ENTRY BIASES IN COURNOT MARKETS WITH FREE ENTRY

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The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

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and hereby certify that, in their opinion, it is worthy of acceptance.

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I would like to take this opportunity to thank my husband for all his support and to thank my children for being there for me.

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TABLE OF CONTENTS

CHAPTERS

LIST OF TABLES

LIST OF FIGURES

Figure Page

CHAPTER 1

INTRODUCTION

This study examines free entry and social optimum in a Cournot market. First, a homogeneous goods market with linear demand is studied. Second, non-linear demand is introduced to the homogeneous goods market. Third, a non-homogeneous goods market is examined under the assumption of identical firms. Lastly, the study is extended to non-homogeneous goods market with non-identical firms.

In the homogeneous goods market, entry biases are considered under the assumption that there are two types of firms with different cost functions. It is established that there are two kinds of entry biases. First, free entry may lead to excess entry relative to the socially optimal level. Second, free entry may lead to the wrong type of firms in the market compared to the socially optimal type.

Entry biases have been well established in the literature. Previous research by Guesnerie and Hart (1985), Mankiw and Whinston (1986), Suzumura and Kiyone (1987) and Cabral (2004) studies free entry from a social welfare point of view. They establish that entry biases exist if firms are not behaving as perfect competitors. A common assumption that has been made in the literature is that firms are identical. For example, Mankiw and Whinston (1986) consider a homogeneous goods market with identical firms that engage in Cournot output competition. They show that free entry entails excessive entry from a social welfare point of view. Cabral (2004) considers a homogenous product industry and answers the question whether free entry leads to the socially optimal number of firms in the market. He discovers that entry costs affect entry biases. When entry costs are low, he examines market competition. When the level of competition in the market increases it implies insufficient entry, whereas softer competition in the market implies excessive entry. On the other hand, when entry costs are high, entry bias depends on the entry game being played in the market.

Obviously, the model will be richer when non-identical firms are introduced. In my study, there are two different types of cost functions, with firms in the market differing based on type. The number of free entry Cournot equilibrium firms and social optimum firms are compared in order to investigate entry biases. Generalization to more than two types of firms seems straightforward, however a model with two types suffices to make the point intended here.

The paper by Gotz (2005) studies a model that is very close to ours in a homogeneous goods market. Gotz (2005) examines the existence of Cournot equilibrium under free entry, adding technological choice to the model. He allows the firms to choose from a set of two technologies, which he calls “small-scale” and “large-scale.” There is a constant marginal cost for small-scale technology and zero marginal cost for large-scale technology. Since a firm in our study has one of the two different cost functions, Gotz’s identification of two different technologies helps us to establish the conditions for the existence of free entry equilibrium.

The importance of Gotz's paper arises from the fact that both the market size and technology choices of the firms are endogenous. His work differs from other studies in the literature about the existence of free entry Cournot equilibrium, since he allows the firms in the market to choose from a set of different technologies. The main difference between this study and Gotz's is a focus on entry biases, while he studies only the free entry equilibrium. Gotz presents sufficient conditions for the existence of free entry Cournot equilibrium but does not discuss the welfare-maximizing number of firms in the market. His model contains a homogeneous product market with linear demand. The model in this study is extended to a non-homogeneous product Cournot market.

In the last two sections, a non-homogeneous goods market is examined. Under the assumption of identical firms, each firm produces one of the two differentiated goods: q_1 and q_2. The number of free entry and social optimum firms are compared to determine entry biases. The main finding is that the number of free entry equilibrium firms is always greater than the number of social optimum firms in a non-homogeneous goods market when each firm has an identical cost function.

When we let the firms differ in the non-homogeneous goods market, two types of firms exist in the market, each type producing one of the two differentiated goods: q_1 and q_2. Again, we are interested in a Cournot market with free entry. It does not look promising to solve for free entry equilibrium strictly by abstract mathematical calculations. There are seven parameters involved in the two first order conditions that are satisfied by the free entry equilibrium number of firms. As a result, first order conditions are obtained through calculation but the number of free entry equilibrium firms is found by solving the first order conditions with Mathematica software. Similarly, the number of social optimum firms is found by maximizing the welfare function over integers with Mathematica.

Three tables showing the number of free entry equilibrium and social optimum firms in the market are constructed when the parameters are assigned different values. The findings from the tables support the results obtained in Chapter 3 and 4 when the degree of product differentiation is close to minimum. Corner solutions are obtained when products become less differentiated. When the degree of product differentiation approaches maximum, non-corner solutions that both types of firms can survive appear since the goods become independent goods.

CHAPTER 2

LITERATURE REVIEW

There exists extensive research on equilibrium in a Cournot market with or without free entry. It would be appropriate to divide the existing theory about Cournot equilibrium and free entry into six sections as follows:

2.1. Existence of Cournot equilibrium with and without free entry into market.

2.2. Properties of free entry Cournot equilibrium.

2.3. Effects of free entry into market.

2.4. Structure of free entry game.

2.5. Comparing Bertrand and Cournot Equilibria.

2.6. Free entry into market from a social welfare point of view and entry biases.

A discussion of the theory and assessment of relevant papers regarding to each of these six sections are provided in the following subtitles.

2.1. Existence of Cournot equilibrium with and without free entry into market:

Frank, Jr. & Quandt (1963), Frank, Jr. (1965), Ruffin (1971), Novshek (1980, 1984, 1985), Bamon & Fraysse (1985), Ushio (1983), Amir & Lambson (2000), Gotz (2005), and Long & Soubeyran (2000) are some of the economic papers that developed ideas about the existence problem of the Cournot equilibrium. Several proofs are known for the existence of the Cournot equilibrium under varying assumptions, which have been improved year after year by interested economists. We can summarize the main assumptions as follows:

- Strict concavity of profit function,
- Existence of identical firms in the market,
- Concavity of inverse demand function,
- Convexity of cost function.

Frank, Jr. & Quandt (1963) present one of the studies about the existence of Cournot equilibrium. They consider more general conditions for the existence of Cournot equilibrium than has been attempted previously. In their model, all firms in the market have the same cost function, which is continuous and monotonically increasing. Their main assumptions are that the demand function and the cost function are continuous and that the profit function is concave. These assumptions are reduced and simplified by other economists such as Novshek and Bamon & Fraysse (1985).

Ruffin (1971) indicates that the conditions for the existence of Cournot equilibrium had not been discovered by year 1971, by stating in his paper that "…we should observe that Cournot model has not yet been thoroughly axiomatized, so that the conditions for the existence of profitable or viable Cournot equilibrium are uncertain." (Ruffin, 1971, p. 494). Ruffin (1971) cites the paper by Frank, Jr. & Quandt (1963) to refer to the existence of Cournot equilibrium.

Ruffin states conditions for the existence of a profitable Cournot equilibrium and indicates that the conditions presented by Frank and Quandt are satisfied by his conditions. He also says regarding to Frank and Quandt that "Unfortunately, they failed to address themselves to the survival problem."(Ruffin, 1971, p. 494). Two of Ruffin's strong assumptions are the existence of identical firms in the market and the strict concavity of profit function.

Frank, Jr. (1965) presents another paper, which focuses on entry in a Cournot market. He makes five sufficient assumptions, which depend on the slopes of both the demand and cost curves, to show the existence of a Cournot equilibrium. One of his assumptions is the strict concavity of profit functions. This assumption had been made by all of the economists who were interested in the existence of Cournot equilibrium until Novshek (1980) came up with the proof of the existence of Cournot equilibrium for any number of identical firms without assuming strict concavity of profit functions.

Novshek is the first economist who considered the existence of Cournot equilibrium with free entry. This paper, Novshek (1980), is mainly about the precise statement and proof of "Folk Theorem" for a homogenous good market with free entry. Folk Theorem states that if the firms in the market are small relative to the market then the Cournot equilibrium exists and the Cournot equilibrium with free entry is approximately competitive.

This paper differs from other papers about the "Folk Theorem" in the way it treats free entry. In previous papers dealing with the "Folk Theorem", free entry is defined as a zero profit condition ignoring the integer problem. Novshek does not treat free entry as a zero profit condition and he also handles the integer problem. The definition of free entry from Novshek's point of view is that an infinite number of firms have access to the same cost function. In his paper, he also makes the strong assumption that all firms in the market are identical.

Novshek explains that firms may become small relative to the market in two ways. First, absolute firm size, which can be defined as the smallest output where minimum average cost is attained, may become small because of changes in technology. Second, the market demand may become large because of it shifts. Ruffin (1971) also presents a verbal statement for the "Folk Theorem" in the conclusion part of his paper. Novshek finds that argument worth mentioning in his paper. Ruffin states the following regarding to the "Folk Theorem":

> "Imagine a Cournot oligopoly under free entry. There is a U-shaped cost curve. For each identical firm, and the number of firms is such that the profit is zero. In equilibrium, the demand curve facing the firm is tangent to the left of the minimum point on the average cost curve. Now let the number of buyers increase indefinitely. This makes the demand curve flatter and flatter (e.g., assume each buyer has the same linear demand curve). Clearly, by increasing the number of buyers the Cournot solution can become arbitrarily close to the competitive solution. The number of firms, in the case will, be large; but it will be the consequence of fundamental economic and technological conditions. In particular, not only must each buyer be a price-taker, but the number of buyers must be sufficiently large to give the market " breadth " relative to the size of the firm. "(Ruffin, 1971, p. 501).

Novshek has a number of other papers where he discusses the existence of Cournot equilibrium. It had not been well-known how to find Cournot equilibria when the number of firms in the market exceeds two, until Novshek published his paper titled "Finding All n-Firm Cournot Equilibria", in 1984. In this paper, Novshek deals with n identical firms and a single homogenous good market. He illustrates the process of finding a n-firm Cournot equilibria with n firms in the market giving two examples and then points out the procedure how it can be carried out in general.

Novshek (1985) examines the existence of Cournot equilibrium in a homogeneous good market. He presents a new existence theorem for n-firm Cournot equilibrium. There are two existence theorems mentioned in his paper. The first one proves the existence of Cournot equilibrium for n identical firms with convex technologies. The second one shows that there exists a Cournot equilibrium even if the firms in the market are not identical when each firm's profit function is concave. The concavity of profit functions is an assumption also made in the paper by Frank and Quandt (1963). Instead of this assumption other assumptions, which imply the concavity of profit functions, are also used in the literature. For instance, the assumption of the concavity of inverse demand function and the assumption of convexity of cost functions.

The difference between the papers Novshek (1984) and Novshek (1985) is that Novshek (1985) is an improved version of Novshek (1984). Novshek (1984) illustrates how to find Cournot equilibria when there exist n identical firms in the market. On the other hand, Novshek (1985) proves the existence of the Cournot equilibrium with "n" not necessarily identical firms when each firms profit function is concave.

The new existence theorem that Novshek (1985) proves does require neither the existence of identical firms nor convexity of technologies. Bamon and Fraysse published a paper only a couple of months later than Novshek's paper in 1985, which is also about the existence of Cournot equilibrium and closely related to the Novshek's work (1985). The authors worked independently and proved the existence of Cournot equilibrium under different assumptions. Bamon and Fraysse's proof is based on the condition that the demand is large with respect to the number of firms in the market, which corresponds to the assumption of Novshek that each firm's marginal revenue is a decreasing function of aggregate output of other firms. Novshek states that his assumption on marginal revenue is commonly imposed in the literature, for instance, Ruffin considers an equivalent assumption in his paper in 1971.

The theorem that Novshek presented in his paper in 1985 for existence of Cournot equilibrium is really important in the sense that it is the first theorem on the existence of Cournot equilibrium without making the common assumptions imposed on the cost functions of the firms. Novshek (1985) points out that his assumption on marginal revenue is implied by another common assumption in literature, which is the concavity of inverse demand function.

Novshek provides two examples in his paper (1985) about the nonexistence of Cournot equilibrium to help the reader to understand the necessity of the assumptions in his existence theorem. He says that the first example is a well-known example in the literature but the second example is a new established example. In the first example, he shows that the Cournot equilibrium does not exist if there are identical firms with non-convex technologies in the market. In the second example, he demonstrates that even if the technologies are convex the Cournot equilibrium may not exist if the firms are not identical.

Novshek (1985) summarizes the most commonly used three assumptions in the literature for the existence of Cournot equilibrium as follows:

- The convexity of each firm's technology,
- Whether every firm has the same technology,
- The concavity of the inverse demand function.

Moreover, he enlightens us about the results of these three assumptions on the existence of Cournot equilibrium. He states the following regarding to the three assumptions:

> "Concavity of inverse demand is enough by itself to guarantee existence of Cournot equilibrium; identical firms and convex technology together guarantee existence of Cournot equilibrium; and without concavity of inverse demand (actually, the marginal revenue condition), if either firms are not identical or technologies are not convex, equilibrium may not exist."(Novshek, 1985, p. 86).

Novshek's paper in 1980 inspired many economists such as Bamon & Fraysse (1985) and Ushio (1983). Bamon and Fraysse are also interested in the existence of Cournot equilibrium under free entry and examine the existence conditions. The difference between the paper by Bamon & Fraysee (1985) and the paper by Novshek (1980) is that Novshek (1980) proves the existence of Cournot equilibrium for any number of identical firms but Bamon and Fraysse deal with non-identical firms and prove the existence of Cournot equilibrium using a new fixed point theorem for non-continuous functions.

Bamon & Fraysse (1985) prove that, even if non-convexity in production is allowed and the firms are not identical, an equilibrium exists whenever the demand in the market is large with respect to the number of firms in the market. As it is mentioned above, Novshek independently obtained similar results in his paper in 1985. Novshek proves the existence of Cournot equilibrium assuming concavity of the inverse demand function and the condition that a firm's marginal revenue must be a decreasing function of aggregate output of other firms. Bamon and Fraysse show that the condition that Novshek (1985) assumed is fulfilled if firms are small relative to the market. Therefore, Bamon & Fraysse (1985) improve the conditions that Novshek (1985) assumed to prove the existence of Cournot equilibrium.

The study of Amir & Lambson (2000) presents results that overlap with the results of other papers about the existence of Cournot equilibrium. In the papers by Frank (1965), Ruffin (1971), Novshek (1980) and Seade (1980), the Implicit Function theorem, which proves the existence of continuously differentiable solutions of a given relation, is widely used and some simplifying assumptions, such as decreasing marginal revenue and differentiability of reaction curves, are made to sign the derivatives. However, Amir and Lambson find these assumptions unnecessary. The goal of Amir and Lambson is to obtain minimal sufficient conditions for the existence of equilibrium in a Cournot industry with symmetric firms.

Amir and Lambson (2000) prove the theorems for the existence of at least one symmetric equilibrium and the existence of two different types of Cournot equilibrium, which are a monopoly equilibrium and a unique symmetric equilibrium. Amir and Lambson (2000) bring a simpler approach to the existence problem of equilibrium in a Cournot industry than the approach of Frank (1965), Ruffin (1971), Novshek (1980) and Seade (1980).

The Gotz (2005) paper is quite related to our study regarding to the model that he presents. This paper examines the existence of Cournot equilibrium under free entry adding technological choice to the model. In the paper of Novshek (1985), there are non-identical firms but the differences of the firms are given exogenously. On the other hand, in Gotz (2005), both market structure and technology are endogenous.

Gotz (2005) considers an industry with a homogenous product. He allows the firms to choose from a set of two technologies, which he calls "small-scale" and "large-scale" technologies. There is a constant marginal cost for small-scale technology and zero marginal cost for large-scale technology. Since we have firms with one of the two different cost functions in our study, Gotz's identification of two different technologies helps us to establish the conditions for the existence of free entry equilibrium.

In Gotz (2005), there are different fixed costs associated with each technology. The inverse demand function contains two parameters that account for the market size. It is quite interesting to see that these parameters define the endogenous market structure. One of the parameters measures the maximum willingness to pay for the product and the other one measures the number of identical consumers in the market.

There is a two-stage game in Gotz's paper. In the first stage, firms decide whether to enter to the market and choose their technologies. In the second stage, firms choose their output levels. Gotz states an additional condition for the equilibrium in his model with technology choice. This additional condition is called the "no-switching" condition since it provides that no firm has an incentive to switch its technology choice in the case of equilibrium.

After deriving the sufficient condition under which a unique Cournot equilibrium exists, Gotz examines the effects of changes both in the two market size parameters and in the parameters associated with the technology choice. Moreover, he considers more general cases such as the case of more than two technology choices in the market and generalizes the conditions of the existence of free entry Cournot equilibrium for "k" different technologies in the market.

Gotz (2005) not only presents sufficient conditions for the existence of free entry Cournot equilibrium but also examines what happens if the sufficient condition is not satisfied. Moreover, he investigates the situations including non-existence, non-uniqueness, and asymmetry of Cournot equilibrium under free entry by proceeding mainly by means of examples. He derives a condition on one of the market size parameters that ensure that a Cournot equilibrium always exists.

The great importance of Gotz's paper arises from the fact that both the market size and technology choices of the firms are endogenous. His work differs form other studies in the literature about existence of free entry Cournot equilibrium, since he allows the firms in the market to choose from a set of different technologies.

Long & Soubeyran (2000) bring a different approach and use contraction mapping theorem to prove existence and uniqueness of Cournot equilibrium. Long & Soubeyran (2000) approach the proof of existence from a different angle by applying the contraction mapping theorem to a function involving marginal costs. The authors refer to many papers from literature including Novshek (1985) to point out why their approach is different than previous studies on existence of Cournot equilibrium. Long & Soubeyran (2000) characterize Cournot equilibrium in terms of marginal costs. This approach is helpful because firms can manipulate their marginal costs in an earlier stage before the Cournot game takes place. As a result Long & Soubeyran (2000) points out a different direction for the studies of a class of two stage Cournot games.

2.2. Properties of free entry Cournot equilibrium:

Ushio (1983) and Corchon & Fradera (2002) contributed to the theory of Cournot equilibrium by examining the properties of equilibrium in a free entry Cournot market. Ushio is also one of the economists who is inspired by the work of Novshek (1980). The paper by Ushio (1983) focuses on the asymptotic properties of Cournot equilibria when there is free entry in the market. There are two main assumptions in the paper that average cost function is always decreasing and the marginal cost function is non-decreasing. Under these assumptions, Ushio investigates some properties of Cournot equilibrium other than the ones that Novshek has already discussed in his paper in 1980.

Ushio (1983) states that as market size increases, equilibrium profits go to zero if average cost function is "U" shaped, on the other hand, the equilibrium output of each firm goes to infinity if average cost function is decreasing. He actually computes the limit points of firms' profits as market size increases and points out the conditions on earning positive profits.

Corchon & Fradera (2002) point out that Novshek (1980) investigated the existence of Cournot equilibrium with free entry first but then no one paid attention to the properties of this kind of equilibrium. Corchon & Fradera's paper completes the missing part of the literature in terms of properties of free entry equilibrium.

Corchon and Fradera (2002) assume that all the firms in the market are identical and the profit function is strongly concave. They investigate not only the effects of a change in the fixed cost but also the effects of a change in demand or marginal cost on the number of firms and the aggregate output. Corchon and Fradera also provide an example to illustrate the effect of an increase in demand on the number of active firms in the market and on the aggregate output. Therefore, Corchon and Fradera present the difference between the Cournot model with free entry and the Cournot model without free entry.

Corchon and Fradera (2002) answer the question what are the values of price, profits and number of active firms that can be generated as a symmetric Cournot equilibrium with free entry. They make some strong assumptions, such as unit elasticity of demand and constant marginal cost, and conclude that any triple {price, profits, number of active firms} can be generated as symmetric Cournot equilibrium with free entry.

2.3. Effects of free entry into market:

Seade (1980) and Ohkawa et al (2005) are interested in the effects of entry. Seade (1980) considers effects of free entry into an oligopolistic industry. Especially the effects of entry on output are examined in his paper. Seade mentions the paper by Ruffin (1971) and states that Ruffin presents sufficient conditions for aggregate output to rise and firm output to fall as entry occurs in a Cournot market. However, Seade points out that Ruffin does not examine what happens when the sufficient conditions are not imposed. Therefore, Seade (1980) focuses on the effects of entry when the sufficient conditions presented by some economists, including Ruffin, do not hold. Seade presents much stronger sufficient conditions for the behavior of aggregate output and proves that output always expands with entry provided the existence of equilibrium. Seade not only examines the effects of entry on aggregate output but also examines the effects of entry on profits at the firm and industry levels.

Ohkawa et al (2005) consider a symmetric Cournot market with free entry. According to the authors in a competitive market with free entry, the market forces inefficient firms out and protects the position of efficient firms. This efficiency does not work under imperfect competition. "In cases of either (i) a concave demand (which implies substitutability) or (ii) strategic complementarity (which implies a convex demand), the type of firms that should remain in the market to achieve social optimality does not necessarily coincide with the type of firms that will survive in the long run." (Ohkawa et al (2005), p. 1143). That is, the market selects the wrong type of firm in the long run. The authors show that such an imperfect selection can occur in the context of an asymmetric Cournot oligopoly.

