

ALTERNATIVE METHODS FOR CALCULATING
OPTIMAL SAFETY STOCK LEVELS

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by
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OPTIMAL SAFETY STOCK LEVELS

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TABLE OF CONTENTS

ACKNOWLEDGEMENTS	ii
LIST OF TABLES	v
LIST OF FIGURES	vi
ABSTRACT	vii
Chapter 1: Introduction and Literature Review	1
Chapter 2: Determination of Safety Stock Levels	5
2-1 Safety stock requirements	5
2-1-1 Cycle time	6
2-1-2 Transit time	6
2-1-3 Service level	7
2-2 Traditional approach to calculate safety stock levels	8
2-2-1 Expected value and standard deviation of lead time	8
2-2-2 Expected value and standard deviation of demand rate	9
2-2-3 Target inventory and actual demand	11
2-3 Alternative safety stock levels calculation	12
2-3-1 Service level adjustment	12
2-3-2 Hybrid method	13
Chapter 3: Case Study Application	16

3-1 Analyzing safety stock requirements	17
3-1-1 Cycle time	17
3-1-2 Transit time	17
3-2 Calculating safety stock levels by traditional approach.....	18
3-2-1 Expected value and standard deviation of lead time.....	18
3-2-2 Expected value and standard deviation of demand rate.....	19
3-2-3 Target inventory and actual demand.....	19
3-3 Alternative safety stock levels	22
3-3-1 Using service level adjustment	22
3-3-2 Using Hybrid method.....	25
Chapter 4: Conclusions and Further Study	33
4-1 Conclusions.....	33
4-2 Further study	34
References.....	36

LIST OF TABLES

Table 1: Shortfalls service level adjustment	24
Table 2: Comparing all methods across all wholesalers	34

LIST OF FIGURES

Figure 1: Direct sourced.....	7
Figure 2: Network sourced.....	7
Figure 3: Service level adjustment.....	13
Figure 4: Gamma distribution.....	14
Figure 5: Shortfall distribution.....	20
Figure 6: Excess distribution	21
Figure 7: Normal vs. empirical	22
Figure 8: Normal service level adjustment.....	23
Figure 9: Normal vs. Gamma.....	25
Figure 10: Hybrid-1 results.....	26
Figure 11: Hybrid-1 vs Normal.....	26
Figure 12: Hybrid-2 results.....	27
Figure 13: Hybrid-2 vs Normal.....	28
Figure 14: Hybrid-3 results.....	29
Figure 15: Hybrid-3 vs Normal.....	29
Figure 16: Hybrid-4 results.....	30
Figure 17: Hybrid-4 vs Normal.....	30
Figure 18: Hybrid method results	31

ABSTRACT

This work considers the problem of safety stock levels for the production of multiple items, each with random demand, across multiple facilities. The traditional methodology for calculating safety stock is discussed and an alternative method for improving service levels is offered. Normal and Gamma distributions are considered to estimate safety stock levels, and the performance of both models, along with a hybrid approach, are tested on a large-scale case study example. The results of this case study indicate that a better inventory policy with less underage and overage cost can be achieved by using the proposed model and solution procedure.

Chapter 1

Introduction and Literature Review

In today's logistics, inventory control plays a crucial role in supply chain management in order to achieve a reliable and efficient supply network. Inventory control is a common problem in different sectors which illustrates how modern life is dependent upon the larger network supplying materials, products and goods. Supply chain variables such as demand, lead time, production time and transportation cost fluctuate due to random perturbations and the nature of the environment. A responsive inventory control can provide a cushion against the variations of these variables. Many different studies have examined different aspects of the relationship between variability and inventory control.

Brander and Forsberg (2006) considered the problem of inventory and scheduling the production of multiple items, each with random demand, on a single facility and presented a model for determination of safety stocks, order-up-to levels and an estimation method for the variance in demand during lead time. Gallego (1990) applied a simulation-based search method for calculating safety stock levels for random demands with constant expected rates. Bourland and Yano (1994) used cyclic schedules for inventory management and developed a planning and control model. They offered a nonlinear mathematical program for determining the cycle length, allocation of idle times, and safety stock levels. Smits et al. (2004) offered a method based on queuing analysis to determine order-up-to levels for a (R, S) system and a fixed cyclic production sequence, where the demand is a compound of renewal process. Sox et al. (1999) presented a method for determination of safety stock levels in models where the production and inventory control are constructed

simultaneously. Doll and Whybark (1973) presented a procedure for determining near optimal frequencies of production for the products and related cycle time. Kelle et al. (1994) extend the procedure of Doll and Whybark (1973) by taking safety stock levels into account and presenting a heuristic for determination of production frequencies and cycle times. They calculated the safety stock by multiplying the safety factor with the standard deviation of the demand during lead time. They used the standard deviation of the one-period ahead forecast and an empirically estimated constant order to estimate the standard deviation of the demand during lead time. Federgruen and Katalan (1996) presented a Poisson or compound Poisson process for each item's demand. They used a fixed rotation sequence and computed base stock levels by polling systems. Madadi et al. (2010) studied an inventory system including a warehouse and multiple suppliers. They pointed out that for each supplier and each warehouse the order size is dependent to the transportation capacity (number of trucks) which is included in the replenishment order size decision. The impact of the order size on the level of transportation capacity is also considered by Alp et al. (2003) under the assumption of certain and random demands as well as random discrete lead times. Yang (2010) provided an algorithm to determine the optimal order quantity, the length of lead time and the number of lots which are delivered from the supplier to the buyer to minimize expected total cost over infinite time horizon. Liao and Shyu (1991) presented a probabilistic inventory model to find the lead time when demand follows a normal distribution. Later, Ben-Daya and Raouf (1994) extended the Liao and Shyu (1991) model by considering both the lead time and the order quantity as decision variables. Ouyang et al. (1996) extended the Ben-Daya and Raouf (1994) model by adding the stock-outs cost when shortages are allowed. Brander and Forsberg (2006) presented safety stock

levels based on estimation of the standard deviation in total demand during a replenishment lead time. If lead time and demand are assumed to be independent random variables the standard deviation in total demand is dependent on the variance and mean of demand and lead time (Ross 1983). Inderfurth and Vogelgesang (2013) considered safety stock problems under stochastic demand and production yield. Braglia et al. (2016) studied a single-vendor single-buyer supply chain problem under continuous review and Gaussian lead-time demand. Moncayo-Martínez and Zhang (2013) considered a heuristic method and Pareto optimality criterion for finding the economic order quantity and safety stock level.

Applications of inventory theory often rely on the normal distribution but there are many other probability density functions (PDF) for statistical description of the demand process. Fortuin (1980) compared five different pdf's (Gaussian, logistic, gamma, log-normal, Weibull) for demand process. Snyder (1984) used the gamma probability distribution in order to cover normal distribution as special cases and also cover the gaps left by this distribution. Chopra et al. (2004) considered safety stocks for gamma lead times and different service levels. Chaturvedi and Martínez-de-Albéniz (2016) found safety stock levels when demand and capacity are gamma distributed.

This work considers the problem of safety stock levels of the production of multiple items, each with random demand, on multiple facilities. The rest of this thesis is organized as follows. In the next chapter, the traditional method for calculating safety stock levels is discussed. The traditional approach, which is referred as the Normal method, assumes that demand during lead time is normally distributed. Moreover, two alternative approaches for non-normal shape probability distributions are introduced in this chapter. In Chapter 3, a case study is described and conducted to demonstrate the performance of the suggested

methods. Chapter 4 presents conclusions from the results of the case study and suggests potential future research extensions.

Chapter 2

Determination of Safety Stock Levels

Inventory control is a problem common to many organizations in different sectors. Inventories are found at manufacturers, wholesalers, retailers, farms, hospitals, universities and governments and are relevant to food, medicines, clothing, and many other areas. Inventory may consist of supplies, raw materials, in-process goods, and finished goods. Generally, the term inventory can be used to describe stock on hand at a given time. Risk and uncertainty impact inventory analysis through many variables, but the most dominant are uncertainty in demand and lead time. Such variations can be absorbed by provision of safety stocks. Safety stocks are inventory in excess of average demand that are kept on hand to avoid stock-outs due to uncertainty in demand and lead time. They are needed to cover the demand during the replenishment lead time in case actual demand exceeds expected demand, or the lead time exceeds the expected lead time. Safety stock calculations are based, in part, on sales forecasts. Since forecasts are seldom exactly correct, the safety stock protects against higher than expected demand levels. Safety stock has two effects on a firm's cost: it decreases the cost of stock-outs, but it increases holding costs (Tersine 1994).

