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DESIGN OF TWO-DIMENSIONAL SHARP-EDGE-THROAT
SUPERSONIC NOZZLE BY DIGITAL COMPUTER

by

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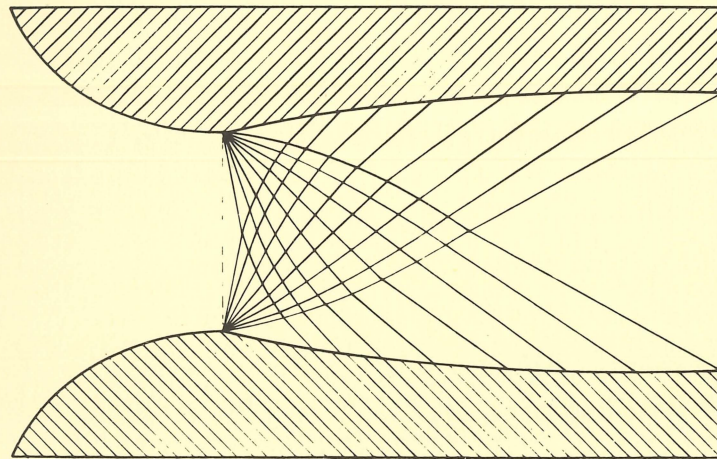
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DESIGN OF TWO-DIMENSIONAL SHARP-EDGE-THROAT SUPERSONIC
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INTRODUCTION

Shames and Seashore¹, (referred to hereafter as Shames only) developed a graphical construction for the design of two-dimensional sharp-edge-throat supersonic nozzles of minimum length using the method of characteristics. They present tables using an expansion "kernel", (zone of wave interaction) from which the nozzle wall contours could be obtained by tedious graphical construction for test section Mach numbers from 1.2 to 10. The method developed by the authors, using the kernel coordinates of Shames, requires no graphical construction, and the program is designed to give accurately the final values of the nozzle wall contour and local Mach numbers, without boundary layer corrections, for nozzle test section Mach numbers of from 1.2 to 5.0 for any height. The program may be easily modified to extend to range to Mach 10. A brief review of the Shames and Seashore method follows:

Two Prandtl-Meyer expansions are assumed to occur at opposite sides of the sharp-edge-throat nozzle, Figure 1, and the expansion waves emitted

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from each corner are divided into a convenient number of characteristics. Because of the simultaneous influence of the expansion waves on the flow in the zone common to both, the characteristics are curved to maintain the Mach angle; this zone is called the "Kernel!" Shames graphically developed the kernel for a Mach 10 nozzle. The characteristics are straight in the zone downstream from the kernel in which only one family of waves are present, Figure 2. When the latter characteristics were determined together with the flow angle and the coordinates of the kernel, the nozzle wall was found by graphical construction. Tables are given for the determination of the coordinates of the kernel, the angle of the characteristics and the flow angle in the single wave zones.

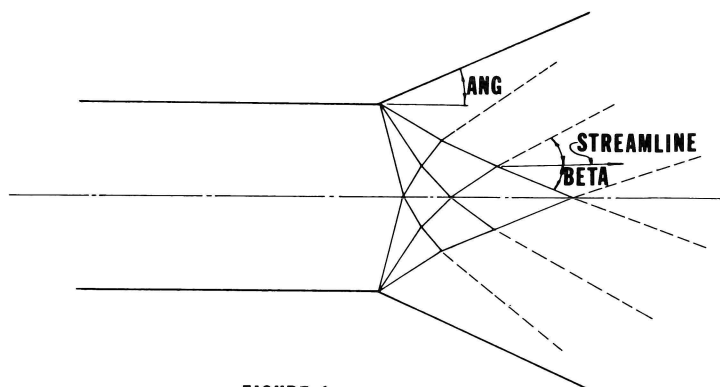
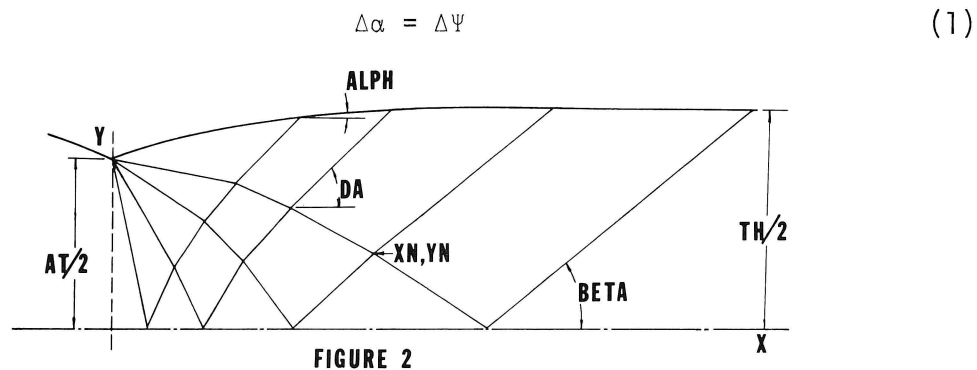


FIGURE 1

An expansion wave incident on a channel wall, in general, requires that a wave be reflected at this point if the flow is to remain on the wall. If the wall is turned through the same angle as a streamline would be turned by the expansion wave, then no reflection occurs and the flow is wave free. The location of point of incidence of the characteristic was obtained graphically. The difference in the characteristics bounding the incident wave gives the change in the wall inclination required to suppress the reflection.

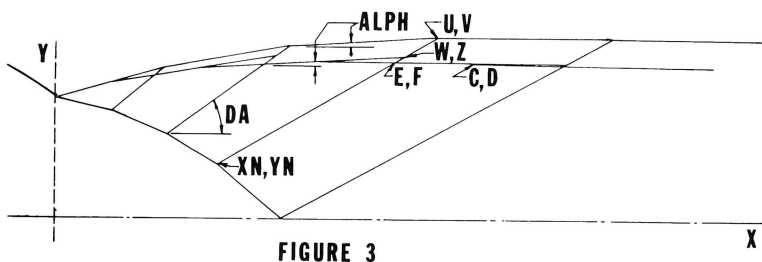


Construction of the nozzle wall starts at the nozzle throat with a straight line segment that makes an angle with the nozzle axis of

$$\text{ANG} \equiv \phi = \frac{1}{2} \left[\sqrt{\frac{k+1}{k-1}} \tan^{-1} \frac{\sqrt{M_f^2 - 1}}{\sqrt{\frac{k+1}{k-1}}} - \tan^{-1} \sqrt{M_f^2 - 1} \right]$$

At the intersection of this segment (nozzle wall) with the first characteristic, the second segment is drawn with the inclination reduced by an amount of $\Delta\alpha$ (equation 1), and the process is repeated until all characteristics have been crossed. When this sequence of straight lines representing the nozzle wall has been completed, a smooth curve approximating these lines is taken as the nozzle contour. An averaging technique was also described that gives a contour that is closer to the true contour that would be obtained if a larger number of characteristics were used, Figure 3. The test section height for a unit depth nozzle is given by,

$$\frac{A_f}{A_t} = \frac{1}{M_f} \left(\frac{1 + \frac{k-1}{2} M_f^2}{\frac{k+1}{2}} \right)^{\frac{k+1}{2(k-1)}} \quad (3)$$



Equation (3) serves as a check on the construction as developed above. Nozzles that have final Mach numbers between the values for which the kernels are given in Shames' tables required a new kernel. It was developed by graphical construction from the kernel of the next higher given Mach number.

The nozzle length was shown to be given by

$$\frac{L}{A_f} = \left(\frac{l_k}{A_t} + \frac{A_f}{2A_t} \tan B \right) \frac{A_t}{A_f} \quad (4)$$

where l_k is the length of the kernel.

NOZZLE DESIGN PROGRAM

The attractive feature of our program is that only the desired final Mach number and the test section height need be supplied by the user. The final output of the program gives the X, Y coordinates of the upper wall of the nozzle and the local Mach number of the flow at that point. Since the nozzle is symmetric, both nozzle blocks may be machined at the same time. The local Mach number may be used to correct for boundary layer if it is desired to do so. We feel that the correction is unnecessary if one machines the contours with a plus tolerance. The program may be modified rather easily to accommodate Mach numbers up to 10 by using Shames' data for the Mach 10 kernel coordinates.

The program is divided into five parts to facilitate the computation of the necessary intermediate kernels from the data given by Shames for the kernels of $M_f = 1.2, 1.4, 1.6, 1.8, 2.0, 3.0,$ and 4.0 , (these will be called standard Mach numbers). The four sub-routines function as follows: PAS 1 generates the characteristic angles, the flow angles, and the local Mach numbers downstream of each wave for the standard Mach numbers given above. PAS 2 selects the next lower Mach number kernel if the final Mach number is anything other than one of the standards. PAS 3 generates the kernel for the next lower standard Mach number and from this generates the correct kernels for all Mach numbers other than standard. Figure 4 illustrates the notation used in PAS 3. PAS 4 completes the computation of the wall coordinates from the final kernel computations and averages the wall angle changes to the mean between that at two adjacent characteristics. The program is written in Fortran IV language for use on the I.B.M. 7040 computer.

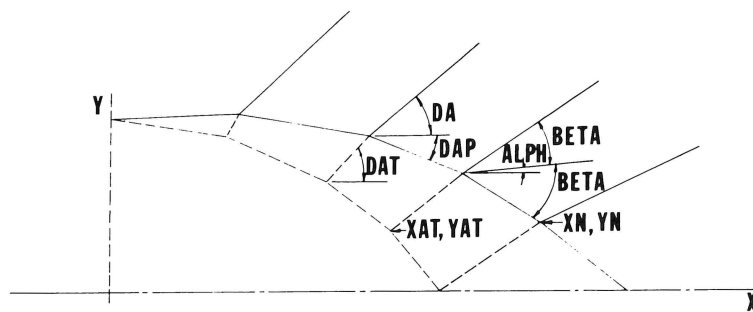


FIGURE 4

If the range given is sufficient for the problem at hand, only one card giving the final Mach number entered in Col. 15 and the test section height entered in Col. 30 need be added to the deck behind the \$ ENTRY card. The total machine time for a non-standard Mach number is 5 minutes and 8 seconds.

Modifications may be made by adding statements after No. 73 and No. 113 to extend the range of final Mach numbers, providing additional data from Shames' tables are inserted. Without modification the program may be used up to $M_f = 6$ to 7 with less accuracy as shown by equation (3).

