DYNAMIC AND EFFICIENCY CHARACTERISTICS OF AN INLET METERING VALVE CONTROLLED FIXED DISPLACEMENT PUMP

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by

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NOMENCLATURE

\( A_s \) Square Cross-Sectional Area of the Valve Inlet
\( A \) Rectangular Cross-Sectional Area of the Valve Outlet
\( A_p \) Static Friction Coefficient
\( A_{pump} \) Cross-Sectional Area of the Pump Inlet
\( A_r \) Cross-Sectional Area of the Spool Valve
\( \hat{A} \) Nondimensional Cross-Sectional Valve Area
\( B_p \) Boundary Lubrication Decay Rate Coefficient
\( c \) Viscous drag coefficient
\( \hat{c} \) Nondimensional Viscous Drag Coefficient
\( C_p \) Hydrodynamic Lubrication Coefficient
\( c_d \) Discharge Coefficient
\( c_r \) Radial Clearance
\( \frac{dQ}{dt} \) Change in Volumetric Flow Rate as a Function of Time
\( D_p \) Starting Torque Coefficient
\( E \) Specific Energy
\( e \) Error term in Levenberg-Marquardt process
\( F_i \) Horizontal Flow Forces
\( F_j \) Vertical Flow Forces
\( F_o \) Initialized Spring Force
\( F_y \) Pressure Induced Valve Force

\( g \) Gravity

\( h \) Enthalpy

\( J \) Jacobian Matrix

\( K \) Leakage Coefficient

\( K_s \) Fluid Compression Coefficient

\( K_1 \) Low Reynolds Number Leakage

\( K_2 \) High Reynolds Number Leakage

\( \hat{K}_1 \) Nondimensional Leakage Coefficient

\( \hat{K}_2 \) Nondimensional Spring Rate Coefficient

\( k \) Valve Spring Constant

\( L \) Length of the Crankshaft

\( m_v \) Mass of the Spool Valve

\( \dot{m} \) Mass Flow Rate

\( \dot{\hat{m}} \) Nondimensional Valve Mass Coefficient

\( P \) Hydraulic Power

\( P_0 \) Pressure at the Valve Inlet

\( P_1 \) Pressure at the Valve Outlet

\( P_2 \) Pressure at the Pump Outlet

\( P_{\text{max}} \) Maximum System Operating Pressure
\( P_p \)  Pressure in the Pump

\( P_{sat} \)  Saturation Pressure of the Fluid

\( P_{ss} \)  Steady State Pressure

\( Q_0 \)  Valve Inlet Volumetric Flow Rate

\( Q_1 \)  Pump Inlet Volumetric Flow Rate

\( Q_2 \)  Pump Outlet Volumetric Flow Rate

\( Q_{sys} \)  Volumetric Flow Rate Demand by the System

\( r \)  Crank Radius

\( s \)  Laplace Operator

\( T \)  Torque on the Crankshaft

\( t \)  Time

\( u \)  Internal Energy

\( \hat{v} \)  Nondimensional Specific Volume Ratio

\( \hat{V} \)  Nondimensional Piston Volume Ratio

\( V_2 \)  Pump Volume

\( V_d \)  Volumetric Displacement of the Pump

\( v_l \)  Liquid Specific Volume

\( v_{lg} \)  Difference between Fluid Liquid Specific Volume and Fluid Vaporous Specific Volume

\( V_{max} \)  Maximum Piston Internal Volume

\( V_n \)  Instantaneous Piston Internal Volume
\( w \)  Weights in Levenberg-Marquardt solution process

\( x \)  Quality

\( y \)  Spool Valve Displacement

\( y_{\text{max}} \)  Maximum Spool Valve Displacement

\( y_{\text{ss}} \)  Steady State Valve Displacement

\( z \)  Vertical Position

\( \alpha \)  Swashplate Angle

\( \beta \)  Fluid Bulk Modulus

\( \eta \)  Efficiency

\( \theta \)  Jet Angle, Crankshaft Displacement

\( \phi \)  Crankshaft Location

\( \tau \)  Time constant

\( \mu_0 \)  Fluid Viscosity

\( \mu \)  Combination Coefficient

\( \psi \)  Nondimensional Fluid Property Group

\( \omega \)  Pump Shaft Speed
CHAPTER 1. INTRODUCTION

1.1 Background and Motivation

The body of this work applies some of the principles at work in a diesel fuel injection system with a fixed displacement hydraulic pump in a way that creates a novel approach to providing varied flow output. In the diesel fuel industry, “metering” refers to the act of controlling the amount of fluid sprayed per charge [1]. For this work, an inlet metering valve is installed upstream from a fixed displacement pump and is used to meter the flow exiting the pump in a manner similar to the way diesel fuel injectors control the amount of partially vaporized fuel injected into a diesel engine. A more thorough examination of diesel fuel injection systems will be conducted in the literature review.

The inlet metering approach differs from traditional hydraulic pump flow rate management. Pump flow is typically managed by one of three approaches: the shaft speed of the pump is varied, the angle of the swashplate within a variable displacement pump is varied, or a fixed displacement pump pushes a constant amount of flow and the excess is simply dumped and recirculated in the system [2] (A detailed diagram of the variable displacement pump is provided in Chapter 4). Fixed displacement pumps are cheaper and more mechanically reliable than variable displacement pumps; however, they contribute to large system inefficiencies since their excess flow is bypassed from actual power supply. In aircraft gas turbine engines, for instance, the ratio of bypass flow to engine flow can be up to twenty-fold [3]. Obviously this level of systemic inefficiency is undesirable. This dissertation will attempt to use a fixed displacement pump but
minimize the losses that result from the bypass of excess flow and instead control the flow at the pump outlet by means of an upstream inlet metering valve.

1.2 System Description

Figure 1-1 shows a schematic of the inlet metering system and the nomenclature used throughout this paper. The inlet metering system is comprised of a charge pump, an inlet metering valve, and a single piston pump which has check valves at both the inlet and outlet. The fluid maintains an initial pressure and flow rate \((P_0, Q_0)\) as established by the charge pump. This pump will not receive further attention in the analysis in the following pages; instead, it will be treated as a constant pressure reservoir upstream from the inlet metering valve. The inlet metering valve is illustrated as a two-way spool valve in Fig. 1-1.

The inlet metering valve serves as the control device for this system, and receives considerable attention in the upcoming analysis. The typical operation of an inlet metering valve can be observed in a diesel fuel injection system.

In a diesel fuel injector, diesel fuel is supplied to a throttling valve, which is controlled by an electronic control device. The amount of fluid distributed is adjusted by changing the orifice diameter of the fluid injector outlet [4]. As the pressure drop across the orifice increases, an increasing percentage of the diesel fuel is vaporized. In diesel fuel injection systems, poppet valves are commonly used [5]. However, for our design purposes, the metering valve that will be modeled is a spool valve. Spool valves have been found to better handle pressure oscillations [6]. Our primary design objective is to
Figure 1-1. System Design and Associated Nomenclature

- Charge Pump
- Downstream Load
- Piston Pump
- Spool Valve

Symbols:
- $P_p$
- $A_i$
- $Q_1$
- $Q_2$
- $P_0$, $Q_0$
- $A_0$
- $L$
- $T$
- $\omega$
- $k$

Equations:

- \[ P_1, Q_1 \]
- \[ P_2, Q_2 \]
- \[ P_0, Q_0 \]

Parameters:
- $A_i$
- $A_0$
- $L$
- $T$
- $\omega$
- $k$
maintain a constant return pressure while the pump’s output flow demand varies. Thus, the spool valve is well suited for our needs.

![Spool Valve Diagram]

Figure 1-2. Spool Valve

The inlet metering valve is shown in Fig. 1-2. We will control the cross-sectional area $A$ of the spool valve and will subsequently maintain control over the pressure drop across the valve. This pressure drop will induce partial vaporization of the working fluid. The predicted phase change will occur as the fluid moves from the fluid supply (indicated with a “0” subscript) into the contracted valve area (indicated with a “1” in the subscript).

The inlet metering valve (shown in Fig. 1-2) is able to vary the discharge flow of the pump by controlling the quality, that is, the mass fraction of vapor in the liquid, of saturated fluid that is entering the pump. As the restriction of the inlet metering valve increases (i.e., as the valve area decreases), the discharge pressure $P_1$ decreases until it reaches saturation pressure and the fluid begins to vaporize. After the upstream check valve opens, the partially vaporized fluid fills the pump. As the fluid is compressed by the pump piston, it condenses due to its exposure to pressure that exceeds the vapor pressure of the fluid. As the amount of vapor increases on the inlet side, the amount of...
condensed liquid decreases on the discharge side. Thus, by varying the area of the inlet metering valve we adjust the discharge flow-rate of the pump in a controlled way.

Figure 1-3. Piston Pump

The piston pump is shown in Fig. 1-3. The partially vaporized fluid from the inlet metering valve accumulates upstream from the first check valve. Once the volume of the line upstream from the check valve has been filled, the check valve opens and the piston volume (denoted with a dashed line in Fig. 1-3) fills with the liquid/vapor mixture. This occurs at the piston downstroke.

As the crankshaft forces the piston head upwards, the size of the control volume illustrated with the dashed line decreases, resulting in an increased pressure. As the pressure increases, the fluid in the vacated piston volume condenses. Once the pressure of the fluid inside the control volume has reached a pressure of $P_2$, the second check
valve opens and the fluid is transmitted downstream at a flowrate of $Q_2$. A small amount of the fluid will remain in the control volume, and as the piston head begins to move downwards (in the negative $z$ direction), the pressure in the cylinder decreases and the fluid will partially vaporize. When the fluid reaches a pressure identical to that of the fluid at the inlet metering valve outlet, the upstream check valve opens and a new volume of partially vaporized fluid enters the control volume. This process continues repeatedly as the crank shaft rotates about its axis. The inlet metering valve is pressure controlled, and so the pressure at the pump outlet, $P_2$, is used to control the valve displacement.

1.3 Experimental Set Up

A prototype of the inlet metering system design was constructed for experimental purposes. This experimental test set up is shown in Fig. 1-4. A 25 hp axial piston variable displacement pressure controlled charge pump was used to establish a fixed inlet pressure for the inlet metering valve system. Temperature, pressure, and flow sensors were placed upstream from the inlet metering valve (referred to as location “0” in the notation throughout this work) and the pump outlet (downstream from the fixed displacement pump, referred to as location “2” in the notation throughout this work). The fluid flow rate, temperature and pressure exiting both the charge pump and the inlet metering pump were recorded for a variety of fixed inlet and discharge pressures. The inlet metering pump shaft speed and inlet metering pump shaft torque were also measured throughout each test. Measurements were taken at 2 ms intervals for each 70 second test. For each test the inlet pressure, outlet pressure, and shaft speed were fixed. The valve position was adjusted at 10 second intervals throughout the test to determine
the effect of changing the valve position on the fluid flow rate. These results will be compared with the analytical predictions from Chapter 3 in Chapter 5.

Figure 1-4. Test rig experimental set up

The 25 hp charge pump was used to fix the pressure upstream from the inlet metering valve. Measurements were recorded for multiple inlet and discharge pressures. A Caterpillar microcontroller was used to adjust the pump pressure setting. Measurements were taken for multiple pump shaft speeds. Each test was conducted for 70 seconds. The experimental conditions studied are listed in Table 1-1. These results will be presented in Chapter 6.
Table 1-1. Experimental Conditions

<table>
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<th>Discharge Pressure [MPa]</th>
<th>Shaft Speed [rpm]</th>
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<tr>
<td>2</td>
<td>2, 5, 10, 20, 25</td>
<td>2500</td>
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1.4 Contribution

This work proposes a novel approach to flow control for a pressure controlled pump. Current approaches to flow control rely either on the use of a variable displacement pump or a fixed displacement pump that dumps excess fluid [2]. This approach allows for the use of a fixed displacement pump without requiring excessive flow leakage. It is our expectation that practical implementation of the inlet metering approach may prove to be cheaper and will offer enhanced control characteristics over existing flow control approaches.

This work derives the linear relationship between inlet metering valve cross-sectional area and pump volumetric flow rate output, which proves to be a valuable control insight. Additionally, this work shows that the inlet metering pump can be designed to behave stably and according to a first order pressure response. This represents a controllability advantage over a traditional variable displacement pump, which demonstrates a second order pressure response, resulting in significant overshoot and oscillation.

This work also compares the efficiency of the inlet metering pump to a variable displacement pump. This will enable those interested in the practical implementation of this technology to weigh the advantages and disadvantages inherent in this design, when determining when an inlet metering pump is an appropriate replacement for an axial...
piston pump. The system of equations developed in the analysis section is solved and modeled in MATLAB / Simulink ®. This modeling illustrates our system’s ability to regulate the system pressure at a constant steady-state value. The equations developed in the theoretical analysis as well as the theoretical efficiency predictions are compared with experimental results. Because this technology is new and untested in the hydraulic world, the analytic results, modeling results, efficiency predictions, and data associated with prototype operation are all novel contributions to the engineering community.

1.5 Dissertation Outline

This dissertation presents a review of the published literature, studying the available information on inlet metering in diesel fuel injection systems, the history of hydraulic systems, the differences between oil and water hydraulic systems, the impacts of cavitation and entrained air on hydraulic systems, and a review of modeling approaches taken for throttling processes like this one. Because the inlet metering approach has already been successfully implemented in contemporary diesel fuel injection systems, it is important to study the literature as it pertains to this technology. Interestingly, there does not appear to be published work on the pressure control features of inlet metering fuel injectors; however, the pressure control associated with this inlet metering pump is a key feature of our design. This work, then, may prove useful to engineers in the diesel fuel industry interested in controllability details.

The reading associated with cavitation and entrained air also becomes very important when we consider the potential damage to our hydraulic system that could result from the fluid vaporization and condensation. This is a nonconventional approach
to controlling hydraulic pump flow, and as such, no historical data exists in regards to the potential impacts of this level of vaporization. What we can learn from the published works available is all we have to inform our predictions about fluid behavior in the system, as well as the direct effects on the fluid itself.

After reviewing the literature, this dissertation derives a system of equations that describe the motion of the valve and the pressure at the pump outlet. These results are used to study the stability of the valve-pump system and develop design criteria for practical use by the engineering community. The efficiency of the inlet metering system is studied, both theoretically and experimentally, and the equations of motion for the system are used to model the behavior of the valve in MATLAB / Simulink ®. These results are presented in the work that follows, and a discussion of their implications accompany their presentation. Conclusions about this system are drawn, and suggestions for future work on the topic will be presented.
CHAPTER 2. LITERATURE REVIEW

2.1 Literature Review Introduction

As was mentioned in the Introduction, this work relies on the combination of existing diesel fuel injection technology and a fixed displacement hydraulic pump. Thus, our search of the literature will aim to gain an understanding of both the approaches taken by the diesel industry and existing hydraulic pump technology. Additionally, the operating fluid selection must be considered. A thorough review of the advantages and disadvantages of hydraulic fluid and water based hydraulic systems will be conducted.

2.2 Inlet Metering Valves

Diesel fuel injectors are ubiquitous in the diesel engine industry. They are designed to inject diesel fuel into the chamber where the fuel can be ignited, which is essential for combustion. The injection of the fuel into the chamber at the requisite high pressure results in the partial vaporization of the fluid, which subsequently improves gas absorption and increases system efficiency [7]. Aspects of this approach will be adapted to our purposes, and thus, a thorough understanding of fuel injectors must be derived from the available literature. Figure 2-1 shows a simplified illustration of a diesel fuel injector, as well as the key terms associated with the design.
The fuel injector is a solenoid operated valve that opens and closes as necessary to allow fuel to flow through the orifice [8, 9]. Fuel injection pumps can also be used to adjust the valve position [10]. Either way, the valve position controls both the quantity (controlled by volume) and quality (controlled by pressure drop) of the fuel discharge [11]. The fuel is driven by the pressure on the cylinder head, which can range from 2,000 to 20,000 psi [11]. As the needle is pushed downwards (based on the orientation illustrated in Figure 2-), fuel is forced through the orifice [12]. The flow inside the injector is impacted by dynamic factors like injection pressure and needle lift, as well as the shape and finish of the orifice [13]. The flow out of the injector is partially vaporized due to the pressure drop across the orifice and saturation pressure of the fuel. The multiphase fluid is sprayed through the orifice and into the cylinder so that combustion can occur during the mixing and evaporation process.

The control of the amount of fluid sprayed per charge is referred to as “metering”, a term that will be applied to this work as well [2]. Fuel injection enables charge stratification and detonation control [11]. This can reduce fuel consumption by approximately 20% compared to chamber engines [15, 16]. They also decrease
emissions and improve engine startability, load acceptance (acceleration), and combustion noise [11, 15]. It should be noted that while the fluid vaporization has positive efficiency impacts in combustion, fluid vaporization can result in cavitation damage in hydraulic systems [14]. This literature review will devote considerable study to the impacts of cavitation on hydraulic systems later. For our application, many of the previously listed efficiency advantages are negated. Instead, we focus on the fuel injector’s ability to control the fluid quality. This control is what allows us to replace a variable displacement pump with a fixed displacement pump.

2.3 Hydraulic Systems, A Brief History

Hydraulic systems have been used since 1795 when Bramah patented a hydraulic press [18]. In 1802 hydraulic cranes were introduced [18]. Shortly after this, with the advent of electricity, fluid power systems were neglected in favor of electrical power. Blackburn provides an extensive comparison of fluid power and electricity, which has been condensed into Table 2-1.

Table 2-1. Comparison of Fluid Power and Electric power [19]

<table>
<thead>
<tr>
<th></th>
<th>Electric Power</th>
<th>Fluid Power</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power transmission over distance</strong></td>
<td>Relatively easy</td>
<td>Relatively difficult</td>
</tr>
<tr>
<td><strong>Controllability</strong></td>
<td>Easy at lower powers</td>
<td>Easy at higher powers</td>
</tr>
<tr>
<td><strong>Power density</strong></td>
<td>Limited by the fact that ferromagnetic materials saturate at low flux density</td>
<td>Very high torque to inertia ratio</td>
</tr>
<tr>
<td><strong>Speed of Response</strong></td>
<td>Mechanically springy, relatively slow response</td>
<td>Mechanically stiff, relatively rapid response</td>
</tr>
</tbody>
</table>
Hydraulic systems maintained their space along the fringe of mechanical technologies until World War II when their ease of control and high torque to weight ratio made them ideal for warship gun turrets [20]. At this point, the United States government began funding hydraulic control research. This influx of funding led to the founding of the Servomechanisms Laboratory of the Massachusetts Institute of Technology in 1939 [19]. Since that time, hydraulic systems have maintained their place in both academia and industry.

2.3 A Comparison of Oil and Water Hydraulic Systems

Until the early 1900’s, all hydraulic systems in use relied on water. However, in 1906 Williams and Janney introduced the idea of replacing water with oil to avoid corrosion and freezing, improve system lubrication, and decrease leakage [21]. This trend of using oil continued until the 1970’s when the oil embargo caused a short term increased interest in water hydraulics [22]. This interest petered out until the mid-nineties when the engineering community began to take a serious look at reintroducing water hydraulics into a variety of applications [21, 23].

Water based hydraulic systems have many advantages over oil. They are fire resistant, low cost, non-toxic, and have lower disposal costs [22]. Water’s thermal conductivity is four to five times greater than oil, so it requires significantly less cooling capacity [21]. Water also contains much less air in solution than oil does, which enhances the rigidity and safety of water hydraulic systems [23]. Water hydraulics have a quicker response and higher efficiency than that of oil hydraulics [24]. Given the focus that this work will devote to managing cavitation, perhaps the most significant
observation about water hydraulic systems is that given the same amount of cavitation in
a system, a system using hydraulic oil as the operating fluid will experience greater mass
loss than the system which uses water as the operating fluid [25]. This concept of mass
loss and erosion resistance will receive greater attention in subsequent sections of the
literature review.

Oil hydraulic systems have substantial environmental, flammability, and
regulatory concerns associated with them. For example, Europe has mandated that no oil
hydraulic systems be permitted in mines [26]. This means that water hydraulic systems
are the default technology in a variety of industries. These industries include nuclear
engineering, coal mining, steel foundries, desalination plants, fire-fighting, food and
beverage production, and plastic molding [21].