2-1 Safety stock requirements

More formally, the term safety stock in supply chain and logistics is used for describing a level of extra stock that is maintained to mitigate risk of stock-outs (shortfall) due to uncertainties in supply and demand. Safety stock act as a buffer stock in case the sales are greater than planned and or the supplier is unable to deliver the additional units at the

expected time. Safety stock is a function of the production cycle time and transportation time, increasing variability in production and delivery time will increase shortfall risk. Given a desired service level, safety stock calculation must consider the trade-off between holding cost and stock-outs cost.

2-1-1 Cycle time

Manufacturing cycle time (C) refers to the time required or spent to convert raw materials into finished goods. In this study, C is a random variable with mean μ_C and standard deviation σ_C which refers to the time from the start of production to the finish of the final products. It is usually composed of process time, move time, inspection time, and queue time.

2-1-2 Transit time

Transportation plays a vital role in supply chain management. This element not only generates cost in a distribution network, but contributes significantly to the quality of customer service (Tyworth and Zeng 1998). The transit time includes the travelling time from origin to destination. The transit time may change, particularly when the destination is not reached directly but via several hubs.

In general, products could be sourced in two ways, first directly from the manufacturer (Figure 1) and second via distribution centers (Figure 2). Although more complicated situations for transporting products that include travel through multiple distribution centers (DCs) are possible, in this study we consider only these two mentioned possibilities. For directly sourced products the transit time is equal to the transit time between manufacturer and wholesaler (T_1) with mean μ_{T_1} standard deviation σ_{T_1} .

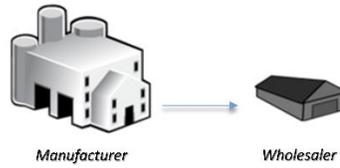


Figure 1: Direct sourced

For products which are sourced through a distribution center, total transit time is equal to transit time from the manufacturer to the distribution center (T_2) plus transit time from distribution center to the wholesaler (T_3), with respective means μ_{T_2} and μ_{T_3} and respective standard deviations σ_{T_2} and σ_{T_3} .

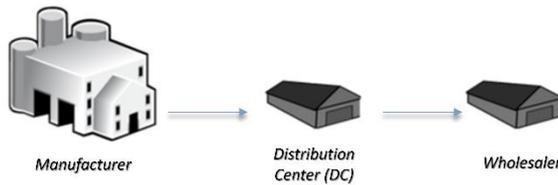


Figure 2: Network sourced

2-1-3 Service level

Service level represents the probability of not observing a stock-out. This percentage is utilized to compute the safety stock. Intuitively, the service level represents a trade-off between the cost of inventory and the cost of stock-outs (which incur missed sales, lost opportunities and client frustration among others). Specifying a service level is an alternative approach to setting inventory levels that does not explicitly account for inventory holding costs. By increasing the service level coefficient, we increase the overage cost due to maintaining excess inventory. The selection of the appropriate service level coefficient is an important decision for supply chain managers. This approach is most often used in applications where stock-outs are considered especially costly to the company.

2-2 Traditional approach to calculate safety stock levels

The traditional approach, which we refer to the Normal method, assumes that demand during lead time is normally distributed, with mean μ_x and standard deviation σ_x (Brander and Forsberg 2006; Silver et al. 1998). The safety stock level of item x (for one certain product in a certain wholesaler) is equal to:

$$SS_x = \sigma_x \Phi^{-1}(\pi_x) \quad (1)$$

where σ_x is the standard deviation of total demand during lead time for item x and $\Phi^{-1}(\pi_x)$ is the standard normal inverse cumulative density function (CDF) at π_x service level. Observe that any inverse CDF could be used here to compute $\Phi^{-1}(\pi_x)$, the normal distribution is just the one typically selected. In order to calculate safety stock levels from (1), we need to estimate σ_x , the standard deviation of the demand during lead time. Ross (1983) showed if lead time and demand are assumed to be independent random variables, the standard deviation of demand during lead time, σ_x , is equal to:

$$\sigma_x = \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2} \quad (2)$$

Where μ_L and σ_L are the expected value and standard deviation of lead time and μ_D and σ_D are the expected value and standard deviation of demand rate for item x .

2-2-1 Expected value and standard deviation of lead time

Expected lead time (μ_L) is equal to the average transit times between sites plus the cycle time for product x at its manufacturer. μ_L is calculated as:

$$\mu_L = \begin{cases} \mu_{T_1} + \mu_C, & \text{if sourced from manufacturer} \\ \mu_{T_2} + \mu_{T_3} + \mu_C + \mu_S, & \text{if sourced from DC} \end{cases} \quad (3)$$

Where μ_S is the mean of the additional time item x spends sitting at the distribution center.

Standard deviation of lead time (σ_L) is equal to square root of the variance of transit times between sites plus the variance of production cycle time. σ_L is calculated by:

$$\sigma_L = \begin{cases} \sqrt{\sigma_{T_1}^2 + \sigma_C^2}, & \text{if sourced from manufacturer} \\ \sqrt{\sigma_{T_2}^2 + \sigma_{T_3}^2 + \sigma_C^2 + \sigma_S^2}, & \text{if sourced from DC} \end{cases} \quad (4)$$

Where σ_S^2 is the variance of the additional time item x spends sitting at the distribution center.

2-2-2 Expected value and standard deviation of demand rate

Assume that forecasts are available for weekly sales. Expected demand rate μ_D is equal to the total expected (forecasted) demand during expected lead time divided by the lead time.

If lead time is computed in days, we can separate the expected lead time into its integer (w) and fractional (r) components as follows:

$$w = \left\lfloor \frac{\mu_L}{7} \right\rfloor \quad (5)$$

$$r = \mu_L - (w * 7) \quad (6)$$

Let $FCST_j$ denote the forecast demand during week j , and let DAY_i denote the percentage of a typical week's demand that is observed on day i . Then, we can compute the percentage of a typical week's demand associated with r days, denoted \mathcal{R} , as follows:

$$\mathcal{R} = \sum_{i=1}^{\lfloor r \rfloor} DAY_i + (r - \lfloor r \rfloor) * DAY_{\lfloor r \rfloor + 1} \quad (7)$$

Assume that the current time is the beginning of week 1. Then expected demand rate μ_D can be computed as:

$$\mu_D = \begin{cases} \frac{1}{\mu_L} (\mathcal{R} * FCST_1), & w = 0 \\ \frac{1}{\mu_L} \left[\sum_{j=1}^w (FCST_j) + \mathcal{R} * FCST_{w+1} \right], & w > 0 \end{cases} \quad (8)$$

Because the forecast demand can vary greatly from week to week, it is typically assumed that the distribution of demand is not stationary across weeks. As a result, in practice it is common to replace the standard deviation of the demand rate σ_D in equation (2) with the root-mean-square error (*RMSE*), a measure of forecast accuracy. Given historical data for the actual sales quantity in week j , denoted SQ_j , the square error for week j , denoted SE_j , is computed as:

$$SE_j = (FCST_j - SQ_j)^2 \quad (9)$$

RMSE can thus be computed over a historical interval of m weeks as follows:

$$RMSE = \sqrt{\frac{1}{m * 7} \sum_{j=1}^m SE_j} \quad (10)$$

We will then substitute $\sigma_D = RMSE$ in all safety stock calculations.

2-2-3 Target inventory and actual demand

The target inventory of item x at the start of an arbitrary week is thus equal to the safety stock level, SS_x , plus the expected demand during the lead time, μ_x . In other words, target inventory TI_x is:

$$TI_x = \mu_x + SS_x \quad (11)$$

Where expected demand during the lead time is equal to the expected demand rate from equation (8) multiplied by the expected lead time from equation (3):

$$\mu_x = \mu_D \mu_L \quad (12)$$

Therefore, if we assume that demand during lead time follows a normal distribution, then target inventory is equal to:

$$TI_x = \mu_x + \sigma_x \Phi^{-1}(\pi_x) = \Phi_{\mu_x, \sigma_x^2}^{-1}(\pi_x) \quad (13)$$

Where $\Phi_{\mu_x, \sigma_x^2}^{-1}(\alpha_x)$ is the normal inverse CDF with mean of μ_x and standard deviation of σ_x , at given service level π_x .

