The main result of this thesis is a generalization of Green’s Theorem:

**Theorem 1.** Suppose $\Omega \subseteq \mathbb{R}^2$ is open, $K \subset \Omega$ is compact with a positively oriented piecewise $C^1$ boundary $\partial K$. Let $P, Q : \Omega \to \mathbb{R}$ be functions such that $P$ and $Q$ are differentiable on $K$ and $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$ is continuous on $K$. Then

$$\int_{\partial K} P \, dx + Q \, dy = \int \int_K \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dx \, dy.$$

After proving the above theorem we prove two further results: a generalization of the Divergence Theorem for $\mathbb{R}^2$ and a generalization of Cauchy’s Integral Theorem. The statements of the two theorems are given below.

**Theorem 2** (Generalization of the Divergence Theorem). Suppose $\Omega \subseteq \mathbb{R}^2$ is open and $K \subset \Omega$ is compact with $C^1$ boundary. For any vector valued function $\vec{f} : \Omega \to \mathbb{R}^2$ differentiable on $K$ with $\text{div} \vec{f}$ continuous on $K$, the following holds

$$\int_{\partial K} < \vec{f}, \vec{\nu}> \, d\sigma = \int \int_K \text{div} \vec{f} \, dx \, dy,$$

where $\vec{\nu}$ is the outward unit normal to $K$.

**Theorem 3** (Generalization of Cauchy’s Integral Formula). Suppose $\Omega \subseteq \mathbb{C}$ is open, and $K \subset \Omega$ is compact with piecewise $C^1$ boundary. Let $f : \Omega \to \mathbb{C}$ be such that $f$ is differentiable at each $z \in K$ and $\frac{\partial f}{\partial \bar{z}}$ is continuous on $K$. Then,

$$f(\zeta) = \frac{1}{2\pi i} \int_{\partial K} \frac{f(z)}{z-\zeta} \, dz - \frac{1}{\pi} \int \int_K \frac{\partial f}{\partial \bar{z}} \frac{1}{z-\zeta} \, dx \, dy \quad \forall \zeta \in K.$$