THE BANDWIDTH OF A SINGLE LAYER ABSORBING MATERIAL

B. M. Sherman
Assistant Instructor
Department of Electrical Engineering
University of Missouri

D. L. Waidelich, P.E.
Professor of Electrical Engineering
University of Missouri

Reprinted from
Volume I of the Proceedings of the Second Annual RADC International RAM Symposium 9, 10, and 11 June 1959, Rome Air Development Center, Griffth Air Force Base, N.Y.
The Engineering Experiment Station was organized in 1909 as a part of the College of Engineering. The staff of the Station includes all members of the Faculty of the College of Engineering, together with Research Assistants supported by the Station Funds.

The Station is primarily an engineering research institution engaged in the investigation of fundamental engineering problems of general interest, in the improvement of engineering design, and in the development of new industrial processes.

The Station desires particularly to co-operate with industries of Missouri in the solution of such problems. For this purpose, there is available not only the special equipment belonging to the Station but all of the equipment and facilities of the College of Engineering not in immediate use for class instruction.

Inquiries regarding these matters should be addressed to:

The Director
Engineering Experiment Station
University of Missouri
Columbia, Missouri
ABSTRACT

The absorber considered in this paper consists of a single layer of lossy dielectric material backed by a good conductor. It is assumed that both the magnetic and the electric dissipation factors of the dielectric may be greater than zero.

Previous work 1, 2 was done on this structure in which the following special cases were considered: (1) zero magnetic dissipation factor (2) zero electric dissipation factor (3) equal electric and magnetic dissipation factors. For these special cases, methods were given for determining possible values of the magnetic and electric parameters and the resulting thickness. Also considered was the possible bandwidth of the structure, and for this purpose an approximate expression for the bandwidth was derived.

This paper extends the above work to include finite but unequal electric and magnetic dissipation factors. A method is given for determining the parameters and thickness of the material. For certain combinations of electric and magnetic dissipation factors a solution is impossible regardless of the values of other design parameters. The region composed of these combinations has been determined and is presented.

There are indications that the bandwidth of this structure may be large. Since the previous expression assumed a small bandwidth, it seemed desirable to examine more closely the relationship between frequency and reflected energy. The power reflection coefficient as a function of frequency is presented for several combinations of design parameters.

INTRODUCTION

About a year ago certain design requirements 1 were developed for a microwave absorber consisting of a single dielectric layer backed by a good conductor. It was assumed that both the magnetic and electric dissipation factors could be greater than zero. The special cases of zero electric dissipation factor, zero magnetic dissipation factor, and equal magnetic and electric dissipation were considered. Also an expression for the bandwidth was developed which was valid as long as the bandwidth was quite narrow.

The purpose of the paper is to extend the previous work by investigating regions of finite, but unequal electric and magnetic dissipation factors. Also, the relationship between reflected energy and frequency is examined to develop a better method of indicating bandwidth when the bandwidth is large.
The structure previously considered was that of Figure 1 where the dielectric was assumed homogeneous and isotropic throughout, and the conductor was assumed to have infinite conductivity. The dielectric was assumed to have a permeability \( \mu = \mu_0 (\mu' - j\mu'') \) where \( \mu_0 \) is the permeability of free space and \( \mu''/\mu' = \tan \delta \mu \) is the magnetic dissipation factor. The permittivity was given as \( \varepsilon = \varepsilon_0 (\varepsilon' - j\varepsilon'') \) where \( \varepsilon_0 \) is the permittivity of free space and \( \varepsilon''/\varepsilon' = \tan \delta \varepsilon \) is the electric dissipation factor. Using these definitions it was shown that

\[
\tan \delta \mu = \frac{1 - (b/a) \tan \theta}{(b/a) + \tan \theta},
\]

\[
\tan \delta \varepsilon = \frac{1 + (b/a) \tan \theta}{(b/a) - \tan \theta},
\]

\[
\theta = \frac{1}{2} (\delta \varepsilon - \delta \mu),
\]

