A Mixed Model for Variance of Successive Difference of Stationary Time Series: Modeling Temporal Instability in Intensive Longitudinal Data

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The undersigned, appointed by the dean of the Graduate School, have examined the thesis entitled

A MIXED MODEL FOR VARIANCE OF SUCCESSIVE DIFFERENCE OF STATIONARY TIME SERIES: MODELING TEMPORAL INSTABILITY IN INTENSIVE LONGITUDINAL DATA

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

A MIXED MODEL FOR VARIANCE OF SUCCESSIVE DIFFERENCE OF STATIONARY TIME SERIES: MODELING TEMPORAL INSTABILITY IN INTENSIVE LONGITUDINAL DATA

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ABSTRACT

Temporal instability of a stochastic process has been of interest in many areas of behavioral and social science. Recent development in data collection techniques in behavioral and health sciences, such as *Ecological Momentary Assessment* (EMA) enables researchers in these areas to get direct assessment on temporal fluctuations over time for many individuals. Although many researchers have used variance and autocorrelation as a temporal instability measure, their utility and interpretation are limited to index temporal instability. I propose variance of successive difference (VSD) of stationary time series as an overall index of temporal instability such that it is a function of variance and first order autocorrelation of time series. A version of variance of successive difference of unequally spaced time series is also presented as well as distinction of within-day and between-day instability measures. Given that VSD is an individual difference measure, it is proposed that group differences on these indices be explored using a mixed variance model proposed by Hedeker et al. (2008). To illustrate, we present EMA data from a study of negative mood in borderline personality disorder (BPD) and major depressive disorder (MDD) patients, resulting that BPD patients showed more negative affective instability than MDD patients.

1. Introduction

Instability of a stochastic process, as a characteristic of a fluctuating time series, is an interesting topic of research in many areas of behavioral and social science. Nevertheless, because measurement and analysis of such temporal instability require time series data, among social sciences instability of a stochastic process has been studied mainly in economics in which a series of repeated observations are available such as monthly price of a product over many years (e.g., Cuddy & Della Valle, 1979). Recent development in data collection techniques in behavioral and health sciences, such as *Ecological Momentary Assessment* (EMA) (Hufford, Shiffman, Paty, & Stone, 2001; Larson & Csikszentmihalyi, 1983; Scollon, Kim-Prieto, & Diener, 2003), however, enables researchers in these areas to get direct assessment on temporal fluctuations over time for many individuals. As such, development of good measures and statistical models of temporal instability of time series becomes more important than ever in behavioral and social sciences.

Temporal instability is commonly conceptualized as *frequent and extreme fluctuations over time* (Larsen, 1987). Several statistical measures have been suggested as an index of temporal instability of a stochastic process. In the area of mood or affect research, commonly used statistical indices of instability include within-individual variance (Eid & Diener, 1999; Hoffman & Meyer, 2006; Larson, Csikszentmihalyi, & Graef, 1980; Penner, Shiffman, Paty, & Fritzsche, 1994), autocorrelation (Cowdry, Gardner, O'Leary, Leibenluft, & Rubinow, 1991; Stein, 1996), and mean square successive difference (Ebner-Priemer et al., 2007; Jahng, Wood, & Trull, 2008; Trull et al, 2008). However, each of these indices represents different feature of temporal instability and some of them, for example variance and autocorrelation, measure limited features of it. It is important to figure out how these indices are related to each other and which index best measures temporal instability.

Unlike traditional longitudinal data in which the number of waves is often less than 10, EMA data often have more than 50 observations for each individual. Accordingly, EMA data are often called intensive longitudinal data (ILD), a set of time series for many individuals. Each of measures of temporal instability can be calculated by individual and compared across individuals. As such, development and application of statistical models that identify factors responsible for the between-individual variation of the temporal instability is also of great interest.

In this thesis, several issues of measurement and analysis of temporal instability of EMA data are discussed. In chapter 2, we discuss the validity of the variance and autocorrelation as a temporal instability measure. As a suitable temporal instability parameter of stationary time series, the variance of successive difference is then proposed. In chapter 3, we discuss a mixed model that models the fixed and random variance across individuals, and suggest that the model be used on successive difference of stationary time series in order to model random temporal instability across individuals. In chapter 4, the suggested measure and statistical model are applied to an EMA data of negative mood, resulting that individuals in borderline personality disorder group showed significantly higher affective instability than individuals in major depressive disorder/dysthymic disorder group as well as significant individual variance in affective instability within each group.

2. Variance of Successive Difference of Stationary Time Series: A Temporal Instability Parameter

Variance and Autocorrelation of Stationary Time Series

As described, ILD or EMA data can be considered as a set of time series data. As such, statistical indices for time series have been suggested as measures of temporal instability. A time series, a collection of n random variables at time $t_1, t_2, \ldots, t_n$, for any positive integer n, is completely described by the joint distribution function

$$F(c_1, c_2, \ldots, c_n) = P(x_{t_1} \le c_1, x_{t_2} \le c_2, ..., x_{t_n} \le c_n). \tag{1}$$

The one-dimensional distribution function of (1) is expressed as

$$F_t(x) = P\left\{x_t \le x\right\} \tag{2}$$

and its corresponding density function is

$$f_t(x) = \frac{\partial F_t(x)}{\partial x}. \tag{3}$$

The mean of a time series x_t is defined as

$$\mu_t = \mu_{xt} = E(x_t) = \int_{-\infty}^{\infty} x f_t(x) dx, \tag{4}$$

where $f_t(x)$ is a density function of x_t, the observation at time t. The autocovariance is defined as the second moment product:

$$\gamma(s,t) = \gamma_x(s,t) = E[(x_s - \mu_s)(x_t - \mu_t)], \tag{5}$$

for all s and t, i.e., the average cross-product relative to the join density $f(x_s, x_t)$. Note that $\gamma(s,t) = \gamma(t,s)$ for all time points s and t, and for $s = t$, the autocovariance reduces to the variance because

$$\gamma(t,t) = E[(x_t - \mu_t)^2]. \tag{6}$$

From (4) and (5), the autocorrelation is defined as

$$\rho(s,t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\gamma(t,t)}}. \tag{7}$$

The definitions of (4), (5), (6), and (7) are completely general for any $f_t(x)$. If a time series is stationary, i.e., a time series is a finite variance process such that the mean value is constant across time and does not depend on time t and the covariance depends on s and t only through their difference |s - t| (week stationarity), the definitions of (4), (5), (6), and (7) can be simplified as

$$\mu_t = \mu, \tag{8}$$

$$\gamma(h) = E[(x_{t+h} - \mu)(x_t - \mu)], \tag{9}$$

$$\gamma(0) = E[(x_t - \mu)^2]. \tag{10}$$

and

$$\rho(h) = \frac{\gamma(t+h,t)}{\sqrt{\gamma(t+h,t+h)\gamma(t,t)}} = \frac{\gamma(h)}{\gamma(0)}, \tag{11}$$

respectively, where a lag $h = s - t$ (or $s = t + h$), because for a stationary time series, $\gamma(s,t) = \gamma(t+h,t) = E[(x_{t+h} - \mu)(x_t - \mu)] = E[(x_h - \mu)(x_0 - \mu)] = \gamma(h,0) = \gamma(h) = \gamma(-h)$.

Using (11), we can describe autocorrelations of a stationary time series simply in terms of lag h (e.g., $\rho(1)$ or $\rho(2)$), also called order h, without specifying all possible values of s and t (e.g., $\rho(1,2)$, $\rho(2,3)$, $\rho(1,3)$, and so forth). For the first order autoregressive process AR(1), i.e., $x_t = \phi x_{t-1} + w_t$ where $w_t \sim N(0, \sigma^2{}_w)$, autocovariance function (9) can be expressed as

$$\gamma(h) = \frac{\sigma_w^2 \phi^h}{1-\phi^2}\,, \tag{12}$$

and autocorrelation function (11) is given by

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)} = \phi^h. \tag{13}$$

In this case, σ^2 and $\phi = \rho(1)$ are all the parameters required to model the covariance of x_t.

