Controlled-Deflection Design Method
For Reinforced Concrete Beams and Slabs

Adrian Pauw
Professor of Civil Engineering

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Controlled-Deflection Design Method for Reinforced Concrete Beams and Slabs

By DONALD G. ALCOCK and ADRIAN PAUW

Describes a design method for reinforced concrete beams and slabs in which the allowable ratio of span to deflection is a criterion. The method may also be used for estimating deflections in given designs including those in which ultimate strength theory is used. Special emphasis is placed on design with lightweight aggregate concretes. The elastic theory only is used and estimation of the elastic modulus of concrete is based on previous work. A short-cut procedure is presented for the design of simply supported beams and slabs subjected to uniformly distributed loads only. Tables and design charts are furnished to aid computation. The problem of deflection caused by creep and shrinkage of the concrete is mentioned but not directly dealt with in this paper. The examples, therefore, deal with short-time deflections only.

In conventional working stress and ultimate strength design procedures, beams and slabs are sized for strength and the designer only checks for allowable deflection when he suspects that deflection may be a problem. When lightweight aggregate concrete is used, deflection may become more of a problem because of the low elastic modulus and high creep characteristics of this material. The use of the so-called "balanced-design" method may then be uneconomical. For such concrete (and where deflection considerations govern the design of normal-weight concrete beams) it has been usual to modify the design by trial and error.

The design method described in this paper permits the designer to limit the theoretical length-deflection ratio to any predetermined value automatically. The theory is based on the flexural rigidity of the transformed cracked section although this specific quantity need not be calculated directly. The designer computes a dimensionless factor Q. This factor is the product of three dimensionless parameters:

1. The steel strain, which is the ratio of the steel working stress to its elastic modulus
2. The desired span to deflection ratio
3. A ratio determined by the type and distribution of the loads.
A table of typical load-distribution coefficients for computing the third ratio in the case of simply supported beams is furnished for convenience.† It is a simple matter to make similar tables for cantilevers and other statically determinate cases. With the $Q$ factor determined, the designer may relate the length to depth ratio of the beam with the neutral-axis depth factor $k$. The relationship is shown to be linear and a chart is provided to aid in the selection of values. Once $k$ has been determined, all the usual design parameters for singly reinforced beams are established. A design chart relating these parameters is also furnished.

The primary purpose of the tables and charts is to provide a rapid design procedure, but they are equally useful for determining deflection in given designs.

**NOTATION**

The notation used is generally that of the American Concrete Institute. The other symbols used in this paper are listed below.

- $\alpha =$ dimensionless coefficient dependent on the type of loading on a beam, defined by Eq. (2)
- $\Delta =$ maximum deflection in same units as span $L$
- $I =$ moment of inertia of transformed section
- $P, P' =$ design parameters defined by Eq. (14) and (15)
- $Q =$ product of three dimensionless ratios defined by Eq. (7)
- $r =$ ratio of steel working stress to concrete working stress, viz., $f_s/f_c$
- $R_e =$ defined by $M = R_e b d^2$ and applies when the concrete stress is critical
- $R_s =$ defined by $M = R_s b d^2$ and applies when the steel stress is critical
- $w =$ unit weight of concrete, in lb per cu ft
- $w_r =$ uniformly distributed load (including self weight) on beam or slab, in kips per ft or kips per sq ft, respectively

† It is possible to modify the factor $Q$, still further if the effect of concrete creep is being investigated. This aspect of deflection is not discussed here, but a paper on the topic is being prepared by the authors.
BASIC ASSUMPTIONS

Essential to the method is the use of a realistic modular ratio \( n \). Elsewhere\(^1 \) it has been shown that the elastic modulus \( E_e \) of concrete is a function of its unit weight \( w \), as well as its cylinder strength \( f_c' \).

The empirical formula

\[
E_e = 33 w^{0.2} \sqrt[3]{f_c'}
\]

has been suggested,\(^1 \) and has been used in this paper along with \( E_s = 30 \times 10^6 \) psi for computing \( E_s/E_e = n \). The curves in Fig. 1 enable the designer to arrive at a suitable value of \( n \) when the strength and unit weight of the concrete are known.

Inherent in the theory presented is the flexural rigidity \( EI \), although the value need never be calculated. With the second moment of area based on the transformed cracked section it follows that \( E \) equals \( E_e \). The assumption of a cracked section throughout the entire length of a simple beam is not, of course, strictly correct, but it is consistent with current practice and gives a conservative estimate of short-time deflection.

![Fig. 1—Values of the modular ratio \( n \) for different values of strength and unit weight](image-url)
Table I—Values of α for Typical Loading Conditions in Simple Beams

<table>
<thead>
<tr>
<th></th>
<th>(\frac{5}{48})</th>
<th>(\frac{1}{10})</th>
<th>Approximately 0.102</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\frac{L}{2}]</td>
<td>[\frac{L}{2}]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[\frac{1}{12}]</td>
<td></td>
<td>[\frac{23}{216}]</td>
<td>[\frac{11}{96}]</td>
</tr>
</tbody>
</table>

† See Eq. (2).

The equations based on the above assumptions are quite general and thus may be used for determining short-time deflections at working loads of beams designed by ultimate strength methods.

**THEORY**

The maximum deflection \(\Delta\) in a simple beam of span \(L\) caused by any system of loading may be expressed as

\[
\Delta = M \alpha \frac{L^2}{EI}
\]

(1)

where \(M\) is the maximum bending moment and \(\alpha\) is a constant.