\[
\mu''/\varepsilon' = \eta^2 \frac{\cos \delta \mu}{\cos \delta \varepsilon},
\]

and

\[
Z_n = \eta \frac{(\cos \theta \sinh 2a - \sin \theta \sin 2b)}{\cosh 2a + \cos 2b} + j \eta \frac{(\cos \theta \sin 2b + \sin \theta \sinh 2a)}{\cosh 2a + \cos 2b}
\]

where \( \sqrt{\mu_0 / \varepsilon_0} \eta / \theta \) is the intrinsic impedance of the dielectric, \( Z_n \) is the normalized impedance at the air-dielectric surface, and \( a + jb \) is the propagation constant for the entire thickness of the dielectric. The conditions which must be fulfilled in order to eliminate reflections were found to be

\[
\sin 2b = - \tan \theta \sinh 2a
\]

and

\[
\eta = \sqrt{\frac{\cosh 2a + \cos 2b}{\cosh 2a - \cos 2b}}
\]

It was also shown that all possible values of \( \delta \varepsilon, \delta \mu \) and \( \theta \) must lie on or within the parallelogram of Figure 2.

**THEORY**

**Permitted Values of Design Parameters**

Equation (6) specifies a condition which must be fulfilled in order to obtain an impedance match at the air-dielectric surface. It may be written in the form,
\[
\sin 2b = - \tan \theta \sinh \left[ \frac{(a/b) 2b}{2} \right] \tag{8}
\]

If \( \theta \) and \((a/b)\) are given, (8) can be solved for \( b \), providing a solution exists. The left side of (8) is a sine curve as shown in Figure 3. The right side of the equation is a curve sloping downward from the origin for positive values of \( \theta \). Every intersection of these curves represents a solution of (8). For a particular value of \( \theta = \theta_0 \) the two curves will intersect at only one point, indicated as "E" in Figure 3. At this point the two curves are equal and have equal slopes. If \( \theta \) is increased beyond \( \theta_0 \), no solutions of (8) will be possible. The conditions at point E may be expressed as

\[
cot \theta_0 \sin 2b_0 = - \sinh \left[ \frac{(a_0/b_0)(2b_0)}{2} \right] \tag{9}
\]

and

\[
cot \theta_0 \cos 2b_0 = - (a_0/b_0) \cosh \left[ \frac{(a_0/b_0)(2b_0)}{2} \right]. \tag{10}
\]

From (9) and (10)

\[
\tan 2b_0 = \frac{1}{(a_0/b_0)} \tanh \left[ \frac{(a_0/b_0)(2b_0)}{2} \right] \tag{11}
\]

and

\[
\tan \theta_0 = \frac{- \sin 2b_0}{\sinh \left[ \frac{(a_0/b_0)(2b_0)}{2} \right]} \tag{12}
\]

If a value of \( b \) is chosen, (11) and (12) can be solved for \( a_0 \) and \( \theta_0 \). With \( a_0 \), \( b_0 \), and \( \theta_0 \) known, \( S_\varepsilon \) and \( S_\mu \) can be found from (1) and (2).

If successive values of \( b \) between \( \pi/2 \) and \( \pi \) are chosen, corresponding values of \( S_\varepsilon \) and \( \theta \) can be found and plotted in the parallelogram of Figure 2. The locus of these points represents a limiting condition for \( S_\varepsilon \) and \( \theta \). For a given \( S_\varepsilon \), the largest value that \( \theta \) may have is located on the locus. Any larger value will not permit a solution of (8). The region in which no solution exists is indicated in Figure 2.

The above method may be applied to values of \( b \) between \( \pi \) and \( 3\pi/2 \) to determine the region in which only one solution exists. This corresponds to finding point G in Figure 3. It is seen that if \( \theta \) is made more negative, only one intersection will occur. In general, this method may be applied to any loop in Figure 3 to determine a region which contains any given number of solutions. Figure 2 indicates six such regions.