Moreover it is shown that in a symmetric Cournot market with free entry, the number of firms in the long run is greater than the socially optimal number of firms. That is, the market selects the wrong number of firms. We also prove in our study that socially optimal number of firms is less than free entry equilibrium number of firms. (excessive entry)

Ohkawa et al (2005) emphasize that the market may select not only the wrong number of firms but also the wrong type of firms in the long run. Authors show that there are two kinds of mistakes can happen when market selects firms. The first one is that the market fails to secure the "good" type of firm in the long run. Good firms are defined as the firms that can achieve a higher level of social welfare than bad firms. Existence of bad firms results in a suboptimal level of social welfare for the economy. The second kind of mistake in the market is that even though the market secures the "good" type firm, it fails to reject the "bad" type of firm in the long run. The second kind mistake is related to the coexistence of more than one type firms in the market. The authors initially show that in a symmetric Cournot market with free entry only one type firm can survive in the long run. Our study and Gotz (2005) also show the same result. The authors state that the coexistence of different types of firms in the long run is negligibly improbable. It depends on whether two or more firm types are located simultaneously on the boundary of survival in the market. It is shown that in almost all cases, only one type can survive in the long run.

The authors suggest that first kind of mistake is a lot worse than the second one because we will never observe good firms in the long run in first kind mistake. We cannot compare good ones to bad ones in the long run since there are no good firms in the long run. This is why the authors call the first kind of market mistake as *hidden market failure.*

Types of firms in Ohkawa et al (2005) are distinguished by the shape of their average cost curves. $AC_1 > AC_2$ implies that type two has stronger increasing returns than type one. Increasing returns are related to the government intervention to control the number of firms in the market to maximize social welfare. Ohkawa at al (2005) show that the government can achieve a higher social welfare in the second-best equilibrium with type-2 firms alone than in the second-best equilibrium with type-1 firms alone because type two has stronger increasing returns than type one. The authors also examine imperfections caused by government intervention. It seems that government can regulate the number of firms by intervening during first stage entry decisions but cannot control each firm's production level once the firm enters to the market. Therefore, there is second best optimization problem. Depending on the government's knowledge of the current state of the production technology, there is a set G of firm types available to the government. Authors show that the numbers of all but one type of firm in G can be set equal to zero in equilibrium.

Ohkawa et al (2005) is quite related to our study in the sense that they also prove that the market may select not only the wrong number of firms but also the wrong type of firms in the long run. Our study differs from their study since we examine both homogeneous and also non-homogeneous good market.

2.4. Structure of free entry game:

The paper by Eaton and Ware (1987) approaches the entry situation in a Cournot market by exploring the structure of the entry game. Eaton and Ware explain the effects of sequential entry into the industry assuming that all the firms in the market have the same cost function and produce an undifferentiated product with decreasing marginal revenue. They develop a theory of market structure under sequential entry.

2.5. Comparing Bertrand and Cournot equilibria:

We are interested in the following papers dealing with the comparison between Bertrand and Cournot equilibria: Boone (2000), Zanchettin (2004) and Singh & Vives (1984). The reason why we are interested in these papers is that these papers contend with differentiated good market. The market is a non-homogeneous product market such that representative consumer's utility is a symmetric quadratic function of two products. We consider the same linear system of inverse demand functions that are generated by the representative consumer's utility in our non-homogeneous good market calculations. Both of the papers Zanchettin (2004) and Singh & Vives (1984) are restricted to differentiated duopoly whereas we generalize Cournot competition to the case when the number of firms in the market more than two. In fact we denote the overall number of firms in the market as $N_1 + N_2$, where N_1 and N_2 are the number of type one and type two firms in the market, respectively.

Singh & Vives (1984) analyze price and quantity competition in a differentiated duopoly proposed by Dixit (1979). They examine Bertrand and Cournot competition according to whether the goods are substitutes, independent or complements. It is shown that if the firms can only compete on quantity or price, the dominant strategy for each firm is to choose quantity when the goods are substitutes. On the other hand, the dominant strategy is to choose price when the goods are complements.

When the firms have constant marginal costs but no fixed costs, and no capacity constraints with a linear demand structure, Singh & Vives (1984) show that Bertrand competition is more efficient that Cournot competition. The reason is that consumer surplus and total surplus are higher in Bertrand competition in equilibrium in spite of whether the goods are substitutes, independent or complements. However, with non-linear demand, the comparison of profits between Bertrand and Cournot competition depends on whether the good are substitutes, independent or complements. The authors show that profits are larger in quantity competition than in price competition when the goods are substitutes but the opposite holds when the goods are complements. Profits in both quantity and price competition are equal when the goods are independent in a differentiated duopoly. To get these results Singh & Vives (1984) use the duality structure of Cournot and Bertrand competition. They show that Cournot competition with substitutes is the dual of Bertrand competition with complements. As a result, they prove propositions for one type of competition, either price or quantity competition, and for one type of good, either substitute or complement good. The other case follows by duality since it could be easily done by interchanging prices and quantities.

Zanchettin (2004) also examines Bertrand and Cournot equilibria in a differentiated duopoly. In fact, Zanchettin (2004) directly refers to Singh & Vives (1984) since he uses the same framework in his model except that he adds a wider range of cost asymmetry to the model of Singh & Vives (1984). He focuses only on substitute goods and shows that with high degrees of cost asymmetry and/or low degrees of product differentiation, the efficient firm and also the industry make higher profits under Bertrand competition than Cournot competition when the goods are substitutes. This result contrasts with Singh & Vives (1984)'s result that profits are larger in quantity competition than in price competition when the goods are substitutes.

The reason for the contrast between Zanchettin (2004)'s result and Singh & Vives (1984)'s result is the assumption of "positive primary outputs". Singh & Vives (1984) make the assumption of positive primary outputs, which means that when both prices are set at marginal costs, both firms sell positive outputs. This assumption restricts the space of parameter values in Singh & Vives (1984). However, Zanchettin (2004) allows the parameters to a wider range by not making the assumption of positive primary outputs. As a result, he shows that Singh & Vives (1984)'s ranking of profits is reversed in a significant area of the parameter space.

Allowing parameters to a wider range helps Zanchettin (2004) to obtain additional results on equilibrium profits when firms are asymmetric in costs. He shows that the inefficient firm's profits always decreases as products become closer substitutes under both price competition and quantity competition. Also, he confirms that the efficient firm's profits are non monotonic in the degree of product differentiation. As a result the efficient firm may have a local incentive to reduce the degree of product differentiation. This result contrasts with the result under symmetric costs that Bertrand and Cournot duopolists are always better off with product differentiation.

Zanchettin (2004) examines non-homogeneous goods market. He points out that this paper is a generalization of the paper by Boone (2000) and the paper on which Zanchettin and Denicolo worked on together in 2003. Zanchettin (2004) generalizes homogeneous goods case by showing that the comparison of equilibrium profits under price and quantity competition depends on both the degree of cost asymmetry and the degree of product differentiation.

Boone (2000) examines the effects of competition on a firm's incentives to innovate. There are two types of innovations considered: product and process innovations. Boone (2000) derives the conditions under which a rise in competitive pressure increases each firm's investments in process innovations to improve efficiency. But if the rise in competitive pressure induces more industry wide process innovation, it reduces industry wide product innovation.

Boone (2000) emphasizes that the results he obtained hold only if firms differ on just efficiency. If firms differ on more dimensions than just efficiency, his results will no longer hold. In fact, Zanchettin (2004) generalizes Boone's (2000) results showing how the equilibrium profits under both price competition model and quantity competition model depends not only on the degree of cost asymmetry but also on the degree of product differentiation.

Zanchettin (2004) is also related to Häckner (2000) in the following manner. Häckner (2000) shows that when there are more than two competitors in the market, price competition provides higher profits to the "high-quality firms[*]" than quantity competition. On the other hand, Zanchettin (2004) shows that no matter what the number of the firms in the market is, profits are receptive to cost asymmetry and horizontal differentiation.

Häckner (2000) concludes that it is uncertain whether price competition or quantity competition is more efficient since profits depend on whether the goods are substitutes or complements. Assuming that there are more than two firms in the market, prices may be higher under price competition than under quantity competition if the goods are complements and quality differences are large. On the other hand, high-quality firms may earn higher profits under price competition than under quantity competition if goods are substitutes.

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[*] Häckner (2000) defines z_k as the ratio between the average quality offered by the rival firms and the quality offered by firm k. If z_k <1 firm k produces a better product than the average rival and vice versa.

2.6. Free entry into market from a social welfare point of view and entry biases:

The papers by Cabral (2004), Guesnerie & Hart (1985), Mankiw & Whinston (1986), Okuno-Fujiwara & Suzumura (1993), Ghosh & Morita (2007), Corchon (2006) and Anderson & Renault (2003) deal with free entry from a social welfare point of view. The recent paper by Cabral (2004) focuses on the welfare properties of free entry under simultaneous entry. There is a common belief that free entry into the market is desirable for social welfare. Cabral (2004) refers to the papers by some authors including Suzumura and Kiyone (1987) that free entry may no longer be good once we remove the assumption of competitive pricing behavior.

Cabral (2004) considers a homogenous product industry and answers the question whether free entry leads to the socially optimal number of firms in the market. He discovers that entry costs affect entry biases. When entry costs are low, he examines market competition. Tough competition in the market implies insufficient entry whereas soft competition in the market implies excessive entry. On the other hand, when entry costs are high, entry bias depends on the entry game being played in the market.

In addition to the model of sequential entry, Cabral (2004) considers two different models of simultaneous entry, which are the "grab-the-dollar" entry and the "war-of-attrition" entry. He defines these simultaneous entry models as follows: "The grab-the-dollar entry model assumes that entry is an instantaneous process and that entry decisions are simultaneously made in each period. The war-of-attrition entry model assumes that entry takes time, i.e., in order to enter firms must spend a certain amount of resources over a period of time.” (Cabral, 2004, p. 944). Then he points out that when entry costs are high, “grab-the-dollar” entry implies insufficient entry however “war-of-attrition” entry implies excessive entry.

Cabral (2004) refers to Suzumura and Kiyono (1987) for the discussion of the connection between welfare and the number of firms in the market and then presents two excess entry theorems. Suzumura & Kiyono (1987) prove the excess entry theorems for identical firms in a quasi-Cournot market with free entry. Therefore, their answer to the popular question “Is free entry into the market desirable all the time?” is “NO” due to their excessive entry theorems.

Guesnerie and Hart (1985) also examine free entry from a social welfare point of view. Guesnerie and Hart (1985) consider a model, which is different from the models that have been used in the literature before. According to their model, trading occurs in many separate markets instead of only one market. The reason why they work on this model is that the model makes firms consider profit maximization as an objective even if there is imperfect competition. Moreover, in this model, the behaviors of firms do not affect incomes of their customers.

Guesnerie and Hart (1985) measure the rate of convergence of Cournot equilibria to Walrasian equilibria in terms of welfare loss. “Hence, the paper relates to another tradition of the literature focusing on the measure of welfare loss due to imperfect competition." (Guesnerie and Hart, 1985, p. 525). They make their analysis general enough so that it covers both the case of free entry and the case of barriers to entry.

Mankiw and Whinston (1986) present a very well organized paper about free entry and social inefficiency and derive the conditions for entry biases in a Cournot market with free entry. Mankiw and Whinston (1986) do not only concentrate on the existence of Cournot equilibrium under free entry but also provide the conditions under which the free entry number of firms represent excessive, insufficient or optimal entry. They help us to understand the fundamental forces related to entry biases.

Mankiw and Whinston (1986) point out that free entry into the market is not desirable all the time by proving the existence of excessive entry under both imperfect competition and what the authors call the "business stealing effect". The definition of "business stealing effect" is that the equilibrium output per firm decreases when a new firm enters to the market.

Mankiw and Whinston consider a simple homogenous product market with free entry and make the following three main assumptions in their paper. First, the equilibrium aggregate output rises as the number of firms in the market increases. Second, "business stealing effect" exists. Third, equilibrium price is not below marginal cost.

Okuno-Fujiwara & Suzumura (1993) is an extension of Mankiw & Whinston (1986) and Suzumura & Kiyono (1987) in the sense that it extends excessive entry result by considering excessive investments. Okuno-Fujiwara & Suzumura (1993) argue that cost reducing R&D investments create incentives towards socially excessive investments. The model is a three stage game. In the first stage firms make the decision of whether to enter to the market. In the second stage firms make commitment to cost reducing R&D investments. In the last stage firms compete in output quantities. Firms produce a homogeneous product in a free entry market.

Okuno-Fujiwara & Suzumura (1993) add a new aspect to the literature about excessive entry by including cost reducing R&D investments. The authors show that strategic R&D investments are socially excessive if the number of firms exceeds a certain critical number. It is shown that there exists a critical number of firms, say n^*, as a function of the elasticity of the inverse demand function such that the sub game perfect equilibrium level of investment exceeds the second best level when there are more than n^* firms in the market.

Okuno-Fujiwara & Suzumura (1993) attempt to answer the question whether there is a welfare improving policy that government can enforce. Their answer is that the welfare improving policy is to restrict the number of firms to either zero or one and require the marginal cost pricing on the operating firm. Since enforcing the marginal cost pricing is almost impossible for an actual government, the authors examine the behavior of second best welfare function with respect to a small change in the number of firms. It is proven that a small reduction in the number of firms at the free entry equilibrium improves the second best social welfare. To prove this result, the authors impose explicit assumptions.

The first assumption is on inverse demand function. All linear inverse demand functions and all constantly elastic inverse demand functions naturally satisfy their assumption on demand. The second assumption is on cost function that needs to be twice continuously differentiable with a negative first derivative and positive second derivative. The third assumption requires that the second stage strategies are strategic substitutes. Moreover, there are other implicit assumptions that are crucial for the excessive entry result such as quantity competition rather than price competition, no uncertainty in R&D, symmetric equilibria, and no product differentiation.

Ghosh & Morita (2007) explore social desirability of free entry by including bargaining to the model. Bargaining takes place between manufacturer and suppliers. Each downstream firm acquires inputs from a supplier with bargaining or each upstream manufacturer sells its product to a distributor in the course of negotiations. The market is a homogeneous final product market with free entry. The authors show that the bargaining power of the suppliers of intermediate products affects the social desirability of free entry. In fact it is shown that free entry in a homogeneous product market leads to a socially insufficient number of firms when the suppliers have sufficiently strong bargaining power. This is in contrast to previous studies in the literature about homogeneous product markets with free entry that the level of entry is always socially excessive. The cause for the contradictory result in Ghosh & Morita (2007) is the bargaining that takes place between manufacturer and suppliers. Regulations in anti-competitive markets are generally rationalized by excessive entry result in the literature.

Ghosh & Morita (2007) point to a different approach by including bargaining in to the model which leads to interesting results especially regarding to entry levels.

Ghosh & Morita (2007) examine a two stage game in a model that contains a large number of potential entrants to the downstream sector each of whom must decide whether to enter by incurring a set up cost. After the entry, each downstream entrant is matched with an upstream supplier. In stage two, Nash bargaining takes place between the entrant and the supplier over the input price and the quantity at the same time. Entrant produces a homogeneous final product by acquiring an intermediate product from the supplier. (An alternative set up can be made in which potential upstream entrants make entry decisions and then each entrant is matched with a downstream firm). The main finding is that the number of downstream firms is socially insufficient in the free entry equilibrium when upstream suppliers have strong bargaining power.

Ghosh & Morita (2007) consider a social planner to determine socially optimal number of firms which is the same approach that we utilized in our study. Social planner can control the number of firms that enter the upstream sector but cannot control their production decisions one they have entered. The objective of the social planner is to select the number of upstream firms that maximizes total surplus, which is defined as gross benefit to the consumers less the sum of production costs and entry costs.

Ghosh & Morita (2007) also consider an alternative model in which entry is free in both upstream and downstream sectors. In the first stage, there are a large number of potential entrants in the downstream and upstream sectors. Firms make the decision of whether to enter. In the second stage, each upstream firm seeks a downstream firm to provide inputs and each downstream firm seeks an input provider upstream firm. Matches occur randomly and firms that that have failed in finding a match exit the market. In the third stage, each pair bargains over input prices and quantities. Ghosh & Morita (2007) find that there is a unique pair (M_f, N_f) such that M_f downstream firms and N_f upstream firms enter in the free entry equilibrium where $M_f \geq 1$ and $N_f \geq 1$.

Ghosh & Morita (2007), as in Mankiw & Whinston (1986), assume that business stealing effect is present in the downstream sector. Output per firm falls as the number of firms in the industry increases under business stealing effect. In fact, if upstream firms have no bargaining power in Ghosh & Morita (2007)'s model, insufficient entry never occurs in the presence of the business-stealing effect, and hence the free entry number of downstream firms is socially excessive. This is the same as the excess-entry result which has previously been demonstrated in the literature.

Ghosh & Morita published another paper about level of entry in free entry equilibrium. They offer a new perspective on the question of "Is free entry desirable for social efficiency?" by analyzing vertical relationships between industries. In the paper Ghosh & Morita (Summer 2007), they examine the number of firms in a vertical oligopoly model. The model is similar to the model in Ghosh & Morita (2007) except that there is no bargaining in this model. There are two sectors: upstream and downstream sector. The number of firms in downstream sector is fixed. However, free entry takes place in upstream sector. The game is a three stage game. In the first stage, firms make their decisions whether to enter to the upstream sector. Second stage involves Cournot competition in the upstream sector. Upstream firms determine the quantities of intermediate product taking rival upstream firms' output as given. Third stage also involves Cournot competition in downstream sector. Downstream firms determine the quantities of the final product taking the input price as given. The input price is determined in the marker clearing level, which makes the demand of downstream firms equal to the total amount of intermediate product supplied by the upstream firms.

Ghosh & Morita (Summer 2007) consider a social planner whose objective is to select the number of upstream firms that maximizes total surplus. The authors show that free entry in an industry that produces a homogeneous product leads to insufficient number of firms, rather than excessive, when we incorporate vertical relationships between industries. Ghosh & Morita contribute to the literature by proving that free entry does not lead to excessive entry when vertical relationship between industries is taken into account or when bargaining takes place between industries.

Let us consider Corchon (2006), which focuses on welfare losses (WL). Especially the percentage of welfare losses (PWL) is calculated to determine what kind of conditions in the market creates higher WL. For instance, with identical firms and no fixed costs, authorities should not be concerned about mergers that do not bring the number of competing firms below four. On the other hand merging from duopoly to monopoly almost doubles PWL. Corchon (2006) assumes that firms are identical with no fixed costs to capture the full size of WL under the methodology in the first part of the paper.

Cournot (1837, chapter 8) shows that oligopolistic behavior can be approximated by perfect competition when the number of firms approaches infinity. He measures WL as the difference between social welfare in the optimal and equilibrium allocation. WL tends to zero when the number of firms approaches infinity. Corchon (2006) asks the question what happens when the number of firms is finite. His answer is that either perfect competition is a good approximation (i.e. WL is quite small) to oligopoly or WL is significant. The paper states the dilemma as either we find environments in which oligopoly produces WL much greater than those found in a linear model or we abandon oligopoly model as a whole. As a result, this paper seeks markets where oligopoly produces large WL.

First, a non-linear inverse demand function, $p = A - bx^{\alpha}$, is considered. It is shown that for some values of α, PWL is arbitrarily small. Corchon (2006) states that in real world markets, α cannot be observed however it can be estimated. The author uses numerical methods to show that maximum PWL is not exceptionally different from the one obtained in the linear case, i.e. $\alpha = 1$. As a result Corchon (2006) is unable to obtain significant WL by using a general class of inverse demand functions. And then the author turns his attention to free entry with a fixed cost.

Corchon (2006) states that one active firm is socially optimal since it yields more social welfare then no firms and economies of scale imply that one firm is optimal when fixed cost is small enough. Thus $PWL = \frac{W^0 - W^*}{W^0}$ where W^0 represents social welfare in the optimal allocation with one active firm and W^* represents welfare in Cournot equilibrium with free entry. As a result the author derives two formulas one of which represents maximal PWL and the other one of which represents minimal PWL depending on upper and lower bounds of fixed cost determined by free entry. Corchon (2006) is able to obtain substantially big PWL when there exists a large number of active firms in the market. In linear case, i.e. $\alpha = 1$, both maximal PWL and minimal PWL tend to zero as number of firms approaches infinity. When α approaches infinity PWL can be made arbitrarily close to 1, i.e. 100% welfare loss. This means that any given price-marginal cost margin, or elasticity of demand, is well-matched with any PWL.

Corchon (2006) considers non-identical firms in the last section. Firms differ by their marginal costs. PWL in this case depends not only on α but also on the market share of the largest firm and on the Hirschman-Herfindahl index, which is defined as the sum of market share squares. Corchon (2006) determines the market structure which maximizes PWL. For instance he shows that when $\alpha < 0$ the market structure that maximizes PWL is monopoly. He also proves that PWL is minimized when firms are identical. As a result Corchon (2006) shows that WL due to asymmetric firms can be quite large.

Corchon (2006) lists important causes of WL as product differentiation, investment, research and development (R&D), and location. However, he left these causes for future research since an analysis of such factors requires fairly complicated models.

Corchon's (2006) conclusion emphasizes the importance of economies of scale and asymmetric firms to calculate WL in real markets. He also determines that oligopoly model cannot be abandoned since he shows that Cournot equilibria yield quite large WL due to asymmetric firms. A fine aspect of this paper is that Corchon provides exact formula for PWL as a function of α and the market structure.

Anderson & Renault (2003) calculates PWL under the assumptions of Corchon (2006) assuming inverse demand function of the form $p = A - bx^{\alpha}$, where x and p represent aggregate output and market price respectively. However Anderson & Renault (2003) do not compare PWL between linear and non-liner models as Corchon (2006) does. Anderson & Renault (2003) consider the degree of curvature of market demand to evaluate the ratios of deadweight loss and consumer surplus to producer surplus under Cournot competition. They conclude that "the more concave is demand, the larger the share of producer surplus in overall surplus, the smaller is consumer surplus relative to producer surplus, and the lower the ratio of deadweight loss to producer surplus."

