The problem of Local Uniformization is the valuation theoretic analogue for the problem of Resolution of Singularities.

This problem was first introduced by O. Zariski in the nineteen thirties, and in 1944, he proved local uniformization over fields of characteristic zero. Zariski’s approach was by constructing special birational extensions dominated by rank 1 valuation rings which he called Perron transforms.

In the process to generalize Local Uniformization, Mathematicians have run into certain formal ideals associated to the valuation, called prime ideals of infinite value; which have proved to be important and interesting. Cutkosky and Ghezzi make essential use of Perron transforms and Zariski’s resolution algorithm in studying such ideals.

We extend Perron transforms to arbitrary rank and prove a strong form of local uniformization, which generalizes Zariski’s theorem. We simultaneously resolve the centers of all the composite valuations and resolve the special ideals associated to the valuation, thus generalizing Cutkosky’s and Ghezzi’s result.