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THE DESIGN OF A SINGLE-LAYER MICROWAVE ABSORBING MATERIAL

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THE DESIGN OF A SINGLE-LAYER MICROWAVE ABSORBING MATERIAL

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Abstract.—The microwave absorbing structure considered here consists of a homogeneous lossy dielectric material backed by a good conductor. Some previous work has been done on obtaining the electric and magnetic parameters of a suitable dielectric assuming that the magnetic dissipation factor was zero. With the widespread use of ferrites at microwave frequencies, design information assuming a finite magnetic dissipation factor is needed and is presented. The special cases of zero electric dissipation factor, zero magnetic dissipation factor and equal electric and magnetic dissipation factors are also considered. Possible values of the electric and magnetic parameters of the absorbing material are obtained and the resulting thickness is presented. A method of choosing the parameters for the smallest thickness of dielectric is given.

Little information has been available as to the possible bandwidth of such a structure. An expression for the bandwidth is derived for a given power reflection coefficient. The parameters of the absorbing material necessary for the greatest bandwidth are indicated.

I. INTRODUCTION

About twenty years ago Dallenbach and Kleinsteuber\(^1\) developed some results for the thickness of a microwave absorbing material consisting of a single dielectric layer backed by a metal reflector. They assumed that the electric dissipation factor was greater than zero while the magnetic dissipation factor was zero. More recent works\(^2-4\) have furthered the development of this structure and have indicated that it is possible to produce materials that have both loss tangents greater than zero. One purpose of this paper is to extend the previous results to cover the case in which either or both loss tangents may be greater than zero.

Theoretically this structure has an advantage in that it can be made to have a zero power reflection coefficient at one frequency and to have a finite bandwidth for a given finite reflection coefficient. Another purpose of this paper then is to present an expression for the bandwidth of this material for a given power reflection coefficient. This will be used to determine what constants of the dielectric layer will produce the greatest bandwidth. Also it is important to determine how the thinnest structure can be made.

II. THEORY

The structure to be considered is that of Fig. 1 where the dielectric is assumed homogeneous and isotropic throughout and the conductor is assumed to have infinite conductivity. A plane electromagnetic wave is incident normally on the surface of the dielectric. The dielectric is assumed to have a permeability of \(\mu_0(\mu' - j\mu'')\) where \(\mu_0\) is the permeability of space and \((\mu''/\mu') = \tan \delta_\mu\) is the magnetic dissipation factor. Similarly the permittivity is \(\varepsilon_0(\varepsilon' - j\varepsilon'')\) where \(\varepsilon_0\) is the permittivity of space and \(\varepsilon''/\varepsilon' = \tan \delta_e\) is the electric dissipation factor. The intrinsic impedance\(^5\) of the dielectric is

\[
\left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{1}{2}} \eta^0 = \left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{1}{2}} \left(\frac{\mu' - j\mu''}{\varepsilon' - j\varepsilon''}\right)^{\frac{1}{2}}
\]
while the propagation constant for the entire thickness of the dielectric is

\[ a + jb = j \omega [\mu_0 \varepsilon_0 (\mu' - j \mu'') (\varepsilon' - j \varepsilon'')]^{\frac{1}{2}} \]

The normalized impedance at the air-dielectric face is obtained and for matching is equal to \((1 + j0)\) as indicated in the Appendix. The results are that the following two equations have to be satisfied.

\[ \sin 2b + \tan \theta \sinh 2a = 0 \]

\[ \eta = \left( \frac{\cosh 2a + \cos 2b}{\cosh 2a - \cos 2b} \right)^{\frac{1}{2}} \]

In (3) and (4) if two of the quantities, \(a, b, \theta, \eta\), are given, the other two may be calculated. Thus if \(a\) and \(\theta\) are chosen, \(b\) can be calculated from (3) and \(\theta\) from (4). It should be noted that there is an infinite number of values of \(b\) which will satisfy (3) for a given \(a\) and \(\theta\). Now from (1) and (2) it is possible to show that

\[ \tan \delta_\mu = \frac{1 - (b/a) \tan \theta}{(b/a) + \tan \theta} \]

\[ \tan \delta_\varepsilon = \frac{1 + (b/a) \tan \theta}{(b/a) - \tan \theta} \]

\[ \theta = (1/2) (\delta_\varepsilon - \delta_\mu) \]

\[ \mu' / \varepsilon' = \frac{\eta^2 \cos \delta_\mu}{\cos \delta_\varepsilon} = \frac{\eta^2 [(b/a) + \tan \theta]}{[(b/a) - \tan \theta]} \]