Anderson & Renault (2003) makes an important remark about socially optimal entry decisions. Of course, a firm's entry decision to a market depends on its profit, but according to Anderson & Renault (2003) the socially optimal entry decision depends on total surplus generated. When demand is very concave, entry is close to optimal. Firms capture almost the entire total surplus. A new entrant will not reduce price much and so its social value will be small. Even so, it may still earn considerable profit by the business stealing effect. However, when demand is convex, much of the surplus generated cannot be captured and entry decisions may be far from optimal.

Dixit (1979) presents a theoretical approach to entry barriers for new firms which are comparable in size to existing ones in the market. Existing firms choose their best strategy keeping in mind the reactions of possible entrants. Dixit (1979) does not consider free entry; instead he focuses on a duopoly with one established firm in the market and one prospective entrant. He examines the effects of product differentiation on entry. He concludes that a greater absolute advantage in demand or in cost for incumbents makes entry harder however lower cross-price effects with potential entrants' products make entry easier.

CHAPTER 3

HOMOGENEOUS GOOD MARKET WITH LINEAR DEMAND

3.1. Statement of the problem

It is established in this chapter that there are two kinds of entry biases. First, free entry may lead to excess entry relative to the socially optimal level. Second, free entry may lead to the wrong type of firms in the market compared to the socially optimal type of firms.

When non-identical firms introduced, the model becomes more sophisticated. In this study, it is assumed that there are two different types of cost functions. Therefore, the firms in the market have one of the two different types of cost functions. The free entry Cournot equilibrium number of firms is compared to the social optimum number of firms in order to investigate entry biases. It seems straight forward to generalize to more than two types of firms. However, a model with two types of firms suffices to make the point intended here.

The remainder of this chapter paper is organized as follows. Section 3.2 introduces the model with two types of firms and Section 3.3 establishes the Cournot equilibrium for this model. Sections 3.4 and 3.5 study respectively the social optimum solution and the free entry equilibrium for the model. Section 3.6 compares the solutions in Sections 3.4 and 3.5 and draws the key conclusions of the paper. Section 3.7 concludes chapter 3.

3.2. The Methods of Our Research

There are two types of firms producing a homogeneous product. The cost function of type 1 and type 2 firms are given by $C_1(q_1) = c_1 \mathrm{q}_1 + \mathrm{k}_1$ and $C_2(q_2) = c_2 \mathrm{q}_2 + \mathrm{k}_2$, respectively. Here, c_i and k_i are respectively the marginal cost and the fixed cost of type i firms. The inverse market demand function is $P(Q) = a - bQ$, where Q is the aggregate output. For convenience, we shall assume that $c_1 < c_2 < a$. That is, type 1 firms have a lower marginal cost than type 2 firms. The total number of firms in the market is $N_1 + N_2$, where N_1 and N_2 are the number of type 1 and type 2 firms, respectively.

3.3. Calculation of equilibrium outputs

In the Cournot equilibrium each firm's output is a best response to all other firms' choices of outputs. The best response of a type 1 firm is the solution of the problem:

$$Max_{q_1} [a - b(q_1 + Q_{-1})]q_1 - c_1 q_1 - k_1,$$

where Q_{-1} denotes the total output of all other type 1 firms and all type 2 firms. Solving this problem yields a type 1 firm's output best response function:

$$q_1 = \frac{a - c_1 - bQ_{-1}}{2b}. \tag{3.1}$$

Similarly, a type 2 firm's best response function is

$$q_2 = \frac{a - c_2 - bQ_{-2}}{2b}, \tag{3.2}$$

where Q_{-2} denotes the total output of all other type 2 firms and all type 1 firms.

The system of equations as represented by (3.1) and (3.2) consists of $N_1 + N_2$ equations. Applying symmetry to all type 1 firms and to all type 2 firms, we can solve for the Cournot (or Cournot-Nash) equilibrium output levels. Assuming that all firms produce a non-negative output level in equilibrium, the equilibrium output level for each type of firms is given by:

$$q_1^* = \frac{a + N_2 c_2 - (N_2 + 1)c_1}{(N_1 + N_2 + 1)b}, \tag{3.3}$$

$$q_2^* = \frac{a + N_1 c_1 - (N_1 + 1)c_2}{(N_1 + N_2 + 1)b}. \tag{3.4}$$

The total equilibrium output is

$$Q^* = N_1 q_1^* + N_2 q_2^* = \frac{(N_1 + N_2)a - N_1 c_1 - N_2 c_2}{(N_1 + N_2 + 1)b}. \tag{3.5}$$

It is straightforward to calculate the equilibrium profit for each type of firms. They are

$$\pi_1^* = b\left(q_1^*\right)^2 - k_1, \tag{3.6}$$

$$\pi_2^* = b\left(q_2^*\right)^2 - k_2. \tag{3.7}$$

It is worthwhile to point out that all these equilibrium values are functions of N_1 and N_2. It follows from (3.3)-(3.7) that in equilibrium each firm's profit is a decreasing function of its marginal cost, its fixed cost, and the number of firms of its type in the market.

3.4. The Social Optimum

In this section we seek to find the social optimal solution. Suppose there is a social planner who can control the number of firms that enter to the market but cannot control the behaviors of firms in the market. The social optimum is determined in two stages. In the first stage, the social planner selects the number of firms of each type. In the second stage, firms in the market compete in output via Cournot.[1] The solution is found by solving the two-stage problem backwards with the second stage solution the same as given in the previous section for any fixed configuration of firms.

The social planner's objective is to maximize the total welfare given by the sum of consumers' surplus and profits of all firms. Equivalently, the total welfare is the total consumer benefits subtracted by total production costs. Formally, the total welfare is a function of (N_1, N_2) and is given by

$$W(N_1, N_2) \equiv \int_0^{Q^*} P(s)ds - N_1(c_1 q_1^* + k_1) - N_2(c_2 q_2^* + k_2), \tag{3.8}$$

where q_1^*, q_2^* and Q^* are the second stage Cournot equilibrium output solutions given by (3.3)-(3.5), respectively. In the right-hand side of (3.8), the first term is the sum of consumer payoffs from consumption, the second term is the total production costs incurred by all type 1 firms, and the last term is the total production costs incurred by all type 2 firms.

[1] This implies that the social optimum here is a second best solution. The first best solution would be when the social planner also chose output levels for all firms.

Our first result is about the solution to the social planner's welfare maximization problem.[2] Let N_1^o denote the socially optimal number of firms when there is only type 1 firms in the market, and let N_2^o denote the socially optimal number of firms when there is only type 2 firms in the market.

Proposition 3.1. The maximization problem of the total welfare function $W(N_1, N_2)$ has a corner solution, which is either $(N_1^o, 0)$ or $(0, N_2^o)$.

Proof: From (3.3)-(3.5), we have

$$\frac{\partial q_1^*}{\partial N_1} = \frac{\partial q_2^*}{\partial N_1} = -\frac{q_1^*}{N_1 + N_2 + 1},$$

$$\frac{\partial q_1^*}{\partial N_2} = \frac{\partial q_2^*}{\partial N_2} = -\frac{q_2^*}{N_1 + N_2 + 1},$$

$$\frac{\partial Q^*}{\partial N_1} = \frac{q_1^*}{N_1 + N_2 + 1}, \quad \text{and}$$

$$\frac{\partial Q^*}{\partial N_2} = \frac{q_2^*}{N_1 + N_2 + 1}.$$

Substituting the demand equation into (3.8) and integrating gives

$$W(N_1, N_2) = aQ^* - \frac{b}{2}(Q^*)^2 - N_1(c_1 q_1^* + k_1) - N_2(c_2 q_2^* + k_2)$$

The partial derivative of W with respect to N_1 is

$$\begin{aligned}\frac{\partial W}{\partial N_1} &= (a - bQ^*)\frac{\partial Q^*}{\partial N_1} - (c_1 q_1^* + k_1) - N_1 c_1 \frac{\partial q_1^*}{\partial N_1} - N_2 c_2 \frac{\partial q_2^*}{\partial N_1} \\ &= (a - bQ^*)\frac{q_1^*}{N_1 + N_2 + 1} - (c_1 q_1^* + k_1) + N_1 c_1 \frac{q_1^*}{N_1 + N_2 + 1} + N_2 c_2 \frac{q_1^*}{N_1 + N_2 + 1}.\end{aligned}$$

[2] Given our assumptions about demand and cost functions, it is fairly easy to establish that the social planner's problem has a solution.

Rewriting gives

$$\frac{\partial W}{\partial N_1}=\frac{q_1^*}{N_1+N_2+1}\left[P^*-(N_1+N_2+1)AC_1^*+N_1c_1+N_2c_2\right], \tag{3.9}$$

where $P^*=a-bQ^*$ denotes the market price in the second stage Cournot equilibrium and $AC_1^* = (c_1q_1^*+k_1)/q_1^* = C_1\left(q_1^*\right)/q_1^*$ is the average cost for type 1 firms. Similarly, the partial derivative of W with respect to N_2 is

$$\frac{\partial W}{\partial N_2}=\frac{q_2^*}{N_1+N_2+1}\left[P^*-(N_1+N_2+1)AC_2^*+N_1c_1+N_2c_2\right], \tag{3.10}$$

where $AC_2^* = (c_2q_2^*+k_2)/q_2^* = C_2\left(q_2^*\right)/q_2^*$ is the average cost for type 2 firms.

Assuming an interior solution for the social optimal market configuration, it must satisfy the first-order conditions:

$$\frac{\partial W}{\partial N_1}=0 \quad \text{and} \quad \frac{\partial W}{\partial N_2}=0.$$

By utilizing the above expressions for the partial derivatives of W, these conditions become

$$\left(AC_1^*-c_1\right)N_1+\left(AC_1^*-c_2\right)N_2=P^*-AC_1^*, \quad \text{and}$$

$$\left(AC_2^*-c_1\right)N_1+\left(AC_2^*-c_2\right)N_2=P^*-AC_2^*.$$

Applying the Cramer's rule to this system of equations leads to

$$N_1=\frac{P^*-c_2}{c_2-c_1} \quad \text{and} \quad N_2=\frac{c_1-P^*}{c_2-c_1}.$$

It follows that $N_1+N_2=-1$ which is impossible in a valid solution for N_1 and N_2. Hence, the social planner's problem must have a corner solution. It is (N_1^o, 0) when the corner solution involves only type 1 firms and is (0, N_2^o) when the corner contains only type 2 firms.

Based on Proposition 3.1. the social optimal market configuration can be found by a direct comparison of the total welfare at the two corner points, (N_1^o, 0) and (0, N_2^o). We next proceed to find $W(N_1^o,0)$ and $W(0,N_2^o)$.

Substituting $N_2 = 0$ into (3.9) and simplifying gives

$$\frac{\partial W}{\partial N_1} = \frac{1}{(N_1+1)^3 b}\left[(a-c_1)^2 - (N_1+1)^3 k_1 b\right].$$

Solving the equation $\partial W / \partial N_1 = 0$ yields the optimal number of type 1 firms when there exist only type 1 firms in the market, is given by

$$N_1^o = \sqrt[3]{\frac{(a-c_1)^2}{k_1 b}} - 1. \tag{3.11}$$

Similarly, the socially optimal number of type 2 firms when there exist only type 2 firms in the market is given by[3]

$$N_2^o = \sqrt[3]{\frac{(a-c_2)^2}{k_2 b}} - 1. \tag{3.12}$$

Direct calculations lead to

$$W(N_1^o,0) = \frac{1}{2b}(a-c_1)^2 - \frac{3}{2b}(a-c_1)^{2/3}(k_1 b)^{2/3} + k_1, \text{ and} \tag{3.13}$$

$$W(0,N_2^o) = \frac{1}{2b}(a-c_2)^2 - \frac{3}{2b}(a-c_2)^{2/3}(k_2 b)^{2/3} + k_2. \tag{3.14}$$

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[3] The solution for the socially optimal number of firms as given by (3.11) or (3.12) matches the finding in Mankiw and Whinston (1986) for the case of identical firms in the market.

Comparing $W(N_1^o, 0)$ and $W(0, N_2^o)$ gives the socially optimal market composition. The result is summarized in the following proposition.

Proposition 3.2. The socially optimal solution consists of N_1^o type 1 firms if and only if

$$W(N_1^o, 0) > W(0, N_2^o),$$

and it consists of N_2^o type 2 firms if and only if

$$W(N_1^o, 0) < W(0, N_2^o).$$

3.5. The Free Entry Equilibrium

The free entry solution is also determined in two stages. In the first stage, firms of either type decide whether to enter the market. In the second stage, firms that entered in the first stage compete in output via Cournot. Similar to the socially optimal solution, the free entry solution is found by backward induction where the second stage solution is the same as the one found for any given market configuration determined in the first stage.

Let N_1^e denote the free entry number of firms when there exist only type 1 firms in the market, and let N_2^e denote the free entry number of firms when there exist only type 2 firms in the market.

Proposition 3.3. The free entry equilibrium solution, if it exists, is either $(N_1^e, 0)$ or $(0, N_2^e)$.

Proof: To find the free entry equilibrium number of firms we proceed as follows. Ignoring the integer problem, the free entry equilibrium market configuration (N_1, N_2) satisfies the following equations:

$$\pi_1^* = 0 \quad \text{and} \quad \pi_2^* = 0,$$

where π_1^* and π_2^* are given by (3.6) and (3.7), respectively. The equation $\pi_1^* = 0$ implies that $q_1^* = \sqrt{\frac{k_1}{b}}$. Applying (3.3) and simplifying gives

$$N_1 = \frac{a + N_2 c_2 - (N_2 + 1)c_1 - (N_2 + 1)\sqrt{k_1 b}}{\sqrt{k_1 b}}.$$

Similarly, the equation $\pi_2^* = 0$ implies

$$N_2 = \frac{a + N_1 c_1 - (N_1 + 1)c_2 - (N_1 + 1)\sqrt{k_2 b}}{\sqrt{k_2 b}}.$$

Solving the above system of equations in (N_1, N_2) gives

$$N_1 = \frac{a - c_2}{c_2 - c_1} \text{ and } N_2 = \frac{c_1 - a}{c_2 - c_1}.$$

It follows that $N_1 + N_2 = -1$ which is impossible in a valid solution. Hence, the free entry equilibrium, if it exists, must be a corner solution. It is either $(N_1^e, 0)$ in which case only type 1 firms exist in equilibrium or $(0, N_2^e)$ in which case only type 2 firms exist in equilibrium.

Proposition 3.3. establishes that the free entry equilibrium must be a corner solution. However, this proposition is silent on whether free entry equilibrium exists. In the following we provide sufficient conditions under which the free entry equilibrium involves type 1 (or type 2) firms only. These conditions, therefore, also ensure that free entry equilibrium exists when they are satisfied. Before proceeding, we first find the free entry equilibrium number of firms when it consists of either type 1 or type 2 firms.

Substituting $N_2 = 0$ into the equation $\pi_1^* = 0$ and solving gives

$$N_1^e = \frac{a - c_1}{\sqrt{k_1 b}} - 1. \tag{3.15}$$

N_1^e in (3.15) is the number of firms when only type 1 firms survive in a free entry equilibrium. Let D_1 denote the corresponding average cost for each type 1 firm. Direct calculation gives

$$\mathrm{D}_1 = \mathrm{c}_1 + \sqrt{\mathrm{k}_1 \mathrm{b}}\ . \tag{3.16}$$

Assumption 1: $\mathrm{D}_1 < \mathrm{a}$.

Assumption 1 says that if only type 1 firms exist in the free-entry equilibrium then each firm's average cost is less than the highest price consumers are willing to pay. It implies immediately that $N_1^e > 0$.

Similarly, substituting $N_1 = 0$ into the equation $\pi_2^* = 0$ and solving gives

$$N_2^e = \frac{a - c_2}{\sqrt{k_2 b}} - 1\ . \tag{3.17}$$

N_2^e in (3.17) is the number of firms when only type 2 firms survive in a free entry equilibrium. Let D_2 denote the corresponding average cost for each type 1 firm. Direct calculation gives

$$\mathrm{D}_2 = \mathrm{c}_2 + \sqrt{\mathrm{k}_2 \mathrm{b}}\ . \tag{3.18}$$

Assumption 2: $D_2 < a$.

Assumption 2 implies immediately that $N_2^e > 0$.

The next proposition shows that the relationship between D_1 and D_2 determines completely which type of firms is present in the free entry equilibrium.

Proposition 3.4. If $D_1 < D_2$ then there is a unique free entry equilibrium given by $(N_1^e, 0)$; if $D_1 > D_2$ then there is a unique free entry equilibrium given by $(0, N_2^e)$.

Proof: In a free entry equilibrium, we have $\pi_1^* = 0$ and $\pi_2^* = 0$, where π_1^* and π_2^* are given by (3.6) and (3.7) respectively. Using (3.3) and (3.4), the equation $\pi_1^* = 0$ is equivalent to

$$N_2 = N_1 \frac{\sqrt{k_1 b}}{c_2 - c_1 - \sqrt{k_1 b}} + \frac{c_1 + \sqrt{k_1 b} - a}{c_2 - c_1 - \sqrt{k_1 b}} .$$

Using the definition of D_1 in (3.16), this equation can be rewritten as

$$N_2 = N_1 \frac{c_1 - D_1}{D_1 - c_2} + \frac{D_1 - a}{D_1 - c_2} . \qquad (3.19)$$

Equation (3.19) indicates a linear relationship between N_1 and N_2, as depicted in Figure 1. Since $c_1 < D_1 < a$, this curve is negatively sloped if $D_1 > c_2$ and is positively sloped if $D_1 < c_2$. In both cases, the intercept on the N_1 axis is positive and is given by $(a - D_1)/(D_1 - c_1)$. On the left side of this curve, $\pi_1^* > 0$ and on the right side of this curve, $\pi_1^* < 0$. ■

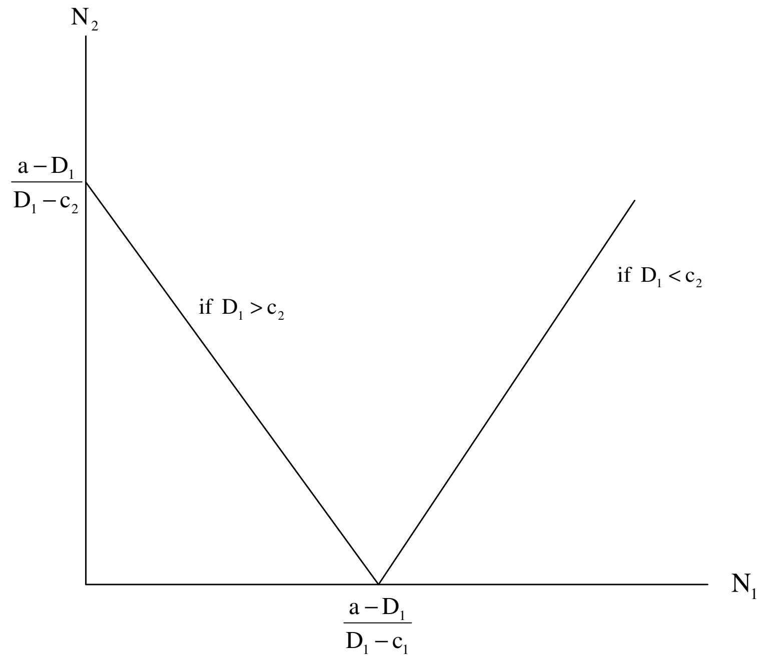

Figure 1: The zero-profit curve for type 1 firms

The equation $\pi_2^* = 0$ is equivalent to

$$N_2 = N_1 \frac{c_1 - c_2 - \sqrt{k_2 b}}{\sqrt{k_2 b}} + \frac{a - c_2 - \sqrt{k_2 b}}{\sqrt{k_2 b}}.$$

Using the definition of D_2 in (3.18), this equation can be rewritten as

$$N_2 = N_1 \frac{c_1 - D_2}{D_2 - c_2} + \frac{a - D_2}{D_2 - c_2}. \tag{3.20}$$

Equation (3.19) implies a linear relationship between N_1 and N_2, as depicted in Figure 2. This curve is negatively sloped since $c_1 < c_2 < D_2 < a$. Its intercept on the N_1 axis is given by $(a - D_2)/(D_2 - c_1)$ which is positive. On the left side of this curve, $\pi_2^* > 0$ and on the right side of this curve, $\pi_2^* < 0$.

