

SECTIONS OF COMPLEX CONVEX BODIES

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ABSTRACT

The main idea of the Fourier analytic approach to sections of convex bodies is to express different parameters of a body in terms of the Fourier transform and then apply methods of Fourier analysis to solve geometric problems. The original Fourier approach applies to convex bodies in \mathbb{R}^n . This thesis is focused at extending this approach to the complex case, where origin symmetric complex convex bodies are the unit balls of norms in \mathbb{C}^n .

In Chapter 2 we present the solution of the complex Busemann-Petty problem which asks whether bodies with smaller volumes of hyperplane sections necessarily have smaller volumes in \mathbb{C}^n . We show that the answer is affirmative if $n \leq 3$ and negative if $n \geq 4$.

Since the answer to this problem is negative in most dimensions, it is natural to ask what conditions on the volumes of hyperplane sections imply the same inequality for volumes in all dimensions. We answer this question in Chapter 3.

Chapter 4 is dedicated to the study of a generalization of the complex Busemann-Petty problem, where the volume is replaced by any measure. This generalizes the result of Zvavitch in the real case.

In chapter 5 we study the extremal sections of complex ℓ_p -balls by complex hyperplanes for $0 < p \leq 2$.

Finally, in Chapter 6, we prove that the complex unit ball, $B_p(\mathbb{C}^n)$, of ℓ_p , $p > 2$ is not a k -intersection body, if $k < 2n - 4$.