

A Foliated Seiberg-Witten Theory

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ABSTRACT

This thesis set out to investigate a generalization of Seiberg-Witten theory from four-dimensional manifolds to four-codimensional Riemannian foliations. We begin by reviewing the fundamental theorems and definitions necessary for studying the transverse space of foliations, in particular Riemannian foliations. To prepare for the Seiberg-Witten theory, the definitions and theory of Spin geometry and how they fit into the context of Riemannian foliations is discussed. A transverse version of the Witten identities are then established for transverse Spin and $\text{Spin}^{\mathbb{C}}$ structures. We then define the Seiberg-Witten equations for the transverse space and determine the necessary bounds on the solutions to those equations. Lastly, we review a theory sufficient for a transverse Sobolev theory for foliations and use it to show the moduli space of the Seiberg-Witten equations are sequentially compact.