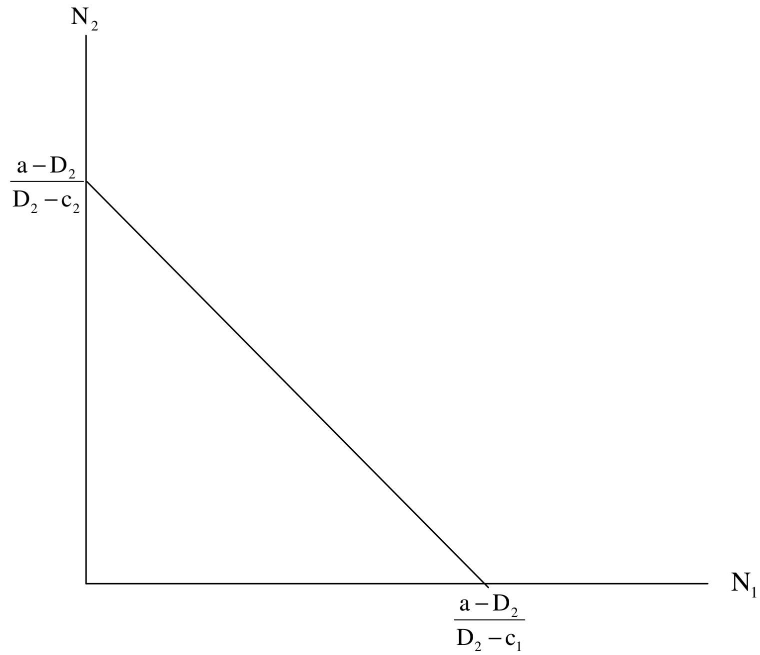

Figure 2: The zero-profit curve for type 2 firms

Consider first the case: $D_1 < D_2$. If $D_1 > c_2$, the two zero-profit curves are shown together in Figure 3. In this case, the curve $\pi_1^* = 0$ has greater intercepts on both axes than the curve $\pi_2^* = 0$. Since the two curves do not intersect, the free entry equilibrium must be on one of the two axes. Hence, the free entry equilibrium corresponds to either the point where the curve $\pi_1^* = 0$ intersects the N_1 axis, or to the point where the curve $\pi_2^* = 0$ intersects the N_2 axis. The latter cannot be equilibrium, since any point to the left curve $\pi_1^* = 0$ means $\pi_1^* > 0$ and type 1 firms would have an incentive to enter. At the point where the curve $\pi_1^* = 0$ intersects the N_1 axis, all type 1 firms have zero profits and no type 2 firm has an incentive to enter. Therefore, the free entry equilibrium corresponds to the point where the curve $\pi_1^* = 0$ intersects the N_1 axis. The equilibrium number of type 1 firms is given by $(a - D_1)/(D_1 - c_1)$, which is equal to N_1^e in (3.15). If $D_1 < c_2$ the analysis is similar and the free entry equilibrium still corresponds to the point where the curve $\pi_1^* = 0$ intersects the N_1 axis.

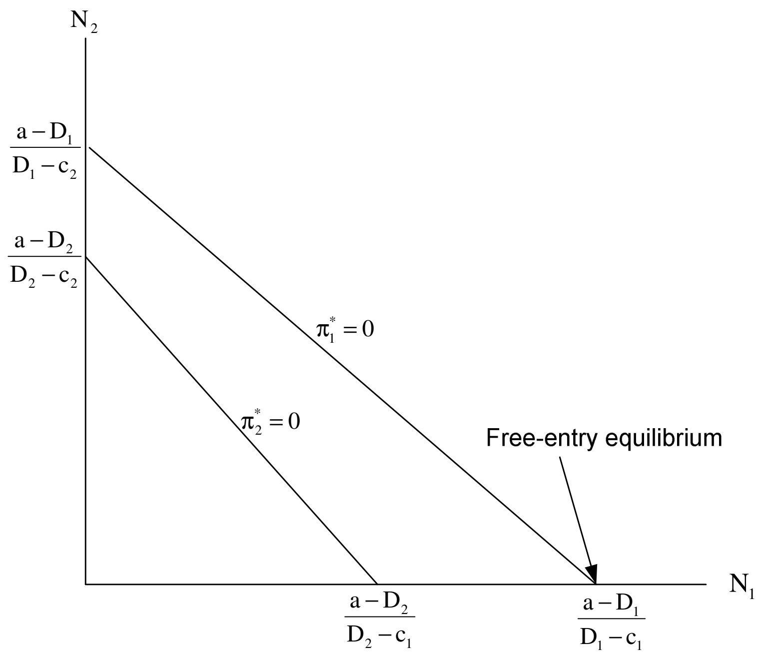

Figure 3: Free-entry equilibrium for the case: $D_2 > D_1 > c_2$

Next, consider the case: $D_1 > D_2$. The two zero-profit curves are shown together in Figure 4. In this case, the curve $\pi_2^* = 0$ has greater intercepts on both axes than the curve $\pi_1^* = 0$. By a similar reason as that given above, the free entry equilibrium corresponds to the point where the curve $\pi_2^* = 0$ intersects the N_2 axis. The equilibrium number of type 2 firms is given by $(a - D_2)/(D_2 - c_2)$, which is equal to N_2^e in (3.17).

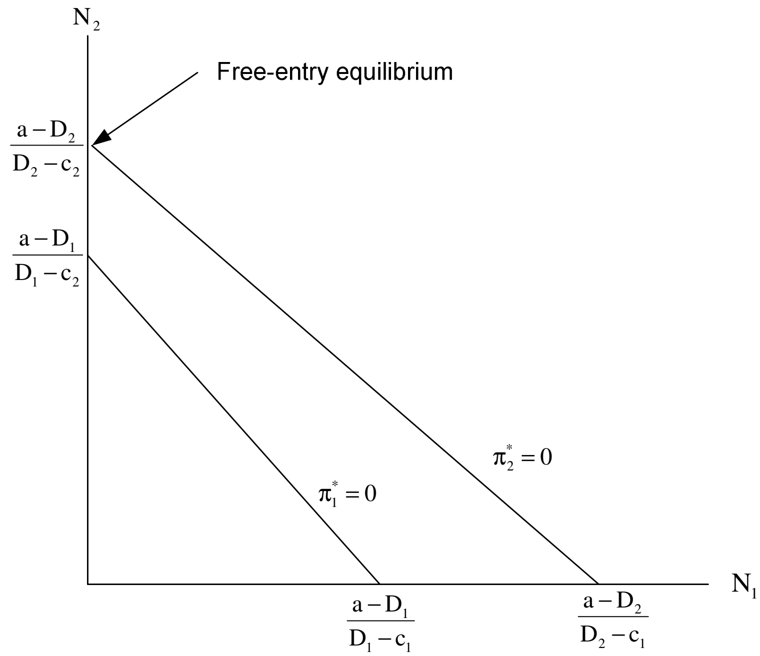

Figure 4: Free-entry equilibrium for the case: $D_2<D_1$

3.6. Entry Biases

In this section we compare the free entry equilibrium market configuration to the socially optimal market configuration. This comparison enables us to conclude that there are two kinds of free entry biases. First, free entry may lead to too many firms relative to the social optimum. Second, free entry may lead to the wrong type of firms in the market as required by the social optimum.

Consider first the case when only type 1 firms are present in both the free entry equilibrium and the social optimum. By (3.11) and (3.15), the socially optimal number and the free entry number of type 1 firms are $N_1^o=\sqrt[3]{\frac{(a-c_1)^2}{k_1 b}}-1$ and $N_1^e=\frac{a-c_1}{\sqrt{k_1 b}}-1$, respectively. It follows that $(N_1^o+1)^3=(N_1^e+1)^2$. Hence, $N_1^o<N_1^e$.

Next, consider the case when only type 2 firms are present in both the free entry equilibrium and the social optimum. By (3.12) and (3.17), the socially optimal number and the free entry number of type 2 firms are $N_2^o=\sqrt[3]{\frac{(a-c_2)^2}{k_2 b}}-1$ and $N_2^e=\frac{a-c_2}{\sqrt{k_2 b}}-1$, respectively. It follows that $(N_2^o+1)^3=(N_2^e+1)^2$. Hence, $N_2^o<N_2^e$. In both cases, we reach the conclusion that free entry leads to excessive entry relative to the social optimum, assuming that the same type of firms are in the market in both solutions.

Finally, we discuss the possibility that free entry and social optimum may involve different types of firms. By Propositions 3.2. and 3.4., the social optimum involves only type 1 firms while the free entry equilibrium involves only type 2 firms if the following conditions hold:

$$\frac{1}{2b}(a-c_1)^2-\frac{3}{2b}(a-c_1)^{2/3}(k_1 b)^{2/3}+k_1>\frac{1}{2b}(a-c_2)^2-\frac{3}{2b}(a-c_2)^{2/3}(k_2 b)^{2/3}+k_2,$$

$$D_2<D_1.$$

The social optimum involves only type 2 firms while the free entry equilibrium involves only type 1 firms if the following conditions hold:

$$\frac{1}{2b}(a-c_1)^2-\frac{3}{2b}(a-c_1)^{2/3}(k_1 b)^{2/3}+k_1<\frac{1}{2b}(a-c_2)^2-\frac{3}{2b}(a-c_2)^{2/3}(k_2 b)^{2/3}+k_2,$$

$$D_2>D_1.$$

As a numerical example, consider a = 1, b = 1, $c_1 = 0$, and $k_2 = 0$. Assumption 1 is satisfied if $k_1 < 1$. Assumption 2 is satisfied if $c_2 < 1$. In Figure 5, the top curve in the square box corresponds to the equation: $D_1 = D_2$, which is reduced to $c_2 = \sqrt{k_1}$. Points above this curve are where the free entry equilibrium has all type 1 firms and points below this curve are where the free entry equilibrium has all type 2 firms. The bottom curve in the square box corresponds to the equation: $W(N_1^o, 0) = W(0, N_2^o)$, which is $c_2 = 1 - \sqrt{1 - 3(k_1)^{2/3} + 2k_1}$. Points above this curve are where the social optimum has all type 1 firms and points below this curve are where the social optimum has all type 2 firms. Hence, in Figure 5, region A contains (k_1, c_2) combinations such both the free entry equilibrium and the social optimum involve type 1 firms only; in region C both the free entry equilibrium and the social optimum involve type 2 firms only; in region B the free entry equilibrium involves all type 2 firms but the social optimum involves type 1 firms only.

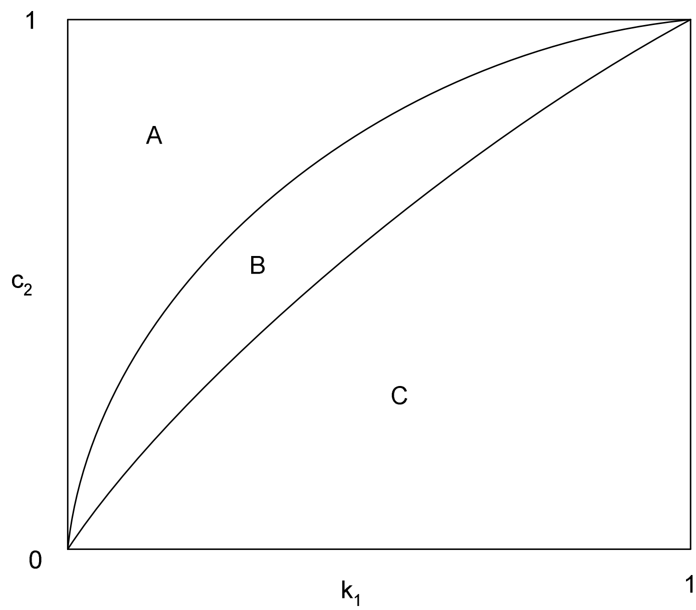

Figure 5: Free-entry equilibrium and social optimum

3.7. Conclusion

In this chapter, we have attempted to examine entry biases and the existence of free entry equilibrium in a Cournot market with two different types of firms, which have different marginal costs and different fixed costs. We have obtained three main results in homogeneous good market with linear demand.

First, we have proven that there is only one type of firms in the market for both the socially optimal solution and the free entry equilibrium. Second, we have shown that when there are the same type of firms in the market for both the social optimum and the free entry equilibrium, excessive entry exists. Moreover, we provide sufficient conditions for the existence of excessive entry regarding to each type of firms. Third, we present sufficient conditions such that there exist only type two firms in the free entry equilibrium but there exist only type one firms in the socially optimal solution. This result arises from the fact that type two firms have lower fixed cost than type one firms has.

CHAPTER 4

HOMOGENEOUS GOOD MARKET WITH NON-LINEAR DEMAND

4.1. Statement of the Problem

There are two different cost functions in a Cournot market with free entry. The firms in the market are divided into two types, each of which has one of the two cost functions. The objective is to investigate the number of free entry equilibrium firms and the number of social optimum firms when there are non-identical firms in a Cournot market with non-linear demand. The method is to compare the number of free entry equilibrium firms to the number of social optimum firms.

4.2. The methods of our research

The model is a two-stage game. In the first stage, there exist two types of firms and each firm must decide whether to enter to the Cournot market. The second stage is the production period. Each of the entrants has one of the two cost functions $C_1(q_1) = c_1 q_1 + k_1$ and $C_2(q_2) = c_2 q_2 + k_2$ such that $C_i(\cdot)$ is continuous and $C_i'(\cdot) \geq 0$, $C_i''(\cdot) \geq 0$ for $q_i \geq 0, i = 1,2$. Firms with the cost functions $C_1(q_1)$ and $C_2(q_2)$ are called type one firms and type two firms, respectively.

The market is a homogeneous product market with non-linear inverse demand function $P(Q) = a - bQ^{\alpha}$, where Q is the aggregate output and $P'(Q) < 0$ at all Q.

Moreover, we assume that $c_1 < c_2 < a$. The overall number of firms in the market is denoted by $N_1 + N_2$, where N_1 and N_2 are the number of type one and type two firms in the market, respectively. q_1^* and q_2^* are the equilibrium output per firm for the first type and second type firms in the market, respectively.

4.3. Calculation of equilibrium outputs

Profit functions are used to obtain the equilibrium outputs per firm as follows:

$$\pi_i = P(Q)q_i - C(q_i)$$

Differentiating $\pi_1 = \left(a - bQ^\alpha\right)q_1 - cq_1 - k_1$ with respect to q_1 gives

$$\frac{\partial \pi_1}{\partial q_1} = a - bQ^\alpha - b\alpha q_1 Q^{\alpha-1} - c_1 = 0$$

$$\Rightarrow Q^\alpha + \alpha q_1 Q^{\alpha-1} = \frac{a - c_1}{b}$$

Similarly,

$$Q^\alpha + \alpha q_2 Q^{\alpha-1} = \frac{a - c_2}{b}$$

As a result,

$$\left(N_1 q_1 + N_2 q_2\right)^\alpha + \alpha q_1 \left(N_1 q_1 + N_2 q_2\right)^{\alpha-1} = \frac{a - c_1}{b}. \tag{4.1}$$

$$\left(N_1 q_1 + N_2 q_2\right)^\alpha + \alpha q_2 \left(N_1 q_1 + N_2 q_2\right)^{\alpha-1} = \frac{a - c_2}{b}. \tag{4.2}$$

Subtracting equation (4.2) from equation (4.1) gives

$$\alpha(N_1q_1 + N_2q_2)^{\alpha-1}(q_1 - q_2) = \frac{c_2 - c_1}{b}. \tag{4.3}$$

Rearranging (4.1) and (4.2) gives

$$(N_1q_1 + N_2q_2)^{\alpha-1}(N_1q_1 + N_2q_2 + \alpha q_1) = \frac{a - c_1}{b}$$

$$(N_1q_1 + N_2q_2)^{\alpha-1}(N_1q_1 + N_2q_2 + \alpha q_2) = \frac{a - c_2}{b}.$$

Dividing these two equations produces

$$\frac{N_1q_1 + N_2q_2 + \alpha q_1}{N_1q_1 + N_2q_2 + \alpha q_2} = \frac{a - c_1}{a - c_2}.$$

After cross multiplication and rearranging terms yields

$$q_1 = \frac{(a - c_1)(N_2 + \alpha) - (a - c_2)N_2}{(a - c_2)(N_1 + \alpha) - (a - c_1)N_1} q_2. \tag{4.4}$$

Plugging (4.4) into equation (4.3) and then solving for q_2 gives the equilibrium output for type two firms as

$$q_2^* = \frac{(a - c_2)(N_1 + \alpha) - (a - c_1)N_1}{\alpha b^{1/\alpha}(N_1 + N_2 + \alpha)^{1/\alpha}[(a - c_1)N_1 + (a - c_2)N_2]^{(\alpha-1)/\alpha}}.$$

Equilibrium output for type one firms can be obtained by substituting q_2^* into equation (4.4) and it is

$$q_1^* = \frac{(a - c_1)(N_2 + \alpha) - (a - c_2)N_2}{\alpha b^{1/\alpha}(N_1 + N_2 + \alpha)^{1/\alpha}[(a - c_1)N_1 + (a - c_2)N_2]^{(\alpha-1)/\alpha}}.$$

4.4. Social Optimum

The social optimum number of firms in the market, which is denoted by N^*, is the pair (N_1, N_2) that solves the following maximization problem, which maximizes the total surplus in the market.

$$\max W(N_1, N_2) \equiv \int_0^{N_1\tilde{q}_1 + N_2\tilde{q}_2} P(s)ds - N_1(c_1\tilde{q}_1 + k_1) - N_2(c_2\tilde{q}_2 + k_2).$$

Proposition 4.1. Social optimum market equilibrium involves only one type of firms. In other words, the maximization problem of the wealth function $W(N_1, N_2)$ has a corner solution, which is either $(N_1^*, 0)$ or $(0, \mathrm{N}_2^*)$.

Proof: The maximization problem of the wealth function $W(N_1, N_2)$ can be solved as follows:

$$\begin{aligned}\max W(N_1, N_2) &\equiv \int_0^{N_1 q_1^* + N_2 q_2^*} P(s)ds - N_1(c_1 q_1^* + k_1) - N_2(c_2 q_2^* + k_2).\\ &= \int_0^{Q^*} (a - bs^\alpha)ds - N_1(c_1 q_1^* + k_1) - N_2(c_2 q_2^* + k_2)\\ &= aQ^* - \frac{b}{\alpha}(Q^*)^{\alpha+1} - N_1(c_1 q_1^* + k_1) - N_2(c_2 q_2^* + k_2)\end{aligned}$$

In the above, Q^* is given by

$$\begin{aligned}Q^* &= N_1 q_1^* + N_2 q_2^*\\ &= N_1 \frac{(a - c_1)(N_2 + \alpha) - (a - c_2)N_2}{\alpha b^{1/\alpha}(N_1 + N_2 + \alpha)^{1/\alpha}[(a - c_1)N_1 + (a - c_2)N_2]^{(\alpha-1)/\alpha}} +\\ &\quad N_2 \frac{(a - c_2)(N_1 + \alpha) - (a - c_1)N_1}{\alpha b^{1/\alpha}(N_1 + N_2 + \alpha)^{1/\alpha}[(a - c_1)N_1 + (a - c_2)N_2]^{(\alpha-1)/\alpha}}\\ &= \frac{\alpha N_1(a - c_1) + \alpha N_2(a - c_2)}{\alpha b^{1/\alpha}(N_1 + N_2 + \alpha)^{1/\alpha}[(a - c_1)N_1 + (a - c_2)N_2]^{(\alpha-1)/\alpha}}\end{aligned}$$

Differentiating Q^* with respect to N_1 gives

$$\frac{\partial Q^*}{\partial N_1} = \frac{\alpha(a-c_1)(N_2+\alpha)-(a-c_2)N_2}{(N_1+N_2+\alpha)\alpha b^{1/\alpha}(N_1+N_2+\alpha)^{1/\alpha}[(a-c_1)N_1+(a-c_2)N_2]^{(\alpha-1)/\alpha}}$$
$$= \frac{q_1^*}{(N_1+N_2+\alpha)}$$

Let us define $B = (N_1+N_2+\alpha)$.

Then, $\frac{\partial Q^*}{\partial N_1} = \frac{q_1^*}{B}$.

Similarly, it can be shown that $\frac{\partial Q^*}{\partial N_i} = \frac{q_i^*}{B}$, i=1,2.

Now, differentiating q_1^* with respect to N_1 gives

$$\frac{\partial q_1^*}{\partial N_1} = -q_1^* \frac{\alpha(a-c_1)N_1+(\alpha a-c_2-(\alpha-1)c_1)N_2+\alpha(\alpha-1)(a-c_1)}{\alpha(N_1+N_2+\alpha)[(a-c_1)N_1+(a-c_2)N_2]}$$

The following is obtained by simplifying the above expression

$$\frac{\partial q_1^*}{\partial N_1} = -\frac{(1+q_1^*(\alpha-1)Q^{*-1})q_1^*}{(\alpha+N_1+N_2)}$$

Let us define $\gamma_i = 1+q_i^*(\alpha-1)Q^{*-1}$. Then $\frac{\partial q_1^*}{\partial N_1} = -\frac{\gamma_1 q_1^*}{B}$.

Similarly, it can be shown that $\frac{\partial q_i^*}{\partial N_j} = -\frac{\gamma_i q_j^*}{B}$.

Differentiating welfare function with respect to N_j gives

$$\frac{\partial W}{\partial N_j} = P^* \frac{\partial Q^*}{\partial N_j} - (c_j q_j^* - k_j) - N_j c_j \frac{\partial q_j^*}{\partial N_j} - N_i c_i \frac{\partial q_i^*}{\partial N_j}$$
$$= P^* \frac{q_j^*}{B} - (c_j q_j^* - k_j) + N_j c_j \gamma_j \frac{q_j^*}{B} + N_i c_i \gamma_i \frac{q_j^*}{B}$$
$$= P^* \frac{q_j^*}{B} - \left(\frac{c_j q_j^* - k_j}{q_j^*}\right)\frac{q_j^*}{B}B + \frac{q_j^*}{B}(N_j c_j \gamma_j + N_i c_i \gamma_i)$$

Finally,

$$\frac{\partial W}{\partial N_j} = \frac{q_j^{\ *}}{B}\left[P^* - AC_j B + \sum_{i=1}^{2} N_i c_i \gamma_i\right]. \tag{4.5}$$

Here, AC_j denotes the average cost of type j firms in the Cournot Equilibrium. When the positive factor $q_j^{\ *}/B$ is ignored, all partial derivatives $\partial W/\partial N_j$ differ only in the term AC_j.

