APPLICATIONS OF FOURIER ANALYSIS TO INTERSECTION BODIES

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ABSTRACT

The concept of an intersection body is central for the dual Brunn-Minkowski theory and has played an important role in the solution of the Busemann-Petty problem. A more general concept of k-intersection bodies is related to the generalization of the Busemann-Petty problem. We are interested in comparing classes of k-intersection bodies. In the first chapter we present the result that was published in J. Schlieper, *A note on k-intersection bodies*, Proc. Amer. Math. Soc., 135 (2007), 2081-2088. The result examines the conjecture that the classes of k-intersection bodies increase with k. In particular, the result constructs a 4-intersection body that is not a 2-intersection body. The second chapter is concerned with the geometry of spaces of Lorentz type. We define a 1-homogeneous functional based on Lorentz type norms. Consider the family of norms $\|x\|_{\pi(a)} = (a_{i_1}x_1^q + \cdots a_{i_n}x_n^q)^{1/q}$ where $a = (a_1, \ldots, a_n)$ with $a_1 \geq \cdots \geq a_n > 0$ and $\pi(a)$ is a permutation of the vector a . Define a 1-homogeneous functional based on this family of norms as follows: for $k \geq 1$,

$$\|x\|_k = \left(\sum_{\pi(a)} \|x\|_{\pi(a)}^{-k}\right)^{-\frac{1}{k}},$$

where the sum is taken over all permutations. We examine the geometric properties of the space $(\mathbb{R}^n, \|\cdot\|_k)$. First, we determine the conditions when the star body $(\mathbb{R}^n, \|\cdot\|_k)$ is a k-intersection body. Second, we find the extremal sections of the star body $(\mathbb{R}^n, \|\cdot\|_k)$. Throughout this work we use the Fourier Analytic methods that were recently developed.