This dissertation is in the area of theoretical and computational physics. Here we study the collective charge and spin excitations of itinerant electronic systems in quasi-two-dimensional semiconductor nanostructures. Our main focus is on the fundamental properties of a two-dimensional electron gas (2DEG) in a doped semiconductor quantum well because it is a paradigm of an electronic many-body system. The 2DEG has been thoroughly studied for many decades, and it still is a subject of great fundamental and practical interest.

Let us begin by discussing some basic concepts of semiconductor quantum wells. In a semiconductor quantum well, material A, with a smaller band gap, is sandwiched between two layers of material B (with a larger band gap). We only consider semiconductors with direct band gaps, such as GaAs, where the energy dispersions of the valence band and conduction band are aligned with maxima and minima, respectively, at the Brillouin zone center; electrons can then directly emit photons when undergoing interband transitions.

Here we are interested in n-doped systems where the electrons live in the conduction band of the quantum well (material A). If only the lowest subband is occupied, and the Fermi energy is not too high (so that the subband remains parabolic), we have the idealized situation of a 2DEG. In the conduction band, the lowest subband is occupied up to the Fermi level, and we have excitations from occupied states to unoccupied states within the first subband. The Fermi surface of a 2DEG is a circle, so these excitations are between filled states within the Fermi circle to states outside. We also refer to these excitations as intrasubband transitions.

In the absence of magnetic fields, spin-up and spin-down subbands are exactly on top of each other. An external in-plane magnetic field causes a Zeeman splitting of the subbands, where spin-up and spin-down subbands will be shifted apart by the effective Zeeman energy which is proportional to an effective magnetic field. The effective magnetic field includes an exchange and correlation contribution caused by Coulomb many-body effects. In this case, we can distinguish between spin-conserving and spin-flip excitations.

In many semiconductor materials, spin-orbit coupling effects are playing an important role. In recent years, spin-orbit coupling has attracted much interest in the context of spintronics, as well as in novel materials such as graphene and topological insulators. Spin-orbit coupling is present in all matter, and with various consequences for the electronic structure. Here, our interest is in those spin-orbit effects that are a consequence of the breaking of inversion symmetry of the system: the Dresselhaus and the Rashba effect.

The central theme of this thesis will be those situations where Coulomb interactions have important consequences, causing new forms of collective behavior. Most notably, Coulomb interactions are responsible for the formation of plasmons. Another example is spin waves in the spin-polarized 2DEG, which can be thought of as a collective spin precession propagating through the system. Electronic spin waves are the itinerant-electron counterpart of magnons, which are collective precessions in lattices of localized spins. Magnon spintronics, a new and promising subfield of spintronics, is based on the idea that information can be encoded and transported by spin waves. The spin-orbit interaction plays an important
role in magnon spintronics since it provides a coupling mechanism between spin dynamics and electrical signals. However, so far, the interplay between spin-orbit coupling and Coulomb many-body effects has been relatively little explored.

In this thesis, we will investigate the following questions:

• What happens when a system of electrons in a quantum well becomes more and more confined, so that it goes through a cross-over between 3D to a 2D state? In particular, can we find theoretical methods to calculate plasmon modes in the system that are robust under such a cross-over?
• How are collective spin-wave excitations in a spin-polarized 2DEG affected by the presence of Rashba and Dresselhaus spin-orbit interactions? Spin waves are well-ordered collective modes of the electron system; will their order be destroyed by the spin-orbit effects, or will it be modified in more subtle ways? How do our theoretical methods hold up under comparison with experimental results?
• There is an intriguing special situation in which the Rashba and Dresselhaus fields have the same strengths. From the recent literature, it is known that this leads to so-called spin-helical states. What happens to collective spin waves in this situation?