Transposing Eq. (1)

\[
\alpha = \frac{\Delta EI}{ML^2}
\]

(2)

Values of \(\alpha\) for several commonly encountered systems of loading on simply supported beams are given in Table 1. Other values can be easily computed from Eq. (2) together with tables of moments and deflections available in many handbooks. A similar procedure will give values of \(\alpha\) for cantilevers and other statically determinate cases. Where the loading system is not symmetrical the maximum moment and deflection will not occur together at the same point on the beam. The value of \(\alpha\) is then approximate but quite adequate for practical purposes.
The deflection of a concrete beam under the action of \( m \) symmetric loading systems, by Eq. (1) and superposition, is

\[
\Delta = \frac{L^2}{E_c I} (M_1 a_1 + M_2 a_2 + \ldots + M_m a_m)
\]

More simply

\[
\Delta = \frac{\sum (M a) L^2}{E_c I}
\]

from which

\[
\frac{\Delta}{L} = \frac{\sum (M a)}{E_c I}
\] (3)

The governing equation for stress in the steel is

\[
f_s = \frac{n \Sigma(M) (1 - k) d}{I}
\] (4)

where \( I \) is based on the transformed cracked section.

Eq. (4) applies to all concrete beams either with or without compressive reinforcement. Combining Eq. (3) and (4) and eliminating \( I \) gives

\[
(1 - k) d = \frac{f_s}{n} \frac{\sum (M a)}{\Sigma (M)} \left\{ \frac{L}{E_c} \right\} \left\{ \frac{L}{\Delta} \right\}
\] (5)

Substituting \( E_s = nE_c \) and rearranging Eq. (5) gives

\[
k = 1 - \frac{\sum (M a)}{\Sigma (M)} \left\{ \frac{f_s}{E_s} \right\} \left\{ \frac{L}{\Delta} \right\} \left\{ \frac{L}{d} \right\}
\] (6)

For simplicity of notation, let the product of the three ratios in braces be \( Q \), viz.,

\[
Q = \frac{\sum (M a)}{\Sigma (M)} \left\{ \frac{f_s}{E_s} \right\} \left\{ \frac{L}{\Delta} \right\} \left\{ \frac{L}{d} \right\}
\] (7)

Then Eq. (6) becomes

\[
k = 1 - Q \left( \frac{L}{d} \right)
\] (8)

Eq. (8) is also general for all reinforced concrete beams. It shows that the values of \( k, Q, \) and \( L/d \) are related linearly so that it is a simple matter to determine one value when the other two are known or specified. Furthermore, when dealing with singly reinforced rectangular beams, a specific value of \( k \) fixes all the necessary design and investigation parameters. Fig. 2 is a design-investigation chart relating the

\[^{\dagger}\text{This equation is given in many texts on reinforced concrete but with } \Sigma(M) \text{ usually denoted simply as } M, \ I = k^2(3 - k) bd^3/6. \text{ By substituting this value for } I, \text{ Eq. (4) may be reduced to the more familiar form involving } j.\]
Fig. 2—Design chart for reinforced concrete beams

\[ k = 1 - Q \frac{L}{d} \]
The design chart is based on the following equations:

- $M_b = R_b d^2$ (Steel critical)
- $M_c = R_c d^2$ (Concrete critical)
- $p = \frac{100 n}{2r(n+r)}$

Fig. 3—Design chart for rectangular beams
three ratios of Eq. (8), viz., \( k, Q, \) and \( L/d \). Fig. 3 relates \( k \) with all the other conventional design parameters, viz., \( n, p, R, \) etc. These parameters are listed in the chart, but space does not warrant their derivation here. Fig. 3 is limited in use to the design of singly reinforced rectangular beams whereas Fig. 2 is quite general.

**Example 1**

The beam shown in Fig. 4 was designed according to ultimate strength theory using load factors of 1.2 and 2.4 for dead load and live load, respectively, and a steel yield stress \( f_y = 50,000 \) psi. It is required to find the ratio of span to deflection under the action of the loading shown, assuming that creep in the concrete may be neglected. The concrete has a compressive strength \( f_c' \) of 3000 psi and a unit weight \( w \) of 110 lb per cu ft.

Now

\[
A_s = 2.37 \text{ sq in. and } A_{c'} = 0.22 \text{ sq in.}
\]

From Fig. 1, \( n = 14 \) to the nearest whole number. The values \( k \) and \( j \) are next computed according to the elastic theory.\(^3\) Tables are generally available to aid computation.\(^3\) In this case \( k \) equals 0.54 and \( j \) is 0.82.

The ratio \( L/d \) is next computed. From Fig. 4

\[
\frac{L}{d} = 13.75 \times \frac{12}{11} = 15
\]

From Fig. 2 or Eq. (8), with \( k = 0.54 \) and \( L/d = 15; Q = 0.03 \).

Dealing first with dead load, and estimating the weight of the beam as 0.1 kips per ft.

\[
M_D = (1.5 + 0.1) \frac{13.75'}{8} = 37.8 \text{ ft-kips}
\]

From Table 1 \( \alpha = 5/48 \). Therefore \( M_D \alpha = 3.94 \text{ ft-kips} \).

Next, dealing with live load,

\[
M_L = \frac{5 \times 13.75'}{4} = 17.2 \text{ ft-kips}
\]

From Table 1 \( \alpha = 1/12 \). Therefore \( M_L \alpha = 1.43 \text{ ft-kips} \).