PROGRAM DETAILS

The program variables are defined as follows, (also see figures):

- ACH = Mach number
- ACHT = Temporary Mach number
- ALPH = Wall or flow angle
- ALRD = ALPH in Radians
- ALSC = Tan (ALRD)
- ALSL = Tan (AIPH)
- ANG = SIFD/2.0 (equation 3)
- BETA = Wave angle
- BET = BETA in Radians
- CHT = $\sqrt{(k+1)/(k-1)}$

DA = Lower Characteristic for desired kernel
DAP = Upper characteristic for desired kernel
DAPS = Tan (DAP)
DAPR = DAP in Radians
DARD = DA in Radians
DASL = Tan (DA)
DAT = Lower characteristic for kernel below desired final
Mach number
DATR = DAT IN RADIANS
DATS = TAN (DATR)
DASC = Tan (DARD)
ETA = $\sqrt{M^2-1}$
SIFD = Final characteristic
SIF = SIFD in Radians
SID = ANG + SIM
SIM = Increment of Prandtl-Meyer angle from Shames' tables
K = Control constant
FM = Final desired Mach number
N = Number of kernel points
X0, Y0 = Coordinates for kernel points for kernel below
next higher standard Mach number
XAT, YAT = Kernel points from Shames' tables
XN, YN = Coordinates of kernel generated from kernel of
next higher standard Mach number
TH = Test section height

FORTRAN IV PROGRAM

```

COMMON DATR(60),DATS(60),DAPS(60),DAP(60),ALRD(60),DARD(60),
1DAPR(60),ALSL(60),DASL(60),ACH(60),BET(60),BETA(60),DA(60),
2XO(60),YO(60),DAT(60),XAT12(60),YAT12(60),XAT14(60),YAT14(60),
3XAT16(60),YAT16(60),XAT18(60),YAT18(60),XAT20(60),YAT20(60),
4XAT30(60),YAT30(60),XAT40(60),YAT40(60),SIM(60),XN(60),YN(60),
5ALPH(60),SIP(60),ALSC(60),DASC(60),ETA,CHT,SIF,SIFD,ANG,K,FM,N12,
6N14,N16,N18,N20,N30,N40,N,ACHT,
7TH,C(60),D(60),X(60),Y(60),U(60),V(60),W(60),Z(60),E(60),F(60)
100 FORMAT (2F15.8)
101 FORMAT (8F10.5)
102 FORMAT (I7)
103 FORMAT (8F10.5)
104 FORMAT (6F15.6)
105 FORMAT (47HMACH NUMBER TOO SMALL FOR USE WITH THIS PROGRAM)
RAT = 1.40
READ 100,FM,TH
READ 101,(SIM(J),J=1,54)
READ 102,N12
READ 103,(XAT12(J),YAT12(J),J=1,N12)
READ 102,N14
READ 103,(XAT14(J),YAT14(J),J=1,N14)
READ 102,N16
READ 103,(XAT16(J),YAT16(J),J=1,N16)
READ 102,N18
READ 103,(XAT18(J),YAT18(J),J=1,N18)
READ 102,N20
READ 103,(XAT20(J),YAT20(J),J=1,N20)
READ 102,N30
READ 103,(XAT30(J),YAT30(J),J=1,N30)
READ 102,N40
READ 103,(XAT40(J),YAT40(J),J=1,N40)
ETA = SQRT(FM**2.0-1.0)
CHT = SQRT((RAT+1.0)/(RAT-1.0))
SIF = CHT*ATAN2(ETA,CHT)-ATAN(ETA)
SIFD = SIF*360.0/(2.0*3.14159)
ANG = SIFD/2.0
IF (FM-1.20)1,69,14
14 IF (FM-1.40)2,8,15
15 IF (FM-1.60)3,9,16
16 IF (FM-1.80)4,10,17
17 IF (FM-2.00)5,11,18
18 IF (FM-3.00)6,12,19
19 IF (FM-4.00)7,13,20
69 K = 1
GO TO 40

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8 K = 2
  GO TO 40
9 K = 3
  GO TO 40
10 K = 4
  GO TO 40
11 K = 5
  GO TO 40
12 K = 6
  GO TO 40
13 K = 7
40 CALL PAS1
  WRITE (6,104) (DA(I),ALPH(I),ACH(I),BETA(I),XN(I),YN(I),I = 1,N)
  GO TO 901
2 K = 1
  GO TO 50
3 K = 2
  GO TO 50
4 K = 3
  GO TO 50
5 K = 4
  GO TO 50
6 K = 5
  GO TO 50
7 K = 6
  GO TO 50
20 K = 7
50 CALL PAS2
  CALL PAS3
901 CALL PAS4
  GO TO 900
  1 WRITE (6,105)
900 CALL EXIT
  END
$IBFTC PAS1
  SUBROUTINE PAS1
  COMMON DATR(60),DATS(60),DAPS(60),DAP(60),ALRD(60),DARD(60),
  1DAPR(60),ALSL(60),DASL(60),ACH(60),BET(60),BETA(60),DA(60),
  2XO(60),YO(60),DAT(60),XAT12(60),YAT12(60),XAT14(60),YAT14(60),
  3XAT16(60),YAT16(60),XAT18(60),YAT18(60),XAT20(60),YAT20(60),
  4XAT30(60),YAT30(60),XAT40(60),YAT40(60),SIM(60),XN(60),YN(60),
  5ALPH(60),SIP(60),ALSC(60),DASC(60),ETA,CHT,SIF,SIFD,ANG,K,FM,N12,
  6N14,N16,N18,N20,N30,N40,N,ACHT,
  7TH,C(60),D(60),X(60),Y(60),U(60),V(60),W(60),Z(60),E(60),F(60)
  GO TO (69,8,9,10,11,12,13),K
69 N = N12
  DO 690 I = 1,N12
  XN(I) = XAT12(I)
690 YN(I) = YAT12(I)
  GO TO 50
  8 N=N14
  DO 80 I=1,N14
  XN(I) = XAT14(I)
80 YN(I) = YAT14(I)

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      GO TO 50
9  N=N16
   DO 90 I=1,N16
   XN(I) = XAT16(I)
90  YN(I) = YAT16(I)
   GO TO 50
10 N=N18
   SIM(N) = 10.36
   DO 99 I=1,N18
   XN(I) = XAT18(I)
99  YN(I) = YAT18(I)
   GO TO 50
11 N=N20
   SIM(N) = 13.19
   DO 110 I=1,N20
   XN(I) = XAT20(I)
110 YN(I) = YAT20(I)
   GO TO 50
12 N=N30
   SIM(N) = 24.88
   DO 120 I=1,N30
   XN(I) = XAT30(I)
120 YN(I) = YAT30(I)
   GO TO 50
13 N=N40
   DO 130 I=1,N40
   XN(I) = XAT40(I)
130 YN(I) = YAT40(I)
   GO TO 50
50  TEST = 1.0
   DO 500 I=1,N
   ALPH(I) = ANG - SIM(I)
   SIP(I) = ANG + SIM(I)
732 APP = SQRT(TEST**2.0-1.0)
   SID = CHT*ATAN2(APP,CHT)-ATAN(APP)
   SIT = SID*360.0/(2.0*3.14159)
   IF (ABS(SIP(I)-SIT)-.05)78,78,79
78  ACH(I) = TEST
   GO TO 460
79  TEST = TEST+.001
   GO TO 732
460 CONTINUE
   BET(I) = ARSIN(1.0/ACH(I))
   BETA(I) = BET(I)*360.0/(6.28316)
500 DA(I) = ALPH(I)+BETA(I)
   DA(I) = ANG
   DO 501 I = 1,N
   DARD(I) = DA(I)*2.0*3.14159/360.0
   ALRD(I) = ALPH(I)*2.0*3.14159/360.0
   ALSC(I) = TAN(ALRD(I))
   DASC(I) = TAN(DARD(I))
501 CONTINUE

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RETURN
END
$IBFTC PAS2
SUBROUTINE PAS2
COMMON DATR(60),DATS(60),DAPS(60),DAP(60),ALRD(60),DARD(60),
1DAPR(60),ALSL(60),DASL(60),ACH(60),BET(60),BETA(60),DA(60),
2XO(60),YO(60),DAT(60),XAT12(60),YAT12(60),XAT14(60),YAT14(60),
3XAT16(60),YAT16(60),XAT18(60),YAT18(60),XAT20(60),YAT20(60),
4XAT30(60),YAT30(60),XAT40(60),YAT40(60),SIM(60),XN(60),YN(60),
5ALPH(60),SIP(60),ALSC(60),DASC(60),ETA,CHT,SIF,SIFD,ANG,K,FM,N12,
6N14,N16,N18,N20,N30,N40,N,ACHT,
7TH,C(60),D(60),X(60),Y(60),U(60),V(60),W(60),Z(60),E(60),F(60)
GO TO (2,3,4,5,6,7,20),K
2 N=N12
ACHT = 1.20
DO 217 I = 1,N12
XO(I) = XAT12(I)
217 YO(I) = YAT12(I)
GO TO 543
3 N=N14
DO 30 I=N14
ACHT = 1.40
XO(I) = XAT14(I)
30 YO(I) = YAT14(I)
GO TO 543
4 N=N16
ACHT = 1.60
DO 40 I=1,N16
XO(I) = XAT16(I)
40 YO(I) = YAT16(I)
GO TO 543
5 N=N18
SIM(N) = 10.36
ACHT = 1.80
DO 51 I=1,N18
XO(I) = XAT18(I)
51 YO(I) = YAT18(I)
GO TO 543
6 N=N20
SIM(N) = 13.19
ACHT = 2.00
DO 60 I=1,N20
XO(I) = XAT20(I)
60 YO(I) = YAT20(I)
GO TO 543
7 N=N30
SIM(N) = 24.88
ACHT = 3.00
DO 70 I=1,N30
XO(I) = XAT30(I)
70 YO(I) = YAT30(I)
GO TO 543
20 N=N40

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    ACHT=4.00
    DO 201 I=1,N40
    XO(I) = XAT40(I)
201 YO(I) = YAT40(I)
543 CONTINUE
    RETURN
    END
$IBFTC PAS3
    SUBROUTINE PAS3
    COMMON DATR(60),DATS(60),DAPS(60),DAP(60),ALRD(60),DARD(60),
    1DAPR(60),ALSL(60),DASL(60),ACH(60),BET(60),BETA(60),DA(60),
    2XO(60),YO(60),DAT(60),XAT12(60),YAT12(60),XAT14(60),YAT14(60),
    3XAT16(60),YAT16(60),XAT18(60),YAT18(60),XAT20(60),YAT20(60),
    4XAT30(60),YAT30(60),XAT40(60),YAT40(60),SIM(60),XN(60),YN(60),
    5ALPH(60),SIP(60),ALSC(60),DASC(60),ETA,CHT,SIF,SIFD,ANG,K,FM,N12,
    6N14,N16,N18,N20,N30,N40,N,ACHT,
    7TH,C(60),D(60),X(60),Y(60),U(60),V(60),W(60),Z(60),E(60),F(60)
    ETT = SQRT(ACHT**2.0-1.0)
    SIFT = CHT*ATAN2(ETT,CHT)-ATAN(ETT)
    SIFTD = SIFT*360.0/(2.0*3.14159)
    ANT = SIFTD/2.0
    TEST = 1.0
    DO 563 I=1,N
    ALPH(I) = ANT - SIM(I)
    SIP(I) = ANT + SIM(I)
722 APP = SQRT(TEST**2-1.0)
    SID = CHT*ATAN2(APP,CHT)-ATAN(APP)
    SIT = SID*360.0/(2.0*3.14159)
    IF(ABS(SIP(I)-SIT)-.05)88,88,89
88 ACH(I) = TEST
    GO TO 46
89 TEST = TEST+.001
    GO TO 722
46 CONTINUE
    BET(I) = ARSIN(1.0/ACH(I))
    BETA(I) = BET(I)*360.0/(6.28318)
563 DAT(I) = ALPH(I)+BETA(I)
    TEST = 1.0
    DAT(1) = ANT
    DO 544 I=1,N
    ALPH(I) = ANG - SIM(I)
    SIP(I) = ANG + SIM(I)
463 APP = SQRT(TEST**2-1.0)
    SID = CHT*ATAN2(APP,CHT)-ATAN(APP)
    SIT = SID*360.0/(2.0*3.14159)
    IF(ABS(SIP(I)-SIT)-.05)208,208,209
208 ACH(I) = TEST
    GO TO 461
209 TEST = TEST+.001
    GO TO 463
461 CONTINUE
    BET(I) = ARSIN(1.0/ACH(I))

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```

      BETA(I) = BET(I)*360.0/(6.28318)
      DA(I) = ALPH(I)+BETA(I)
544 DAP(I) = BETA(I)-ALPH(I)
      DA(1) = ANG
      YN(1) = YO(1)
      XN(1) = XO(1)
      DO 721 I=1,N
      ALRD(I) = ALPH(I)*2.0*3.14159/360.
      DARD(I) = DA(I)*2.0*3.14159/360.0
      DAPR(I) = DAP(I)*3.14159/180.0
      ALSC(I) = TAN(ALRD(I))
      DASC(I) = TAN(DARD(I))
      DAPS(I) = TAN(DAPR(I))
      DATR(I) = DAT(I)*2.0*3.14159/360.0
721 DATS(I) = TAN(DATR(I))
      DO 177 I=1,N
      IF(I-1)177,177,139
139 A = XO(I)-YO(I)/DATS(I)
      B = YN(I-1)+XN(I-1)*DAPS(I-1)
      G = 1.0+DAPS(I-1)/DATS(I)
      XN(I) = (A+B/DATS(I))/G
      YN(I) = (B-A*DAPS(I-1))/G
177 CONTINUE
      YN(N+1) = 0.0
      XN(N+1) = YN(N)/DAPS(N)+XN(N)
      ACH(N+1) = FM
      BET(N+1) = ARSIN(1.0/ACH(N+1))
      BETA(N+1) = BET(N+1)*180.0/3.14159
      ALPH(N+1) = 0.0
      ALRD(N+1) = 0.0
      ALSC(N+1) = 0.0
      DA(N+1) = BETA(N+1)
      DARD(N+1) = BET(N+1)
      DASC(N+1) = TAN(DARD(N+1))
      N = N+1
104 FORMAT (6F15.6)
700 WRITE (6,104) (DA(I),ALPH(I),ACH(I),BETA(I),XN(I),YN(I),I = 1,N)
      RETURN
      END
$IBFTC PAS4
      SUBROUTINE PAS4
      COMMON DATR(60),DATS(60),DAPS(60),DAP(60),ALRD(60),DARD(60),
1DAPR(60),ALSL(60),DASL(60),ACH(60),BET(60),BETA(60),DA(60),
2XO(60),YO(60),DAT(60),XAT12(60),YAT12(60),XAT14(60),YAT14(60),
3XAT16(60),YAT16(60),XAT18(60),YAT18(60),XAT20(60),YAT20(60),
4XAT30(60),YAT30(60),XAT40(60),YAT40(60),SIM(60),XN(60),YN(60),
5ALPH(60),SIP(60),ALSC(60),DASC(60),ETA,CHT,SIF,SIFD,ANG,K,FM,N12,
6N14,N16,N18,N20,N30,N40,N,ACHT,
7TH,C(60),D(60),X(60),Y(60),U(60),V(60),W(60),Z(60),E(60),F(60)
      AT = TH*(216.0/125.0)*FM*(1.0/(1.0+(FM**2)/5.0))**3
40 FORMAT (2F18.8//)
50 FORMAT (5X,4HX(I),11X,4HY(I),10X,33H CONTOUR OF WALL BEFORE AVERAG
ing)

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60 FORMAT (5X,4HX(I),11X,4HY(I),10X,19H FINAL WALL CONTOUR)
70 FORMAT (3F18.8//)
DO 1 I = 1,N
  X(I) = XN(I)*AT/2.0
  Y(I) = YN(I)*AT/2.0
1 CONTINUE
  U(1) = X(1)
  V(1) = Y(1)
  M = N-1
  DO 2 I = 1,M
    CA = V(I)-ALSC(I)*U(I)
    CB = Y(I+1)-DASC(I+1)*X(I+1)
    DN = ALSC(I)-DASC(I+1)
    V(I+1) = (CB*ALSC(I)-CA*DASC(I+1))/DN
2 U(I+1) = (CB-CA)/DN
  WRITE (6,50)
  WRITE (6,40) (U(I),V(I),I = 1,N)
  W(1) = U(1)
  Z(1) = V(1)
  C(1) = (U(1)+U(2))/2.0
  D(1) = (V(1)+V(2))/2.0
  DO 3 I = 2,M
    DN = ALSC(I)-DASC(I)
    CA = D(I-1)-ALSC(I)*C(I-1)
    CB = Y(I)-DASC(I)*X(I)
    Z(I) = (CB*ALSC(I)-CA*DASC(I))/DN
    W(I) = (CB-CA)/DN
    CA = D(I-1)-C(I-1)*ALSC(I)
    CB = Y(I+1)-X(I+1)*DASC(I+1)
    DN = ALSC(I) - DASC(I+1)
    F(I-1) = (CB*ALSC(I)-CA*DASC(I+1))/DN
    E(I-1) = (CB-CA)/DN
    C(I) = (W(I)+E(I-1))/2.0
3 D(I) = (Z(I)+F(I-1))/2.0
  CA = D(N-1)-C(N-1)*ALSC(N)
  CB = Y(N)-X(N)*DASC(N)
  DN = ALSC(N)-DASC(N)
  Z(N) = (CB*ALSC(N)-CA*DASC(N))/DN
  W(N) = (CB-CA)/DN
  WRITE (6,60)
  WRITE (6,70) (W(I),Z(I),ACH(I),I = 1,N)
  RETURN
  END