Obviously oil hydraulics would not be so ubiquitous in their use if water held all
of the advantages. Oil hydraulics are superior in respect to lubricity and rust resistance
[27]. The reduced lubrication of water means that the valve spools in those systems tend
to stick to the valve bodies, which causes more wear than would be experienced in a
traditional oil hydraulic system [28]. One way that this is frequently mitigated is by the
introduction of hydrostatic bearings into the spool valve [28, 29]. While this has been
shown to be an effective way of managing decreased lubrication, it requires additional
parts and increased machining. This is just one of many reasons why water hydraulic
systems can cost between 30 and 200% more than their oil hydraulic counterparts [22,
30]. Oil hydraulic systems are also known to be quieter than water hydraulics due to
their increased viscosity [31]. Further, lower viscosity of water results in increased
leakage when compared to oil based hydraulic systems [32].
Although oil based hydraulic systems fill the majority of fluid power applications, water based systems have their place. Environmental concerns and increasing oil prices should serve to increase the market share of water hydraulic systems. For our analysis, we have elected to focus on a water hydraulic system given the readily available thermodynamic properties and their small but significant presence in industry. It is more difficult to find data on hydraulic oil systems since much of their material data is proprietary.

The most significant differentiating factor between water and oil based hydraulic systems for the purposes of our study is the discrepancy in saturation pressures. For fluids with lower vaporization pressures, like water, gasoline, or diesel fuel, cavitation is a crucial issue facing inlet metering valve implementation. Hydraulic oils have vaporization pressures orders of magnitude greater than those fluids [33]. As a result, the primary source of specific volume change for the hydraulic fluid is entrained air in the system. Here the reader should recognize the significance of what has been said: working fluid selection matters immensely in the inlet metering approach. Water based systems will see part of the water vaporized as the result of the pressure drop, while oil based systems will see air bubbles forming as the air comes out of solution. Both of these topics will receive careful study in the subsequent literature review, although we anticipate that cavitation will be of greatest concern for the system we have elected to study.
2.4 Cavitation

Cavitation is the result of the rapid vaporization and subsequent condensation of a liquid and can either be caused by an increase in temperature (e.g. boiling) or a decrease in pressure. Nucleation sites, that is, the site where cavitation begins, can exist in two different forms: dissolved gas or entrained gas. Dissolved gas does not affect the volume or compressibility of the liquid at all; however, entrained gas is dispersed throughout the liquid as bubbles and does impact the volume and fluid bulk modulus of the liquid [35].

Gaseous cavitation occurs when the fluid’s pressure drops below the saturation pressure of the noncondensable gas (typically air) that is dissolved in the fluid [36]. It occurs at a much slower rate than hydrodynamic cavitation, and can result in noise and vibrations [37, 38]. This type of cavitation can be expected to appear with greater frequency in hydraulic oil based systems, as hydraulic oil contains much more entrained air than water does [23].

Hydrodynamic cavitation (also known as vaporous cavitation) is induced by a rapid reduction in pressure from a higher pressure to the fluid’s saturation pressure. If the local pressure remains depressed, the small cavity will continue to grow, and when the pressure increases again, the cavity bubble will become unstable and collapse [35, 36]. This collapse is the cause of cavitation damage [37]. There are two means by which cavitation damage can occur. The collapse can induce a shock wave in the liquid or an asymmetric collapse can cause the bubble to lose its spherical shape and create a high speed liquid jet which damages the solid surface [35]. This type of cavitation can be expected to be more prevalent in water based hydraulic systems since water has a saturation pressure several orders of magnitude smaller than hydraulic oil.
In 1965 the concept of erosion strength was first introduced in the literature. Thiruvengadam defined erosion strength as the volume of material fractured on the surface of the parent material by the work from an external force [39]. Since this introduction, it has become the standard means by which to discuss a material’s ability to resist cavitation. Erosion strength and erosion resistance are two terms used to describe a numeric property that provides designers with a way to choose the appropriate material to withstand cavitating conditions.

Substantial effort has been exerted to uncover relationships between well-known material properties like hardness and the lesser understood erosion resistance. Hattori et al found that the erosion resistance of carbon steel, excluding stainless steel, increases proportionally (to the power of 2.4) with the Vickers hardness [40], and later published a correlation for the erosion resistance of stainless steel that accounts for the work hardening that occurs during cavitation [41]. Heymann observed that erosion resistance is strongly correlated (to the power of 2.5) with material hardness for nine other materials [42]. Varga conducted similar studies and developed relationships between material properties and the normalized erosion resistance for cast bronzes, wrought bronzes, nickel alloys, aluminum alloys, and titanium alloys [43].

Other numerically expressed properties have been identified to help us talk about cavitation. For example, the cavitation index is a metric that has been developed in order to predict the formation of cavitation, and it has been directly correlated to performance breakdown, noise and erosion [44]. When considering insertion of cavitation resistant materials as a means to protect the pump from cavitation damage, investigation of the material’s cavitation index will be a useful metric.
2.5 Machine Design Considerations

Cavitation can occur in both spool valves and pumps. Large pressure differences across valve chambers and high frequency motion of valve-controlled actuators can induce cavitation in spool valves [45]. In pumps, cavitation occurs when the pressure in the piston cylinder drops rapidly at the beginning and end of the pump’s suction stroke due to the small opening area of the cylinder port [38]. Because of the aforementioned undesirable effects, one of the primary objectives of hydraulic system design is often to avoid cavitation in the pump [41]. In addition to the noise, vibrations, and damage, serious reduction of the pump’s filling performance can result from cavitation [46]. This is another reason to actively design against gaseous cavitation.

The literature is full of examples of engineers working to avoid cavitation. Wang redesigned the valve plate in the axial piston pump to reduce the development of cavitation in the piston barrels [51]. Suzuki and Urata designed a valve with two serial throttles to mitigate cavitation in a pressure reducing valve for a water hydraulic system [47]. Their work was derivative of the work of Liu who also utilized multiple throttles to cause a stepwise pressure drop rather than a rapid decrease in pressure [48]. Manring explored different flow passage geometries to cause a gradual pressure reduction rather than a steep pressure drop [49].

The breadth of literature in regards to designing pumps and valves to avoid cavitation indicates the emphasis that has been placed on cavitation avoidance in the scientific community. In addition to geometry design, another approach frequently taken is to simply install inserts in the pumps that are highly resistant to cavitation damage [52]. It should be noted that, particularly for water hydraulic systems, stainless steel and
ceramic materials have been found to be highly cavitation-resistant [29, 30, 34]. Metallic composite interfaces have been observed to be the best option in axial piston water pumps and motors [18].

2.6 Entrained Air

Entrained air is an issue that applies specifically to hydraulic oil based systems. Water hydraulic systems are able to use the inlet metering technique to induce fluid vaporization. Therefore, this section is less likely to apply to them. The saturation pressure of the air in hydraulic oil systems is much lower than that of the oil, thus the behavior of entrained air becomes highly significant.

Air becomes entrained in the hydraulic fluid when a volume of air is trapped in the fluid and subsequently breaks into a set of bubbles [53]. These bubbles are then transported along by the fluid flow and can either continue as entrained air or, if pressurized, shift to dissolved air, which has no impact on the fluid volume [54]. The process by which air becomes entrained is also known as aeration [55].

Entrained air results in a significant reduction of the effective fluid bulk modulus, and subsequently slows the system responsiveness [56]. It should be noted here that although entrained air has this negative impact on the system, dissolved air does not impact fluid bulk modulus [57]. Entrained air can also result in pressure pulsations, air binding, increased noise, poor pump suction performance [58], filter blocking [59], and increased throttling losses [60]. Whether the hydraulic system experiences cavitation or aeration, similar damage can result from either [55]. However, for all these negative impacts, substantial effort in the literature has been devoted to intentionally adding
entrained air to the system. When there is less than 5% but more than 0% (more specifically, between 0.3% and 1.0%) of air entrained in the system, it has been showed to cushion the implosive effects of cavitation and actually improve pump performance [52, 58, 61].

After aeration has occurred, the best way to change the volumetric ratio of liquid to air in the system is done by adjusting the inlet pressure [58]. Temperature adjustment is the main means of controlling the aeration within the fluid [59]. Further consideration of both of these approaches would be necessary if introducing inlet metering valves to a hydraulic system which uses hydraulic oil.

The inlet metering valve relies on fluid vaporization to change the volume of the working fluid in order to maintain control of the pump output flow. Due to the high vaporization pressure of hydraulic oil, vaporizing the fluid in order to adjust its specific volume is not an option. Instead, hydraulic fluid specific volume adjustments would be made by dissolved and entrained air modulation. The inlet metering valve in a water hydraulic system, on the other hand, is able to rely on fluid vaporization as discussed previously.

2.7 Fluid Modeling Justification

In addition to the similarities the inlet metering valve in our hydraulic system design has to the diesel fuel injector, our inlet metering design shows similarities to aspects of a refrigeration system. A typical vapor-compression refrigeration process consists of four major components: a condenser, a compressor, an evaporator, and an expansion valve [62]. This expansion valve is the location where the refrigeration fluid makes the transition from saturated liquid to two-phase liquid-vapor mixture. Similarly,
our inlet metering valve marks the location in the system where the working fluid, in our case, water, transitions from saturated liquid to liquid-vapor mix.

A throttling process always occurs when a flowing fluid experiences an abrupt pressure decrease when passing through a restricted opening [63]. Thus, throttling occurs in both the expansion valve associated with refrigeration cycles and in our inlet metering valve. Since our inlet metering valve is novel and no literature exists to suggest the best approach to modeling the phase change that occurs at the valve restriction, the best approximation to be found is to study the literature that exists on refrigeration cycles.

A survey of the literature finds that a particular area of focus in refrigeration cycle studies has been on the increase of efficiency. This is unsurprising; the search for greater efficiency is prevalent in virtually every developed technology. The question we are most interested in, though, is how does the scientific community theoretically model the refrigeration systems that they are studying?

Evidence of usage of both the first law of thermodynamics and the second law of the thermodynamics can be found. In his review of the literature on vapor compression refrigeration systems, Ahamed et al asserts that the first law analysis is the most common approach to the analysis of thermal systems, and that the energy performance of heating, ventilation and airconditioning (HVAC) systems are most frequently evaluated with the first law [64]. Examples of this are plentiful. Hasan et al uses a first law analysis to study the amount of energy converted to a useful energy output in a refrigeration cycle that relies on a solar heat source [65]. Khaliq et al use a first law analysis to study the efficiency of a refrigeration cycle that takes advantage of an ejector to increase efficiency [66].
The exergy analysis made possible by use of the second law of thermodynamics is also incredibly prevalent in the refrigeration literature [67-69]. The second law appears to be most effective at revealing system irreversibilities [64, 70]. After the irreversibilities are identified, they can be reduced using Bejan’s exergy minimization principle [71]. The minimization of exergy generation will improve system performance. Rakopoulos et al recommends coupling a first law analysis with a second law analysis for system optimization, in order to identify the maximum system performance point [70].

For our preliminary modeling, then, we will perform a first law based analysis to study the efficiency of our inlet metering system. This efficiency analysis will be presented in Chapter 4. The exergy analysis appears to be most useful when considering a system dealing with multiple working fluids (e.g. water and air as in an HVAC system). In this initial work on an inlet metering system, we will not perform an exergy analysis; however, it may be a topic of interest in the future.

In 1950, Joseph Keenan published “An investigation of ejector design by analysis and experiment.” This work appears to be fundamental to much of the refrigeration cycle analysis done today. In his study, he introduces the one-dimensional constant pressure flow model. This model assumes that the pressures of both the liquid and vapor flowing in the system are perfectly mixed and exist at identical pressures [72]. Notably, he acknowledges the risk of choking in his system, but elects to neglect this issue even though it had been observed in experiments that he had knowledge of. Further development of this model was conducted by Huang, who addresses choking in the nozzle [73].
Much more recently, in 2007, Guo-liang Ding published a review of the recent approaches to simulations of refrigeration systems. Homogeneous flow, an assumption that we will be applying to our work, is still a commonly accepted model that can be used to simplify calculations associated with multiphase flow [74]. This assumption implies that the system changes phase under thermodynamic equilibrium, there is no slip between the liquid and vapor phases, and there is complete mixing between the liquid and vapor phase. This will lead to a lower entropy production value than we would otherwise generate; the irreversible thermodynamic transitions that occur during two phase flows are a source of entropy production [75]. While this omission of one of the sources of entropy in our system will be present in our model, this assumption is prevalent in much of contemporary refrigeration literature and thus we will apply it here as well. Ding also recommends using look up tables for thermodynamic properties of the refrigerant [74]. We will also rely heavily on published and established thermodynamic property tables for water in our system evaluation.

An example of the application of the one-dimensional pressure flow model can be seen in the work by Wang et al. In this work, the authors use thermodynamic look up tables, the first law of thermodynamics, and principles from the conservation of mass and conservation of energy to analyze their system [76]. We will also apply all of these tools to our study of the inlet metering system. Most notably, in all of the refrigeration literature that was read in preparation for this dissertation, no theoretical handlings of the effects of air mixing with the refrigerants selected as working fluids were observed [65, 69, 72]. Instead the authors performed their analysis under the assumption that their working fluid was pure and unchanged by entrained or dissolved air. For our analysis,
too, then we will proceed as though our working fluid (water) is pure and unaffected by entrained or dissolved air. Having made this assumption, we will then be able to apply the principle of fluid quality, that is, the mass fraction of the vapor present in a given volume of working fluid relative to the total mass of that volume of working fluid [62]. This will prove to be a key parameter in Chapter 3’s analysis.

2.8 Literature Conclusions

The literature search has revealed that cavitation effects in hydraulic systems have been studied quite thoroughly, as have the cavitating abilities of inlet metering valves in fuel injection systems. However, no results have been found to suggest that inlet metering technology has been applied to hydraulic systems, whether oil or water based. This suggests that our work is novel.
3.1 Analysis Introduction

The purpose of this analysis is to develop a theoretical model of an inlet metering pump. Here we will develop a computationally inexpensive method to determine the percent of fluid that is vaporized during the operation of this pump, illustrate the control related impacts of vaporized fluid on the operation of the pump, and derive a coupled set of differential equations that describe the dynamics of the valve and pressure output of the pump. In Section 3.14, we will nondimensionalize the coupled set of differential equations. At the end of this chapter, the theoretical model for a variable displacement pump will be introduced. This model will be used for comparison’s sake in Chapter 6.

3.2 Transient Pressure Analysis

The pressure rise rate equation is given as Eq. (3.1) [20]. As we seek to control the output pressure of the pump, a cursory glance at Eq. (3.1) reveals that we must also know the pump output flow \( Q_2 \), the fluid volume at the pump outlet (note that this is not to be confused with the fluid specific volume), \( V_2 \), the leakage associated with the pump, \( K \), and the fluid bulk modulus \( \beta \).

\[
\dot{P}_2 = \frac{\beta}{V_2} \left[ Q_2 - KP_2 - Q_{sys} \right] \tag{3.1}
\]

We will assume a constant leakage value. The literature suggests that 10% is reasonable for a water hydraulic system, so a value of \( K = \frac{Q_{max}}{10P_{max}} \) will be used [47].
The fluid specific volume at the pump outlet and the fluid bulk modulus will also be treated as constant. There are a significant number of published thermodynamic property tables that provide specific volume of water at a given pressure and temperature. These will be used throughout the analysis. The subsequent section will deal with determining the pump output flow, $Q_2$.

3.3 Pump Discharge Flow

As mentioned in the introduction, variation of the inlet metering valve cross-sectional area results in a change in the pump discharge flow rate. This section will explore how the pump output flow is affected by the valve area. Figure 3.1 shows the piston pump, crankshaft and check valves associated with the pump design, as well as the nomenclature that will be used.

![Piston pump diagram](image)

Figure 3-1. Piston pump
In a typical configuration for a piston pump, \( L \) will be much larger than the shaft radius \( r \). Thus, a small angle approximation can be made for the crank shaft angle \( \alpha \). This approximation yields an expression for the vertical displacement of the piston that is shown in Eq. (3.2).

\[
z = r (1 - \cos \theta)
\]

(3.2)

The pump will oscillate up and down as a function of the angle \( \theta \). An illustration of the mechanics of motion is provided in the strip chart shown in Figure 3.2.

![Strip chart illustrating piston pressure and displacement](image)

Figure 3-2. Strip chart illustrating piston pressure and displacement

Starting from the bottom dead center position, the internal volume of the piston is at its maximum value \( (V_r = 2A_pr) \), the piston is at its minimum displacement \( (z = 0) \), and the liquid inside the piston is at its lowest pressure over the course of the cycle. For our system, we have designed it such that the fluid enters the piston chamber at its
saturation pressure. This fluid will be a mixture of liquid and vapor, the specific percentages of which are dictated by the inlet metering valve cross-sectional area.

Once the piston begins to move upwards, (the angle $\theta$ increases from 0 to a specific value $\phi_1$), the pressure inside the piston chamber increases. As the pressure increases to a value of $P_2$, the vaporized fluid condenses until all of the fluid in the piston is liquid. Once the fluid is fully condensed, the check valve downstream from the pump (shown in Fig. 3.1) opens. This occurs at piston position $\phi_1$. From piston position $\phi_1$ to position $\pi$, the piston forces the fluid out of the chamber and through the check valve downstream from the pump. Once the piston reaches its maximum height ($z = 2r$), all of the fluid exiting the piston chamber for that given stroke will have been forced out of the piston. A small volume of fluid will remain in the chamber. As the piston moves from position $\pi$ to position $\phi_2$, the remaining fluid will vaporize until it reaches a pressure level equal to that of the fluid which is waiting upstream from the first check valve. At position $\phi_2$ the first check valve will open; and as the piston moves downward, the space that it vacates will be filled by the partially vaporized fluid coming from the inlet metering valve. This will continue until the piston chamber is again completely filled with a vapor/liquid mix at position $\theta = 0$. Then the piston will move upwards, and the cycle continues.

The contracted valve area induces partial fluid vaporization for the fluid entering the pump. Our means of talking about fluid vaporization throughout this paper will rely on a discussion of fluid quality, which refers to the percentage of vaporized fluid in the working fluid on a per mass basis.
The fluid specific volume $v$ for the pump inlet will be modeled as shown in Eq. (3.3).

The quality of the fluid will be denoted as $x$. When $x$ is zero, the fluid is entirely liquid. When $x$ is unity, the fluid is in an entirely vaporized state of gas. In our design, the fluid will condense inside the pressurized pump environment and will exit the pump in an entirely liquid phase, also shown in Eq. (3.3). The Tait Equation [77] justifies the assumption that the fluid outflow will be entirely liquid. This assumption is reasonable because the fluid will exit the piston pump at a significantly higher pressure than the fluid’s saturation pressure.

It has been assumed that the system experiences a constant mass flow rate. This assumption results in the relationship shown in Eq. (3.4), where $Q_1$ is the volumetric flow rate into the piston pump, $v_1$ is the specific volume of the fluid entering the piston pump (which may be partially vaporized), $Q_2$ is the volumetric flow rate out of the piston pump, and $v_{l_2}$ is the completely liquid specific volume of the fluid leaving the piston pump.

$$\frac{Q_1}{v_1} = \frac{Q_2}{v_{l_2}} \quad (3.4)$$

As was just mentioned in the description of Eq. (3.4), the fluid flowing into the piston pump could be partially vaporized. A more explicit description of the fluid flowing out of the piston pump in terms of the fluid specific volume, fluid quality at the pump inlet, and pump volumetric flow rate at the pump inlet is shown in Eq. (3.5).

$$Q_2 = \frac{v_{l_2}}{v_1 + xv_{lg_1}} Q_1 \quad (3.5)$$
Equation (3.5) clearly illustrates that the pump discharge flow is a function of the inlet fluid quality. For example, consider the case where the inlet fluid is fully condensed \( x = 0 \). If we recognize that the outlet flow specific volume \( v_{l2} \) is nearly identical to the inlet flow specific volume \( v_{l1} \) (the outlet flow specific volume will be slightly, though insignificantly, higher than the inlet flow specific volume due to the higher outlet pressure), we can observe that the discharge flow rate is simply equal to the pump inlet flow rate. (The assumption that \( v_{l0} \approx v_{l1} \approx v_{l2} \) will be applied throughout this analysis.)

If, however, we consider a fully vaporized fluid \( x = 1 \) and recognize that \( v_{lg1} \gg v_{l1} \), we can see that the discharge flow rate of the pump becomes very small. From this, we find that we have created a variable-displacement pump utilizing a fixed displacement pump, and an inlet metering valve that allows us to exert control over the fluid quality at the inlet.

The pump experiences a different flow rate for each of the phases described by Fig. 3-2. The following analysis will explore the pump behavior for each of these conditions. Then an analysis of the relationship between the valve’s cross sectional area and the fluid quality will be completed.