Stationary time series characterize a marginal stable process in the sense that their mean and (auto)covariance do not change over time. We assume the stationarity of time series here and discuss temporal instability in terms of stationary covariance, i.e., time-invariant variance and autocorrelations. In mood studies using EMA data, for example, the (within-individual) variance or standard deviation has been most widely used as an index of affective or mood instability (Eid & Diener, 1999; Farmer et al., 2004; Gorman & Wessman, 1974; Hoffman & Meyer, 2006; Larson et al., 1980; Penner et al., 1994; Stein, 1996; Zeigler-Hill & Abraham, 2006). As seen in (10), the variance of a time series process ($\gamma(0) = \sigma^2$) measures variability or the general dispersion of scores but does not take into account the sequence or the order of a process over time.

On the other hand, autocorrelation measures temporal dependency of order h between scores at time $t + h$ and at time t (i.e., sequences of measurements taken at equally spaced time intervals). An h_{th} order autocorrelation provides an index of how well scores at $t + h$ correlate with scores at time t, i.e., the magnitude of the *persistency* of states between measurement points over the same time interval (Cowdry et al., 1991;

Stein, 1996). Although autocorrelation is a good measure of temporal dependency, it does not reflect extremity or degree of amplitude of fluctuations.

The terms *instability* and *variability* have been used interchangeably in the literature (Larsen, 1987; see also Eid & Diener, 1999; Farmer et al., 2004; Hoffman & Meyer, 2006; Stein, 1996; Woyshville, Lackamp, Eisengart, & Gilliland, 1999). Failing to distinguish temporal instability from variability, however, may lead to confusion of two different characteristics of a time series process. The variability, as quantified by the variance of time series, is the degree of variation of scores around the overall average score and does not necessarily represent temporal instability in and of itself. Instead, the variability of a time-series is a component of temporal instability. Another important component of temporal instability is the temporal dependency of a process, as evidenced by the autocorrelation. High temporal dependency of scores corresponds to low temporal instability. Accordingly, a high value of instability requires not only a high level of variability but also a low level of temporal dependency.

Variance of Successive Difference of Stationary Time Series

Unlike the variance or the autocorrelation, measures based on successive change quantify temporal instability in terms of both variability and temporal dependency over time. We propose the variance of successive difference of stationary time series as a temporal instability parameter. Because $\delta^2 = E\left[(x_t - x_{t-1}) - E(x_t - x_{t-1})\right]^2 = E(x_t - x_{t-1})^2$ for stationary time series, the variance of successive difference (δ^2) can be expressed as a function of the population variance σ^2 and the population first order autocorrelation $\rho(1)$:

$$\begin{aligned}\delta^2 &= E\left[(x_t - x_{t-1}) - E(x_t - x_{t-1})\right]^2 = E(x_t - x_{t-1})^2 = E\left[(x_t - \mu) - (x_{t-1} - \mu)\right]^2 \\ &= E\left[(x_t - \mu)^2 + (x_{t-1} - \mu)^2 - 2(x_t - \mu)(x_{t-1} - \mu)\right] \\ &= \gamma(0) + \gamma(0) - 2\gamma(1) = 2\gamma(0)\left(1 - \frac{\gamma(1)}{\gamma(0)}\right) = 2\sigma^2(1 - \rho(1))\end{aligned} \quad . \tag{14}$$

In (14), the temporal instability parameter δ^2 is explicitly expressed as a function of variability (σ^2) and temporal dependency ($\rho(1)$): A high value of temporal instability of a time series is obtained when the series has high variability and low temporal dependency.

The idea of variance of successive difference is strongly tied with the *mean square successive difference* (MSSD), originally proposed by von Neumann, Kent, Bellinson, and Hart (1941). MSSD is, as its name implies, the average of the squared difference between successive observations at time t+1 and t. The MSSD for a time series of n measurement occasions is given by

$$MSSD = \frac{1}{n-1}\sum_{t=2}^{n}(x_t - x_{t-1})^2 \ . \tag{15}$$

Indeed, (15) is a sample estimator of $E(x_t - x_{t-1})^2$ and if time series are stationary, it is an estimator of the variance of successive difference. von Neumann himself used δ^2 to denote MSSD and used the ratio of δ^2 to sample variance s^2 to detect existing trend or independence of time series (von Neumann, 1941). Notice that if successive observations are stationary and independent, i.e., $\rho(1) = 0$, δ^2 is twice the value of σ^2, as noted by von Neumann et al. (1941).

It is also related to the Durbin-Watson statistic (Durbin & Watson, 1950, 1951), a test statistic used to detect the presence of autocorrelation in the residuals from a regression analysis, which is given by

$$d = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2} . \tag{16}$$

As adopted from von Neumann (1941), d is related to $\frac{\delta^2}{s^2}$ by $\frac{nd}{n-1}$ and ranges from 0 to 4, as expected from (14). Here we propose δ^2 as a parameter of temporal instability to denote the parameter of variance of successive difference, not to denote MSSD. Because of its functional relation to variance (i.e., variability) and first order autocorrelation with which temporal dependency is fully represented for the stationary Markov or first order autoregressive process, δ^2 has usefulness as both statistical and theoretical parameter in itself, not as a supplement for other parameters.

Three estimators of δ^2 are available: (i) the sample variance of successive difference (VSD),

$$\begin{aligned}\hat{\delta}_1^2 &= \frac{1}{n-2}\sum_{t=2}^{n}\left[\left(x_t - x_{t-1}\right) - \frac{1}{n-1}\sum_{t=2}^{n}\left(x_t - x_{t-1}\right)\right]^2 \\ &= \frac{1}{n-2}\sum_{t=2}^{n}\left[\left(x_t - x_{t-1}\right) - \frac{x_n - x_1}{n-1}\right]^2\end{aligned} , \tag{17}$$

(ii) MSSD as in (15),

$$\hat{\delta}_2^2 = \frac{1}{n-1}\sum_{t=2}^{n}(x_t - x_{t-1})^2 , \tag{18}$$

and (iii) a function of the estimates of variance and first order autocorrelation of x_t,

$$\hat{\delta}_3^2 = 2\hat{\sigma}^2(1 - \hat{\rho}(1)) , \tag{19}$$

where $\hat{\sigma}^2 = \frac{1}{n-1}\sum_{t=1}^{n}(x_t - \bar{x})^2$ and $\hat{\rho}(1) = \frac{\sum_{t=2}^{n}(x_t - \bar{x})(x_{t-1} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2}$. The three calculations are identical in population but not necessarily the same in sample. $\hat{\delta}_1^2$ estimates δ^2 in terms of the variance (of successive difference) while $\hat{\delta}_2^2$ estimates δ^2 in terms of the expected value (of squared successive difference). $\hat{\delta}_3^2$ requires a two-step estimation in which variance and autocorrelation are estimated first and then δ^2 is calculated based on the previous estimations. These differences in estimation typify applicable statistical models for individual differences in temporal instability using ILD as discussed in chapter 3.

Adjustment of Successive Differences with Random Time Interval

Indexing temporal instability by (14) assumes that time intervals between consecutive observations are uniform (i.e., $t_i - t_{i-1}$ is the same for all *i*s, where t_i is time at occasion *i*). If this condition is met, the successive difference has the same meaning across all occasions. In some studies using EMA data, however, observations are irregularly spaced over time (e.g., at randomly selected times within a day or as occurs when gaps in the series occur when a participant is unable to respond). Assuming a positive serial autocorrelation between observations at two consecutive time points, that is, the value of an observation at one time point is similar to the value at the next time point, a successive difference, in absolute or squared value, over a longer time interval tends to be greater than that over a shorter time interval.

The autocorrelation function (13) can be alternatively written by

$$\rho(h) = e^{-\theta h} \tag{20}$$

where $\phi = e^{-\theta}$, $\theta > 0$. The autocorrelation function (20) has the advantage that it may be used in continuous time models with random time interval. Here the lag h represents time interval between two consecutive observations which does not necessarily be an integer. By definition, h should be positive and $0 < \rho(h) < 1$. Sample variogram or semi-variogram, for example, can be used to estimate θ and $\rho(h)$ in (20) (see Diggle (1990) or Diggle, et al. (2002) for a brief description of sample variogram).