Note that, by assuming a constant lead time (value of zero for standard deviation of lead time) and given actual sales quantities SQ_j for week j we can compute the actual demand during the lead time, denoted AD_x , as follows:

$$AD_x = \begin{cases} \mathcal{R} * SQ_j, & w = 0 \\ \left(\sum_{j=1}^w SQ_j \right) + \mathcal{R} * SQ_{w+1}, & w > 0 \end{cases} \quad (14)$$

Observe that if AD_x is greater than TI_x , then a shortfall (stock-out) would have occurred under target inventory TI_x .

2-3 Alternative safety stock levels calculation

The traditional (Normal) method could be a practical way to set target inventories for problems if demand follows a normal probability distribution, but it is unclear if the Normal approach would work well in situations where demand does not follow a normal distribution. In this thesis, we proposed two methods to find a better-performing safety stock calculation for problems with non-normally distributed demand. The first is Service level adjustment and the other one is Hybrid method.

2-3-1 Service level adjustment

Consider a notional set of demand data as presented in the histogram of Figure 3. We can compute the mean and standard deviation of these data, and have overlaid a curve showing the probability density function (PDF) of a normal distribution with this same mean and standard deviation. Suppose our desired service level was $\pi_x = 0.95$. In Figure 3 the blue line represents the demand level associated with a 95 percent service level, based on the PDF of the normal distribution ($\Phi^{-1}(0.95) = 5.15$). The red line in the figure shows the actual demand level of the empirical data associated with service level $\pi_x = 0.95$ ($F^{-1}(0.95) = 5.53$). Observe that basing the safety stock on the Normal method would lead to stock-outs more frequently than the target $1 - \pi_x = 5$ percent of the time, since demand level 5.15 is associated with empirical cumulative distribution function (CDF) value of 0.93. Because the demand level of 5.53 was associated with 5% stock-outs in the empirical data, the service level adjustment approach would simply use the Normal method, but increase the target service level parameter π_x to be equal to the CDF value

$\Phi(5.53) = 0.97$. That is, we would aim to achieve 5% stock-outs by basing our target inventory on $\pi_x = 0.97$.

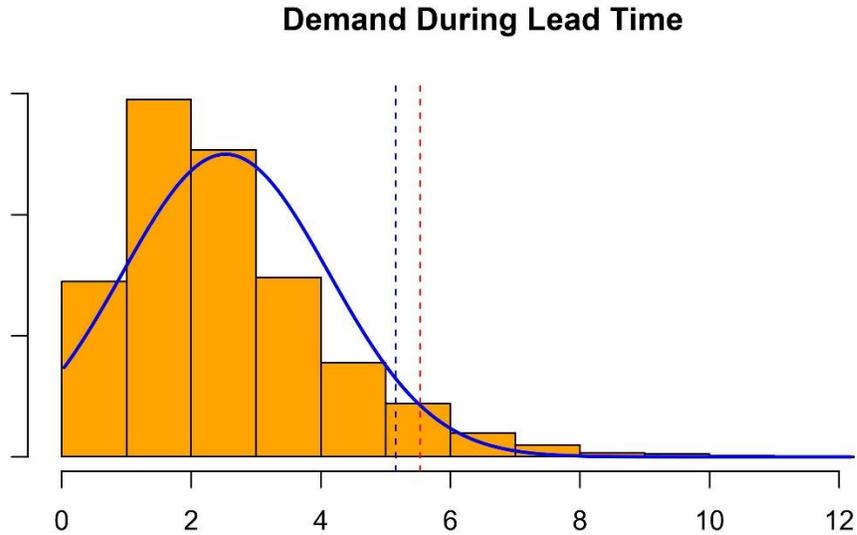


Figure 3: Service level adjustment

2-3-2 Hybrid method

The Hybrid method utilizes a combination of historical and forecast-based data. Instead of assuming that the demand during lead time follows a normal distribution, we instead assume that this demand can be represented by a Gamma distribution, which is much more flexible. The Gamma distribution is specified by two parameters, the shape (α) and rate (β). The mean and standard deviation of a Gamma distribution are given by $\frac{\alpha}{\beta}$ and $\frac{\sqrt{\alpha}}{\beta}$, respectively. Figure 4 shows a Gamma distribution with the same mean and standard deviation as the empirical data, overlaid on the empirical histogram. Observe that the Gamma distribution appears to fit the data much better than does the normal distribution.

Demand During Lead Time

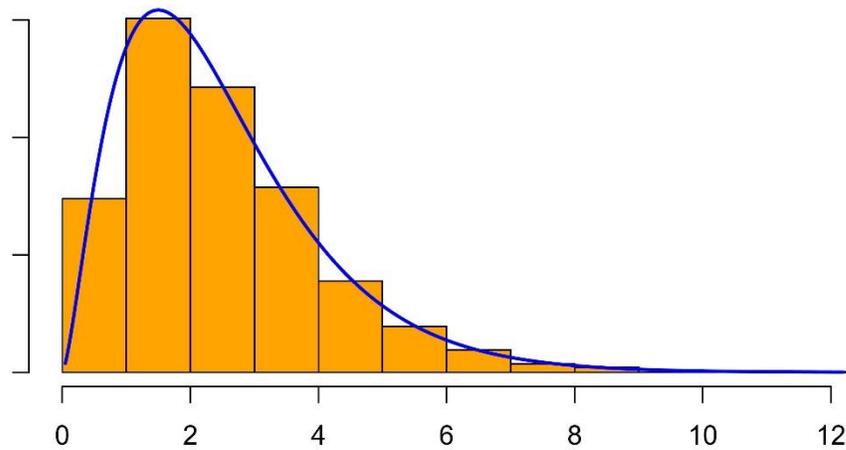


Figure 4: Gamma distribution

Because of the distribution of demand across time is assumed to not be stationary across weeks, we may not want to base our estimates of the mean and standard deviation of demand solely on historical data. Instead, must like the Normal method (which uses a forecast-based mean and a historical-based *RMSE* for the standard deviation), our Hybrid method will fit the Gamma distribution shape and rate parameters based on a combination of historical and forecast-based data. We will consider all four possible combinations:

- Historical-based shape and historical-based rate
- Forecast-based shape and forecast-based rate
- Historical-based shape and forecast-based rate
- Forecast-based shape and historical-based rate

As an example, consider again the data presented in Figures 3 and 4. This notional data has a mean of $\mu = 2.5$ and standard deviation of $\sigma = 1.56$. Assume further that the forecast demand during lead time μ_x is equal to 5 and the *RMSE* estimate of σ_D was equal to 2. Our proposed approach would calculate the Gamma distribution parameters as follows:

- Historical-based shape and historical-based rate:

$$\alpha = \frac{\mu^2}{\sigma^2} = 2.43; \quad \beta = \frac{\mu}{\sigma^2} = 1.03$$

- Forecast-based shape and forecast-based rate:

$$\alpha = \frac{\mu_x^2}{RMSE_x^2} = 6.25; \quad \beta = \frac{\mu_x}{RMSE_x^2} = 1.25$$

- Historical-based shape and forecast-based rate

$$\alpha = \frac{\mu^2}{\sigma^2} = 2.43; \quad \beta = \frac{1}{2} \left(\frac{\alpha}{\mu_x} + \sqrt{\frac{\alpha}{RMSE_x^2}} \right) = 0.63$$

- Forecast-based shape and historical-based rate

$$\beta = \frac{\mu}{\sigma^2} = 1.03; \quad \alpha = \frac{1}{2} (\mu_x \beta + RMSE_x^2 \beta^2) = 4.70$$

Under each combination, we can specify the PDF of the demand during lead time by using the Gamma distribution PDF: $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, where $\Gamma(\alpha)$ is the Gamma function $\int_0^\infty x^{\alpha-1} e^{-x} dx$. The target inventory can then be calculated from equation (13), assuming that TI_x is equal to the inverse CDF of the appropriate Gamma distribution.

Chapter 3

Case Study Application

We partnered with a liquid consumer packaged goods company to examine a case study applying these alternative inventory requirement calculations. This liquid consumer packaged goods company operates a set of production facilities companies which are supporting a set of more than 500 wholesalers. Each wholesaler has an inventory of products supporting its sales to retail locations. For each product that is carried at a given wholesaler, the service level target is to have stock-outs no more than 0.22% of the time. In practice, the company observed that stock-outs occur more frequently than this target level.

