II-Nitrogen as a Factor in Corn Production

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THE FILLING OF AN EAR ON A CORN PLANT
AS A BIOPHYSICAL FUNCTION

I. Theoretical

Favorable conditions for growth bring corn to antheses with a well developed sturdy plant. Thereafter it functions to fill the kernels of an ear. This analysis concerns the contribution of nitrogen to the filling process.

Number 2 shelled corn contains 8.7% protein ("Feeds and Feeding," Henry and Morrison). This converts to 1.4% nitrogen. 56 pounds of shelled corn contains 0.78 pounds of nitrogen per bushel. The reciprocal expresses an efficiency $E$ of 1.28 bushels of corn per pound of nitrogen. The question to be resolved is the fraction of $E$ as a supply of nitrogen to a plant that reaches the grain.

The supply of nitrogen filling an ear ranges from zero, designated as $x_0$, through increasing amounts $x$ producing yield $y$, to $X$ for which yield $Y$ is a maximum at stand densities limiting ear formation to one per plant. The yield response function, $y = f(x)$, is delineated by the parameters $x_0$, $X$, $Y$, and $E$.

The synthetic system of a corn plant is complete at antheses. Nitrogen assimilated by a plant after antheses is divided between two phases of the system: one, the transport system buffered through wet-dry cycles by storage cells of the pith; the other, the protein accumulating in kernels on the ear. The transport system includes above and below ground parts of the plant along with soil solution moving nitrogen to the root. Delivery of the last molecule of protein to the ear occurs with an amount of nitrogen in the supply system equal to that stored in the seed. The supply system as a generator is matched to its load $Y$ for maximum transfer of power.

The division of an increment of nitrogen $dx$ between the supply system $ds$ and the grain $dy$ at any position $x$ between $x_0$ and $X$ is $dx = ds + dy$. The fraction-producing grain $\frac{dy}{dx} = E \frac{X - x}{X - x_0}$, where $E$ is the bushels of grain produced by one pound of nitrogen reaching the grain.
"The Law of Diminishing Returns," a familiar expression describing returns from applied fertilizers, recognizes that not all applied fertilizers get into the grain that is produced. A yield response function is a physical expression of the law of diminishing returns.

Integrating the derivative \( \frac{dy}{dx} \) yields

\[
y = -\frac{E}{2(X - x_0)} (X - x)^2 + C
\]

When \( x = X \), \( X - X = 0 \), and \( y = Y \), giving \( C = Y \). Also, \( \frac{E}{2(X - x_0)} \) contains all constants for which a simple single one \( c \) will suffice producing, \( y = Y - c(X - x)^2 \)

Upon squaring and collecting terms, equation (2) produces

\[
y = (Y - cX^2) + (2cX)x - cx^2
\]

from which,

\[
y = a + bx - cx^2
\]

The second degree polynomial or quadratic, accepted by many as an empirical expression depicting a relationship, is used with no attempt to assign biological significance to the coefficients. The derivation of equation (3) provides coefficients that establish physical attributes for the coefficients \( a \), \( b \) and \( c \) of equation (4). That of \( c \) which appears throughout includes three parameters in \( c = \frac{E}{2(X - x_0)} \).

The physical quantities of the coefficients of \( x \) in equation (3), which are represented by single coefficients in equation (4), require explanations to give them meanings that provide comprehension. The variable \( x \) also requires explanations if it is to be used properly.

Initially, the values \( x \) and \( X \) will be designated as amounts of nitrogen entering a plant during the grain producing interval. This then becomes a theoretical ideal situation not encountered in practice. The procedure establishes a base or state of reference by which to judge the performance of practical systems.

The coefficients of equation (4) take on additional meaning from the components they represent in equation (3). The meaning of the parameters \( X \) and \( Y \) are straightforward, \( X \) being the pounds per acre of nitrogen required after antheses to bring the yield to a
maximum of $Y$ bushels per acre. The meaning of the constant $c$ that appears in all three coefficients was lost when the quantity it represents was replaced. That quantity $\frac{E}{2(X - x_0)}$ of equation (1) contains $x_0$ which has a value of zero with respect to the grain producing interval. It does not represent zero with respect to the supply of nitrogen serving the plant growing from a seed. It is not a quantity subject to evaluation by direct measurement.

Equation (1) with $C = Y$ and $x = x_0 = 0$, for which $y = 0$ give $E = \frac{2Y}{X}$ and $Y = \frac{EX}{2}$. Either way, only half the nitrogen $X$ required to produce $Y$ ends up in the grain. $X = \frac{2Y}{E}$ and $\frac{1}{E} = 0.78$ from which $X = 2(0.78Y)$.