Note that temporal instability parameter δ^2 in (14) can be generalized to temporal instability of order h, given by

$$\begin{aligned}\delta^2(h) &= E\left[(x_t - x_{t-h}) - E(x_t - x_{t-h})\right]^2 = E(x_t - x_{t-h})^2 = E\left[(x_t - \mu) - (x_{t-h} - \mu)\right]^2 \\ &= E\left[(x_t - \mu)^2 + (x_{t-h} - \mu)^2 - 2(x_t - \mu)(x_{t-h} - \mu)\right] \\ &= \gamma(0) + \gamma(0) - 2\gamma(h) = 2\gamma(0)\left(1 - \frac{\gamma(h)}{\gamma(0)}\right) = 2\sigma^2(1 - \rho(h))\end{aligned} \tag{21}$$

Assuming (20), temporal instability over any continuous time interval h is written by

$$\delta^2(h) = E\left[(x_t - x_{t-h}) - E(x_t - x_{t-h})\right]^2 = 2\sigma^2(1 - e^{-\theta h}). \tag{22}$$

Because h is not fixed, it is difficult to characterize a time series using (22). As such, expression of temporal instability over a certain time interval (e.g., 30 minutes or two hours) may be useful, especially when comparing several time series. For an interval $h = h_0$, temporal instability over time interval h_0 is given by

$$\delta^2(h_0) = 2\sigma^2(1 - e^{-\theta h_0}). \tag{23}$$

Placing (22) into (23) reveals

$$\delta^2(h_0) = \frac{1-e^{-\theta h_0}}{1-e^{-\theta h}} E\left[(x_t - x_{t-h}) - E(x_t - x_{t-h})\right]^2$$
$$= E\left(\sqrt{\frac{1-e^{-\theta h_0}}{1-e^{-\theta h}}}(x_t - x_{t-h}) - E\left[\sqrt{\frac{1-e^{-\theta h_0}}{1-e^{-\theta h}}}(x_t - x_{t-h})\right]\right)^2, \quad (24)$$

meaning that temporal instability over time interval h_0 can be estimated as a variance of weighted successive difference $\sqrt{\frac{1-e^{-\theta h_0}}{1-e^{-\theta h}}}(x_t - x_{t-h})$. The weighted successive difference standardizes a successive difference with random time interval h in terms of the successive difference with a fixed time interval h_0. By weighting, successive differences with different time intervals become comparable and calculable to each other. Possible values for an interval h_0 include mean or median of the time intervals, or other theoretically significant time interval for fluctuations of a response under study. As noted, θ and $\rho(h_0)$ can be estimated using sample variogram, for example.

Corresponding to (17), (18), and (19), $\delta^2(h_0)$ can be estimated by

$$\hat{\delta}_1^2(h_0) = \frac{1}{n-2}\sum_{i=2}^{n}\left[w_i\left(x_i - x_{i-1}\right) - \frac{1}{n-1}\sum_{i=2}^{n} w_i\left(x_i - x_{i-1}\right)\right]^2, \quad (25)$$

$$\hat{\delta}_2^2(h_0) = \frac{1}{n-1}\sum_{i=2}^{n} w_i (x_i - x_{i-1})^2, \quad (26)$$

and

$$\hat{\delta}_3^2(h_0) = 2\hat{\sigma}^2(1-\rho(h_0)), \quad (27)$$

respectively, where x_i is the observation at time t_i, $w_i = \sqrt{\frac{1-e^{-\theta h_0}}{1-e^{-\theta h_i}}}$, $h_i = t_i - t_{i-1}$, and $\rho(h_0) = e^{-\hat{\theta} h_0}$. (17), (18), and (19) are special cases of (25), (26), and (27), respectively.

For a time series with an equally spaced time interval, (25), (26), and (27) are equal to (17), (18), and (19), respectively, because $h_0 = h_i$ and $w_i = 1$, for all i.

Within-day versus Between-day Instability

In EMA study, responses are often collected several times within days for many days. In such a design, researchers may be interested in both within-day and between-day fluctuations of responses. The distinction between within-day and between-day fluctuations enables us to focus on within-day and between-day instability, both conceptually and analytically. Some individuals may be characterized by within-day instability reflecting fluctuations of relatively short duration within days (e.g., hourly or diurnal affect instability), whereas others may be typified by between-day instability producing fluctuations across days.

Following this distinction, within-day $\hat{\delta}_W$ and between-day $\hat{\delta}_B$ can be calculated as indices of within- and between-day instability, respectively. The calculations are based on the within-day successive difference, $x_{ij} - x_{(i-1)j}$, and the between-day successive difference, $\bar{x}_j - \bar{x}_{j-1}$, where x_{ij} is the observation at time t_{ij} for i_{th} occasion within j_{th} day, and $\bar{x}_j$ is the averaged score of observations within j_{th} day, or $\bar{x}_j = \frac{1}{n_j}\sum_{i=1}^{n_j} x_{ij}$ (n_j is the number of observations within j_{th} day). Notice that in such cases the entire time series of x_{ij} may not be stationary because of between-day fluctuations. If this is the case, by separating a whole time series into a set of subseries of within-day fluctuations, we may construct a longitudinal panel data in which within-day covariance structure is stationary.

The within-day instability, therefore, can be thought of as a function of within-day variance and within-day first order autocorrelation that typifies the residual within-day covariance structure of the panel data. Accordingly, $\rho(1)$ (for fixed time interval) and $\rho(h_0)$ (i.e., θ) (for random time interval) of within-day fluctuations can be estimated by modeling residual within-day covariance structure of the panel data.

Because responses are often collected at randomly prompted time within days, (25), (26), and (27) can be applied to estimate within-day temporal instability over time interval h_0, i.e., $\hat{\delta}^2_{1W}(h_0)$, $\hat{\delta}^2_{2W}(h_0)$, and $\hat{\delta}^2_{3W}(h_0)$, such that

$$\hat{\delta}^2_{1W}(h_0) = \frac{1}{N-J-1}\sum_{j=1}^{J}\sum_{i=2}^{n_j}\left[w_{ij}\left(x_{ij} - x_{(i-1)j}\right) - \frac{1}{N-J}\sum_{j=1}^{J}\sum_{i=2}^{n_j} w_{ij}\left(x_{ij} - x_{(i-1)j}\right)\right]^2 , \quad (28)$$

$$\hat{\delta}^2_{2W}(h_0) = \frac{1}{N-J}\sum_{j=1}^{J}\sum_{i=2}^{n_j} w_{ij}(x_{ij} - x_{(i-1)j})^2 , \quad (29)$$

and

$$\hat{\delta}^2_{3W}(h_0) = 2\hat{\sigma}^2_W(1-\rho_W(h_0)) , \quad (30)$$

where N is the total number of observations, J is the number of days, $w_{ij} = \sqrt{\frac{1-e^{-\theta h_0}}{1-e^{-\theta h_{ij}}}}$, $h_{ij} = t_{ij} - t_{(i-1)j}$, $\hat{\sigma}^2_W = \frac{1}{N-J}\sum_{j=1}^{J}\sum_{i=1}^{n_j}(x_{ij} - \bar{x}_j)^2$, and $\rho_W(h_0) = e^{-\hat{\theta} h_0}$. On the other hand, between-day temporal instability $\hat{\delta}^2_{1B}$, $\hat{\delta}^2_{2B}$, and $\hat{\delta}^2_{3B}$ can be estimated from (17), (18), and (19) such that

$$\hat{\delta}^2_{1B} = \frac{1}{J-2}\sum_{j=2}^{J}\left[\left(\bar{x}_j - \bar{x}_{j-1}\right) - \frac{1}{J-1}\sum_{j=2}^{J}\left(\bar{x}_j - \bar{x}_{j-1}\right)\right]^2 \quad (31)$$

$$\hat{\delta}^2_{2B} = \frac{1}{J-1}\sum_{j=2}^{J}(\bar{x}_j - \bar{x}_{j-1})^2 , \quad (32)$$

and

$$\hat{\delta}_{3B}^2 = 2\hat{\sigma}_B^2(1-\hat{\rho}_B(1)), \qquad (33)$$

where $\hat{\sigma}_B^2 = \frac{1}{J-1}\sum_{j=1}^{J}(\overline{x}_j - \overline{x})^2$, $\hat{\rho}_B(1) = \frac{\sum_{j=2}^{J}(\overline{x}_j - \overline{x})(\overline{x}_{j-1} - \overline{x})}{\sum_{j=1}^{J}(\overline{x}_j - \overline{x})^2}$, and $\overline{x} = \frac{1}{J}\sum_{j=1}^{J}\overline{x}_j$.