Problem context:

Three-week-out distribution plans are made, aiming to keep wholesaler inventory at the target inventory level three weeks from the current date, based on forecast demands, such that a service level target is achieved. Note that different production facilities produce various products with different cycles, for example, some products are only produced every four weeks and others produced every week. Moreover, some wholesalers receive some products directly shipped from the company, while other products are received from a distribution center.

Approach:

Identify frequency of shortfall occurrence using historical data, had target inventories been computed using Normal method. Calculate how much additional safety stock would have

been needed to achieve desired performance. Determine if alternative Hybrid method could have achieved better performance than the Normal method, with respect to service level achieved, relative to target service level.

Output:

Alternative safety stock requirements that can be implemented in existing inventory management systems.

3-1 Analyzing safety stock requirements

We obtained files from the company providing sales quantity and forecasted demand for certain products, at each wholesaler, for each week over a 15-month period. The database includes records across a set of nearly 250,000 unique product-wholesaler pairs.

3-1-1 Cycle time

In order to compute safety stocks, we need to identify the production cycle associated with each product-production facility pair. The available data provided a single cycle time (C), in weeks, for each pair. A constant standard deviation of 2.5 days was assumed for all product-production facility pairs cycle times.

3-1-2 Transit time

The company's products are sourced in two ways, first directly from production facilities (Figure 1) and second, via distribution centers (Figure 2). For directly sourced pairs the transit time is equal to the average transit time between a production facility-wholesaler pair (T_1). The data also provided the transit time standard deviation (σ_{T_1}) for production facility-wholesaler pairs. When transit times were missing from the data, we estimated the

values from other shipments from the production facility to other wholesalers in the same state.

For network sourced pairs, total transit time is equal to the transit time from the production facility to the utilized distribution center (T_2) plus the transit time from the DC to the wholesaler (T_3). This same data set also contained means and standard deviations for transit times between production facilities and DCs, and between DCs and wholesalers. Due to limitations on available data for production cycle frequencies and transit times, our analysis was able to consider approximately 93% of the total sales, as measured by volume, across this data set.

3-2 Calculating safety stock levels by traditional approach

Based on the traditional approach, demand during the lead time is assumed to be distributed normally with standard deviation σ_x . Given desired service level $\pi_x = 99.78\%$ for all products, the safety stock $SS_{x,k}$ for product x in week k is equal to:

$$SS_{x,k} = \sigma_{x,k} \Phi^{-1}(0.9978) = 2.85\sigma_{x,k} \quad (15)$$

where $\sigma_{x,k}$ is the standard deviation of the total demand during lead time for item x in week k , as calculated by equation (2).

3-2-1 Expected value and standard deviation of lead time

We assumed a single production cycle for each product-production facility pair, and a single transit time average between each pair of locations, these values are not assumed to change across different weeks. Also, we assumed the mean of the additional time which item x spends sitting at the distribution center is zero ($\mu_S = 0$). We can then compute the value of μ_L for each product-wholesaler pair using equation (3).

Because we assume that the standard deviation of production cycle time is equal to 2.5 days for all product-production facility pairs, and the standard deviation of the additional delay due to sitting at a DC is 2 days, we can calculate σ_L using equation (4), simplified as:

$$\sigma_L = \begin{cases} \sqrt{\sigma_{T_1}^2 + 6.25}, & \text{sourced from manufacturer} \\ \sqrt{\sigma_{T_2}^2 + \sigma_{T_3}^2 + 6.25 + 4}, & \text{sourced from DC} \end{cases} \quad (16)$$

3-2-2 Expected value and standard deviation of demand rate

The expected demand rate for each product-wholesaler pair at a certain date is calculated by equation (8). Note that because the company utilizes three-week-out distribution plans, $FCST_1$ here refers to the forecast demand for the week three weeks from the current time, $FCST_2$ here refers to the forecast demand for the week four weeks from the current time, etc.

Since three-week-out distribution plans are made and historical sales quantity and forecasted demand are available, we compute σ_D as equal to the root-mean-square error (*RMSE*) of three-week-out forecast versus actual sales in week k over the previous 52 weeks for each product-wholesaler pair using equation (10), with $m = 52$.

3-2-3 Target inventory and actual demand

Given the expected lead times for each product-wholesaler pair, we computed the actual demand during the lead time for each week from historical sales records, as in equation (14). We first evaluated the performance of the Normal method, computing target inventory levels based on equation (13), where, as noted previously, $\Phi^{-1}(0.9978) = 2.85$

for all product-wholesaler pairs. Across the set of more than 5 million records analyzed, we observed the following performance.

At 99.78% service level (i.e., $Z = 2.85$):

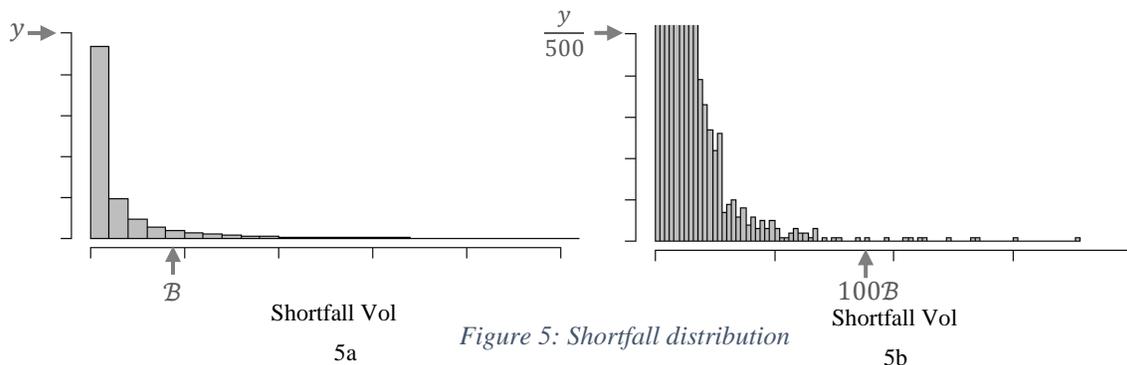
- Records with Shortfall = 0.75%
- Records with Excess = 99.16%
- Records with Equal actual demand and target inventory = 0.09%

Examining the quantities of shortfall and excess:

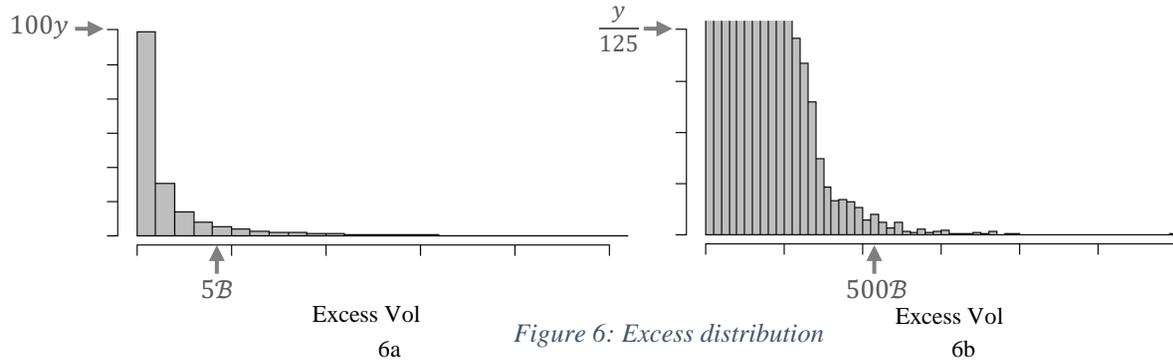
- Average of Shortfall = \mathcal{B} liters
- Average of Excess = $3.42\mathcal{B}$ liters

Observe that the desired 99.78% service level implies stock-outs should occur 0.22% of the time, but our analysis suggests stock-outs would have occurred more than three times as often (0.75%), across this set of records.

The following histograms show the distribution of shortfalls based on the volume of shortage that we compute would have occurred. Figure 5a shows that the preponderance of shortfalls occurred for deficit of \mathcal{B} liters or less and Figure 5b reveals that there are a few records in the tail of the distribution with rare frequencies and high volume.



The histograms in the figure below show the distribution of excess based on the volume of excess that we compute would have occurred across this record set.