**Determination of Design Parameters**

If \( \mu'/\varepsilon' \), \( S_\varepsilon \), and \( S_\mu \) are known, then the electric and magnetic parameters, \( \varepsilon' \), \( \varepsilon'', \mu' \), and \( \mu'' \), will be determined when a particular value is given to any one of them. The relationships among \( \mu'/\varepsilon' \), \( S_\varepsilon \), and \( S_\mu \) may be conveniently represented by plotting \( \mu'/\varepsilon' \) vs. \( S_\mu \) for constant values of \( S_\varepsilon \). In Figure 2 the condition of \( S_\mu = \text{constant} \) would be represented by a straight line parallel to the right hand boundary of the parallelogram and intersecting the vertical axis at
\( \delta_e = \delta_{\mu'} \). The procedure followed in this analysis for a particular curve is to choose successive values of \( \delta_e \) which lie on a constant \( \delta_{\mu} \) line and then calculate the resulting \( \mu'/\epsilon' \). From (4) and (7) when a match is obtained,

\[
\mu'/\epsilon' = \frac{\cosh 2a + \cos 2b}{\cosh 2a - \cos 2b} \cdot \frac{\cos \delta_{\mu}}{\cos \delta_e} \tag{13}
\]

For simplicity let \( 2b = \mu \) and \( a/b = \tan \alpha \). It may be shown that

\[
\delta_{\mu} = \alpha - \theta \tag{14}
\]

and

\[
\delta_e = \alpha + \theta \tag{15}
\]

Then

\[
\mu'/\epsilon' = \left[ \frac{\cosh \left( \frac{u \tan \alpha}{u \tan \alpha} \right) + \cos u}{\cosh \left( \frac{u \tan \alpha}{u \tan \alpha} \right) - \cos u} \right] \cdot \frac{\cos \delta_{\mu}}{\cos \delta_e} \tag{16}
\]

and

\[
\tan \theta = \frac{-\sin u}{\sinh (u \tan \alpha)} \tag{17}
\]

If \( \delta_{\mu} \) and \( \delta_e \) are chosen, then \( \theta, \alpha, \) and \( u \) may be determined from (3), (14), \( \mu' \) and (17). These values may be used in (16) to determine \( \mu'/\epsilon' \).

In Figure 2 it is seen that when \( \delta_e \) is large, there is only one solution of (17) over most of the region of a constant \( \delta_{\mu} \) line. For this case \( \mu'/\epsilon' \) is a single valued function of \( \delta_e \) except for points near the vertical axis where \( \delta_{\mu} = \delta_e \). For points on and very near the axis there are many solutions of (17), and in this small region \( \mu'/\epsilon' \) is multivalued. When \( \delta_{\mu} \) is small, an appreciable part of the operating region is in areas of multiple solutions. For this case \( \mu'/\epsilon' \) is multivalued for most allowed values of \( \delta_e \).

Each curve in Figure 4 approaches the point where \( \mu'/\epsilon' = 1 \) and \( \delta_{\mu} = \delta_e \) after spiralling around it many times. If \( \delta_{\mu} \) is large, the spiral cannot be seen because of its extremely small amplitude. However, for small values of \( \delta_{\mu} \) the spiral is plainly visible as shown for the case when \( \delta_{\mu} = 30^\circ \).

**Reflected Energy**

Previous analytical work on this structure included an approximate expression for the bandwidth when the change in frequency was very small. Since there are indications that a large bandwidth is possible, it is of interest to examine more closely the effect of frequency upon reflected energy.

If \( Z_n \) is the normalized impedance at the air-dielectric surface, then \( \rho \), the voltage reflection coefficient, is given by

\[
\rho = \frac{Z_n - 1}{Z_n + 1} \tag{18}
\]
Figure 1. The Structure of the Absorbing Material

Figure 2. Parallelogram Containing the Possible Values of $\delta_\mu$, $\delta_\varepsilon$, and $\theta$
Figure 3. Intersection Points for Three Values of $\theta$

Figure 4. Curves of Design Parameters for Several Values of $\delta_\mu$
Using the expression for \(Z_n\) given in (5),

\[
\rho = \frac{A-B+jC}{A+B+jC} \tag{19}
\]

where

\[
A = \gamma (\cos \theta \sinh 2a - \sin \theta \sin 2b)
\]

and

\[
C = \gamma (\cos \theta \sin 2b + \sin \theta \sinh 2a)
\]

The power reflection coefficient, \(R\), can be found by squaring the magnitude of \(\rho\).