Hence, as soon as \(a, b, \theta\) and \(\eta\) are known, the dielectric constants, \(\tan \delta_\mu, \tan \delta_\varepsilon\) and \((\mu' / \varepsilon')\) may be calculated from (5), (6), (7) and (8). If now either \(\mu'\) or \(\varepsilon'\) is chosen, the remaining quantity may be calculated from (8). From (2) it is possible to show then that the ratio of the thickness \(l\) of the dielectric to the wavelength \(\lambda\) of a plane wave in free space is

\[ \frac{l}{\lambda} = \frac{a \cos \theta [(b/a)^2 - \tan^2 \theta]^{\frac{1}{2}}}{2 \pi (\mu' \varepsilon')^{\frac{1}{2}}} \]
The band width $B$ is obtained by assuming that $\eta$ and $\theta$ of (1) are independent of frequency and that $a$ and $b$ of (2) are directly proportional to the frequency. The details are presented in the Appendix. If $R$ is defined as the power reflection coefficient, the band width from (41) is

$$B = \frac{2R^4 \cos (\delta_e - \theta) \sinh 2a}{b \cos \theta}$$

Since $\delta_e$ and $\delta_\mu$ may range from zero to ninety degrees, by the use of (7) the possible values of $\theta$ must lie on or within the parallelogram of Fig. 2. Before considering a general point anywhere within the parallelogram, several important special cases will be considered.

### III. SPECIAL CASES

The first special case to be considered is that of zero magnetic dissipation factor or $\delta_\mu = 0$. From (3), (5), and (7)

$$\sin 2b = - \tan \theta \sinh (2b \tan \theta)$$

If $\theta$ is chosen as some fixed value in the first quadrant, the left hand side of (11) can be plotted as a sinusoidal wave as shown in Fig. 3. Since $\tan \theta = \tan (\delta_e/2)$ and since $\tan \theta$ is positive, the right-hand side of (11) can be plotted in Fig. 3 as a curve sloping downward from the origin. For $\tan \theta > 0$ there are a finite number of intersection points of the two curves labeled A, B, C, D. For $\tan \theta$ close to zero, there are many of these intersection points, whereas for $\tan \theta$ sufficiently large there are no intersection points. If a value of $\tan \theta$ is assumed, the corresponding value of $b$ for point A of Fig. 3 can be determined from (11) and then

$$a = b \tan \theta$$

The corresponding electric dissipation factor, $\tan \delta_e$ can be calculated from (6) and is shown in Fig. 4 as a function of $b$ for the points A and B of Fig. 3. The possible range of $\delta_e$ is between zero and 41.3 degrees. The dielectric ratio $\mu'/\varepsilon'$ may be calculated by the use of (4) and (8) and is shown in Fig. 4. The ratio $\mu'/\varepsilon'$ is plotted for $b$ between 90° and 124° while the ratio $\varepsilon'/\mu'$ is plotted for the remaining range of $b$. The thickness parameter $l(\mu'\varepsilon')^{1/2}/\lambda$ may be calculated from (9) and is given in Fig. 4. The range of the thickness parameter
is from 0.25 to 0.5. The band width parameter $B/R^{1}$ may be calculated from (10) and the results are shown in Fig. 4. The maximum value of the bandwidth parameter is 2.28. The limiting values of these parameters occur when the two curves of Fig. 3 become tangent to one another as shown in Fig. 5. This is equivalent to the points A and B of Fig. 3 becoming the one point AB of Fig. 5. This occurs at a limiting value of $\theta = \theta_0$ and no $\theta > \theta_0$ will produce a solution. A method of obtaining $\theta_0$ is outlined in the Appendix. One such point occurs at $\theta_0 = 20.65^\circ$ and $b_0 = 123.93^\circ$ and a second such point is at $\theta_0 = 11.73^\circ$ and $b_0 = 309^\circ$. It is possible to show that $\mu'/\epsilon' = 1.0$ at these points.

The corresponding results for the points C and D of Fig. 3 are shown in Fig. 6. The range of $\delta$, is between zero and 23.5 degrees. The dielectric ratio $\mu'/\epsilon'$ is plotted for $b$ between 270$^\circ$ and 309$^\circ$, while the ratio $\epsilon'/\mu'$ is plotted for the rest of the range of $b$. The range of the thickness parameter is from

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**Fig. 4**—Curves of parameters for the special case $\delta_\mu = 0$ with $90^\circ \leq b \leq 180^\circ$.