Hence the ratio

\[
\frac{\sum (M \alpha)}{\sum (M)} = \frac{3.94 + 1.43}{37.8 + 17.2} = 0.098
\]

The steel strain\(^\dagger\)

\[
\varepsilon_s = \frac{f_s}{E_s} = \frac{M}{A_{c'} \varepsilon_d E_s}
\]

\(^\dagger\) This equation results from the use of the well-known relationship, \( M = f_s A_{c'} j d \).
CONTROLLED-DEFLECTION DESIGN METHOD

Then

\[
\varepsilon_s = \frac{12,000 \times (37.8 + 17.2)}{2.37 \times 0.82 \times 11 \times 30 \times 10^5} = 1.03 \times 10^{-5}
\]

From Eq. (7)

\[
\frac{L}{\Delta} = \frac{Q}{\varepsilon_s \frac{\Sigma (M a)}{\Sigma (M)}}
\]

Then

\[
\frac{L}{\Delta} = \frac{0.03 \times 10^3}{1.03 \times 0.098} = 297
\]

This value is a little low and indicates a 0.56-in. deflection in 13 ft 9 in.

**Example 2**

Using concrete with \(f' = 3750 \text{ psi}\), \(w = 130 \text{ lb per cu ft}\) and the span and loading condition of Example 1, design (on the basis of the elastic theory) a singly reinforced beam 9 in. wide and a span to deflection ratio of at least 540.

\[
\frac{L}{\Delta} = 540 \quad \text{and} \quad \frac{\Sigma (M a)}{\Sigma (M)} = 0.098
\]

This value is the same as that used in Example 1 since it may be assumed that the beams of Examples 1 and 2 will have much the same dead load.

If the steel is at working stress

\[
\varepsilon_s = \frac{20}{30 \times 10^5}
\]

From Eq. (7) and the previous ratios, \(Q\) is 0.035.
The concrete may, by the 1956 ACI Code be stressed to 0.45 $f'_c$ and the steel to 20,000 psi. Therefore

$$r = \frac{20,000}{0.45 \times 3750} = 11.9$$

From Fig. 1 $n = 10$. Fig. 3 shows that $k$ is 0.46. From Fig. 3 $R_c/f'_c = 0.19$. Therefore $R_c = 324$. Now taking $b$ equal to 9 in.

$$d = \sqrt{\frac{12,000 M}{R_c b}} = \sqrt{\frac{12,000 \times 55}{324 \times 9}} = 15\text{ in.}$$

Then

$$\frac{L}{d} = \frac{165}{15} = 11$$

From Eq. (8)

$$Q = \frac{1 - 0.46}{11} = 0.049$$

and from Eq. (7)

$$\frac{L}{\Delta} = \frac{0.049}{0.67 \times 10^3 \times 0.098} = 750$$

Since 540 is less than 750 the 15-in. depth is acceptable. To compute the steel area, Fig. 3 is used to obtain $p = 1.9\%$.

Then

$$A_s = 0.019 \times 9 \times 15 = 2.57\text{ sq in.}$$

Hence, use one #9 and two #8 bars which gives 2.58 sq in. The beam is shown in Fig. 5.

In most cases it is required to adjust the values of $d$ and $A_s$ so that convenient bar sizes and formwork dimensions may be used. The resulting $L/\Delta$ ratio may then be obtained by recomputing $p$ and determining the new $Q$ via Fig. 3 and 2. The ratio $L/\Delta$ is, by Eq. (7), proportional to $Q$ and inversely proportional to the steel stress.
Beams with uniformly distributed loading only

In the case of simply supported and singly reinforced rectangular beams it may be shown that

\[ R_s = \frac{M}{bd^2 f_c} = \frac{n/r(2n/r + 3)}{6(n/r + 1)^2} \]  

(9)

When the beam supports uniformly distributed loading only

\[ M = \frac{1000 w_t L^2}{96} \text{ in.-lb} \]  

(10)

where \( w_t \) is the total load in kips per ft and \( L \) is the span in inches.

Combining Eq. (9) and (10) gives

\[ \left( \frac{L}{d} \right)^2 = \frac{16 f_s b}{1000 w_t} \times \frac{n/r(2n/r + 3)}{(n/r + 1)^2} \]  

(11)

It may also be shown that

\[ k = \frac{n/r}{n/r + 1} \]  

(12)

Then from Eq. (8) and (12)

\[ \left( \frac{L}{d} \right)^2 = \frac{1}{Q^2(n/r + 1)^2} \]  

(13)

Combining Eq. (11) and (13), multiplying both sides by \( n \) and substituting \( f_s = r f_c \) gives

\[ P = \frac{1000 w_t n}{16 b Q^2 f_c} = \left( \frac{n}{r} \right)^2 (2n/r + 3) \]  

(14)

Computation of the parameter \( P \) enables the designer to enter the design chart, Fig. 3, and determine the values of \( k \) and \( p \). With \( k \) known, Fig. 2 gives the required values of \( L/d \).

It should be emphasized that the value of \( P \) is an upper limit. The designer is free to select a lower value of \( L/d \) than the value indicated, in which case the deflection and stresses will be less than the specified maxima. Example 4 illustrates how this limitation may prevent the use of balanced design.

Example 3

The beam designed in Example 2 and shown in Fig. 5 is to be loaded with a uniformly distributed load. Neglecting creep under dead load, find the maximum uniformly distributed load (including its own weight) that the beam can support and the resulting \( L/\Delta \) ratio.

