciences involved. A careful consideration of the assumptions and limitations of the different functions must be made before choosing one for estimation purposes. The specific equation form used to express production phenomena automatically imposes certain restraints or assumptions with respect to the production relationships involved and the economic optimum levels of resources used and quantities produced.

After considering the characteristics of the linear, Cobb-Douglas, quadratic and square root functions and experimental data, it was evident that either the quadratic or square root function would provide the best fit. After testing both functions, it was found that both provided a good fit to the data expressed for the high magnitudes of the coefficients of multiple determination (R^2). Thus, with both providing an equally good fit to the data, the quadratic function was chosen on the basis of the ease of calculations.

One would not expect the production function originating with zero fertility to have linear isoclines that intersect the axis as is the case with the quadratic function. However, with fertilizer production function studies there are applied nutrients and nutrients inherent in the soil. Thus, as shown in Figure 1 the experiments is not starting with a void of nutrients in the soil as indicated by point 0 but with some level of soil nutrients. This is shown as Q in the diagram which indicates ON units of Nitrogen and OP units of phosphorus available in the soil before addition of fertilizer. These levels of inherent nutrients can be obtained by a soil test. In this case the fertilizer response function would start from point Q rather than from point 0. Estimated isoclines which are linear and intersect at point M, the maximum expected output, would then provide a close estimate of actual isoclines for the total nutrient surface.

The quadratic function used was of the general form shown in equation 1:[1]

$$1.\ Y = a + bX - cX^2$$

Here Y represents the output; a is a constant, indicating the level of output when X is zero; and b is the amount of change in output for a one-unit change in X. The squared, or second degree term of the function serves as an adjustment factor in the production function which allows either diminishing (negative squared term) or increasing (positive squared term) marginal products.

In this study the amount of topdress nitrogen and phosphorus (N and P) will be considered as the variable inputs. Extension of equation 1 to two resources (N and P) results in production surface equation 2.[1]

$$2.\ Y = a + b_1N + b_2P - b_3N^2 - b_4P^2 + b_5NP$$

[1]These equations are not in statistical form. To be in statistical form there would need to be an error term in each equation or it would need to be indicated that Y is estimated through $\hat{Y}$.

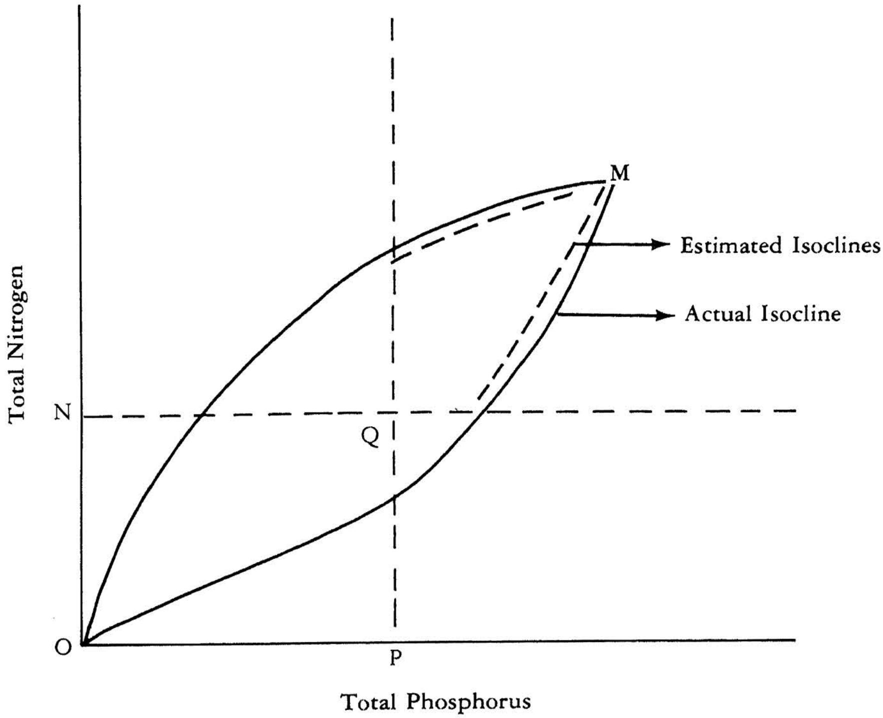

Figure 1. Alternative Isoclines for Production Surfaces.

where

Y = yield of potatoes in tons (1000 kg) per Ha

N = kilograms of topdressed nitrogen applied per Ha[2]

P = kilograms of topdressed phosphorus applied per Ha[2]

and $b_1 \ldots b_5$ are the respective coefficients as explained previously. This equation indicates that diminishing marginal returns exist for each factor when considered independently but that there is a positive interaction between the two factors. As previously pointed out, the equation could be presented with positive signs for all b's. However, in most production responses it is logical that the signs for b_3 and b_4 are negative with the sign for the interaction term (b_5) either positive or negative depending on the degree of complementary between the two production factors.

Tests for Goodness of Fit

To estimate the production function, multiple regression techniques are used. In using these techniques the relationship between the dependent and independent variable can be estimated through the use of the least squares principle (2). After the regression or estimating equation has been derived, it is desirable to know how closely it fits the "true

[2]Kilograms per hectare is almost the same as pounds per acre.

relation." There are a number of statistical tests used to measure the "goodness of fit" of the estimated equation.

A t-ratio is used to test whether the partial regression coefficients are significantly different from zero at a given probability level. If the test fails for a given regression coefficient then the conclusion is that there is no significant relation between Y and the particular independent variable in question. Hence, the variable can be omitted without significantly affecting the predictability of Y. If, however, the regression coefficient is significant then the predictability of Y is improved by including the independent variable in the equation. Assuming that the errors are normally distributed with mean of zero and variance σ^2 the value of t for each b_i is given by equation 3.

$$3.\quad t = \frac{b_i}{\text{var}\,(b_i)}$$

Although the individual terms of the regression equation may all be statistically significant, it is still desirable to test the entire equation for significance. This can be done by using the F - test shown in equation 4.

$$4.\quad F = \frac{\text{Mean square due to Regression}}{\text{Mean square from Regression}}$$

Another measure of the "goodness of fit" of the regression equation is the coefficient of multiple determination, R^2, which measures the percent of total variation in Y explained by the regression equation (4).

Average measurements for each experiment were available for analysis. Therefore, a "goodness-of-fit" test which compares the mean square from regression with the lack of fit sum of squares could not be made. Experiments with replication are needed for an analysis of this nature (5). If these data were available it could be determined if the production function variation is any better or worse than ordinary experimental error.

Analysis of Variance, LSD, and Formation of Groups for Production Function Estimating Purposes with the Farms in Study

The experimental data came from seven different farms which were strategically located in the Ecuador potato production area. Thus, weather and soil conditions, as well as the varieties used, can change from one farm to another. For this reason not all farms may belong to a homogenous population. If this is true, then, there is no basis for an estimate of a single production function covering all farms. To test for a homogenous population of the null hypothesis that the population (yields) means are statistically the same for all farms the analysis of variance was used.[3] The variance ratio shown in equation 5 was the criterion for testing the null hypothesis.

$$5.\quad F = \frac{\text{Treatments Mean Square}}{\text{Error Mean Square}} = \frac{\text{Mean Square Between Classes}}{\text{Mean Square Within Classes}}$$

The calculated F value is then compared with the value shown in the appropriate table. If the calculated F value is bigger than the F value from this table, the null

[3]The analysis of variance table is presented in Table 4.

hypothesis that all the means are the same is rejected and it is concluded that at least two of the farms have different means.

If the null hypothesis is rejected, the Least Significant Difference (LSD) is utilized to tell which of the means are significantly different at specified levels of importance. Thus, through LSD the farms for which there are no significant difference in means can be put into one group.

Estimation of Isoquants and Marginal Rates of Substitution

The isoquant is derived from the production function equation by expressing input of one factor as a function of a stated output level and quantity of other resources. The corresponding isoquant of equation 3 is given in equation 6.

$$6.\quad N = \frac{b_1 + b_5P \pm [\,(b_1 + b_5P)^2 - 4b_3\,(Y + b_4P^2 - b_2P - a)\,]^{.5}}{2b_3}$$

The marginal rate of substitution can be measured as the "inverse" ratio of marginal products with a minus sign attached. The equation for the marginal rates of substitution corresponding to equation 6 is equation 7.

$$7.\quad \frac{dN}{dP} = -\frac{b_2 - 2b_4P + b_5N}{b_1 - 2b_3N + b_5P}$$

Economic Analysis

Production functions, when used for economic analysis and subsequent recommendations, provide one of the two sets of information needed for choice and decision-making. The other set of information needed is the price and cost data which serve as the economic criteria.

The various quantities and relationships derived provide a basis for specifying selected economic optima. One economic optimum is to select the superior level of one nutrient to apply when the level of the other is considered to be fixed. For example, suppose that N (nitrogen) is variable with the quantity of phosphorus being fixed at a specified level. Under these conditions the optimum level of nitrogen applied depends on the marginal products of nitrogen and the nitrogen-to-potato price ratio. In solving for this optimum level, the marginal product of nitrogen is equated to the price ratio of nitrogen (pN) and potatoes (pY) (equation 8).

$$8.\quad \frac{\delta Y}{\delta N} = \frac{pN}{pY}$$

When solving equation 8 the most profitable quantity of nitrogen for fixed levels of phosphorus is obtained. Similarly, the same methodology is used when varying the phosphorus application with nitrogen at fixed levels (equation 9).

$$9.\quad \frac{\delta Y}{\delta P} = \frac{pP}{pY}$$

In addition, the combinations of nitrogen and phosphorus that will minimize fertilizer costs for specified yield levels and price ratios can be calculated. The least-cost resource combination for a given yield is obtained when the marginal rate of substitution of the resource (equation 7. with yield at stated levels) is equal to the inverse price ratio as shown in equation 10.

$$10.\quad \frac{\delta N}{\delta P} = \frac{pP}{pN}$$

In determining the economic optimum fertilizer usage, optimum combinations of nutrients and their level of application must be simultaneously determined. This combination is solved by setting the marginal products for both nutrients equal to the price ratio and solving simultaneously equations 11 and 12 for the quantity of the nutrients to apply for maximum profits.

$$11.\quad \frac{\delta Y}{\delta N} = \frac{pP}{pY}$$

$$12.\quad \frac{\delta Y}{\delta P} = \frac{pP}{pY}$$

These optima are attained when the partial derivatives (marginal products) for both nutrients are equal to the respective nutrient to potatoes price ratio (5).