Then

\[
R = \frac{1 - D}{1 + D} \tag{20}
\]

where

\[
D = \frac{2AB}{A^2 + B^2 + C^2}
\]

It can be shown that

\[
a = \left[ \ell / \mu_o \epsilon_o \mu' \epsilon' \sec \delta_{\mu} \sec \delta_{\epsilon} \sin \frac{1}{2} \left( \delta_{\epsilon} + \delta_{\mu} \right) \right] \omega \tag{21}
\]

and

\[
b = \left[ \ell / \mu_o \epsilon_o \mu' \epsilon' \sec \delta_{\mu} \sec \delta_{\epsilon} \cos \frac{1}{2} \left( \delta_{\epsilon} + \delta_{\mu} \right) \right] \omega \tag{22}
\]

If \(\mu', \epsilon', \delta_{\mu},\) and \(\delta_{\epsilon}\) are constant for all frequencies, then \(a\) and \(b\) are proportional to frequency. While this is probably not true over an extremely wide band of frequencies, it is nearly true over a narrower band and it is assumed to be true in this analysis. Therefore, if a suitable value of \(\mu'/\epsilon'\) is chosen, then (3), (4), (20), (21), and (22) can be used to obtain \(R\) as a function of \((\epsilon' \ell \ell')\) for any given point in the parallelogram of Figure 2.

**Special Case of \(\delta_{\epsilon} = \delta_{\mu}\)**. If \(\delta_{\epsilon} = \delta_{\mu}\) the above equations are considerably simplified. From (3), (4), (19), and (20)

\[
\rho = \frac{\mu'/\epsilon' \sinh 2a - (\cosh 2a + \cos 2b) + j \mu'/\epsilon' \sin 2b}{\mu'/\epsilon' \sinh 2a + (\cosh 2a + \cos 2b) + j \mu'/\epsilon' \sin 2b} \tag{23}
\]

and

\[
R = \left[ \mu'/\epsilon' \sinh 2a - (\cosh 2a + \cos 2b) \right]^2 + \mu'/\epsilon' \sin^2 2b
\]

\[
\left[ \mu'/\epsilon' \sinh 2a + (\cosh 2a + \cos 2b) \right]^2 + \mu'/\epsilon' \sin^2 2b \tag{24}
\]

From (21) and (22)

\[
a = \left[ \ell / \mu_o \epsilon_o \mu' \epsilon' \tan \delta_{\epsilon} \right] \omega \tag{25}
\]

and

\[
b = \left[ \ell / \mu_o \epsilon_o \mu' \epsilon' \right] \omega \tag{26}
\]
As indicated in Figure 4 when $\delta_\mu$ is small, there are several values of $\mu'/\varepsilon'$ which will permit solutions of (8). Each of these values will produce a solution at a different value of $(\varepsilon'/f)$. Figure 5 indicates how the reflection coefficient is affected by changes in $\mu'/\varepsilon'$.

When the frequency is very large, $a$ and $b$ are also large. Then from (24)

$$\frac{1}{R^2} = \left| \frac{\mu'/\varepsilon' - 1}{\mu'/\varepsilon' + 1} \right|^2. \quad (27)$$

If $\mu'/\varepsilon'$ is chosen very close to unity, $R^2$ will approach zero for a sufficiently large frequency and will remain at zero for all larger frequencies.

Effect of Changing $\delta_\varepsilon$ and $\delta_\mu$. Calculations have shown that the effect of increasing $\delta_\varepsilon$ and $\delta_\mu$ is to lower the frequency at which an impedance match is obtained. This can be seen by comparing Figures 5 and 6. The curves of Figure 6, calculated for $\delta_\varepsilon = 60^\circ$, also show the effects of allowing $\delta_\varepsilon$ and $\delta_\mu$ to take unequal values. For each curve $\mu'/\varepsilon'$ was given the value which would cause an impedance match at some frequency. While reflections may often be eliminated at some frequency when $\delta_\varepsilon \neq \delta_\mu$, the results indicate that it is desirable to make $\delta_\varepsilon$ and $\delta_\mu$ equal and large in order to reduce reflections over a large frequency range.