**Fig. 5**—Limiting solution for $\delta_\mu = 0$ showing the coincidence of points A and B.
0.75 to 1.0. The maximum value of the band width parameter is 1.73. In comparing Figs. 4 and 6, the range of \( b \) from 90° to 180° has a larger electric dissipation factor, smaller thickness and larger band width than the range from 270° to 360°. Larger ranges of \( b \) could be calculated in a similar fashion but the increase in thickness and decrease in bandwidth probably would not make them useful.

The second special case to be considered is that of zero electric dissipation factor or \( \delta_e = 0 \). From (3), (5) and (7)

\[
\sin 2b = -\tan \theta \sinh (-2b \tan \theta)
\]

Now \( \tan \theta = -\tan (\delta_e/2) \) and \( \tan \theta \) is negative. If \( \theta \) is chosen as some fixed value in the fourth quadrant, the left-hand side of (13) can be plotted as a sinusoidal wave as shown in Fig. 7. The right-hand side of (13) can be plotted as a curve sloping upwards from the origin. Again there are a finite number

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**Fig. 6**—Curves of parameters for the special case \( \delta_\mu = 0 \) with 270° ≤ \( b \) ≤ 360°.

**Fig. 7**—Intersection points for the special case, \( \delta_e = 0 \).
of intersection points, A, B, C, as shown in Fig. 7. When the magnitude of \( \tan \theta \) is small, there are many of these intersection points, but if the magnitude of \( \tan \theta \) is made large enough, no intersection occurs. The limiting value of \( \theta \) is \(-45^\circ\) and thus \( \theta > -45^\circ \). If a value of \( \tan \theta \) is assumed, the corresponding value of \( b \) can be determined from (13) and then

(14) \[ a = -b \tan \theta \]

The magnetic dissipation factor, \( \tan \delta_\mu \), may be calculated by the use of (5) and is shown in Fig. 8 as a function of \( b \) for the point A of Fig. 7. The range of \( \delta_\mu \) is zero to ninety degrees. The dielectric ratio \( \mu'/\epsilon' \) may be found from (4) and (8) and is shown in Fig. 8. This ratio approaches a value of one-third as \( b \) approaches zero degrees. The thickness parameter \( l(\mu'/\epsilon')^{1/2}/\lambda \) may be obtained from (9) and is plotted in Fig. 8. The range of the parameter is from zero to 0.25. The band width parameter \( B/R^{1/2} \) is calculated by use of (10) and is shown in Fig. 8. The maximum value of four occurs for \( b \) approaching zero degrees. The case illustrated in Fig. 8 has the advantages of giving the greatest bandwidth and the smallest thickness although the magnetic dissipation factor has to be quite high particularly for small values of \( b \).

The corresponding results for the points B and C of Fig. 7 are shown in Fig. 9. The range of \( \delta_\mu \) is between zero and 29.5 degrees. The dielectric ratio \( \epsilon'/\mu' \) is plotted for \( b \) between 180 and 217 degrees, while the ratio \( \mu'/\epsilon' \) is plotted for the rest of the range of \( b \). The thickness parameter ranges from 0.5 to 0.75. The maximum of the band width parameter is 1.92. Comparison of Figs. 8 and 9 indicates that the range of \( b \) from zero to 90° has a larger magnetic
Fig. 9—Curves of parameters for the special case \( \delta_e = 0 \) with \( 180^\circ \leq b \leq 270^\circ \).

dissipation factor, smaller thickness and larger bandwidth than the range from \( 180^\circ \) to \( 270^\circ \). Larger ranges of \( b \) would result in larger thicknesses and smaller bandwidths.

The last special case to be mentioned is that in which the angle \( \theta \) equals zero. From (7), the electric dissipation factor is then equal to the magnetic dissipation factor. This is similar to the distortionless case in transmission line theory. From (3) and (5)

\[
\begin{align*}
(15) & \quad b = \pi/2, \pi, 3\pi/2, 2\pi, \ldots \\
(16) & \quad a = b \tan \delta_\mu
\end{align*}
\]

and

\[
\begin{align*}
(17) & \quad \mu'/\varepsilon' = \coth^2 a \\
(18) & \quad l(\mu'/\varepsilon')^{1/\lambda} = b/2\pi
\end{align*}
\]

Also from (10):

\[
(19) \quad B/R^4 = (2/b) \cos \delta_e \sinh 2a
\]