CHAPTER III

RESULTS

The Formation of Farm Groups

An analysis of variance was performed to test if the yield means among the various farms were the same. The calculated sum of observations (ΣX), means ($\overline{X}$), and the sum of squares of deviations (ΣX^2) for each farm and the totals needed for the analysis of variance comparison are presented in Table 3.

Table 4 is the usual analysis of variance table with general computing instruction for (a) classes of farms with (n) observations per class. The symbol (T) denotes a typical class total, while $G = \Sigma T = \Sigma\Sigma X$ (summed over both rows and columns) is the grand total. The first step is to calculate the correction of the mean (c) which is equal to G^2/an, (8). The remaining steps, like the calculations of the mean squares and the sum of squares between and within the farms, can be followed from Table 4.

With this information, the null hypothesis that the yield means among the farms are the same can be tested. The calculated F value is the result of the mean square between the farms (1998.85) divided by the mean square within the farms (33.00). This value of 60.57 is larger than 2.92, the F-Table value at 1 per cent level of probability, so the null hypothesis is rejected and it is concluded that at least two of the farms have significantly different means.

TABLE 3

Sum of Observations, Means and Sum of Squares for Each Farm and Totals, Potato Production, Ecuador, 1975

	X	$\overline{X}$	X^2
Farm 1	716.18	32.55	24277.79
Farm 2	913.94	41.54	39026.76
Farm 3	553.29	25.15	14173.74
Farm 4	403.81	18.36	7795.16
Farm 5	773.30	35.15	28433.37
Farm 6	378.86	17.22	6644.83
Farm 7	826.79	37.58	31882.37
Total	4566.17=G	207.55	152234.02

Table 4

Analysis of Variance Table

Source of Variation	Degrees of Freedom	Sum of Squares (SS)	Mean Square (MS)
Between farms	a-1 = 6	(T^2/n) - c = 11993.1	1998.85
Within farms	a(n-1)=147	Subtract = 4851.9	33
	an-1 = 153	X^2 - c = 16845.0	—

a = 7
n = 22

Through the use of the LSD technique in equation 13 the significantly different means can be calculated.

$$13.\ LSD = t(\pounds\, d_f) \sqrt{MS\ (\text{within terms})\ \frac{.2}{n}}$$

Plugging into equation 13 yields a least significance difference level of 3.4. Thus, when comparing two farm means, if the difference between the means is greater than 3.4 the means are significantly different and come from two different populations. For example, the highest mean, 41.54 (Farm 2), is significantly greater or different from the means of the other farms, as the respective differences are larger than 3.4. Thus, the farm size forms one group as shown in Table 5. In the grouping process, Farm Group I is formed by Farm 3; Farm Group II by Farm 2; Farm Group III by farms 7 and 5; Farm Group IV by Farms 1 and 5, and Farm Group V by Farms 6 and 4.

TABLE 5

Least Significant Difference (LSD) Table
Used to Group Farms for Estimating
Potato Production Functions, Ecuador, 1975

	Farm 6	Farm 4	Farm 3	Farm 1	Farm 5	Farm 7	Farm 2	LSD
Mean Yields	17.22	18.36	25.15	32.55	35.15	37.58	41.54	3.4

The lines below the table join the farms which have means that are not significantly different.

The Estimated Production Function for Each Group

The estimated production functions for farm groups I through V are presented in Equations 14 through 18 respectively.

$$14.\quad \hat{Y} = 17.40555 + 0.087389N + .0068367P - 0.00024556N^2 - 0.0000266P^2 + 0.0000447NP$$

$$15.\quad \hat{Y} = 27.97 + 0.065029N + 0.1019294P - 0.0003871N^2 - 0.00026109P^2 + 0.0002493NP$$

$$16.\quad \hat{Y} = 19.6184 + 0.125185N + 0.0297899P - 0.0002676N^2 - 0.00005834P^2 + 0.00004078NP$$

$$17.\quad \hat{Y} = 17.13386 + 0.162173N - 0.017756P - 0.0004753N^2 - 0.00006464P^2 + 0.0001415NP$$

$$18.\quad \hat{Y} = 10.21 + 0.073265N + 0.005442P - 0.0001933N^2 - 0.0000186P^2 + 0.00004366NP$$

Regression coefficients and their respective standard errors are presented in Table 6. Nitrogen was significantly different from zero at the 10 per cent level for all groups. Although the regression coefficients for phosphorus in some equations are not significant at the 10 per cent level they were included in the function in order to account for the interaction between nitrogen and phosphorus.

The coefficients of multiple determination R^2 (Table 6) describes the proportion of total variation in potato yield explained by the regression equations, and they are 77.4, 80.13, 73.0, 85.0, and 74.0 per cent, for Farm Groups I through V, respectively. The remaining percentages not explained are attributed to measurement errors, specification errors, or sampling errors.

The level of significance of the overall regression equation is tested by means of the F-ratio. Under this test all equations were significant at the 5 per cent level. Thus, the null hypothesis that all of the betas are simultaneously equal to zero is not accepted.

The predicted potato yields for varying nitrogen and phosphorus combinations for Farm Groups I through V are presented in Tables 7 through 11 respectively.

TABLE 6

Regression Coefficients, Standard Errors, Coefficients of Multiple Determination (R^2) and F Values for Potato Production for Each Farm Group, Ecuador, 1975

Farm Group	Coefficient							
	a	b_1	b_2	b_3	b_4	b_5	Value Of R^2	Value of F
I	17.4055*	0.08739*	0.006837	-0.000245*	-0.0000266	0.0000447	.77422	10.970**
	(1.63818)***	(0.01599)	(0.010270)	(0.0000583)	(0.00002485)	(0.0000457)		
II	27.97*	0.06503*	0.10193*	-0.000387*	-0.000261*	0.0000249*	.80135	12.908**
	(3.10896)	(0.03035)	(0.01949)	(0.0001107)	(0.00004716)	(0.00008675)		
III	19.6184*	0.125285*	0.02979*	0.0002676*	-0.00005834	0.00004078	.72924	20.469**
	(2.36018)	(0.023043)	(0.01479)	(0.000084)	(0.00003580)	(0.00006585)		
IV	17.13386*	0.16217*	0.017756	-0.000475*	-0.0000646*	0.0001415*	.85065	43.289**
	(1.81845)	(0.017754)	(0.011401)	(0.000064)	(0.0000275)	(0.0000507)		
V	10.2100*	0.07326*	0.005442	-0.000193*	-0.0000186	0.0000436	.74001	21.631**
	(1.14112)	(0.011141)	(0.007154)	(0.0000406)	(0.0000173)	(0.0000318)		

*Significant at 10 percent level.

**Significant at 5 percent level.

***Numbers in parenthesis are the standard errors.

TABLE 7

Predicted Potato Yield for Different Levels of Nitrogen and Phosphorus, Ecuador, 1975 Tons/Ha

Farm Group I

Kg of Phosphorus per Ha	Kilograms of Nitrogen per Hectare												
	0	25	50	75	100	125	150	175	200	225	250	275	300
0	17.41	19.44	21.16	22.58	23.69	24.49	24.99	25.18	25.06	24.64	23.91	22.87	21.52
25	17.56	19.62	21.37	22.82	23.95	24.79	25.31	25.53	25.44	25.04	24.34	23.33	22.01
50	17.68	19.77	21.55	23.02	24.19	25.05	25.60	25.84	25.78	25.41	24.74	23.76	22.47
75	17.77	19.88	21.69	23.19	24.39	25.27	25.85	26.13	26.09	25.75	25.11	24.15	22.89
100	17.83	19.97	21.80	23.33	24.55	25.47	26.08	26.38	26.37	26.06	25.44	24.51	23.28
125	17.84	20.02	21.88	23.44	24.69	25.63	26.27	26.60	26.62	26.33	25.74	24.84	23.64
150	17.83	20.03	21.92	23.51	24.79	25.76	26.42	26.78	26.83	26.57	26.01	25.14	23.96
175	17.79	20.01	21.93	23.55	24.85	25.85	26.54	26.93	27.01	26.78	26.24	25.40	24.25
200	17.71	19.96	21.91	23.55	24.89	25.91	26.63	27.05	27.15	26.95	26.44	25.63	24.51
225	17.60	19.88	21.86	23.52	24.89	25.94	26.69	27.13	27.26	27.09	26.61	25.82	24.73
250	17.45	19.76	21.77	23.46	24.85	25.94	26.71	27.18	27.34	27.20	26.75	25.99	24.92
275	17.27	19.61	21.64	23.37	24.79	25.90	26.70	27.20	27.39	27.27	26.85	26.12	25.08
300	17.06	19.43	21.49	23.24	24.69	25.83	26.66	27.18	27.40	27.31	26.91	26.21	25.20
325	16.82	19.21	21.30	23.08	24.55	25.72	26.58	27.13	27.38	27.32	26.95	26.27	25.29
350	16.54	18.96	21.08	22.89	24.39	25.58	26.47	27.05	27.32	27.29	26.95	26.30	25.35
375	16.23	18.68	20.82	22.66	24.19	25.41	26.37	26.93	27.24	27.23	26.92	26.30	25.37
400	15.88	18.36	20.53	22.40	23.96	25.21	26.15	26.79	27.12	27.14	26.85	26.26	25.36
425	15.51	18.01	20.21	22.10	23.69	24.97	25.94	26.60	26.96	27.01	26.76	26.19	25.32
450	15.10	17.63	19.86	21.78	23.39	24.70	25.70	26.39	26.77	26.85	26.62	26.09	25.25
475	14.65	17.21	19.47	21.42	23.06	24.39	25.42	26.18	26.55	26.66	26.46	25.95	25.14