3.4 Pump Flow Phase I: \( 0 < \theta < \phi_1 \)

When the pump begins its upstroke, from \( \theta = 0 \to \phi_1 \), the fluid is condensing. The maximum volume of the pump occurs at \( \theta = 0 \) and the mass of the fluid that fits inside the piston volume can be given as Eq. (3.6).

\[
m = \frac{V_{max}}{v_l + v_{lg}x} \tag{3.6}
\]
At the bottom of the piston stroke, the fluid is a mixture of vapor and liquid and will vary as a function of the fluid quality. Once the crankshaft reaches the position \( \phi_1 \) the fluid is fully condensed \( (v = v_l) \). At this point, we can calculate the exact position of the crankshaft as a function of our piston internal maximum volume and fluid quality. This derivation is shown in Eq. (3.7).

\[
\frac{V_{\text{max}}}{v_i + v_{lg}x} = \frac{V_{\text{max}} - 2A_p r [1 - \cos \phi_1]}{v_i} \tag{3.7}
\]

The nondimensional groups shown in Eq. (3.8) are used to nondimensionalize Eq. (3.7). The specific volume ratio, \( \hat{v} \), and pump volume ratio, \( \hat{V} \), will both be key parameters considered throughout the analysis.

\[
\hat{v} = \frac{v_{lg}}{v_i} \quad \text{and} \quad \hat{V} = \frac{A_p r}{V_{\text{max}}} \tag{3.8}
\]

\[
\cos \phi_1 = 1 - \frac{\hat{v}x}{\hat{V}(1 + \hat{v}x)} \tag{3.9}
\]

A nondimensional expression for the crankshaft position at the point where our fluid is fully liquid, \( \phi_1 \), is given as Eq. (3.9). This is derived by applying the specific volume ratio and pump volume ratio to Eq. (3.7). From this expression, it is evident that the rotation required to condense our fluid is a function of the thermodynamic properties of the fluid \( (v_i \text{ and } v_{lg}) \), the dimensions of the piston itself \( (A_p, r, \text{ and } V_{\text{max}}) \), and the quality of the fluid entering the piston \( (x) \).
3.5 Pump Flow Phase II: $\phi_1 < \theta < \pi$

As the pump continues its upstroke, from $\theta = \phi_1 \rightarrow \pi$, the liquid is pushed to the discharge side of the pump. The flowrate out of the pump is a function of the changing volume of the piston chamber, as shown in Eq. (3.10). Our analysis will be conducted for a single piston pump, however, the results may be generalized for $N$ pistons as illustrated by Eqs. (3.10) and (3.11). The instantaneous volume of the piston can be derived from Eq. (3.7), and is shown as Eq. (3.11).

$$Q_n = -\frac{dV_n}{dt}$$

$$V_n = V_{\text{max}} - 2A_p r (1 - \cos \theta)$$

Equations (3.10) and (3.11) lead to the instantaneous expression for the fluid’s volumetric flow rate, which is shown as Eq. (3.12).

$$Q_n = -\frac{dV_n}{dt} = 2A_p r \omega \sin \theta$$

The average flow rate, then, can be shown as Eq. (3.13).

$$\bar{Q}_n = \frac{A_p r \omega}{\pi} \sin \theta d\theta = \frac{A_p r \omega}{\pi} [1 + \cos \phi]$$

The pump volumetric displacement, $V_d$, is simply a function of the piston geometry, where $A_p$ is the cross-sectional area of the piston and $r$ is the distance the piston is displaced within the cylinder. Equation (3.14) shows this relationship.

$$V_d = \frac{2A_p r}{2\pi}$$
Equation (3.15) shows the average result associated with Eq (3.13), utilizing the groups provided earlier in Eq. (3.8) and the expression for volumetric displacement shown in Eq. (3.14). From this, we can observe that the volumetric flow rate remains a function of the thermodynamic properties of the fluid, the dimensions of the piston itself, and the quality of the fluid entering the piston. It is also affected by the speed of the crank shaft ($\omega$), which was not the case for the condensation process.

$$\bar{Q}_2 = V_\omega \left[ 1 - \frac{\hat{v}_x}{2\dot{V} (1 + \hat{v}_x)} \right] \quad (3.15)$$

3.6 Pump Flow Phase III: $\pi < \theta < \phi_2$

As the pump begins its downstroke (from $\theta = \pi \rightarrow \phi_2$), the small amount of fluid that remains in the piston chamber (the fluid that was not expelled at the maximum height of the piston upstroke) is vaporized back to the quality associated with the fluid waiting upstream of the check valve. The exact location of $\phi_2$ can be found in a method similar to the derivation provided in the Pump Flow Phase I calculation. It again relies on the nondimensional groups provided in Eq. (3.8). The result is shown in Eq. (3.16).

$$\cos \phi_2 = -1 - \frac{(2\dot{V} - 1)\hat{v}_x}{\dot{V}} \quad (3.16)$$

Equation (3.16) shows that $\phi_2$ is a function of the thermodynamic properties of the fluid, the dimensions of the piston itself, and the quality of the fluid entering the piston. These are the same parameters associated with the identification of $\phi_1$ in Eq. (3.9).
3.7 Pump Flow Phase IV: $\phi_2 < \theta < 2\pi$

As the pump continues its downstroke, the upstream check valve opens and the partially vaporized fluid mixture is drawn into the pump. The volumetric flow rate of this fluid is still dictated by the relation given as Eq. (3.12) and can be expressed as shown in Eq. (3.17). It is a function of the thermodynamic properties of the fluid, the dimensions of the piston itself, the quality of the fluid entering the piston, and the speed of the crank shaft, which is the same list of parameters that shapes the average flow rate exiting the pump as described in Eq. (3.15).

$$\bar{Q}_i = V_d \omega \left[ 1 - \frac{(1 - 2\dot{V}) \dot{I}_x}{2\dot{V}} \right]$$  \hspace{1cm} (3.17)

3.8 Pump Flow Summary

Table 3-1 shows the volumetric flow rate entering or exiting the pump at each point along the cycle illustrated by Figs. 3-1 and 3-2. The locations of $\phi_1$ and $\phi_2$ are a function of the fluid quality leaving the inlet metering valve, and are given by Eqs. (3.9) and (3.16) respectively. From Table 3-1 it can be observed that a derivation of the fluid quality as a function of our valve geometry would be extremely useful. This will be done in Section 3.9.
Table 3-1. Summary of pump flow behavior at throughout the cycle

<table>
<thead>
<tr>
<th>Pump Flow Phase</th>
<th>Crankshaft Location</th>
<th>Volumetric Flow Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump Flow Phase I</td>
<td>$\theta = 0 \rightarrow \phi_1$</td>
<td>$Q = 0$</td>
</tr>
<tr>
<td>Pump Flow Phase II</td>
<td>$\theta = \phi_1 \rightarrow \pi$</td>
<td>$\bar{Q}_z = V_d \omega \left[ 1 - \frac{\hat{v}_x}{2\dot{V}(1 + \hat{v}_x)} \right]$</td>
</tr>
<tr>
<td>Pump Flow Phase III</td>
<td>$\theta = \pi \rightarrow \phi_2$</td>
<td>$Q = 0$</td>
</tr>
<tr>
<td>Pump Flow Phase IV</td>
<td>$\theta = \phi_2 \rightarrow 2\pi$</td>
<td>$\bar{Q}_i = V_d \omega \left[ 1 - \frac{(1 - 2\dot{V})\hat{v}_x}{2\dot{V}} \right]$</td>
</tr>
</tbody>
</table>

3.9 Inlet Metering Valve

Figure 3.3 shows the inlet metering valve, where $A_o$ is the cross-sectional area at the valve inlet, and $A_I$ is the cross-sectional area at the valve outlet. For simplicity of calculations, it will be assumed that the valve openings are identical squares. The dimensions of the fully open orifice are $y_{max} \times y_{max}$. $A_o$ is fixed, and $A_I$ varies linearly with the valve displacement $y$. This is written explicitly in Eq. (3.18).

\[
\begin{align*}
A_o &= y_{max}^2 \\
A_I &= yy_{max}
\end{align*}
\]  

(3.18)
For the purposes of our analysis, we will assume a constant mass flow rate given by

\[
\dot{m} = \frac{Q_0}{v_{i0}} = \frac{Q_1}{v_{i1} + x(v_{e1})} = \frac{Q_2}{v_{i2}}. \tag{3.19}
\]

Relying on our definitions for volumetric flow rate derived in the Pump Flow Phase II and IV analyses [Eqs. (3.15) and (3.17) or Table 3.1], the conservation of mass provided as Eq. (3.19), and the orifice equation which is presented in Eq. (3.20), a nondimensional expression for the cross sectional area of the valve as a function of the fluid quality can be derived. The orifice equation is a classical relation that can be found in a standard hydraulic control textbook, like that of Manring [20]. It describes the fluid volumetric flow rate, \( Q_0 \), as a function of the valve cross-sectional area, \( A_i \), the discharge coefficient, \( c_d \) which is a function of the valve geometry, the fluid specific volume, \( v_o \), and the pressure difference across the valve.

\[
Q_0 = A c_d \sqrt{2v_f (P_0 - P_1)} \tag{3.20}
\]

Figure 3-3. Inlet metering valve and associated nomenclature
Substitution of Eqs. (3.17) and (3.20) into Eq. (3.19) yields Eq. (3.21).

\[
Ac_d \sqrt{2v_t (P_0 - P_1)} = \frac{1}{1 + \hat{\nu}_x} V_r \omega \left[ 1 - \frac{(1 - 2\hat{V}) \hat{\nu}_x}{2\hat{V}} \right] \tag{3.21}
\]

We then define the valve dimensions according to Eq. (3.22). This convenient definition will enable us to solve Eq. (3.21), which results in Eq. (3.23).

\[
A = \hat{A} \frac{V_r \omega}{c_d \sqrt{2v_t (P_0 - P_1)}} \tag{3.22}
\]

\[
\hat{A} = \frac{2\hat{V} - (1 - 2\hat{V}) \hat{\nu}_x}{2\hat{V} (1 + \hat{\nu}_x)} \tag{3.23}
\]

Equation (3.23) can be simplified due to the relative magnitudes of the nondimensional groups from Eq. (3.8). This simplification is shown as Eq. (3.24).

\[
\hat{A} \approx 1 - \frac{\hat{\nu}_x}{2\hat{V}} \tag{3.24}
\]

Rearranging Eq. (3.24) provides an expression for the fluid quality in terms of valve outlet area. This result is shown as Eq. (3.25).

\[
x = \frac{(1 - \hat{A})2\hat{V}}{\hat{v}} \tag{3.25}
\]

The volumetric flow rate can be nondimensionalized using the pump volumetric displacement and shaft speed, as shown in Eq. (3.26).

\[
Q_2 = V_r \omega \hat{Q}_2 = \frac{A_p r \omega}{\pi} \hat{Q}_2 \tag{3.26}
\]

38
Recall the definition for the fluid outlet volumetric flow rate given by Eq. (3.15).

Substitution of the relation for fluid quality from Eq. (3.25) into Eq. (3.15) and the nondimensional expression for volumetric flow rate in Eq. (3.26) yields a simplified approximation for the volumetric flow rate. Observe that Eq. (3.27) matches the result for the nondimensional valve cross sectional area, \( \hat{A} \), which was presented in Eq. (3.23).

\[
\hat{Q}_2 = 1 - \frac{\hat{v}_x}{2V (1 + \hat{v}_x)}
\]  

(3.27)

Normalizing the volumetric flow rate about the condition where \( x=0 \) leads to a simplified expression for the nondimensional volumetric flow rate.

\[
\hat{Q}_2 \approx 1 - \frac{\hat{v}_x}{2V}
\]  

(3.28)

Recall the definition for the nondimensional valve cross-sectional area given in Eq. (3.24). A comparison between Eqs. (3.24) and (3.28) (or Eqs. (3.23) and (3.27)) reveals a direct relationship between the volumetric flow rate and valve cross-sectional area.

\[
\hat{Q}_2 = \hat{A}
\]  

(3.29)

Equation (3.29) shows that the discharge flow of the pump is directly proportional to the cross-sectional area of the inlet metering valve which provides a very nice relationship from a control point of view.

In order to apply the pressure rise rate equation from Eq. (3.1), a second expression for the flow exiting the pump must be developed. The pressure rise rate equation will rely on the previously presented orifice equation (Eq. (3.20)). Recall the conservation of mass (Eq. (3.19)). By definition, then, \( Q_0 \approx Q_2 \) if we assume the specific
volume of all purely liquid quantities of the working fluid are constant \( (v_{i0} = v_{i1} = v_{i2}) \).

This result is provided in Eq. (3.30)

\[
Q_2 = A_l c_d \sqrt{2v_{i0} (P_0 - P_1)}
\]

(3.30)

Recall Eq. (3.18) which shows the relationship between valve areas and valve displacement. Thus, the volumetric flow rate can be rewritten in terms of valve displacement \( y \). This result is shown in Eq. (3.31).

\[
Q_2 = y_{\text{max}} c_d \sqrt{2v_{i0} (P_0 - P_1)} y
\]

(3.31)

Revisiting the pressure rise rate equation [Eq. (3.1)] and substituting the leakage coefficient and pump outlet expression into it yields the pressure rise rate equation in terms of valve displacement \( y \). This is given as Eq. (3.32).

\[
\dot{P}_2 = \frac{\beta}{V_2} \left[ y_{\text{max}} c_d \sqrt{2v_{i0} (P_0 - P_1)} y - KP_2 - Q_{\text{sys}} \right]
\]

(3.32)

Equation (3.32) will then be normalized about the steady state volumetric flow rate. This will be done by solving the steady state system. The result is given as Eq. (3.33)

\[
Q_{\text{sys,ss}} = y_{\text{max}} c_d \sqrt{2v_{i0} (P_0 - P_1)} y_{\text{ss}} - KP_{\text{ss}}
\]

(3.33)

Substitution of Eq. (3.33) into Eq. (3.32) yields a normalized expression about the steady system scenario where \( y_{\text{ss}} \) and \( P_{\text{ss}} \) are the steady-state valve displacement and steady-state pressure respectively. This result is given as Eq. (3.34).

\[
\dot{P}_2 = \frac{\beta}{V_2} \left[ y_{\text{max}} c_d \sqrt{2v_{i0} (P_0 - P_1)} (y - y_{\text{ss}}) - K (P_2 - P_{\text{ss}}) \right]
\]

(3.34)
In the following section, a second expression for valve displacement will be derived. This is necessary for control law development.

3.10 Valve Flow Force Considerations

A free body diagram of the inlet metering valve is shown as Fig. 3-4. The valve experiences stabilizing forces from the spring and downstream pressure (the resulting force from $P_2$ is called $F_y$), as well as a destabilizing force from the internal flow forces associated with the fluid slug inside the valve.

![Free body diagram of the inlet metering valve](image)

Figure 3-4. Inlet metering valve free body diagram

The dynamic equation for the valve, based on Fig. 3-4, is given in Eq. (3.35), where $m_v$ is the mass of the spool valve, $F_o$ is the initialized spring force, $k$ is the spring constant, $c$ is the drag coefficient associated with the valve displacement, $F_y$ is the force on the valve that results from application of pressure, $P_2$, on the cross sectional area of the valve spool, $A_v$, and $F_i$ is the summation of all internal horizontal flow forces within the valve.

$$m_v \ddot{y} = F_o - ky - F_y + F_i - c \dot{y}$$ (3.35)
3.11 Flow Force Analysis

In order to find the internal flow force, \( F_i \), the Reynolds Transport Theorem will be used. A derivation of the Reynolds Transport Theorem can be found in a typical fluids textbook [78]. The Reynolds Transport Theorem is given in Eq. (3.36). It may be helpful to observe that vector quantities are denoted by boldface type.

\[
\mathbf{F} = \frac{\partial}{\partial t} \int_{CV} \rho \mathbf{u} dV + \int_{CS} \rho \mathbf{u} (\mathbf{u} \cdot \hat{n}) dA 
\]  

(3.36)

![Diagram of flow forces and inlet metering valve](attachment:diagram.png)

Figure 3-5. Inlet metering valve flow at the outlet

Expanding the Reynolds Transport Theorem for the geometry in Fig. 3-5 yields Eq. (3.37).

\[
\mathbf{F} = \frac{\partial}{\partial t} \int_{V_0} \rho \mathbf{u}_0 dV_0 + \frac{\partial}{\partial t} \int_{V_1} \rho \mathbf{u}_1 dV_1 + \int_{A_0} \rho \mathbf{u}_0 (\mathbf{u}_0 \cdot \hat{n}_0) dA_0 + \int_{A_1} \rho \mathbf{u}_1 (\mathbf{u}_1 \cdot \hat{n}_1) dA_1 
\]  

(3.37)

The vector expressions associated with the Reynolds Transport Theorem and based on the geometry in Fig. 3-5 are given in Eq. (3.38). The fluid velocity is described by the vector quantity \( \mathbf{u} \) and the normal vector component is written as \( \mathbf{n} \).
\[ \mathbf{u}_0 = \frac{Q_l}{A_0} \mathbf{i} + 0 \mathbf{j} \quad \mathbf{u}_1 = \frac{Q_l}{A_1} \cos \theta \mathbf{i} - \frac{Q_l}{A_1} \sin \theta \mathbf{j} \]

(3.38)

\[ \hat{\mathbf{n}}_0 = -\mathbf{i} + 0 \mathbf{j} \quad \hat{\mathbf{n}}_1 = \cos \theta \mathbf{i} - \sin \theta \mathbf{j} \]

Observe that the force described by the Reynolds Transport Theorem is simply a vector as illustrated in Eq. (3.39).

\[ \mathbf{F} = F_i \mathbf{i} + F_j \mathbf{j} \]

(3.39)

The force can be broken up into components as shown in Eq. (3.39). Substituting the vector quantities from Eq. (3.38) into Eq. (3.37) and then separating the result into components yields Eq. (3.40). The application of the Reynolds Transport Theorem is covered extensively by Manring [20].

\[ F_i = -L \frac{dQ}{v} \frac{dQ^2}{dt} - \frac{Q_o^2}{A_1 v_o} \cos \theta \]

\[ F_j = \frac{Q_o^2}{A_1 v_1} \sin \theta \]

(3.40)

The orifice equation was given earlier in Eq. (3.31). The transient volumetric flow force can then be written as Eq. (3.41).

\[ \frac{dQ}{dt} = c_d \sqrt{2v_o (P_0 - P_1)} \frac{dA_i}{dt} \]

(3.41)

Applying Eqs. (3.18), (3.40), and (3.41) yields an expression for the internal horizontal flow force in terms of valve displacement. This equation elicits a description of the force that results from the changing momentum of the fluid slug within the valve.
\[ F_i = -\frac{L}{v_o} y_{\text{max}} c_d \sqrt{2v_o (P_0 - P_i)} \frac{dy}{dt} - 2(P_0 - P_i)c_d^2 y_{\text{max}} \cos \theta y \quad (3.42) \]

For simplicity, the leading coefficients in Eq. (3.42) will be defined by the groupings in Eq. (3.43)

\[ \xi_1 = \frac{L}{v_o} c_d \sqrt{2v_o (P_0 - P_i)} y_{\text{max}}, \quad (3.43) \]

\[ \xi_2 = 2(P_0 - P_i)c_d^2 y_{\text{max}} \cos \theta \]

which then produces the following expression for the horizontal force associated with the fluid slug momentum

\[ F_i = -\xi_1 \frac{dy}{dt} - \xi_2 y. \quad (3.44) \]

Thus the expression for the changing fluid momentum can be rewritten as Eq. (3.44) utilizing the groups defined in Eq. (3.43).

The angle \( \theta \) in Eqs. (3.42) and (3.43) (and shown in Figure 3.5) is the angle at which the fluid exits the valve outlet. This is called the jet angle, which was classically derived by Von Mises in 1917 [79]. The jet angle solution from Von Mises is given in Eq. (3.45) [80], where \( c_r \) is the radial clearance within the valve and \( y \) is the valve displacement.
In order to approximate Eq. (3.45) for the purposes of our analysis, a constant jet angle was assumed. Merritt states that this is an entirely realistic approximation, particularly for a valve that operates typically between 75% and 100% of the maximum valve opening [54]. Merrit suggests that $\theta = 69^\circ$ is an accurate approximation; however, Yang et al makes a compelling argument that specifically for water hydraulics, $\theta = 64^\circ$ is better [24, 54]. In his work, "Water hydraulics-A novel design of spool-type valves for enhanced dynamic performance", he engages in an extensive CFD analysis of a variety of spool valve geometries and for multiple fluid viscosities (simulating both water and mineral oil) and validates Merritt’s assertion for mineral oil. However, he notes that the efflux angle is impacted by the reduced fluid viscosity of water, and that a slightly smaller jet angle is more representative [24]. His CFD results clearly illustrate this point, and thus, $\theta = 64^\circ$ will be used throughout the analysis.

