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# ADDITIONS TO SAMPLE-DATA THEORY

G. V. LAGO

Associate Professor of Electrical Engineering

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# ADDITIONS TO SAMPLED-DATA THEORY

G. V. LAGO

University of Missouri, Columbia, Missouri

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# ADDITIONS TO SAMPLED-DATA THEORY[a]

G. V. LAGO

University of Missouri, Columbia, Missouri

*Abstract.*—In the current literature, two different approaches to sampled-data theory prevail. One approach emphasizes the z-transformation methods for design and synthesis. The other approach emphasizes the Laplace transformation methods and studies stability problems through the frequency response of the system in conjunction with the Nyquist diagram.

In this paper the basic equations for the two different approaches are examined and it is shown that the forms of these equations that are accepted in the literature as being equivalent give identical results only under special conditions. The equation based on the frequency response approach is corrected in order that the two equations are equivalent. An open-loop example is presented showing that the corrected equation gives the proper results. A closed-loop example is considered in conjunction with the Nyquist diagram in the complex plane and the maximum velocity constant for absolute stability is derived. This result is checked by a similar development based on z-transformation methods to show that it is correct and that the result found by methods accepted in the literature is incorrect.

## I. INTRODUCTION

One approach to sampled-data tends to emphasize the z-transformation methods for analysis and the article by Ragazzini and Zadeh[1] is an excellent summary of these methods. A second approach emphasizes the Laplace transformation methods and studies stability problems through response of the system in conjunction with the Nyquist diagram. The work of Linvill[2] is concerned primarily with this approach.

In this paper, the derivations of the basic equations for the two different approaches are examined and it is shown that the forms of these equations that have been accepted as being equivalent, give identical results only under special conditions. The equation based on the frequency response approach is corrected in order that the two equations are equivalent. To demonstrate the correctness of the modified equation, two examples are presented. The first is an open-loop example in which the transient response of a sampler followed by a network is determined by the use of both the modified equation and the original equation. The actual output of the network is determined by using the impulse-response approach[3]. It is shown that the output obtained by using the modified equation agrees with the actual output whereas the output obtained by using the original equation does not. The second is a closed-loop example in which the maximum velocity constant for absolute stability is determined by use of the frequency response of the system in conjunction with the Nyquist diagram. The maximum velocity constant is determined by using both the modified equation and the original equation. Then the maximum velocity is determined by use of z-transformation methods to show that the modified equation leads to the correct result.

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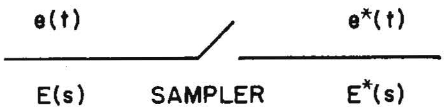
[a]The material of this paper is part of a thesis presented to the Faculty of Purdue University in partial fulfillment of the requirements for a Ph.D. degree.

## II. DERIVATION OF ALTERNATE FORMS FOR BASIC EQUATIONS

The sampler is the component that distinguishes a sampled-data system from a continuous system. The usual schematic method of showing the sampler is by means of a switch as in Fig. 1. The input signal to the sampler $e(t)$ is con-

*Fig. 1—Schematic method of showing sampler.*




tinuous, whereas the output signal $e^*(t)$ is a train of pulses with a width equal to the time the switch is closed and a height proportional to the amplitude of $e(t)$ at the sampling instant. In this paper the sampling period $T$ is assumed constant. As long as the time the switch is closed is small, compared with the sampling period $T$ and the time constants of the remainder of the system, the analysis can be simplified by approximating the train of pulses by a train of impulses with areas representing the sample amplitudes of $e(t)$. Under these conditions, $e^*(t)$ can be written as

$$e^*(t) = e(t) \cdot \sum_{n=0}^{\infty} u_0 (t - nT) \tag{1}$$

where $u_0(t - nT)$ is a unit impulse at $t = nT$. Since the impulse duration is zero, (1) can be written as

$$e^*(t) = \sum_{n=0}^{\infty} e(nT) \cdot u_0(t - nT) \tag{2}$$

The Laplace transform of (2) is

$$E^*(s) = \sum_{n=0}^{\infty} e(nT)\, \varepsilon^{-nTs} \tag{3}$$

An alternate equation for $E^*(s)$ can be derived in a different manner. The sampled signal $e^*(t)$ can be written as

$$e^*(t) = e(t) \cdot i(t) \tag{4}$$

where $i(t)$ is a train of unit impulses occurring every $T$ seconds. The impulse train occurs for both positive and negative values of time whereas $e(t)$ is assumed to be zero for negative values of time. The complex Fourier series form for $i(t)$ is

$$i(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} \varepsilon^{jn\omega_s t} \tag{5}$$

where

$$(6) \qquad \omega_s = 2\pi f_s = \frac{2\pi}{T}$$

When (5) is substituted for $i(t)$ in (4), the result is

$$(7) \qquad e^*(t) = e(t) \cdot \frac{1}{T} \sum_{n=-\infty}^{+\infty} \varepsilon^{jn\omega_s t}$$