TABLE 8

Predicted Potato Yields for Different Levels of Nitrogen and Phosphorus, Ecuador, 1975 Tons/Ha

Farm Group II

Kg of Phosphorus per Ha	Kilograms of Nitrogen per Hectare												
	0	25	50	75	100	125	150	175	200	225	250	275	300
0	27.97	29.35	30.25	30.67	30.60	30.05	29.00	27.49	25.49	23.00	20.00	16.57	12.60
25	30.35	31.89	32.95	33.52	33.61	33.21	32.33	30.97	29.12	26.79	23.98	20.68	16.90
50	32.41	34.11	35.32	36.05	36.29	36.05	35.33	34.12	32.43	30.25	27.59	24.45	20.80
75	34.15	35.60	37.36	38.25	38.65	38.56	37.99	36.94	35.40	33.39	30.88	27.89	24.40
100	35.55	37.60	39.08	40.12	40.68	40.75	40.34	39.44	38.06	36.20	33.85	31.01	27.70
125	36.63	38.79	40.47	41.67	42.38	42.61	42.35	41.61	40.38	38.68	36.49	33.81	30.60
150	37.38	39.70	41.54	42.89	43.76	44.14	44.04	43.46	42.39	40.84	38.80	36.28	33.30
175	37.81	40.29	42.28	43.78	44.81	45.35	45.40	44.97	44.06	42.67	40.79	38.42	35.57
200	37.91	40.54	42.70	44.35	45.53	46.23	46.44	46.16	45.41	44.17	42.44	40.24	37.58
225	37.69	40.47	42.77	44.59	45.93	46.78	47.15	47.03	46.43	45.35	43.78	41.73	39.10
250	37.13	40.07	42.54	44.51	46.00	47.00	47.53	47.57	47.13	46.20	44.78	42.89	40.50
275	36.26	39.35	41.97	44.10	45.75	46.90	47.59	47.78	47.49	46.72	45.46	43.72	41.50
300	35.05	38.30	41.08	43.36	45.16	46.48	47.32	47.66	47.54	46.92	45.82	44.23	42.10
325	33.52	36.93	39.86	42.30	44.26	45.73	46.72	47.23	47.25	46.79	45.85	44.42	42.50
350	31.66	35.22	38.31	40.91	43.02	44.65	45.80	45.46	46.64	46.34	45.55	44.27	42.50
375	29.47	33.20	36.44	39.19	41.46	43.25	44.55	45.37	45.70	45.55	47.92	43.80	42.20
400	26.97	30.84	34.24	37.15	39.57	41.52	42.98	43.95	44.44	44.44	43.97	43.00	41.56
425	24.13	28.16	31.71	34.78	37.36	39.46	41.07	42.20	42.85	43.01	42.69	41.89	40.60
450	20.97	25.16	28.86	32.08	34.82	37.08	38.85	40.13	40.93	41.25	41.09	40.44	39.30
475	17.48	21.82	25.68	29.06	31.96	34.37	36.29	37.73	38.69	39.16	39.15	38.66	37.60

TABLE 9

Predicted Potato Yields for Different Levels of Nitrogen and Phosphorus, Ecuador, 1975 Tons/Ha

Farm Group III

Kg of Phosphorus per Ha	Kilograms of Nitrogen per Hectare												
	0	25	50	75	100	125	150	175	200	225	250	275	300
0	19.62	22.58	25.20	27.50	29.46	31.08	32.38	33.33	33.95	34.23	34.19	33.81	33.00
25	20.33	23.31	25.97	28.29	30.27	31.92	33.23	34.22	34.86	35.17	35.15	34.80	34.10
50	20.96	23.98	26.65	29.00	31.00	32.68	34.02	35.03	35.70	36.04	36.04	35.71	35.00
75	21.52	24.56	27.27	29.64	31.67	33.37	34.74	35.77	36.50	36.83	36.86	36.55	35.90
100	22.01	25.07	27.81	30.20	32.26	33.99	35.38	36.44	37.16	37.55	37.60	37.32	36.70
125	22.43	25.52	28.27	30.70	32.78	34.53	35.95	37.03	37.78	38.20	38.28	38.02	27.40
150	22.77	25.89	28.67	31.12	33.23	35.00	36.45	37.56	38.33	38.77	38.87	38.65	38.00
175	23.04	26.18	28.99	31.46	33.60	35.40	36.87	38.00	38.81	39.27	39.40	39.20	38.60
200	23.24	26.41	29.24	31.74	33.90	35.73	37.22	38.38	39.21	39.70	39.85	39.67	39.00
225	23.37	26.56	29.42	31.94	34.13	35.98	37.50	38.68	39.54	40.05	40.23	40.08	39.50
250	23.42	26.64	29.52	32.07	34.28	36.16	37.71	38.92	39.79	40.33	40.54	40.41	39.90
275	23.40	26.64	29.55	32.12	34.36	36.27	37.84	39.07	39.97	40.54	40.77	40.67	40.20
300	23.30	26.57	29.51	32.10	34.37	36.30	37.90	39.16	40.08	40.68	40.93	40.86	40.40
325	23.14	26.43	29.39	32.01	34.31	36.26	37.88	39.17	40.12	40.74	41.02	40.97	40.50
350	22.90	26.22	29.20	31.85	34.17	36.15	37.80	39.11	40.09	40.73	41.03	41.01	40.60
375	22.59	25.93	28.94	31.62	33.96	35.96	37.64	38.97	39.98	40.65	40.98	40.98	40.00
400	22.20	25.57	28.61	31.31	33.67	35.71	37.40	38.77	39.79	40.49	40.85	40.87	40.00
425	21.74	25.14	28.20	30.92	33.32	35.37	37.10	38.49	39.54	40.26	40.64	40.69	40.40
450	21.21	24.63	27.72	30.50	32.89	34.97	36.72	38.13	39.21	39.96	40.37	40.44	40.10
475	20.61	24.05	27.16	29.94	32.39	34.49	36.27	37.71	38.81	39.58	40.02	40.12	39.80

TABLE 10

Predicted Potato Yields for Different Levels of Nitrogen and Phosphorus, Ecuador, 1975 Tons/Ha

Farm Group IV

Kg of Phosphorus per Ha	Kilograms of Nitrogen per Hectare												
	0	25	50	75	100	125	150	175	200	225	250	275	300
0	17.13	20.89	24.05	26.62	28.60	29.98	30.77	30.96	30.56	29.56	27.97	25.79	23.00
25	17.54	21.38	24.63	27.29	29.39	30.82	31.70	31.98	31.67	30.76	29.26	27.16	24.40
50	17.86	21.79	25.13	27.88	30.03	31.59	32.55	32.92	32.70	31.88	30.47	28.46	25.86
75	18.10	22.12	25.55	28.39	30.63	32.27	33.73	33.78	33.65	32.92	31.59	29.67	27.10
100	18.26	22.37	25.89	28.81	31.14	32.88	34.02	34.56	34.52	33.87	32.64	30.81	28.30
125	18.34	22.54	26.15	29.11	31.58	33.40	34.63	35.26	35.30	34.75	33.60	31.86	29.50
150	18.34	22.63	26.32	29.42	31.93	33.84	35.16	35.88	36.01	35.54	34.49	32.83	30.50
175	18.26	22.64	26.42	29.61	32.20	34.20	35.61	36.42	36.64	36.26	35.29	33.72	31.50
200	18.10	22.56	26.43	29.71	32.39	34.48	35.98	36.88	37.18	36.89	36.01	34.54	32.41
225	17.86	22.41	26.37	29.73	32.50	34.68	36.26	37.25	37.65	37.45	36.65	35.27	33.20
250	17.53	22.17	26.22	29.68	32.53	34.80	36.47	37.55	38.03	37.92	37.21	35.91	32.00
275	17.13	21.86	25.99	29.54	32.48	34.84	36.60	37.76	38.33	38.31	37.69	36.48	34.60
300	16.64	21.46	25.69	29.32	32.35	34.79	36.64	37.90	38.56	38.62	38.09	36.97	35.20
325	16.08	20.98	25.30	29.01	32.14	34.67	36.60	37.95	38.70	38.85	38.41	37.38	35.70
350	15.43	20.43	24.83	28.63	31.85	34.47	36.49	37.92	38.75	39.00	38.65	37.70	36.16
375	14.70	19.79	24.28	28.17	31.47	34.18	36.29	37.81	38.74	39.07	38.81	37.95	36.50
400	18.89	19.07	23.64	27.63	31.02	33.81	36.02	37.62	38.64	39.06	38.88	38.11	36.75
425	13.00	18.27	22.93	27.00	30.48	33.37	35.66	37.35	38.45	38.96	38.88	38.19	36.90
450	12.03	17.38	22.14	26.30	29.87	32.84	35.22	37.00	38.19	38.79	38.79	38.20	37.01
475	10.98	16.42	21.26	25.51	29.17	32.23	34.70	36.57	37.85	38.53	38.62	38.12	37.00

TABLE 11

Predicted Potato Yields for Different Levels of Nitrogen and Phosphorus, Ecuador, 1975 Tons/Ha