```

\$ENTRY

CARD HERE TO GIVE FM(DEC. AT COL. 7) AND TH(DEC. AT COL. 22)

0.00	.01	.04	.07	.10	.13	.16	.19
.25	.31	.37	.46	.55	.64	.73	.82
.91	1.00	1.20	1.40	1.60	1.80	2.00	2.40
2.80	3.20	3.60	4.00	4.50	5.00	5.50	6.00
6.50	7.00	7.50	8.00	9.00	10.00	11.00	12.00
13.00	14.00	15.00	16.00	17.00	18.00	19.00	21.00
23.00	25.00	27.00	29.00	31.00	32.89		
22							
0.000	1.000	.213	.614	.277	.500	.305	.448
.328	.408	.343	.380	.356	.357	.368	.336
.386	.305	.404	.274	.417	.253	.433	.223
.448	.198	.463	.176	.475	.155	.487	.136
.498	.118	.507	.103	.527	.072	.543	.046
.559	.022	.574	0.00				
29							
0.000	1.000	.257	.698	.335	.607	.369	.567
.397	.535	.417	.513	.432	.495	.447	.477
.469	.452	.491	.427	.507	.407	.527	.384
.547	.363	.564	.342	.580	.325	.594	.308
.608	.293	.620	.279	.645	.252	.667	.227
.687	.205	.706	.185	.722	.167	.752	.133
.783	.101	.808	.075	.832	.050	.856	.025
.880	0.000						
35							
0.000	1.000	.292	.751	.381	.676	.421	.642
.451	.616	.472	.597	.492	.582	.508	.567
.533	.546	.557	.525	.577	.508	.600	.489
.622	.471	.643	.454	.661	.439	.679	.425
.694	.412	.708	.399	.738	.375	.764	.353
.787	.333	.810	.315	.828	.300	.867	.268
.903	.239	.934	.213	.963	.190	.992	.167
1.022	.142	1.054	.117	1.083	.093	1.115	.067
1.145	.045	1.172	.022	1.202	0.000		
39							
0.000	1.000	.320	.790	.417	.725	.460	.697
.494	.675	.517	.659	.537	.646	.557	.633
.583	.615	.612	.597	.632	.583	.658	.567
.682	.550	.704	.535	.725	.522	.743	.510
.762	.498	.777	.488	.811	.467	.841	.447
.867	.430	.892	.413	.912	.400	.957	.371
.997	.345	1.034	.321	1.067	.300	1.102	.277
1.137	.254	1.175	.231	1.208	.208	1.247	.183
1.282	.162	1.315	.139	1.350	.117	1.379	.097
1.443	.055	1.507	.012	1.528	0.000		
42							
0.000	1.000	.345	.824	.449	.770	.496	.746
.533	.728	.557	.715	.580	.703	.601	.692
.630	.678	.660	.662	.683	.650	.710	.637
.737	.622	.762	.609	.784	.598	.804	.588
.824	.578	.842	.569	.878	.550	.911	.533
.939	.519	.968	.504	.992	.492	1.041	.467
1.088	.443	1.128	.422	1.167	.403	1.206	.382

1.246	.362	1.288	.339	1.328	.318	1.373	.295
1.413	.274	1.453	.253	1.493	.232	1.528	.213
1.603	.173	1.679	.133	1.753	.092	1.827	.052
1.898	.012	1.918	0.000				
50							
0.000	1.000	.439	.953	.573	.939	.633	.933
.681	.928	.713	.925	.742	.921	.768	.918
.805	.914	.845	.910	.875	.906	.909	.903
.946	.898	.980	.895	1.010	.892	1.038	.888
1.063	.887	1.088	.883	1.138	.878	1.183	.873
1.222	.868	1.263	.863	1.298	0.858	1.368	.850
1.434	.842	1.496	.833	1.555	.826	1.616	.819
1.678	.810	1.745	.800	1.812	.792	1.883	.780
1.949	.771	2.015	.761	2.085	.750	2.147	.739
2.281	.717	2.417	.693	2.554	.667	2.695	.639
2.835	.611	2.982	.578	3.138	.543	3.289	.508
3.457	.467	3.618	.423	3.796	.375	4.167	.272
4.584	.144	5.023	0.000				
54							
0.000	1.000	.507	1.056	.661	1.073	.731	1.080
.785	1.086	.822	1.090	.857	1.094	.887	1.097
.931	1.102	.977	1.107	1.012	1.112	1.053	1.116
1.096	1.120	1.137	1.125	1.172	1.129	1.204	1.132
1.235	1.135	1.264	1.138	1.326	1.145	1.380	1.150
1.427	1.155	1.479	1.160	1.522	1.162	1.608	1.172
1.694	1.180	1.772	1.187	1.849	1.192	1.927	1.199
2.011	1.205	2.099	1.212	2.187	1.217	2.285	1.223
2.374	1.228	2.467	1.233	2.563	1.237	2.650	1.241
2.842	1.247	3.039	1.252	3.242	1.255	3.453	1.256
3.669	1.255	3.897	1.252	4.143	1.244	4.387	1.235
4.660	1.221	4.932	1.204	5.227	1.183	5.866	1.129
6.607	1.050	7.445	.939	8.402	.792	9.518	.587
10.770	.329	12.179	0.000				

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BIBLIOGRAPHY

1. Shames, Harold, and Seashore, Ferris L., "Design Data for Graphical Construction of Two-Dimensional Sharp-Edge-Throat Supersonic Nozzles, NACA RM E8J12, December, 1948.

APPENDIX

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUMDESIGN DATA FOR GRAPHICAL CONSTRUCTION
OF TWO-DIMENSIONAL SHARP-EDGE-THROAT
SUPERSONIC NOZZLES

By Harold Shames and Ferris L. Seashore

SUMMARY

Design data are presented for the graphical construction of two-dimensional sharp-edge-throat supersonic nozzles of minimum length for test-section Mach numbers from 1.20 to 10.00. The method of characteristics used in the design is briefly reviewed.

INTRODUCTION

A general discussion of the method of characteristics as applied to supersonic-nozzle design is given in references 1 to 3. The application of the method of characteristics to the design of minimum-length sharp-edge-throat nozzles is described in reference 3.

By means of charts and tables presented herein for designing such nozzles using an expansion "kernel," nozzle-wall contours for wind-tunnel test-section Mach numbers from 1.20 to 10.00 may be obtained with a minimum of graphical construction. The principles of the method of characteristics used in the design are reviewed. The nomenclature of reference 1 was found to be more convenient than the speed-index or pressure-number systems of references 2 and 3, and is therefore used in this report.

