EXPLORATIONS OF GEOMETRIC COMBINATORICS
IN VECTOR SPACES OVER FINITE FIELDS
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ABSTRACT

Let $\mathbb{F}^d_q$ be the $d$-dimensional vector space over the finite field with $q$ elements. We study various geometric combinatorics problems in vector spaces over finite fields as well as their arithmetic implications. For example:

- **The Erdős-Falconer distance problem:** How large does $E \subset \mathbb{F}^d_q$ need to be to ensure that

  $$|\Delta(E)| = |\{||x - y|| = (x_1 - y_1)^2 + \cdots + (x_d - y_d)^2 : x, y \in E\}| \gtrsim q?$$

- **The dot product problem:** How large does $E \subset \mathbb{F}^d_q$ need to be to ensure that

  $$|\Pi(E)| = |\{x \cdot y : x, y \in E\}| \gtrsim q?$$

- **The $k$-point configuration problem:** How large does $E \subset \mathbb{F}^d_q$ need to be to ensure that a congruent copy of every non-degenerate $k$-point configuration is contained in $E$?

- **The sum-product problem:** Can the sumset $A + A = \{a + a' : a, a' \in A\}$ and the product set $A \cdot A = \{aa' : a, a' \in A\}$ for $A \subset \mathbb{F}_q$ both be small?