Farm Group V

Kg of Phosphorus per Ha	Kilograms of Nitrogen per Hectare												
	0	25	50	75	100	125	150	175	200	225	250	275	300
0	10.21	11.92	12.39	14.62	15.60	16.35	16.85	17.11	17.13	16.91	16.44	15.74	14.70
25	10.33	12.07	13.57	14.82	15.84	16.61	17.14	17.43	17.47	17.28	16.84	16.16	15.20
50	10.44	12.20	13.72	15.01	16.05	16.85	17.40	17.72	17.79	17.63	17.22	16.57	15.60
75	10.51	12.31	13.86	15.17	16.23	17.06	17.65	17.99	18.09	17.95	17.57	16.98	16.00
100	10.57	12.39	13.97	15.30	16.40	17.25	17.86	18.23	18.36	18.25	17.89	17.30	16.46
125	10.60	12.45	14.05	15.42	16.54	17.42	18.06	18.46	18.61	18.53	18.20	17.63	16.80
150	10.61	12.48	14.12	15.51	16.66	17.56	18.23	18.66	18.84	18.78	18.48	17.94	17.10
175	10.59	12.49	14.15	15.57	16.75	17.69	18.38	18.83	19.04	19.01	18.74	18.22	17.40
200	10.55	12.48	14.17	15.62	16.82	17.78	18.50	18.98	19.22	19.22	18.97	18.49	17.76
225	10.49	12.45	14.16	15.64	16.87	17.86	18.61	19.11	19.38	19.40	19.18	18.72	18.02
250	10.41	12.39	14.13	15.63	16.89	17.91	18.69	19.22	19.51	19.56	19.37	18.94	18.26
275	10.30	12.31	14.08	15.61	16.89	17.94	18.74	19.30	19.62	19.70	19.54	19.13	18.40
300	10.17	12.21	14.00	15.56	16.87	17.94	18.77	19.36	19.71	19.81	19.68	19.30	18.60
325	10.01	12.08	13.90	15.49	16.83	17.93	18.78	19.40	19.77	19.90	19.80	19.45	18.80
350	9.84	11.93	13.78	15.39	16.76	17.88	18.77	19.41	19.81	19.97	19.89	19.57	19.00
375	9.64	11.76	13.63	15.27	16.67	17.82	18.73	19.40	19.83	20.02	19.96	19.67	19.13
400	9.41	11.56	13.46	15.13	16.65	17.73	18.67	19.37	19.82	20.04	20.01	19.74	19.23
425	9.16	11.34	13.27	14.96	16.41	17.62	18.59	19.31	19.80	20.04	20.08	19.80	19.30
450	8.89	11.09	13.05	14.77	16.25	17.49	18.48	19.23	19.74	20.01	20.04	19.82	19.37
475	8.60	10.83	12.82	14.56	16.07	17.33	18.35	19.13	19.67	19.96	20.02	19.83	19.40

Marginal Product Calculations

The Marginal Product for Nitrogen (MPPn) for farm groups I through V respectively are given in equations 19, 21, 23, 25 and 27. Similarly, equations 20, 22, 24, 26 and 28 are the marginal products or partial derivatives $\frac{\delta Y}{\delta P}$ of potato yield with respect to phosphorus (MPPp) for each respective farm group.

19. $\delta Y/\delta N = 0.087389 - 0.000491N + 0.0000447P$
20. $\delta Y/\delta P = 0.006836 - 0.0000532P + 0.0000447N$
21. $\delta Y/\delta N = 0.065029 - 0.0007742N + 0.0002494P$
22. $\delta Y/\delta P = 0.10193 - 0.000552P + 0.0002494N$
23. $\delta Y/\delta N = 0.12518 - 0.0005352N + 0.00004078P$
24. $\delta Y/\delta P = 0.02979 - 0.0001167P + 0.00004078N$
25. $\delta Y/\delta N = 0.16217 - 0.00095N - 0.0001415P$
26. $\delta Y/\delta P = 0.01775 - 0.0001293P + 0.0001415N$
27. $\delta Y/\delta N = 0.07326 - 0.000386N + 0.00004366P$
28. $\delta Y/\delta P = 0.005442 - 0.0000372P + 0.00004366N$

To find the input levels required to produce the maximum estimated yield, both equations $\delta Y/\delta N$ and $\delta Y/\delta P$ for each farm group are set equal to zero. The maximum yield is estimated by replacing the input levels required to produce maximum estimated yields in the corresponding production function equation. These results are presented in Table 12.

TABLE 12

Estimation of Nitrogen and Phosphorus Input Levels Needed to Attain Maximum Yields for the Five Farm Groups of Potato Cultivation, Ecuador, 1975

Farm Group	Estimated Maximum Yield tons/Ha	Input Level to Produce Maximum Yield	
		Nitrogen Kg/Ha	Phosphorus Kg/Ha
I	27.4	206.5	313.2
II	47.8	173.6	278.2
III	41.1	263.8	392.8
IV	39.1	228.0	385.0
V	20.1	247.0	505.7

Yield Isoquants and Marginal Rates of Substitution

Isoquant equations 29 through 33 are derived from each respective production function or equation 14 through 18 respectively. In the isoquant equations presented below, phosphorus is expressed as a function of nitrogen and yield.

$$29.\quad P = \frac{[0.006837 + 0.0000447N] \pm \left[[0.0068367 + 0.0000447N]^2 - 4\,(0.0000266)\,[Y - 17.405 - 0.08739N + 0.000246N^2]\right]^{.5}}{2(0.0000266)}$$

$$30.\quad P = \frac{[0.1019 + 0.000249N] \pm \left[[0.10193 + 0.000249N]^2 - 4\,(0.0002611)\,[Y - 27.97 - 0.065029N + 0.000387N^2]\right]^{.5}}{2(0.0002611)}$$

$$31.\quad P = \frac{[0.02979 + 0.00004078N] \pm \left[[0.02979 + 0.00004078N]^2 - 4\,(0.0000583)\,[Y - 19.62 - 0.1252N + 0.0002676N^2]\right]^{.5}}{2(0.0000583)}$$

$$32.\quad P = \frac{[0.01775 + 0.0001415N] \pm \left[[0.01775 + 0.0001414N]^2 - 4\,(0.00006464)\,[Y - 17.1336 - 0.1622N + 0.000475N^2]\right]^{.5}}{2\,(0.00006464)}$$

$$33.\quad P = \frac{[0.005442 + 0.0000436N] \pm \left[[0.005442 + 0.0000436N]^2 - 4\,(0.0000186)\,[Y - 10.21 - 0.07326N + 0.0000193N^2]\right]^{.5}}{2(0.0000186)}$$

The input combinations which produce selected yield levels, and the marginal rates of substitution for these combinations, are presented in Tables 13 and 14 for equations 29 to 33 respectively. The respective isoquants are presented in Figures 2 through 6.

TABLE 13

Combinations of Nitrogen and Phosphorus Needed to Produce Specified Potato Yields and Corresponding Estimated Marginal Rates of Substitution for Farm Groups I, II, Ecuador, 1975

Kilograms of Nitrogen per Ha	Kilograms of Phosphorus per Ha	MRS $\frac{dP}{dN}$
	GROUP I	
	Y=25 Tons	
100	106.85	−6.37
120	60.74	−3.51
140	13.49	−1.56
220	15.44	+1.27
	Y=26 Tons	
140	117.56	−3.55
160	74.75	−1.23
180	61.84	−0.15
200	66.92	+0.65
	Y=27 Tons	
175	188.87	−2.21
185	175.13	−0.70
195	172.08	+0.11
215	188.33	+1.55
	GROUP II	
	Y=41 Tons	
60	124.90	−0.96
80	110.97	−0.48
100	104.45	−0.19
120	102.93	+0.03
140	105.34	+0.21
	Y=43 Tons	
80	147.82	−0.89
100	135.63	−0.38
120	131.22	−0.076
140	132.07	+0.15
160	137.15	+0.35
	Y=45 Tons	
100	180.87	−1.01
120	168.57	−0.32
140	165.97	+0.04
160	169.55	+0.31

TABLE 14

Combinations of Nitrogen and Phosphorus Needed to Produce Specified Potato Yields and Corresponding Estimated Marginal Rates of Substitution for Farm Groups III, IV, V, Ecuador, 1975

Kilograms of the Nitrogen per Ha	Kilograms of Phosphorus per Ha	MRS $\frac{dP}{dN}$
	GROUP III	
	Y=37 Tons	
180	115.85	-1.42
200	93.90	-0.82
220	82.23	-0.37
240	78.62	+0.0001
260	82.11	+0.34
	Y=39 Tons	
200	186.65	-1.59
220	164.25	-0.72
240	153.85	-0.15
260	157.89	+0.34
280	169.54	+0.83
	Y=41 Tons	
250	316.23	-1.41
260	311.24	+0.30
270	322.27	+1.91
	GROUP IV	
	Y=32 Tons	
120	75.32	-2.34
140	42.44	-1.91
160	28.16	-0.38
180	25.86	+0.13
200	32.86	+0.56
	Y=34 Tons	
140	115.55	-2.00
160	89.10	-0.78
180	80.92	-0.07
200	84.88	+0.46
220	98.62	+0.92
	Y=36 Tons	
170	159.89	-1.09
180	151.90	-0.53
190	148.80	-0.10
200	149.60	+0.26
210	153.74	+0.57

TABLE 14
continued

GROUP V

	Y=18 Tons	
140	151.16	−4.33
160	95.37	−1.75
180	72.38	−0.64
200	67.24	+0.10
220	75.29	+0.70
	Y=19 Tons	
190	176.86	−1.05
200	169.61	−0.42
210	167.91	+0.07
220	170.79	+0.50
	Y=20 Tons	
220	375.31	−0.30
230	360.48	−0.04
240	368.29	+1.51

