TOROIDALIZATION OF LOCALLY TOROIDAL MORPHISMS

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ABSTRACT

Let $X$ and $Y$ be nonsingular varieties over an algebraically closed field $k$ of characteristic zero. A toroidal structure on $X$ is a simple normal crossing divisor $D_X$ on $X$.

Suppose that $D_X$ and $D_Y$ are toroidal structures on $X$ and $Y$ respectively. A dominant morphism $f: X \to Y$ is toroidal (with respect to the toroidal structures $D_X$ and $D_Y$) if for all closed points $p \in X$, $f$ is isomorphic to a toric morphism of toric varieties specified by the toric charts at $p$ and $f(p)$.

A dominant morphism $f: X \to Y$ of nonsingular varieties is toroidalizable if there exist sequences of blow ups with nonsingular centers $\pi : Y_1 \to Y$ and $\pi_1 : X_1 \to X$ so that the induced map $f_1 : X_1 \to Y_1$ is toroidal.

Let $f : X \to Y$ be a dominant morphism. Suppose that there exist finite open covers $\{U_i\}$ and $\{V_i\}$ of $X$ and $Y$ respectively such that $f(U_i) \subset V_i$ and the restricted morphisms $f : U_i \to V_i$ are toroidal for all $i$. $f$ is then called locally toroidal.

It is proved that a locally toroidal morphism from an arbitrary variety to a surface is toroidalizable.