SYMBOLS

The following symbols are used in this report:

A_f	area of nozzle bearing uniform flow at M_f (equal to height for nozzle of unit width)
A_t	area of nozzle at throat (equal to height for nozzle of unit width)
L	length of nozzle from throat to test section
l_k	length of kernel
M	Mach number

M_f	final Mach number
x	abscissa of point of intersection of ψ_f^+ characteristic with ψ^- characteristic
y	ordinate of point of intersection of ψ_f^- characteristic with ψ^- characteristic
α	angle of wall to nozzle axis
β	Mach angle, $(\sin^{-1} \frac{1}{M})$
β_f	final Mach angle
γ	ratio of specific heat at constant pressure to specific heat at constant volume
θ	angle of inclination of streamline to nozzle axis
λ^+	angle that ψ^+ characteristic makes with x axis ($\beta - \theta$) (λ^+ is positive number when drawn below horizontal)
λ^-	angle that ψ^- characteristic makes with x axis ($\beta + \theta$) (λ^- is positive number when drawn above horizontal)
ϕ	angle of corner in wall at nozzle throat
ψ	equivalent Prandtl-Meyer turning angle
ψ^+	characteristics (Mach waves) originating at upper nozzle wall
ψ^-	characteristics (Mach waves) originating at lower nozzle wall
ψ_f	value of ψ at nozzle exit
ψ_f^+	downstream characteristic bounding expansion wave originating at upper nozzle wall
ψ_f^-	downstream characteristic bounding expansion wave originating at lower nozzle wall

METHOD OF NOZZLE DESIGN

System of Characteristics in Sharp-Edge-Throat Nozzles

A two-dimensional nozzle with a sharp-edge throat is shown in figure 1. The increase in flow Mach number with displacement downstream of the throat is obtained from the system of expansion waves generated at the angular turn of the wall at the nozzle throat (fig. 2(a)). The expansion waves, as shown in figure 2(a), turn the flow toward the adjacent nozzle wall downstream of the corner with a consequent increase in

stream-tube cross-sectional area and Mach number. The system of expansion waves from each corner is identical with that developed in an infinite uniform sonic flow constrained to flow around a sharp corner in a single two-dimensional wall. The solution for this case is discussed in reference 4. The expansion waves are propagated into the flow along straight lines radiating from the corner in the case for the flow along only one wall in an infinite flow. Along any given radial line, the flow direction, the Mach number, and the physical state of the gas is the same for all points on that line (fig. 2(a)). Each of these radial lines can be assigned a number in degrees or radians that corresponds to the angular deviation of the flow crossing the line from the direction of the undisturbed sonic flow. A line so numbered is called a characteristic. The angular deviation of the flow between two characteristics is equal to the difference of the numbers assigned to these characteristics. At each characteristic, the flow makes the Mach angle $\beta = \sin^{-1} 1/M$ with the characteristic. The characteristics are therefore coincident with the Mach lines in the flow.

Two separate walls in the flow (fig. 2(a)) result in two separate systems of intersecting expansion waves originating at the respective wall corners. If the characteristics from the upper and lower walls are designated ψ^+ and ψ^- , respectively, every point in the flow traversed by both expansion waves is crossed by a characteristic from the upper and lower walls. Because of the simultaneous influence on the flow of the expansion waves from the corner on the upper and lower walls in the zone common to both sets of waves, the characteristics are curved to maintain the Mach angle with the flow (zone I, fig. 2 (b)). The characteristics are straight in zones occupied by only one set of expansion waves (zones II and III, fig. 2(b)).

By means of the characteristics in zones II and III, the graphical construction of the nozzle-wall contour required to give wave-free flow in the test section can be made. Tables I and II provide the information for obtaining the characteristics in zones II and III without involved plotting or computation. The construction of the wave pattern from which the information in tables I and II was obtained is described in the following section.

Development of Kernel

From references 2 and 4, the value of the flow Mach number at a point in the flow, crossed by characteristics having values of ψ^+ and ψ^- , respectively, is given by

$$\psi = \psi^+ + \psi^- = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \frac{\sqrt{M^2 - 1}}{\sqrt{\frac{\gamma + 1}{\gamma - 1}}} - \tan^{-1} \sqrt{M^2 - 1} \quad (1)$$

The flow direction with respect to the nozzle axis is

$$\theta = \psi^+ - \psi^- \quad (2)$$

For an isentropic flow of known uniform total pressure and temperature the flow at any point is completely specified by the local values of the intersecting pair of characteristics.

A wave pattern for a pair of opposite corners at the nozzle throat is established by dividing the wave emitted by each corner into a convenient number of characteristics, and by determining the resulting wave pattern due to the interaction of both sets of waves by means of the foregoing principles; that is, the local Mach number is given by equation (1), the flow direction is given by equation (2), and the local Mach angle is determined from the relation $\beta = \sin^{-1} 1/M$.

The resulting system of characteristics in the zone of the flow traversed by waves from both corners (zone I, fig. 2(b)) is shown schematically in figure 2(c). Such a pattern is called a kernel. In order to obtain the tables giving the pertinent design parameters for sharp-edge nozzles ranging in test-section Mach number from 1.20 to 10.00, a kernel was graphically developed for two opposing corners of equal angle (51.16°) corresponding to $M = 10.00$ at the test section with the following increments in ψ^+ and ψ^- :

ψ^+ and ψ^- limits (deg)		ψ^+ and ψ^- increments (deg)
0	- 0 .01	0.01
	.01 - .19	.03
	.19 - .37	.06
	.37 - 1.00	.09
	1.00 - 2.00	.20
	2.00 - 4.00	.40
	4.00 - 8.00	.50
	8.00 - 19.00	1.00
	19.00 - 51.16	2.00

In the range of low values of ψ^+ and ψ^- , where the construction is sensitive to small changes in these values, small increments in ψ^+ and ψ^- were used, as indicated in the preceding table.

Because the corners at the nozzle throat were chosen equal, the resultant wave pattern is symmetrical and only the half above the nozzle axis need be considered. The wave pattern at any point in the kernel is not influenced by the wave pattern downstream of that point. Consequently, the kernel for any corner less than the maximum of 51.16° can be obtained from the kernel for 51.16° by neglecting the characteristic of value greater than the desired corner angle. This principle is illustrated in figure 2(c).

The bounding characteristic separating zone I from zone II (fig. 2(b)) is designated as ψ_{f+} . The points of intersection of the ψ^- characteristics with the ψ_{f+} characteristic, and the slopes of the ψ^- characteristics at these points, are all that is required to determine the nozzle contour.

The constructed kernel for $M = 10.00$ provided data for the design of nozzles for final Mach numbers M_f from 1.20 to 2.00 in increments of 0.20 and from 2.00 to 10.00 in increments of 1.00.

The coordinates $(\frac{x}{A_t/2}, \frac{y}{A_t/2})$ of the points of intersection of the bounding characteristic ψ_{f+} with the ψ^- characteristics are tabulated with other pertinent data in table I.

For Mach numbers up to 4.00, a kernel of 12-inch half throat height $A_t/2$ was graphically developed and for Mach numbers from 5.00 to 10.00 a half throat height of 6 inches was used. For the 6-inch kernel, however, the scale was reduced at intervals as the height of the kernel increased in order to maintain the construction within the physical limit of the drawing board. This scale reduction accounts for the decreasing number of decimal places for the coordinates in table I in the high Mach number range. Turning-angle increments in ψ^+ and ψ^- , as given in the preceding table, were used for both kernels. Construction was performed with a drafting machine capable of setting to ± 2.5 minutes.

Wall Contour

An expansion wave incident on a channel wall will, in general, require that a secondary wave be emitted at the point of incidence in order to keep the flow against the wall. If the wall is curved in the way a streamline would be turned under the influence of the incident wave, however, no secondary wave arises to keep the flow along the wall. This method of suppression of secondary waves is the principle used to obtain uniform wave-free flow in the test section. The graphical construction is required to locate the point of incidence of the waves on the nozzle wall. The difference in value of the characteristics bounding the incident wave gives the change of wall inclination required to suppress secondary waves (fig. 2(d)); that is, for the upper wall,

$$\Delta\alpha = \Delta\psi^- \quad (3a)$$

or for the lower wall,

$$\Delta\alpha = \Delta\psi^+ \quad (3b)$$

where $\Delta\alpha$ is the required change of wall inclination. The accuracy of the wall contour obtained improves as the number of characteristics drawn to represent the incident expansion wave is increased. Only the upper nozzle wall need be developed if the nozzle is symmetrical about the center line.

Symmetrical two-dimensional sharp-edge-throat nozzles are produced by making the angle of the turn at both walls at the throat equal in magnitude. If ϕ represents the angle of turn for the upper and lower walls, the downstream characteristics ψ_f^+ and ψ_f^- that bound the respective expansion waves will have this value. Because of the symmetry of the wave pattern about the nozzle axis, a ψ^+ characteristic will intersect a ψ^- characteristic of the same magnitude at the nozzle axis. In particular, the ψ_f^+ and ψ_f^- characteristics intersect on the nozzle axis (fig. 2(c)). The flow along the streamline on the nozzle axis will have the final Mach number M_f at the intersection of these bounding characteristics. From equation (1),

$$\psi_f^+ + \psi_f^- = \psi_f = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \frac{\sqrt{M_f^2-1}}{\sqrt{\frac{\gamma+1}{\gamma-1}}} - \tan^{-1} \sqrt{M_f^2-1}$$

Because $\psi_f^+ = \psi_f^-$ are equal in magnitude and represent the angle through which each wall is turned at the throat,

$$\phi = \psi_f^+ = \psi_f^- = \frac{\psi_f}{2} = \frac{1}{2} \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \frac{\sqrt{M_f^2-1}}{\sqrt{\frac{\gamma+1}{\gamma-1}}} - \tan^{-1} \sqrt{M_f^2-1} \quad (4)$$

Equation (4) gives the value of the wall angle at the throat that corresponds to the desired test-section Mach number M_f . These values are presented in table II, columns 1 and 2.

The method of using the kernel that is schematically shown in figure 3(a) to obtain the nozzle-wall contour of two-dimensional sharp-throat nozzles of minimum length (fig. 3(b)) is illustrated by application to a specific problem. Assume that it is desired to design a nozzle of this class with a test-section Mach number of 4.00 and a throat height of 6 inches.

The throat-corner angle ϕ and the value of the downstream bounding characteristics ψ_f^+ are obtained from equation (4) or table II, column 2, with M_f equal to 4.00, column 1:

$$\phi = \psi_f^+ = \psi_f^- = 32.89^\circ$$

The wall contour is obtained by plotting the zone II characteristics of the ψ^- set (fig. 3(b)), which are straight lines that make the angle λ^- with the nozzle axis at the intersection of the ψ^- characteristics and the bounding characteristic ψ_f^+ . All that is required to obtain the zone II

plot are the coordinates of the points of intersection of the ψ^- set of characteristics with the bounding characteristic ψ_f^+ and the local slopes λ^- of the ψ^- characteristics. Columns 4 and 5 of table I give the coordinates of intersection in terms of the half throat height $A_t/2$ and column 6 gives the angle of inclination λ^- of the ψ^- characteristic at the intersection. For example, the $\psi^- = 12.00^\circ$ characteristic intersects the ψ_f^+ characteristic at $\frac{x}{A_t/2} = 3.453$ and $\frac{y}{A_t/2} = 1.256$, which gives $x = 10.359$ and $y = 3.768$ for a

nozzle of 6-inch throat. The inclination of the ψ^- characteristic in zone II is $\lambda^- = 42.14^\circ$. The complete plot of the zone II characteristics has the form schematically illustrated in figure 4(a).