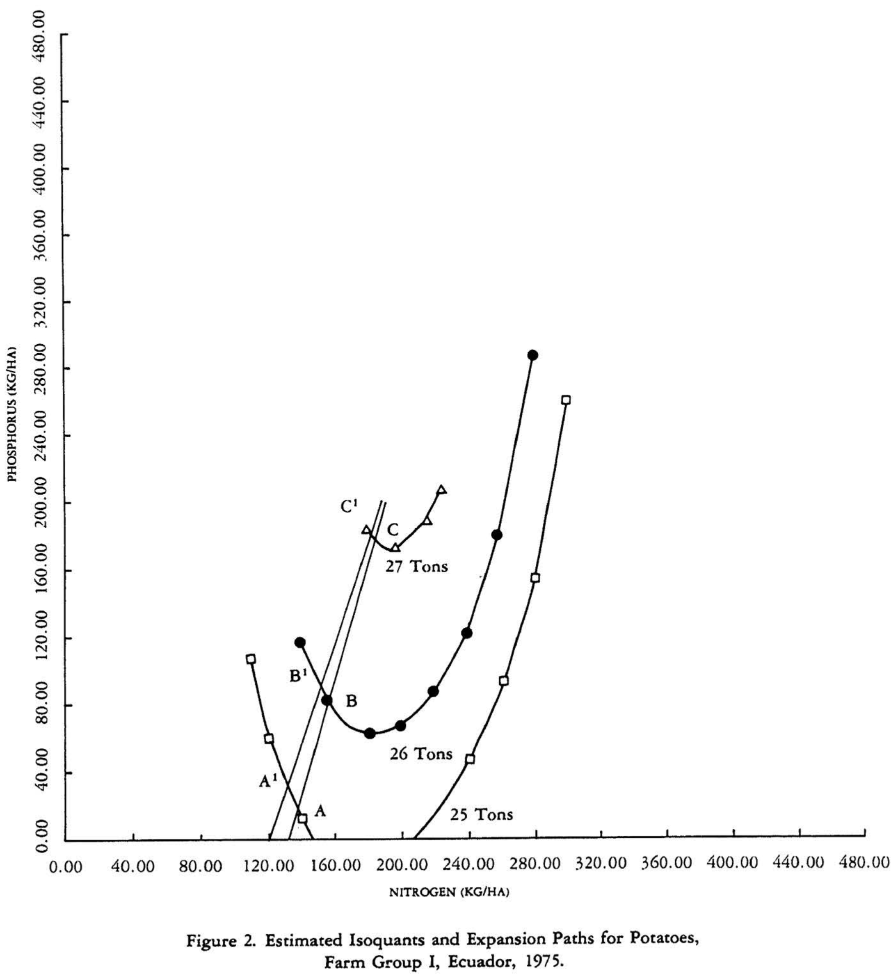

Figure 2. Estimated Isoquants and Expansion Paths for Potatoes, Farm Group I, Ecuador, 1975.

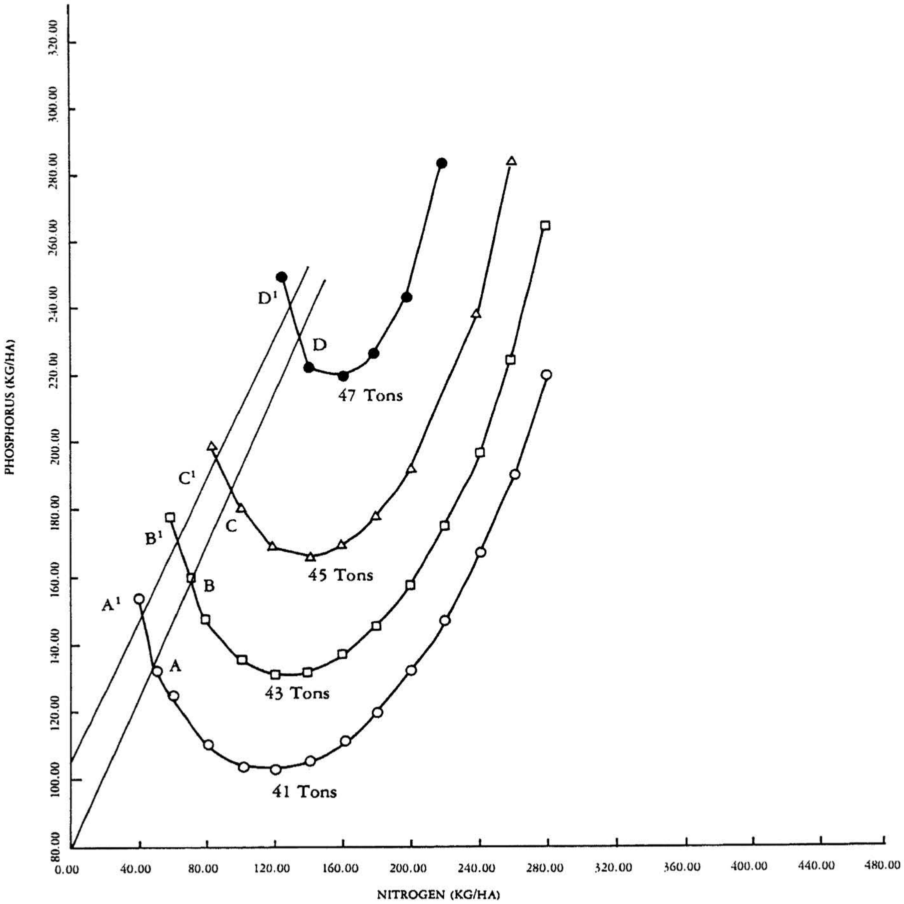

Figure 3. Estimated Isoquants and Expansion Paths for Potatoes, Farm Group II, Ecuador, 1975.

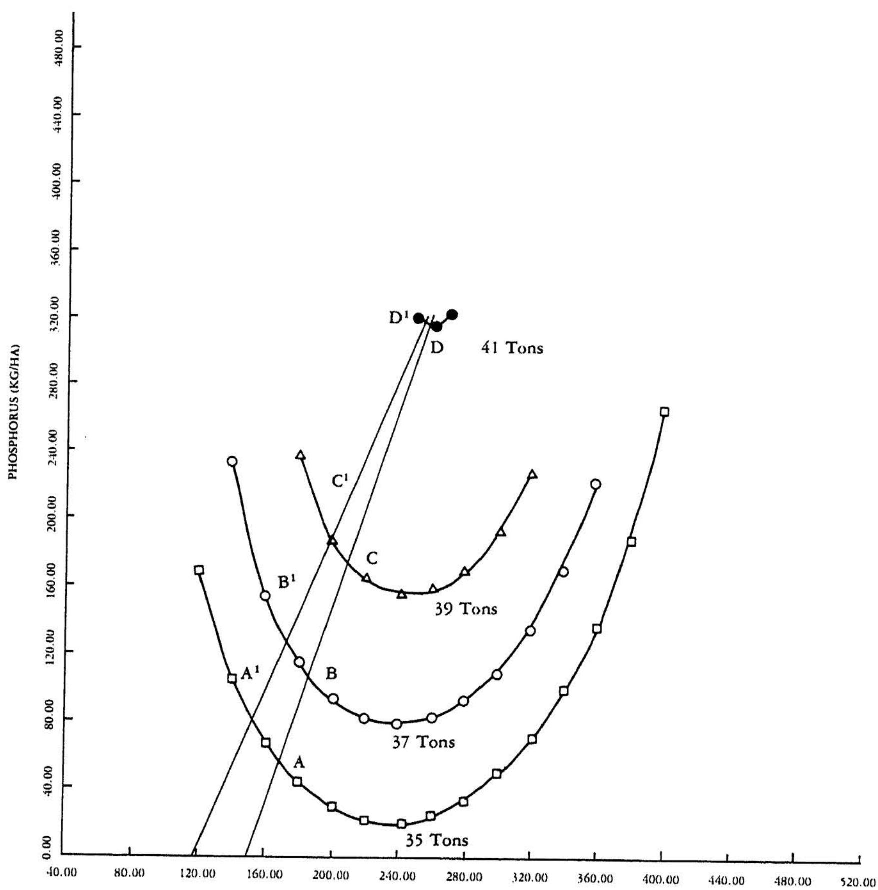

Figure 4. Estimated Isoquants and Expansion Paths for Potatoes, Farm Group III, Ecuador, 1975.

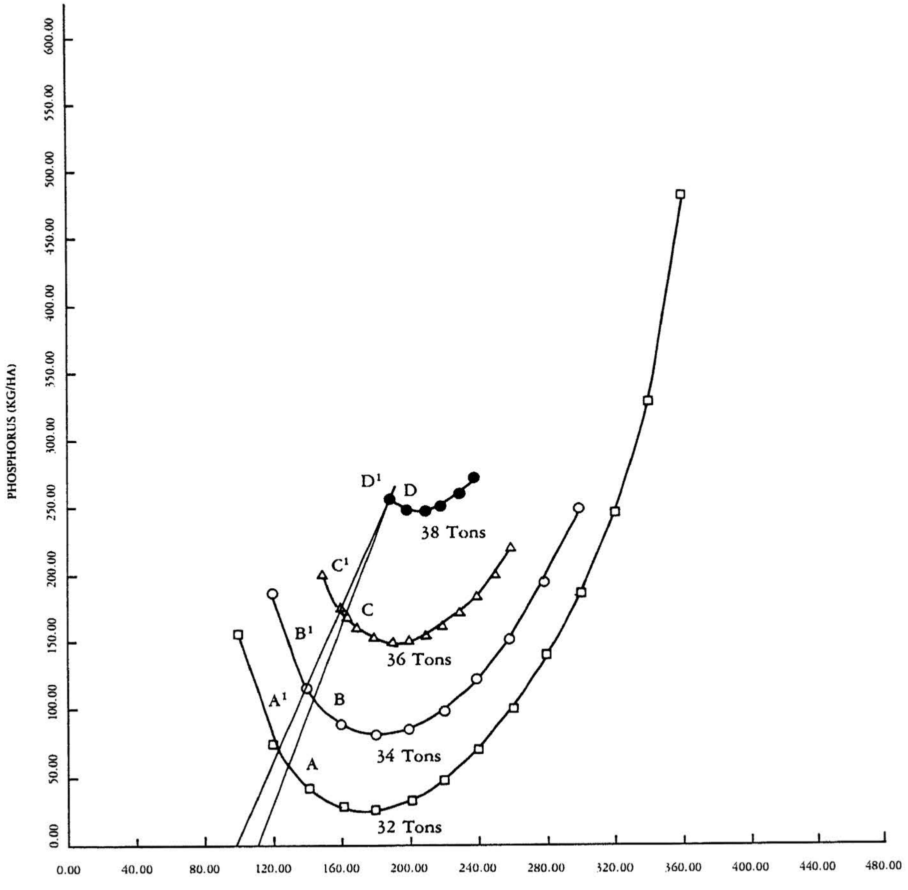

Figure 5. Estimated Isoquants and Expansion Paths for Potatoes, Farm Group IV, Ecuador, 1975.