The value $c$ replaced $\frac{E}{2(X - x_0)}$. With $x_0 = 0$, and $E = \frac{2Y}{X}$, then $c = \frac{Y}{X^2}$. Introduced into equation (3) when $x = X$ giving $y = Y$ there results, $Y = (Y - \frac{YX^2}{2X^2}) + \left(\frac{2Y}{X^2}\right)X - \left(\frac{Y}{X^2}\right)X^2$, or $Y = (Y - Y) + 2Y - Y$. The value of $a$ in the ideal condition of equation (3), when represented by equation (4), is zero.

By equation (3), $b$ of equation (4) equals $2cX$, with $c = \frac{Y}{X^2}$, $b = \frac{2Y}{X}$.

Hence, the coefficient $b$ of equation (4) is the efficiency $E$ with a numerical value of 1.28 bushels of corn per pound of nitrogen in the grain of an ideal system. Finally, there is $c$ in the last coefficients of equations (3) and (4). Taking its value as $\frac{Y}{X^2}$, replacing $X^2$ by $[2(0.78Y)]^2$ converts $\frac{Y}{X^2}$ to $\frac{0.411}{Y}$. As a consequence, an ideal response function may be derived for any value of $Y$ attained through trial and error methods of plant populations, fertilizer kinds and amounts, management practices and tests of hybrid varieties. The Ideal reference state for a yield function for which $x$ represents the quantity of nitrogen assimilated by the crop becomes,

$$y = 0 + 1.28x - \frac{0.411x^2}{Y}.$$
II. Generating an Ideal Response Function for a Single Yield Y.

Minor, Morris, Mason, Knerr and Lawman, 1989. CORN, Special Report 404, Agricultural Experiment Station, College of Agriculture, University of Missouri, Columbia, Missouri.

Of 77 hybrid varieties tested under irrigation on the Agronomy Research Center, Columbia, Mo. all yielded in excess of 200 bushels per acre. The best yielded 268 bushels per acre. For this one yield, equation (5) provides a response function of,
\[ y = 0 + 1.28x - 0.00153x^2 \]
the value 0.00153 =0.411/268. The first derivative set equal to zero and solved for \( x \) required to produce yield \( Y \), namely
\[ \frac{dy}{dx} = 1.28 - 2(0.00153)x = 0, \]
gives
\[ x = \frac{1.28}{0.00306} \text{ or } 418 \text{ pounds of nitrogen per acre.} \]
Of this, 325 pounds were applied as fertilizer, the remainder, 93 coming from the soil and or loss after application. Such high yields seldom are obtained unless a large turnover is provided by adequate crop treatments in previous seasons.

The yield function solved for the following values of \( x \) reflect the responses to be expected provided the plant populations are adjusted to accommodate the yields. A supply \( x \) of half the maximum has been included that substantiates the diminishing returns from treatments approaching the maximum. The first 209 pounds of nitrogen produces 200 bushels per acre. The second 209 pounds adds only 68 bushels.

Values of \( y \) for five amounts of nitrogen obtained by equation (5) were:

<table>
<thead>
<tr>
<th>N-lbs./acre.</th>
<th>y-bu./acre.</th>
<th>0.6 lb. ears/ac.</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>113</td>
<td>13,200</td>
</tr>
<tr>
<td>200</td>
<td>193</td>
<td>22,800</td>
</tr>
<tr>
<td>300</td>
<td>246</td>
<td>28,700</td>
</tr>
<tr>
<td>400</td>
<td>267</td>
<td>31,200</td>
</tr>
</tbody>
</table>
Equation (5) provides a standard by which to interpret results of fertilizer investigations. For fertilizer data to be valid, it should be obtained from soil with a history of uniform past treatment. Otherwise carryover of nitrogen from previous variable treatments confuse the amounts furnished by the soil.

Amounts of fertilizer applied to the soil are not a measure of the amount that reaches the crop. Losses by vaporization, leaching, weeds, runoff and erosion and fixation by organisms if not by minerals may occur. A yield response function relates yields to amounts of fertilizer that are applied. A bar above $x$ and $X$ distinguishes the variable $x$ of a fertilizer amount from $x$ and $X$ as suppliers of nitrogen assimilated by a crop filling ears.

Amounts of nitrogen applied to soils by fertilization are supplemented by amounts from other sources, carryover from previous treatments, releases from crop residues, liberation from decomposing humus and even from precipitation containing nitrogen fixed by lightening discharges. Management practices of applying, times of applying, incorporation in soil and chemical forms have received attention from investigators. Establishing the amounts of nitrogen assimilated by a crop from soil and fertilizer are ascertained best from the yield of a crop as a consequence of the biophysical processes that function to fill kernels on an ear of corn.

Equation (5), as its development has been explained, is an attempt to provide a state of reference based on the physical mechanisms operating in the grain producing interval of a corn plant. A viable standard provides a means of estimating amounts of nitrogen reaching a plant from applied fertilizer and soil.