As in Example 2 \( n = 10, r = 11.9 \) and \( L/d = 11 \). From Fig. 2 or Eq. (8) \( Q = 0.049 \). Using Fig. 3 or Eq. (14) \( P = 3.3 \) and from Eq. (14) and
fully stressing the steel

\[ w_1 = \frac{16 \times 9.0 \times (0.049)^2 \times 20,000 \times 3.3}{10 \times 1000} = 2.28 \text{ kips per ft} \]

The design is "balanced" and the concrete stress, \( f_c = 0.45f'_c = 1688 \) psi. It is evident that the ratio \( \Sigma (M_\alpha)/\Sigma (M) \) is always 5/48 under uniformly distributed loading provided that creep is neglected. Therefore from Eq. (7)

\[ \frac{L}{\Delta} = 0.049 \times \frac{30 \times 10^3}{20} \times \frac{48}{5} = 706 \]

One-way slabs

The theory may be further simplified in the design of slabs by fixing the value of \( Q \). Say \( L/\Delta \) is limited to 300. Neglecting creep, \( \Sigma (M_\alpha)/\Sigma (M) = 5/48 \) and \( f_s/E_s = 20/(30 \times 10^9) \). Then \( Q = 1/48 \). Since \( b = 12 \) in. for slabs, \( P \) reduces to

\[ P' = 0.6 \times n \times w \]  

where \( w \) is in kips per sq ft.

The assumption of different values for \( L/\Delta, \Sigma (M_\alpha)/\Sigma (M) \) and \( f_s \) will result in different values of \( Q \) and \( P' \). It is not the purpose of this paper to suggest any specific values but only to present a method whereby such values may be conveniently used. For purposes of illustration the line for \( Q = 1/48 \) is shown in Fig. 2.

Example 4

Using the same concrete as in Example 2, design a one-way slab with a 15-ft span to support a uniformly distributed load of 250 lb per sq ft. Limit \( L/\Delta \) to a minimum of 300.

Let the superimposed load \( w_1 = 0.25 \) kips per sq ft; self load, assume \( w_2 = 0.08 \) kips per sq ft; therefore, total load \( w_1 = 0.33 \) kips per sq ft.

Using Eq. (15) \( P' = 0.6 \times 10 \times 0.33 = 1.98 \) and Fig. 3 \( k = 0.40 \). From Fig. 2, with \( Q = 1/48 \) \( L/d = 28.8 \) from which \( d = 6.2 \) in.

Also, using Fig. 3, \( p = 1.36 \) percent from which

\[ A_s = 0.136 \times 12 \times 6.2 = 1.01 \text{ sq in. per ft} \]

The design is satisfactory from the point of view of deflection and strength, but is not balanced. The value of \( r \) for balanced design, referring to Example 2, is 11.9 whereas with \( P' = 1.98 \), the value of \( r \) is 14.8. This higher value indicates that in this particular case the deflection criterion demands an under-reinforced slab; the concrete stress being only

\[ f_s = \frac{20,000}{14.8} = 1350 \text{ psi} \]
CONCLUSIONS

The examples are intended to illustrate the simplicity with which the designer may use deflection as a criterion in design whatever limitations he may be called on to apply. Nothing but straightforward elastic theory is used and this does not exclude the investigation of deflection in beams designed by ultimate strength theories. The authors are extending the present theory to cover the estimation of deflections caused by inelastic creep of the concrete and some promising results have been obtained.

REFERENCES

3. ACI Committee 317, Reinforced Concrete Design Handbook, American Concrete Institute, Detroit, Mich., 1955, pp. 54-55.

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Discussion of this paper should reach ACI headquarters in triplicate by Aug. 1, 1962, for publication in December 1962 JOURNAL.

Método de Diseño de Deflexión Controlada para Vigas y Losas de Concreto Armado

Se describe un método de diseño de vigas y losas de concreto armado, en el cual se usa como criterio la razón permisible de luz a flecha. El método también se puede usar para estimar deflexiones en diseños dados, incluyendo aquellos en los cuales se usa la teoría plástica. Se hace hincapié en los diseños con concretes ligeros. Se usa sólo la teoría elástica, y los cálculos del módulo elástico del concreto se basan en trabajos anteriores. Se presenta un modo abreviado para el diseño de vigas y losas apoyadas simplemente, sometidas sólo proporcionan a cargas distribuidas uniformemente.

Se proporcionan tablas y gráficas de diseño para ayudar los cálculos. El problema de deflexiones causadas por flujo plástico y contracción del concreto se menciona también, pero no está tratado directamente en este artículo. Los ejemplos, por lo tanto, se refieren sólo a flechas debidas a cargas de corta duración.
Méthode de Calcul des Poutres et des Dalles de Béton Armé par Déflection Contrôlée

Description d'une méthode de calcul pour les dalles et les poutres en béton armé dans laquelle la proportion admise de la flèche par rapport à la portée est un critère. La méthode peut aussi s'appliquer pour l'estimation des déflections dans des études données, y compris celles basées sur la théorie de résistance ultime. Une attention spéciale est donnée aux études de béton d'agrégat léger. Seule la théorie élastique est employée et l'estimation du module d'élasticité du béton est basé sur des ouvrages précédents. Un processus en raccourci est donné pour le calcul des poutres et des dalles en portée simple à chargement uniforme seulement. Des tables et des abaques sont fournis comme aide aux calculs. Le problème de la déflexion due au fluage et au retrait du béton est mentionné mais n'est pas traité directement dans ce mémoire. Conséquemment les examples ne traitent que des fléchissements à court terme.

Entwurfsmethode für Träger und Platten aus Stahlbeton mit vorbestimmter Durchbiegung

Discussion of a paper by Donald G. Alcock and Adrian Pauw:

Controlled-Deflection Design Method for Reinforced Concrete Beams and Slabs

By L. S. MULLER and AUTHORS

By L. S. MULLER†

The authors’ approach to the problem is to be commended. Their achievement, however, would have been greater if they could have applied their method directly to ultimate strength design, where deflections influence and limit the dimensioning more critically than by a working stress design. The writer is well aware of the difficulties, as he has tried to develop the deflection equations using ultimate strength.