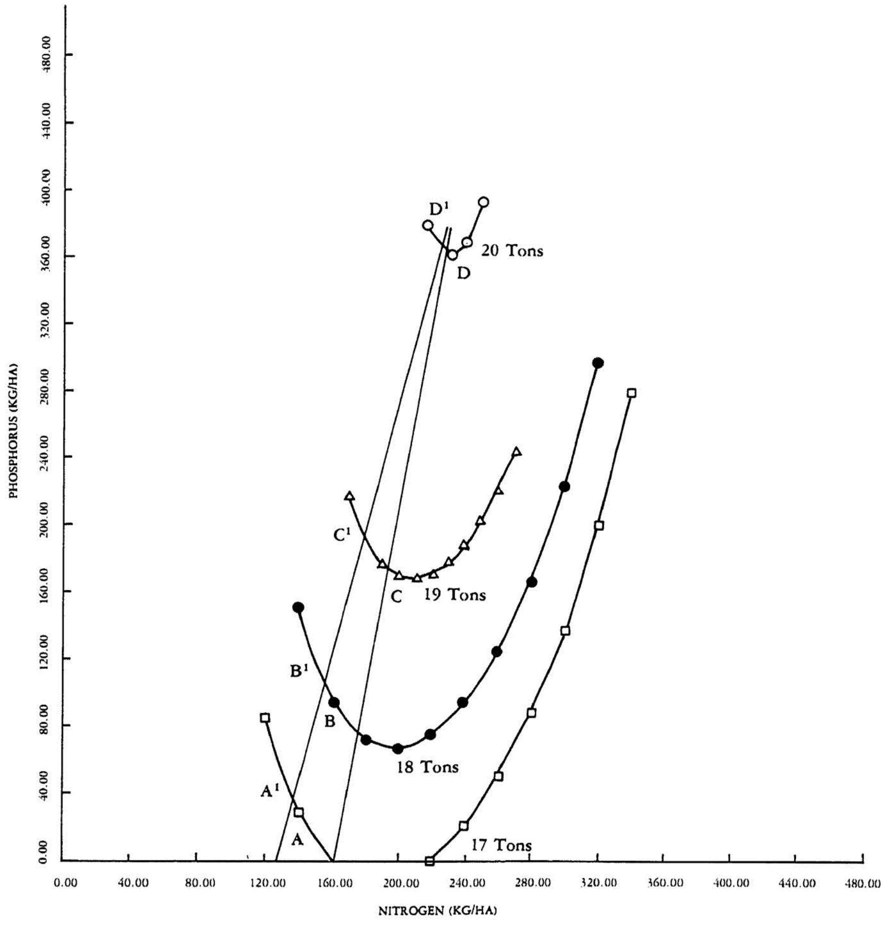

Figure 6. Estimated Isoquants and Expansion Paths for Potatoes, Farm Group V, Ecuador, 1975.

Economic Optima

In order to calculate economic optima, potato prices and the cost of nitrogen and phosphorus are needed. Potato prices used for this analysis were acquired from the Statistics Department of the Agricultural Ministry of Ecuador and the National Statistics Department. Due to deficiencies in price data, a relatively wide range of potato prices of 1,000, 2,250, and 3,500 sucres per ton were used. Urea and superphosphate prices were acquired from the Quito stores of the National Bank of Development of Ecuador. On the basis of past and future tendencies, prices of five, ten, and fifteen sucres per kilogram of urea and seven, fourteen and twenty-one sucres per kilogram of superphosphate are used.

Optimum level of Applying One Nutrient When The Level of the Other is Considered to be Fixed

To solve for the optimum level of nitrogen application the marginal physical product equation: $\delta Y/\delta N$, for each of the five groups is equated to the price ratio of Nitrogen (pN) to potatoes (pY) as shown in equation 8. Table 15 provides the optimum level of applying nitrogen for each farm group when phosphorus is held constant at selected levels.

Minimum Costs for Specific Yields

The combination of nitrogen and phosphorus which would be the least cost and hence the optimum application level for specific yields (Y*) and nutrient price ratios (C) is obtained by solving equation 33 which is the isocline or least-cost expansion path and equation 34 or the production function.

$$34.\quad N = \frac{Cb_1 - b_2}{b_5 + 2Cb_3} + \left(\frac{Cb_5 + 2b_4}{b_5 + 2Cb_3}\right) P$$

$$35.\quad Y^* = a + b_1N + b_2P - b_3N^2 - b_4P^2 + b_5NP$$

Because of the quadratic nature of the response function (thereby allowing both positive and negative marginal products), equation 34 has two solutions: one which is the maximum cost solution and the other the minimum cost solution, where the price of nitrogen multiplied by its amount plus the amount of phosphorus multiplied by its price is at a minimum (1).

Table 16 shows the combination of nitrogen and phosphorus needed to minimize fertilizer costs for selected yield levels and two different price ratios. These points are marked in Figures 2 to 6 as A, B, C, and in some cases D, for the first nutrient price ratio that appears in Table 16 and A^1, B^1, C^1, D^1 for the second nutrient price ratio for each respective farm group. These lines represent the expansion paths for the respective input price ratios.

Solution for the Two Variable Nutrients

The exact fertilizer combination and output level which provides the profit maximizing input-output combination are solved by setting the marginal physical products or partial derivatives for both nutrients ($\delta Y/\delta N$ and $\delta Y/\delta P$) equal to the price ratio of the nutrient to potatoes and simultaneously solving for the quantity of the nutrients (N and P) to apply (equation 11 and 12). These optima are reported by group for varying nitrogen, phosphorus and potato prices in Tables 17 and 18.

TABLE 15
Profit Maximizing Rates of Topdressed Nitrogen by Farm Group for Various Potato and Nitrogen Prices Under Specified Levels of Phosphorus Applications, Ecuador, 1975

Potato Prices (Sucres/ton)	Nitrogen Prices (Sucres/Kg)	Levels Phosphorus (Kg/Ha)	Profit Maximizing Quantities of Nitrogen (Kg/Ha)				
			Group I	Group II	Group III	Group IV	Group V
1000	5	0	167.8	77.5	224.6	165.3	176.6
1000	5	100	176.9	109.7	232.2	180.2	187.9
1000	5	200	186.0	142.0	239.8	195.1	199.2
1000	10	0	157.6	71.1	215.2	160.1	163.6
1000	10	100	166.7	103.3	222.8	175.0	174.9
1000	10	200	175.8	135.5	230.5	189.9	186.2
1000	15	0	147.4	64.6	205.9	154.8	150.7
1000	15	100	156.5	96.8	213.5	169.7	162.0
1000	15	200	165.6	129.0	221.1	184.6	173.3
2250	5	0	173.4	81.1	229.8	168.3	183.8
2250	5	100	182.6	113.3	237.4	183.2	195.1
2250	5	200	191.7	145.6	245.0	198.0	206.4
2250	15	0	164.4	75.4	221.4	163.6	172.3
2250	15	100	173.5	107.6	229.1	178.5	183.6
2250	15	200	182.6	139.8	236.7	193.4	194.8
3500	5	0	175.1	82.1	231.2	169.1	185.8
3500	5	100	184.2	114.4	238.9	184.0	197.1
3500	5	200	193.3	146.6	246.5	198.9	208.4
3500	10	0	172.2	80.3	228.6	167.6	182.1
3500	10	100	181.3	112.5	236.2	182.5	193.4
3500	10	200	190.4	144.7	243.8	197.3	204.7
3500	15	0	169.3	78.5	225.9	166.1	178.4
3500	15	100	178.4	110.7	233.5	181.0	189.7
3500	15	200	187.5	142.9	241.1	159.9	201.0

TABLE 16

Combinations of Nitrogen and Phosphorus Needed to Minimize Fertilizer Costs for Specified Potato Yield Levels by Farm Group at Different Factor Price Ratios, Ecuador, 1975

Yield Level (tons/Ha)	Price Ratio PN/PP	Optimum Kilograms of Nitrogen	Optimum Kilograms of Phosphorus
		GROUP I	
25	0.650	140.00	13.79
26	0.650	155.70	81.38
27	0.650	178.70	183.10
25	0.476	131.90	23.31
26	0.476	148.50	85.84
27	0.476	172.68	176.25
		GROUP II	
41	2.0	43.00	149.67
43	2.0	65.13	171.43
45	2.0	91.89	197.76
47	2.0	132.86	238.00
41	1.5	48.00	137.51
43	1.5	68.20	160.00
45	1.5	92.79	187.75
47	1.5	130.94	229.75
		GROUP III	
37	0.65	120.44	177.29
39	0.65	187.75	202.19
41	0.65	324.50	352.79
37	1.07	97.97	195.36
39	1.07	169.20	213.95
41	1.07	314.03	251.75
		GROUP IV	
32	0.650	54.42	131.20
34	0.650	105.47	146.06
36	0.650	168.75	164.40
32	0.476	71.736	123.77
34	0.476	100.43	133.20
36	0.476	177.90	158.76
		GROUP V	
18	1.07	77.35	173.71
19	1.07	176.17	191.85
20	1.07	360.00	225.75
18	0.476	103.85	155.69
19	0.476	194.44	178.75
20	0.476	367.00	222.00