When (7) is expanded and the Laplace transformation is taken term by term the result can be summed as

$$(8) \qquad E^*(s) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} E(s+jn\omega_s)$$

In the past it has been said that (8) is the equivalent of (3). This is true only in certain cases. Equation (5) is derived on the basis that $i(t)$ is an even function. This means that the impulse at the origin is even and hence has as much area to the left of $t = 0$ as to the right. When $e(t)$, which is zero for negative values of time, multiplies this infinite series, the result is that (8) yields an impulse at $t = 0$ of weight $e(0)/2$ whereas (3) yields an impulse at $t = 0$ of weight $e(0)$. Equation (3) is the correct interpretation of the physical problem; therefore, for (8) to be used correctly in sampled-data systems, it should be written as

$$(9) \qquad E^*(s) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} E(s + jn\omega_s) + \frac{e(0)}{2}$$

## III. OPEN-LOOP EXAMPLE

The system shown in Fig. 2 is analyzed by using both (8) and (9) to demonstrate that (9) gives the correct result. In the example considered, the system transfer function is defined by

$$(10) \qquad G(s) = \frac{1}{s+1} \text{ and } T = 1 \text{ sec}$$

The input $e(t)$ is a unit step function. It follows that

$$(11) \qquad E(s) = \frac{1}{s}$$

The Laplace transformation of the output of the system is

$$(12) \qquad C(s) = G(s)\, E^*(s)$$

When $E^*(s)$ of (9) is substituted into (12), it becomes

$$(13) \qquad C(s) = G(s) \left[\frac{1}{T} \sum_{n=-\infty}^{+\infty} E(s + jn\omega_s) + \frac{e(0)}{2}\right]$$

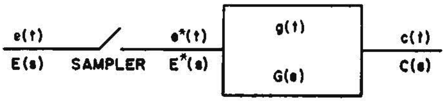



*Fig. 2—Sampler followed by network.*

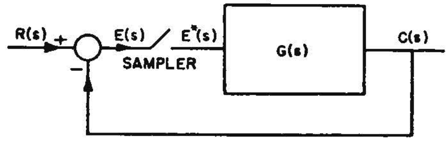



*Fig. 3—Responses of the system of Figure 2 obtained by using Equations (15) and (16).*

When this infinite series is approximated by three terms and the appropriate values for the example are inserted, the result becomes

$$C(s) \approx \frac{1}{s+1}\left[\frac{1}{s-j2\pi}+\frac{1}{s}+\frac{1}{s+j2\pi}+\frac{1}{2}\right] \tag{14}$$

The inverse Laplace transform of (14) is

$$c(t) \approx 1 - \varepsilon^{-t}\left[\frac{1}{2}+\frac{2}{1+(2\pi)^2}\right]+\frac{2}{1+(2\pi)^2}\cos 2\pi t + \frac{4}{1+(2\pi)^2}\sin 2\pi t \tag{15}$$

Equation (15) is plotted in Fig. 3A for four sampling periods as Curve *a*. Curve *b* is the correct solution as obtained by the impulse-response approach[3]. As more terms are used in (13), Curve *a* approaches Curve *b*.

If (8) is used in (12) and the infinite series is approximated by three terms, the result is

$$c(t) \approx 1 - \varepsilon^{-T}\left[1+\frac{2}{1+(2\pi)^2}\right]+\frac{2}{1+(2\pi)^2}\cos 2\pi t + \frac{4}{1+(2\pi)^2}\sin 2\pi t \tag{16}$$

Equation (16) is plotted as Curve *a* in Figure 3B. Again Curve *b* is the correct solution. As more terms are used, Curve *a* does not approach Curve *b*.

## IV. CLOSED-LOOP EXAMPLE

Before the closed-loop example is considered, the following development is needed. The development is first presented as found in the current literature. Equation (12) is the Laplace transformation of the output of the system of Fig. 2. Based on (8), the transform of the values of $c(t)$ at the sampling instants is

$$C^*(s) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} C(s+jn\omega_s) \tag{17}$$

which can be written as

$$(18) \qquad C^*(s) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} G(s + jn\omega_s) \quad \cdot \quad E^*(s + jn\omega_s)$$

Since $E^*(s)$ is periodic, (18) becomes

$$(19) \qquad C^*(s) = E^*(s) \frac{1}{T} \sum_{n=-\infty}^{+\infty} G(s + jn\omega_s)$$

By using the notation

$$(20) \qquad G^*(s) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} G(s + jn\omega_s)$$

equation (19) becomes

$$(21) \qquad C^*(s) = E^*(s)\, G^*(s)$$

Equation (20) indicates that the $G^*(s)$ is obtained from $G(s)$ in the same manner that $E^*(s)$ is obtained from $E(s)$.

