APPROXIMATE ISOMETRIES AND DISTORTION ENERGY FUNCTIONALS

Oksana Bihun

Professor Carmen Chicone, Dissertation Supervisor

ABSTRACT

A fundamental problem in Riemannian Geometry and related areas is to determine whether two diffeomorphic compact Riemannian manifolds (M, g_M) and (N, g_N) are isometric; that is, if there exists a diffeomorphism $h : M \to N$ such that $h^*g_N - g_M = 0$, where h^*g_N denotes the pullback of g_N by h. If no such diffeomorphism exists, it is important to know whether there exists a diffeomorphism that most closely resembles an isometry. This is accomplished by minimization of the deformation energy functional

$\Phi(h) = \int_M \|h^*g_N - g_M\|^2$.

We also propose other measures of the distortion produced by some classes of diffeomorphisms and isotopies between two isotopic Riemannian n-manifolds and, with respect to these classes, prove the existence of minimal distortion morphs and diffeomorphisms. In particular, we prove the existence of minimal diffeomorphisms and morphs with respect to distortion due to change of volume. Also, we consider the class of time-dependent vector fields (on an open subset Ω of $\mathbb{R}^{n+1}$ in which the manifolds are embedded) that generate morphs between two manifolds M and N via an evolution equation, define the bending and the morphing distortion energies for these morphs, and prove the existence of minimizers of the corresponding functionals in the set of time-dependent vector fields that generate morphs between M and N and are L^2 functions from $[0, 1]$ to the Sobolev space $W_0^{k,2}(\Omega; R^{n+1})$.