3.12 Full Dynamic Valve Equation

The valve dynamics can be fully depicted by applying Eq. (3.42) to Eq. (3.35). The full valve equation is given as Eq. (3.46).

$$m_v \ddot{y} + (c + \xi_1) \dot{y} + (k + \xi_2) y = F_o - P_z A_{valve}$$  (3.46)
The spring initialized force, $F_o$, can be determined by solving Eq. (3.46) for a steady system. The spring force is shown as Eq. (3.47).

$$F_o = A_{valve} P_{ss} + \left( k + \xi_2 \right) y_{ss}$$ (3.47)

Substitution of Eq. (3.47) into Eq. (3.46) yields a normalized expression about the steady system scenario where $y_{ss}$ and $P_{ss}$ are the steady-state valve displacement and steady-state pressure respectively. This is given as Eq. (3.48).

$$m v \ddot{y} = -(c + \xi_1) \dot{y} - \left( k + \xi_2 \right) \left( y - y_{ss} \right) - A_{valve} \left( P - P_{ss} \right)$$ (3.48)

3.13 Summary of Dimensional Equations

Eq. (3.49) shows the coupled set of differential equations that describe the motion of the inlet metering spool valve. The derivation of the pressure rise rate equation reached its conclusion in Eq. (3.34), and the derivation of outlet pressure as a function of the valve was concluded in Eq. (3.48).

$$\begin{align*}
1. \quad & \dot{P} = \frac{\beta}{V_2} \left[ y_{max} c_d \sqrt{2 v_{r0} (P_0 - P_1)} \left( y - y_{ss} \right) - K \left( P - P_{ss} \right) \right] \\
2. \quad & \frac{m v}{A_{valve}} \ddot{y} + \left( \frac{c + \xi_1}{A_{valve}} \right) \dot{y} + \left( \frac{k + \xi_2}{A_{valve}} \right) \left( y - y_{ss} \right) = P_{ss} - P
\end{align*}$$ (3.49)

3.14 Nondimensionalization

Several methods exist to identify the most significant terms in a dynamic equation. Numerical regression can be used [81]. Development of nondimensional groupings for applied sensitivity analysis is another option [82, 83]. This second
approach seems to be a more expedient process which is easily adjusted to varying system parameters. As such, this will be the method employed in this analysis.

Taking a nondimensional approach to Eq. (3.49) enables simplification of our system of equations. By identifying nondimensional groupings in our key equations, we will be able to compare the relative magnitudes of the nondimensional groups and determine which terms contribute most significantly to the system dynamic behavior. After we identify the insignificant terms, we can eliminate the higher order differential terms in our expression. A lower order system will behave with less overshoot and oscillation; it is helpful to determine if our system can be reduced. Eqs. (3.50) and (3.51) show the important nondimensional term definitions that will be used in our analysis.

\[ y = y_{ss} \hat{y} \quad P = P_{ss} \hat{P} \quad t = \tau \hat{t} \quad \tau = \frac{V_2}{\beta K} \]  

\( \hat{K}_1 = \frac{y_{ss} c_d \sqrt{2y_{10} (P_0 - P_1)}}{KP_{ss}} \quad \hat{K}_2 = \frac{(k + \xi_2) y_{ss}}{A_{valve} P_{ss}} \quad \hat{c} = \frac{(c + \xi_1) y_{ss}}{\tau A_{valve} P_{ss}} \quad \hat{m} = \frac{m_v y_{ss}}{\tau^2 A_{valve} P_{ss}} \)  

The nondimensional groups given in Eqs. (3.50) and (3.51) can be applied to Eq. (3.49) in order to yield the nondimensional description of the system given as Eq. (3.52).

1. \( \hat{P} + \hat{P} = \hat{K}_1 \hat{y} + \left[1 - \hat{K}_1 \right] \)
2. \( \hat{m} \ddot{\hat{y}} + \hat{c} \dot{\hat{y}} + \hat{K}_2 \hat{y} = -\hat{P} + \left(1 + \hat{K}_2 \right) \)  

Observe that for steady-state operations, the nondimensional pressure, \( \hat{P} \), must be equivalent to the nondimensional valve displacement, \( \hat{y} \). In order for this to be true,
design guidelines should be established such that the nondimensional leakage coefficient $\hat{K}_i$ is unity. The design guidelines requisite for this scenario will be discussed further in Chapter 5, but for now it is sufficient to assert that the nondimensional leakage coefficient can be designed to achieve this aim. The resulting set of nondimensional equations, then can be shown in Eq. (3.53).

\[
\begin{align*}
1. & \quad \hat{P} + \hat{P} = \hat{y} \\
2. & \quad \hat{m}\ddot{y} + \hat{c}\dot{y} + \hat{K}_2\dot{y} = -\hat{P} + \left(1 + \hat{K}_2\right)
\end{align*}
\] (3.53)

The nondimensional set of equations in Eq. (3.53) comprises the most important contribution of this analysis. From this result, we will be able to investigate the efficiency of the system (Chapter 4), study the stability of the system (Chapter 5), develop design criteria and rules of thumb for engineers looking to apply this technology (Chapter 5), and model the dynamic behavior of our system (Chapter 6). In Chapter 6 we will solve Eq. (3.53) for a variety of design scenarios. This will show the behavior of the inlet metering system and illustrate its ability to maintain a downstream steady-state pressure.

3.15 Variable Displacement Pump Model

Because we are interested in comparing the dynamic characteristics of the inlet metering system design to the more traditional approach to flow control in a hydraulic system, it is helpful at this time to introduce a model of an axial piston pump (A labeled diagram of a variable displacement pump is provided in Chapter 4). Dynamic modeling of axial piston pumps are prevalent in hydraulic systems literature. Zeigers and Akers
developed a model in 1985 that many current models expand on [84]. The dynamic model that we will rely on was produced by Manring, and the governing simplified nondimensional equations are shown in Eq. (3.54) [85]. (For comprehensive coverage of the derivation of axial piston pump dynamics see the appendix of Schoenau et al’ s paper on variable displacement pump dynamics [86].)

\[
\begin{align*}
\dot{\hat{P}} &= -\varphi_1 \hat{P} + \hat{\alpha} + (\varphi_1 - 1) \\
\dot{\alpha} &= -\chi_1 \hat{P} - \chi_2 \hat{\alpha} + (\chi_1 + \chi_2)
\end{align*}
\]  

Equation (3.54) describes the dynamics of the system pressure, \( \hat{P} \), and swash plate angle, \( \hat{\alpha} \). These system characteristics are affected by the pump leakage, \( \varphi_1 \), the combined effects of the discharge pressure moment exerted on the swash plate, open – centered valve flow into the control actuator, volumetric change of the control actuator, spool valve spring rate, and pressure induced flow-force on the spool valve, \( \chi_1 \), and the combined effects of the swash-plate spring rate, open-centered valve flow into the control actuator, volumetric change of the control actuator, spool valve spring rate, and pressure induced flow-force on the spool valve, \( \chi_2 \).

The transfer function of the system described by Eq. (3.54) is given in Eq. (3.55).

The process of utilizing the transfer function in the determination of the bandwidth frequency for the variable displacement pump is identical to the process that will be described in greater depth in the context of the inlet metering system in Chapter 5.

\[
\hat{P}(s) = \frac{\chi_1 + \varphi_1 \chi_2}{s^2 + (\varphi_1 + \chi_2)s + \varphi_1 \chi_2} 
\]  

Equations (3.54) and (3.55) will be applied later for the purposes of comparison.
CHAPTER 4. SYSTEM EFFICIENCY

4.1 Efficiency Introduction

The efficiency of the inlet metering system is of paramount importance when considering implementation of this technology. In this chapter an analysis based on experimental data will be conducted to determine the efficiency associated with a traditional axial piston variable displacement pump. Then, a theoretical analysis of the inlet metering valve system will be presented. A comparison between the two systems will be shown. The efficiency analysis based on experimental data from the inlet metering system prototype will be conducted in Chapter 6. It will rely heavily on the analysis presented here, and will compare our theoretical predictions with the real measured values.

4.2 Traditional Pump Efficiency

Frequent comparisons will be made between an inlet metering pump and a variable displacement pump. The variable displacement axial piston pump is traditionally used in the applications for which the inlet metering system is being considered. Figure 4-1 shows the standard design of a variable displacement pump and the associated terminology. The pump contains several pistons arranged in a circle about the cylindrical block. The block is positioned against a valve plate and held there by the resultant force from the cylinder-block spring. A ball-and-socket joint is used to connect each piston to a slipper. The reader should note that these ball-and-socket joints are
expensive and require tight machining tolerances. The implementation of the inlet metering valve system removes the requirement of ball-and-socket joints from the design. The slippers are separated from the swashplate by a bearing. As the swash plate angle, $\alpha$, is adjusted, the ball-and-socket joints on the pistons enable rotation of the plate. When the valve plate is held in a set position for a given swash plate angle, the cylinder block is driven at a constant angular velocity, $\omega$. As the cylinder block is driven, each piston passes over the discharge and intake ports that are cut into the valve plate. As a piston rotates over the intake port, the piston moves out of the cylinder block and draws fluid into the piston bore. When a piston rotates over the discharge port, the piston pushes into the cylinder block, causing the fluid to be pushed out of the piston bore. This motion continues repeatedly throughout the operation of the pump.

Figure 4-1. Variable displacement pump diagram [20]
An experimental study of the variable displacement pump was conducted in order to evaluate the pump efficiency [87]. Equation (4.1) shows the method by which the pump efficiency was determined. As shown in Eq. (4.1), the power supplied to the pump is the product of the nondimensional pump torque, \( \hat{T}_p \), and the nondimensional angular velocity of the shaft, \( \hat{\omega}_p \), where the nondimensional angular velocity of the shaft is defined as \( \hat{\omega} \omega_{\text{max}} = \omega \). The power transmitted by the pump is the product of the nondimensional pump discharge pressure, \( \hat{P} \), and the nondimensional pump discharge flow rate, \( \hat{Q}_p \).

\[
\eta_p = \frac{\hat{P} \hat{Q}_p}{\hat{T}_p \hat{\omega}_p}
\]

(4.1)

Figure 4-2. Schematic of the pump, showing mass and volumetric flow rates

Observe that the nondimensional angular velocity of the pump shaft, \( \hat{\omega}_p \), is unity. The derivation of expressions for nondimensional flowrate and torque are also covered by
Manring in his work on hydrostatic transmission efficiency mapping [87]. These results are shown as Eq. (4.2) and (4.4) respectively. For the variable displacement pump, the swashplate angle \((\hat{\alpha})\) will vary between -1 and 1. The nondimensional volumetric flow rate, then, is expressed as:

\[
\hat{Q}_p = \hat{\alpha} \hat{\omega} - K_0 \hat{\alpha} \hat{\omega} \hat{P} - K_1 \frac{\hat{P}}{\mu} - K_2 \sqrt{\hat{P}} \tag{4.2}
\]

where

\[
K_0 = \frac{P_{\text{max}}}{\beta}, \quad K_1 = \frac{K_1' P_{\text{max}}}{V_d \omega \mu}, \quad K_2 = \frac{K_2' \sqrt{P_{\text{max}}}}{V_d \omega}.
\tag{4.3}
\]

The nondimensional torque is written as:

\[
\hat{T} = \hat{\alpha} \hat{\dot{P}} + A_p \hat{\alpha} \hat{P} \text{Exp}\left(-B_p \frac{\hat{\mu} \hat{\omega}}{\hat{\alpha} \hat{P}}\right) + C_p \sqrt{\hat{\alpha} \hat{\omega} \hat{P}} \hat{\mu} + D_p \tag{4.4}
\]

where

\[
B_p = B'_p \frac{\mu \omega}{P_{\text{max}}}, \quad C_p = C'_p \sqrt{\frac{\mu \omega}{P_{\text{max}}}}, \quad D_p = \frac{D'_p}{V_d \frac{P_{\text{max}}}{\mu}}.
\tag{4.5}
\]

Each of the coefficients in Eqs. (4.3) and (4.5) are non-dimensional and determined experimentally. A complete definition of the 145cc pump efficiency is expressed in terms of the swashplate position, pressure, and experimental coefficients. Substitution of Eqs. (4.2) and (4.4) into Eq. (4.1) yields

\[
\eta = \frac{\hat{\alpha} \hat{\dot{P}} \left( \hat{\alpha} - K_0 \hat{\alpha} \hat{P} - K_1 \frac{\hat{P}}{\mu} - K_2 \sqrt{\hat{P}} \right)}{\hat{\alpha} \hat{\dot{P}} + A \hat{\alpha} \hat{P} \text{Exp}\left(-B \frac{\hat{\mu} \hat{\omega}}{\hat{\alpha} \hat{P}}\right) + C \sqrt{\hat{\alpha} \hat{\omega} \hat{P}} \hat{\mu} + D}.
\tag{4.6}
\]

In order to derive the coefficients associated with the efficiency calculation, 840 efficiency measurements were taken throughout the operation of the variable
displacement pump. This experiment was conducted by Noah Manring and Viral Mehta and is beyond the scope of this dissertation. For a more complete discussion of experimental methods, please see their work [87]. The coefficients for the pump flow in Eq. (4.2) were derived from applying the following matrix to the experimentally measured quantities:

\[
\begin{pmatrix}
\hat{\alpha}_1 \hat{P}_1 & \hat{P}_1 & \sqrt{\hat{P}_1} \\
\hat{\alpha}_2 \hat{P}_2 & \hat{P}_2 & \sqrt{\hat{P}_2} \\
\vdots & \vdots & \vdots \\
\hat{\alpha}_n \hat{P}_n & \hat{P}_n & \sqrt{\hat{P}_n}
\end{pmatrix}
\begin{pmatrix}
K_0 \\
K_1 \\
K_2 \\
\vdots \\
\hat{\alpha}_n - \hat{Q}_{p1} \\
\hat{\alpha}_2 - \hat{Q}_{p2} \\
\vdots \\
\hat{\alpha}_n - \hat{Q}_{pm}
\end{pmatrix}
\begin{pmatrix}
C
\end{pmatrix}
\]

\(\text{(4.7)}\)

where each row in matrix \(A\) and column vector \(F\) represents an experiment that has been conducted in the laboratory and where there are a total of \(n\) experiments shown. The quantities in \(A\) and \(F\) are known from experiments. The only unknowns in Eq. (4.7) are the coefficients found in the column vector \(C\). These, however, may be determined using the theory of least squares:

\[
C = (A^T A)^{-1} A^T F
\]

\(\text{(4.8)}\)

which produces the best solution for the coefficients given the data in \(A\) and \(F\) [88]. This method can also be used to determine most of the experimental coefficients found in Eq. (4.4); however, the decay constant \(B\) inside the exponential function is not able to be evaluated in this way. Instead the decay constant was selected by a trial and error method that maximized the \(R^2\) value of the curve fit, where the \(R^2\) value measures the ability of the theoretical curve to match the trajectory of the experimental data.

The coefficients presented in Table 4-1 were produced for the 145cc axial piston pump, using the 840 efficiency measurements and the least squares method previously
described. These coefficients produced a $R^2$ value of 0.925 for the pump volumetric efficiency, and a $R^2$ value of 0.864 for the pump torque efficiency, where the volumetric efficiency was evaluated as

$$\eta_v = \frac{Q_d}{V_d \omega} \quad \tag{4.9}$$

and the torque efficiency was evaluated as

$$\eta_t = \frac{V_d P_d}{T} \quad \tag{4.10}$$

Table 4-1. Pump coefficients for Eqs. (4.3) and (4.5) [87]

<table>
<thead>
<tr>
<th>Physical Meaning</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid compression</td>
<td>$K_0$</td>
<td>0.0204</td>
</tr>
<tr>
<td>Low Reynolds-number leakage</td>
<td>$K_1$</td>
<td>0.0151</td>
</tr>
<tr>
<td>High Reynolds-number leakage</td>
<td>$K_2$</td>
<td>0.0109</td>
</tr>
<tr>
<td>Static friction</td>
<td>$A_p$</td>
<td>0.1257</td>
</tr>
<tr>
<td>Decay rate for boundary lubrication</td>
<td>$B_p$</td>
<td>10</td>
</tr>
<tr>
<td>Hydrodynamic lubrication</td>
<td>$C_p$</td>
<td>0.0077</td>
</tr>
<tr>
<td>Starting torque</td>
<td>$D_p$</td>
<td>0.0147</td>
</tr>
</tbody>
</table>

Recall the nondimensional expression for pump efficiency given in Eq. (4.1). The values for nondimensional torque and flow rate were already given in Eqs. (4.2) and (4.4), and can be evaluated based on the numerical values provided in Table 4-1. We will assume the pressure in the pump is constant and normalized about a maximum pressure $P_f$
42 MPa so that it is on the order of unity. (While the outlet pressures may vary slightly, the objective of the pump is to hold the downstream pressure constant and for this analysis we will assume that the pump achieves its goal.) Then it becomes clear that the only varying parameter in Eqs. (4.1), (4.2), and (4.4) is the swash plate angle, which is what is used to control the pump flowrate. Thus we can evaluate the theoretical efficiency of the variable displacement axial piston pump as a function of pump volumetric flow rate. This result is presented in Chapter 6, and shows how the efficiency of the variable displacement pump varies as a function of volumetric flow rate out of the axial piston pump. A similar analysis will now be performed for the inlet metering pump.

4.3 Inlet Metering Pump Efficiency

The previous analysis reported the efficiency associated with a traditional variable displacement pump. This portion of the chapter will find the theoretical efficiency for the inlet metering system that we have designed. The system inputs will be the torque on the shaft, $T$, and the power associated with the valve inlet pressure, $P_o$, and volumetric flow rate entering the valve, $Q_o$. The power leaving the system is defined by the product of the pump outlet pressure, $P_2$, and the volumetric flow rate out of the pump, $Q_2$. There will be leakage losses in the inlet metering pump similar to the losses associated with the variable displacement pump. New leakage coefficients for the inlet metering system will be identified in a manner similar to that describe in Section 4.2.
The power output of the piston pump is a function of the output pressure and volumetric flow rate, which is, in turn, a function of the inlet metering valve cross sectional area. This is shown in Eq. (4.11). Equation (4.11) relies on the relationship between valve displacement and pump volumetric flow rate that was determined in Eq. (3.23).

\[ P_{out} = P_2 Q_2 = P_2 V_s \omega \hat{A} \]  

(4.11)

The torque transmitted by the shaft depends on the stroke position. Recall Table 3-1 and the earlier determination of the key crankshaft positions. An energy input will be required to condense the fluid as the shaft rotates from \( \theta = 0 \) to \( \theta = \phi \); the specific energy associated with the fluid compression is shown in Eq. (4.12).

\[ e_{0-\phi} = h_0 - h_{\phi} \]  

(4.12)

Enthalpy is a function of pressure, and the pressure within the piston chamber varies as shown in Fig. 4-4.
Figure 4-4. Pressure as a Function of Crankshaft Position

The fluid at the pump inlet, what is referred to as state 1, will be partially vaporized. The fluid will be entirely condensed at state 2, when it exits the pump. This is reflected in the expanded definition of enthalpy,

\[ h_0 = u_i + u_{lg} x + P_i \left( v_i + v_{lg} x \right) \]

Substitution of Eq. (4.13) into Eq. (4.12) yields a new expression for the fluid compression specific energy requirement,

\[ e_{p-s} = \left( u_{lg} + P v_{lg} \right) x \]  \hspace{1cm} (4.14)

Observe that the specific energy value is on a per mass basis. Thus, the mass within the piston can be calculated by

\[ m = \frac{V_{\text{max}}}{v_i + v_{lg} x} \]  \hspace{1cm} (4.15)
The energy associated with the fluid condensation, then, can be calculated by finding the product of Eqs. (4.14) and (4.15). This result is given in Eq. (4.16). Observe that Eq. (4.16) takes advantage of the nondimensional specific volume ratio \( \hat{v} \) defined in Eq. (3.8) of the analysis.

\[
E_{0-\phi} = \frac{V_{\max} \left( u_{lg} + P_{ig} \right) x}{v_i + v_{lg} x} = \frac{V_{\max} \left( \frac{u_{lg}}{v_{lg}} + P_i \right) \hat{v}x}{1 + \hat{v}x}
\]  

Equation (4.16) can be simplified if it is normalized about the scenario where \( x = 0 \). This yields the compression energy expression

\[
E_{0-\phi} \approx V_{\max} \left( \frac{u_{lg}}{v_{lg}} + P_i \right) \hat{v}x. \tag{4.17}
\]

Equation (4.17) can be rewritten in terms of the valve cross-sectional area utilizing our earlier definition of fluid quality. The compression energy in terms of valve position then is

\[
E_{0-\phi} \approx 2A_p r \left( \frac{u_{lg}}{v_{lg}} + P_i \right) \left( 1 - \hat{A} \right). \tag{4.18}
\]

Equation (4.18) evaluates the amount of energy required per revolution of the crankshaft. This can be translated to a per radian basis by dividing Eq. (4.18) by \( 2\pi \) radians, and recognizing that \( V_p = \frac{A_p r}{\pi} \). The energy required on a per radian basis is given as Eq. (4.19).

\[
E_{0-\phi} \approx V_d \left( \frac{u_{lg}}{v_{lg}} + P_i \right) \left( 1 - \hat{A} \right) \tag{4.19}
\]

The power required to condense the fluid, then, is
\[ P_{\phi \cdot \Phi} \approx V_u \omega \left( \frac{h_{lg}}{v_{lg}} + P_1 \right) (1 - \hat{A}). \]  

(4.20)

For the next phase of the crankshaft cycle, from \( \theta = \phi_1 \) to \( \theta = \pi \), the upstream check valve is open and the power output is simply the discharge power. It can be written as

\[ P_{\phi \cdot \pi} = P_2 V_u \omega \hat{A}. \]  

(4.21)

Note that the pressure in the piston chamber remains constant from \( \theta = \phi_1 \) to \( \theta = \pi \).