The authors have treated simply supported beams and slabs only, while in practice beams and slabs are usually continuous; or at least they have some degree of fixity at one, or at both ends. Nevertheless, there is little difficulty in extending the method to these cases, if in the authors’ equation (p. 649)

$$\Delta = \frac{L^2}{EJ} (M_1a_1 + M_2a_2 + \ldots + M_na_n)$$

the support moments are also included but with a negative sign. For instance, in the case of two equal support moments $M$:

$$\Delta = -\frac{ML^2}{EL} \frac{x}{2L} \left(1 - \frac{x}{L}\right)$$

where

$$\alpha = -\frac{x}{2L} \left(1 - \frac{x}{L}\right)$$

and for the case of one support moment $M$:

$$\Delta = -\frac{ML^2}{EL} \frac{x}{6L} \left[1 - \left(\frac{x}{L}\right)^2\right]$$

where

$$\alpha = -\frac{x}{6L} \left[1 - \left(\frac{x}{L}\right)^2\right]$$

and the distances of the $x/L$ ratio are measured from the opposite support.

†Member American Concrete Institute, Engineer, The Israel Land Development Co., Ltd., Jerusalem, Israel.
Similar equations are arrived at using an asymmetrical load. For instance, in the case of one concentrated load at a distance $a$, and a critical moment at a distance $x$ from the left support:

$$
\Delta = \frac{PL^2}{6EI} \left( \frac{L-a}{L} \right) x \left[ \frac{a}{L} \left( 2 - \frac{a}{L} \right) - \left( \frac{x}{L} \right)^2 \right]
$$

Since

$$
\frac{P(L-a)}{L} x = M_c
$$

Then

$$
\Delta = \frac{ML^2}{6EI} \left[ \frac{a}{L} \left( 2 - \frac{a}{L} \right) - \left( \frac{x}{L} \right)^2 \right]
$$

where $x/L \leq a/L$. If $x/L > a/L$, values at $(1 - x/L)$ and $(1 - a/L)$ should be taken.

With the above additions the authors’ method may prove to be more useful in the design of reinforced concrete beams and slabs. The writer congratulates the authors for their ingenious approach.

**AUTHORS’ CLOSURE**

The authors appreciate Mr. Muller’s interest and his proposed extension of this method to include continuous beams. However, this extension should be applied with caution.

The theory assumes that there is negligible variation in flexural rigidity along the beam and that the crack distribution is fairly uniform. Thus a continuous beam having a rectangular cross section with equal steel top and bottom, presents no difficulty. The increased rigidity of the uncracked section near the points of contraflexure will not reduce the deflection materially. But in an unsymmetrical member, such as a T-beam, a section in the region of negative moment, in a continuous beam, may have markedly different flexural properties from those of a section in the positive-moment region. It would therefore be wise to treat the positive moment region of the beam between calculated (or assumed) points of contraflexure as simply supported.

Continuity applied to beams with cracked sections introduces the problem of nonlinear behavior. This is rather beyond the scope of the authors’ paper and requires further study.
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The design method described in this paper permits the designer to limit the theoretical length-deflection ratio to any predetermined value automatically. The theory is based on the flexural rigidity of the transformed cracked section although this specific quantity need not be calculated directly. The designer computes a dimensionless factor \(Q\). This factor is the product of three dimensionless parameters:

1. The steel strain, which is the ratio of the steel working stress to its elastic modulus
2. The desired span to deflection ratio
3. A ratio determined by the type and distribution of the loads.
A table of typical load-distribution coefficients for computing the third ratio in the case of simply supported beams is furnished for convenience.† It is a simple matter to make similar tables for cantilevers and other statically determinate cases. With the $Q$ factor determined, the designer may relate the length to depth ratio of the beam with the neutral-axis depth factor $k$. The relationship is shown to be linear and a chart is provided to aid in the selection of values. Once $k$ has been determined, all the usual design parameters for singly reinforced beams are established. A design chart relating these parameters is also furnished.

The primary purpose of the tables and charts is to provide a rapid design procedure, but they are equally useful for determining deflection in given designs.

**NOTATION**

The notation used is generally that of the American Concrete Institute. The other symbols used in this paper are listed below.

- $a = \text{dimensionless coefficient dependent on the type of loading on a beam, defined by Eq. (2)}$
- $\Delta = \text{maximum deflection in same units as span } L$
- $I = \text{moment of inertia of transformed section}$
- $P, P' = \text{design parameters defined by Eq. (14) and (15)}$
- $Q = \text{product of three dimensionless ratios defined by Eq. (7)}$
- $r = \text{ratio of steel working stress to concrete working stress, viz., } f_s/f_c$
- $R_c = \text{defined by } M = R_c bd^2 \text{ and applies when the concrete stress is critical}$
- $R_s = \text{defined by } M = R_s bd^2 \text{ and applies when the steel stress is critical}$
- $w = \text{unit weight of concrete, in lb per cu ft}$
- $w_u = \text{uniformly distributed load (including self weight) on beam or slab, in kips per ft or kips per sq ft, respectively}$

† It is possible to modify the factor $Q$, still further if the effect of concrete creep is being investigated. This aspect of deflection is not discussed here, but a paper on the topic is being prepared by the authors.
BASIC ASSUMPTIONS

Essential to the method is the use of a realistic modular ratio $n$. Elsewhere it has been shown that the elastic modulus $E_e$ of concrete is a function of its unit weight $w$, as well as its cylinder strength $f'_c$.

