

Public Abstract

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Graduation Term: SP 2008

Department: Mathematics

Degree: PhD

Title: Sections of Complex Convex Bodies

The main idea of the Fourier analytic approach to sections of convex bodies is to express different parameters of a body in terms of the Fourier transform and then apply methods of Fourier analysis to solve geometric problems. The original Fourier approach to sections of convex bodies applies to convex bodies in \mathbb{R}^n to the power of n .

This thesis is focused at extending this approach to the complex case, where origin symmetric complex convex bodies are the unit balls of norms in \mathbb{C}^n . If considered as convex bodies in \mathbb{R}^{2n} complex convex bodies acquire the property of invariance with respect to certain rotations. This crucial observation arises from the nature of the norm of the bodies. Also complex hyperplanes correspond to only few of $(2n - 2)$ -dimensional subspaces of \mathbb{R}^{2n} . These facts motivated the study of the complex analogs of certain results on sections of real convex bodies.

In Chapter 2 we present the solution of the complex Busemann-Petty problem which asks whether bodies with smaller volumes of hyperplane sections necessarily have smaller volumes in \mathbb{C}^n . We show that the answer is affirmative if n is less than or equal to 3 and negative if n is greater than or equal to 4.

Since the answer is negative in most dimensions, it is natural to ask what conditions on the volumes of hyperplane sections imply the same inequality for volumes in all dimensions. We answer this question in Chapter 3.

Chapter 4 is dedicated to the study of a generalization of the complex Busemann-Petty problem, where the volume is replaced by any measure. This generalizes the result of Zvavitch in the real case.

In chapter 5 we study the extremal sections of complex l_p -balls by complex hyperplanes for $0 < p < 2$ less than or equal to 2.

Finally, in Chapter 6, we prove that the complex unit ball, $B_p(\mathbb{C}^n)$, of l_p , $p > 2$ is not a k -intersection body, if k is less than $2n - 4$.