3. A Mixed Model for Variance of Successive Difference

Although the discussion so far has focused on the operationalization of individual-level measures of instability, the focus of many EMA studies is on between-individual factors that account for variation of temporal instability. Researchers may, for example, be interested in whether differences in mood instability exist between individuals with and without a certain diagnosis. Such questions can be addressed via a two-step or two-stage approach: Indices of instability (e.g., $\hat{\delta}_3^2$) are calculated first for each individual, and then the significance of individual covariates that predict individual temporal instability are tested using regression or ANOVA. For example, we may first obtain $\hat{\delta}_3^2$ from $\hat{\sigma}^2$ and $\hat{\rho}(1)$ for each individual and then test group difference in $\hat{\delta}_3^2$ using ANOVA.

This two-step approach, however, is not without limitations: First, error in parameter estimation for each individual (e.g., $\hat{\delta}_{3k}^2$, for individual k) is not taken into account in the two-step approach. Initially, parameters for each individual are estimated based on observations at the individual level. Each estimate, however, has its own error of estimation. In the second step, these estimates are used as observations in the model and group mean difference is tested only against the standard error of estimation of the group mean difference (of the estimates). This ignores the fact that the estimates, rather than the "real" data, are being used. If the second step estimation is conducted without considering error of estimation at the first step, the standard error of estimation of group mean differences will be downwardly biased, the test statistic will therefore be too large and the probability of making a type I error will be inflated. In addition, the two-step approach does not take into account the different number of observations across individuals, a situation which often occurs in EMA data. Individuals in a study often vary in the number of measurement occasions assessed (e.g., due to missing data) and therefore, an individual's temporal instability score should contribute differentially to the estimation of the group mean, for example.

To address the problems listed, linear mixed models (Laird & Ware, 1982; Goldstein, 1986) or multilevel regression models (Goldstein, 1995; Raudenbush & Bryk, 1986, 2002) can be used in general. A set of time series or intensive longitudinal data (e.g., EMA data) discussed so far have a two-level structure: Observations are nested within individuals. Each measurement occasion is viewed as a sample from a random distribution of observations for an individual, and each individual coefficient or parameter is viewed as a draw from a random distribution of individual parameters. Linear mixed models estimate variance (and covariance) of the random intercept and regression effects (i.e., the parameters of regression function) as well as the fixed parameters of regression function. In case of modeling variability or temporal instability, as the variance of successive difference, we are interested in systematic and random variation of variance across individuals. In such cases, variance should be modeled as a function of covariates and random error. Recently, Hedeker, Mermelstein, and Demirtas (2008) proposed a mixed variance model that models both the fixed and the random variance as described below.

Variance Function Models and the Mixed Variance Model

Modeling variance as a function of within-individual or between-individual covariates has been suggested by many researchers (Goldstein, 1995; Hedeker et al., 2008; Hedeker & Mermelstein, 2007; Pinheiro & Bates, 2000; Raudenbush & Bryk, 1987; Raudenbush & Bryk, 2002). A basic form of variance function is suggested in the context of the single level linear regression model (Carroll & Ruppert, 1988; Davidian & Giltinan, 1995; Harvey, 1976). Suppose there is a response variable y_i whose expected value is a function of a vector of covariates $\mathbf{x_i}$ and its corresponding parameter $\boldsymbol{\beta}$. A general specification for variance function g of y_i is

$$Var(y_i) = \sigma^2 g^2(\mu_i, \mathbf{v_i}, \boldsymbol{\eta}) \tag{34}$$

where $\mu_i = E(y_i) = f(\mathbf{x_i}, \boldsymbol{\beta})$, $\mathbf{v_i}$ is a vector of covariates predicting the variance of y_i, and $\boldsymbol{\eta}$ is a parameter vector relating μ_i and $\mathbf{v_i}$ to the variance of y_i. Initially, the variance functions were suggested to relate mean and variance for heteroscedastic regression models. Examples of variance function g in this context include the power-of-the-mean model

$$g(\mu_i, \boldsymbol{\eta}) = \mu_i^{\eta}, \tag{35}$$

the exponential model

$$g(\mu_i, \boldsymbol{\eta}) = \exp(\mu_i \eta), \tag{36}$$

and a two-component model

$$g^2(\mu_i, \boldsymbol{\eta}) = \eta_1 + \mu_i^{2\eta_2}. \tag{37}$$

Notice that if $\eta = 1$ in (35), σ^2 is the coefficient of variation. Variance of y_i is sometimes thought to depend on a covariate. In this case, an exponential model can be used as

$$g(\mathbf{v_i}, \boldsymbol{\eta}) = \exp(\eta v_i) \tag{38}$$

This single level variance function has been extended to the mixed models by several researchers (Davidian & Giltinan, 1995; Goldstein, 1995; Hedeker & Mermelstein, 2007; Pinheiro & Bates, 2000; Raudenbush & Bryk, 2002). Assume a two-level regression model for y_{ik}, given by

$$y_{ik} = f(\mathbf{z_{ik}}, \mathbf{w_k}, \boldsymbol{\gamma}, \mathbf{u_k}) + e_{ik}, \tag{39}$$

where $\mathbf{z_{ik}}$ is a vector of within-individual covariates at occasion i for individual k, $\mathbf{w_k}$ is a vector of between individual covariates for k, $\boldsymbol{\gamma}$ is a vector of fixed effects and $\mathbf{u_k}$ is a vector of random effects for k. The variance function g of this model is expressed as

$$Var(y_{ik}) = \sigma^2 g^2(\mu_{ik}, \mathbf{v_{ik}}, \boldsymbol{\eta}) \tag{40}$$

where $\mu_{ik} = E(y_{ik}) = f(\mathbf{z_{ik}}, \mathbf{w_k}, \boldsymbol{\gamma}, \mathbf{u_k})$, $\mathbf{v_{ik}}$ is a vector of within- and/or between-individual covariates predicting the variance of y_{ik}, and $\boldsymbol{\eta}$ is a parameter vector relating μ_{ik} and $\mathbf{v_{ik}}$ to the variance of y_{ik}. $\mathbf{v_{ik}}$ may or may not include $\mathbf{z_{ik}}$ and $\mathbf{w_k}$. As in a single level case, variance function g may have the form of either the power-of-the-mean model or the exponential model or a combination of both (Pinheiro & Bates, 2000). A major difference between the variance functions of the single level regression and the multilevel linear mixed model is that $\mathbf{v_{ik}}$ may include one or more between-individual covariates to explain the heterogeneity of variance in a mixed variance function but not in a single level variance function. Notice that the parameter $\boldsymbol{\eta}$ in variance function (40) is also fixed as in the single level variance function.

The variance function (40) can be, however, further extended to a function with random parameters $\boldsymbol{\delta}_\mathbf{k}$. Hedeker et al. (2008) suggested one such model. Consider a multilevel model (39) where $f(\mathbf{z_{ik}}, \mathbf{w_k}, \boldsymbol{\gamma}, \mathbf{u_k}) = \mathbf{z}'_\mathbf{ik}\mathbf{W_k}\boldsymbol{\gamma} + \mathbf{z}'_\mathbf{ik}\mathbf{u_k}$, written by

$$y_{ik} = \mathbf{z}'_\mathbf{ik}\mathbf{W_k}\boldsymbol{\gamma} + \mathbf{z}'_\mathbf{ik}\mathbf{u_k} + e_{ik} \tag{41}$$

where, $\mathbf{W_k} = \mathbf{I}_P \otimes \mathbf{w'_k}$, and P is the number of random effects. Hedeker et al. (2008) suggested a mixed variance function for the variance of y_{ik} such that

$$\sigma^2_{e_{ik}} = \exp(\mathbf{v'_{ik}}\boldsymbol{\eta} + \delta_k), \qquad (42)$$

where $\mathbf{v_{ik}}$ is a vector of within- and/or between-individual covariates predicting the variance of y_{ik}, $\boldsymbol{\eta}$ is a vector of fixed effects of $\mathbf{v_{ik}}$, and δ_k is a random intercept distributed as normal (across individuals) with mean 0 and variance σ^2_δ. Taking logs both side of (42) yields $\log(\sigma^2_{e_{ik}}) = \mathbf{v'_{ik}}\boldsymbol{\eta} + \delta_k$, meaning that the logarithm of variance is a linear function of $\mathbf{v_{ik}}$, i.e., (42) is a loglinear mixed model. Note that the estimated variances of (42) are guaranteed to be positive. Because δ_k is normally distributed, the variance follows lognormal distribution across individuals. The regression model (41) and the variance model (42) can be jointly written as

$$y_{ik} = \mathbf{z'_{ik}}\mathbf{W_k}\boldsymbol{\gamma} + \mathbf{z'_{ik}}\mathbf{u_k} + \exp\left\{\frac{1}{2}(\mathbf{v'_{ik}}\boldsymbol{\eta} + \delta_k)\right\}\varepsilon_{ik} \qquad (43)$$

where ε_{ik} is a standard normal. The model (43) can be estimated by (marginalized) maximum likelihood (ML) method and the ML estimates can be easily obtained by the NLMIXED procedure in SAS, for example.