The empirical formula

$$E_e = 33 \frac{w^{3/2} \sqrt{f'_c}}{f'_c}$$

has been suggested, and has been used in this paper along with $E_s = 30 \times 10^6$ psi for computing $E_s/E_e = n$. The curves in Fig. 1 enable the designer to arrive at a suitable value of $n$ when the strength and unit weight of the concrete are known.

Inherent in the theory presented is the flexural rigidity $EI$, although the value need never be calculated. With the second moment of area based on the transformed cracked section it follows that $E$ equals $E_e$. The assumption of a cracked section throughout the entire length of a simple beam is not, of course, strictly correct, but it is consistent with current practice and gives a conservative estimate of short-time deflection.

Fig. 1—Values of the modular ratio $n$ for different values of strength and unit weight
The equations based on the above assumptions are quite general and thus may be used for determining short-time deflections at working loads of beams designed by ultimate strength methods.

**THEORY**

The maximum deflection $\Delta$ in a simple beam of span $L$ caused by any system of loading may be expressed as

$$\Delta = M \alpha \frac{L^4}{EI}$$

(1)

where $M$ is the maximum bending moment and $\alpha$ is a constant.

Transposing Eq. (1)

$$\alpha = \frac{\Delta EI}{ML^4}$$

(2)

Values of $\alpha$ for several commonly encountered systems of loading on simply supported beams are given in Table 1. Other values can be easily computed from Eq. (2) together with tables of moments and deflections available in many handbooks. A similar procedure will give values of $\alpha$ for cantilevers and other statically determinate cases. Where the loading system is not symmetrical the maximum moment and deflection will not occur together at the same point on the beam. The value of $\alpha$ is then approximate but quite adequate for practical purposes.
The deflection of a concrete beam under the action of \( m \) symmetric loading systems, by Eq. (1) and superposition, is

\[
\Delta = \frac{L^2}{E_e I} (M_1 \alpha_1 + M_2 \alpha_2 + \ldots + M_m \alpha_m)
\]

More simply

\[
\Delta = \frac{\sum (M \alpha) L^2}{E_e I}
\]

from which

\[
\frac{\Delta}{L'} = \frac{\sum (M \alpha) L}{E_e I}
\] (3)

The governing equation for stress in the steel is

\[
f_s = \frac{n \sum (M \alpha) (1 - k) d}{I}
\] (4)

where \( I \) is based on the transformed cracked section.

Eq. (4) applies to all concrete beams either with or without compressive reinforcement. Combining Eq. (3) and (4) and eliminating \( I \) gives

\[
(1 - k) d = \frac{f_s \{ \frac{\sum (M \alpha)}{\sum (M)} \} \{ \frac{L}{E_e} \} \{ \frac{L}{\Delta} \}}{n \{ \sum (M) \} \{ \frac{1}{E_e} \} \{ \frac{1}{\Delta} \}}
\] (5)

Substituting \( E_e = n E_e \) and rearranging Eq. (5) gives

\[
k = 1 - \frac{\{ \sum (M \alpha) \} \{ f_s \} \{ \frac{L}{E_e} \} \{ \frac{L}{\Delta} \}}{\{ \sum (M) \} \{ \frac{1}{E_e} \} \{ \frac{1}{\Delta} \} \{ d \}}
\] (6)

For simplicity of notation, let the product of the three ratios in braces be \( Q \), viz.,

\[
Q = \frac{\{ f_s \} \{ \frac{L}{E_e} \} \{ \frac{\sum (M \alpha)}{\sum (M)} \}}{\{ \frac{1}{E_e} \} \{ \frac{1}{\Delta} \} \{ \sum (M) \}}
\] (7)

Then Eq. (6) becomes

\[
k = 1 - \frac{L}{d} \left( \frac{L}{d} \right)
\] (8)

Eq. (8) is also general for all reinforced concrete beams. It shows that the values of \( k \), \( Q \), and \( L/d \) are related linearly so that it is a simple matter to determine one value when the other two are known or specified. Furthermore, when dealing with singly reinforced rectangular beams, a specific value of \( k \) fixes all the necessary design and investigation parameters. Fig. 2 is a design-investigation chart relating the

\[\text{footnote 1}\] This equation is given in many texts on reinforced concrete but with \( \sum (M) \) usually denoted simply as \( M \), \( I = k^2(3-k)b_d^2/6 \). By substituting this value for \( I \), Eq. (4) may be reduced to the more familiar form involving \( j \).
Fig. 2—Design chart for reinforced concrete beams

\[ k = 1 - Q \left( \frac{L}{d} \right) \]
Fig. 3—Design chart for rectangular beams

The design chart is based on the following equations:

\[ n = \frac{E_s}{E_c} \]

1. Based on (Steel critical)
2. Based on (Concrete critical)

The design chart is based on the following conditions:

\[ f = \frac{f_s}{f_c} \]
three ratios of Eq. (8), viz., \( k \), \( Q \), and \( L/d \). Fig. 3 relates \( k \) with all the other conventional design parameters, viz., \( n \), \( p \), \( R \), etc. These parameters are listed in the chart, but space does not warrant their derivation here. Fig. 3 is limited in use to the design of singly reinforced rectangular beams whereas Fig. 2 is quite general.

**Example 1**

The beam shown in Fig. 4 was designed according to ultimate strength theory using load factors of 1.2 and 2.4 for dead load and live load, respectively, and a steel yield stress \( f_y = 50,000 \) psi. It is required to find the ratio of span to deflection under the action of the loading shown, assuming that creep in the concrete may be neglected. The concrete has a compressive strength \( f_c' \) of 3000 psi and a unit weight \( w \) of 110 lb per cu ft.