Under the Normal method, safety stock is equal to 2.85 times the standard deviation of demand during the lead time. The actual demand during lead time should be less than target inventory 99.78% of the time, if demand during lead time follows a normal distribution. Figure 7 shows a histogram of all empirical data for demand during lead time versus the corresponding normal distribution with the same mean and standard deviation. Observe that the normal distribution does not show a good fit for the data set. In particular, the fit for the right-tail of the distribution, which is the area of greatest interest, since we are most concerned with stock-outs, is poorly the by the normal distribution. Assuming the CDF of the normal distribution implies a target inventory of \mathcal{L} liters ($\Phi_{\mu\sigma^2}^{-1}(0.9978) = \mathcal{L}$) while under the empirical distribution's CDF, a service level of 99.78% would require $2.67\mathcal{L}$ liters ($F^{-1}(0.9978) = 2.67\mathcal{L}$). A substantial increase in the target inventory levels would be needed for this example to achieve the desired service level.

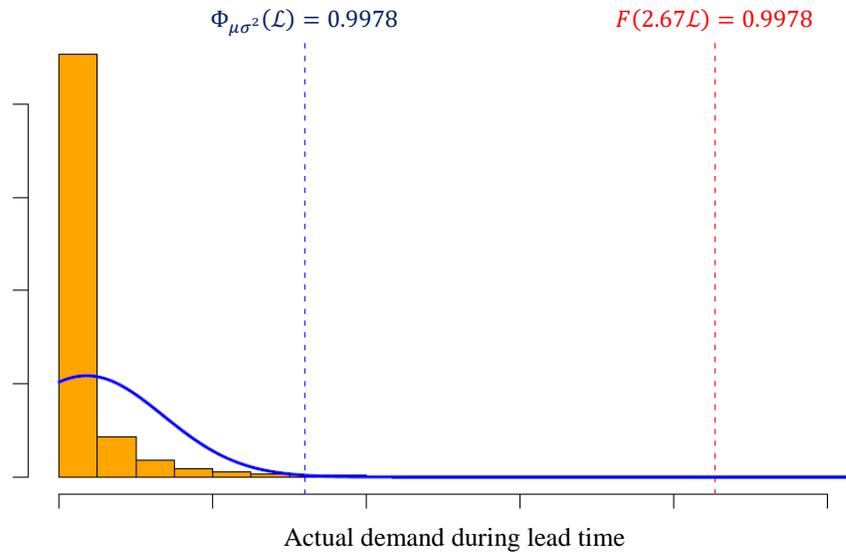


Figure 7: Normal vs. empirical

3-3 Alternative safety stock levels

We observed that the Normal method, assuming a normal distribution for the demand during lead time, can perform poorly when the demand is not normally-distributed. This study offers two methods to try to improve the performance of target inventories. The first method to attempt to reach the company's desired service level is service level adjustment, which is discussed under section 3-3-1. We will also examine a Hybrid method under section 3-3-2.

3-3-1 Using service level adjustment

The company's goal is satisfying a desired service level which implies that stock-outs occur no more than 0.22% of the time. Increasing the target service level under an assumed normal distribution could potentially make up this deficiency. We first examined the performance of uniform changes to the Z -value, $Z = \Phi^{-1}(\pi_x)$, across all records in our data set. As presented in Figure 8 the Normal method assumes $Z = 2.85$, and achieves an overall performance of 0.75% shortfalls, as discussed in section 3-2-3. If we increase the

Z-value to 3.25 for all safety stock calculations, the percentage of shortfalls decreases to 0.50%. In order to achieve the desired service level of 0.22% shortfalls, we need to increase the Z-value to 4.3. Note that, from the normal distribution CDF, $\Phi^{-1}(0.999991) = 4.3$, which suggests that the Normal method would expect that such a large inventory level would only observe 0.0009% stock-outs. The results under this $Z = 4.3$ service level adjustment are as follows:

- Records with Shortfall = 0.22%
- Records with Excess = 99.76%
- Average of Shortfall = 1.40B liters
- Average of Excess = 5.1B liters (average of excess inventory has increased by 49%, relative to current baseline with $Z = 2.85$)

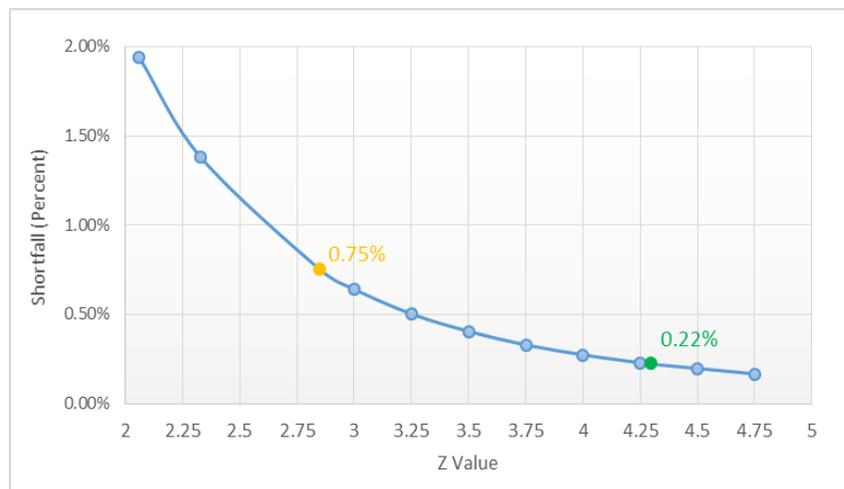


Figure 8: Normal service level adjustment

Performing this analysis across all product-wholesaler combinations masks the variance between combinations. It is possible that the necessary increases in inventory might differ across different types of product-wholesaler pairs. The company has an existing classification that separates products into eleven families of products (segments). We can

perform a similar analysis for each segment at each wholesaler, identifying the Z-value necessary to achieve 0.22% shortfalls for all data records in that segment. Table 1 shows the shortfall percentage at Z-value = 2.85 in a certain wholesaler and the Z-value necessary to achieve no more than 0.22% shortfalls for all data records these segments.

Table 1: Shortfalls service level adjustment

<i>Segment</i>	<i>%Shortfall Z=2.85</i>	<i>Z-value New</i>	<i>%Shortfall Z New</i>
<i>Segment A</i>	1.30%	4.05	0.00%
<i>Segment C</i>	0.38%	3.95	0.19%
<i>Segment F</i>	2.07%	4.15	0.19%
<i>Segment G</i>	1.79%	6.05	0.18%
<i>Segment H</i>	0.51%	3.35	0.17%
<i>Segment I</i>	1.79%	5.65	0.00%
<i>Segment J</i>	0.61%	3.25	0.00%
<i>Segment K</i>	0.26%	3.05	0.13%

The metric the company is using accepts an overall shortfall percentage of 0.22%, it would potentially allow all the shortfalls to occur in one segment, and no shortfalls in any other segment, as long as the aggregate metrics are met.

Unfortunately, this Service level adjustment process is very time-consuming, since one has to examine a range of potential Z-values, updating all calculations for each new Z-value, in order to identify the smallest Z-value that achieves the desired service level (as in Figure 8). The desire to find a procedure that is less computationally expensive, and which can thus be easily extended to any number of segments, led to the creation of the Hybrid method, examined in the next section.

3-3-2 Using Hybrid method

As discussed in chapter 2, our Hybrid method uses a Gamma distribution to estimate the PDF of demand during lead time. Figure 9 shows the histogram of demand during lead time for all records in our data set, along with the normal distribution (red line) and Gamma distribution (purple line) having the same mean and standard deviation.

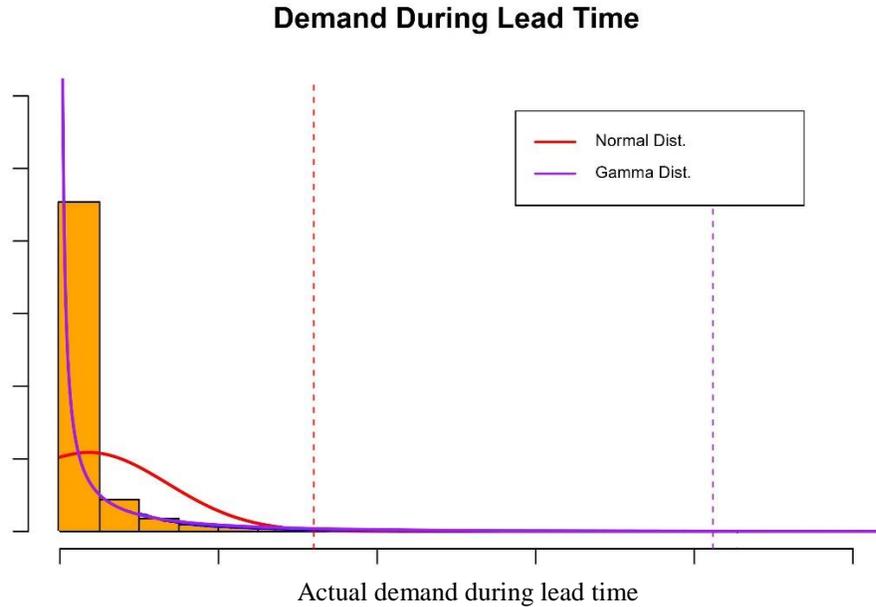


Figure 9: Normal vs. Gamma

Observe that the Gamma distribution fits these data much better than does the normal distribution. At a target 99.78% service level, the Gamma distribution CDF $F_{\text{gamma}}^{-1}(0.9978)$ is equal to 0.969 times the $F_{\text{empirical}}^{-1}(0.9978)$ value.