The necessary condition for welfare maximization is that $\frac{\partial W}{\partial N_i} = 0$ for i=1,2. This condition combined with equation (4.5) implies that the only way to have an interior solution is that average costs for type one and type two firms at the Cournot equilibrium should be the same. Since the latter is a probability zero event, the conclusion is that the optimal market structure entails only one type of firms. Please see Section 4.6 where average cost functions at Cournot equilibrium are examined and it is shown that $AC_1^{\ *} = AC_2^{\ *}$ has measure zero.

4.5. The Condition that Social Optimum Number of Firms Satisfies

Now, let us derive a condition that is satisfied by socially optimal number of firms. Substituting $N_2 = 0$ into the equation $\frac{\partial W}{\partial N_1} = 0$ yields the optimal number of type 1 firms when there exist only type 1 firms in the market.

$$\frac{\partial W}{\partial N_j}\Big|_{N_2=0} = \frac{q_j^*}{B}\left[P^* - AC_j B + \sum_{i=1}^{2} N_i c_i \gamma_i\right]\Big|_{N_2=0}$$

$$= \frac{q_j^*}{B}\{a - b\left(\frac{N_1(a-c_1)}{b^{1/\alpha}(N_1+\alpha)^{1/\alpha}(a-c_1)^{(\alpha-1)/\alpha} N_1^{(\alpha-1)/\alpha}}\right)^{\alpha}$$

$$-(N_1+\alpha)c_1 - \frac{(N_1+\alpha)k_1 b^{1/\alpha}(N_1+\alpha)^{1/\alpha}(a-c_1)^{(\alpha-1)/\alpha} N_1^{(\alpha-1)/\alpha}}{(a-c_1)}$$

$$+ N_1 c_1\left(1 + \frac{(\alpha-1)(a-c_1)b^{1/\alpha}(N_1+\alpha)^{1/\alpha}(a-c_1)^{(\alpha-1)/\alpha} N_1^{(\alpha-1)/\alpha}}{b^{1/\alpha}(N_1+\alpha)^{1/\alpha}(a-c_1)^{(\alpha-1)/\alpha} N_1^{(\alpha-1)/\alpha} N_1(a-c_1)}\right)\}$$

Since $\frac{q_j^*}{B} \neq 0$, the other factor is set to be equal to zero. Simplifying and rearranging terms give the following:

$$(a-c_1)\frac{\alpha}{k_1}\left(\frac{(a-c_1)}{b}\right)^{1/\alpha} = N_1^{(\alpha-1)/\alpha}(N_1+\alpha)^{(2\alpha+1)/\alpha}.$$

Therefore, the socially optimal number N_1^* of type 1 firms when there exists only type 1 firms in the market has to satisfy the following condition:

$$\frac{(a-c_1)^{\alpha+1}}{b}\left(\frac{\alpha}{k_1}\right)^{\alpha} = \left(N_1^*\right)^{\alpha-1}\left(N_1^*+\alpha\right)^{2\alpha+1}$$

Proposition 4.2. The free entry equilibrium solution, if exists, involves only one type of firms. In other words, free entry equilibrium is a corner solution, which is either $(N_1^0, 0)$ or $(0, \mathrm{N}_2^0)$.

Proof: The free entry market configuration satisfies the following equations:

$\pi_1^* = 0$ and $\pi_2^* = 0$

$\pi_1^* = P(Q^*)q_1^* - C(q_1^*) = 0$ implies that $P(Q^*) = \frac{C(q_1^*)}{q_1^*} = AC_1^*$, where AC_1^* represents average cost of type 1 firms in Cournot equilibrium.

Similarly, $\pi_2^* = P(Q^*)q_2^* - C(q_2^*) = 0$ implies that $P(Q^*) = \frac{C(q_2^*)}{q_2^*} = AC_2^*$, where AC_2^* represents average cost of type 2 firms in Cournot equilibrium. Therefore, $\pi_1^* = \pi_2^* = 0$ requires that $AC_1^* = AC_2^*$, which is a probability zero event given that there are two **<u>different</u>** types of firms in the market. Please refer to Section 4.6 where average cost functions at Cournot equilibrium are examined and it is shown that $AC_1^* = AC_2^*$ has measure zero.

4.6. Examining Average Cost Functions at Cournot Equilibrium

To examine whether it is possible to have the same average cost at Cournot equilibrium for two different types of firms, set average cost functions equal as follows

$$AC_1 = \frac{c_1 q_1^* + k_1}{q_1^*} = \frac{c_2 q_2^* + k_2}{q_2^*} = AC_2$$

$$\Rightarrow c_1 + \frac{k_1}{q_1^*} = c_2 + \frac{k_2}{q_2^*}$$

$$\Rightarrow \frac{k_1}{q_1^*} = c_2 - c_1 + \frac{k_2}{q_2^*}$$

$$\Rightarrow k_1 = \left(c_2 - c_1 + \frac{k_2}{q_2^*} \right) q_1^*$$

One can easily observe that the equation $AC_1 = AC_2$ is linear in terms of k_1 and k_2 since q_1^* and q_2^* do not include the fixed costs k_1 or k_2 in them. Thus, the equation $AC_1 = AC_2$ can be represented by a straight line in coordinate plane k_1, k_2. As a result the equation $AC_1 = AC_2$ has measure zero in two dimensional k_1, k_2 space for any parameters c_1, c_2. The conclusion is that average cost functions cannot be equal to each other at Cournot equilibrium when there are two different types of firms in the market. Note that k_1, k_2 & c_1, c_2 are parameters involved in cost functions to define the two different types of firms. Firms are different as long as these parameters do not coincide with each other. Since $AC_1 = AC_2$ is a measure zero event it is not possible to find $c_1 \neq c_2$ and $k_1 \neq k_2$ such that average cost functions are equal to each other at Cournot equilibrium for two different types of firms.

4.7. The Condition that Free Entry Equilibrium Number of Firms Satisfies

Zero profit condition is used to derive the condition that is satisfied by the free entry equilibrium number of firms.

$$\pi_1^* = P(Q^*)q_1^* - C(q_1^*) = 0$$

$$\Rightarrow \left(a - bQ^{*^{\alpha}} \right) q_1^* - c_1 q_1^* - k_1 = 0$$

$$\Rightarrow \left(a - b\left[\frac{\alpha N_1 (a - c_1) + \alpha N_2 (a - c_2)}{\alpha b^{1/\alpha} (N_1 + N_2 + \alpha)^{1/\alpha} \left[(a - c_1) N_1 + (a - c_2) N_2 \right]^{(\alpha - 1)/\alpha}} \right]^{\alpha} - c_1 \right) \times$$
$$\frac{(a - c_1)(N_2 + \alpha) - (a - c_2) N_2}{\alpha b^{1/\alpha} (N_1 + N_2 + \alpha)^{1/\alpha} \left[(a - c_1) N_1 + (a - c_2) N_2 \right]^{(\alpha - 1)/\alpha}} - k_1 = 0$$

It is shown that free entry equilibrium is a corner solution. Therefore, only one type of firms exist in free entry equilibrium. Without loss of generality, it can be assumed that free entry involves only type one firms. As a result, substituting $N_2 = 0$ into the zero profit condition above gives the condition that free entry equilibrium number of firms satisfies:

$$\left(a - b\left[\frac{\alpha N_1 (a - c_1)}{\alpha b^{1/\alpha} (N_1 + \alpha)^{1/\alpha} [(a - c_1) N_1]^{(\alpha-1)/\alpha}} \right]^{\alpha} - c_1 \right) \frac{(a - c_1)\alpha}{\alpha b^{1/\alpha} (N_1 + \alpha)^{1/\alpha} [(a - c_1) N_1]^{(\alpha-1)/\alpha}} - k_1 = 0$$

Simplifying gives

$$\Rightarrow \frac{\alpha (a - c_1)}{(N_1 + \alpha)} \frac{(a - c_1)^{1/\alpha}}{b^{1/\alpha} (N_1 + \alpha)^{1/\alpha} [N_1]^{(\alpha-1)/\alpha}} - k_1 = 0$$

As a result, the condition that is satisfied by the free entry equilibrium number of firms is obtained as follows under the assumption that there is only type one firms in the market:

$$\frac{\alpha^{\alpha} (a - c_1)^{\alpha+1}}{b k_1^{\alpha}} = \left(N_1^{0} + \alpha \right)^{(\alpha+1)} \left(N_1^{0} \right)^{(\alpha-1)}.$$

When this condition is compared to the following condition that social optimum number of firms satisfies,

$$\frac{(a - c_1)^{\alpha+1}}{b} \left(\frac{\alpha}{k_1} \right)^{\alpha} = \left(N_1^{*} \right)^{\alpha-1} \left(N_1^{*} + \alpha \right)^{2\alpha+1}$$

it is obvious that $N_1^{0} > N_1^{*}$, which means that there is excessive entry when the same type of firms exist in the market both in free entry equilibrium and social optimum.

Proposition 4.3. The level of entry is excessive when there is the same type of firms in the market in both social optimum and free entry equilibrium.

4.8. Conclusion

In this chapter, a homogeneous product market with non-linear inverse demand function $P(Q) = a - bQ^{\alpha}$ is examined. There are two types of firms in the market. It is shown that both free entry equilibrium and social optimum are corner solutions that only one type firms exist. The main conclusion is that when there is the same type of firms in free entry equilibrium as well as in social optimum, the entry is excessive.

CHAPTER 5

NON-HOMOGENEOUS GOOD MARKET WITH IDENTICAL FIRMS

5.1. Statement of the Problem

This chapter contains a non-homogenous goods market assuming that the firms in the market are identical. The main interest is a Cournot market with free entry when the firms in the market are producing a different good, i.e. there exist two differentiated goods, q_1 and q_2, in the market. We would like to investigate the number of free entry equilibrium firms and social optimum firms under free entry when there is non-homogeneous goods in the market. Therefore, we hope to contribute to the literature in terms of examination of entry biases in a Cournot market with free entry and non-homogenous goods.

5.2. **The methods of our research**

The model is a two-stage game. In the first stage, each firm must decide whether to enter to the Cournot market. The second stage is the production period. Each of the entrants has identical cost function $C(q) = c\mathbf{q} + \mathbf{k}$ such that $C(\cdot)$ is continuous and $\mathbf{C}'(\cdot) \geq 0$, $\mathbf{C}''(\cdot) \geq \mathbf{0}$ **for** $q_i \geq \mathbf{0}, \mathbf{i} = \mathbf{1,2}$.

The market is a non-homogeneous product market such that representative consumer's utility is a symmetric quadratic function of the two products q_1 and q_2:

$U = \alpha(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right)$ where the parameter γ measures the degree of product differentiation such that $0 \le \gamma \le 1$. In addition, $\gamma = 0$ and $\gamma = 1$ correspond to independent goods (the maximum degree of differentiation) and homogeneous goods (the minimum degree of differentiation), respectively. Note that $0 \le \gamma < 1$ in this chapter since the market is a non-homogeneous goods market.

The following linear system of inverse demand functions are generated by this utility function when maximized with respect to budget constraint:

$$p_1 = \alpha - q_1 - \gamma q_2$$

$$p_2 = \alpha - q_2 - \gamma q_1$$

5.3. Calculation of equilibrium outputs

First, let us assume that there are only two firms in a non-homogeneous product market containing two differentiated products. Firm i is producing good i, i=1,2. Under Cournot competition, firm 1's objective is to maximize its profits with respect to q_1:

$$\max_{q_1}\left[p_1 q_1 - C(q_1)\right]$$

$$\Rightarrow \max_{q_1}\left[(\alpha - q_1 - \gamma q_2)q_1 - cq_1 - k\right]$$

First order condition determines the following best response functions:

$$\alpha - q_1 - \gamma q_2 - q_1 - c = 0$$

$q_1 = \dfrac{\alpha - \gamma q_2 - c}{2}$ and similarly $q_2 = \dfrac{\alpha - \gamma q_1 - c}{2}$.

The equilibrium outputs $\overset{\circ}{q}_1$ **and** $\overset{\circ}{q}_2$ are obtained from the best response functions:

$\overset{\circ}{q}_1 = \dfrac{\alpha - c}{2+\gamma}$, and similarly $\overset{\circ}{q}_2 = \dfrac{\alpha - c}{2+\gamma}$

Calculation of equilibrium outputs can be generalized when the number of firms in the market are more than two. The overall number of firms in the market are denoted by $N_1 + N_2$, where N_1 and N_2 are the number of type one and type two firms in the market, respectively. In this case, demand functions turn to be the following linear system:

$$p_1 = \alpha - \sum_{i=1}^{N_1} q_{1i} - \gamma \sum_{j=1}^{N_2} q_{2j}$$

$$p_2 = \alpha - \sum_{j=1}^{N_2} q_{2j} - \gamma \sum_{i=1}^{N_1} q_{1i}$$

where q_{1i} and q_{2j} represent output of firm i from type 1 and output of firm j from type 2, respectively.

The optimization problem of type one firms and type two firms are solved to find equilibrium outputs q_1^* and q_2^*. Firm i from type one maximizes its profits as follows:

$$\max_{q_{1i}} p_1 q_{1i} - C(q_{1i})$$

$$\Rightarrow \max_{q_{1i}} \left(\alpha - \sum_{t=1}^{N_1} q_{1t} - \gamma \sum_{j=1}^{N_2} q_{2j} \right) q_{1i} - c q_{1i} - k$$

First order conditions:

$$\alpha - \sum_{t=1}^{N_1} q_{1t} - \gamma \sum_{j=1}^{N_2} q_{2j} - q_{1i} - c = 0$$

$$\Rightarrow q_{1i} = \frac{1}{2}(\alpha - c) - \frac{\gamma}{2} \sum_{j=1}^{N_2} q_{2j} - \frac{1}{2} \sum_{\substack{t=1 \\ t \neq i}}^{N_1} q_{1t} - \gamma - c = 0 , \quad i = 1,2,3,\ldots, N_1 .$$

There are N_1 equations for type one firms. Similarly, there are N_2 equations corresponding to type two firms:

$$q_{2j} = \frac{1}{2}(\alpha - c) - \frac{\gamma}{2} \sum_{i=1}^{N_1} q_{1i} - \frac{1}{2} \sum_{\substack{t=1 \\ t \neq j}}^{N_2} q_{2t} - \gamma - c = 0 , \quad j = 1,2,3,\ldots, N_2$$

As a result, $N_1 + N_2$ equations are obtained from maximization problem.

Because of symmetry, in equilibrium $q_{1i} = q_1$ for $i = 1,2,3,\ldots, N_1$ and $q_{2j} = q_2$ for $j = 1,2,3,\ldots, N_2$.

First order conditions are reduced to the following system of equations:

$$q_1 = \frac{1}{2}(\alpha - c) - \frac{\gamma}{2} N_2 q_2 - \frac{N_1 - 1}{2} q_1$$

$$q_2 = \frac{1}{2}(\alpha - c) - \frac{\gamma}{2} N_1 q_1 - \frac{N_2 - 1}{2} q_2$$

By rearranging terms,

$$(N_1 + 1) q_1 = \alpha - c - \gamma N_2 q_2$$

$$(N_2+1)q_2 = \alpha - c - \gamma N_1 q_1$$

Equilibrium outputs q_1^* and q_2^* are solved from the two equations above.

$$q_1^* = \frac{(N_2+1-\gamma N_2)(\alpha - c)}{(N_1+1)(N_2+1)-\gamma^2 N_1 N_2}$$

$$q_2^* = \frac{(N_1+1-\gamma N_1)(\alpha - c)}{(N_1+1)(N_2+1)-\gamma^2 N_1 N_2}$$

Note that $0 \le c < \alpha$ in order to avoid the trivial case in which neither firm has an incentive to produce.

Equilibrium outputs q_1^* and q_2^* are consistent with $\mathring{q}_1$ **and** $\mathring{q}_2$ that are obtained for the case $N_1 = N_2 = 1$ earlier in this section.

5.4. **The Free Entry Equilibrium**

Let us denote the free entry equilibrium number of firms in the market by (N_1^e, N_2^e).

Proposition 5.1. Free entry equilibrium number of type one firms is equal to free entry equilibrium number of type two firms in the market:

$$N_2^e = N_1^e = N^e \Rightarrow (N_1^e, N_2^e) = (N^e, N^e)$$

Proof: Let π_1^* and π_2^* represent equilibrium profits. Under free entry $\pi_1^* = \pi_2^* = 0$.

$$\pi_1^* = p_1^* q_1^* - C(q_1^*)$$

$$\Rightarrow \pi_1^* = (\alpha - N_1 q_1^* - \gamma N_2 q_2^*)q_1^* - c q_1^* - k$$

$$\Rightarrow \pi_1^* = \left(\alpha - c - N_1 q_1^* - \gamma N_2 q_2^*\right) q_1^* - k = 0$$

By substituting q_1^* and q_2^*,

$$\left(\alpha - c - N_1 \frac{(N_2 + 1 - \gamma N_2)(\alpha - c)}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2} - \gamma N_2 \frac{(N_1 + 1 - \gamma N_1)(\alpha - c)}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2}\right) \frac{(N_2 + 1 - \gamma N_2)(\alpha - c)}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2} = k$$

$$\Rightarrow k = \frac{(N_2 + 1 - \gamma N_2)(\alpha - c)^2}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2} \left(1 - \frac{N_1 (N_2 + 1 - \gamma N_2)}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2} - \frac{\gamma N_2 (N_1 + 1 - \gamma N_1)}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2}\right)$$

Simplifying and rearranging gives

$$k = \frac{(N_2 + 1 - \gamma N_2)^2 (\alpha - c)^2}{\left((N_1+1)(N_2+1) - \gamma^2 N_1 N_2\right)^2}.$$

Similarly, $k = \dfrac{(N_1 + 1 - \gamma N_1)^2 (\alpha - c)^2}{\left((N_1+1)(N_2+1) - \gamma^2 N_1 N_2\right)^2}$ is obtained from $\pi_2^* = 0$

Consequently, the following equations are obtained from $\pi_1^* = \pi_2^* = 0$:

$$k = \left\{\frac{(N_2 + 1 - \gamma N_2)(\alpha - c)}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2}\right\}^2$$

$$k = \left\{\frac{(N_1 + 1 - \gamma N_1)(\alpha - c)}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2}\right\}^2$$

Therefore,

$$\frac{\left(N_2^e + 1 - \gamma N_2^e\right)(\alpha - c)}{\left(N_1^e + 1\right)\left(N_2^e + 1\right) - \gamma^2 N_1^e N_2^e} = \sqrt{k} = \frac{\left(N_1^e + 1 - \gamma N_1^e\right)(\alpha - c)}{\left(N_1^e + 1\right)\left(N_2^e + 1\right) - \gamma^2 N_1^e N_2^e}$$

$$\Rightarrow \left(N_2^e + 1 - \gamma N_2^e\right)(\alpha - c) = \left(N_1^e + 1 - \gamma N_1^e\right)(\alpha - c)$$

As a result, $N_2^e = N_1^e$. Hence, free entry equilibrium number of type one firms is equal to free entry equilibrium number of type two firms in the market: $N_2^e = N_1^e = N^e$

■

Next, let us calculate the free entry equilibrium number of firms N^e. The condition that N^e must satisfy to be the free entry equilibrium number of firms is the zero profit condition:

$$\pi_2^* = 0 \Rightarrow \frac{\left(N^e + 1 - \gamma N^e\right)(\alpha - c)}{\left(N^e + 1\right)\left(N^e + 1\right) - \gamma^2 \left(N^e\right)^2} - \sqrt{k} = 0$$

$$\Rightarrow \frac{\left(N^e + 1 - \gamma N^e\right)(\alpha - c)}{\left(N^e + 1\right)^2 - \gamma^2 \left(N^e\right)^2} = \sqrt{k}$$

Solving for N^e gives $N^e = \dfrac{\alpha - c - \sqrt{k}}{(1+\gamma)\sqrt{k}}$

As a result, free entry equilibrium number of firms is

$$\left(N_1^{\,e}, N_2^{\,e}\right) = \left(N^e, N^e\right) = \left(\frac{\alpha - c - \sqrt{k}}{(1+\gamma)\sqrt{k}}, \frac{\alpha - c - \sqrt{k}}{(1+\gamma)\sqrt{k}}\right).$$

5.5. **The Social Optimum**

We consider a welfare maximizing social planner who can control the number of firms that enter to the market but cannot control the behaviors of firms in the market. Hypothetically, the objective of the planner is to choose the welfare maximizing number of firms, which is also called the number of social optimum firms and denoted by $\left(N_1^{\,*}, N_2^{\,*}\right)$.