Now

\[ A_\times = 2.37 \text{ sq in. and } A_\times' = 0.22 \text{ sq in.} \]

From Fig. 1, \( n = 14 \) to the nearest whole number. The values \( k \) and \( j \) are next computed according to the elastic theory.\(^3\) Tables are generally available to aid computation.\(^3\) In this case \( k \) equals 0.54 and \( j \) is 0.82.

The ratio \( L/d \) is next computed. From Fig. 4

\[ \frac{L}{d} = 13.75 \times \frac{12}{11} = 15 \]

From Fig. 2 or Eq. (8), with \( k = 0.54 \) and \( L/d = 15; Q = 0.03 \).

Dealing first with dead load, and estimating the weight of the beam as 0.1 kips per ft.

\[ M_\theta = (1.5 + 0.1) \frac{13.75^2}{8} = 37.8 \text{ ft-kips} \]

From Table 1 \( \alpha = 5/48 \). Therefore \( M_\theta \alpha = 3.94 \text{ ft-kips} \).

Next, dealing with live load,

\[ M_\lambda = \frac{5 \times 13.75}{4} = 17.2 \text{ ft-kips} \]

From Table 1 \( \alpha = 1/12 \). Therefore \( M_\lambda \alpha = 1.43 \text{ ft-kips} \).

Hence the ratio

\[ \frac{\sum (M \alpha)}{\sum (M)} = \frac{3.94 + 1.43}{37.8 + 17.2} = 0.098 \]

The steel strain†

\[ \varepsilon_\lambda = \frac{f_y}{E_y} = \frac{M}{A_\times jd\ E_\times} \]

\( \dagger \) This equation results from the use of the well-known relationship, \( M = f_y A_\times jd \).
Then

\[ \varepsilon_s = \frac{12,000 \times (37.8 + 17.2)}{2.37 \times 0.82 \times 11 \times 30 \times 10^3} = 1.03 \times 10^{-3} \]

From Eq. (7)

\[ \frac{L}{\Delta} = \frac{Q}{\varepsilon_s \sum (Ma) / \sum (M)} \]

Then

\[ \frac{L}{\Delta} = \frac{0.03 \times 10^3}{1.03 \times 0.098} = 297 \]

This value is a little low and indicates a 0.56-in. deflection in 13 ft 9 in.

**Example 2**

Using concrete with \( f'_c = 3750 \) psi, \( w = 130 \) lb per cu ft and the span and loading condition of Example 1, design (on the basis of the elastic theory) a singly reinforced beam 9 in. wide and a span to deflection ratio of at least 540.

\[ \frac{L}{\Delta} = 540 \text{ and } \frac{\sum (Ma)}{\sum (M)} = 0.098 \]

This value is the same as that used in Example 1 since it may be assumed that the beams of Examples 1 and 2 will have much the same dead load.

If the steel is at working stress

\[ \varepsilon_s = \frac{20}{30 \times 10^3} \]

From Eq. (7) and the previous ratios, \( Q \) is 0.035.
The concrete may, by the 1956 ACI Code be stressed to 0.45 $f_c'$ and the steel to 20,000 psi. Therefore

$$ r = \frac{20,000}{0.45 \times 3750} = 11.9 $$

From Fig. 1 $n = 10$. Fig. 3 shows that $k$ is 0.46. From Fig. 3 $R_e/f_c' = 0.19$. Therefore $R_e = 324$.

Now taking $b$ equal to 9 in.

$$ d = \sqrt{\frac{12,000 M}{R_e b}} = \sqrt{\frac{12,000 \times 55}{324 \times 9}} = 15 \text{ in.} $$

Then

$$ \frac{L}{d} = \frac{165}{15} = 11 $$

From Eq. (8)

$$ Q = \frac{1 - 0.46}{11} = 0.049 $$

and from Eq. (7)

$$ \frac{L}{\Delta} = \frac{0.049}{0.67 \times 10^{-3} \times 0.098} = 750 $$

Since 540 is less than 750 the 15-in. depth is acceptable. To compute the steel area, Fig. 3 is used to obtain $p = 1.9$ percent.

Then

$$ A_s = 0.019 \times 9 \times 15 = 2.57 \text{ sq in.} $$

Hence, use one #9 and two #8 bars which gives 2.58 sq in. The beam is shown in Fig. 5.

In most cases it is required to adjust the values of $d$ and $A_s$ so that convenient bar sizes and formwork dimensions may be used. The resulting $L/\Delta$ ratio may then be obtained by recomputing $p$ and determining the new $Q$ via Fig. 3 and 2. The ratio $L/\Delta$ is, by Eq. (7), proportional to $Q$ and inversely proportional to the steel stress.
**Beams with uniformly distributed loading only**

In the case of simply supported and singly reinforced rectangular beams it may be shown that

\[
R_c = \frac{M}{bd^2f_c} = \frac{n/r(2n/r + 3)}{6(n/r + 1)^2} \quad \text{(9)}
\]

When the beam supports uniformly distributed loading only

\[
M = \frac{1000 w_1 L^2}{96} \quad \text{in.-lb} \quad \text{(10)}
\]

where \(w_1\) is the total load in kips per ft and \(L\) is the span in inches.