In order to find the best hybrid approach in a reasonable time, we tried to test all hybrid scenarios on a small scale. For this reason, seven wholesalers were chosen for the preliminary analysis. We next test the four approaches to the Hybrid method that were presented in section 2-3-2 for the seven selected wholesalers.

Hybrid-1

The first approach, denoted Hybrid-1, assumes a historical-based shape and a historical-based rate. Separate Gamma distributions were fit for each of the 11 different product segments in the seven chosen wholesalers, the results for this approach appear in Figure 10. Also, Figure 11 shows inventory excess changes related to the Normal method level.

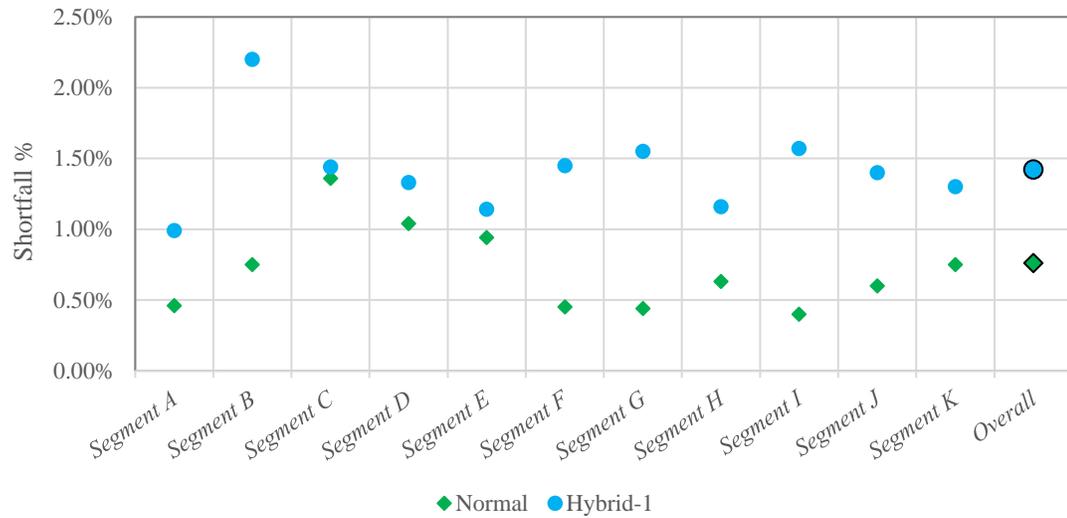


Figure 10: Hybrid-1 vs Normal, Shortfalls

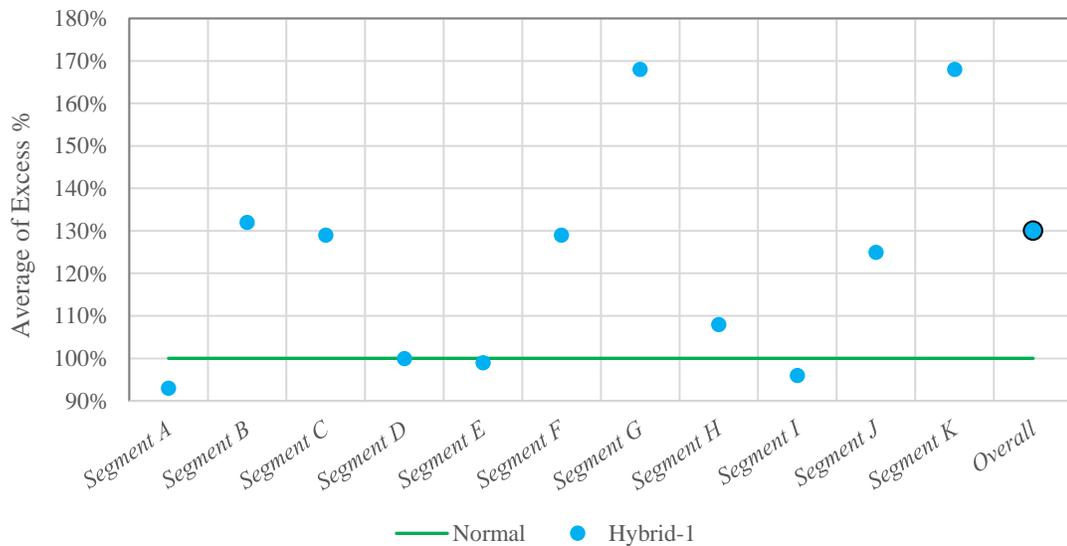


Figure 11: Hybrid-1 vs Normal, Inventory Levels

Observe that approach Hybrid-1 performs quite poorly. In every segment, Hybrid-1 has a greater percentage of shortfalls than does the Normal method. In fact, Hybrid-1 is dominated by the Normal method for many segments, such as segment B, for which Hybrid-1 has a greater shortfall percentage and also an increased inventory level. This poor performance is likely due to the fact that the demand levels are not stationary over time, but instead are forecast to vary significantly over time, due to seasonal effects.

Hybrid-2

The second approach, denoted Hybrid-2 assumes a forecast-based shape and a forecast-based rate. Separate distributions were again fit for each of the 11 product segments in these seven wholesalers, the results of this strategy are presented in Figure 12.

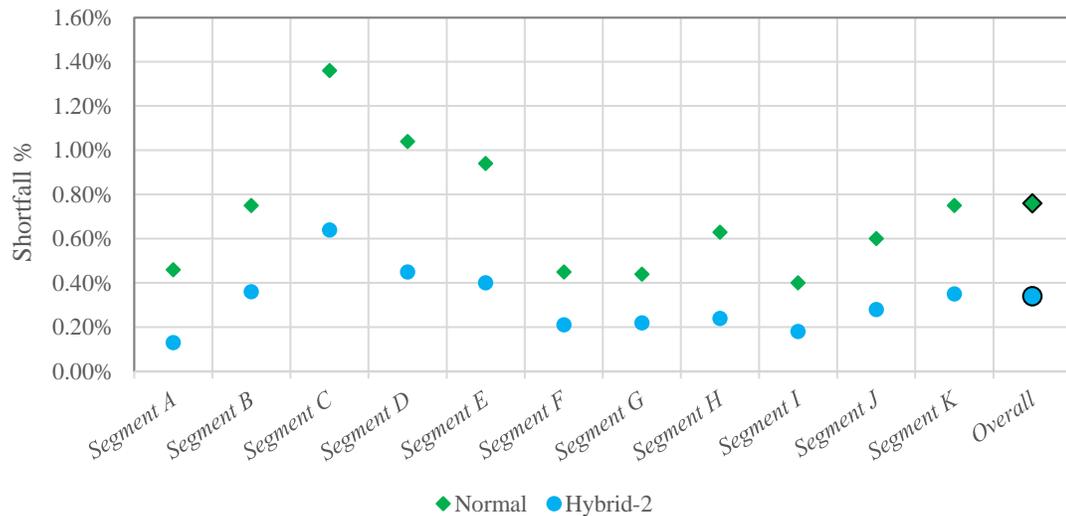


Figure 12: Hybrid-2 vs Normal, Shortfalls

Figure 13 reveals how much excess inventory increases under the Hybrid-2 approach with respect to the Normal method.