Although Hedeker et al. (2008) restricted (43) to one random effect, this model can easily be extended to a model with two or more random effects by replacing (43) with

$$\sigma^2_{e_{ti}} = \exp(\mathbf{v'_{ti}}\boldsymbol{\eta} + \mathbf{s'_{ti}}\boldsymbol{\delta_i}), \qquad (44)$$

where $\boldsymbol{\delta_i}$ is a vector of random effects of $\mathbf{s_{ti}}$.

Mixed Variance Model of Successive Difference

To model individual difference in temporal instability, (43) can be applied to successive difference. Assuming stationarity of y_{ijk}, that is the observation at i_{th} occasion within j_{th} day for individual k, we may model within-day temporal instability over time interval h_0 by fitting

$$w_{ijk}(y_{ijk} - y_{(i-1)jk}) = \exp\left\{\frac{1}{2}(\mathbf{v}'_{\mathbf{k}}\boldsymbol{\eta} + \delta_k)\right\}\varepsilon_{ijk} \tag{45}$$

where $w_{ijk} = \sqrt{\dfrac{1-e^{-\theta h_0}}{1-e^{-\theta h_{ijk}}}}$, $h_{ijk} = t_{ijk} - t_{(i\text{-}1)jk}$, and ε_{ijk} is a standard normal. $\mathbf{v_k}$ is a vector of between-individual covariates predicting the variance of $w_{ijk}(y_{ijk} - y_{(i-1)jk})$, $\boldsymbol{\eta}$ is a vector of fixed effects of $\mathbf{v_k}$, and δ_k is a random intercept distributed as normal (across individuals) with mean of 0 and variance of σ_δ^2. By stationarity assumption, we can drop mean function from (43), because the expected value of (weighted) successive difference in stationary time series is zero. In addition, subscript i and j in covariate $\mathbf{v}$ can be dropped out because variance does not change in stationary time series. Notice that, for within-day observations with fixed time interval, $w_{ijk} = 1$ because $h_{jik} = h_0$ = constant. (45) models temporal instability that varies as a function of individual covariates $\mathbf{v_k}$ as well as random variation δ_k.

In the same fashion, between-day temporal instability may be modeled as

$$(\overline{y}_{jk} - \overline{y}_{(j-1)k}) = \exp\left\{\frac{1}{2}(\mathbf{q}'_{\mathbf{k}}\boldsymbol{\lambda} + \upsilon_k)\right\}\xi_{jk}, \tag{46}$$

where ξ_{jk} is a standard normal. $\mathbf{q_k}$ is a vector of between-individual covariates predicting the variance of $(\bar{y}_{jk} - \bar{y}_{(j-1)k})$, $\boldsymbol{\lambda}$ is a vector of fixed effects of $\mathbf{q_k}$, and υ_k is a random intercept distributed as normal (across individuals) with mean of 0 and variance of σ_{υ}^2.

When stationary mean assumption is not realistic in an EMA data, we may fit linear or nonlinear trend by individual and model (45) and (46) on the residuals from the fitted trends. When modeling within-day trend by individual, residuals may be obtained from a linear mixed model where within-day intercept, trend or both randomly vary across days. The following section will present a real-life example of the analysis of negative affective instability. The study collected ecological momentary assessment (EMA) data from psychiatric outpatients with borderline personality disorder or major depressive disorder, respectively. This study focused on quantifying and comparing affective instability in the two groups of psychiatric outpatients.

4. Negative Affect Instability in Borderline Personality Disorder and Major Depressive Disorder Patients: An Example Analysis

Method

The data described are from an ongoing study of affective instability in borderline personality disorder (BPD) patients and discussed in more detail in Trull et al. (2008). Affective instability, especially negative affect, is considered as a core feature of BPD -- one that distinguishes this disorder from other disorders like depressive disorders. Participants were recruited from community mental health outpatient clinics through flyers. Potential participants were screened through chart review, and final eligibility for the study was determined through administration of DSM-IV-TR (APA, 2000) Axis I and Axis II structured interviews. Two groups of outpatients were entered into the study, following determination of eligibility into one of two patient groups. The data presented in this paper are based on 84 participants: 46 who met DSM-IV-TR (APA, 2000) diagnostic criteria for BPD and who endorsed the diagnostic feature of affective instability; and 38 who met DSM-IV-TR diagnostic criteria for current major depressive disorder or for current dysthymic disorder (MDD/DYS) and did not report affective instability.

All participants were assigned an electronic diary programmed to prompt them to rate a series of mood descriptors from the Positive and Negative Affect Scales-Extended version (PANAS-X; Watson & Clark, 1999). Each mood item was rated on a 1-5 scale, where 1 = not at all and 5 = extremely. Each participant was asked to rate his or her mood as it had been since the last assessment completed on the electronic diary (i.e., the last prompted response). Here we discuss only the negative affect composite score based on 21 items, for illustrative purposes. Items came from PANAS 10-item negative affect scale, supplemented by additional items tapping fear, hostility, and sadness (Watson & Clark, 1999). The mean of the 21 items was used as the negative affect score. The study design called for six randomly prompted assessments per each day over approximately 4 weeks of consecutive days. However, as expected, the number of days of assessments per each person and the number of assessments per each day differed (Days: Median (Mdn) = 29, Interquartile Range (IQR) = 2, per each person; Assessments: Mdn = 5, IQR = 1, per each day). In total, 76 to 186 assessments (Mdn = 153, IQR = 24) per each person were conducted. The prompted time for assessment (and thus time intervals between successive assessments) within a day varied randomly across days and across people.

Of primary interest is whether the BPD group demonstrates more temporal negative mood instability than the MDD/DYS group, as predicted. Before investigating group differences using a linear mixed variance model of successive difference, however, we illustrate how the issues discussed previously were addressed in this data set, including: (1) within-day vs. between-day instability, (2) adjustment of successive difference for random time interval, (3) characteristics of measures for variability, temporal dependency, and instability.

Results

Calculations of variance, autocorrelation, and successive difference. Because the current EMA data were collected several time a day over many days for many individuals and we are interested in group difference of both between- and within-day temporal instability, we calculated between- and within-day variance, autocorrelation, and variance of successive difference. Before we calculate those indices, within-day and between-day polynomial trends are subtracted from the raw scores of negative affect by individual in order to make the series of scores stationary. Linear and quadratic trends were fitted on daily mean scores of affect (i.e., $\bar{y}_{jk} = \frac{1}{n_{jk}} \sum_{i=1}^{n_{jk}} y_{ijk}$, where y_{ijk} is the negative affect score at occasion i on j_{th} day and n_{jk} is the number of observations within day j, for individual k) by regression for each individual, and then residuals of daily mean ($\dot{r}_{jk}$) were obtained. Subsequently, between-day variance and autocorrelation are calculated for individual k as $\hat{\sigma}^2_{Bk} = \frac{1}{J_k - 1} \sum_{j=1}^{J_k} (\dot{r}_{jk} - \ddot{r}_k)^2$ and $\hat{\rho}_B(1)_k = \frac{\sum_{j=2}^{J_k} (\dot{r}_{jk} - \ddot{r}_k)(\dot{r}_{(j-1)k} - \ddot{r}_k)}{\sum_{j=1}^{J_k} (\dot{r}_{jk} - \ddot{r}_k)^2}$, respectively, where $\ddot{r}_k = \frac{1}{J_k} \sum_{j=1}^{J_k} \dot{r}_{jk}$ and J_k is the number of days for individual k, while between-day variance of successive difference was calculated as in (33), i.e., $\hat{\delta}^2_{3Bk} = 2\hat{\sigma}^2_{Bk}(1 - \hat{\rho}_B(1)_k)$.