Construction of the nozzle wall starts at the nozzle throat with a straight-line segment ab (fig. 4(b)) that makes the angle with the nozzle axis $\phi = 32.89^\circ$ which was previously computed for $M_f = 4.00$. At the intersection of the nozzle wall with the first ψ^- characteristic ($\psi_1^- = 0.01^\circ$), the inclination of the wall is reduced according to equation (3a) by an amount $(\psi_1^- - \psi_0^-)$ corresponding to the angle through which the flow is turned clockwise by the expansion wave between ψ_0^- and ψ_1^- . As previously discussed, no wave emission occurs at the wall turned in this way. At every intersection of the wall with a characteristic, the wall inclination to the nozzle axis is reduced by the angle of turning produced by the wave between ψ^- and ψ_{n-1}^- . The angle of the

wall α at each characteristic is given in table I, column 7. For example, at point b , $\psi^- = 0.01^\circ$ and $\alpha = 32.88^\circ$; similarly at point c , $\psi^- = 0.04^\circ$ and $\alpha = 32.85^\circ$. When the sequence of straight-line segments representing the nozzle nozzle wall is completed, a smooth curve approximating the shape of the sequence of straight lines is taken as the effective nozzle-wall contour.

The accuracy of the final wall contour increases with the number of characteristics used to represent the expansion waves from the wall corners at the nozzle throat.

An averaging method for attaining a contour that is closer to the true contour will be described for a nozzle with a test-section Mach number M_f of 4.00 as an example, as shown in figure 4(b). As before, construction starts at the nozzle throat with a straight line ab making the corner angle with the nozzle axis ($\phi = 32.89^\circ$). Line ab is then bisected by point c , and line cd is drawn at the wall angle $= 32.88^\circ$, corresponding to $\psi^- = 0.01^\circ$, (table I, column 7) until it intersects the $\psi^- = 0.04^\circ$ characteristic. Point B , the wall coordinate point lying along $\psi^- = 0.01^\circ$, is located by the intersection of line cd and $\psi^- = 0.01^\circ$. Line Bd is then bisected by point e , and line eg is drawn at the wall angle $\alpha = 32.85^\circ$ corresponding to $\psi^- = 0.04^\circ$. Point D is located by the intersection of line eg and $\psi^- = 0.04^\circ$. The preceding process is continued until the design is complete. The nozzle contour is taken as the smooth curve through points A, B, D, \dots , tangent to construction lines ab, cd, eg, \dots .

The test-section height of the nozzle (numerically equal to A_f), which is obtained by either of the graphical processes described, should be related to the throat height by the expression

$$\frac{A_f}{A_t} = \frac{1}{M_f} \left(\frac{1 + \frac{\gamma-1}{2} M_f^2}{\frac{\gamma+1}{2}} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (5)$$

These area-ratio values are presented in table II, column 4. For example,

$$M_f = 4.00, \quad \frac{A_f}{A_t} = 10.719.$$

The design of a nozzle that has a Mach number intermediate between values given in table I requires the determination of the shape of the ψ_f^+ characteristic of the kernel corresponding to the desired Mach number. This design is accomplished by using the coordinates of the ψ_f^+ characteristics given in table I that are closest to the desired Mach number and then establishing by construction the points of intersection of the ψ_f^+ characteristics that correspond to the desired Mach number with the ψ^- characteristics, as shown in figure 5. For example, the kernel for $M = 4.30$ is established with the kernel for $M = 4.00$ as a base.

The bounding characteristic ψ_{f+} and the zone II plot of ψ - characteristics for $M_f = 4.00$ are established as previously described. These characteristics are dashed in figure 5. The bounding characteristic and the zone II plot of ψ - characteristics for $M_f = 4.30$ are established according to the following procedure:

The value of the bounding characteristic ψ_{f+} for $M_f = 4.30$ is obtained from equation (4) or table II, column 2, ($\psi_{f+} = 34.77^\circ$). The angle that the ψ_{f+} characteristic makes with the nozzle axis at any point is designated λ^+ (fig. 5) and is determined by the relation

$$\lambda^+ = \beta - \theta \quad (6)$$

where β is the Mach angle determined by the local Mach number corresponding to the local equivalent Prandtl-Meyer turning angle ψ , given by equation (1), and θ is the angle of inclination of the flow to the nozzle axis, given by equation (2). (Note that positive values of λ^+ are drawn with a negative slope.) Thus at point A at the throat (fig. 5): from equation (1),

$$\psi = \psi^+ + \psi^- = 34.77 + 0 = 34.77^\circ$$

from table II, columns 3 and 5, for $\psi = 34.77$,

$$\beta = 25.53^\circ$$

from equation (2),

$$\theta = \psi^+ - \psi^- = 34.77 - 0 = 34.77^\circ$$

Consequently,

$$\lambda^+ = \beta - \theta = 25.53 - 34.77 = -9.24^\circ$$

The negative sign indicates that λ^+ is drawn with positive slope, as shown at point A of figure 5. The bounding ψ_f^+ characteristic is drawn at the angle $\lambda^+ = -9.24^\circ$, until it intersects the first ψ^- characteristic $\psi^- = 0.01^\circ$ at point B. At point B the new λ^+ value for ψ_f^+ is determined by repeating the aforementioned procedure using $\psi^+ = 34.77^\circ$ and $\psi^- = 0.01^\circ$. The ψ_f^+ characteristic is drawn at this new λ^+ value until it intersects the next ψ^- characteristic at point B, λ^- , is determined by the relation (fig. 5)

$$\lambda^- = \beta + \theta \quad (7)$$

with the same values for β and θ as were used to determine λ^+ . In this manner the entire zone II plot of ψ^- characteristics is obtained for $M_f = 4.30$.

The wall contour is then developed by the method previously described for $M_f = 4.00$. The entire procedure is expedited if columns 1 to 3, 6, and 7, ϕ and ψ_f^+ , of table I, and λ^+ are determined for the ψ_f^+ characteristic for $M = 4.30$ before the drawing is initiated.

Nozzle Length

The nozzle length from the throat to the test section may be calculated from the length of the kernel and the projection of the last characteristic on the nozzle axis, as shown in figure 3(b). The projection may be determined from the final Mach angle and the final area ratio. The expression for the ratio of the nozzle length to the nozzle test-section height

$$\frac{L}{A_f} = \left(\frac{l_k}{A_t} + \frac{A_f}{2A_t \tan \beta} \right) \frac{A_t}{A_f}$$

is plotted in figure 6 for Mach numbers up to 10.

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REFERENCES

1. Pinkel, I. Irving: Equations for the Design of Two-Dimensional Supersonic Nozzles. NACA RM No. E8B02, 1948.
2. Puckett, A. E.: Supersonic Nozzle Design. Jour. Appl. Mech., vol. 13, no. 4, Dec. 1946, pp. A265-A270.
3. Shapiro, Ascher H., and Edelman, Gilbert M.: Method of Characteristics for Two-Dimensional Supersonic Flow - Graphical and Numerical Procedures. Jour. Appl. Mech., vol. 14, no. 2, June 1947, pp. A154-A162.
4. Taylor, G. I., and Maccoll, J. W.: The Two-Dimensional Flow Around a Corner; Two-Dimensional Flow Past a Curved Surface. Vol. III of Aerodynamic Theory, div. H, ch. IV, secs. 5-6, W. F. Durand, ed., Julius Springer (Berlin), 1935, pp. 243-249.

