

RESOLUTION OF FORMAL IDEALS
ALONG A VALUATION

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ABSTRACT

Let X be a possibly singular complete algebraic variety, defined over a field k of characteristic zero. X is nonsingular at $p \in X$ if $O_{X,p}$ is a regular local ring. The problem of resolution of singularities is to show that there exists a nonsingular complete variety X , which birationally dominates X . Resolution of singularities (in characteristic zero) was proven by Hironaka in 1964. The valuation theoretic analogue to resolution of singularities is local uniformization.

Let ν be a valuation of the function field of X , ν dominates a unique point p , on any complete variety Y , which birationally dominates X . The problem of local uniformization is to show that, given a valuation ν of the function field of X , there exists a complete variety Y , which birationally dominates X such that the center of ν on Y , is a regular local ring. Zariski proved local uniformization (in characteristic zero) in 1944. His proof gives a very detailed analysis of rank 1 valuations, and produces a resolution which reflects invariants of the valuation.

We extend Zariski's methods to higher rank to give a proof of local uniformization which reflects important properties of the valuation. We simultaneously resolve the centers of all the composite valuations, and resolve certain formal ideals associated to the valuation.