Linear and quadratic within-day trends were fitted by the following linear mixed model.

$$y_{ijk} = \beta_{0k} + \beta_{1k} t_{ijk} + \beta_{2k} t^2_{ijk} + \gamma_{0jk} + e_{ijk} , \tag{47}$$

where $\gamma_{0jk} \sim N(0, \sigma^2_{\gamma 0k})$, $e_{ijk} \sim N(0, \sigma^2_{ek})$, and $COV(e_{ijk}, e_{i'jk}) = e^{-\theta_k h_{(i-i')jk}}$ for $h_{(i-i')jk} = | t_{ijk} - t_{i'jk} |$, $i \neq i'$, meaning that within-day trend of negative affect for individual

k is a function of time of the day (t_{ijk}) and quadratic time of the day whose level (intercept) may vary day by day (i.e., $\sigma^2_{\gamma 0k} \neq 0$). (47) was fitted for each individual k and then within-day variance, autocorrelation, and variance of successive difference were obtained from σ^2_{ek} , $e^{-\theta_k h_0}$ and $\hat{\delta}^2_{3Wk} = 2\sigma^2_{ek}(1 - e^{-\theta_k h_0})$. The time interval h_0 was set to 2.339 hours, the median of the total within-day time interval across all individuals.

Comparison among indices. Means and standard deviations of the six indices, i.e., within- and between-day variance of successive difference (VSD), variance (VAR), and autocorrelation (ACORR), are presented in Table 1 and correlations among the six indices are presented in Table 2.

Individuals in BPD group showed higher temporal instability and variability and lower temporal dependency than individuals in MDD/DYS group in both within- and between-day fluctuations (Table 1). Although t-tests suggest significant mean difference in between-day affective instability ($t_{65.7} = 2.67$, $p < .01$) and variability ($t_{60.4} = 2.30$, $p < .05$) and no other significant difference, tests on the difference of the mean of estimates, as a two-step approach, are likely to be biased. We leave our conclusion on the mean difference of affective instability open until we apply a mixed model of variance.

Table 1
Means and Standard Deviations for Within-Day and Between-Day Indices of Negative Affect Instability, Variability, and Temporal Dependency

		Within-Day			Between-Day		
Group	N	VSD	VAR	ACORR	VSD	VAR	ACORR
MDD/DYS	38	0.19(0.20)	0.12(0.11)	0.21(0.17)	0.12(0.13)	0.09(0.10)	0.19(0.29)
BPD	46	0.28(0.27)	0.18(0.16)	0.17(0.18)	0.25(0.29)	0.18(0.25)	0.14(0.27)
Total	84	0.24(0.24)	0.15(0.14)	0.19(0.18)	0.19(0.14)	0.14(0.20)	0.16(0.28)

Note. VSD = Variance of Successive Difference; VAR = Variance; ACORR = Autocorrelation.

Table 2
Correlations among Within-Day and Between-Day Indices of Negative Affect Instability, Variability, and Temporal Dependency

	Within-Day			Between-Day		
Variables	VSD	VAR	ACORR	VSD	VAR	ACORR
Within-Day						
VSD	-----					
VAR	0.93**	-----				
ACORR	-0.16	0.12	-----			
Between-Day						
VSD	0.75**	0.81**	0.14	-----		
VAR	0.63**	0.70**	0.15	0.91**	-----	
ACORR	0.10	0.12	0.04	0.08	0.32*	-----

Note. VSD = Variance of Successive Difference; VAR = Variance; ACORR = Autocorrelation.
* $p < .01$, ** $p < .001$.

Scatter plots for values of selected individual instability measures are presented in Figure 1. Although VSD and VAR were positively correlated with each other, several individuals showed a high score on one measure but a low or moderate score on the other measure. For example, an individual, marked as P1 in Figure 1, showed a relatively higher within-day VAR of negative affect over time (.41, sixth highest among all individuals) than within-day VSD (.47, 14th highest).

The original and detrended within-day time series plots of negative affect over the course of the study for this person are presented in Figure 2A and 2B, respectively, showing the characteristics of the individual’s mood change. Although individual P1 had several days with high within-day dispersion of negative mood, resulting in a large variance, the within-day changes are more sequential than random, captured in high within-day temporal dependency (ACORR = .43, see Figure 1B). Consequently, this person produced a lower rank in VSD scores than in VAR.

In contrast, individual P2 in Figure 1 produced high scores on both variability and

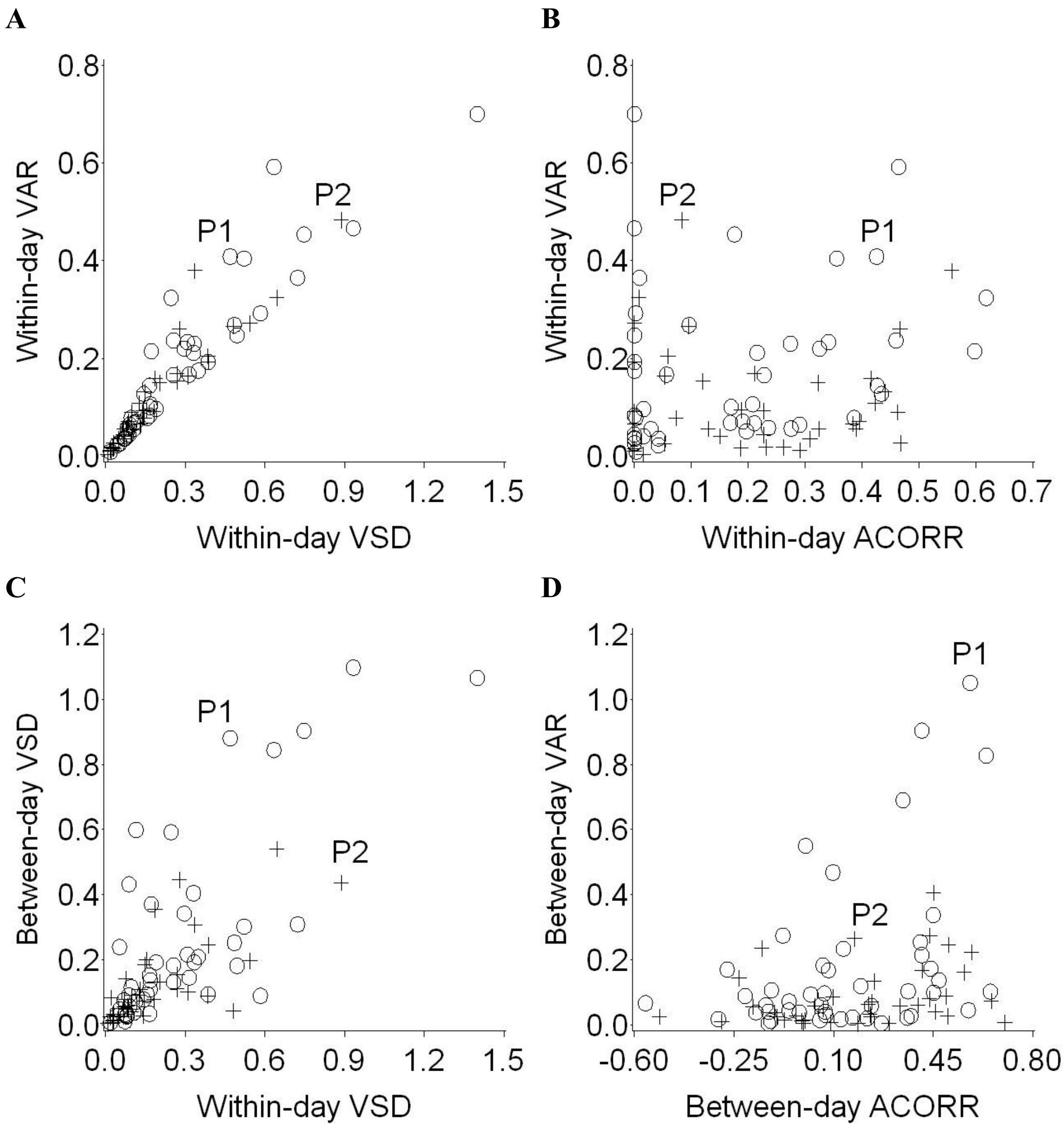

Figure 1. Selected scatter plots among individual instability scores. A: Within-day variance (VAR) by within-day variance of successive difference (VSD). B: Within-day VAR by within-day autocorrelation (ACORR). C: Between-day VSD by within-day VSD. D: Between-day VAR by Between-day ACORR. In the scatter plots, circles indicate individuals in borderline personality disorder group and plus signs indicate individuals in major depressive disorder/dysthymic disorder group. P1 and P2 refer to specific individuals whose scores are discussed further in the text and displayed in Figure 2 and Figure 3.

instability measures: within-day VAR (.48) and within-day VSD (.89), both ranked at the third highest. This person showed not only high dispersion of negative mood scores within-days but also frequent fluctuations over time of the day, quantified by a low within-day temporal dependency (ACORR = .08, see Figure 1B and Figure 3B). On the other hand, individual P1 had a higher score in between-day VSD (.88) in comparison to within-day VSD (see Figure 1C). As expected, moderate within-day instability does not necessarily imply moderate instability in general. Instead, individual P1 can be characterized as a person with high instability in daily negative mood even with not-as-much-as high within-day instability. In contrast, P2 showed high within-day affective instability but moderate or low between-day affective instability, relative to P1. Figure 2C and 3C, the time series plot of daily mean negative mood for P1 and P2, shows the characteristic of between-day fluctuations.