TABLE I. - DETAILED NOZZLE DESIGN PARAMETERS - Continued
 $[\gamma = 1.400]$

1	2	3	4	5	6	7	1	2	3	4	5	6	7
ψ - (deg)	ψ (deg)	β (deg)	$\frac{x}{A_t}$ /2	$\frac{y}{A_t}$ /2	λ - (deg)	α (deg)	ψ - (deg)	ψ (deg)	β (deg)	$\frac{x}{A_t}$ /2	$\frac{y}{A_t}$ /2	λ - (deg)	α (deg)
$M_f, 7.00; \phi$ and $\psi_{f+}, 45.49^\circ$							$M_f, 8.00; \phi$ and $\psi_{f+}, 47.81^\circ$						
0	45.49	21.02	0	1.000	-----	45.49	0	47.81	20.17	0	1.000	-----	47.81
.01	45.50	21.02	.627	1.280	66.50	45.48	.01	47.82	20.16	.653	1.340	67.96	47.80
.04	45.53	21.01	.817	1.367	66.46	45.45	.04	47.85	20.15	.853	1.443	67.92	47.77
.07	45.56	21.00	.903	1.407	66.42	45.42	.07	47.88	20.14	.940	1.487	67.88	47.74
.10	45.59	20.98	.973	1.440	66.37	45.39	.10	47.91	20.13	1.013	1.527	67.84	47.71
.13	45.62	20.97	1.020	1.460	66.33	45.36	.13	47.94	20.12	1.063	1.553	67.80	47.68
.16	45.65	20.96	1.063	1.480	66.29	45.33	.16	47.97	20.10	1.107	1.573	67.75	47.65
.19	45.68	20.95	1.100	1.493	66.25	45.30	.19	48.00	20.09	1.147	1.593	67.71	47.62
.25	45.74	20.93	1.157	1.523	66.17	45.24	.25	48.06	20.07	1.207	1.627	67.63	47.56
.31	45.80	20.91	1.210	1.543	66.09	45.18	.31	48.12	20.04	1.260	1.653	67.54	47.50
.37	45.86	20.89	1.260	1.567	66.01	45.12	.37	48.18	20.03	1.313	1.680	67.47	47.44
.46	45.95	20.85	1.313	1.590	65.88	45.03	.46	48.27	19.99	1.367	1.710	67.34	47.35
.55	46.04	20.81	1.367	1.613	65.75	44.94	.55	48.36	19.96	1.423	1.740	67.22	47.26
.64	46.13	20.78	1.420	1.637	65.63	44.85	.64	48.45	19.93	1.480	1.767	67.10	47.17
.73	46.22	20.75	1.463	1.657	65.51	44.76	.73	48.54	19.89	1.527	1.790	66.97	47.08
.82	46.31	20.72	1.510	1.677	65.39	44.67	.82	48.63	19.86	1.573	1.817	66.85	46.99
.91	46.40	20.69	1.550	1.697	65.27	44.58	.91	48.72	19.83	1.617	1.837	66.73	46.90
1.00	46.49	20.65	1.587	1.713	65.14	44.49	1.00	48.81	19.80	1.657	1.857	66.61	46.81
1.20	46.69	20.57	1.670	1.750	64.86	44.29	1.20	49.01	19.73	1.743	1.903	66.34	46.61
1.40	46.89	20.50	1.743	1.780	64.59	44.09	1.40	49.21	19.66	1.820	1.940	66.07	46.41
1.60	47.09	20.42	1.807	1.807	64.31	43.89	1.60	49.41	19.59	1.887	1.973	65.80	46.21
1.80	47.29	20.35	1.880	1.840	64.04	43.69	1.80	49.61	19.52	1.963	2.013	65.53	46.01
2.00	47.49	20.28	1.940	1.867	63.77	43.49	2.00	49.81	19.45	2.027	2.043	65.26	45.81
2.40	47.89	20.13	2.060	1.920	63.22	43.09	2.40	50.21	19.31	2.157	2.110	64.72	45.41
2.80	48.29	19.98	2.183	1.970	62.67	42.69	2.80	50.61	19.18	2.287	2.173	64.19	45.01
3.20	48.69	19.84	2.297	2.017	62.13	42.29	3.20	51.01	19.06	2.410	2.230	63.67	44.61
3.60	49.09	19.70	2.410	2.067	61.59	41.89	3.60	51.41	18.91	2.533	2.290	63.12	44.21
4.00	49.49	19.56	2.533	2.117	61.05	41.49	4.00	51.81	18.77	2.663	2.353	62.58	43.81
4.50	49.99	19.38	2.663	2.167	60.37	40.99	4.50	52.31	18.60	2.803	2.417	61.91	43.31
5.00	50.49	19.21	2.800	2.220	59.70	40.49	5.00	52.81	18.44	2.953	2.487	61.25	42.81
5.50	50.99	19.06	2.940	2.277	59.05	39.99	5.50	53.31	18.28	3.107	2.553	60.59	42.31
6.00	51.49	18.87	3.100	2.337	58.36	39.49	6.00	53.81	18.11	3.283	2.633	59.92	41.81
6.50	51.99	18.70	3.247	2.390	57.69	38.99	6.50	54.31	17.94	3.447	2.707	59.25	41.31
7.00	52.49	18.53	3.393	2.443	57.02	38.49	7.00	54.81	17.78	3.613	2.777	58.59	40.81
7.50	52.99	18.38	3.553	2.503	56.37	37.99	7.50	55.31	17.62	3.790	2.853	57.93	40.31
8.00	53.49	18.21	3.710	2.557	55.70	37.49	8.00	55.81	17.46	3.960	2.923	57.27	39.81
9.00	54.49	17.87	4.023	2.667	54.36	36.49	9.00	56.81	17.14	4.300	3.060	55.95	38.81
10.00	55.49	17.55	4.357	2.773	53.04	35.49	10.00	57.81	16.82	4.673	3.207	54.63	37.81
11.00	56.49	17.24	4.727	2.890	51.73	34.49	11.00	58.81	16.52	5.093	3.360	53.33	36.81
12.00	57.49	16.92	5.113	3.007	50.41	33.49	12.00	59.81	16.22	5.527	3.520	52.03	35.81
13.00	58.49	16.61	5.537	3.130	49.10	32.49	13.00	60.81	15.92	6.003	3.683	50.73	34.81
14.00	59.49	16.31	5.983	3.253	47.80	31.49	14.00	61.81	15.62	6.517	3.857	49.43	33.81
15.00	60.49	16.01	6.477	3.383	46.50	30.49	15.00	62.81	15.32	7.083	4.037	48.13	32.81
16.00	61.49	15.71	6.980	3.510	45.20	29.49	16.00	63.81	15.04	7.667	4.183	46.85	31.81
17.00	62.49	15.41	7.553	3.643	43.90	28.49	17.00	64.81	14.76	8.327	4.410	45.57	30.81
18.00	63.49	15.12	8.150	3.777	42.51	27.49	18.00	65.81	14.48	9.023	4.603	44.29	29.81
19.00	64.49	14.84	8.800	3.913	41.33	26.49	19.00	66.81	14.19	9.790	4.810	43.00	28.81
21.00	66.49	14.28	10.323	4.207	38.77	24.49	21.00	68.81	13.65	11.560	5.243	40.46	26.81
23.00	68.49	13.73	12.107	4.510	36.22	22.49	23.00	70.81	13.11	13.707	5.720	37.92	24.81
25.00	70.49	13.19	14.243	4.813	33.68	20.49	25.00	72.91	12.59	16.293	6.213	35.40	22.81
27.00	72.49	12.66	16.843	5.110	31.15	18.49	27.00	74.81	12.08	19.480	6.747	32.89	20.81
29.00	74.49	12.15	19.980	5.387	28.64	16.49	29.00	76.81	11.56	23.367	7.293	30.37	18.81
31.00	76.49	11.64	23.720	5.617	26.13	14.49	31.00	78.81	11.07	28.017	7.813	27.88	16.81
33.00	78.49	11.14	28.353	5.780	23.63	12.40	33.00	80.81	10.58	34.00	8.30	25.39	14.81
35.00	80.49	10.65	33.73	5.82	21.14	10.49	35.00	82.81	10.12	40.97	8.73	22.93	12.81
37.00	82.49	10.18	40.87	5.70	18.67	8.49	37.00	84.81	9.63	50.30	9.03	20.44	10.81
39.00	84.49	9.69	49.83	5.32	16.18	6.49	39.00	86.81	9.16	62.22	9.10	17.97	8.81
41.00	86.49	9.23	61.43	4.51	13.72	4.49	41.00	88.81	8.70	76.65	8.78	15.51	6.81
43.00	88.49	8.77	76.05	3.07	11.26	2.49	43.00	90.81	8.25	98.17	7.73	13.06	4.81
45.00	90.49	8.32	95.23	.67	8.81	.49	45.00	92.81	7.81	125.13	5.77	10.62	2.81
45.49	90.98	8.21	100.01	0	8.21	0	47.81	94.81	7.36	164.43	1.80	8.17	.81
										179.47	0	7.18	0

TABLE II. - OVER-ALL NOZZLE DESIGN PARAMETERS

[$\gamma = 1.400$]

1	2	3	4	5	1	2	3	4	5
M_f	ϕ and ψ_f, ψ ψ_f^+	ψ_f, ψ	A_f/A_t	β_f, β	M_f	ϕ and ψ_f, ψ ψ_f^+	ψ_f, ψ	A_f/A_t	β_f, β
	(deg)	(deg)		(deg)		(deg)	(deg)		(deg)
1.00	0	0	1.0000	90.000	1.80	10.362	20.725	1.4390	33.749
1.02	.063	.126	1.0003	78.635	1.82	10.652	21.304	1.4610	33.329
1.04	.175	.351	1.0013	74.058	1.84	10.939	21.878	1.4836	32.921
1.06	.318	.637	1.0029	70.630	1.86	11.224	22.450	1.5069	32.523
1.08	.484	.968	1.0051	67.808	1.88	11.510	23.020	1.5307	32.135
1.10	.668	1.336	1.0079	65.380	1.90	11.793	23.586	1.5553	31.757
1.12	.867	1.735	1.0113	63.234	1.92	12.076	24.152	1.5804	31.388
1.14	1.080	2.160	1.0153	61.306	1.94	12.356	24.713	1.6062	31.028
1.16	1.304	2.607	1.0198	59.550	1.96	12.635	25.270	1.6326	30.677
1.18	1.537	3.074	1.0248	57.936	1.98	12.913	25.827	1.6597	30.335
1.20	1.779	3.558	1.0304	56.443	2.00	13.190	26.380	1.6875	30.000
1.22	2.028	4.057	1.0366	55.052	2.02	13.464	26.929	1.7160	29.673
1.24	2.285	4.570	1.0432	53.751	2.04	13.738	27.476	1.7451	29.353
1.26	2.546	5.093	1.0504	52.528	2.06	14.011	28.022	1.7750	29.041
1.28	2.814	5.627	1.0581	51.375	2.08	14.281	28.562	1.8056	28.736
1.30	3.085	6.170	1.0663	50.285	2.10	14.548	29.097	1.8369	28.437
1.32	3.360	6.721	1.0750	49.251	2.12	14.815	29.631	1.8690	28.145
1.34	3.635	7.279	1.0842	48.268	2.14	15.080	30.161	1.9018	27.859
1.36	3.922	7.844	1.0940	47.332	2.16	15.344	30.688	1.9354	27.578
1.38	4.206	8.413	1.1042	46.439	2.18	15.606	31.213	1.9698	27.304
1.40	4.493	8.987	1.1149	45.585	2.20	15.866	31.732	2.0050	27.036
1.42	4.782	9.565	1.1262	44.767	2.22	16.125	32.250	2.0409	26.773
1.44	5.073	10.146	1.1379	43.983	2.24	16.381	32.763	2.0777	26.515
1.46	5.365	10.730	1.1502	43.230	2.26	16.637	33.274	2.1154	26.262
1.48	5.663	11.327	1.1629	42.507	2.28	16.889	33.778	2.1538	26.014
1.50	5.953	11.906	1.1762	41.810	2.30	17.141	34.283	2.1931	25.772
1.52	6.248	12.495	1.1899	41.140	2.32	17.391	34.782	2.2333	25.533
1.54	6.542	13.085	1.2042	40.493	2.34	17.639	35.279	2.2744	25.300
1.56	6.837	13.675	1.2190	39.868	2.36	17.885	35.771	2.3164	25.070
1.58	7.135	14.270	1.2344	39.265	2.38	18.131	36.262	2.3593	24.845
1.60	7.430	14.860	1.2502	38.682	2.40	18.373	36.746	2.4031	24.624
1.62	7.726	15.452	1.2666	38.118	2.42	18.615	37.230	2.4479	24.407
1.64	8.021	16.043	1.2835	37.572	2.44	18.854	37.708	2.4936	24.195
1.66	8.316	16.633	1.3010	37.043	2.46	19.092	38.184	2.5403	23.985
1.68	8.611	17.223	1.3190	36.530	2.48	19.327	38.655	2.5880	23.780
1.70	8.905	17.810	1.3376	36.032	2.50	19.562	39.124	2.6367	23.578
1.72	9.198	18.397	1.3567	35.549	2.52	19.794	39.589	2.6864	23.380
1.74	9.490	18.981	1.3764	35.080	2.54	20.025	40.050	2.7372	23.185
1.76	9.783	19.566	1.3967	34.624	2.56	20.254	40.508	2.7891	22.993
1.78	10.073	20.146	1.4175	34.180	2.58	20.481	40.963	2.8420	22.805

TABLE II. - OVER-ALL NOZZLE DESIGN PARAMETERS - Continued
 [$\gamma = 1.400$]