During the next phase of the crankshaft cycle, from \( \theta = \pi \) to \( \theta = \hat{\phi}_2 \), the check valves are both closed and the fluid in the system is vaporizing. The specific energy associated with fluid vaporization is described by

\[ e_{\pi \cdot \hat{\phi}_2} = h_{\pi} - h_{\hat{\phi}_2}. \]  

(4.22)

The specific energy calculation for the vaporization phase of the crankshaft cycle can be conducted in a way very similar to the calculation of specific energy associated with the condensation phase of the crankshaft cycle. The enthalpy for this phase of the cycle is described by

\[ h_{\pi} = u_i + P_1 v_i \]

\[ h_{\hat{\phi}_2} = u_i + u_{lg} x + P_1 (v_i + v_{lg} x). \]  

(4.23)

The specific energy required to vaporize the fluid, then, is

\[ e_{\pi \cdot \hat{\phi}_2} = - \left( u_{lg} + P_1 v_{lg} \right) x. \]  

(4.24)

The energy required to vaporize the fluid can be calculated in the same manner of the calculations associated with Eq. (4.16). Then the mass of the fluid to be vaporized is given by
\[
m = \frac{V_{\text{max}} - 2A_pr}{v_l}.
\] (4.25)

The energy required to vaporize the fluid (on a per-revolution basis) is

\[
E_{\pi-\phi_2} = \frac{-\left(V_{\text{max}} - 2A_pr\right)}{v_l}\left(u_{lg} + P_{l}v_{lg}\right)x.
\] (4.26)

The energy required to vaporize the fluid on a per-radian basis is

\[
E_{\pi-\phi_2} = \frac{-A_pr\left(1 - 2\hat{V}\right)}{\pi}\left(\frac{u_{lg}}{v_{lg}} + P_{l}\right)\left(1 - \hat{A}\right).
\] (4.27)

Therefore, the power required to vaporize the fluid is

\[
P_{\pi-\phi_2} = -V_d\omega\left(1 - 2\hat{V}\right)\left(\frac{u_{lg}}{v_{lg}} + P_{l}\right)\left(1 - \hat{A}\right),
\] (4.28)

where the nondimensional group \(\hat{V}\) is the volume ratio developed during the analysis.

For the final phase of the crankshaft cycle, from \(\theta = \phi_2\) to \(\theta = 2\pi\), the downstream check valve is open and the power output is simply the discharge power. It is

\[
P_{\phi_2-2\pi} = -PV_d\omega\dot{Q}_m = -PV_d\omega\left[1 - \left(1 - 2\hat{V}\right)\left(1 - \hat{A}\right)\right].
\] (4.29)

A summary of the power demand through each of the crankshaft cycle locations is shown in Table 4-2. This table will be helpful as a reference later in the efficiency analysis.
Table 4-2. Summary of Pump Flow Behavior at Throughout the Cycle

<table>
<thead>
<tr>
<th>Pump Flow Phase</th>
<th>Crankshaft Location</th>
<th>Power Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump Flow Phase I</td>
<td>( \theta = 0 \rightarrow \phi_1 )</td>
<td>( P_{0-\phi} \approx V_d \omega \left( \frac{\mu g}{v_{lg}} + P_1 \right) \left( 1 - \hat{A} \right) )</td>
</tr>
<tr>
<td>Pump Flow Phase II</td>
<td>( \theta = \phi_1 \rightarrow \pi )</td>
<td>( P_{\phi_1-\pi} = P V_d \omega \hat{A} )</td>
</tr>
<tr>
<td>Pump Flow Phase III</td>
<td>( \theta = \pi \rightarrow \phi_2 )</td>
<td>( P_{\pi-\phi_2} = -V_d \omega \left( 1 - 2\hat{V} \right) \left( \frac{\mu g}{v_{lg}} + P_1 \right) \left( 1 - \hat{A} \right) )</td>
</tr>
<tr>
<td>Pump Flow Phase IV</td>
<td>( \theta = \phi_2 \rightarrow 2\pi )</td>
<td>( P_{\phi_2-2\pi} = -P V_d \omega \left[ 1 - \left( 1 - 2\hat{V} \right) \left( 1 - \hat{A} \right) \right] )</td>
</tr>
</tbody>
</table>

The nondimensional definitions implemented by Manring in his work on mapping hydrostatic transmission efficiency are provided in Eq. (4.30) [87].

\[
Q_p = \hat{Q}_p V_d \omega \quad P = \hat{P} P_{max} \quad T = \hat{T} V_d P_{max} \quad \hat{\mu} = \frac{\mu_0 \times \text{Exp} \left( -\lambda \left[ T - T_0 \right] \right)}{\mu_0} \quad \quad (4.30)
\]

The nondimensional fluid viscosity uses a baseline fluid viscosity, \( \mu_0 \), for SAE grade fluids presented in *Hydraulic Control Systems* [20], as well as the intrinsic fluid property, \( \lambda \), and the measured temperature of the working fluid in the system, as compared to the baseline temperature \( T_0 = 80^\circ C \). Use of the expression for nondimensional torque from Eq. (4.4) as well as the nondimensional definitions in Eq. (4.30) yields an additional useful definition. The varying inlet metering valve cross-sectional area is analogous to the varying swashplate angle and thus the idealized torque requirement for the inlet metering pump is
\[ T_{\text{ideal}} = V_d \hat{A} P_2. \]  
(4.31)

There will also be an additional shaft power requirement associated with the vaporization of the fluid. This requirement can be found by summing the power demands from Table 4-2 and yields

\[ T_{\text{vaporization}} = V_d \left( \frac{\mu \omega}{v_{lg}} \right) (1 - \hat{A}) (2\hat{V}). \]  
(4.32)

It is assumed that the inlet metering valve system will experience losses similar to those depicted in the axial piston pump analysis. Thus, the entire torque requirement can be written as

\[ T = V_d \hat{A} P_2 + V_d \left( \frac{\mu \omega}{v_{lg}} \right) (1 - \hat{A}) (2\hat{V}) + V_d \hat{A} P_2 \left[ A_p \text{Exp} \left( -B_p' \frac{\mu \omega_p}{AP} \right) + C_p' \frac{\mu \omega_p}{AP} \right] + D_p', \]  
(4.33)

where the coefficients \( A_p, B_p, C_p, \) and \( D_p \) in Eq. (4.33) come from the inlet metering valve system experiments.

In order to nondimensionalize Eq. (4.33), the nondimensional grouping

\[ \psi = \frac{\mu \omega}{P_{\text{max}} v_{lg}} \]  
(4.34)

will be used in addition to the nondimensional groups already given in Eq. (4.5).

Equation (4.33) can be rewritten as Eq. (4.35).

\[ \hat{T} = \hat{A} \hat{P}_2 + 2\hat{V}\psi (1 - \hat{A}) + A_p \hat{A} P \text{Exp} \left( -B_p' \frac{\hat{\mu} \hat{\omega}}{\hat{A} \hat{P}} \right) + C_p' \frac{\hat{\mu} \hat{\omega}}{\hat{A} \hat{P}} + D_p' \]  
(4.35)
In Eq. (4.35) the first term encapsulates the ideal torque, and the second term contains the additional torque required to vaporize and compress the fluid. The remaining terms pertain to the energy losses that result from the typical use of a pump.

Recall from Eq. (3.26) that

$$\dot{Q}_{2,\text{ideal}} = \hat{A}. \quad (4.36)$$

This derivation was based on an idealized pump performance. In order to account for the inevitable leakage that will occur during operation of the single piston pump, loss coefficients analogous to those identified in Eq. (4.3) and quantified in Table 4-1 will be applied to Eq. (4.36). The pump volumetric flow rate, accounting for leakage losses, out of the fixed displacement piston pump can be written as

$$\dot{Q}_2 = \hat{A}\hat{\omega} - K_0\hat{A}\hat{\omega}\hat{P} - K_1\frac{\dot{P}}{\mu} - K_2\sqrt{\hat{P}}. \quad (4.37)$$

The efficiency originally written in Eq. (4.1) can be rewritten in terms of Eqs. (4.35) and (4.37) as Eq. (4.38).

$$\eta = \frac{\hat{P}\left(\hat{A}\hat{\omega} - K_0\hat{A}\hat{\omega}\hat{P} - K_1\frac{\dot{P}}{\mu} - K_2\sqrt{\hat{P}}\right)}{\hat{A}\hat{P} + \dot{\hat{Q}}_0\hat{P}_0 + 2\hat{V}\psi(1 - \hat{A}) + A_p\hat{A}\hat{P}\exp\left(-B_p\frac{\hat{\mu}\hat{\omega}}{A_P}\right) + C_p\sqrt{\hat{A}\hat{P}\hat{\mu}\hat{\omega}} + D_p} \quad (4.38)$$

Two additional nondimensional parameters,

$$Q_0 = \dot{Q}_0\psi, \quad \text{and} \quad P_0 = \hat{P}_0\psi, \quad (4.39)$$

are employed. The addition of the system inlet power to the denominator of Eq. (4.38) is a result of the increased inlet pressure necessary for inlet metering system operation. For operation of an axial piston variable displacement pump, the inlet pressure is simply atmospheric pressure. This value is so small relative to the outlet pressure that it is
neglected. In contrast, the charge pump pressure is elevated over atmospheric pressure and thus requires a greater energy input. From this point we can infer that it would be prudent to keep the charge pressure, $P_o$, as small as possible to maximize efficiency while still providing a sufficient pressure drop across the valve to generate fluid vaporization. The valve inlet volumetric flow rate, $Q_0$, is dependent on the charge pump selection. A fixed displacement charge pump will always supply the maximum volumetric flow rate ($\hat{Q}_0 = 1$); however, a variable displacement charge pump could be used to reduce the inlet flowrate requirement to a level proportional to the valve cross-sectional area ($\hat{Q}_0 = \hat{A}$). The other differences between the efficiency in Eq. (4.6) and Eq. (4.38) are the use of swashplate angle in the axial piston pump evaluation as compared to the use of the inlet metering valve cross-sectional area as a means to control the pump volumetric flow rate and the addition of the torque associated with fluid vaporization in the denominator of Eq. (4.38).

In order to determine the experimentally derived leakage coefficients, the measurements corresponding to the data collected for all inlet pressures (2.0, 2.5, 3.0, and 3.5 MPa), discharge pressures (2, 5, 10, 20, and 25 MPa), and two shaft speeds (1000 and 1500 rpm) were used. This data was collected by another research team, led by Dr. Roger Fales, and discussion of the experimental methods employed is beyond the scope of this dissertation. After these measurements were recorded, the torque, discharge flow rate, and outlet pressure were averaged for each set outlet pressure and valve position over the 10 second interval for which the valve was held in position. The relevant averages were taken from the experimental measurements and then the Levenberg-
Marquardt algorithm was applied [89]. (The Appendix contains all the averaged data employed throughout the analysis.)

The Levenberg-Marquardt algorithm is a numerical solution designed to minimize a nonlinear function, such as the torque model in Eq. (4.35). This numerical approach combines the steepest descent numerical method, which converges quite slowly but with excellent stability, and the Gauss-Newton method, which converges on a solution rapidly, but is prone to solution divergence [90]. Equation (4.40) shows the iterative equation employed, and Fig. 4-5 illustrates the process.

\[ w_{k+1} = w_k - \left( J_k^T J_k + \mu I \right)^{-1} J_k e_k \]  

(4.40)

In Eq. (4.40), \( W \) represents the “weights” or in our case, the coefficients (e.g. \( A, B, C, \) and \( D \)). The Jacobian matrix, \( J \), is derived from the matrix provided in Eq. (4.41) in the search for coefficients \( A, B, C, \) and \( D \), as well as the nondimensional thermodynamic parameter \( \psi \). It is similarly derived from the matrix representation of Eq. (4.7) and employed to determine the leakage coefficients \( K_0, K_1, \) and \( K_2 \). Note that increasing the combination coefficient, \( \mu \), decreases the step size, and decreasing the combination coefficient increases the step size. This iterative adjustment of step size is what leads to rapid solution convergence. The error is quantified in the term \( e_k \).

\[
\begin{bmatrix}
2\dot{V} (1 - \hat{A}_1) \hat{A}_1 \hat{P}_{d1} \exp \left( -B \hat{\mu}_1 \hat{\phi}_1 / \hat{A}_1 \hat{P}_{d1} \right) \sqrt{\hat{A}_1 \hat{P}_{d1} \hat{\mu}_1 \hat{\phi}_1} & 1 \\
\vdots & \vdots & \vdots \\
2\dot{V} (1 - \hat{A}_n) \hat{A}_n \hat{P}_{dn} \exp \left( -B \hat{\mu}_n \hat{\phi}_n / \hat{A}_n \hat{P}_{dn} \right) \sqrt{\hat{A}_n \hat{P}_{dn} \hat{\mu}_n \hat{\phi}_n} & 1 \\
\end{bmatrix}
\begin{bmatrix}
\psi \\
A \\
C \\
D \\
\end{bmatrix} = \begin{bmatrix}
\hat{T}_1 - \hat{A}_1 \hat{P}_{d1} \\
\vdots \\
\hat{T}_n - \hat{A}_n \hat{P}_{dn} \\
\end{bmatrix}
\] 

(4.41)
Figure 4-5: Levenberg-Marquardt Solution Process [90]

The Levenberg-Marquardt solution process was applied to both the torque and flow equations [Eq. (4.35) and (4.37)]. The resulting coefficients are provided in Table 4-3. Each of these coefficients has a physical meaning, as Eq. (4.35) and (4.37) are models derived from the actual physical principles at work in operation of the inlet metering pump.
Table 4-3. Inlet metering pump coefficients

<table>
<thead>
<tr>
<th>Physical Meaning</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid compression</td>
<td>$K_0$</td>
<td>8.607E-02</td>
</tr>
<tr>
<td>Low Reynolds-number leakage</td>
<td>$K_1$</td>
<td>1.000E-02</td>
</tr>
<tr>
<td>High Reynolds-number leakage</td>
<td>$K_2$</td>
<td>5.063E-07</td>
</tr>
<tr>
<td>Static friction</td>
<td>$A_p$</td>
<td>Effectively 0</td>
</tr>
<tr>
<td>Decay rate for boundary lubrication</td>
<td>$B_p$</td>
<td>N/A</td>
</tr>
<tr>
<td>Hydrodynamic lubrication</td>
<td>$C_p$</td>
<td>1.305E-02</td>
</tr>
<tr>
<td>Starting torque</td>
<td>$D_p$</td>
<td>7.182E-02</td>
</tr>
<tr>
<td>Thermodynamic parameters</td>
<td>$\psi$</td>
<td>0.0140</td>
</tr>
</tbody>
</table>

These parameters yield a $R^2$ value of 0.9822 with respect to flow evaluation, and a $R^2$ value of 0.9780 with respect to torque evaluation, suggesting that this method of nonlinear coefficient determination is effective. The negligible magnitude of the static friction coefficient, $A_p$, yields a slightly modified equation to describe the torque requirement for the inlet metering system. The finalized model for torque, then, is presented in Eq. (4.42), where the relevant coefficients are obtained from Table 4-3.

$$
\hat{T} = \hat{A} \hat{P}_2 + 2\hat{V} \psi \left(1 - \hat{A} \right) + C_p \sqrt{\hat{A} \hat{P}_2 \hat{\mu}} + D_p
$$

(4.42)

A comparison between Tables 4-1 and 4-3 yields insight into the operation of the inlet metering pump. The effect of fluid compression, $K_0$, is of the same magnitude in operation of the two pumps. The working fluid in the two pumps was the same, and it appears to have translated to the behavior of the fluid flow within the pump. Several
local minima exist which makes determining the leakage coefficients somewhat challenging. After comparing multiple combinations of initial guesses and constraints supplied to the iterative process used for solution, it was concluded that the result in Table 4-3 yielded an excellent fit while simultaneously making physical sense. From this result, we see that the high Reynolds number leakage is very near zero, but the low Reynolds number leakage is of a similar magnitude to that of the variable displacement pump [87].

Static friction was incredibly small for the inlet metering pump when compared to the variable displacement pump. It is presumed that pump construction is the largest contributing factor to this result. Operation of the variable displacement pump generates substantial friction due to the metal-to-metal contact associated with its swashplate position. In contrast, the inlet metering pump utilized a rolling element bearing for its operation. From this, it was concluded that dropping the term containing the static friction coefficient, $A$, would lead to the most physically appropriate model. This negated a need to determine the decay rate for boundary lubrication, $B$. The hydrodynamic lubrication, $C$, and starting torque, $D$, coefficients were both highly similar to the 145cc variable displacement results.

One additional item of interest is the thermodynamic parameter, $\psi$, shown earlier as Eq. (4.34). For all of the theoretical modeling conducted in this work, it was assumed that the working fluid of the system was water ($\psi = 0.0298$). Operation of the experimental apparatus required use of CAT HYDO Advanced 10 hydraulic oil, a proprietary hydraulic oil about which little is known in terms of thermodynamic properties. The specific volume of the fluid is easily obtained [33], but the internal
energy is unknown. For this reason, the iterative process undertaken to determine the other coefficients was also employed to estimate the thermodynamic property group associated with the compression torque requirement. The magnitude of the thermodynamic parameter determined for the inlet metering operating system is comparable with that of water.

4.4 Efficiency Model Validation

The results presented in Table 4-3 were validated by evaluating the ability of the efficiency model to predict the torque requirement and discharge flow for the pump’s operation at higher shaft speeds. Data was taken for the pump operating at 2 MPa inlet pressure, 2, 2.5, 5, 10, 20, and 25 MPa discharge pressure, and 2000 and 2500 rpm. When these actual results were compared with the model predictions, the torque model was found to have very good agreement with the higher speed data ($R^2 = 0.9835$). The flow model correlation coefficient was substantially lower ($R^2 = 0.7505$); however, the flow data associated with the higher pump speed operations showed significantly greater variation than any of the other data recorded during pump operation. For comparison, this information is presented in Table 4-4. Thus, it is assumed that the coefficients associated with the flow model are reasonably good. Comparison of the new data with the previously introduced model leads us to conclude that the model described by the coefficients of Table 4-3 is valid.

Table 4-4. Standard Deviation of Collected Data

<table>
<thead>
<tr>
<th>Standard Deviation of Averaged Flow at 1000 and 1500 rpm</th>
<th>0.2390</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation of Averaged Flow at 2000 and 2500 rpm</td>
<td>0.3963</td>
</tr>
<tr>
<td>Standard Deviation of Averaged Torque at 1000 and 1500 rpm</td>
<td>0.2098</td>
</tr>
<tr>
<td>Standard Deviation of Averaged Torque at 2000 and 2500 rpm</td>
<td>0.2030</td>
</tr>
</tbody>
</table>
4.5 Discussion of the Variable Displacement and Inlet Metering System Analyses

In this chapter, governing equations describing the torque requirement and discharge flow for the two pumps and their associated coefficients were presented. High \( R^2 \) values indicate that these coefficients provide reasonably good models for the systems. These coefficients will be applied in Chapter 6 to facilitate further comparison between the two designs, and actual measured efficiency values will also be presented.
5.1 Introduction to System Design and Modeling

In this chapter a variety of numerical constraints will be developed. These constraints will apply to the nondimensional groups presented in Chapter 3 and will be used to aid the design process associated with the practical implementation of an inlet metering valve system. Design requirements that will guarantee the system behaves in a stable fashion will be introduced in Section 5.2. Additional design constraints will be presented in Section 5.3. Application of these design constraints and the resulting simplified model will be discussed in Section 5.4. Section 5.5 will analyze the dynamic response of the system of equations, and Section 5.6 will illustrate our design process. Section 5.7 will enumerate the sample systems that were designed for the purposes of our study and assign numerical values to the constants that have been used throughout. A discussion of our approach to modeling the behavior of this system will conclude the chapter in Section 5.8.