The number of social optimum firms in the market is the pair $\left(N_1^*, N_2^*\right)$ that maximizes total surplus in the market. Let us define $W\left(N_1, N_2\right)$ as welfare function to represent total surplus. From the utility function $W\left(N_1, N_2\right)$ can be written as follows:

$$W\left(N_1, N_2\right) = (\alpha - c)\left(N_1 q_1^* + N_2 q_2^*\right) - \left[\frac{1}{2}\left(N_1 q_1^* + N_2 q_2^*\right)^2 - (1-\gamma) N_1 q_1^* N_2 q_2^*\right] - \left(N_1 + N_2\right)k$$

Proposition 5.2. Social optimum number of type one firms is equal to social optimum number of type two firms in the market:

$$N_2^* = N_1^* = N^* \Rightarrow \left(N_1^*, N_2^*\right) = \left(N^*, N^*\right)$$

Proof: The maximization problem of the wealth function $W\left(N_1, N_2\right)$ can be solved as follows to obtain the solution $\left(N_1^*, N_2^*\right)$:

$$\begin{aligned} &\max_{N_1, N_2} W\left(N_1, N_2\right) \\ &= \max_{N_1, N_2} \left\{ (\alpha - c)\left(N_1 q_1^* + N_2 q_2^*\right) - \left[\frac{1}{2}\left(N_1 q_1^* + N_2 q_2^*\right)^2 - (1-\gamma) N_1 q_1^* N_2 q_2^*\right] - \left(N_1 + N_2\right)k \right\} \end{aligned}$$

Let us consider the first order condition with respect to N_1:

$$\begin{aligned} &(\alpha - c) q_1^* + (\alpha - c) N_1 \frac{\partial q_1^*}{\partial N_1} + (\alpha - c) N_2 \frac{\partial q_2^*}{\partial N_1} - \left(N_1 q_1^* + N_2 q_2^*\right)\left(q_1^* + N_1 \frac{\partial q_1^*}{\partial N_1} + N_2 \frac{\partial q_2^*}{\partial N_1}\right) \\ &+ (1-\gamma) q_1^* N_2 q_2^* + (1-\gamma) N_1 \frac{\partial q_1^*}{\partial N_1} N_2 q_2^* + (1-\gamma) N_1 q_1^* N_2 \frac{\partial q_2^*}{\partial N_1} - k = 0 \end{aligned} \quad .(5.1)$$

$\frac{\partial q_1^*}{\partial N_1}$ and $\frac{\partial q_2^*}{\partial N_1}$ are calculated first.

$$\frac{\partial q_1^*}{\partial N_1} = \frac{-(\alpha - c)[(1-\gamma)N_2 + 1][(1-\gamma^2)N_2 + 1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2}$$

$$\frac{\partial q_2^*}{\partial N_1} = \frac{(\alpha - c)(1-\gamma)[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2] - (\alpha - c)[(1-\gamma)N_1 + 1][(1-\gamma^2)N_2 + 1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2}$$

Rewriting and simplifying gives

$$\frac{\partial q_2^*}{\partial N_1} = \frac{(\alpha - c)\gamma(\gamma N_2 - N_2 - 1)}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2}.$$

Rearranging (5.1) gives

$$\left[(\alpha - c) - \gamma N_2 q_2^* - \gamma N_1 N_2 \frac{\partial q_2^*}{\partial N_1}\right] q_1^* + \left[(\alpha - c)N_1 - \gamma N_1 N_2 q_2^* - N_1^2 q_1^*\right]\frac{\partial q_1^*}{\partial N_1}$$
$$+ \left[(\alpha - c)N_2 - N_2^2 q_2^*\right]\frac{\partial q_2^*}{\partial N_1} - N_1 q_1^{*2} - k = 0 \qquad (5.2)$$

It makes calculations easier if (5.2) is simplified in pieces. Let us split (5.2) into three pieces, simplify each piece separately and then put the pieces together:

$$\left[(\alpha - c) - \gamma N_2 q_2^* - \gamma N_1 N_2 \frac{\partial q_2^*}{\partial N_1}\right] q_1^* \Rightarrow \text{piece 1}$$

$$\left[(\alpha - c)N_1 - \gamma N_1 N_2 q_2^* - N_1^2 q_1^*\right]\frac{\partial q_1^*}{\partial N_1} \Rightarrow \text{piece 2}$$

$$\left[(\alpha - c)N_2 - N_2^2 q_2^*\right]\frac{\partial q_2^*}{\partial N_1} - N_1 q_1^{*2} - k \Rightarrow \text{piece 3}$$

First, simplifying piece 1:

$$\left[(\alpha-c)-\gamma N_2 q_2^{*}-\gamma N_1 N_2\frac{\partial q_2^{*}}{\partial N_1}\right]q_1^{*}$$

$$=\left[(\alpha-c)-\gamma N_2\frac{(\alpha-c)[(1-\gamma)N_1+1]}{(N_1+1)(N_2+1)-\gamma^2 N_1N_2}-\frac{\gamma^2 N_1N_2(\alpha-c)(\gamma N_2-N_2-1)}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^2}\right]q_1^{*}$$

$$=\frac{(\alpha-c)^2[(1-\gamma)N_2+1]}{(N_1+1)(N_2+1)-\gamma^2N_1N_2}-\frac{\gamma N_2(\alpha-c)^2[(1-\gamma)N_1+1][(1-\gamma)N_2+1]}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^2}$$
$$-\frac{\gamma^2N_1N_2(\alpha-c)^2[(1-\gamma)N_2+1][(\gamma-1)N_2-1]}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^3}$$

Second, piece 2 is simplified as follows:

$$\left[(\alpha-c)N_1-\gamma N_1N_2q_2^{*}-N_1^{2}q_1^{*}\right]\frac{\partial q_1^{*}}{\partial N_1}$$

$$=\frac{-(\alpha-c)^2N_1[(1-\gamma)N_2+1]\left[(1-\gamma^2)N_2+1\right]}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^2}+\frac{\gamma N_1N_2(\alpha-c)^2[(1-\gamma)N_1+1][(1-\gamma)N_2+1]\left[(1-\gamma^2)N_2+1\right]}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^3}$$
$$+\frac{N_1^{2}(\alpha-c)^2[(1-\gamma)N_2+1]^2\left[(1-\gamma^2)N_2+1\right]}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^3}$$

$$=\frac{-(\alpha-c)^2N_1[(1-\gamma)N_2+1]\left[(1-\gamma^2)N_2+1\right]}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^2}+\frac{N_1(\alpha-c)^2\left[(1-\gamma^2)N_2+1\right][(1-\gamma)N_2+1]\left[(1-\gamma^2)N_1N_2+\gamma N_2+N_1\right]}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^3}$$

Lastly, simplifying piece 3:

$$\left[(\alpha-c)N_2-N_2^{2}q_2^{*}\right]\frac{\partial q_2^{*}}{\partial N_1}-N_1q_1^{*2}-k$$

$$=\left[(\alpha-c)N_2-\frac{N_2^{2}(\alpha-c)[(1-\gamma)N_1+1]}{(N_1+1)(N_2+1)-\gamma^2N_1N_2}\right]\frac{(\alpha-c)\gamma[(\gamma-1)N_2-1]}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^2}$$
$$-\frac{N_1(\alpha-c)^2[(1-\gamma)N_2+1]^2}{\left[(N_1+1)(N_2+1)-\gamma^2N_1N_2\right]^2}-k$$

$$= \frac{(\alpha - c)^2 N_2 \gamma [(\gamma - 1)N_2 - 1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2} - \frac{N_2{}^2 (\alpha - c)^2 \gamma [(1-\gamma)N_1 + 1][(\gamma - 1)N_2 - 1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^3}$$
$$- \frac{N_1 (\alpha - c)^2 [(1-\gamma)N_2 + 1]^2}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2} - k$$

Putting all three pieces together gives a simplified version of (5.2) as follows:

$$\frac{(\alpha - c)^2 [(1-\gamma)N_2 + 1]}{(N_1+1)(N_2+1) - \gamma^2 N_1 N_2} - \frac{\gamma N_2 (\alpha - c)^2 [(1-\gamma)N_1 + 1][(1-\gamma)N_2 + 1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2}$$

$$+ \frac{\gamma^2 N_1 N_2 (\alpha - c)^2 [(1-\gamma)N_2 + 1]^2}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^3} - \frac{(\alpha - c)^2 N_1 [(1-\gamma)N_2 + 1][(1-\gamma^2)N_2 + 1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2}$$

$$+ \frac{N_1 (\alpha - c)^2 [(1-\gamma^2)N_2 + 1][(1-\gamma)N_2 + 1][(1-\gamma^2)N_1 N_2 + \gamma N_2 + N_1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^3} + \frac{(\alpha - c)^2 N_2 \gamma [(\gamma - 1)N_2 - 1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2}$$

$$+ \frac{N_2{}^2 (\alpha - c)^2 \gamma [(1-\gamma)N_1 + 1][(1-\gamma)N_2 + 1]}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^3} - \frac{N_1 (\alpha - c)^2 [(1-\gamma)N_2 + 1]^2}{[(N_1+1)(N_2+1) - \gamma^2 N_1 N_2]^2} - k = 0$$

Note that $0 \le c < \alpha$ to avoid the trivial case in which neither firm has an incentive to produce. Since $\alpha - c \ne 0$, dividing both sides by $(\alpha - c)^2$ gives (5.3).

$$\frac{[(1-\gamma)N_2+1]}{(N_1+1)(N_2+1)-\gamma^2 N_1 N_2}-\frac{\gamma N_2[(1-\gamma)N_1+1][(1-\gamma)N_2+1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2}$$

$$+\frac{\gamma^2 N_1 N_2[(1-\gamma)N_2+1]^2}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3}-\frac{N_1[(1-\gamma)N_2+1][(1-\gamma^2)N_2+1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2}$$

$$+\frac{N_1[(1-\gamma^2)N_2+1][(1-\gamma)N_2+1][(1-\gamma^2)N_1N_2+\gamma N_2+N_1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3}+\frac{N_2\gamma[(\gamma-1)N_2-1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2}$$

$$+\frac{N_2{}^2\gamma[(1-\gamma)N_1+1][(1-\gamma)N_2+1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3}-\frac{N_1[(1-\gamma)N_2+1]^2}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2}=\frac{k}{(\alpha-c)^2} \qquad (5.3)$$

Since the calculations are quite long, we start simplifying (5.3) with the terms containing $[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2$ in the denominator first:

$$\frac{1}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2}\left\{\begin{array}{l}-\gamma N_2[(1-\gamma)N_1+1][(1-\gamma)N_2+1]-N_1[(1-\gamma)N_2+1][(1-\gamma^2)N_2+1]\\-\gamma N_2[(1-\gamma)N_2+1]-N_1[(1-\gamma)N_2+1]^2\end{array}\right\}$$

$$=\frac{[(1-\gamma)N_2+1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2}\{-\gamma N_2[(1-\gamma)N_1+1]-N_1[(1-\gamma^2)N_2+1]-\gamma N_2-N_1[(1-\gamma)N_2+1]\}$$

Simplifying more leads to (5.4).

$$\frac{[(1-\gamma)N_2+1]2[(\gamma^2-1)N_1N_2-\gamma N_2-N_1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2}. \qquad (5.4)$$

Next, we continue simplifying (5.3) with the terms containing $[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3$ in the denominator as follows:

$$\frac{1}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3}\left\{\begin{array}{l}\gamma^2 N_1 N_2[(1-\gamma)N_2+1]^2+N_1[(1-\gamma^2)N_2+1][(1-\gamma)N_2+1][(1-\gamma^2)N_1N_2+\gamma N_2+N_1]\\+\gamma N_2{}^2[(1-\gamma)N_1+1][(1-\gamma)N_2+1]\end{array}\right\}$$

$$= \frac{[(1-\gamma)N_2+1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3}\left\{\begin{array}{l}\gamma^2 N_1 N_2[(1-\gamma)N_2+1]+N_1[(1-\gamma^2)N_2+1][(1-\gamma^2)N_1 N_2+\gamma N_2+N_1] \\ +\gamma N_2{}^2[(1-\gamma)N_1+1]\end{array}\right\}$$

Simplifying more leads to

$$\frac{[(1-\gamma)N_2+1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3}\left\{\begin{array}{l}2N_1 N_2{}^2(1-\gamma^2)\gamma+\gamma N_2{}^2+N_1{}^2+\gamma(1+\gamma)N_1 N_2+2(1-\gamma^2)N_1{}^2 N_2 \\ +(1-\gamma^2)^2 N_1{}^2 N_2{}^2\end{array}\right\}\cdot(5.5)$$

Substituting (5.4) and (5.5) into (5.3) gives

$$\frac{[(1-\gamma)N_2+1]}{(N_1+1)(N_2+1)-\gamma^2 N_1 N_2}+\frac{[(1-\gamma)N_2+1]2[(\gamma^2-1)N_1 N_2-\gamma N_2-N_1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2}$$

$$+\frac{[(1-\gamma)N_2+1]}{[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3}\left\{\begin{array}{l}2N_1 N_2{}^2(1-\gamma^2)\gamma+\gamma N_2{}^2+N_1{}^2+\gamma(1+\gamma)N_1 N_2+2(1-\gamma^2)N_1{}^2 N_2 \\ +(1-\gamma^2)^2 N_1{}^2 N_2{}^2\end{array}\right\}$$

$$=\frac{k}{(\alpha-c)^2}$$

Here, $1-\gamma>0$, since $0\le\gamma<1$ in a non-homogeneous goods market. Then $(1-\gamma)N_2{}^*+1>0$. As a result dividing both sides by $(1-\gamma)N_2{}^*+1$ and additional simplifying leads to

$$[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2+[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]2[(\gamma^2-1)N_1 N_2-\gamma N_2-N_1]$$

$$+2N_1 N_2{}^2(1-\gamma^2)\gamma+\gamma N_2{}^2+N_1{}^2+\gamma(1+\gamma)N_1 N_2+2(1-\gamma^2)N_1{}^2 N_2+(1-\gamma^2)^2 N_1{}^2 N_2{}^2$$

$$=\frac{k[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^3}{(\alpha-c)^2[(1-\gamma)N_2+1]}$$

Calculating $[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2]^2$ and substituting into the equation above leads to the following equation:

$$(1-\gamma)^2 N_2^{\,3} + \left(1-\gamma+2(1-\gamma)^2\right)N_2^{\,2} + 3(1-\gamma)N_2 - \gamma(1-\gamma)^2 N_1 N_2^{\,2} - \gamma(1-\gamma)N_1 N_2 + 1$$
$$= \frac{k\left[(N_1+1)(N_2+1)-\gamma^2 N_1 N_2\right]^3}{(\alpha-c)^2}$$

Finally, the first order conditions with respect to N_1 and N_2 respectively are given by (5.6) and (5.7).

$$(1-\gamma)^2 N_2^{*3} + (1-\gamma)(3-2\gamma)N_2^{*2} + 3(1-\gamma)N_2^{*} - \gamma(1-\gamma)^2 N_1^{*} N_2^{*2} - \gamma(1-\gamma)N_1^{*} N_2^{*} + 1$$
$$= \frac{k\left[\left(N_1^{*}+1\right)\left(N_2^{*}+1\right)-\gamma^2 N_1^{*} N_2^{*}\right]^3}{(\alpha-c)^2} \cdot \quad (5.6)$$

$$(1-\gamma)^2 N_1^{*3} + (1-\gamma)(3-2\gamma)N_1^{*2} + 3(1-\gamma)N_1^{*} - \gamma(1-\gamma)^2 N_2^{*} N_1^{*2} - \gamma(1-\gamma)N_1^{*} N_2^{*} + 1$$
$$= \frac{k\left[\left(N_1^{*}+1\right)\left(N_2^{*}+1\right)-\gamma^2 N_1^{*} N_2^{*}\right]^3}{(\alpha-c)^2} \cdot (5.7)$$

Since right hand sides are identical, setting left hand sides equal to solve for $\left(N_1^{*}, N_2^{*}\right)$ gives

$$(1-\gamma)\left[(1-\gamma)N_2^{*3} + (3-2\gamma)N_2^{*2} + 3N_2^{*} - \gamma(1-\gamma)N_1^{*}N_2^{*2}\right]$$
$$= (1-\gamma)\left[(1-\gamma)N_1^{*3} + (3-2\gamma)N_1^{*2} + 3N_1^{*} - \gamma(1-\gamma)N_2^{*}N_1^{*2}\right]$$

Since $0 \le \gamma < 1$ in a non-homogeneous goods market, simplifying $1-\gamma > 0$ gives

$$\left(N_2^{*} - N_1^{*}\right)\left[\left(N_2^{*2} + N_1^{*2}\right)(1-\gamma) + \left(N_2^{*} + N_1^{*}\right)(3-2\gamma) + N_2^{*}N_1^{*}(1-\gamma)^2 + 3\right] = 0 \,. (5.8)$$

Second factor in (5.8) is positive since $0 \le \gamma < 1 \Rightarrow 1-\gamma > 0$ and $0 \le \gamma < 1 \Rightarrow 2\gamma < 2 < 3 \Rightarrow 3-2\gamma > 0$. Consequently, (5.8) is equal to zero if and only if $N_2^{*} - N_1^{*} = 0 \Rightarrow N_2^{*} = N_1^{*} = N^{*} \Rightarrow \left(N_1^{*}, N_2^{*}\right) = \left(N^{*}, N^{*}\right)$.

■

Next, let us calculate the social optimum number of firms N^*. A condition that N^* must satisfy to be the socially optimal number of firms can be found by setting $N_2^* = N_1^* = N^*$ in (5.6) as follows:

$$(1-\gamma)^2 N^{*3} + (1-\gamma)(3-2\gamma)N^{*2} + 3(1-\gamma)N^* - \gamma(1-\gamma)^2 N^{*3} - \gamma(1-\gamma)N^{*2} + 1 = \frac{k\left[(N^*+1)^2 - \gamma^2 N^{*2}\right]^3}{(\alpha-c)^2}$$

Simplifying gives

$$((1-\gamma)N^*+1)^3 = \frac{k((1-\gamma)N^*+1)^3((1+\gamma)N^*+1)^3}{(\alpha-c)^2}$$

Here, $0 \le \gamma < 1 \Rightarrow 1-\gamma > 0$. Therefore, $(1-\gamma)N^* + 1 > 0$, which allows us to simplify $((1-\gamma)N^*+1)^3$ on both sides. As a result,

$$1 = \frac{k((1+\gamma)N^*+1)^3}{(\alpha-c)^2}.$$

Solving for N^* gives $N^* = \dfrac{(\alpha-c)^{2/3} - k^{1/3}}{k^{1/3}(1+\gamma)}$

As a result, social optimum number of firms is given by

$$\left(N_1^*, N_2^*\right) = \left(N^*, N^*\right) = \left(\frac{(\alpha-c)^{2/3} - k^{1/3}}{k^{1/3}(1+\gamma)}, \frac{(\alpha-c)^{2/3} - k^{1/3}}{k^{1/3}(1+\gamma)}\right).$$

5.6. **Entry Biases**

Entry biases are examined by comparing N^e to N^*.

$$N^e = \frac{\alpha - c - \sqrt{k}}{(1+\gamma)\sqrt{k}} = \frac{\alpha-c}{(1+\gamma)\sqrt{k}} - \frac{1}{(1+\gamma)}$$

$$N^* = \frac{(\alpha-c)^{2/3} - k^{1/3}}{k^{1/3}(1+\gamma)} = \frac{(\alpha-c)^{2/3}}{k^{1/3}(1+\gamma)} - \frac{1}{(1+\gamma)}$$

$N^e > N^*$ if and only if $\alpha - c > \sqrt{k}$:

$$N^e > N^* \Leftrightarrow \frac{\alpha - c}{\sqrt{k}} > \left[\frac{(\alpha - c)^2}{k}\right]^{1/3} \Leftrightarrow \frac{(\alpha - c)^3}{k^{3/2}} > \frac{(\alpha - c)^2}{k}$$

$$\Leftrightarrow (\alpha - c)^3 > (\alpha - c)^2 \sqrt{k}$$

Note that $0 \le c < \alpha$ to avoid the trivial case in which neither firm has an incentive to produce. Since $\alpha - c \ne 0$, dividing both sides by $(\alpha - c)^2$ gives

$$\alpha - c > \sqrt{k} \, .$$

As a result, $N^e > N^*$ if and only if $\alpha - c > \sqrt{k}$.

In fact, since $N^e = \dfrac{\alpha - c - \sqrt{k}}{(1+\gamma)\sqrt{k}} = \dfrac{\alpha - c}{(1+\gamma)\sqrt{k}} - \dfrac{1}{(1+\gamma)} > 0$

$$\Rightarrow \frac{\alpha - c}{(1+\gamma)\sqrt{k}} > \frac{1}{(1+\gamma)}$$

$$\Rightarrow \alpha - c > \sqrt{k}$$

As a result, $\alpha - c > \sqrt{k}$ is always true, which means that the number of free entry equilibrium firms is always greater than the number of social optimum firms. In other words, the entry is excessive.

Proposition 5.3. Free entry equilibrium number of firms is always greater than social optimum number of firms in the market:

$$N^e > N^*$$

5.7.Conclusion

In this chapter, a non-homogenous goods market with identical firms is examined. It is shown that in a non-homogeneous good market when each of the entrant firms has identical cost function, free entry equilibrium number of type one firms is equal to free entry equilibrium number of type two firms in the market. Moreover, it is found that social optimum number of type one firms is equal to social optimum number of type two firms.

When entry biases are examined the entry is found to be excessive. It is proven that, the number of free entry equilibrium firms is greater than the number of social optimum firms if and only if $\alpha - c > \sqrt{k}$. On the other hand $\alpha - c > \sqrt{k}$ always holds since free entry equilibrium number of firms in the market are positive, i.e. $N^e > 0$. Therefore, the main conclusion is that free entry equilibrium number of firms is always greater than the social optimum number of firms in a non-homogeneous good market when each of the entrant firms has identical cost function.