1	2	3	4	5	1	2	3	4	5
M_f	ϕ and ψ_f^+ (deg)	ψ_f, ψ (deg)	A_f/A_t	β_f, β (deg)	M_f	ϕ and ψ_f^+ (deg)	ψ_f, ψ (deg)	A_f/A_t	β_f, β (deg)
2.60	20.707	41.415	2.8960	22.620	3.30	27.611	55.222	5.6286	17.640
2.62	20.931	41.863	2.9511	22.438	3.32	27.782	55.564	5.7358	17.530
2.64	21.154	42.308	3.0073	22.259	3.34	27.952	55.904	5.8448	17.422
2.66	21.374	42.749	3.0647	22.082	3.36	28.120	56.241	5.9558	17.315
2.68	21.593	43.187	3.1233	21.909	3.38	28.288	56.576	6.0687	17.209
2.70	21.810	43.621	3.1830	21.738	3.40	28.454	56.908	6.1837	17.105
2.72	22.026	44.053	3.2440	21.571	3.42	28.619	57.238	6.3007	17.002
2.74	22.240	44.481	3.3061	21.405	3.44	28.782	57.564	6.4198	16.900
2.76	22.453	44.906	3.3695	21.243	3.46	28.944	57.888	6.5409	16.799
2.78	22.664	45.328	3.4342	21.082	3.48	29.105	58.210	6.6642	16.700
2.80	22.873	45.746	3.5001	20.925	3.50	29.265	58.530	6.7896	16.602
2.82	23.080	46.161	3.5674	20.770	3.52	29.423	58.847	6.9172	16.504
2.84	23.286	46.573	3.6359	20.617	3.54	29.581	59.162	7.0470	16.409
2.86	23.491	46.982	3.7058	20.466	3.56	29.737	59.474	7.1791	16.314
2.88	23.694	47.388	3.7771	20.318	3.58	29.892	59.784	7.3135	16.220
2.90	23.895	47.790	3.8498	20.171	3.60	30.045	60.091	7.4501	16.128
2.92	24.095	48.190	3.9238	20.027	3.62	30.198	60.397	7.5891	16.036
2.94	24.293	48.586	3.9993	19.885	3.64	30.350	60.700	7.7304	15.946
2.96	24.490	48.980	4.0763	19.745	3.66	30.500	61.000	7.8742	15.856
2.98	24.685	49.370	4.1547	19.607	3.68	30.649	61.299	8.0204	15.768
3.00	24.878	49.757	4.2346	19.471	3.70	30.797	61.595	8.1690	15.680
3.02	25.071	50.142	4.3160	19.337	3.72	30.944	61.889	8.3202	15.594
3.04	25.261	50.523	4.3989	19.205	3.74	31.090	62.181	8.4739	15.508
3.06	25.451	50.902	4.4835	19.074	3.76	31.235	62.471	8.6302	15.424
3.08	25.638	51.277	4.5696	18.946	3.78	31.379	62.758	8.7891	15.340
3.10	25.825	51.650	4.6573	18.819	3.80	31.522	63.044	8.9506	15.258
3.12	26.010	52.020	4.7467	18.694	3.82	31.663	63.327	9.1148	15.176
3.14	26.193	52.386	4.8377	18.570	3.84	31.804	63.608	9.2817	15.095
3.16	26.375	52.750	4.9304	18.449	3.86	31.943	63.887	9.4513	15.015
3.18	26.556	53.112	5.0248	18.328	3.88	32.082	64.164	9.6237	14.936
3.20	26.735	53.470	5.1210	18.210	3.90	32.220	64.440	9.7990	14.857
3.22	26.913	53.826	5.2189	18.093	3.92	32.356	64.713	9.9771	14.780
3.24	27.089	54.179	5.3186	17.977	3.94	32.492	64.984	10.1580	14.703
3.26	27.265	54.530	5.4201	17.863	3.96	32.626	65.253	10.3420	14.627
3.28	27.438	54.877	5.5234	17.751	3.98	32.760	65.520	10.5288	14.552

TABLE II. - OVER-ALL NOZZLE DESIGN PARAMETERS - Continued
 $[\gamma = 1.400]$

1	2	3	4	5	1	2	3	4	5
M_f	ϕ and ψ_f^+	ψ_f, ψ	A_f/A_t	β_f, β	M_f	ϕ and ψ_f^+	$\psi_f,$	A_f/A_t	β_f, β
	(deg)	(deg)		(deg)		(deg)	(deg)		(deg)
4.00	32.892	65.785	10.719	14.478	5.50	40.622	81.244	36.869	10.476
4.05	33.219	66.439	11.207	14.295	5.55	40.821	81.643	38.281	10.380
4.10	33.542	67.085	11.715	14.117	5.60	41.016	82.032	39.741	10.287
4.15	33.857	67.714	12.243	13.943	5.65	41.209	82.418	41.246	10.195
4.20	34.167	68.334	12.791	13.774	5.70	41.397	82.795	42.796	10.104
4.25	34.472	68.945	13.363	13.609	5.75	41.585	83.171	44.400	10.015
4.30	34.770	69.541	13.955	13.448	5.80	41.768	83.537	46.050	9.928
4.35	35.064	70.128	14.571	13.290	5.85	41.950	83.900	47.754	9.842
4.40	35.353	70.707	15.210	13.137	5.90	42.128	84.257	49.507	9.758
4.45	35.637	71.274	15.874	12.986	5.95	42.303	84.607	51.318	9.675
4.50	35.916	71.833	16.562	12.840	6.00	42.477	84.955	53.178	9.594
4.55	36.190	72.380	17.277	12.696	6.05	42.649	85.299	55.101	9.514
4.60	36.459	72.919	18.018	12.556	6.10	42.817	85.634	57.077	9.435
4.65	36.724	73.448	18.787	12.419	6.15	42.984	85.968	59.114	9.358
4.70	36.984	73.969	19.583	12.284	6.20	43.148	86.296	61.210	9.282
4.75	37.241	74.483	20.409	12.153	6.25	43.309	86.618	63.370	9.207
4.80	37.493	74.986	21.263	12.025	6.30	43.469	86.938	65.589	9.133
4.85	37.741	75.483	22.151	11.899	6.35	43.625	87.251	67.877	9.061
4.90	37.985	75.970	23.067	11.776	6.40	43.780	87.561	70.228	8.989
4.95	38.225	76.451	24.018	11.655	6.45	43.934	87.868	72.647	8.919
5.00	38.460	76.921	25.000	11.537	6.50	44.084	88.169	75.134	8.850
5.05	38.691	77.383	26.018	11.421	6.55	44.233	88.466	77.695	8.782
5.10	38.920	77.841	27.069	11.308	6.60	44.379	88.759	80.323	8.715
5.15	39.146	78.293	28.159	11.197	6.65	44.525	88.051	83.027	8.649
5.20	39.367	78.735	29.283	11.088	6.70	44.668	89.336	85.804	8.584
5.25	39.585	79.170	30.446	10.981	6.75	44.809	89.618	88.661	8.520
5.30	39.799	79.599	31.649	10.876	6.80	44.947	89.895	91.594	8.457
5.35	40.008	80.017	32.893	10.773	6.85	45.085	90.170	94.609	8.394
5.40	40.216	80.433	34.174	10.672	6.90	45.221	90.442	97.700	8.333
5.45	40.422	80.844	35.501	10.573	6.95	45.355	90.710	100.880	8.273

TABLE II. - OVER-ALL NOZZLE DESIGN PARAMETERS - Concluded
 $[\gamma = 1,400]$

1	2	3	4	5	1	2	3	4	5
M_f	ϕ and ψ_f^+ (deg)	ψ_f, ψ (deg)	A_f/A_t	β_f, β (deg)	M_f	ϕ and ψ_f^+ (deg)	ψ_f, ψ (deg)	A_f/A_t	β_f, β (deg)
7.00	45.487	90.974	104.143	8.213	8.50	48.786	97.573	251.086	6.756
7.05	45.618	91.237	107.492	8.155	8.55	48.878	97.757	257.974	6.717
7.10	45.746	91.492	110.931	8.097	8.60	48.969	97.938	265.014	6.677
7.15	45.873	91.746	114.459	8.040	8.65	49.059	98.118	272.211	6.639
7.20	45.999	91.999	118.080	7.984	8.70	49.147	98.294	279.567	6.600
7.25	46.122	92.244	121.794	7.928	8.75	49.234	98.469	287.084	6.562
7.30	46.245	92.491	125.605	7.873	8.80	49.321	98.643	294.766	6.525
7.35	46.365	92.731	129.513	7.820	8.85	49.407	98.814	302.615	6.488
7.40	46.485	92.971	133.520	7.766	8.90	49.491	98.983	310.633	6.451
7.45	46.603	93.206	137.629	7.714	8.95	49.576	99.153	318.823	6.415
7.50	46.720	93.441	141.842	7.662	9.00	49.660	99.320	327.190	6.379
7.55	46.835	93.671	146.159	7.611	9.05	49.741	99.483	335.733	6.344
7.60	46.949	93.898	150.585	7.561	9.10	49.823	99.647	344.458	6.309
7.65	47.061	94.122	155.120	7.511	9.15	49.904	99.808	353.368	6.274
7.70	47.172	94.345	159.770	7.462	9.20	49.983	99.967	362.463	6.240
7.75	47.283	94.567	164.527	7.414	9.25	50.063	100.127	371.749	6.206
7.80	47.391	94.738	169.403	7.366	9.30	50.141	100.282	381.228	6.173
7.85	47.499	94.998	174.418	7.319	9.35	50.219	100.438	390.902	6.140
7.90	47.604	95.209	179.511	7.272	9.40	50.295	100.591	400.775	6.107
7.95	47.708	95.417	184.744	7.226	9.45	50.371	100.742	410.851	6.074
8.00	47.813	95.627	190.109	7.181	9.50	50.445	100.891	421.131	6.042
8.05	47.916	95.832	195.597	7.136	9.55	50.520	101.041	431.620	6.011
8.10	48.016	96.033	201.215	7.092	9.60	50.594	101.188	442.322	5.979
8.15	48.117	96.234	206.964	7.048	9.65	50.667	101.334	453.236	5.948
8.20	48.215	96.431	212.846	7.005	9.70	50.733	101.476	464.370	5.917
8.25	48.312	96.625	218.865	6.962	9.75	50.811	101.623	475.725	5.887
8.30	48.410	96.821	225.022	6.920	9.80	50.882	101.764	487.304	5.857
8.35	48.506	97.013	231.320	6.878	9.85	50.951	101.903	499.112	5.827
8.40	48.599	97.199	237.763	6.837	9.90	51.021	102.042	511.152	5.797
8.45	48.694	97.388	244.350	6.796	9.95	51.090	102.180	523.425	5.768
					10.00	51.158	102.317	535.938	5.739

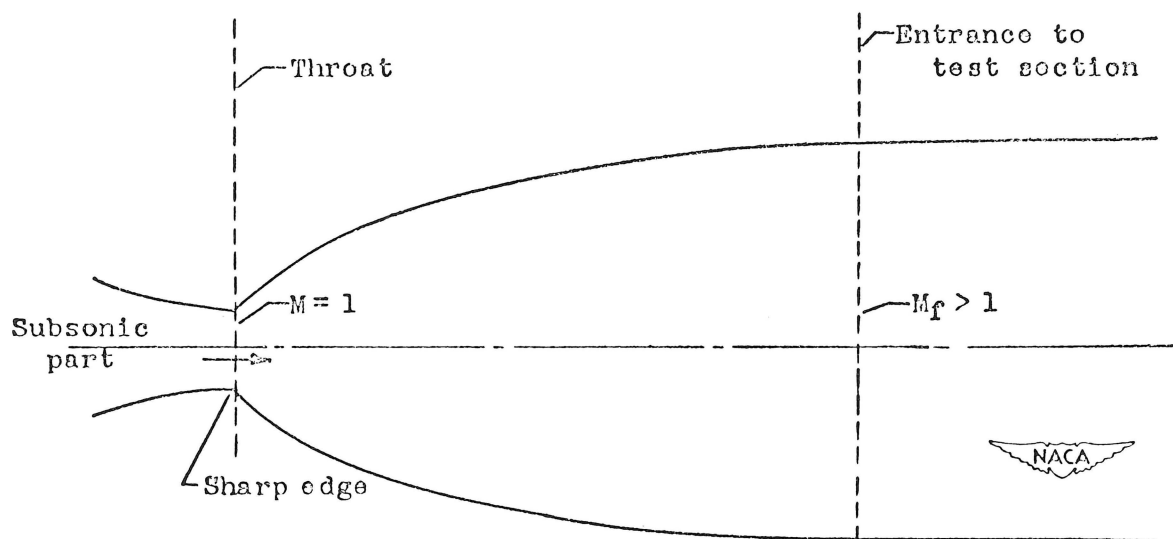
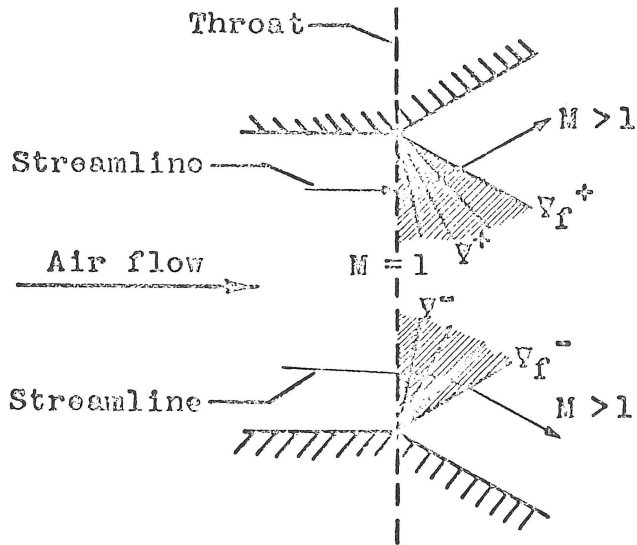
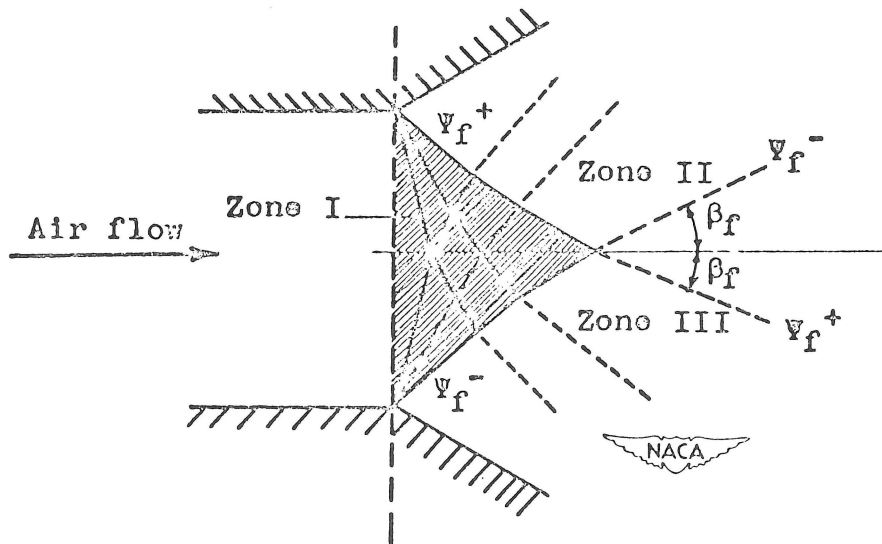