5.2 System Stability

When the valve is disturbed from its initial resting state, it will either oscillate and slowly return to its initial condition, or it will grow in oscillations until it fails or reaches a physical limit. The desirable condition is for our valve to be stable, that is, to gradually return to its initial resting state. We must determine if, in fact, our valve is stable. This question of stability has been studied since 1856, when Hermite formally presented the problem of finding the stability of a lumped parameter, linear, time-invariant system [93].
Gantmacher assesses Hermite’s solution as only partially complete; however, it was later completely solved independently by both Routh and Hurwitz [94]. J. C. Maxwell was also interested in the stability problem presented by Hermite. He was the first to publish a dynamic analysis of a feedback system using differential equations in 1868, while attempting to develop a speed governor [95]. In 1877 he served on a committee to determine the winner of the Adams Prize, a prestigious award distributed by Cambridge University to recognize original research in the field of mathematics [96]. This call for papers was answered by E. J. Routh, who was awarded the Adams Prize for his essay on the Stability of Motion [97, 98]. Routh’s paper showed that the number of roots in the characteristic polynomial can be determined from the coefficients of the polynomial [98].

Eighteen years later, Hurwitz solved what was essentially the same problem. He was working on controlling the speed of a high pressure water turbine, and was unaware of the work done by Routh [99]. Their results are known collectively as the Routh-Hurwitz stability criterion.

The Routh-Hurwitz stability criterion provides an algebraic procedure for determining if the polynomial has any zeros in the right half plane [100]. If zeros exist in the right half plane, the system is unstable. The zeros of the polynomial only lie entirely in the left hand plane when a particular set of algebraic combinations of the coefficients have the same sign [101]. A practical description for the application of the Routh-Hurwitz criteria can be found in a variety of control texts, including *Hydraulic Control Systems* by Noah Manring [20].

While the proof of the Routh-Hurwitz problem has existed since the late 1800’s, considerable attention was lavished on the stability criterion in the 1990’s. The proofs
offered by Routh and Hurwitz both rely on advanced math techniques [98, 99], but in 1990 Chapellat et al applied the Hermite-Biehler theorem to stability criterion [102]. Since then, the academic community has produced multiple efforts to prove the Routh Hurwitz criteria by elementary means in order to make the stability proof more accessible to undergraduate students [103, 104]. While these proofs are valuable for deeper understanding, ultimately we are concerned with practical application, and the coverage in a standard control text [100] proves sufficient for the analysis conducted here.

The classic Routh-Hurwitz stability criterion can be used to consider system stability. The Routh Hurwitz stability criteria will be applied to the nondimensional equations presented as Eq. (3.51) in Chapter 3 of this work. Equation (3.51) is presented in state space form here:

\[
\begin{pmatrix}
\dot{\hat{P}} \\
\dot{\hat{y}} \\
\dot{\hat{y}}
\end{pmatrix} = \begin{pmatrix}
-\hat{K}_1 & 1 & 0 \\
0 & 0 & 1 \\
-\frac{1}{\hat{m}} & -\hat{K}_2 & -\hat{c}
\end{pmatrix} \begin{pmatrix}
\hat{P} \\
\hat{y} \\
\hat{y}
\end{pmatrix} + \begin{pmatrix}
\hat{K}_1 & -1 \\
0 & 0 \\
\frac{1}{\hat{m}} & \hat{K}_2
\end{pmatrix}.
\]

The characteristic equation is then derived from Eq. (5.1) as follows:

\[
\det \begin{pmatrix}
-\hat{K}_1 & 1 & 0 \\
0 & 0 & 1 \\
-\frac{1}{\hat{m}} & -\hat{K}_2 & -\hat{c}
\end{pmatrix} - s \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} = 0,
\]

where the resulting characteristic equation can be written as

\[
\hat{m}s^3 + (\hat{m} + \hat{c})s^2 + (\hat{c} + \hat{K}_2)s + \hat{K}_1 + \hat{K}_2 = 0.
\]
For the system to be stable, the design criterion given by Eq. (5.4) must be satisfied. Additionally, all the nondimensional groups must be real and positive values. This will always be true given practical real world constraints, so this second requirement will not be discussed further.

\[
(\hat{m} + \hat{c})(\hat{c} + \hat{K}_2) > \hat{m}(\hat{K}_1 + \hat{K}_2)
\]  

(5.4)

Expansion of the terms in Eq. (5.4) reveals that neglecting the drag coefficient without neglecting valve mass will always yield an unstable system. We can also observe from Eq. (5.4) that the damping effects associated with the drag coefficient contribute to stability. A third observation from Eq. (5.4) suggests that increasing the spring constant, \( \hat{K}_2 \), will always increase system stability.

The most significant observation associated with Eq. (5.4) relies on the assertion in Chapter 3 that the nondimensional leakage coefficient \( \hat{K}_1 \) is unity. If this design criteria is applied to Eq. (5.4), we can see that the system will always be stable as long as the coefficients are real and positive, which they must be. Thus, our system design will always be stable.

5.3 Design Criteria Development

The stability criterion given in Eq. (5.4) provides the first guideline useful in the design process. In order for the nondimensional leakage coefficient to be unity, the design criteria shown in Eq. (5.5) must be satisfied.

\[
y_{ss} = \frac{\text{KP}_{ss}}{c_d y_{max} \sqrt{2V_{ref}P_0}}
\]  

(5.5)
The maximum value of the valve displacement, $y_{\text{max}}$, can also be derived from the equations that come from the analysis in Chapter 3. By setting Eqs. (3.27) and (3.29) equal to each other, and recalling the nondimensional definitions in Eq.(3.8), we can show

$$y_{\text{max}} c_d \sqrt{2v_{\text{t}_0} (P_0 - P_1)} y = V_d \omega \left[ 1 - \frac{\dot{v}_x}{\frac{A_p r}{V_{\text{max}}}(1 + \dot{v}_x)} \right].$$  \hspace{1cm} (5.6)

We can solve Eq. (5.6) and find the maximum valve opening dimension to be as defined as

$$y_{\text{max}} = \sqrt{\frac{V_d \omega}{c_d \sqrt{2v_{\text{t}_0} (P_0)}}}.$$  \hspace{1cm} (5.7)

This maximum valve displacement is a function of the pressure supplied by the charge pump, $P_0$. Recall from the efficiency analysis in Chapter 4 that significant system inefficiencies result from an elevated pressure at the charge pump outlet, and thus $P_0$ should be kept as small as possible (but still higher than the saturation pressure, $P_1$, of the working fluid) while permitting reasonable design dimensions for the valve.

The mass of the valve can be designed by considering the dimensions of the cross-sectional valve area, $A_{\text{valve}}$, and the length of the valve, $L$, as well as the anticipated valve material density ($\rho_{\text{steel}} \sim 7850 \frac{kg}{m^3}$). This result is shown in Eq. (5.8).

$$m_v = \rho_{\text{steel}} A_{\text{valve}} L$$  \hspace{1cm} (5.8)
An additional design criteria can be derived from consideration of the nondimensional spring constant coefficient. In order to minimize the spring preload requirement,

\[
\frac{k_{y_{ss}}}{A_{valve} P_{ss}} \approx 1.
\]  
(5.9)

The spring preload force requirement can be evaluated using

\[
F_{preload} = \left( k + \frac{A_{valve}}{L} \right) y_{ss} + P_{ss} A_{valve}
\]

(5.10)
to check for a reasonable magnitude.

The damping coefficient is also designed in terms of the valve dimensions. It is calculated by

\[
c = \frac{0.02\pi L}{3(2E-05)} \sqrt{\frac{4A_{valve}}{\pi}}.
\]  
(5.11)

The origins of Eq. (5.11) can be found in published hydraulic texts [20].

Further, the system dynamic response is governed by the time scale, \( \tau \), which is scaled by

\[
\tau = \frac{V_2}{\beta K}.
\]  
(5.12)

Interestingly, the dynamic response is improved by increasing the system leakage. Increased system leakage will also have a negative impact on the system efficiency. A balance must be struck between achieving a fast time response while minimizing energy losses. The time response can also be improved by increasing the volume of fluid at the pump outlet.
The final design criteria that can be developed relies on the nondimensional groups from Eq. (3.51). If the nondimensional spring constant coefficient, $\hat{K}_2$, is several orders of magnitude greater than the nondimensional mass coefficient, $\hat{m}$, and the nondimensional damping coefficient, there can be a highly useful dynamic result.

Equation (5.13) compares the expanded nondimensional spring constant [originally defined in Eq. (3.49)] to, first, the nondimensional mass constant [defined in Eq. (3.49)], and then to the nondimensional damping constant [defined in Eq. (3.49)]. The factor of 100 has been introduced to require that the nondimensional spring constant be several orders of magnitude greater than the nondimensional mass and damping coefficients.

Simplification of Eq. (5.13) yields the additional design criteria,

\[
\frac{(k + \xi_2) y_{ss}}{A_{valve} P_{ss}} > 100 \frac{m_y y_{ss}}{\tau A_{valve} P_{ss}}
\]

\[
\frac{(k + \xi_2) y_{ss}}{A_{valve} P_{ss}} > 100 \frac{(c + \xi_1) y_{ss}}{\tau A_{valve} P_{ss}}
\]

These design criteria are notable because they enable a reduction of the order of the nondimensional dynamic equations [see Eq. (3.51)]. The parameters in Eq. (5.14) can be satisfied using physically reasonable design criteria. If this is done, the insignificant terms in Eq. (3.51) can be neglected. We then find the simplified system of equations which describes our system. This is shown here:
1. \( \dot{P} + \hat{P} = \hat{y} \)
2. \( \hat{K}_2 \hat{y} = -\hat{P} + \left(1 + \hat{K}_2 \right) \).

(5.15)

Equation (5.15) contains a significant result. We have developed a first order pressure control system. This is different from a typical variable displacement pump design, which is a second order system [105]. Equation (5.15) can be solved by hand; this will be done in Section 5.4. This will enable “back of the envelope” calculations, and ease the design process. These design criteria, as well as the stability criteria introduced in Section 5.2, will be helpful to engineers who seek to develop and implement this technology.

5.4 Solution of the System of Equations

The simplified system of equations given by Eq. (5.15) can be solved by hand using basic differential equation solution techniques [106]. This solution is given in Eq. (5.16).

\[
\begin{align*}
\hat{P} &= 1 - \exp \left( -i \left( \frac{1 + \hat{K}_2}{\hat{K}_2} \right) \right) \\
\hat{y} &= \frac{1}{\hat{K}_2} \exp \left( -i \left( \frac{1 + \hat{K}_2}{\hat{K}_2} \right) \right) + 1 
\end{align*}
\]

(5.16)

Examination of Eq. (5.16) illustrates the effects of changing the nondimensional spool spring rate coefficient, \( \hat{K}_2 \) on the behavior of the system output pressure and valve position.

While solution of Eq. (5.15) was relatively simple, the full dynamic set of equations, provided at the conclusion of Chapter 3, cannot be solved by hand. Instead, it
must be solved numerically. The use of Eq. (5.16) is convenient; however, it is important to make sure this simplified solution is equivalent to the results indicated by the full system. In Chapter 6, a comparison of the simplified model and the full order model will be made. At that point, we will be able to determine the ability of Eq. (5.16) to actually describe our system behavior.

5.5 Dynamic System Response

A variety of information about the behavior of the valve can be derived from the dynamic equations found in Chapter 3 of this work. The bandwidth frequency, \( \omega_{bw} \), and time constant, \( \tau \), can be found using Eqs. (5.17) through (5.21) [20].

The bandwidth frequency for the higher order system is the dynamic response characteristic we are most interested in. The transfer function of the higher order system is

\[
\hat{P}(s) = \frac{1 + \hat{K}_2}{\hat{m}s^3 + (\hat{m} + \hat{\xi})s^2 + (\hat{c} + \hat{K}_2)s + 1 + \hat{K}_2}, \tag{5.17}
\]

and the nondimensional bandwidth frequency can be found by numerically evaluating the frequency where the amplitude of

\[
\hat{P}(j\omega) = \frac{1 + \hat{K}_2}{-\hat{m}\omega^3 - (\hat{m} + \hat{\xi})\omega^2 + (\hat{c} + \hat{K}_2)j\omega + 1 + \hat{K}_2} \tag{5.18}
\]

is equal to \( \frac{\sqrt{2}}{2} \). This solution must be determined numerically. The bandwidth frequency for a first order system is also relevant, as it will apply to our simplified
models. The first order bandwidth frequency, in hertz, can be evaluated based on the nondimensional pressure rise rate equation,

\[
\begin{bmatrix}
\frac{1}{1 + \frac{1}{K_2}}
\end{bmatrix} \hat{\dot{P}} + \hat{P} = 1.
\]  

(5.19)

Based on Eq. (5.19), the nondimensional bandwidth frequency is

\[
\hat{\omega}_{bw} = \tau \omega_{bw} = 1 + \frac{1}{K_2}.
\]  

(5.20)

This result is in radians. Thus, the first order bandwidth frequency in hz is evaluated as

\[
\omega_{bw} = \frac{\beta K}{V_2} \left( 1 + \frac{1}{K_2} \right) \left( \frac{1}{2\pi} \right)
\]  

(5.21)

and is a much more simple calculation than the numerical solution required by Eq. (5.18). It can be completed quickly without the use of a computer.

Equations (5.18) and (5.21) give the maximum frequency of the pressure at which the system output of the pressure will sufficiently track the input behavior of a third and first order system respectively [20]. We are interested in the inlet metering valve’s ability to control the downstream pressure output, and thus the higher the bandwidth frequency, the more rapidly our inlet metering system will be able to respond to pressure fluctuations. Similarly, a lower time constant will be desirable in generating rapid system response. Further evaluation of the system bandwidth frequencies will occur in Section 5.8 and Chapter 6.
5.6 Design Process

For the purposes of exploring the dynamic behavior of our system, we will examine two different high order systems (Models A and B) based on Eq. (3.51) and two simplified systems (Models A’ and B’) based on Eq. (5.15). While the process to arrive at each of these systems is not difficult, many parameters are interrelated, and thus, a thorough explanation of the design process will be introduced here.

All of the stability and design criteria [Eqs. (5.4) through (5.14)] rely on the parameters encompassed by the four nondimensional groups \( \left( \hat{K}_1, \hat{K}_2, \hat{c}, \hat{m} \right) \) and the time constant, \( \tau \). Some of these parameters are fixed based on the working fluid selection, some of the values are products defined by other parameters, and the remaining values are design parameters which we are free to choose, subject to reasonable physical constraints. Table 5-1 explicitly lists the parameters and the amount of control designers retain over them.

<table>
<thead>
<tr>
<th>Fluid Properties (Fixed Value)</th>
<th>Design Choice (Free to Choose)</th>
<th>Product of Design Choices (Result of Third Column)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{\max} )</td>
<td>( V_d \omega )</td>
<td>( P_{ss} )</td>
</tr>
<tr>
<td>( v_i )</td>
<td>( P_o )</td>
<td>( A_{pump} ) ( y )</td>
</tr>
<tr>
<td>( v_{lg} )</td>
<td>( V_2 )</td>
<td>( A_{valve} ) ( y )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( L )</td>
<td>( A_{valve} ) ( y )</td>
</tr>
<tr>
<td>( c_d )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In order to satisfy the stability and design criteria, the designer can select the parameters listed as Design Choices. Selection of these values will impact the
magnitudes of the nondimensional groups. The relative magnitudes of the
nondimensional groups can be compared according to Eqs. (5.4) and (5.14) to determine
system stability and the relevance of the first order model to our higher order system.
Additionally, the equations describing the system dynamic response [Eqs. (5.17) through
Eq. (5.21)] can be used to predict certain aspects of our design’s response.

5.7 Illustrative Model Selection

Four models have been selected to explore the behavior of the inlet metering
system. These models have been selected to illustrate the variations in system behavior
as a result of differences in nondimensional group sizing. These models will be called
Model A, Model A’, Model B, and Model B’. Table 5-2 contains the design parameters
associated with the two baseline models based on Eq. (3.51). The models denoted with a
prime are the simplified models based on the design criteria given in Eq. (5.14) and the
resulting dynamic equations given by Eq. (5.15). Model A’ is the first order model of the
full order system represented by Model A. Similarly, Model B’ is the first order model of
the full order system represented by Model B.
Table 5-2. Model design parameter selections

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model A</th>
<th>Model B</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{d,0} )</td>
<td>3.33E-03</td>
<td>3.33E-03</td>
<td>m(^3)/s</td>
</tr>
<tr>
<td>( L )</td>
<td>6</td>
<td>6</td>
<td>cm</td>
</tr>
<tr>
<td>( m_v )</td>
<td>3.70E-02</td>
<td>1.33E-02</td>
<td>kg</td>
</tr>
<tr>
<td>( K )</td>
<td>363</td>
<td>48.4</td>
<td>kN/m</td>
</tr>
<tr>
<td>( c_d )</td>
<td>0.62</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>( A_{valve} )</td>
<td>7.85E-05</td>
<td>2.83E-05</td>
<td>m(^2)</td>
</tr>
<tr>
<td>( c )</td>
<td>0.628</td>
<td>0.377</td>
<td></td>
</tr>
<tr>
<td>( A_p r / V_{max} )</td>
<td>0.10</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>( Q_{max} )</td>
<td>3.33E-03</td>
<td>3.33E-03</td>
<td>m(^3)/s</td>
</tr>
<tr>
<td>( y_{max} )</td>
<td>10.9</td>
<td>4</td>
<td>mm</td>
</tr>
<tr>
<td>( y_{ss} )</td>
<td>0.81</td>
<td>2.21</td>
<td>mm</td>
</tr>
<tr>
<td>( P_{ss} )</td>
<td>15</td>
<td>15</td>
<td>MPa</td>
</tr>
<tr>
<td>( K )</td>
<td>1.67E-11</td>
<td>1.67E-11</td>
<td>m(^3)/s/kg</td>
</tr>
<tr>
<td>( p_o )</td>
<td>0.10</td>
<td>0.10</td>
<td>MPa</td>
</tr>
<tr>
<td>( p_{max} )</td>
<td>20</td>
<td>20</td>
<td>MPa</td>
</tr>
<tr>
<td>( v_l )</td>
<td>1.04E-03</td>
<td>1.04E-03</td>
<td>m(^3)/kg</td>
</tr>
<tr>
<td>( v_{lg} )</td>
<td>1.67</td>
<td>1.67</td>
<td>m(^3)/kg</td>
</tr>
<tr>
<td>( \beta )</td>
<td>2.20</td>
<td>2.20</td>
<td>GPa</td>
</tr>
</tbody>
</table>

Models A, A’, B, and B’ will be compared with the variable displacement pump introduced in Section 3.15. The numerical values associated with the nondimensional groups in Eqs. (3.52) and (3.53) are provided in Table 5-3.

Table 5-3. Numerical values associated with the nondimensional parameters in Eq. (3.52) [105]

<table>
<thead>
<tr>
<th>Nondimensional Group</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_1 )</td>
<td>9.85E-02</td>
</tr>
<tr>
<td>( \chi_1 )</td>
<td>88.03</td>
</tr>
<tr>
<td>( \chi_2 )</td>
<td>2.934</td>
</tr>
</tbody>
</table>
Table 5-4 shows the numerical values of the nondimensional parameters and differentiating characteristics of each of the models discussed in Chapter 5. These values are the results of Eqs. (3.49) and (3.51), as well as Eqs. (5.4) and (5.14). It also provides the dynamic characteristics found using Eqs. (5.17) through (5.21).

Table 5-4. Models of interest and their associated dynamic characteristics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model A</th>
<th>Model A’</th>
<th>Model B</th>
<th>Model B’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$\tau$ [ms]</td>
<td>27.3</td>
<td>27.3</td>
<td>20.6</td>
<td>20.6</td>
</tr>
<tr>
<td>$\hat{K}_1$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\hat{K}_2$</td>
<td>0.253</td>
<td>0.253</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\hat{c}$</td>
<td>4.64E-04</td>
<td>0</td>
<td>1.74E-03</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{m}$</td>
<td>3.44E-05</td>
<td>0</td>
<td>1.63E-04</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{\omega}_{bw}$</td>
<td>4.99</td>
<td>4.94</td>
<td>5.08</td>
<td>4.85</td>
</tr>
<tr>
<td>$\omega_{bw}$ [hz]</td>
<td>29.14</td>
<td>28.82</td>
<td>39.19</td>
<td>37.41</td>
</tr>
<tr>
<td>Displacement</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Examination of Table 5-4 permits a preliminary understanding of the behavior of the inlet metering system. Models A’ and B’ are not governed by Eqs. (5.17) or (5.18). Those equations are only relevant for the higher order systems; instead, Models A’ and B’ are the first order systems described by Eq. (5.15). Their first order nature is emphasized by the “zero” value applied to the values of their nondimensional damping coefficient, $\hat{c}$, and nondimensional mass coefficient, $\hat{m}$. Observe that although Models A’ and B’ are simplified approximations of the true system response described by Models A and B, the first order model frequency calculation given by Eq. (5.21) results in only 0.9% relative error between Models A and A’ and 4.3% relative error between Models B and B’.