An example of a fertilizer investigation will be presented as an approach to evaluating the percentage of nitrogen in applied fertilizer that is assimilated by the crop and the amount contributed by the soil that ended up in the harvested yield. The results, reported in special report 13 for data collected in 1961, reflect an unusually high recovery of nitrogen from the fertilizer that was added.
Special Report 13 was the first of seven reports from 1961 to 1968 inclusive involving four locations in the state. The results for the first season were obtained from soils that did not contain carryover from variable treatments of previous seasons.

The results reported herewith were from a sandy loam soil cropped previously to cotton at the Southeast Missouri Research Center, Portageville, Missouri. Corn yields as bushels per acre are a function of four plant populations and seven amounts of applied nitrogen. Yields of the various plant populations were regressed against amounts of applied nitrogen. Figure I.

\[ y = 58.5 + 0.794x - 0.001946x^2. \quad R^2 = 0.963. \]

The bar above \( x \) signifies amount of nitrogen applied to distinguish it from \( \bar{x} \) of the combined amounts of nitrogen from both fertilizer and soil and from \( x \) the amount assimilated into the grain.

The solution of the regression equation reveals the dominant effect of nitrogen with respect to the impact of the populations of plants that were employed. Expressing yield as a function of applied nitrogen, then the primary effect of population was to partition the nitrogen to the number of ears produced. Low plant populations combined with adequate nitrogen completely filled the ears. High plant populations with limited amounts of nitrogen produced ears too small to be acceptable. Evidence from experience suggests a fair compromise between number of ears and size ears at 0.6 pounds per ear. Although somewhat higher yields may be achieved with 0.5 pound ears, the sturdiness of the plant, when harvest is delayed, mitigates against such high populations of plants. The maximum yield of 139.5 bu./acre provided ear weights of 0.80, 0.64, 0.51 and 0.48 pounds at the respective populations. The minimum yield of 58.5 bu./acre provided 0.33, 0.27, 0.21 and 0.20 pound ears at the same respective populations. These ear weights are based upon 70 lbs. of ear corn per bushel.
A. The amount of nitrogen $X$ as fertilizer required for maximum yield $Y$.

Setting the first derivative equal to zero and solving for $X$.

$$\frac{dY}{dX} = 0.794 - 2(0.001946)X = 0.$$  

$$X = \frac{0.794}{0.003892} = 204.$$

204 pounds of nitrogen as fertilizer produced maximum yield $Y$. Solving the response function at 204 pounds of added nitrogen-

$$Y = 58.5 + (0.794)204 - (0.001946)(204^2) = 139.5$$

B. The fertilizer equivalent of the amount of nitrogen contributed by the soil.

The contribution of nitrogen as fertilizer equivalent from the soil may be estimated in two ways.

1. Extrapolating the response function to zero by setting $y = 0$ and solving the quadratic for $X$ using the binomial reduction formula:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -64.$$  

The 64 pounds extrapolated below zero is the fertilizer equivalent of nitrogen contributed by the soil.

2. Shifting the axis of the response function to cover the spectrum from zero to the total supply giving a maximum yield. The shift combines both soil and fertilizer in the response function with origin at zero.

The circumflex will be used to identify the variables with the axis shifted to bring the origin to zero The value of $a$ in equation (4) is zero when the variable $\hat{X}$ is zero. $a$ represents the quantity $Y - c\hat{X}^2$ of equation (3) of which the maximum yield $Y = 139.5$ bushels,

$$c = 0.001946 \text{ and } Y - c\hat{X}^2 = 0.$$  

Solving, $\hat{X} = \sqrt{\frac{Y}{c}} = \sqrt{\frac{139.5}{0.001946}} = 268$ pounds of nitrogen as fertilizer equivalent. Subtracting 204 pounds for $\hat{X}$ from 268 pounds for $X$ yields 64 pounds contributed by the soil.
Completing the extended form of the response function for nitrogen applied in fertilizer by evaluating the coefficient \( b \) for the variable \( \hat{x} \) and the parameter \( \hat{X} \). The value of the coefficient \( c \) obtained from the response function for added fertilizer in terms of \( \hat{x}_0 \) remains unchanged in shifting the origin from \( \hat{x}_0 \) to \( \hat{x} = 0 \).

The value of the coefficient \( b \) as derived from equation (3), \( b = 2c\hat{X} \) or \( 2(0.001946)(268) = 1.043 \). The yield response function for nitrogen from both soil and fertilizer as measured by that added in fertilizer becomes, \( y = 0 + 1.043\hat{x} - 0.001946\hat{x}^2 \).

Compare with, \( y = 0 + 1.28x - 0.002946x^2 \) for the reference state. The value of \( c \) for \( Y = 139.5 \) from equation (5) is \( 0.411/139.5 = 0.002946 \). Solving the equation of the reference state for \( X \), the amount of nitrogen responsible for the yield \( Y \), where \( X = \sqrt{\frac{Y}{c}} \) from equation (3) is 217 pounds per acre.