CHAPTER 6

NON-HOMOGENEOUS GOOD MARKET WITH NON-IDENTICAL FIRMS

6.1. Statement of the Problem

A non-homogenous goods market is examined under the assumption of non-identical firms in the market. There are two types of firms in the market each of which producing a different good, i.e. there exist two differentiated goods, q_1 and q_2, in the market. We are interested in a Cournot market with free entry. The number of free entry equilibrium firms and social optimum firms are examined under free entry when there are two different types of firms in a Cournot market. The main goal is to compare the number of free entry equilibrium firms to the number of social optimum number of firms.

6.2. **The methods of our research**

The model is a two-stage game. In the first stage, there exist two types of firms and each firm must decide whether to enter to the Cournot market. The second stage is the production period. Each of the entrants has one of the two cost functions $C_1(q_1) = c_1\mathrm{q}_1 + \mathrm{k}_1$ and $C_2(q_2) = c_2\mathrm{q}_2 + \mathrm{k}_2$ such that $C_i(\cdot)$ is continuous and $\mathrm{C}_i^{\prime}(\cdot) \geq 0$, $\mathrm{C}_i^{\prime\prime}(\cdot) \geq 0$ for $q_i \geq 0, \mathrm{i} = 1,2.$ Firms with the cost functions $C_1(q_1)$ and $C_2(q_2)$ are called type one firms and type two firms, respectively.

The market is a non-homogeneous product market such that representative consumer's utility is a symmetric quadratic function of the two products q_1 and q_2:

$U = \alpha(q_1 + q_2) - \frac{1}{2}(q_1{}^2 + q_2{}^2 + 2\gamma q_1 q_2)$, where the parameter γ measures the degree of product differentiation such that $0 \le \gamma \le 1$. In addition, $\gamma = 0$ and $\gamma = 1$ correspond to independent goods (the maximum degree of differentiation) and homogeneous goods (the minimum degree of differentiation), respectively. Note that $0 \le \gamma < 1$ since non-homogeneous goods market is examined in this study.

The following linear system of inverse demand functions are generated by this utility function when maximized with respect to budget constraint:

$$p_1 = \alpha - q_1 - \gamma q_2$$

$$p_2 = \alpha - q_2 - \gamma q_1$$

The overall number of firms in the market is denoted by $N_1 + N_2$, where N_1 and N_2 are the number of type one and type two firms in the market, respectively. In this case demand functions turn to be the following linear system:

$$p_1 = \alpha - \sum_{i=1}^{N_1} q_{1i} - \gamma \sum_{j=1}^{N_2} q_{2j}$$

$$p_2 = \alpha - \sum_{j=1}^{N_2} q_{2j} - \gamma \sum_{i=1}^{N_1} q_{1i}$$

, where q_{1i} and q_{2j} represent output of firm i from type 1 and output of firm j from type 2, respectively.

6.3. Calculation of equilibrium outputs

To find equilibrium outputs q_1^* and q_2^*, the optimization problem of type 1 firms and type two firms is solved. Firm i from type one maximizes its profits as follows:

$$\max_{q_{1i}} p_1 q_{1i} - C_1(q_{1i})$$

$$\Rightarrow \max_{q_{1i}} \left(\alpha - \sum_{t=1}^{N_1} q_{1t} - \gamma \sum_{j=1}^{N_2} q_{2j} \right) q_{1i} - c_1 q_{1i} - k_1$$

First order conditions:

$$\alpha - \sum_{t=1}^{N_1} q_{1t} - \gamma \sum_{j=1}^{N_2} q_{2j} - q_{1i} - c_1 = 0$$

$$\Rightarrow q_{1i} = \frac{1}{2}(\alpha - c_1) - \frac{\gamma}{2} \sum_{j=1}^{N_2} q_{2j} - \frac{1}{2} \sum_{\substack{t=1 \\ t \neq i}}^{N_1} q_{1t} - \gamma - c_1 = 0, \quad i = 1,2,3,\ldots,N_1 .$$

There are N_1 equations for type one firms and N_2 equations for type two firms:

$$q_{2j} = \frac{1}{2}(\alpha - c_2) - \frac{\gamma}{2} \sum_{i=1}^{N_1} q_{1i} - \frac{1}{2} \sum_{\substack{t=1 \\ t \neq j}}^{N_2} q_{2t} - \gamma - c_2 = 0, \quad j = 1,2,3,\ldots,N_2$$

As a result, there are $N_1 + N_2$ equations obtained from maximization problem.

Because of symmetry, in equilibrium $q_{1i} = q_1$ for $i = 1,2,3,\ldots,N_1$ and $q_{2j} = q_2$ for $j = 1,2,3,\ldots,N_2$.

First order conditions are reduced to the following system of equations:

$$q_1 = \frac{1}{2}(\alpha - c_1) - \frac{\gamma}{2} N_2 q_2 - \frac{N_1 - 1}{2} q_1$$

$$q_2 = \frac{1}{2}(\alpha - c_2) - \frac{\gamma}{2} N_1 q_1 - \frac{N_2 - 1}{2} q_2$$

By rearranging terms,

$$(N_1 + 1) q_1 = \alpha - c_1 - \gamma N_2 q_2$$

$$(N_2 + 1) q_2 = \alpha - c_2 - \gamma N_1 q_1$$

Equilibrium outputs q_1^* and q_2^* are solved from the first order conditions.

$$\Rightarrow q_1^* = \frac{(\alpha - c_1)(N_2 + 1) - \gamma N_2 (\alpha - c_2)}{(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2}. \qquad (6.1)$$

$$\Rightarrow q_2^* = \frac{(\alpha - c_2)(N_1 + 1) - \gamma N_1 (\alpha - c_1)}{(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2}. \qquad (6.2)$$

Note that $(\alpha - c_1)(N_2 + 1) > \gamma N_2 (\alpha - c_2)$ and $(\alpha - c_2)(N_1 + 1) > \gamma N_1 (\alpha - c_1)$ to avoid the trivial case in which neither firm has an incentive to produce. Moreover we assume that $0 \le c_1 < c_2 < \alpha$.

6.4. **The Free Entry Equilibrium and Social Optimum Number of Firms**

The number of free entry equilibrium firms in the market is denoted by (N_1^e, N_2^e). Under free entry $\pi_1^* = \pi_2^* = 0$, where π_1^* and π_2^* represent equilibrium profits.

$$\pi_1^* = p_1^* q_1^* - C_1(q_1^*)$$

$$\Rightarrow \pi_1^* = (\alpha - N_1 q_1^* - \gamma N_2 q_2^*) q_1^* - c_1 q_1^* - k_1$$

By substituting q_1^* and q_2^* from (6.1) and (6.2),

$$\left(\alpha - c_1 - N_1 \frac{(\alpha - c_1)(N_2 + 1) - \gamma N_2 (\alpha - c_2)}{(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2} - \gamma N_2 \frac{(\alpha - c_2)(N_1 + 1) - \gamma N_1 (\alpha - c_1)}{(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2}\right) \times$$
$$\frac{(\alpha - c_1)(N_2 + 1) - \gamma N_2 (\alpha - c_2)}{(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2} = k_1$$

Simplifying leads to

$$\Rightarrow [(\alpha - c_1)(N_2 + 1) - \gamma N_2 (\alpha - c_2)]^2 = k_1 [(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2]^2 . \tag{6.3}$$

Similarly,

$$[(\alpha - c_2)(N_1 + 1) - \gamma N_1 (\alpha - c_1)]^2 = k_2 [(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2]^2 . \tag{6.4}$$

Last two equations, (6.3) and (6.4), are called first order conditions that are satisfied by the free entry equilibrium number of firms. The goal is to solve $N_1{}^e$ and $N_2{}^e$ from (6.3) and (6.4)

$$(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2 = \frac{(\alpha - c_1)(N_2 + 1) - \gamma N_2 (\alpha - c_2)}{\sqrt{k_1}}$$

$$(N_1 + 1)(N_2 + 1) - \gamma^2 N_1 N_2 = \frac{(\alpha - c_2)(N_1 + 1) - \gamma N_1 (\alpha - c_1)}{\sqrt{k_2}}$$

By setting right hand sides equal to each other:

$$\frac{(\alpha - c_1)(N_2 + 1) - \gamma N_2 (\alpha - c_2)}{\sqrt{k_1}} = \frac{(\alpha - c_2)(N_1 + 1) - \gamma N_1 (\alpha - c_1)}{\sqrt{k_2}}$$

$$\Rightarrow \frac{N_2[\alpha(1-\gamma)+\gamma c_2-c_1]}{\sqrt{k_1}}+\frac{\alpha-c_1}{\sqrt{k_1}}=\frac{N_1[\alpha(1-\gamma)+\gamma c_1-c_2]}{\sqrt{k_2}}+\frac{\alpha-c_2}{\sqrt{k_2}}$$

$N_2{}^e$ can be solved in terms of $N_1{}^e$. Note that $\alpha(1-\gamma)+\gamma c_2-c_1>0$ always holds since $0\le\gamma\le 1<\frac{c_1-\alpha}{c_2-\alpha}$.

As a result, $N_2{}^e$ is obtained in terms of $N_1{}^e$ as follows:

$$\begin{aligned} N_2{}^e = & \frac{\sqrt{k_1}}{[\alpha(1-\gamma)+\gamma c_2-c_1]}\frac{[\alpha(1-\gamma)+\gamma c_1-c_2]}{\sqrt{k_2}}N_1{}^e+\frac{\sqrt{k_1}}{[\alpha(1-\gamma)+\gamma c_2-c_1]}\frac{\alpha-c_2}{\sqrt{k_2}} \\ & -\frac{\sqrt{k_1}}{[\alpha(1-\gamma)+\gamma c_2-c_1]}\frac{\alpha-c_1}{\sqrt{k_1}} \end{aligned} \quad (6.5)$$

The graph of (6.5) on N_1, N_2 coordinate plane is a straight line with the following slope, x-intercept (intercept on N_1 axis), and y-intercept (intercept on N_2 axis):

Slope: $\frac{[\alpha(1-\gamma)+\gamma c_1-c_2]}{[\alpha(1-\gamma)+\gamma c_2-c_1]}\frac{\sqrt{k_1}}{\sqrt{k_2}}$

x-intercept: $\frac{1}{\alpha(1-\gamma)+\gamma c_1-c_2}\left[\frac{\sqrt{k_2}}{\sqrt{k_1}}(\alpha-c_1)-(\alpha-c_2)\right]$

y-intercept: $\frac{1}{[\alpha(1-\gamma)+\gamma c_2-c_1]}\left[\frac{\sqrt{k_1}}{\sqrt{k_2}}(\alpha-c_2)-(\alpha-c_1)\right]$

It follows

- the slope is negative (positive) if $[\alpha(1-\gamma)+\gamma c_1-c_2]$ is negative (positive),

■ the two intercepts have the same sign (opposite signs) if $\left[\alpha(1-\gamma)+\gamma c_1 - c_2\right]$ is negative (positive).

Hence, there are the following 4 possible cases:

(i) $\frac{\alpha - c_2}{\alpha - c_1} < \gamma$ and $\frac{\sqrt{k_2}}{\sqrt{k_1}} < \frac{\alpha - c_2}{\alpha - c_1}$: slope is negative, both intercepts are positive.

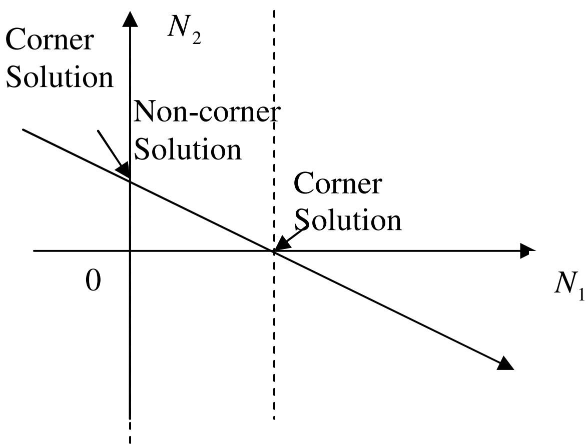

Figure 6: Free entry equilibrium when slope and both intercepts are positive.

(ii) $\frac{\alpha - c_2}{\alpha - c_1} < \gamma$ and $\frac{\sqrt{k_2}}{\sqrt{k_1}} > \frac{\alpha - c_2}{\alpha - c_1}$: slope is negative, both intercepts are negative.

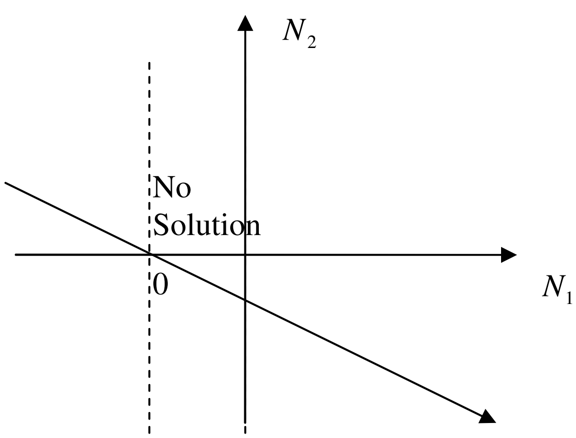

Figure 7: Free entry equilibrium when slope and both intercepts are negative.

(iii) $\frac{\alpha - c_2}{\alpha - c_1} > \gamma$ and $\frac{\sqrt{k_2}}{\sqrt{k_1}} < \frac{\alpha - c_2}{\alpha - c_1}$: slope is positive, x-intercept is negative, y-intercept is positive.

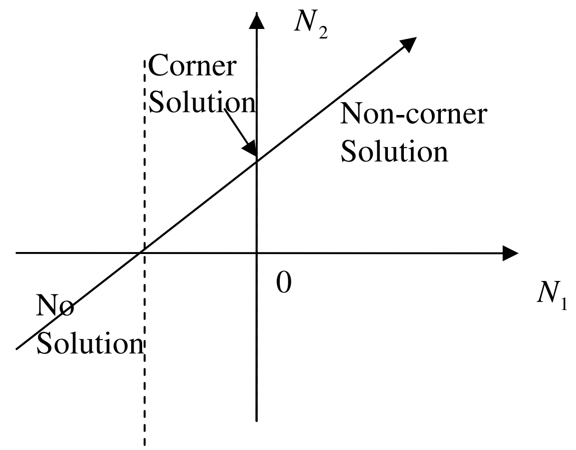

Figure 8: Free entry equilibrium when slope and y-intercept are positive and x-intercept is negative.

(iv) $\frac{\alpha - c_2}{\alpha - c_1} > \gamma$ and $\frac{\sqrt{k_2}}{\sqrt{k_1}} > \frac{\alpha - c_2}{\alpha - c_1}$: slope is positive, x-intercept is positive, y-intercept is negative.

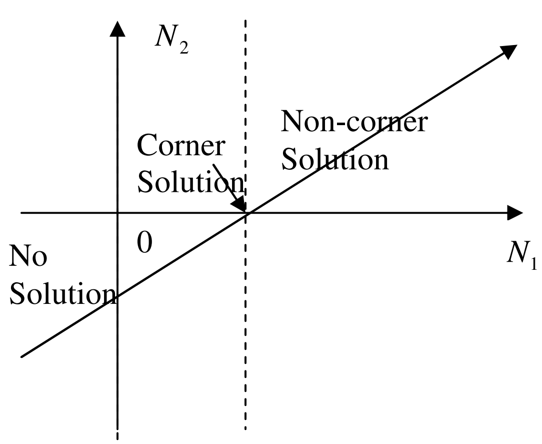

Figure 9: Free entry equilibrium when slope and x-intercept are positive and y-intercept is negative.

There are a few observations that follow from the four possible cases. First of all, it is obvious that there is no free entry solution in case (ii) when $\frac{\alpha - c_2}{\alpha - c_1} < \gamma$ and $\frac{\sqrt{k_2}}{\sqrt{k_1}} > \frac{\alpha - c_2}{\alpha - c_1}$. On the other hand, the only possible corner solution is the one that only type two firms survive, $\left(0, N_2^{\,e}\right)$, in case (iii) when $\frac{\alpha - c_2}{\alpha - c_1} > \gamma$ and $\frac{\sqrt{k_2}}{\sqrt{k_1}} < \frac{\alpha - c_2}{\alpha - c_1}$. Similarly, in case (iv); the only possible corner solution is the one that only type one firms survive, $\left(N_1^{\,e}, 0\right)$, when $\frac{\alpha - c_2}{\alpha - c_1} > \gamma$ and $\frac{\sqrt{k_2}}{\sqrt{k_1}} > \frac{\alpha - c_2}{\alpha - c_1}$. Both corner solutions are possible in case (i) when $\frac{\alpha - c_2}{\alpha - c_1} < \gamma$ and $\frac{\sqrt{k_2}}{\sqrt{k_1}} < \frac{\alpha - c_2}{\alpha - c_1}$.

Free entry equilibrium number of firms is the pair $\left(N_1^{\,e}, N_2^{\,e}\right)$ that satisfies (6.3) and (6.4). Social optimum number of firms is the pair $\left(N_1^{\,*}, N_2^{\,*}\right)$ that solves the maximization problem of wealth function $W\left(N_1, N_2\right)$ as follows:

$$\max_{N_1, N_2} W\left(N_1, N_2\right)$$

$$= \max_{N_1, N_2} \left\{ \begin{aligned} &\alpha\left(N_1 q_1^{\,*} + N_2 q_2^{\,*}\right) - c_1 N_1 q_1^{\,*} - c_2 N_2 q_2^{\,*} \\ &- \left[\frac{1}{2}\left(N_1 q_1^{\,*} + N_2 q_2^{\,*}\right)^2 - (1-\gamma) N_1 q_1^{\,*} N_2 q_2^{\,*}\right] - N_1 k_1 - N_2 k_2 \end{aligned} \right\}$$

Analytical solutions to free entry equilibrium and to social optimum require quite complex calculations due to the number of parameters involved in the calculations. To obtain free entry equilibrium solution, the pair of numbers $\left(N_1^{\ e}, N_2^{\ e}\right)$ that satisfies (6.3) and (6.4) needs to be calculated algebraically. Such a calculation leads to two second degree polynomials one of which is in terms of $N_1^{\ e}$ and the other one of which is in terms of $N_2^{\ e}$. Calculations become fairly complicated since each of these equations contains six parameters. As a result, simulations method is preferred rather than algebraic calculations in non-homogeneous market with non-identical firms.

The following tables show the free entry equilibrium number of firms $\left(N_1^{\ e}, N_2^{\ e}\right)$ and social optimum number of firms $\left(N_1^{\ *}, N_2^{\ *}\right)$ in the market when the parameters are assigned different values by Mathematica. When both N_1 and N_2 are negative the solution is set at (0,0). It is assumed that there is a corner solution when one of N_1, N_2 is negative in a pair $\left(N_{1,}, N_2\right)$. If N_1 is negative we let N_1=0 in the profit function for type two firms and solve for N_2 when profit is zero. If N_2 is found as a negative number as well then the solution is set at (0,0).

Social optimum number of firms is found by maximizing the welfare function over integers. However, free entry solution is the pair of positive real numbers that solve (6.3) and (6.4). To work out the integer problem, four possible integer pairs of $\left(N_{1,}, N_2\right)$ are considered. The integer pair that gives $\pi_1 > 0$ and $\pi_2 > 0$ is taken as the solution. For instance, if (3.2,1.8) is a free entry solution then the integer pairs (3,1), (3,2), (4,1), (4,2) are considered. However when one of the N_1, N_2 is in [0,1) then corner solution appears to be present.