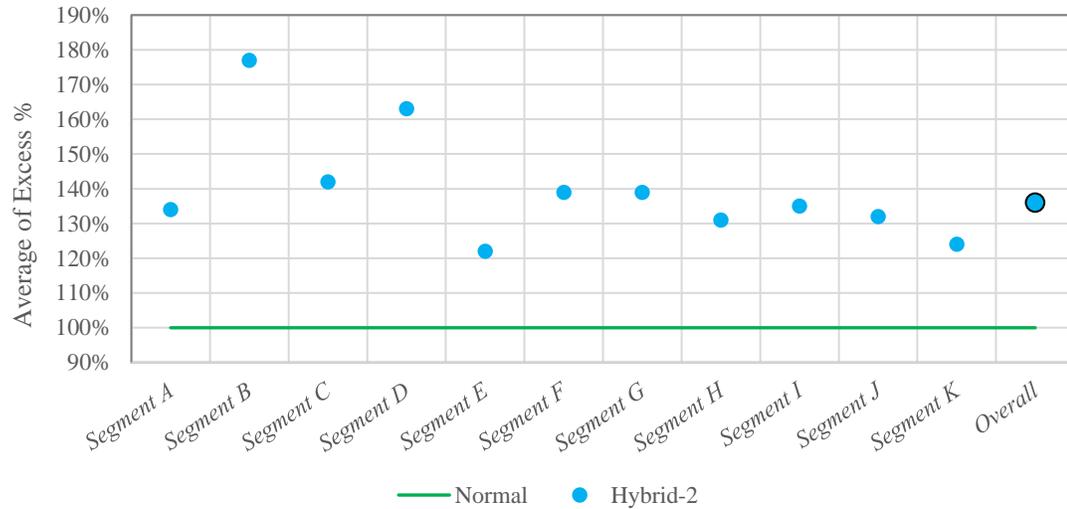


Figure 13: Hybrid-2 vs Normal, Inventory Levels

The company’s goal is a 99.78% service level which implies stock-outs no more than 0.22% of the time. Hybrid-2 method shows better performance in comparison to the traditional Normal method. By this method, the shortfall percentage decreased for every segment, and the service level of four segments are at a value less than or equal to the target level of 0.22%. Moreover, these stock-outs reductions were achieved without imposing large increases to the current inventory level, with the average excess inventory only increased by 40%.

Hybrid-3

The third method uses a Gamma distribution is denoted Hybrid-3, this approach utilizes a historical-based shape parameter and a forecast-based rate parameter. Separate Gamma distributions were fit for each of the 11 different product segments in these seven wholesalers, the results for this approach appear in Figure 14. Figure 15 reveals how much excess inventory increases under the Hybrid-3 approach in comparison with the Normal method.

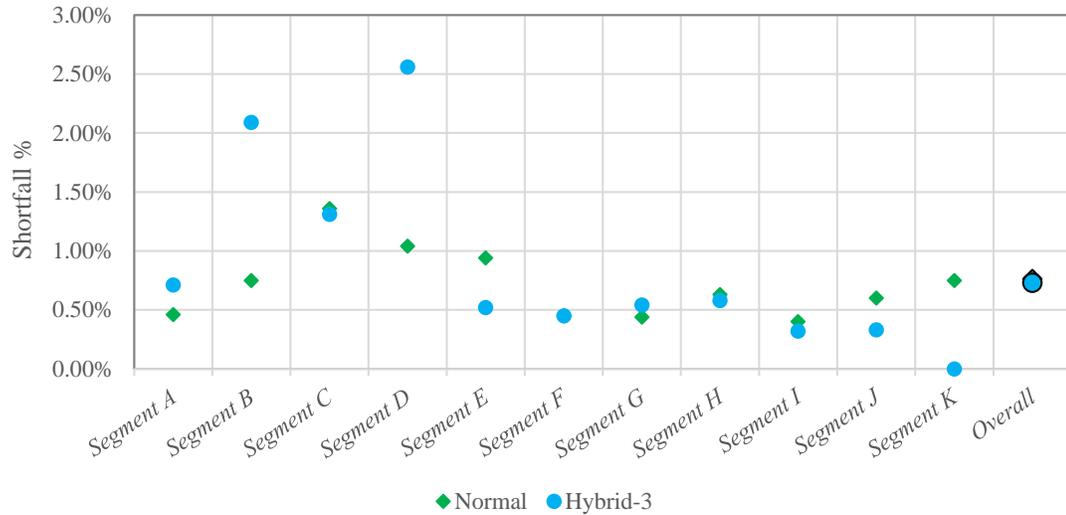


Figure 14: Hybrid-3 vs Normal, Shortfalls

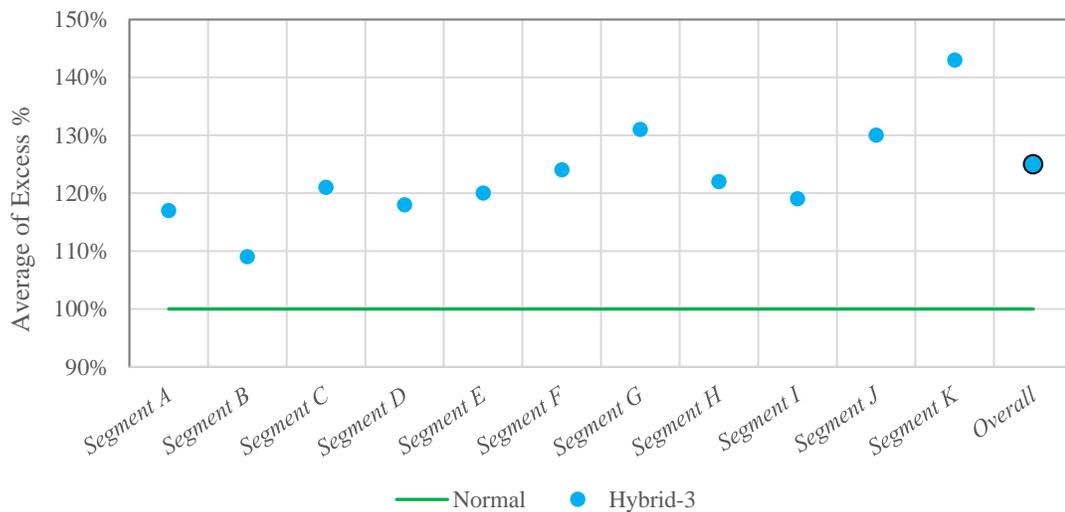


Figure 15: Hybrid-3 vs Normal, Inventory Levels

The results for the Hybrid-3 approach are somewhat mixed. For 6 of the 11 segments, Hybrid-3 generates fewer stock-outs than does the Normal method, another segment has equal performance across the two methods. However, for every segment considered, Hybrid-3 generates increased excess inventory, although these increases are in many cases not terribly large: only 3 segments show an increase in excess inventory of greater than 25% under the Hybrid-3 approach for inventory requirements.

Hybrid-4

The final method examined, denoted Hybrid-4, uses a Gamma distribution with a forecast-based shape parameter and a historical-based rate parameter. Separate Gamma distributions were again fit for each of the 11 different product segments in these seven wholesalers, the results for this approach appear in Figure 16.

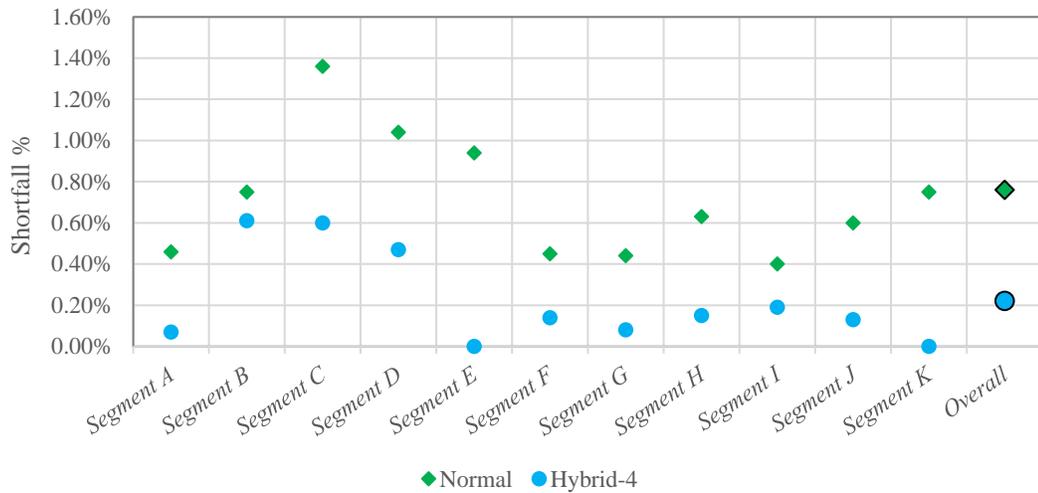


Figure 16: Hybrid-4 vs Normal, Shortfalls

Figure 17 reveals how much excess inventory increases under the Hybrid-4 approach with respect to the Normal method.