REFERENCES


Figure 5. Variation of the Reflection Coefficient with $\mu'/\varepsilon'$ for the Special Case of $\delta_\varepsilon = \delta_\mu = 30^\circ$

Frequency in KMC/s
$\lambda$ in Centimeters

Figure 6. Variation of Reflection Coefficient with $\theta$ for $\delta_\varepsilon = 60^\circ$

Frequency in KMC/s
$\lambda$ in Centimeters
33. Stability of Laminar Flow in Curved Channels by Chia-Shun Yih, Associate Professor of Engineering Mechanics, University of Michigan and W. M. Sangster, Associate Professor of Civil Engineering, University of Missouri. Reprinted from The Philosophical Magazine, Volume 2, Eighth Series, Page 305, March 1957.

34. Viscosity of Suspensions of Spherical and Other Isodimensional Particles in Liquids by Andrew Pusheng Ting, Chemical Construction Corporation and Ralph H. Luebbers, Professor of Chemical Engineering, University of Missouri. Reprinted from the American Institute of Chemical Engineers Journal, Volume 3, Page 111, March, 1957.


39. Compensation of Sampled-Data Systems by L. M. Benningfield, Assistant Professor of Electrical Engineering, and G. V. Lago, Associate Professor of Electrical Engineering, Reprinted from the National Electronics Conference, Volume XIII, Hotel Sherman, Chicago, Illinois, October 7, 8, 9, 1957, pages 888-897.


43. Network Analyzer Measurement of the Mesh Equivalent of a Complex Circuit by J. C. Hogan, Professor of Electrical Engineering, University of Missouri and V. E. Verrall, Electrical Engineer, Central Illinois Public Service Company.


45. Pressure Changes at Open Junctions in Conduits by William M. Sangster, Associate Professor of Civil Engineering, University of Missouri; Horace W. Wood, Professor and Chairman of Civil Engineering, University of Missouri; Ernest T. Smerdon, Instructor in Civil Engineering, University of Missouri; and Herbert G. Bossy, Highway Research Engineer, U. S. Bureau of Public Roads, Washington, D.C. Reprinted from the Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, 2Q57, June, 1959, HY6.


47. The Bandwidth of a Single Layer Absorbing Material by B. W. Sherman, Assistant Instructor, Department of Electrical Engineering, University of Missouri, and D. L. Waidelich, P.E., Professor of Electrical Engineering, University of Missouri. Reprinted from Volume I of the Proceedings of the Second Annual RADC International RAM Symposium 9, 10, and 11 June 1959, Rome Air Development Center, Griffith Air Force Base, N.Y. *Out of Print
The University of Missouri
SCHOOLS AND COLLEGES

College of Arts and Science
   School of Social Work
College of Agriculture
   School of Forestry
   School of Home Economics
School of Business and Public Administration
College of Education
College of Engineering
   Engineering Experiment Station
Graduate School
School of Journalism
School of Law
School of Medicine
   School of Nursing
School of Veterinary Medicine
Local Identifier Sherman1959

Capture information

Date captured 2018 January

Scanner manufacturer Ricoh
Scanner model MP C4503
Scanning software
Optical resolution 600 dpi
Color settings Grayscale, 8 bit; Color 24 bit
File types Tiff

Source information

Format Book
Content type Text
Notes Digitized duplicate copy not retained in collection.

Derivatives - Access copy

Compression LZW
Editing software Adobe Photoshop
Resolution 600 dpi
Color Grayscale, 8 bit; Color, 24 bit
File types Tiffs converted to pdf
Notes Greyscale pages cropped and canvassed. Noise removed from background and text darkened.
Color pages cropped.