TABLE 17

Optimum Combination of Nitrogen and Phosphorus for Farm Groups I, II, III, for Specific Potato, Nitrogen and Phosphorus Price Relationships, Ecuador, 1975

Potatoes Price (Sucres/Ton)	Nitrogen Price (Sucres/Kg)	Phosphorus Price (Sucres/Kg)	Optimum Fertilizer Use: Group I Kg of N per Ha	Group I Kg of P per Ha	Group II Kg of N per Ha	Group II Kg of P per Ha	Group III Kg of N per Ha	Group III Kg of P per Ha
1000	5	7	181.4	149.3	160.8	258.6	246.0	281.3
1000	10	7	170.4	140.1	153.2	255.0	236.4	277.9
1000	15	7	159.3	130.8	145.6	251.3	226.8	274.6
2250	5	14	189.1	170.8	165.7	262.5	251.9	290.2
2250	5	21	183.3	107.7	163.4	255.4	249.8	262.9
2250	10	14	184.2	166.7	162.3	260.9	247.7	288.7
2250	10	21	178.5	103.6	160.1	253.8	245.6	261.4
2250	15	14	179.2	162.5	158.8	259.2	243.3	287.2
2250	15	21	173.4	99.4	156.6	252.2	241.2	259.9
3500	5	7	198.5	257.7	169.9	272.5	256.2	327.7
3500	5	14	194.8	217.0	168.9	268.0	254.9	310.1
3500	10	7	195.4	255.1	167.8	271.5	253.5	326.8
3500	10	14	191.7	214.4	166.3	267.0	252.1	309.1
3500	15	7	184.8	171.0	162.7	261.4	250.7	325.8
3500	15	14	188.5	211.7	164.1	265.9	249.4	308.2

TABLE 18

Optimum Combination of Nitrogen and Phosphorus for Farm Groups IV, V For Specific Potato, Nitrogen and Phosphorus Price Relationships, Ecuador, 1975

Potato Price (Sucres/Ton)	Nitrogen Price (Sucres/Kg)	Phosphorus Price (Sucres/Kg)	Optimum Fertilizer Use: Group IV Kg of N per Ha	Group IV Kg of P per Ha	Group V Kg of N per Ha	Group V Kg of P per Ha
1000	5	7	212.3	315.6	198.1	190.6
1000	10	7	206.0	308.7	183.2	173.1
1000	15	7	199.7	301.8	168.3	155.6
2250	5	14	216.9	326.8	209.3	225.5
2250	5	21	212.7	298.1	198.4	129.2
2250	10	14	214.2	323.8	202.7	217.5
2250	10	21	209.9	295.1	191.8	121.5
2250	15	14	211.3	320.6	195.9	209.6
2250	15	21	207.1	292.0	185.1	113.5
3500	5	7	223.7	366.7	226.2	358.1
3500	5	14	220.9	348.2	219.3	296.1
3500	10	7	221.9	364.7	222.0	353.1
3500	10	14	219.1	346.2	215.0	291.1
3500	15	7	220.1	362.7	217.7	348.1
3500	15	14	217.3	344.2	210.7	286.1

CHAPTER IV

CONCLUSIONS

After considering numerous algebraic production function forms it was concluded that the quadratic form provided a good fit to potato input-output relations in the highlands in Ecuador. The two independent variables considered were nitrogen (N) and phosphorus (P). Predicted production functions for each farm group indicated that diminishing returns existed for each factor, but that there was a positive interaction between the two factors (N and P).

As expected, estimated yields first increased and then decreased as application levels of topdress nitrogen and phosphorus were increased. In addition, the magnitude of yield increases were the largest when the level of nitrogen and/or phosphorus applied was low. The level of N and P to apply in maximizing profit depends upon input (N and P) and output (potato) prices. Economically it is not practical to increase the level of topdress nitrogen or phosphorus beyond those quantities needed to reach maximum yields (Table 12) even if inputs are a free resource. However, profits are usually maximized at an input level which produces a yield level short of yield maximization.

In solving for the economic optimum level of fertilizer application three basic analyses were performed: (a) calculation of the optimum level of applying N or P when the level of one is considered to be fixed; (b) calculation of the minimum cost combination of N and P for given yield levels; and (c) calculation of the optimum N and P combination under varying N,P and potato prices.

Considering the optimum level of applying one nutrient when the level of the other is considered fixed leads to the optimum level of applying one nutrient after considering the magnitude of the marginal products and the nutrients to potato price ratio. Examination of Table 15 shows that when the price of nitrogen is raised, with all else held constant, profits are maximized by restricting the use of this input. Conversely, when the nitrogen price is reduced profits are maximized by using more nitrogen. The same is true with respect to the use of phosphorus. Also, the profit-maximizing quantities of the nitrogen increase as the levels at which phosphorus is fixed are increased. This is explained by the positive nitrogen phosphorus interaction coefficient appearing in the production function for all farm groups.

The combinations of nitrogen and phosphorus, which provides the lowest cost for given yield levels, are shown in Tables 13 and 14. The slopes of the yield isoquants, and hence the marginal rates of substitution between nutrients change with higher yield levels. In essence, the relative combination of N and P changes as higher yield levels are obtained. The mixture to minimize costs contains relatively more nitrogen than phosphorus as yield levels increase. This distinction must be made in fertilizer recommendations as in some instances the same fertilizer mix is recommended for a given soil under varying productivity situations. As shown here, the yield levels as well as the soil type needs to be accounted for. In addition, the marginal rates of substitution ($\delta P/\delta N$) as shown in Tables 13 and 14 are negative and decreasing over a range of yields for each farm type. This is also shown by the convexity of the isoquants in Figures 2 through 6.

The negative marginal rates of substitution mean that for a given yield level if the amount of nitrogen applied is increased the amount of phosphorus necessary to maintain that yield level decreases. The opposite is true for the uneconomic application levels or where the marginal rates of substitution are positive.

The isoquants in Figures 2 through 6 all indicate that there are decreasing returns to fertilizer as an input as yield levels increase. This is evident in that the successive isoquants get further and further apart.

With respect to the optimum combination of nutrient input and respective output levels (Tables 17, 18), it is seen that a reduction in potato prices not only reduces the total quantity of fertilizer used on potatoes but also may call for a change in the fertilizer grade. From these tables it can also be concluded that the best operating conditions imply: a) increased input levels as input becomes cheaper relative to output; b) increased usage of an input when its price decreases relative to the price of the other input; c) increased profit as input prices decline relative to the price of output; and d) the ratios of input and output prices and not the absolute price levels affect the optimum operating conditions.

Limitations of the Study

One of the limitations of this study was the absence of soil test values on the level of existing soil nutrients. This limitation precluded use of the soil test values as independent variables in the prediction equation. These levels affect crop yields being studied but were unavailable for each experimental plot. Soil test values are acknowledged to be particularly important to estimate effects of topdress phosphorus on yields. With respect to this point, it should be pointed out that the Ecuadorian highland soils, being of volcanic origin, are on the average phosphorus deficient. Thus, a lower level of phosphorus availability exists in the soils as contrasted to other soils in the world. Due to this low level of inherent phosphorus availability, the lack of phosphorus soil test values should not have a significant effect on the prediction equation.

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APPENDIX

The data used for corn and wheat analysis were experimental, cross sectional and collected under the block experimental design. The nutrients studied were topdress nitrogen and phosphorus. Tables A.1 and A.2 show the different levels of nitrogen and phosphorus applications and the observed yields for corn and wheat respectively. The analysis of variance performed showed that differences between the yield means for wheat and corn are significant.

The least significance difference permitted the formation of the following groups: For corn, farm group one is formed by farm four; farm group two: by farms one, five and eight; farm group three: by farms five and nine; farm group four: by farms three, seven and nine; farm group five: by farms six and two. For wheat, farm group one is formed by farms one and four; farm group two: farms two, five and four; and farm group three by farm three.

For each farm group an estimation of the production function was made. The only variables considered were nitrogen and phosphorus. For all farm groups the quadratic equation gave the best fit. The estimated corn production functions for farm groups one through five are presented in equation C1 through C5 respectively. Estimated wheat production functions for the three wheat farm groups are given in equations W1 through W3 respectively.

Tables A.3 and A.4 show the optimum combination of nutrients, (nitrogen and phosphorus) for specific nitrogen, phosphorus and corn prices. For wheat, the optimum combination of N and P under specific N, P and wheat prices are presented in Table A.5.

$$(C1)\quad Y = 78.98358 + 0.65486N + 0.341538P - 0.00224297N^2 - 0.0012315P^2 + 0.00102155NP$$

$$(C2)\quad Y = 80.065328 + 0.568085N + 0.07375P - 0.0014434N^2 - 0.00031589P^2 + 0.00029308NP$$

$$(C3)\quad Y = 70.62852 + 392608N + 0.21456P - 0.00080268N^2 - 0.0005941P^2 + 0.0000301NP$$

$$(C4)\quad Y = 71.167993 + 0.30523746N + 0.11754P - 0.0007333N^2 - 0.000040107P^2 + 0.0002078NP$$

$$(C5)\quad Y = 31.579388 + 0.322515N + 0.156275P - 0.0008449N^2 - 0.000593P^2 + 0.0003247NP$$

$$(W1)\quad Y = 1633.2956 + 9.30435N + 6.31386P - 0.05671yN^2 - 0.027363P^2 + 0.042665NP$$

$$(W2)\quad Y = 1185.943 + 12.4172N + 4.00486P - 0.042369N^2 - 0.01255P^2 + 0.01465NP$$

$$(W3)\quad Y = 2765.4698 + 7.58252N - 3.72215P - 0.029297N^2 - 0.01614P^2 + 0.014965NP$$