TABLE 1

Free entry and social optimum when $\alpha = 50000, c_1 = 800, c_2 = 15000, \sqrt{k_1} = 20000$, γ and k_2 varies**:**

$\sqrt{k_2}$	γ	Case	Free Entry $\left(N_1^{\ e}, N_2^{\ e}\right)$	Social optimum $\left(N_1^{\ *}, N_2^{\ *}\right)$	$\sqrt{k_2}$	γ	Case	Free Entry $\left(N_1^{\ e}, N_2^{\ e}\right)$	Social optimum $\left(N_1^{\ *}, N_2^{\ *}\right)$
4000	0.1	iii	(1, 7)	(1,3)	12000	0.1	iii	(1, 1)	(1,1)
	0.2	iii	(1, 6)	(1,3)		0.2	iii	(1,1)	(1,1)
	0.3	iii	(1, 6)	(1,2)		0.3	iii	(1, 1)	(1,1)
	0.4	iii	(1, 5)	(1,2)		0.4	iii	(1, 1)	(1,1)
	0.5	iii	(0, 7)	(1,2)		0.5	iii	(1, 1)	(1,0)
	0.6	iii	(0, 7)	(1,1)		0.6	iii	(1, 0)	(1,0)
	0.75	i	(0, 7), (1, 0)	(1,1)		0.75	i	(1, 0)	(1,0)
	0.8	i	(0, 7), (1, 0)	(1,1)		0.8	i	(1, 0)	(1,0)
	0.85	i	(0, 7), (1, 0)	(0,3)		0.85	i	(1, 0)	(1,0)
	0.9	i	(0, 7), (1, 0)	(0,3)		0.9	i	(1, 0)	(1,0)
	0.95	i	(0, 7), (1, 0)	(0,3)		0.95	i	(1, 0)	(1,0)
6000	0.1	iii	(1, 4)	(1,2)	14600	0.1	iv	(1, 1)	(1,1)
	0.2	iii	(1, 4)	(1,2)		0.2	iv	(1, 1)	(1,1)
	0.3	iii	(1, 3)	(1,2)		0.3	iv	(1, 0)	(1,1)
	0.4	iii	(1, 3)	(1,1)		0.4	iv	(1, 0)	(1,0)
	0.5	iii	(0, 4)	(1,1)		0.5	iv	(1, 0)	(1,0)
	0.6	iii	(0, 4)	(1,1)		0.6	iv	(1, 0)	(1,0)
	0.75	i	(0, 4), (1, 0)	(1,1)		0.75	ii	(1, 0)	(1,0)
	0.8	i	(0, 4), (1, 0)	(1,0)		0.8	ii	(0, 0)	(1,0)
	0.85	i	(0, 4), (1, 0)	(1,0)		0.85	ii	(0, 0)	(1,0)
	0.9	i	(0, 4), (1, 0)	(1,0)		0.9	ii	(0, 0)	(1,0)
	0.95	i	(0, 4), (1, 0)	(1,0)		0.95	ii	(0, 0)	(1,0)
10000	0.1	iii	(1, 2)	(1,1)	15500	0.1	iv	(1, 1)	(1,1)
	0.2	iii	(1, 2)	(1,1)		0.2	iv	(1, 0)	(1,1)
	0.3	iii	(1, 1)	(1,1)		0.3	iv	(1, 0)	(1,1)
	0.4	iii	(1, 1)	(1,1)		0.4	iv	(1, 0)	(1,0)
	0.5	iii	(1, 1)	(1,1)		0.5	iv	(1, 0)	(1,0)
	0.6	iii	(1, 1)	(1,0)		0.6	iv	(1, 0)	(1,0)
	0.75	i	(1, 0)	(1,0)		0.75	ii	(0, 0)	(1,0)
	0.8	i	(1, 0)	(1,0)		0.8	ii	(0, 0)	(1,0)
	0.85	i	(1, 0)	(1,0)		0.85	ii	(0, 0)	(1,0)
	0.9	i	(1, 0)	(1,0)		0.9	ii	(0, 0)	(1,0)
	0.95	i	(1, 0)	(1,0)		0.95	ii	(0, 0)	(1,0)

$\sqrt{k_2}$	γ	Case	Free Entry (N_1^e, N_2^e)	Social optimum (N_1^*, N_2^*)	$\sqrt{k_2}$	γ	Case	Free Entry (N_1^e, N_2^e)	Social optimum (N_1^*, N_2^*)
16000	0.1	iv	(1, 1)	(1,1)	18000	0.1	iv	(1, 0)	(1,1)
	0.2	iv	(1, 0)	(1,1)		0.2	iv	(1, 0)	(1,0)
	0.3	iv	(1, 0)	(1,0)		0.3	iv	(1, 0)	(1,0)
	0.4	iv	(1, 0)	(1,0)		0.4	iv	(1, 0)	(1,0)
	0.5	iv	(1, 0)	(1,0)		0.5	iv	(1, 0)	(1,0)
	0.6	iv	(1, 0)	(1,0)		0.6	iv	(1, 0)	(1,0)
	0.75	ii	(0, 0)	(1,0)		0.75	ii	(0, 0)	(1,0)
	0.8	ii	(0, 0)	(1,0)		0.8	ii	(0, 0)	(1,0)
	0.85	ii	(0, 0)	(1,0)		0.85	ii	(0, 0)	(1,0)
	0.9	ii	(0, 0)	(1,0)		0.9	ii	(0, 0)	(1,0)
	0.95	ii	(0, 0)	(1,0)		0.95	ii	(0, 0)	(1,0)

Table 1 depicts the changes in free entry and social optimum number of firms when parameters γ and k_2 have been changed holding all the other parameters constant. It seems that the number of type two firms decreases with the increase in the fixed cost k_2 in both free entry equilibrium and social optimum. Also, as the goods become homogeneous corner solutions start to appear. In other words, when the degree of product differentiation is close to zero the goods become to be independent goods and both type of firms can survive in the market due to the maximum degree of product differentiation. Of course when fixed cost k_2 is too high only type one firms survives even though the goods are highly differentiated.

In Table 1, as fixed cost k_2 increases, social optimum turns out to be a corner solution such that only type one firms survive even when the goods are highly differentiated. For very high values of k_2, free entry solution develops into no solution when the goods become less and less differentiated. Free entry solution and social optimum start to coincide for highly differentiated goods as fixed cost k_2 increases.

When fixed cost k_1 and the degree of product differentiation γ are changed, the results will be similar to the results obtained from Table 1.

TABLE 2

Free entry and social optimum when $\alpha = 50000, c_1 = 800, \gamma = 0.5, \sqrt{k_1} = 20000$, c_2 and k_2 varies:

c_2	$\sqrt{k_2}$	Case	Free Entry (N_1^e, N_2^e)	Social optimum (N_1^*, N_2^*)
11500	12000	iii	(1,1)	(1,1)
	10000	iii	(0,2)	(1,1)
	2000	iii	(0,18)	(1,3)
	16000	iv	(1,0)	(1,0)
	18000	iv	(1,0)	(1,0)
	30000	iv	(1,0)	(1,0)
	60000	iv	(1,0)	(1,0)
15000	12000	iii	(1,1)	(1,0)
	10000	iii	(1,1)	(1,1)
	2000	iii	(0,16)	(1,3)
	16000	iv	(1,0)	(1,0)
	18000	iv	(1,0)	(1,0)
	30000	iv	(1,0)	(1,0)
	60000	iv	(1,0)	(1,0)
18000	12000	iii	(1,0)	(1,0)
	10000	iii	(1,1)	(1,0)
	6000	iii	(1,2)	(1,1)
	2000	iii	(0,15)	(1,2)
	16000	iv	(1,0)	(1,0)
	18000	iv	(1,0)	(1,0)
	30000	iv	(1,0)	(1,0)
	60000	iv	(1,0)	(1,0)
30000	12000	ii	(0,0)	(1,0)
	10000	ii	(0,0)	(1,0)
	6000	i	(1,0)	(1,0)
	2000	i	(1,3),	(1,0)
	16000	ii	(1,0)	(1,0)
	18000	ii	(0,0)	(1,0)
	30000	ii	(0,0)	(1,0)
	60000	ii	(0,0)	(1,0)
			(0,0)	
32500	12000	ii	(0,0)	(1,0)
	10000	ii	(0,0)	(1,0)
	6000	i	(1,0)	(1,0)
	2000	i	(1,1),	(1,0)
	16000	ii	(1,0)	(1,0)
	18000	ii	(0,0)	(1,0)
	30000	ii	(0,0)	(1,0)
	60000	ii	(0,0)	(1,0)
			(0,0)	
40000	12000	ii	(0,0)	(1,0)
	10000	ii	(0,0)	(1,0)
	6000	ii	(0,0)	(1,0)
	2000	i	(1,0)	(1,1)
	16000	ii	(0,0)	(1,0)
	18000	ii	(0,0)	(1,0)
	30000	ii	(0,0)	(1,0)
	60000	ii	(0,0)	(1,0)

Table 2 demonstrates the free entry equilibrium and social optimum number of firms when c_2 and k_2 are changed holding all the other parameters constant. In this case, the increase in the marginal cost of type two firms decreases the number of type two firms in the market for both social optimum and free entry equilibrium. Increasing both marginal cost and fixed cost of type two firms drives type two firms out of the market. Type one firms continues to survive in social optimum but they also exit in free entry equilibrium when product differentiation is held constant at medium level.

According to Table 2, when marginal cost of type two firms is a lot higher than marginal cost of type one firms, type two firms cannot survive. As a result, both social optimum and free entry equilibrium turns out to be corner solutions that only type one firms survives.

TABLE 3

Free entry and social optimum when $\alpha = 50000, c_2 = 35000, \gamma = 0.5, \sqrt{k_2} = 12000$, c_1 and k_1 varies:

c_1	$\sqrt{k_1}$	Case	Free Entry (N_1^e, N_2^e)	Social optimum (N_1^*, N_2^*)
800	20000	ii	(0,0)	(1,0)
	30000	ii	(0,0)	(1,0)
	40000	i	(0,0)	(0,0)
	120000	i	(0,0)	(0,0)
	15000	ii	(0,0)	(1,0)
	13000	ii	(0,0)	(1,0)
	8000	ii	(0,0)	(2,0)
	4000	ii	(0,0)	(4,0)
10000	20000	ii	(0,0)	(1,0)
	30000	ii	(0,0)	(0,0)
	40000	i	(0,0)	(0,0)
	120000	i	(0,0)	(0,0)
	15000	ii	(0,0)	(1,0)
	13000	ii	(0,0)	(1,0)
	8000	ii	(0,0)	(2,0)
	4000	ii	(0,0)	(4,0)

c_1	$\sqrt{k_1}$	Case	Free Entry $\left(N_1^{\ e}, N_2^{\ e}\right)$	Social optimum $\left(N_1^{\ *}, N_2^{\ *}\right)$	c_1	$\sqrt{k_1}$	Case	Free Entry $\left(N_1^{\ e}, N_2^{\ e}\right)$	Social optimum $\left(N_1^{\ *}, N_2^{\ *}\right)$
15000	20000	ii	(0,0)	(1,0)	30000	20000	iii	(0,0)	(0,0)
	30000	i	(0,0)	(0,0)		30000	iii	(0,0)	(0,0)
	40000	i	(0,0)	(0,0)		40000	iii	(0,0)	(0,0)
	120000	i	(0,0)	(0,0)		120000	iii	(0,0)	(0,0)
	15000	ii	(0,0)	(1,0)		15000	iv	(0,0)	(0,0)
	13000	ii	(0,0)	(1,0)		13000	iv	(0,0)	(0,0)
	8000	ii	(0,0)	(2,0)		8000	iv	(1,0)	(1,0)
	4000	ii	(0,0)	(3,0)		4000	iv	(4,0)	(2,0)
22000	20000	iv	(0,0)	(0,0)	33000	20000	iii	(0,0)	(0,0)
	30000	iii	(0,0)	(0,0)		30000	iii	(0,0)	(0,0)
	40000	iii	(0,0)	(0,0)		40000	iii	(0,0)	(0,0)
	120000	iii	(0,0)	(0,0)		120000	iii	(0,0)	(0,0)
	15000	iv	(0,0)	(1,0)		15000	iii	(0,0)	(0,0)
	13000	iv	(1,0)	(1,0)		13000	iv	(0,0)	(0,0)
	8000	iv	(2,0)	(1,0)		8000	iv	(1,0)	(1,0)
	4000	iv	(6,0)	(3,0)		4000	iv	(3,0)	(2,0)

Table 3 displays the effects of parameters c_1 and k_1 on free entry and social optimum number of firms. According to Table 3, as the marginal cost of type one firms increases type one firms start to exit the market. Type one firms still survives for low fixed cost k_1. Both free entry and social optimum are corner solutions that only type one firms survives when marginal cost c_1 is less than marginal cost c_2 and fixed cost k_1 is small enough.

6.5. Conclusion

In this chapter, non-homogeneous good market with non-identical firms is examined. Due to the complexity of calculations, free entry and social optimum number of firms are not calculated analytically. Instead, solutions are calculated by Mathematica by assigning different values to parameters.

There are two main conclusions derived in this chapter. First, corner solutions start to come into sight when the degree of product differentiation is close to minimum. As the product differentiation parameter γ approaches 1, products become less differentiated. In a homogeneous good market both free entry and social optimum number of firms is corner solutions as it is shown in Chapters 3 and 4.

Second, when the degree of product differentiation is close to zero the goods become to be independent goods and both type of firms can survive in the market due to the maximum degree of product differentiation. Evidently, if fixed cost of type two firms is too high, only type one firms survives even though the goods are highly differentiated.

Other than these two main conclusions, effects of changes in parameters cause expected changes in solutions such as the number of type two firms decreases with the increase in the fixed cost k_2 in both free entry equilibrium and social optimum. An increase in the marginal cost of type two firms decreases the number of type two firms in the market for both social optimum and free entry equilibrium. When both marginal cost and fixed cost of type two firms go up type two firms start to exit the market.

CHAPTER 7

CONCLUSION

The main focus of this study is entry biases in Cournot markets with free entry. Entry biases are calculated by comparing the number of free entry equilibrium firms to the number of social optimum firms. Both homogeneous and non-homogeneous good markets are examined. In homogeneous good market, linear and non-linear demands are studied separately. In non-homogeneous good market, first it is assumed that the firms in the market are identical. Second, non-identical firms in non-homogeneous good market are considered.

There are three main conclusions obtained in homogeneous good market with linear demand. First, it is shown that that there exist only one type of firms in the market for both the social optimum and free entry equilibrium. In other words, both free entry equilibrium and social optimum are corner solutions. Second, when there are the same type of firms in the market for both the social optimum and the free entry equilibrium, excessive entry happens. Moreover, sufficient conditions for the existence of excessive entry regarding to each type of firms are provided. Third, it is possible to have one type of firms in social optimum and the other type of firms in free entry equilibrium. The sufficient conditions such that there exist only type two firms in free entry equilibrium but only type one firms in social optimum are presented.

When non-linear demand is introduced in homogenous demand market, similar results are obtained. It is shown that both free entry equilibrium and social optimum are corner solutions that only one type of firms exist. The main conclusion is that when there is the same type of firms exist in free entry equilibrium as well as in social optimum, the entry is excessive.

Non-homogenous goods market is examined with identical firms as well as non-identical firms. Firms are identical when each of the entrant firms has identical cost function. It is shown that in the case of identical firms, free entry equilibrium number of type one firms is equal to free entry equilibrium number of type two firms in the market. Moreover, it is found that social optimum number of type one firms is equal to social optimum number of type two firms. When it comes to entry biases, the entry is again found to be excessive since it is proven that the number of free entry equilibrium firms is always greater than the number of social optimum firms.

Lastly, non-homogeneous market with non-identical firms is examined. Due to the number of parameters involved in the calculations, analytical solutions to free entry equilibrium and to social optimum are rather complex. Therefore non-homogeneous market with non-identical firms is inspected by simulations.

Simulations by Mathematica shows that as goods become homogeneous corner solutions start to appear, which is consistent with the findings in Chapters 3 and 4 in homogeneous good market. When the degree of product differentiation is close to zero the goods become independent goods and both type of firms can survive in the market due to the maximum degree of product differentiation. Evidently, if fixed cost of type two firms is too high, only type one firms survive even though the goods are highly differentiated.

In conclusion, entry biases in Cournot markets with free entry in both homogeneous and non-homogeneous markets are examined. It is shown that both free entry and social optimum are corner solutions and entry decision is excessive in homogeneous good market. In non-homogeneous good market with identical firms, both free entry equilibrium and social optimum are solutions on the 45^0 line, in other words the number of type one firms is the same as the number of type two firms. Entry is yet again found to be excessive. The findings in non-homogeneous good market with non-identical firms support the results obtained in homogeneous good market. As the degree of product differentiation increases, non-corner solutions start to appear. Both types of firms may survive in non-homogeneous good market with non-identical firms depending on the fixed costs and the degree of product differentiation.

BIBLIOGRAPHY

AMIR, RABAH AND LAMBSON, VAL E. "On The Effects of Entry in Cournot Markets" *The Review of Economic Studies*, Vol. 67, No. 2 (Apr., 2000), pp. 235-254.

ANDERSON, S. P. AND RENAULT, R. "Efficiency and Surplus bounds in Cournot Competition" *Journal of Economic Theory*, Vol. 113, No. 2 (2003), pp. 253-264.

BAMON, RODRIGO AND FRAYSSE, JEAN "Existence of Cournot Equilibrium in Large Markets" *Econometrica*, Vol. 53, No. 3 (May, 1985), pp. 587-598.

BOONE, J. "Competitive pressure: the effects on investments in product and process innovation" *Rand Journal of Economics*, Vol. 31, (2000), pp. 549-569.

CABRAL, LUIS M. B. "Simultaneous Entry and Welfare" *European Economic Review*, Vol. 48, (2004), pp. 943-957.

CORCHON, LUIS C. "Welfare Losses Under Cournot Competition" (Oct. 5, 2006) working paper, accepted for publication in *International Journal of Industrial Organization*.

CORCHON, LUIS C. AND FRADERA, ISABEL "Comparative Statistics in Cournot Free Entry Equilibrium" *Mathematical Social Sciences*, Vol. 44, No. 2 (Nov., 2002), pp. 155-168.

COURNOT, A. Researches into the Mathematical Principles of the Theory of Wealth. English edition of Cournot (1938). Translated by N. T. Bacon, New York: Kelley 1960.

DIXIT, A. K. "A Model of Duopoly Suggesting a Theory of Entry Barriers" *Bell Journal of Economics*, Vol. 10, (1979), pp. 20-32.

EATON, B. CURTIS AND WARE, ROGER "A Theory of Market Structure with Sequential Entry" *The RAND Journal of Economics*, Vol. 18, No. 1 (Spring, 1987), pp. 1-16.

FRANK, JR., CHARLES R. “Entry in a Cournot Market” *The Review of Economic Studies*, Vol. 32, No. 3 (Jul., 1965), pp. 245-250.

FRANK, JR., CHARLES R. AND QUANDT, RICHARD E. “On The Existence of Cournot Equilibrium” *International Economic Review*, Vol. 4, No. 1 (Jan., 1963), pp. 92-96.

GHOSH, ARGHYA AND MORITA, HODAKA “Free Entry and Social Efficiency Under Vertical Oligopoly” *The Rand Journal of Economics*, Vol. 38, No. 2 (Summer 2007), pp. 541-554.

GHOSH, ARGHYA AND MORITA, HODAKA “Social Desirability of Free Entry: A Bilateral Oligopoly Analysis” *International Journal of Industrial Organization*, Vol. 25 (2007), pp. 925-934.

GOTZ, GEORG “Market Size, Technology Choice, and the Existence of Free-Entry Cournot Equilibrium” *Journal of Institutional and Theoretical Economics*, Vol. 161 (2005), pp. 503-521.

GUESNERIE, ROGER AND HART, OLIVER “Welfare Losses Due to Imperfect Competition: Asymptotic Results for Cournot Nash Equilibria with and without Free Entry” *International Economic Review*, Vol. 26, No. 3 (Oct., 1985), pp. 525-545.

HÄCKNER, JONAS “A Note on Price and Quantity Competition in Differentiated Oligopolies” *Journal of Economic Theory*, Vol. 93 (2000), pp. 233-239.

LONG, NGO VAN AND SOUBEYRAN, ANTOINE “Existence and Uniqueness of Cournot Equilibrium: A Contraction Mapping Approach.” *Economics Letters*, Vol. 67 (2000), pp. 345-348.

MANKIW, N. G. AND WHINSTON, M. D. “Free Entry and Social Inefficiency” *The RAND Journal of Economics*, Vol. 17, No. 1 (Spring, 1986), pp. 48-58

NOVSHEK, WILLIAM "Cournot Equilibrium with Free Entry" *The Review of Economic Studies*, Vol. 47, No. 3 (Apr., 1980), pp. 473-486.

NOVSHEK, WILLIAM "Finding All n-Firm Cournot Equilibria" *International Economic Review*, Vol. 25, No. 1 (Feb., 1984), pp. 61-70.

NOVSHEK, WILLIAM "On the Existence of Cournot Equilibrium" *The Review of Economic Studies*, Vol. 52, No. 1 (Jan., 1985), pp. 85-98.

OHKAWA, TAKAO; OKAMURA, MAKOTO; NAKANISHI, NORITSUGU AND KIYONO, KAZUHARU "The Market Selects The Wrong Firms in the Long Run" *International Economic Review*, Vol. 46, No. 4 (Nov., 2005), pp. 1143-1165.

OKUNO-FUJIWARA, MASAHIRO AND SUZUMURA, KOTARO "Symmetric Cournot Oligopoly and Economics Welfare: A Synthesis" *Economic Theory*, Vol. 3, (1993), pp. 43-59.

RUFFIN, R. J. "Cournot Oligopoly and Competitive Behavior" *The Review of Economic Studies*, Vol. 38, No. 4 (Oct., 1971), pp. 493-502.

SALOP, S. "Monopolistic Competition with Outside Goods" *Bell Journal of Economics*, Vol. 10, (1979), pp. 141-156.

SEADE, JESUS "On the Effects of Entry" *Econometrica*, Vol. 48, No. 2 (Mar., 1980), pp. 479-490.

SING, NIRVIKAR. AND VIVES, XAVIER "Price and Quantity Competition in a Differentiated Duopoly" *The RAND Journal of Economics*, Vol. 15, No. 4 (Winter, 1984), pp. 546-554.

SUZUMURA, KOTARO AND KIYONO, KAZUHARU "Entry Barriers and Economic Welfare" *The Review of Economic Studies*, Vol. 54, Issue. 1 (Jan., 1987), pp. 157-167.

TIROLE, JEAN The Theory of Industrial Organization, The MIT Press, 14^{th} printing, 2003. ISBN 0-262-20071-6

USHIO, YOSHIAKI "Cournot Equilibrium with Free Entry: The Case of Decreasing Average Cost Functions" *The Review of Economic Studies*, Vol. 50, No. 2 (Apr., 1983), pp. 347-354.

ZANCHETTIN, PIERCARLO "Differentiated Duopoly with Asymmetric Costs" *Journal of Economics & Management Strategy* , Vol. 15, No. 4 (Dec., 2006), pp. 999-1015.

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