Combining Eq. (9) and (10) gives

\[
\left(\frac{L}{d}\right)^2 = \frac{16 f_c b}{1000 w_1} \times \frac{n/r(2n/r + 3)}{(n/r + 1)^2} \quad \text{(11)}
\]

It may also be shown that

\[
k = \frac{n/r}{n/r + 1} \quad \text{(12)}
\]

Then from Eq. (8) and (12)

\[
\left(\frac{L}{d}\right)^2 = \frac{1}{Q^2(n/r + 1)^2} \quad \text{(13)}
\]

Combining Eq. (11) and (13), multiplying both sides by \(n\) and substituting \(f_c = rf_c\) gives

\[
P = \frac{1000 w_1 n}{16 b Q^2 f_c} = \left(\frac{n}{r}\right)^2 (2n/r + 3) \quad \text{(14)}
\]

Computation of the parameter \(P\) enables the designer to enter the design chart, Fig. 3, and determine the values of \(k\) and \(p\). With \(k\) known, Fig. 2 gives the required values of \(L/d\).

It should be emphasized that the value of \(P\) is an upper limit. The designer is free to select a lower value of \(L/d\) than the value indicated, in which case the deflection and stresses will be less than the specified maxima. Example 4 illustrates how this limitation may prevent the use of balanced design.

**Example 3**

The beam designed in Example 2 and shown in Fig. 5 is to be loaded with a uniformly distributed load. Neglecting creep under dead load, find the maximum uniformly distributed load (including its own weight) that the beam can support and the resulting \(L/d\) ratio.

As in Example 2 \(n = 10, r = 11.9\) and \(L/d = 11\). From Fig. 2 or Eq. (8) \(Q = 0.049\). Using Fig. 3 or Eq. (14) \(P = 3.3\) and from Eq. (14) and
fully stressing the steel

\[ w_r = \frac{16 \times 9.0 \times (0.049)^2 \times 20,000 \times 3.3}{10 \times 1000} = 2.28 \text{ kips per ft} \]

The design is "balanced" and the concrete stress, \( f_c = 0.45f'_c = 1688 \) psi. It is evident that the ratio \( \Sigma(Ma)/\Sigma(M) \) is always 5/48 under uniformly distributed loading provided that creep is neglected. Therefore from Eq. (7)

\[ \frac{L}{\Delta} = 0.049 \times \frac{30 \times 10^3}{20} \times \frac{48}{5} = 706 \]

**One-way slabs**

The theory may be further simplified in the design of slabs by fixing the value of \( Q \). Say \( L/\Delta \) is limited to 300. Neglecting creep, \( \Sigma(Ma)/\Sigma(M) = 5/48 \) and \( f_s/E_s = 20/(30 \times 10^3) \). Then \( Q = 1/48 \). Since \( b = 12 \) in. for slabs, \( P \) reduces to

\[ P' = 0.6n w_r \]

where \( w_r \) is in. kips per sq ft.

The assumption of different values for \( L/\Delta, \Sigma(Ma)/\Sigma(M) \) and \( f_s \) will result in different values of \( Q \) and \( P' \). It is not the purpose of this paper to suggest any specific values but only to present a method whereby such values may be conveniently used. For purposes of illustration the line for \( Q = 1/48 \) is shown in Fig. 2.

**Example 4**

Using the same concrete as in Example 2, design a one-way slab with a 15-ft span to support a uniformly distributed load of 250 lb per sq ft. Limit \( L/\Delta \) to a minimum of 300.

Let the superimposed load \( w_1 = 0.25 \) kips per sq ft; self load, assume \( w_2 = 0.08 \) kips per sq ft; therefore, total load \( w_t = 0.33 \) kips per sq ft.

Using Eq. (15) \( P' = 0.6 \times 10 \times 0.33 = 1.98 \) and Fig. 3 \( k = 0.40 \). From Fig. 2, with \( Q = 1/48 \) \( L/d = 28.8 \) from which \( d = 6.2 \) in.

Also, using Fig. 3, \( p = 1.36 \) percent from which

\[ A_s = 0.136 \times 12 \times 6.2 = 1.01 \text{ sq in. per ft} \]

The design is satisfactory from the point of view of deflection and strength, but is not balanced. The value of \( r \) for balanced design, referring to Example 2, is 11.9 whereas with \( P' = 1.98 \), the value of \( r \) is 14.8. This higher value indicates that in this particular case the deflection criterion demands an under-reinforced slab; the concrete stress being only

\[ f_c = \frac{20,000}{14.8} = 1350 \text{ psi} \]
CONCLUSIONS

The examples are intended to illustrate the simplicity with which the designer may use deflection as a criterion in design whatever limitations he may be called on to apply. Nothing but straightforward elastic theory is used and this does not exclude the investigation of deflection in beams designed by ultimate strength theories. The authors are extending the present theory to cover the estimation of deflections caused by inelastic creep of the concrete and some promising results have been obtained.

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Méthode de Calcul des Poutres et des Dalles de Béton
Armé par Déflection Contrôlée

Description d’une méthode de calcul pour les dalles et les poutres en béton armé dans laquelle la proportion admise de la flèche par rapport à la portée est un critère. La méthode peut aussi s’appliquer pour l’estimation des déflections dans des études données, y compris celles basées sur la théorie de résistance ultime. Une attention spéciale est donnée aux études de béton d’agrégat léger. Seule la théorie élastique est employée et l’estimation du module d’élasticité du béton est basé sur des ouvrages précédents. Un processus en raccourci est donné pour le calcul des poutres et des dalles en portée simple à chargement uniforme seulement. Des tables et des abaques sont fournis comme aide aux calculs. Le problème de la déflection due au fluage et au retrait du béton est mentionné mais n’est pas traité directement dans ce mémoire. Con­séquemment les examples ne traitent que des fléchissements à court terme.

Entwurfmethode für Träger und Platten aus Stahlbeton mit vorbestimmter Durchbiegung

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