

# Subsequences of frames and their operators

Richard G. Lynch

Dr. Peter G. Casazza, Dissertation Supervisor

## ABSTRACT

The purpose of this dissertation is to investigate the explicit behavior of subcollections of frames. The first chapter is an introduction to the necessary background required for the thesis, as well as a brief summary of the results. The second chapter is devoted to studying the precise unconditional behavior of the frame expansions of vectors. We show that the unconditional constants are bounded above by  $\sqrt{B/A}$ , and the main result is that a Bessel sequence is 1-unconditional if and only if it is an orthogonal sum of tight frames. In the third chapter, we introduce a new topic which we deem *weaving frames*. Two frames  $\Phi := \{\phi_i\}_{i \in \Lambda}$  and  $\Psi := \{\psi_i\}_{i \in \Lambda}$  for a Hilbert space  $\mathbb{H}$  are *woven* if there are positive constants  $A \leq B$  so that for every subset  $\sigma \subset \Lambda$ , the family  $\{\phi_i\}_{i \in \sigma} \cup \{\psi_i\}_{i \in \sigma^c}$  is a frame for  $\mathbb{H}$  with frame bounds  $A$  and  $B$ . Fundamental properties of woven frames are developed and key differences between weaving Riesz bases and weaving frames are considered. In particular, it is shown that a Riesz basis cannot be woven with a redundant frame. We also introduce an apparently weaker form of *weaving* but show that it is equivalent to weaving. Weaving frames has potential applications in wireless sensor networks that require distributed processing under different frames, as well as preprocessing of signals. The last chapter is dedicated to extending the results of weaving Hilbert space frames to the Banach space setting by using approximate Schauder frames.