When \( b = (2n-1)\pi/2 \) radians with \( n = 1, 2, 3, \ldots \), \( a \) is given by (16) and

\[
(20) \quad \mu'/\varepsilon' = \tanh^2 a
\]

while the results for the thickness and bandwidth parameters are given by (18) and (19). This case is that indicated by the vertical axis of Fig. 2 on which
\( \delta_\epsilon = \delta_\mu. \) From (16) \( a \) is proportional to the magnetic or electric dissipation factor. With the aid of (19) it is possible to show that for large \( a \)
\[
B / R^4 \approx e^{2a} / a
\]
It thus appears that the bandwidth can be made very wide provided that a material can be made with electric and magnetic dissipation factors large and approximately equal. Such a material would very likely have a large relative permittivity \( \epsilon' \) and a large relative permeability \( \mu' \) and from (18) would have a small thickness.

**IV. GENERAL CASE**

Any point in the parallelogram of Fig. 2 may be obtained by the following procedure. From (3)
\[
\sin 2b = - \tan \theta \sin 2a
\]
The lefthand side of (22) is the sine wave of Fig. 10. Assume that values of \( \theta \) and \( a \) are given. Then the right-hand side of (22) is a horizontal line as shown in Fig. 10. The intersections A, B, C, D of the straight line and the sine wave are possible solutions of this general case. Notice that given values of \( a \) are limited to those satisfying the inequality
\[
\sinh 2a \leq [\cot \theta]
\]
The value of \( b \) may be determined from a plot similar to Fig. 10 or by solving for \( b \) from (22). The magnetic and electric dissipation factors then may be obtained from (5) and (6). The dielectric constant, thickness and band width are found from (8), (9) and (10). Other limitations on the possible choices of \( \theta \) and \( a \) are that the magnetic and electric dissipation factors must be greater than or equal to zero. As an example, the following values may be calculated assuming \( \theta = 10^\circ \) and \( a = 1.15 \). Using point A of Fig. 10, the values \( b = 2.084 \), \( \tan \delta_\mu = 0.342 \), \( \tan \delta_\epsilon = 0.806 \), \( \mu' / \epsilon' = 0.997 \), \( l(\mu' \epsilon')^1 / \lambda = 0.326 \) and \( B / R^4 = 4.21 \) are found. This appears to indicate that larger bandwidths would result if both the electric and magnetic dissipation factors are quite a bit greater than zero.

**APPENDIX**

The normalized impedance is
\[
(24) \quad \eta (\cos \theta + j \sin \theta) \tanh (a + jb) = \eta (\cos \theta \sin 2a - \sin \theta \sin 2b) / \cosh 2a + \cos 2b + j \eta (\cos \theta \sin 2b + \sin \theta \sinh 2a) / \cosh 2a + \cosh 2b
\]
For matching, the impedance of (24) must be equal to \( (1 + j0) \) or
\[(25) \quad \sin 2b + \tan \theta \sinh 2a = 0\]
\[(26) \quad \eta = \left( \frac{\cosh 2a + \cos 2b}{\cosh 2a - \cos 2b} \right)\]

From (25) and (26) the following expressions may be obtained. Let
\[\delta = [(\eta^4 + 1) - 2\eta^2 \cos 2\theta]^2\]
then
\[(27) \quad \sinh 2a = 2\eta \cos \theta /\delta\]
\[(28) \quad \cosh 2a = (\eta^2 + 1) /\delta\]
\[(29) \quad \sin 2b = -2\eta \sin \theta /\delta\]
\[(30) \quad \cos 2b = (\eta^2 - 1) /\delta\]

For the band width derivation assume
\[(31) \quad a = c(\omega_0 + \Delta \omega), \quad b = d(\omega_0 + \Delta \omega)\]
where \(\omega_0\) is the angular frequency at which matching occurs and \(\Delta \omega\) is the change in angular frequency measured from the frequency of matching. From (24)
\[(32) \quad \eta \left( \frac{\sinh 2c\omega_0 \cos \theta - \sin 2d\omega_0 \sin \theta}{\cosh 2c\omega_0 + \cos 2d\omega_0} \right) = 1\]
\[(33) \quad \eta \left( \frac{\sinh 2c\omega_0 \sin \theta + \sin 2d\omega_0 \cos \theta}{\cosh 2c\omega_0 + \cos 2d\omega_0} \right) = 0\]