The results of the analysis of the fertilizer investigation provide three values for the amount of nitrogen involved in producing yield \( Y \). Figure 2.

1. The amount of nitrogen in the fertilizer that was added, 204 pounds.
2. The amount of nitrogen in the fertilizer that was added extended to include that provided by the soil, 268 pounds as fertilizer equivalent.
3. The amount of nitrogen assimilated by the plant in grain production, 217 pounds.

81% of the nitrogen supplied by soil and fertilizer was assimilated \( 268 - 204 = 64 \) pounds from the soil as fertilizer equivalent. 81% of this represents the actual amount delivered by the soil, namely 54 lbs.
CORN YIELD AS A FUNCTION OF APPLIED N
AT FOUR PLANT POPULATIONS

\[ y = 58.453 + 0.79444x - 1.9463e^{-3}x^2 \quad R^2 = 0.963 \]

Figure 1.

CORN YIELD, \( y = f(N) \).

Figure 2.
Some Considerations In Fertilizing Corn With Nitrogen

Fertilizing corn with nitrogen involves considerations of costs and returns, forms of nitrogen, time and methods of applications, maintaining soil fertility, carryover of nitrogen from previous crops, controlling pollution of ground water, and variability of the environment over a period of years with and without irrigation. Each of these is a topic unto itself. Each must be evaluated in its own sphere of influence before any attempt may be made to assemble the lot into a whole.

An acceptable yield function for nitrogen provides a base from which to deal with costs and returns; specifically, identifying the intensive margin where costs and returns are in balance. The production function was generated about a parameter \( E \), an efficiency term expressing the bushels of corn containing one unit of nitrogen. This was followed by a fraction partitioning the portions of \( E \) to each of two phases of the production system; one being the grain, the other being the service system by which nitrogen moves from soil through the plant to the synthetic mechanism in the leaf.

The production function as expressed by the quadratic \( y = a + bx - cx^2 \) identifies the coefficient \( b \) as the effective efficiency \( E' \) operating at \( x_0 \) where the prior supply of nitrogen was provided by the soil in creating the yield \( a \) with no applied nitrogen as fertilizer. The effective efficiency \( E' \) at any value of \( x \) between \( x_0 \) and \( X \) is the derivative \( \frac{dy}{dx} = b - 2cx \). The value of the derivative declines linearly from the value \( b \) at \( x_0 \) to zero where \( 2cx = b \), \( x \) being the value of \( x \) for which yield \( y \) is a maximum. Figure 3.
The Law of Diminishing Returns

Accumulation of Nitrogen in Service System

Accumulation of Nitrogen in Grain

Figure 3.

The Intensive Margin, \( x \)

Grain

Service System

Figure 4.

\[
\frac{dy}{dx} = E' \\
$/bu. \times dy = $/lb. \times d \times \\
E' = b - 2cx \\
\frac{dy}{dx} = \frac{$/lb.}{$/bu.} = E'_i \\
x = \frac{b - E'}{2c} \\
x_i = \frac{b - E_i}{2c}.
\]
The Amount of Applied Nitrogen at the Intensive Margin.

The intensive margin is identified as the value of an increment of yield $/bu. x dy equal to the cost of an increment $/lb.x dx. producing

$$\frac{dy}{dx} = \frac{$/lb.}{$/bu.}$$

The experiment at Portageville, Mo. as reported by Kroth and Doll provided the function, (Fig. 1), $y=58.5 + 0.794x - 0.001946x^2$.

Current prices of $0.25 per pound for nitrogen and $2.50 per bushel for corn place the intensive margin at 178.3 pounds of nitrogen per acre. Solving for $x$ at the intensive margin,

$$\frac{0.794 - 0.1}{2*0.001946} = 178.3 \text{ lbs/acre of applied nitrogen.}$$

By comparison the quantity of applied nitrogen required for the maximum yield, for which $x = \frac{0.794}{2*0.001946} = 204.0 \text{ lbs./acre.}$

The saving of 25.7 pounds of nitrogen valued at $6.42 was accompanied by a reduction in yield from 81.0 to 79.7 or 1.3 bu./acre valued at $3.25. Note - the yields are those developed by the fertilizer. They do not include the 58.5 bushels produced by the soil with no applied nitrogen. Operating at the intensive margin, the savings are within the experimental error of yield measurements.

IV. Conclusion.

The consequences of the developments in this study support the concept that the coefficients of a second degree polynomial, as they occur in a yield response function, do possess a biophysical significance. The concept also establishes a standard or reference state for grain formation by corn as a function of nitrogen that is assimilated. It permits evaluating the percentage of added nitrogen that is effective. It provides a means of estimating the amount of nitrogen contributed by sources other than that added in fertilizer.