Group difference of BPD and MDD/DYS patients in negative affect instability.

To compare mean difference in negative affective instability between BPD and MDD/DYS groups, we used a mixed variance model on successive difference of negative affect. As in the preliminary analysis, all analyses were conducted using detrended residuals to ensure stationarity of individual EMA data. Because we intend to use mixed models for valid tests and model variance instead of mean, variants of $\hat{\delta}_1^2$ were used instead of those of $\hat{\delta}_2^2$ or $\hat{\delta}_3^2$. As such, successive differences were directly modeled as the first level observations nested within individuals.

Because within-day fluctuations were measured with unequally spaced time, weighted successive differences were fitted in order to model within-day negative affective instability, as given by

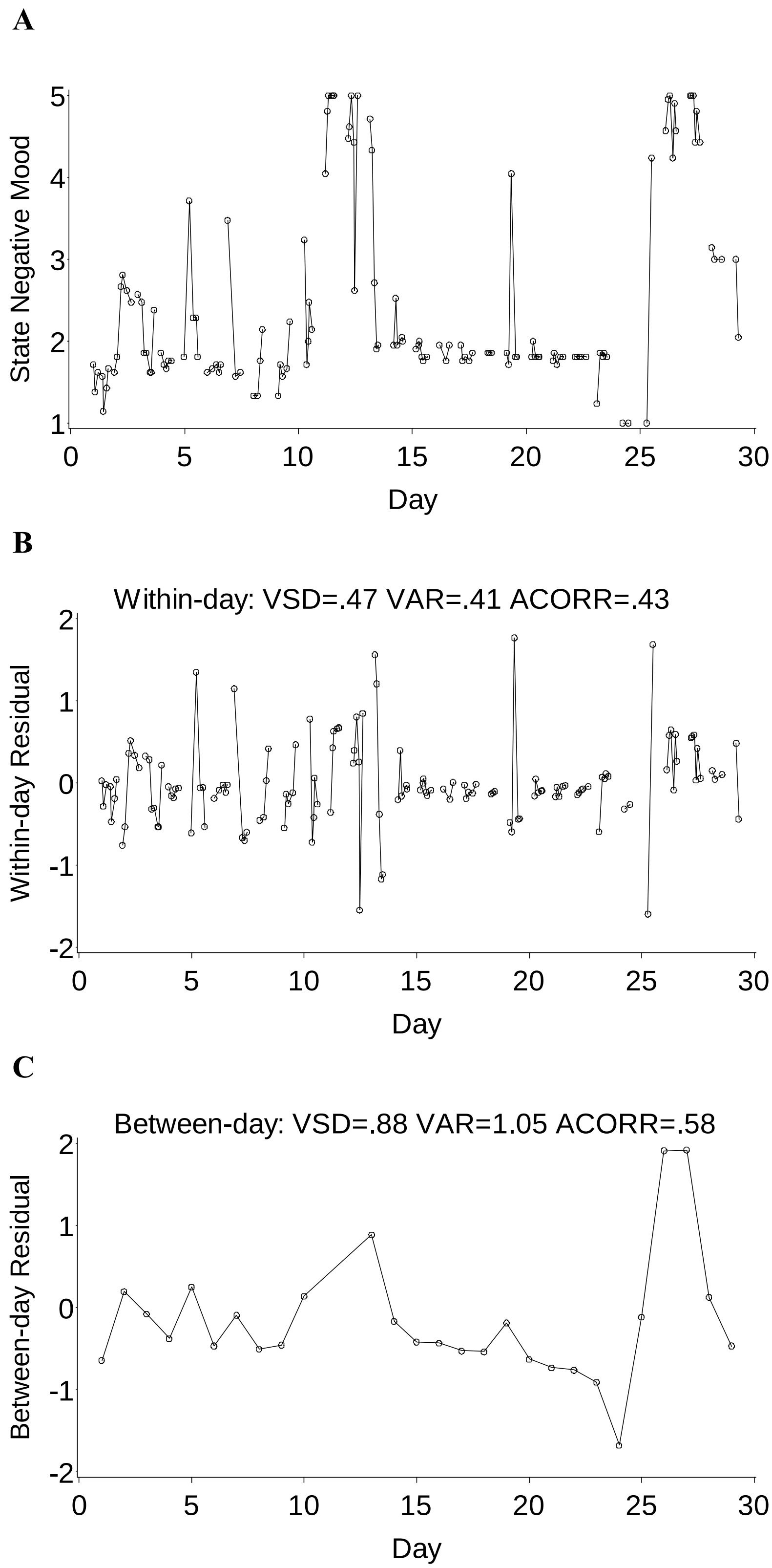

Figure 2. Negative mood fluctuations for the individual P1 in Figure 1. A: State negative mood score. B: Detrended residual fluctuations of within-day negative mood score. C: Detrended residual fluctuations of daily mean across days.

$$w_{ijk}(e_{ijk} - e_{(i-1)jk}) = \exp\left\{\frac{1}{2}(\beta_0 + \beta_1 BPD_k + \delta_k)\right\}\varepsilon_{ijk} \quad (48)$$

or alternatively written by

$$VAR\left[w_{ijk}(e_{ijk} - e_{(i-1)jk})\right] = \exp(\beta_0 + \beta_1 BPD_k + \delta_k) \quad (49)$$

where e_{ijk} is the residual obtained from (47), $w_{ijk} = \sqrt{\frac{1-e^{-\theta h_0}}{1-e^{-\theta h_{ijk}}}}$, $h_{ijk} = t_{ijk} - t_{(i\text{-}1)jk}$, h_0 = 2.399, and ε_{ijk} is a standard normal. BPD_k is a group variable for individual k ($BPD_k = 0$ for MDD/DYS and $BPD_k = 1$ for BPD) and e^{β_0} is the geometric mean of variance of (residual) successive difference, i.e., within-day negative affective instability, for MDD/DYS group and e^{β_1} is the ratio of the geometric mean of VSD for BPD to that of MDD/DYS group. A positive value of β_1 suggests that BPD group has higher negative affective instability than MDD/DYS group. The variance of δ_k is the variance of the log of affective instability across individuals within groups.

For between-day affective instability, daily mean residuals were fitted as given by

$$(\dot{r}_{jk} - \dot{r}_{(j-1)k}) = \exp\left\{\frac{1}{2}(\beta_0 + \beta_1 BPD_k + \delta_k)\right\}\varepsilon_{jk} \quad (50)$$

or alternatively written by

$$VAR(\dot{r}_{jk} - \dot{r}_{(j-1)k}) = \exp(\beta_0 + \beta_1 BPD_k + \delta_k) \quad (51)$$

where $\dot{r}_{jk}$ is the daily mean residual obtained from previous analysis and ε_{jk} is a standard normal. Meaning and interpretation of other parameters are the same as above, except that we model between-day negative affective instability in this model.