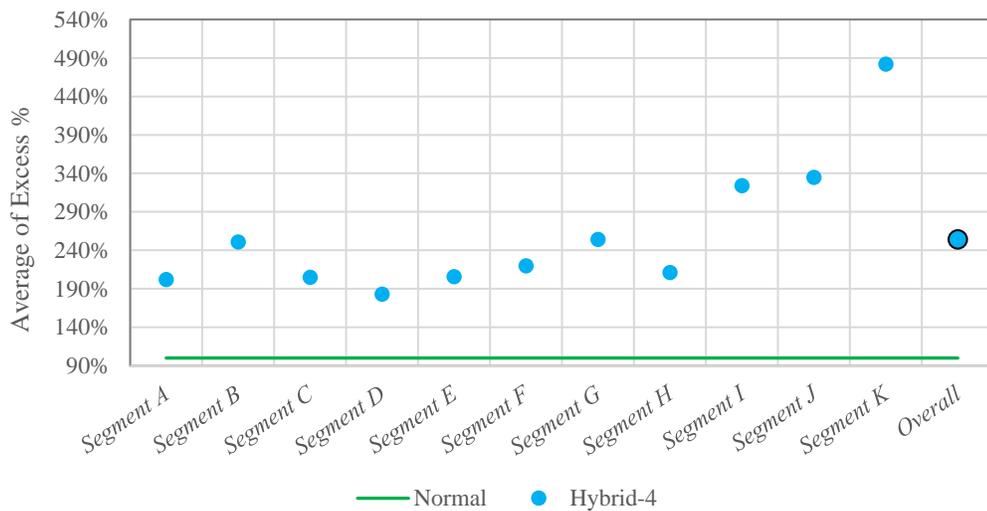


Figure 17: Hybrid-4 vs Normal, Inventory Levels

Examining the performance of Hybrid-4, we observe that it generates very infrequent stock-outs, in fact, for 8 of the 11 segments, the shortfall rate is less than the target of 0.22%. However, these reductions in shortfalls are accomplished via an extremely large increase in average excess inventory, with every segment but one increasing the average excess inventory by more than 100%, relative to the Normal method.

Comparing performance across approaches

Figure 18 illustrates the performance of the different approaches for all segments.

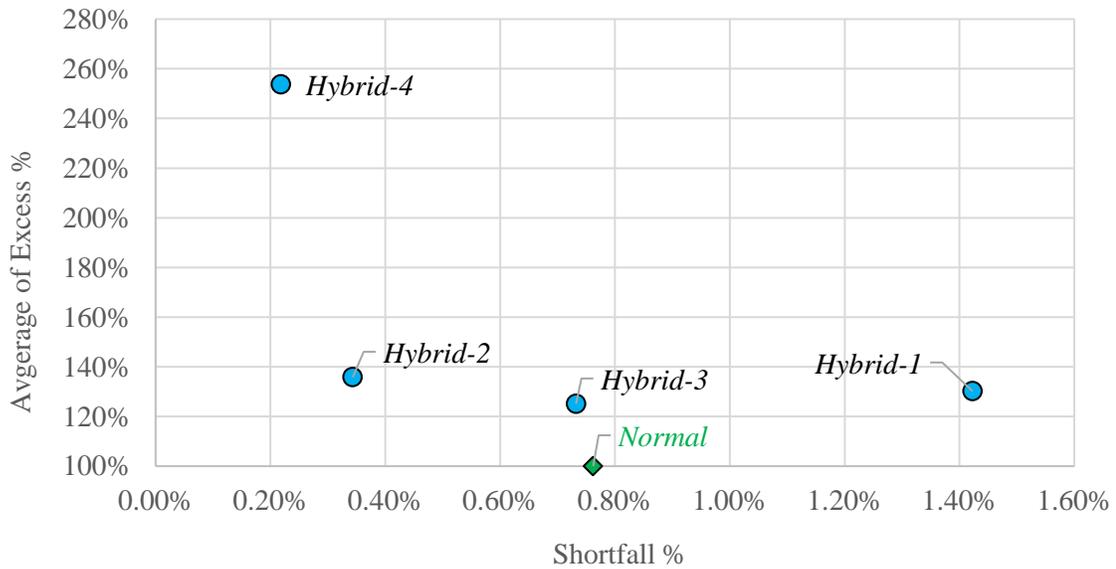


Figure 18: Overall Hybrid method results

Hybrid-1 is a dominated solution, as the Normal method generates fewer stock-outs and reduced excess inventory. Hybrid-3 performs somewhat better, with a shortfall percentage that is very slightly less than the shortfall percentage of the Normal method, but with an average increase in excess inventory of greater than 20%. While Hybrid-4 generates the lowest overall average shortfall percentage, it achieves the performance via an increase of greater than 150% in excess inventory. While the use of a service level parameter suggests

that shortfalls are viewed as much more troubling than inventory holding costs, an increase of this magnitude is so large as to suggest that approach Hybrid-4 is likely infeasible, due to storage space limitations at the wholesalers. Approach Hybrid-2, which utilizes forecast-based values for both the assumed Gamma distribution's shape and rate parameters, is arguably the best-performing approach for this test case data set. It achieves an overall average shortfall rate of 0.38%, while only increasing the average excess inventory by 37%, relative to the baseline Normal approach.

Chapter 4

Conclusions and Further Study

4-1 Conclusions

In this work, we offered an alternative solution for safety stock and inventory level problems. The traditional methodology assumes that demand during lead time follows a normal distribution. A case study, based on data from a liquid consumer packaged goods company, was used to evaluate alternative techniques for setting the target inventory levels. In the first alternative approach studied, referred to as the service level adjustment method, the Z-value associated with the normal distribution was increased until the observed performance in the data set achieved the desired service level of 0.22% stock-outs. This required use of a Z-value equal to 4.3, which, according to the theoretical normal distribution, should generate 0.0009% stock-outs. A drawback to this approach is that it requires extensive computational testing, evaluating the performance of alternative Z-values until one is encountered that achieves the desired service level in the test data set. Such an approach could be very difficult to implement in a situation with a large number of items and suppliers.

A hybrid approach, utilizing the Gamma probability density function, was examined next. A variety of historical-based and forecasted-based combinations was utilized to fit the Gamma distribution's shape and rate parameters. The Hybrid-2 approach, which utilized a forecast-based approach for estimating both the Gamma distribution shape and rate parameters, was observed to perform best for our test case example.

Table 6 shows the results related to the three selected approaches across all wholesalers. As presented in Table 6, the Hybrid-2 approach is able to generate a shortfall rate that is half the value of the Normal method, with a 37% average increase in excess inventory. The service level adjustment technique finds that the shortfall can be reduced to the target level of 0.22%, but only by accepting an average increase of 49% in excess inventory. Since the service level adjustment technique cannot be easily adopted for use in general settings with many segments and data points, we view Hybrid-2, which is a non-dominated solution, as the preferred method, since it can easily and quickly be implemented for large data sets with many product segments.

Table 2: Comparing all methods across all wholesalers

	<i>Normal</i>		<i>Normal: Service level adjustment</i>		<i>Hybrid-2</i>	
	<i>Shortfall</i>	<i>Avg. Excess</i>	<i>Shortfall</i>	<i>Avg. Excess</i>	<i>Shortfall</i>	<i>Avg. Excess</i>
Among All Records	0.75%	<i>B</i>	0.22%	1.49 <i>B</i>	0.38%	1.37 <i>B</i>

In other words, we can use the Gamma distribution with forecast-based parameters to find inventory levels for problems whose demand during lead time distributions have long tail distributions and find solutions that outperform the Normal approach.

4-2 Further study

The current study mainly focuses on Gamma and Normal distributions, therefore the offered approaches are limited to these two distributions. It is possible that by choosing different distributions the results could be improved. Furthermore, quantile forecasting method (Ghodsypour and O'brien 2001) could be another alternative method for finding

safety stock levels in such situations, since quantile regression estimates are often more robust against outliers in the response measurements than the Normal method.

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