It should be noted that (21) was derived from (8) instead of (9). Equation (9) suggests that $C^*(s)$ to be used correctly in sampled-data systems should be written as

$$(22) \qquad C^*(s) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} C(s) + jn\omega_s) + \frac{c(0)}{2}$$

and since $G^*(s)$ is to be obtained from $G(s)$ in the same manner as $E^*(s)$ is obtained from $E(s)$, $G^*(s)$ should also have the form

$$(23) \qquad G^*(s) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} G(s + jn\omega_s) + \frac{g(0)}{2}$$

The closed-loop system shown in Fig. 4 is analyzed to demonstrate that (23)

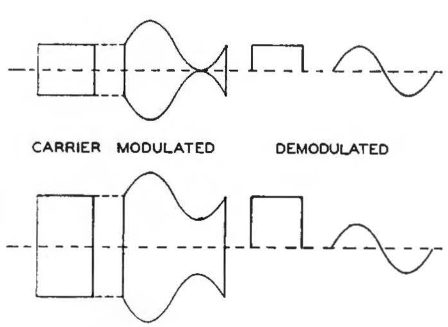



*Fig. 4—Simple sampled-data system.*

is the correct form for $G^*(s)$. In the example considered, the transfer function $G(s)$ is defined by

$$G(s) = \frac{K(s + 3.5)}{s(s + 1)} \quad \text{and } T = 1 \text{ sec} \tag{24}$$

For this system, the Laplace transformation of the output can be shown to be[1]

$$C(s) = \frac{G(s)\, R^*(s)}{1 + G^*(s)} \tag{25}$$

The maximum value of $K$ for the system before it becomes unstable is first found by $z$-transformation methods in order to check later results. The $z$-transformation is obtained by substituting $z$ for $\varepsilon^{sT}$ in the Laplace transform of the sequence of samples[1]. $G(s)$ can be expanded as

$$G(s) = \frac{K(s + 3.5)}{s(s + 1)} = K\left[\frac{3.5}{s} - \frac{2.5}{s + 1}\right] \tag{26}$$

The $z$-transform of (26) is

$$G(z) = K\left[\frac{3.5\, z}{z - 1} - \frac{2.5\, z}{z - 0.36788}\right] = \frac{K\, z\, (z - 1.212)}{(z - 1)\,(z - 0.3679)} \tag{27}$$

The $z$-transform of the $c(t)$ is

$$C(z) = \frac{G(z)\, R(z)}{1 + G(z)} \tag{28}$$

In performing the $z$-transformation, the change of variables

$$z = \varepsilon^{sT} \tag{29}$$

maps the left half of the $s$-plane to the interior of the unit circle in the $z$-plane. As $K$ is increased, the system becomes unstable when the first pole (or poles) of $G(z) \,/\, [1 + G(z)]$ leaves the unit circle.

For this example

$$\frac{G(z)}{1 + G(z)} = \frac{Kz(z + 1.212)}{(K + 1)\, z^2 + (1.212\, K - 1.3679)\, z + 0.3679} \tag{30}$$

Therefore, stability conditions require that the zeros of the test polynomial

$$p(z) = (K + 1)z^2 + (1.212K - 1.3679)z + 0.3679 \tag{31}$$

lie within the unit circle. The geometry of the zeros of a polynomial can be tested by methods given by Marden[4]. The tests in this case are

$$p(0) < 1 \tag{32}$$

$$p(1) > 0 \tag{33}$$

$$p(-1) > 0 \tag{34}$$

Conditions (32) and (33) are satisfied by all values of $K$ but from (34) the value of

$$K_{max} = 12.88 \tag{35}$$

can be found.

The value of $K_{max}$ is now found by using the Laplace transform in conjunction with the Nyquist diagram. As $K$ is increased, the system becomes unstable when the first pole (or poles) of $\dfrac{G(s)}{1 + G^*(s)}$ leaves the left half $s$-plane. This occurs when $1 + G^*(s)$ equals zero or when

$$G^*(s) = -1 \tag{36}$$

$G^*(s)$ is given by an infinite series which can be written as

$$G^*(s) = \frac{g(0)}{2} + \frac{1}{T}\Big[G(s) + G(s - j\omega_s) + G(s + j\omega_s) + G(s - j2\omega_s) + G(s + j2\omega_s) + \dots\Big] \tag{37}$$