Models A and A’ adhere to the design criteria developed in Section 5.3. Observing a
relative error of less than 1% in regards to the difference between the first order bandwidth frequency response and the higher order bandwidth frequency response indicates that Model A’ is indeed a good approximation of Model A. The ability to quickly estimate the bandwidth frequency of the inlet metering design is an attractive feature for engineers.

The four models described by the parameters in Table 5-4, as well the variable displacement pump, will be modeled in Chapter 6, allowing us to gain further confidence that the simplified models are reasonable approximations of the higher order models. These models will solve Eqs. (3.51), (3.52), or (5.15) depending on their equations’ relevance to the model design based on the nondimensional parameters listed in Table 5-3.

5.8 Modeling Introduction

As was mentioned earlier, the solution to the full dynamic set of equations presented at the end of Chapter 3 cannot be found by hand. Instead, the solution must be found numerically through the use of a computer driven solution. Modeling our system in MATLAB/Simulink provides us with valuable insights in regards to how the system will behave dynamically.

A block diagram of the dynamic system represented by Eq. (3.51), including all terms whether significant or numerically insignificant (due to relatively small orders of magnitude), is shown in Fig. 5-1. The simplified system [based on Eq. (5.15)] is shown in Fig. 5-2. The system in Fig. 5-2 adheres to the design criteria developed in Section 5.3 of this work, and behaves in a first order fashion. Recall that the solution to Eq. (5.8) was
already found. It was provided as Eq. (5.9). Thus, it is unnecessary to actually solve the first order model through use of the computer; however, it is convenient to solve Eq. (5.8) by the same means as Eq. (3.51) purely for comparative efforts. The system models in Figs. 5-1 and 5-2 will both be employed to solve Eqs. (3.51) and (5.15) respectively.

Figure 5-1. Block diagram of the full order inlet metering valve system
Comparison of Figs. 5-1 and 5-2 reveals that the introduction of the simplified model permits removal of the higher order valve displacement terms. In the higher order model, the valve position is controlled by the pressure \( \hat{P} \), the nondimensional mass term \( \hat{m} \), the nondimensional damping coefficient \( \hat{c} \), and the nondimensional spring constant coefficient \( \hat{K}_2 \). In the simplified model, the valve position is only shaped by the nondimensional spring constant. Figures 5-1 and 5-2 also show that there is no direct change to the pressure behavior of the simplified model as compared to the higher order model.

The results associated with modeling Eqs. (3.51), (3.52), and (5.8) will be presented in Chapter 6 of this work. A discussion of these results and their physical significance will also be presented in Chapter 6.
CHAPTER 6. RESULTS AND DISCUSSION

6.1 Results Introduction

This chapter will first present the experimental results and compare them with our theoretical predictions. It will then present the results of the efficiency analysis presented in Chapter 4. It will compare the efficiencies of the axial piston variable displacement pump to the system efficiency of the inlet metering design. This comparison will first employ the theoretical derivations and then present the efficiency measurements associated with experimental results. It will then give the solutions to Eqs. (3.51) and (5.8). These solutions vary with time, and plots of nondimensional valve displacement, \( \hat{y} \), and nondimensional downstream pressure, \( \hat{P} \), as a function of nondimensional time, \( \hat{t} \), will serve as a means to illustrate the results. Following the presentation of the system dynamic behavior, a discussion of system stability will be conducted in Section 6.5. Section 6.6 will be used to explore the importance of following the design criteria presented in Chapter 5, and the final two sections of this chapter will compare the dynamic response of the inlet metering system models that were developed to the behavior of a variable displacement pump.

6.2 Experimental Results

The inlet metering system prototype introduced in Chapter 1 of this work was employed to gather performance data that will be used to validate both the dynamic equations developed in Chapter 3 as well as the efficiency predictions from Chapter 4.
For the experiments, the valve voltage supply was increased from 0 V to 0.5 V and then from 1 to 5 V in 1 volt increments. This was done for the pressure and speed points listed in Table 6-1. The valve voltage was adjusted at 10 second intervals. A hydraulic schematic of the system was presented in Chapter 1, and a photograph of the actual setup is presented as Fig. 6-1. Figure 6-2 shows the results for only one of the test conditions, but it is representative of all results.

Table 6-1. Data Collection Points Associated with Experiments

<table>
<thead>
<tr>
<th>Pump Inlet Pressure [MPa]</th>
<th>Pump Outlet Pressures [MPa]</th>
<th>Pump Shaft Speeds [rpm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>2.0, 5.0, 10.0, 20.0, 25.0</td>
<td>1000, 1500</td>
</tr>
<tr>
<td>2.5</td>
<td>2.0, 5.0, 10.0, 20.0, 25.0</td>
<td>1000, 1500</td>
</tr>
<tr>
<td>3.0</td>
<td>2.0, 5.0, 10.0, 20.0, 25.0</td>
<td>1000, 1500</td>
</tr>
<tr>
<td>3.5</td>
<td>2.0, 5.0, 10.0, 20.0, 25.0</td>
<td>1000, 1500</td>
</tr>
</tbody>
</table>

Figure 6-1. Inlet Metering System Experimental Set Up [107]
A variety of information can be derived from this recorded data. The pump discharge flow rate as a function of valve cross-sectional area can be found and compared with Eq. (6.1), which was derived in the analysis covered in Chapter 3.

\[ \hat{Q}_2 = \hat{A} \]  

(6.1)

It is difficult to evaluate Eq. (6.1) by examination of Fig. 6-2. It is easier to see the flow rate as a function of valve position if averages of volumetric flow rate at each valve position are taken and plotted as a function of valve position. This result is shown in Fig. 6-3. For these results, the data collected at a 2 MPa inlet pressure and 1500 rpm
shaft speed was averaged for all measured values of outlet pressure (2, 5, 10, 20, and 25 MPa) for a specified valve position.

![Graph](image)

**Figure 6-3.** Averaged nondimensional outlet flow as a function of measured valve position

From Fig. 6-3 it can be seen that the volumetric flow rate and valve position appear to be linearly related, similar to the prediction made by Eq. (6.1) for valve positions of 0.5 to 2.0 V. For larger valve openings (from valve positions corresponding to valve voltages of greater than 2.0 V), the valve position has no effect on the outlet flow. From this, it is assumed that the experimental inlet metering valve is not sized in the way our design suggests: where the maximum valve cross-section is the cross-section at which fluid begins to vaporize and thus the pump outlet flow begins to be reduced by any reduction in valve cross-sectional area. For the work presented in this dissertation, all comparisons between our analytical predictions and the experimental results relies on
the assumption that the valve voltages of 2 – 5 V all correspond to the maximum valve displacement \( \hat{A} = 1 \). Then, on the region between 0 and 2 V it appears that the flow out of the fixed displacement pump varies linearly, as predicted by Eq. (6.1). In order to consider this more deeply, let us consider the nondimensional expressions for the inlet metering valve cross sectional area and the flow rate out of the pump which were derived in the Chapter 3 analysis. For convenience, they have been rewritten here as Eqs. (6.2) and (6.3).

\[
\hat{A} = \frac{2\hat{V} - (1 - 2\hat{V})\hat{v}_x}{1 + \hat{v}_x} \tag{6.2}
\]

\[
\hat{Q}_2 = 1 - \frac{\hat{v}_x}{2\hat{V}(1 + \hat{v}_x)} \tag{6.3}
\]

Note that Eqs. (6.2) and (6.3) rely on the nondimensional terms for the piston volume, \( \hat{V} \), specific volume of the working fluid, \( \hat{v} \), and the fluid quality, \( \hat{x} \). In order for the direct linear relationship between inlet metering valve opening and volumetric flow rate out of the pump (shown in Eq. (6.1)) to be true, the fluid quality in Eq. (6.2) must be equal to the fluid quality in Eq. (6.3) for all valve positions and flows. Solution of Eqs. (6.2) and (6.3) for equivalent fluid qualities yields Eq. (6.4).

\[
\hat{Q}_2 = \left[ \frac{1}{2\hat{V}} \right] \hat{A} \tag{6.4}
\]

Recall the definition of the piston volume ratio in Eq. (3.8). The piston volume ratio, \( \hat{V} \), is \( \frac{1}{2} \) and thus, Eq. (6.4) matches the previously derived Eq. (6.1).

To evaluate the accuracy of the piston volume ratio prediction, a comparison of the averaged volumetric flow rate as a function of nondimensionalized valve cross-
sectional area (on the region from 0 to 2 V) was plotted for each outlet pressure and pump shaft speed. The pump inlet pressure appears to have had minimal effect on the flow rate. Figures 6-4 and 6-5 show a comparison between the predicted Eq. (6.1) and the averaged volumetric flow rate as a function of nondimensional valve cross-sectional area for the measurements recorded for a 3 MPa inlet pressure.

Figure 6-4. Averaged non-dimensional pump outlet volumetric flow as a function of nondimensional valve position for 3 MPa pump inlet pressure, 1000 rpm shaft speed.
These results demonstrate a good correlation between our prediction in Eq. (6.1) and experimental results. Figures 6-4 and 6-5 show the results for the 3 MPa inlet pressure, but this analysis can be conducted for all measured inlet pressure systems. A comparison of the goodness of fit, as represented by $R^2$ values, for the experimental results depicted in Figs. 6-4 through 6-5 are provided in Table 6-2. These results were consistent with the other inlet pressure measurement values: a nondimensional piston volume ratio of 0.5 displays a reasonably good correlation with the results for shaft speeds of 1000 and 1500 rpm, which aligns with the definition presented in Chapter 3. Observe that the linear relationship between valve position and flow rate neglects leakage losses. A better model will be presented in the upcoming sections of this work.
Table 6-2. Goodness of fit of predicted linear relationship as compared to experimental data

<table>
<thead>
<tr>
<th>Inlet Pressure</th>
<th>Discharge Pressure</th>
<th>Shaft Speed</th>
<th>R$^2$ Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0 MPa</td>
<td>Averaged, all values</td>
<td>1000 rpm</td>
<td>0.8156</td>
</tr>
<tr>
<td>3.0 MPa</td>
<td>Averaged, all values</td>
<td>1500 rpm</td>
<td>0.9637</td>
</tr>
</tbody>
</table>

The working fluid in the experimental test setup introduced in Chapter 1 is CAT HYDO Advanced 10. From the materials safety datasheet, it can be determined that the nondimensional specific volume ratio, $\hat{\nu}$, is approximately 330 [33]. This is substantially smaller than the nondimensional specific volume ratio for saturated water at 100°C ($\hat{\nu} = 1600$), which was used in all of the analytical predictions.

Now that the nondimensional pump piston volume and working fluid specific volume ratios have been determined, Eq. (6.3) can be used to predict the fluid quality at any given pump flow rate. Projected results for fluid quality as a function of flow rate and valve position are provided in Table 6-3. Recall that a fluid quality of 0 refers to a fully liquid fluid, and a fluid quality of 1 refers to a fully vaporized fluid. Fluid quality is determined on a per mass basis. While this property is frequently employed in thermodynamic calculations, it may be more intuitive to the reader to think about percent vaporization on a per volume basis. The righthand column in Table 6-3 is intended to accommodate this. These projections exhibit the behavior of an exponential decay function. This is significant, as this means that minimal vaporization is required to operate the valve between 70% and 100% of its maximum opening. This could be helpful for reducing cavitating behavior in the system.
Table 6-3. Predicted fluid quality as a function of nondimensional flow and valve position

<table>
<thead>
<tr>
<th>Nondimensional Pump Outlet Flow Rate ($\dot{Q}_2$)</th>
<th>Nondimensional Valve Position ($\hat{A}$)</th>
<th>Fluid Quality ($x$)</th>
<th>Percentage of Fluid that is Liquid by Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>2.73E-02</td>
<td>84.7%</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>1.21E-02</td>
<td>86.2%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>7.07E-03</td>
<td>87.7%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>4.55E-03</td>
<td>89.3%</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>3.03E-03</td>
<td>90.9%</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6</td>
<td>2.02E-03</td>
<td>92.6%</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7</td>
<td>1.30E-03</td>
<td>94.3%</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>7.58E-04</td>
<td>96.2%</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>3.37E-04</td>
<td>98.0%</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>0.00</td>
<td>100%</td>
</tr>
</tbody>
</table>

6.3 System Efficiency Results

In Chapter 4, extensive coverage was given to the derivation of the theoretical models and associated coefficients for both the inlet metering system and the variable displacement pump. Recall the derivation of the system efficiencies for both the axial piston variable displacement pump and the inlet metering pump that took place in Chapter 4. The axial piston variable displacement pump efficiency was written as

$$\eta = \frac{\dot{P} \left( \hat{\alpha} - K_o \hat{\alpha} \dot{P} - K_i \frac{\dot{P}}{\mu} - K_i \sqrt{\dot{P}} \right)}{\hat{\alpha} \dot{P} + A \dot{P} \exp \left( -B \frac{\hat{\mu} \hat{\omega}_p}{\hat{P}} \right) + C \sqrt{\alpha \hat{\dot{P}}} \hat{\mu} \hat{\omega}_p + D},$$

(6.5)

where $\hat{\alpha}$ is the normalized swashplate angle, $\dot{P}$ is the nondimensional pump discharge pressure, $\hat{\mu}$ is the nondimensional fluid viscosity and $\hat{\omega}_p$ is the nondimensional pump shaft angular velocity (This was previously introduced as Eq. (4.20) in the efficiency
analysis). The coefficients $A$, $B$, $C$, and $D$ are experimentally derived constants. The expression for the inlet metering pump efficiency differs in five ways:

1. The initial volumetric flow rate entering the inlet metering system must be accounted for as it is transmitted at a pressure much higher than atmospheric pressure.

2. Instead of a swashplate angle, the pump is controlled by the nondimensional cross-sectional area, $\hat{A}$.

3. There is an additional term in the denominator of the efficiency expression associated with the torque required to vaporize the fluid.

4. Experimentation was done at a variety of shaft speeds, so the nondimensional shaft speed, $\hat{\omega}$, cannot be uniformly treated as unity and thus excluded from the efficiency equation. It must be considered.

5. Experimental results indicate that the static friction term (that is, the term following the coefficient $A_p$) makes a negligible contribution to the behavior of the system. This term can be neglected in the evaluation of the inlet metering system.

The inlet metering pump efficiency in Eq. (6.6) makes use of new experimental coefficients that were presented in Table 4-3.

$$
\eta = \frac{\hat{P} \left( \hat{A} \hat{\omega} - K_0 \hat{A} \hat{\omega} \hat{P} - K_1 \frac{\hat{P}}{\mu} - K_2 \sqrt{\hat{P}} \right)}{\hat{A} \hat{P} + \hat{Q}_0 \hat{P}_0 + 2 \hat{V} \psi \left(1 - \hat{A}\right) + C_p \sqrt{\hat{A} \hat{P} \hat{\omega}} + D_p}
$$

(6.6)
A comparison of the behavior of the two pumps is presented in Figs. 6-5 through 6-7. These figures have all been produced by implementing the coefficients provided in Tables 4-1 and 4-3. Note that Model A and Model B (as defined in Chapter 5) have identical theoretical efficiencies, and Figs. 6-6 through 6-8 can be taken to represent either model.

Figure 6-6. Comparison of fluid flow to valve position
The idealized discharge flow rate of the pump, absent of leakage, would result in a 45° line across Fig. 6-6. Leakage losses affect the inlet metering pump and variable displacement pump differently. The flow output of the inlet metering pump is slightly higher than the flow output of the variable displacement pump for low flow requirement scenarios, but this changes quickly as the valve displacement increases, and for much of the operational range of the two pumps, the inlet metering pump appears to be subject to greater leakage losses. The torque requirement, illustrated by Fig. 6-7, is greater for the inlet metering system than for the variable displacement pump for all flow rates. This is due to the additional energy requirement associated with compressing the partially vaporized fluid.

Figure 6-7. Torque requirement as a function of pump flow output
Equation (6.6), coupled with the coefficients in Table 4-3, is plotted in Fig. 6-8 and shown with the solid line as the inlet metering system efficiency result. This calculation takes into account the additional energy requirement associated with the charge pump. Equation (6.5) and the coefficients in Table 4-1 provide the curve for the variable displacement pump (indicated with the coarse dashed line). Equation (6.7) is also applicable to studying the behavior of the inlet metering pump apart from the additional charge pump energy requirement. This result is shown with the finely dashed line, also in Fig. 6-8.
Figure 6-8 reveals the decreased efficiency of the inlet metering system. The primary weakness of the inlet metering design is in its low efficiency.

\[ \eta = \frac{\hat{P} \left( \hat{\omega} - K_0 \hat{\omega} \hat{P} - K_1 \frac{\hat{P}}{\hat{\mu}} - K_2 \sqrt{\hat{P}} \right)}{\hat{A} \hat{P} + A_p \hat{P} \exp \left( -B_p \frac{\hat{\mu} \hat{\omega}}{\hat{A} \hat{P}} \right) + C_p \sqrt{\hat{A} \hat{P}} \hat{\mu} \hat{\omega} + D_p} \]  

(6.7)

Figure 6-9 contains the efficiency map for the inlet metering system design. This mapping further emphasizes the decreased efficiency observed in operation of the inlet metering pump as compared to previously published works [87, 107, 108]. Practitioners may find this map useful in determining optimal operating points for the inlet metering pump in the event that this design sees practical use.
Figure 6-10 presents the actual measured experimental data (as opposed to data modeled using the experimentally derived coefficients), and compares the efficiency of the inlet metering system for a variety of inlet and outlet pressures. The measurements encapsulated in Fig. 6-10 were obtained for a range of inlet metering valve positions.

Figure 6-10. Measured efficiency results for the inlet metering system

Figure 6-10 reveals that the inlet metering system efficiency is largely impacted by the discharge pressure selection. The inlet pressure, on the other hand, has minimal effect on the system efficiency. The maximum measured efficiency for the inlet metering pump is found to be 89.4%; however, once the charge system requirement is taken into account, the inlet metering system maximum efficiency is recorded at 76.2%. The maximum efficiency for a variable displacement pump is typically on the order of 85-95% [110].
6.4 A Qualitative Discussion about the Inlet Metering System Efficiency

Much of the analysis in this document disparages the efficiency of the inlet metering system. The maximum efficiency of the inlet metering system is only 76.2%, which is significantly lower than the maximum efficiency (94.1%) presented by the variable displacement pump that serves as our consistent frame of reference throughout this paper. Even when the inlet metering pump is considered apart from the charge pump energy requirement, efficiency of the inlet metering pump never reaches 90%. However, this negative comparison is not entirely fair.

It is important to keep in mind that we are comparing results from the operation of two very different types of pumps. The variable displacement pump studied is a 145cc model that can be purchased “off the shelf” for operation in a hydraulic system. The inlet metering system pump is a 9 cc diesel fuel pump that was retrofit to operate in a hydraulic system. The machining tolerances and construction of these two pumps are very different. A more fair comparison might be represented by the result presented as Fig. 6-11.
In Fig. 6-11 the efficiency of the variable displacement pump was normalized about the maximum operating efficiency of the variable displacement pump that has been studied throughout this paper, as shown in Eq. (6.8)

\[ \hat{\eta}_{145cc} = \frac{\eta_{145cc}}{\max(\eta_{145cc})} \]  \hspace{1cm} (6.8)

Similarly, Eq. (6.9) shows how the inlet metering system and inlet metering pump were both normalized about the maximum operating efficiency of the inlet metering pump.

\[ \hat{\eta}_{IMV\text{-system}} = \frac{\eta_{IMV\text{-system}}}{\max(\eta_{IMV\text{-pump}})} \quad \text{and} \quad \hat{\eta}_{IMV\text{-pump}} = \frac{\eta_{IMV\text{-pump}}}{\max(\eta_{IMV\text{-pump}})} \]  \hspace{1cm} (6.9)

The maximum operating efficiency of the inlet metering pump did not consider the energy required by the charge pump. This consideration is reflected by the slightly
decreased efficiency of the inlet metering system (shown with the solid line) as compared to the efficiency of the inlet metering pump on its own (shown with the finely dashed line).