The NLMIXED procedure in SAS 9.1™ was used to fit the suggested models. The estimates of the fixed parameters and variance components, and their standard errors

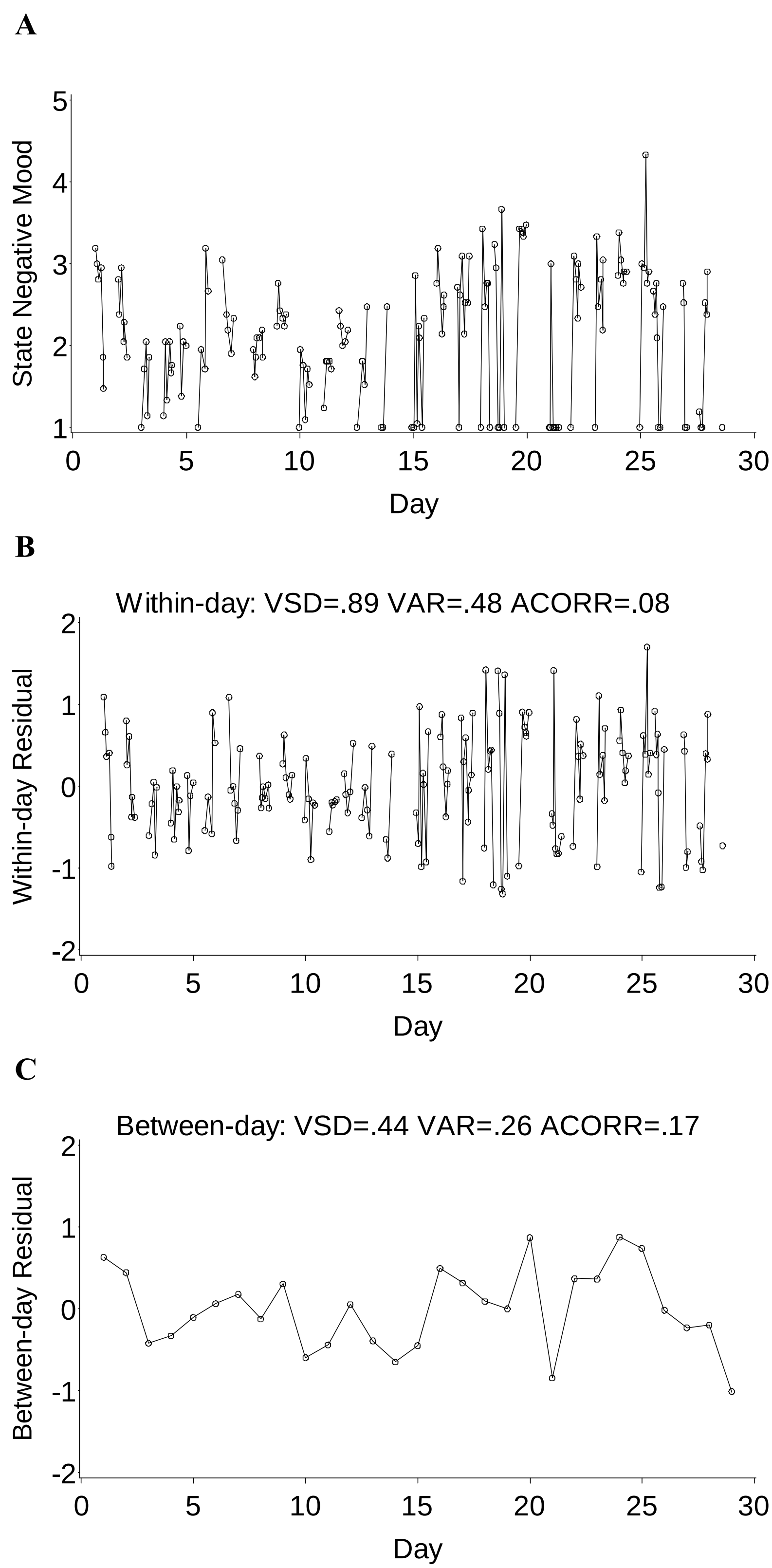

Figure 3. Negative mood fluctuations for the individual P2 in Figure 1. A: State negative mood score. B: Detrended residual fluctuations of within-day negative mood score. C: Detrended residual fluctuations of daily mean across days.

are presented in Table 3. Results showed that the BPD group had a significantly greater within-day negative affective instability (β_1 = .48, t_{83} = 2.17, p < .05) and between-day negative affective instability (β_1 = .62, t_{83} = 2.39, p < .05) than did the MDD/DYS group. The BPD group is expected to have 1.61 (= $e^{.48}$) times greater (geometric mean of) within-day VSD and 1.86 (= $e^{.62}$) times greater (geometric mean of) between-day VSD of negative affect than those of MDD/DYS group (within-day VSD: geometric mean of MDD/DYS = $e^{-2.22}$ = .11, geometric mean of BPD = $e^{-1.74}$ = .18; between-day VSD: geometric mean of MDD/DYS = $e^{-2.75}$ = .06, geometric mean of BPD = $e^{-2.13}$ = .12). Significant variance was found in both log of within-day VSD (σ_δ^2 = 1.01, t_{83} = 6.37, p < .0001) and that of between-day VSD (σ_δ^2 = 1.33, t_{83} = 6.11, p < .0001). In summary, the BPD group can be characterized by high negative affective instability in both within- and between-day fluctuations, in comparison with MDD/DYS group.

Table 3
Estimates of Fixed effects and Level 1 Variance of Mixed Variance Model of Successive Difference

	Within-day VSD		Between-day VSD	
Variable	Estimate	SE	Estimate	SE
Intercept	-2.22**	0.16	-2.75**	0.19
Group[a]	0.48*	0.22	0.62*	0.26
Level 1 variance	1.01**	0.16	1.33**	0.22

Note. SE = Standard Error. VSD = Variance of Successive Difference
[a]Group variable was coded as 1 for BPD group and 0 for MDD/DYS group.
* p < .05, ** p <.0001.

V. Discussion

Although within-person variance is the most-widely used measure of variability/instability over time, the variance of successive difference (VSD) is more conceptually appealing measures of temporal instability. Variance only represents dispersion of states, without considering temporal order or serial correlation. In contrast, autocorrelation does not take into account the variability of a process. Both variance and autocorrelation tap only one component of temporal instability and can not be used as a stand-alone index of temporal instability. However, we do not argue that the variance and the autocorrelation are useless for characterizing fluctuations of a time series. Instead, it is believed that VSD might be used as a global index of temporal instability of fluctuations, and the variance and autocorrelation can be used as supplements of VSD to fully characterize fluctuations of a time series.

On the other hand, because VSD captures only short-term fluctuations, longer-period effects (whether caused by long-term fluctuations or systematic trends) may be overlooked. By examining within- and between-day instability (using within- and between-day VSD), it is possible to determine whether the overall instability is caused by within-day fluctuations or between-day changes. Identifying the source of temporal variability/instability is important so that factors influencing variability/instability can be explored. If overall variability is heavily influenced by between-day changes, the unknown influences on the variation may also vary across days. If variability is more influenced by within-day fluctuations, the variation is possibly influenced by short-term fluctuating factors.

The linear and nonlinear mixed model is one of the most useful and widely used tools to analyze intensive longitudinal data, such as EMA data used in this paper (Walls & Schafer, 2006; Schwartz & Stone, 2007). Because ILD or EMA data often consist of numerous observations within a number of individuals, effects of both time varying (within-individual) and time invariant (between-individuals) covariates are of great interest in the studies with ILD. Although we restricted our discussion of the suggested model for stationary time series, time varying temporal instability and its predictors are also of interest. The effects of individual differences in impulsivity as well as situational change in personal relationship, for example, may be investigated as factors of affective instability using the suggested mixed model with EMA data.

Although I believe many of our suggestions are useful in the analysis of temporal instability, especially in affective instability studies using EMA, several limitations of this paper are also acknowledged. We made a conceptual and analytical distinction between temporal variability and instability and proposed the use of VSD as a stand-alone single index of temporal instability. However, the ultimate utility of this distinction may require additional real-data studies to ensure that those two conceptually different processes indeed exist distinctively in psychological phenomena. As seen in our example, VAR and VSD were highly correlated with each other, and analyses of both reached the same conclusion of group difference in mood fluctuations. If a group of people is distinguished from another group of people by temporal instability but not by variability, for example, then we can say that the distinction between temporal instability and variability is helpful to understand individual process of interest. This is also true for the distinction of within-day and between-day instability. Additionally, the question of whether instability may be more profitably assessed via the fluctuations of a single variable over time or via a more general latent variable model are also not examined in the present manuscript. However, no particular barriers exist to such multivariate extensions of the proposed technique.

The operationalizations of instability and the analysis of such instability measures via a mixed variance model provide a useful way for researchers to investigate whether systematic inter-individual differences exist in patterns of intra-individual change over time (Molenaar, Huizenga, & Nesselroade, 2003). Although the relevant research questions under investigation may suggest a general linear model (e.g., t-test or ANOVA) of such individual differences as outlined above, the proposed mixed models hold much promise. The importance of measurement and analysis of temporal instability increases as theoretical interest of affective instability (or other varieties of "instability") and use of ILD or EMA data increase and I believe that the present work provides many contributions on this field of research.

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