The term $\dfrac{g(0)}{2}$ can be found as

$$\frac{g(0)}{2} = \frac{K}{2} \tag{38}$$

by applying the initial value theorem to $G(s)$. When the equation for $G(s)$ is substituted into (37), the result becomes

$$G^*(s) = \frac{K}{2} + K\Bigg[\frac{s + 3.5}{s(s + 1)} + \frac{s - j2\pi + 3.5}{(s - j2\pi)(s - j2\pi + 1)} + \frac{s + j2\pi + 3.5}{(s + j2\pi)(s + j2\pi + 1)} + \frac{s - j4\pi + 3.5}{(s - j4\pi)(s - j4\pi + 1)} + \frac{s + j4\pi + 3.5}{(s + j4\pi)(s + j4\pi + 1)} + \dots\Bigg] \tag{39}$$

$K_{max}$ can be determined by setting $G^*(s)$ equal to minus unity when

$$s = + \frac{j\omega_s}{2} = + j\pi \;. \tag{40}$$

When these conditions are imposed on (39), the result is

$$G^*(s) = \frac{K_{\max}}{2} + K_{\max}\Bigg[\frac{j\pi + 3.5}{j\pi(j\pi + 1)} + \frac{-j\pi + 3.5}{-j\pi(-j\pi + 1)} \tag{41}$$

$$+ \frac{j3\pi + 3.5}{j3\pi(j3\pi + 1)} + \frac{-j3\pi + 3.5}{-j3\pi(-j3\pi + 1)} + \cdots \Bigg]$$

The first two terms inside the brackets of (41) are a complex conjugate pair and can be combined as

$$\frac{+j\pi + 3.5}{j\pi(j\pi + 1)} + \frac{-j\pi + 3.5}{-j\pi(-j\pi + 1)} = \frac{5}{1 + \pi^2} \tag{42}$$

The next two terms, etc., can be combined and by inspection it can be seen that (41) can be written as

$$\mathbf{G}^*(s) = \frac{K_{\max}}{2} - K_{\max}\, 5 \left[\frac{1}{1 + \pi^2} + \frac{1}{1 + 3^2\pi^2} + \frac{1}{1 + 5^2\pi_2} + \frac{1}{1 + 7^2\pi^2} + \cdots\right] = -1 \tag{43}$$

The series inside the brackets of (43) converges slowly but can be summed approximately by use of the series

$$\frac{B_{2n-1}}{(2n)!} = \frac{2}{(2^{2n} - 1)\pi^{2n}} \left[1 + \frac{1}{3^{2n}} + \frac{1}{5^{2n}} + \frac{1}{7^{2n}} + \cdots\right] \tag{44}$$

when $n = +1$ and

$$B_{2n-1} = B_1 = \frac{1}{6} \tag{45}$$

where $B_1$ is one of Bernoulli's numbers. Equation (44) becomes

$$\frac{1}{8} = \frac{1}{\pi^2} + \frac{1}{3^2\pi^2} + \frac{1}{5^2\pi^2} + \frac{1}{7^2\pi^2} + \cdots \tag{46}$$

When the first seven terms inside the brackets are summed, (43) can be written as

$$\frac{K_{\max}}{2} - 5\, K_{\max} \left[0.11191 + \frac{1}{1 + (15)^2\pi^2} + \frac{1}{1 + (17)^2\pi^2} + \cdots\right] = -1 \tag{47}$$

When the first seven terms of (46) are summed, the result is

$$0.125 = 0.12139 + \frac{1}{(15)^2\pi^2} + \frac{1}{(17)^2\pi^2} + \cdots \tag{48}$$

The remaining terms of (48) have the sum

$$\frac{1}{(15)^2\pi^2} + \frac{1}{(17)^2\pi^2} + \cdots = 0.125 - 0.12139 = 0.00361 \tag{49}$$

and this is approximately equal to the sum of the remaining terms of (47) Therefore, (47) becomes

(50) $$\frac{K_{max}}{2} - K_{max}\, 5[0.11191 + 0.00361\,] = -1$$

from which $K_{max}$ can be found as

(51) $$K_{max} = 12.88$$

This result checks the value found by the $z$-transformation method. If $K_{max}$ had been found by using (20) instead of (23), the result would be

(52) $$K_{max} = 1.73$$

which is incorrect.

## V. CONCLUSIONS

Other examples besides the two presented in this paper have also shown the correctness of the modified equations. The reason the equations now accepted have not led to errors in the literature is that they have been applied only to cases in which $e(0)$ [or $r(0)$ or $g(0)$] is zero. These restrictions limit the situations in which solutions similar to those outlined above are possible. For example, if the approximate output of the closed-loop system of Fig. 4 is desired, the restriction to situations in which $r(0)$ is zero is quite severe. This paper develops the general case. The examples presented in the literature become special cases of this general case.

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