Examination of Fig. 6-12 suggests that the inlet metering system efficiency may not be in as dire of shape as we had previously stated. If we were comparing identical pump constructions, it appears that operation of the inlet metering system would still be less efficient; however, the discrepancy between the traditional variable displacement approach and the inlet metering approach is significantly smaller. This assertion is clarified with Fig. 6-12.

Figure 6-12 Discrepancy Between the Normalized Efficiencies of the Variable Displacement Pump and Inlet Metering System
The charge pump energy requirement means that, at minimum, the inlet metering system is 2.4% less efficient than the variable displacement approach. This is significantly less than the 18% decrease in efficiency predicted by Fig. 6-8. For operating conditions at 60% or greater of the pump discharge flow rate, the inlet metering system is less than 10% less efficient than the variable displacement pump. This loss in efficiency may prove to be insignificant when the dynamic characteristics (which will be presented in the following section) of the inlet metering system are considered.

6.5 Valve and Pressure Response Results

Our system is modeled in Matlab/Simulink based on the block diagrams from Figs. 5-1 and 5-2. For all of the modeling simulations, it is helpful to note that the simplified system (all models denoted with a prime) is denoted with a dashed line, and the full order model (models denoted simply by a capital letter) is indicated with a solid line. The simplified system was solved in Eq. (5.9) and can be plotted by hand in lieu of solving it via a computing tool such as Matlab/Simulink. The higher order model (Eq. 3.51) cannot be solved by hand; however, and thus it necessitates our use of software. Figs. 6-13 and 6-14 show the results for Models A and A’ and Figs. 6-14 and 6-15 show the results for Models B and B’.
Figure 6-13. Models A and A’ valve displacement

Figure 6-14. Models A and A’ discharge pressure behavior
Figure 6-15. Models B and B’ valve displacement

Figure 6-16. Models B and B’ discharge pressure behavior
6.6 Stability Discussion

As discussed in Section 5.2, it is impossible to design an inlet metering system with reasonable physical parameters that is not stable. This represents another attractive aspect of the inlet metering system design. For an unstable system, the pressure would oscillate and continue to increase ad infinitum. This sort of system design would eventually fail as the pressure surpassed the system’s ability to contain it.

This unstable behavior stands in stark contrast to the damping behavior illustrated by Models A and B. The pump discharge pressure over an extended timescale does not noticeably oscillate for Models A and A’ or B and B’. For the higher order models, the valve oscillations are not noticeably dampened within the small time interval shown in Figs. 6-13 and 6-14. The damping effects of the design in use can be seen more clearly in Figs. 6-17 and 6-18.
6.7 Design Parameter Adherence

The design parameters provided as Eq. (5.7) have been selected in order to guarantee that the relative order of magnitude of the nondimensional spring constant coefficient is much greater than the nondimensional mass coefficient and the nondimensional damping coefficient. If this is so, then the system can effectively be modeled as a first order system as indicated by Eq. (5.9).

Models A and A’ obey the design criteria in Eq. (5.7); however, Models B and B’ do not. The significance of our design criteria development and findings is best embodied by a comparison of Figs. 6-12 and 6-14: using this design, the inlet metering pump can effectively produce a first order pressure response. Variable displacement pumps display a second order pressure response [111]. The advantages associated with the absence of pressure overshoot and oscillation may be significant, depending on the...
application of this technology, and include reduction in system noise [112] and reduced stress on design components. Evidence of efforts to minimize pressure overshoot cross far-reaching industries, from biomedical applications [113] to residential well pumping [114] to hydraulic applications [116, 117]. This system design, then, may have broad application.

Figure 6-19. Magnified view of Models B and B’ pressure response

Figure 6-19 has been provided to show the magnitude of the higher order model’s departure from the simplified first order model expressed as Model B’. From Fig. 6-19 it can be seen that the simplified model never results in more than 10% error. As such, the first order model may prove to be a convenient tool for back of the envelope calculations, even when the design criteria developed in Eq. (5.7) are not adhered to.
6.8 System Dynamics Comparison

Evaluation of the system dynamics was first introduced in Chapter 5. Bode plots will be employed in this section to compare the inlet metering system design to a variable displacement pump (introduced in Section 5.6). The Bode plot was invented in 1938 by Hendrik Bode, and combines two separate plots \cite{117}. One plot depicts the magnitude of the frequency response of the system, and the other shows the phase \cite{118}. These figures become a highly useful tool when analyzing the frequency response of a dynamic system. As such, they will be applied here.

The Bode diagrams in Figs. 6-20 and 6-21 compare the dynamic response of the full order inlet metering system (as described by the transfer function in Eq. (5.17), and illustrated with a solid black line in the plots), the simplified inlet metering system (as evaluated in Eq. (5.20) and illustrated with a dashed black line in the plots), and the comparative variable displacement pump (described by the transfer function in Eq. (3.53) and shown with a solid red line in the plots). Observe that the higher order model and variable displacement pump both exhibit higher order response characteristics. This is indicated by the slight overshoot. This occurs near a nondimensional bandwidth frequency of 10 for the variable displacement pump, 30 for Model B, and 100 for Model A. Note that the conversion from nondimensional frequency to a frequency in hertz is

\[
\omega_{bw} = \left( \frac{\omega_{bw}}{2\pi} \right).
\] (6.8)

The simplified model bandwidth predictions of Models A’ and B’ exhibit first order behavior, as expected. This is displayed as the straight dashed line followed by a drop off on the Bode diagrams of Figs. 6-20 and 6-21.
Figure 6-20. Models A and A’ Bode Plot

Figure 6-21. Models B and B’ Bode Plot
Observe that the simplified system gives a reasonably good prediction of bandwidth frequency for bandwidth frequencies of less than 200 hz (or near 35 on the nondimensional plots). Because hydraulic systems rarely operate at frequencies above even 25 hz, the simplified bandwidth calculations of Eqs. (5.20) and (5.21) are valid design tools.

Further, observe that the higher order response values occur at greater frequencies for the inlet metering system design than for the variable displacement pump design. This means that for high speed hydraulic system designs that surpass 60 hz (e.g. electro-hydraulically controlled injection machines in advanced manufacturing that can approach 200 hz [119]), our ability to delay the frequency at which the system experiences the overshoot associated with the higher order response could be a valuable asset for high speed applications.

The final notable characteristic of the dynamic response of the inlet metering system can be shown in the step response plots provided in Figs. 6-22 and 6-23. Figure 6-22 compares Models A and A’ with the variable displacement pump model, and Figure 6-23 compares Models B and B’ with the same variable displacement pump model. Note that for both inlet metering system designs, the inlet metering system shows negligible pressure overshoot, while the variable displacement pump exhibits significant pressure overshoot. It is desirable to minimize pressure overshoot, thus, the inlet metering system design is attractive in this respect.
Figure 6-22. Models A and A’ Step Response Plot

Figure 6-23. Models B and B’ Step Response Plot
6.9 Discussion of Valve Dynamic System Response

The frequencies of our designs could present some issues associated with thermodynamic flashing if the system undergoes phase change at a nonequilibrium rate. Considerations of the thermodynamic modeling issues associated with flashing are beyond the scope of this dissertation; however, work exists in the literature that suggests that this issue is being considered by the academic community [119, 120]. It would be valuable at some future point in time to conduct CFD simulations to gain greater insight into the thermodynamic behavior of the system.

The efficiency comparisons presented in Figs. 6-7 and 6-8 reveal one of the main limitations of the inlet metering system design: reduced efficiency. The normalized efficiency comparison presented in Fig. 6-10 suggests, however, that the inlet metering system may not represented a significant drop in efficiency, depending on the engineers’ goals for dynamic performance. This project would be greatly enhanced by a future true head-to-head performance comparison, as well as design efforts focused on improving the inlet metering system’s efficiency.

As discussed in Sections 6.7 and 6.8, one of the primary advantages associated with the inlet metering system design is the ability to design a pump that produces a discharge pressure governed by first order behavior. In contrast, the inlet metering valve will always overshoot its designed steady-state valve position as the system approaches its fixed steady-state outlet pressure. This does not present a physical issue as the maximum valve displacement, which is a product of other design criteria (see Chapter 5 for more details), is several times greater than that of the steady-state valve displacement. It does, however, point to another limitation of the first order model: it can only be used
to predict the behavior of the discharge pressure. The positioning of the inlet metering valve cannot be exactly described by Eq. (5.9). Figures 6-17 and 6-18 illustrate this point quite clearly.

Ultimately, our results show us that the inlet metering system results in a less efficient pump design. However, this design results in a first order pressure response. The controllability advantages associated with this may outweigh the energy costs of the inlet metering system, depending on the application.
CHAPTER 7. CONCLUSION

7.1 Conclusion Introduction

This research initially set out to design a pump that could hold a constant downstream pressure for a varying volumetric flow demand without requiring the use of a variable displacement swashplate. Our goals in the pursuit of this design were to improve pump efficiency and decrease system cost. As we developed the inlet metering system design it became clear that we could shape the downstream pressure of the system (and, in fact, get it to behave nearly according to a first order response, which significantly improves upon the current second order pressure response associated with a variable displacement pump) utilizing the inlet metering approach, thus satisfying our original design criteria.

The inlet metering pump was initially proposed because of its anticipated efficiency improvements. We expected the pump to require less torque to move the partially vaporized fluid through the system (for a given volume we would be moving less mass), which would decrease the amount of power that would need to be supplied. However, our study revealed that the inlet metering approach was less efficient (about 70-90% efficient) when compared to the traditional variable displacement pump (about 95% efficient). Thoughtful consideration of the designs of the two pump considered suggests that the efficiency discrepancy may not be as dire as suggested by these results (see Section 6.4), but it is difficult to say for certain, as no data beyond that presented in this dissertation is available.
As stated, cost considerations were another factor considered in this project. Fixed displacement pumps are much cheaper than variable displacement pumps, and our ability to design an inlet metering valve that permits the use of a fixed displacement pump will definitely have a positive impact on the cost of manufacturing; however, our investigation of the available literature has led us to recommend the installation of cavitation-resistant inserts [29, 30, 33, 52]. This will increase the construction costs of a fixed displacement pump; however, it seems protection against cavitation damage could be prudent in light of the typical lifecycle and operation of a pump.

The biggest revelation that came from this theoretical analysis of the inlet metering pump was that it could be designed to exhibit a first order pressure response. We have noted in several places how unique this is when compared to a variable displacement pump (e.g. [85, 105]). As long as both the design criteria and the Routh Hurwitz stability criteria presented in Chapter 5 are satisfied, the simplified first order solution (shown in Eq. 5.16) can accurately approximate the pressure behavior. The ability to solve a system of dynamic equations that describe our pump system by hand should ease the design process, as well as eliminate pressure overshoot and oscillation.

Following the construction of a theoretical model, a physical prototype was constructed. Experimental data was obtained for assorted fixed inlet and outlet pressures, as well as four different pump speeds. The information from these experiments was used to determine the experimental coefficients, $K_0$, $K_1$, $K_2$, $A$, $B$, $C$, and $D$ that are used in the estimation of downstream pressure and torque. These results were then applied to an efficiency calculation, and we compared these projections to the calculated system
average efficiencies. Extremely good agreement ($R^2 > 0.97$ for all predictions) was found for both the flow and torque equations.

As the inlet metering system has been explored in the previous chapters, we have found that we were able to satisfy our original design goal and learned a variety of things about the anticipated behavior of the inlet metering system. Section 7.2 will present a list of all conclusions drawn in this work. Section 7.3 will present a brief list comparing the merits and drawbacks associated with the inlet metering design with the traditional variable displacement pump. This dissertation will end with a discussion of the work to be done on this topic for the future.

7.2 CONCLUSIONS

The following conclusions are supported by the analysis and results of this document:

1) Cavitation resistant inserts are recommended to mitigate the potential for pump damage from the vaporization of the fluid [52].

2) Stainless steel or ceramic inserts are likely to be the most cavitation-resistant for a water hydraulic system [29, 30, 33].

3) The theoretical prediction in regards to pump flow rate control is that there is a direct linear relationship between the volumetric flow rate and valve cross-sectional area.

4) A coupled set of equations, comprised of one first order differential equation and one second order differential equation can be used to describe the inlet metering valve and the discharge pressure of the single piston fixed displacement pump.
5) Nondimensionalization is an effective technique to compare the relative orders of magnitude of the system parameters, and reveals that the valve mass and valve damping coefficient have minimal impact on system dynamics.

6) Neglecting the insignificant terms in the coupled set of differential equations yields a simplified first order system.

7) The simplified differential equations describing the inlet metering system can be solved by hand, thus easing the design process.

8) For volumetric flow rates between 50 and 100% of the maximum pump outlet flow rate, a 145cc pump (a representative variable displacement pump) exhibits efficiencies ranging between 87 and 95%.

9) For volumetric flow rates between 50 and 100% of the maximum pump outlet flow rate, the inlet metering pump yields a theoretical efficiency prediction ranging between 77 and 83%.

10) For volumetric flow rates between 50 and 100% of the maximum pump outlet flow rate, an inlet metering system yields a theoretical efficiency prediction ranging between 72 and 81%.

11) The inlet metering valve system has additional power requirements of an elevated valve inlet pressure and a vaporizing torque. These quantities both contribute to the decreased efficiency of the inlet metering valve system.

12) Maintaining the minimum possible charge pump pressure requirement is necessary to maximize the inlet metering valve system efficiency.
13) The inlet metering system will always result in a stable design as evaluated by the Routh Hurwitz stability criterion if the system is designed such that the nondimensional leakage coefficient $\hat{K}_1$ is unity.

14) Neglecting viscous drag without neglecting valve mass will always yield an unstable system.

15) Viscous drag is a crucial stabilizing force in the operation of this valve.

16) Increasing the nondimensional spring constant, $\hat{K}_2$, will always increase system stability.

17) The dynamic response of the inlet metering system is improved by increasing the system leakage.

18) Increasing the system leakage will have a negative impact on system efficiency.

19) Satisfaction of the design criteria supplied in Chapter 5 enables a system design that adheres to first order pressure response.

20) Satisfaction of the design criteria supplied in Chapter 5 permits the use of a very basic equation requiring only addition and the knowledge of the nondimensional spring constant to estimate the bandwidth frequency of the inlet metering system.

21) Experimental results reveal that the volumetric flow rate and valve position are linearly related for valve positions corresponding to $0 - 2.0 \, V$, and volumetric flow rate is unrelated for valve positions corresponding to $3.0 - 5.0 \, V$.

22) The piston volume ratio $\hat{V}$ is approximately 0.5 for the inlet metering system prototype.

23) The nondimensional specific volume ratio, $\hat{\nu}$, is approximately 330 for the working fluid in the inlet metering system prototype.
24) Fluid quality in the inlet metering system can be predicted utilizing equations developed in this work.

25) A theoretical model for the efficiency of an inlet metering pump was developed.

26) The theoretical model for the inlet metering pump discharge flow rate shows excellent correlation with experimental results for all measured values.

27) The theoretical model for the inlet metering pump torque requirement shows excellent correlation with experimental results for all measured values.

28) System efficiency always increases for an increased discharge pressure.

29) System efficiency is not significantly changed by the inlet pressure.

30) The inlet metering system design is less efficient than the variable displacement design for all flow rates.

31) The inlet metering system pressure response can be adequately predicted by the simplified model if the design criteria from Chapter 5 are adhered to.

32) The inlet metering system can be designed such that the pressure exhibits a first order response with negligible overshoot and oscillation if the design criteria from Chapter 5 are adhered to.

33) The simplified system gives a reasonably good prediction of the inlet metering system frequency for bandwidth frequencies of less than 200 hz.

34) The inlet metering system design delays the onset of higher order response from about 70 hz (as occurs for the variable displacement pump modeled in this work) to nearly 300 hz.

35) The inlet metering system exhibits minimal pressure overshoot, while the variable displacement model exhibits significant overshoot.
36) The first order model can only be used to predict the behavior of the discharge pressure. It fails to detail the valve oscillations.

37) An inlet metering valve combined with a fixed displacement pump can be used in place of a variable displacement pump, thus introducing cost savings and removing a mechanically complex piece of machinery.

7.3 Comparative Analysis

The following are considerations that suggest utilization of the inlet metering valve system could display significant improvements over the traditional variable displacement pump approach to flow control:

1. The inlet metering valve system eliminates the mechanically complex pieces necessary for operation of a variable displacement swashplate

2. Fixed displacement pumps cost about 20% of the cost of a variable displacement pump.

3. The inlet metering system can be designed such that the downstream pressure of the pump displays a first order response.

   a. The pressure from the inlet metering system does not exhibit overshoot and oscillation.

   b. The coupled set of differential equations that describes the inlet metering system can be solved by hand, which simplifies the design process.

   c. The simplified system of equations allows for a quick prediction of system bandwidth frequency. It displays reasonably good accuracy and does not require a numerical solution like that of the higher order system.
The following are issues that suggest the traditional variable displacement pump approach retains advantages over the inlet metering approach to flow control:

1. The inlet metering approach ensures cavitation will occur in the hydraulic system.
   a. Expensive erosion-resistant inserts may be required.
   b. Cavitation driven noise could prove unappealing to consumers.
2. The inlet metering system consistently operates at efficiencies less than that of the variable displacement pump for a given pump outlet flow.
3. Flashing and nonequilibrium thermodynamic issues are complex, could impact the operation of this design, and require further investigation.

7.4 Future Work

The author is aware of the simplifications that were made in regard to the working fluid in the inlet metering system. The simplicity of the assumption of pure water was necessary for this preliminary work and is representative of contemporary approaches found in the literature; however, it is not the most accurate depiction of realistic operating conditions. If this work could be expanded to consider the thermodynamic properties of hydraulic oil and account for the dissolved air that inevitably exists, that would mark a considerable improvement in the modeling approach.

A Computational Fluid Dynamics (CFD) study of the valve and the phase change occurring within the valve would provide valuable insight into the cavitation process, and could expand the body of knowledge available to designers who are trying to implement this technology. Further testing of the inlet metering system prototype could also provide useful information. Evaluation of the long term effects of cavitation could provide
valuable insights on the potentially destructive results of fluid vaporization and the life of the pump. This testing could aid designers in determining the importance and/or placement of cavitation resistant inserts.

A second law thermodynamic efficiency evaluation, like the one developed by Rakopoulos and Giakoumis that is discussed in Section 2.7, could supplement the first order analysis that already occurred [70]. Exergy analyses are valuable for studying individual components in the system, and could provide greater insight into both the valve and pump independently. Further, exergy analyses are employed in optimization activities [121, 124] and could be useful in design of inlet metering systems.

A final item that would be of great interest would be the experimental analysis of a more directly comparable variable displacement pump and inlet metering system. Figure 6-11 suggests that the efficiency losses associated with the inlet metering system may be relatively small, and experimental validation of this assertion would be highly valuable with respect to consideration of the marketability of the inlet metering design.
REFERENCES


[99] A. Hurwitz, "On the conditions under which an equation has only roots with negative real parts," Mathematische Annalen, vol. 46, pp. 273-284, 1895.


**APPENDIX**

Table A.1  Averaged Dimensional Data Obtained by the Fales Team

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VITA

Julie Wisch was born in Jefferson City, Missouri in 1989 to Kevin and Karen Wisch. She has one brother, Stephen Wisch. She attended Immaculate Conception for kindergarten through eighth grade, and then attended Helias High School, where she graduated in 2007. Upon graduation, she matriculated to the University of Iowa, where she briefly studied biomedical engineering before determining that mechanical engineering required significantly fewer chemistry classes. She earned a Bachelor of Science degree in Mechanical Engineering in 2011 from the University of Iowa.

Following her graduation from the University of Iowa, Julie joined Accenture in Chicago, Illinois. From June 2011 to January 2013, Julie was staffed on projects at Caterpillar, BP, and Exelon. She was a consultant for a year and a half, and is still incapable of explaining exactly what it is that a consultant does.

Julie completed her doctoral work at the University of Missouri in hydraulic systems under the guidance of Dr. Noah Manring. She greatly enjoyed the opportunity to work on an industry-driven project, and is pleased to have had the chance to collaborate with engineers at Caterpillar. Outside of her graduate work, she served as the assistant softball coach at Tolton Catholic High School from 2013 - 2015, and is deeply grateful to the Tolton community for having a place to play and do something entirely unrelated to her academic life. She defended her dissertation in April of 2016, and plans to move to Fayetteville, Arkansas following graduation to work in the Fayetteville Public School District and launch a STEM program at the Agee-Lierly Life Preparation Services Center.