EXPLORATIONS OF GEOMETRIC COMBINATORICS
IN VECTOR SPACES OVER FINITE FIELDS

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ABSTRACT

Let $\mathbb{F}_q^d$ be the d-dimensional vector space over the finite field with q elements. We study various geometric combinatorics problems in vector spaces over finite fields as well as there arithmetic implications. For example:

- **The Erdős-Falconer distance problem:** How large does $E \subset \mathbb{F}_q^d$ need to be to ensure that

$$|\Delta(E)| = |\{||x-y|| \equiv (x_1-y_1)^2 + \cdots + (x_d-y_d)^2 : x, y \in E\}| \gtrsim q?$$

- **The dot product problem:** How large does $E \subset \mathbb{F}_q^d$ need to be to ensure that

$$|\Pi(E)| = |\{x \cdot y : x, y \in E\}| \gtrsim q?$$

- **The k-point configuration problem:** How large does $E \subset \mathbb{F}_q^d$ need to be to ensure that a congruent copy of every non-degenerate k-point configuration is contained in E?

- **The sum-product problem:** Can the sumset $A + A = \{a + a' : a, a' \in A\}$ and the product set $A \cdot A = \{aa' : a, a' \in A\}$ for $A \subset \mathbb{F}_q$ both be small?