if \(z = \eta^2 \tanh (a + jb)\) as in (24), then by the use of (27) through (33)
\[(34) \quad D \Re e(z - 1) = (\eta^2 - 1 - 2\eta^2 \sin^2 \theta)(\cosh 2c \Delta \omega - \cos 2d \Delta \omega)\]
\[+ \eta(\eta^2 - 1) \cos \theta \sinh 2c \Delta \omega - \eta(\eta^2 + 1) \sin \theta \sin 2d \Delta \omega\]
\[(35) \quad D \Im m(z - 1) = \eta^2 \sin 2 \theta (\cosh 2c \Delta \omega - \cos 2d \Delta \omega)\]
\[+ \eta(\eta^2 + 1) \sin \theta \sinh 2c \Delta \omega + \eta(\eta^2 - 1) \cos \theta \sin 2d \Delta \omega\]
\[(36) \quad D = (\eta^2 + 1) \cosh 2c \Delta \omega + 2\eta \cos \theta \sinh 2c \Delta \omega\]
\[+ (\eta^2 - 1) \cos 2d \Delta \omega + 2\eta \sin \theta \sin 2d \Delta \omega\]

Expand \(\Re e(z - 1)\) and \(\Im m(z - 1)\) in powers of \(\Delta \omega\). Then by the use of (34), (35) and (36)
\[(37) \quad \frac{1}{4}|z - 1|^2 = \frac{1}{4}[\Re e(z - 1)]^2 + \frac{1}{4}[\Im m(z - 1)]^2\]
\[= A_2(\Delta \omega /\omega_0)^2 + A_3(\Delta \omega /\omega_0)^3 + \ldots\]
where \(4\eta^2 A_2 = \omega_0^2 (c^2 + d^2) (\eta^4 + 1 - 2\eta^2 \cos 2\theta), 2\eta^2 A_3 = \omega_0^3 (c^2 + d^2) \left\{ c \cos \theta \right\}\]
\[\left[ \eta^4 - 1 + 2\eta^2 \cos 2\theta \right] - d \sin \theta \left[ \eta^4 + 1 - 2\eta^2 \cos 2\theta \right]\]
The power reflection coefficient \(R\) is
\[(38) \quad R = \frac{|z - 1|^2}{|z + 1|^2} = \left| \frac{z - 1}{z + 1} \right|^2\]

When \(\omega \approx \omega_0, \Delta \omega \ll \omega_0, |z - 1| \ll 2\) and from (37 and (38)
\[R = \left| \left( \frac{z - 1}{2} \right)^2 - 2\left( \frac{z - 1}{2} \right)^3 + 3\left( \frac{z - 1}{2} \right)^4 - 4\left( \frac{z - 1}{2} \right)^5 + \ldots \right|\]
\[(39) \quad R \approx \frac{1}{4} |z - 1|^2 = A_2 \left( \frac{\Delta \omega}{\omega_0} \right)^2 + A_3 \left( \frac{\Delta \omega}{\omega_0} \right)^3 + \ldots . \]

From (5) and (6) and (31):

\[(40) \quad \cos^2 (\delta_e - \theta) = \frac{1}{1 + (a/b)^2} = \frac{1}{1 + (c/d)^2} \]

Using (27), (40) and the first term of the series for \(R\) in (39)

\[R = \left( \frac{2\Delta \omega}{\omega_0} \right)^2 \frac{\omega_0 d^2 \cos^2 \theta}{4 \cos^2 (\delta_e - \theta) \sinh^2 2a} \]

Let the band width \(B\) be defined as

\[(41) \quad B = \left( \frac{2\Delta \omega}{\omega_0} \right) = \frac{2R^{\frac{1}{2}} \cos (\delta_e - \theta) \sinh 2a}{b \cos \theta} \]

where \(b = d(\omega_0 + \Delta \omega) \approx d\omega_0\)

To obtain \(\theta_0\) and \(b_0\) for a point of tangency as shown in Fig. 5, the two following equations derived from (11) must be solved simultaneously:

\[(42) \quad \sin 2b_0 = -\tan \theta_0 \sinh (2b_0 \tan \theta_0) \]

\[(43) \quad \cos 2b_0 = -\tan^{-2} \theta_0 \cosh (2b_0 \tan \theta_0) \]

From (42) and (43)

\[(44) \quad \cos 2b_0 = -\tan \theta_0 \]

or

\[(45) \quad \cos 2b_0 \cosh (-2b_0 \cos 2b_0) + 1 = 0 \]

Equation (45) is solved for \(b_0\) and \(\theta_0\) is then obtained from (44).

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