TABLE A.1

Observed Corn Yields Resulting From Different Levels of Topdress Nitrogen and Phosphorus for Each Farm, Ecuador, 1975

Treatments in Kg/Ha		Observed Corn Yields in Kg Per Hectare								
Nitrogen	Phosphorus	Farm 1 Antuntagui	Farm 2 Cotacachi	Farm 3 Ureugui	Farm 4 Imantag	Farm 5 Tababel	Farm 6 Conocoto	Farm 7 Checa	Farm 8 Cutuglagua	Farm 9 El Valle
0	200	67.91	24.79	60.21	92.50	86.46	52.96	86.67	82.92	83.77
50	200	110.21	57.29	93.54	151.67	130.42	55.65	93.13	110.21	94.80
100	200	121.04	72.92	107.71	151.04	135.42	65.77	121.46	120.00	103.62
150	200	133.96	83.34	117.70	188.54	137.92	61.96	120.83	145.00	116.84
200	200	161.25	93.75	92.71	185.83	148.33	72.94	122.71	143.00	143.30
250	200	159.0	88.96	130.83	160.83	133.75	63.54	119.58	126.25	141.09
300	200	147.50	83.12	110.83	149.79	145.63	82.85	122.09	142.29	143.30
200	0	102.50	38.13	102.50	122.29	128.34	65.96	103.54	155.42	103.62
200	50	166.66	70.00	110.00	139.17	151.88	68.04	115.21	130.21	101.42
200	100	159.80	76.88	112.71	176.04	140.00	90.73	117.71	149.79	121.25
200	150	155.41	88.96	104.37	163.13	135.42	71.31	106.46	162.71	123.46
200	250	139.37	94.38	120.83	148.33	145.84	69.81	114.17	150.21	116.84
200	300	173.12	92.92	113.12	173.96	130.21	61.72	118.75	161.67	141.09
0	0	46.67	23.96	62.08	62.69	74.58	39.38	86.88	90.21	61.73
50	50	138.33	53.96	93.75	146.25	117.50	30.83	96.07	117.92	90.39
150	150	140.62	79.16	111.88	168.96	138.54	88.33	116.67	141.04	123.46
250	250	147.91	81.25	111.04	190.83	115.63	82.25	121.67	148.62	127.87
300	300	163.96	92.01	108.54	167.92	123.73	55.96	120.84	151.67	127.87

TABLE A.2

Observed Wheat Yield Resulting From Different Levels of Topdress Nitrogen and Phosphorus for Each Farm, Ecuador, 1975

Treatments in Kg/Ha		Observed Wheat Yields in Kg/Per Hectare				
Nitrogen	Phosphorus	Farm 1 Aychapieho	Farm 2 S'Javier	Farm 3 Remonta	Farm 4 Paquistancia	Farm 5 12a Vieja
0	0	1813.00	1070.89	2789.22	1426.75	719.50
50	160	2713.25	2078.67	3177.80	2140.50	2386.25
100	120	2884.50	2354.13	3700.07	2667.25	2234.75
150	180	3343.50	2793.02	3771.53	3294.25	2320.00
200	240	3122.75	2349.28	3567.54	3137.00	2793.50
250	300	3020.50	2548.69	3703.79	3097.50	2935.50
150	0	1883.00	2311.11	3050.18	1299.50	2594.50
150	60	2883.00	2466.03	3845.87	2430.25	2462.00
150	120	3106.50	2002.23	3782.85	2916.25	2879.00
150	240	3090.74	2567.80	3406.98	3286.75	2812.50
150	300	3267.75	2435.57	3784.88	3144.75	3011.25
0	180	2235.25	1475.46	2821.16	1591.25	1212.00
50	180	2847.00	2381.23	3637.77	2243.00	1903.25
100	180	2782.50	2456.24	3409.02	3030.75	2689.25
100	180	2825.50	2687.06	3967.44	2911.50	2623.25
150	180	2270.00	2244.47	3587.82	2978.25	2983.00
150	180	3172.75	2509.20	3865.74	2958.75	2556.75

Tables A.3 and A.4 show the optimum combination of nutrients, (nitrogen and phosphorus) for specific nitrogen, phosphorus and corn prices. For wheat, the optimum combination of N and P under specific N, P and wheat prices are presented in Table A.5.

TABLE A.3

Optimum Combination of Nitrogen and Phosphorus for Farm Group I, II, III, for Specific Corn, Nitrogen and Phosphorus Price Relationships, Ecuador, 1975

			OPTIMUM FERTILIZER USE					
			GROUP I		GROUP II		GROUP II	
Corn Price (Sucres/Kg)	Nitrogen Price (Sucres/Kg)	Phosphorus Price (Sucres/Kg)	Kg of N per Ha	Kg of P per Ha	Kg of N per Ha	Kg of P per Ha	Kg of N per Ha	Kg of P per Ha
150	5	7	183.1	195.7	198.9	135.1	224.1	141.6
150	5	14	178.3	174.7	191.1	57.6	224.0	102.4
150	10	7	174.9	192.3	186.8	129.5	203.3	141.6
150	10	14	170.1	171.3	179.0	52.0	203.3	102.3
150	15	7	166.7	188.9	174.7	123.9	182.6	141.5
150	15	14	161.9	167.9	166.8	46.4	182.5	102.2
225	5	7	187.4	203.8	205.6	162.8	231.1	154.7
225	5	14	184.3	189.8	200.4	111.2	231.0	128.6
225	10	7	182.0	201.5	197.5	159.1	217.2	154.7
225	10	21	175.6	173.6	192.3	107.4	217.1	102.3
225	15	7	176.5	199.2	189.4	155.3	203.4	154.7
225	15	14	173.3	185.3	184.2	103.7	203.3	128.5
300	5	7	189.6	207.8	208.9	176.7	234.5	161.3
300	5	14	187.2	197.4	205.0	137.9	234.5	141.7
300	5	21	184.8	186.9	201.1	99.2	234.5	122.0
300	10	14	183.1	195.7	198.9	135.1	224.1	141.6
300	10	21	180.7	185.2	195.0	96.4	224.1	122.0
300	15	14	179.0	194.0	192.9	132.3	213.7	141.6
300	15	21	176.6	183.5	189.0	93.6	213.7	122.0

TABLE A.4

Optimum Combination of Nitrogen and Phosphorus for Farm Groups IV, V for Specific Corn, Nitrogen and Phosphorus Price Relationships, Ecuador, 1975

			OPTIMUM FERTILIZER USE			
			GROUP IV		GROUP V	
Corn Price (Sucres/Kg)	Nitrogen Price (Sucres/Kg)	Phosphorus Price (Sucres/Kg)	Kg of N per Ha	Kg of P per Ha	Kg of N per Ha	Kg of P per Ha
150	5	7	218.0	166.6	199.4	147.0
150	5	14	205.8	104.0	191.4	105.5
150	10	7	193.6	157.8	178.5	141.3
150	10	14	181.3	95.2	170.6	99.8
150	15	7	169.1	149.1	157.7	135.6
150	15	14	156.9	86.5	149.7	94.1
225	5	7	230.3	190.4	209.0	162.7
225	5	14	222.1	148.7	203.6	135.1
225	10	7	214.0	184.6	195.1	158.9
225	10	21	197.6	101.1	184.4	103.6
225	15	7	197.7	178.7	181.2	155.1
225	15	14	189.5	137.0	175.9	127.5
300	5	7	236.4	202.3	213.8	170.6
300	5	14	230.3	171.0	209.8	149.8
300	5	21	224.1	139.7	205.8	129.1
300	10	14	218.0	166.6	199.4	147.0
300	10	21	211.9	135.3	195.4	126.2
300	15	14	205.8	162.2	188.9	144.1
300	15	21	199.7	130.9	185.0	123.4

TABLE A.5

Optimum Combination of Nitrogen and Phosphorus by Farm Group for Specific Wheat, Nitrogen and Phosphorus Price Relationships, Ecuador, 1975

			OPTIMUM FERTILIZER USE					
			GROUP I		GROUP II		GROUP III	
Wheat Price (Sucres/Kg)	Nitrogen Price (Sucres/Kg)	Phosphorus Price (Sucres/Kg)	Kg of N per Ha	Kg of P per Ha	Kg of N per Ha	Kg of P per Ha	Kg of N per Ha	Kg of P per Ha
3.3	5	7	137.9	184.1	157.5	167.0	131.8	110.7
3.3	5	14	117.3	129.3	141.3	73.0	112.8	36.2
3.3	10	7	119.0	169.4	137.6	155.4	102.5	97.1
3.3	10	14	98.4	114.6	121.4	61.4	83.5	22.6
3.3	15	7	100.1	154.7	117.8	143.8	73.2	83.5
3.3	15	14	79.5	99.8	101.5	49.8	54.1	9.0
4.4	5	7	147.8	201.5	166.6	193.4	143.9	132.7
4.4	5	14	132.3	160.4	154.4	122.9	129.6	76.8
4.4	5	21	116.9	119.3	142.2	52.4	115.4	20.9
4.4	10	14	118.2	149.4	139.5	114.2	107.6	66.6
4.4	10	21	102.7	108.2	127.3	43.7	93.4	10.7
4.4	15	14	104.0	138.3	124.5	105.5	85.6	56.4
4.4	15	21	88.5	97.2	112.4	35.0	71.4	0.5
5.5	5	7	153.7	212.0	172.0	209.2	151.2	146.0
5.5	5	14	141.4	179.1	162.2	152.8	139.7	101.2
5.5	5	21	129.0	146.2	152.5	96.4	128.3	56.5
5.5	10	7	142.4	203.1	160.0	202.3	133.6	137.8
5.5	10	14	130.0	170.2	150.3	145.9	122.2	93.1
5.5	15	7	131.1	194.3	148.1	195.3	116.0	129.6
5.5	15	14	118.7	161.4	138.4	138.9	104.6	84.9