Figure 1. - Sharp-edge-throat supersonic nozzle of minimum length.

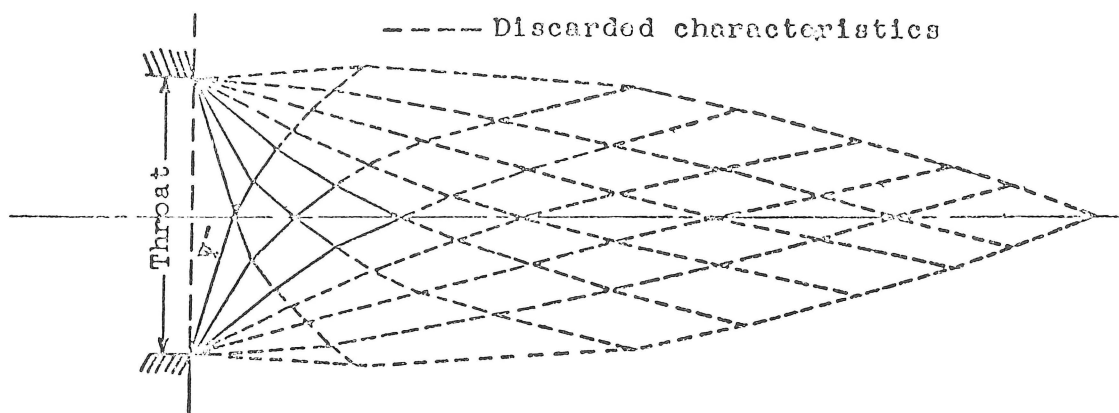


(a) Expansion waves represented by a finite number of characteristics.

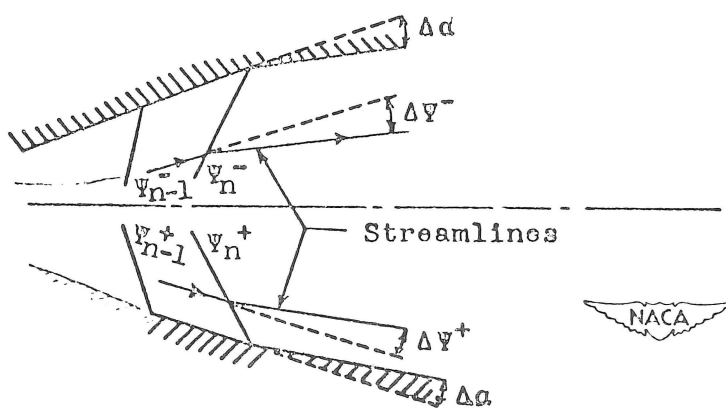


(b) Wave pattern formed by interaction of two expansion waves.

Figure 2. - Schematic representation of expansion waves by characteristics.

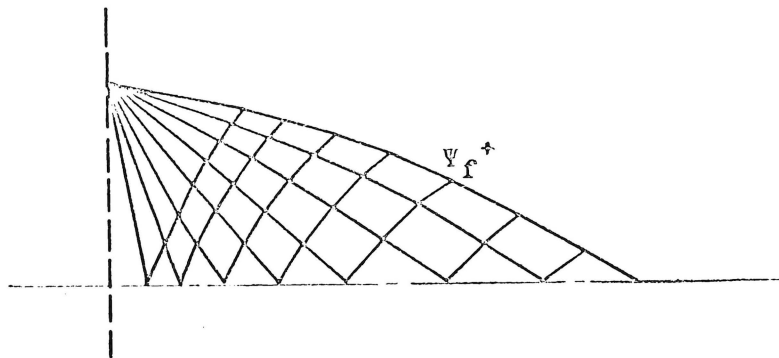


(c) Kernel formed from kernel corresponding to higher Mach number.

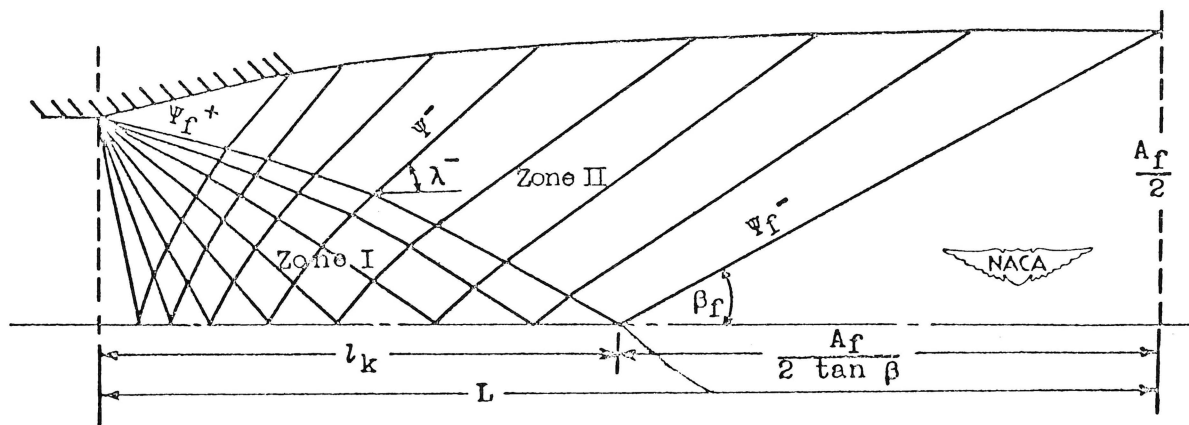


(d) Suppression of expansion wave by bending wall.

Figure 2. - Concluded. Schematic representation of expansion waves by characteristics.

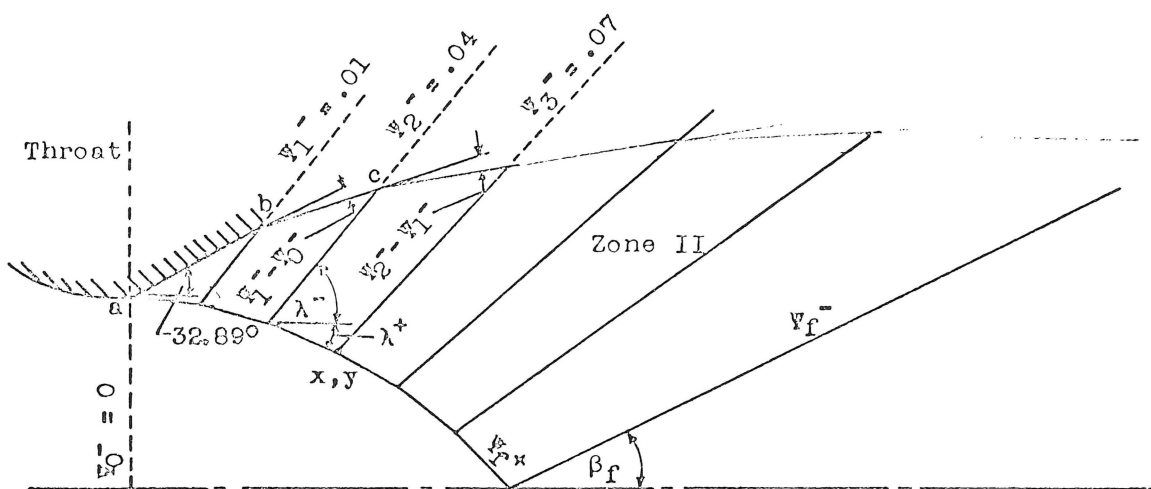


(a) Kernel.

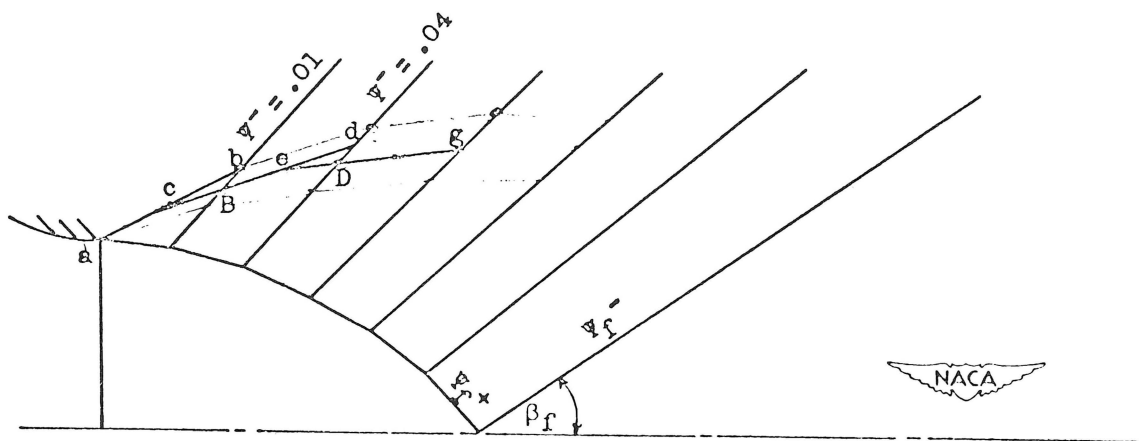


(b) Wave pattern and wall contour.

Figure 3. - Complete wave pattern and wall contour of graphically designed nozzle with sharp-edge throat.



(a) Conventional development.



(b) Averaging development.

Figure 4. - Development of wall contour from bounding Ψ_f^+ characteristic of kernel.

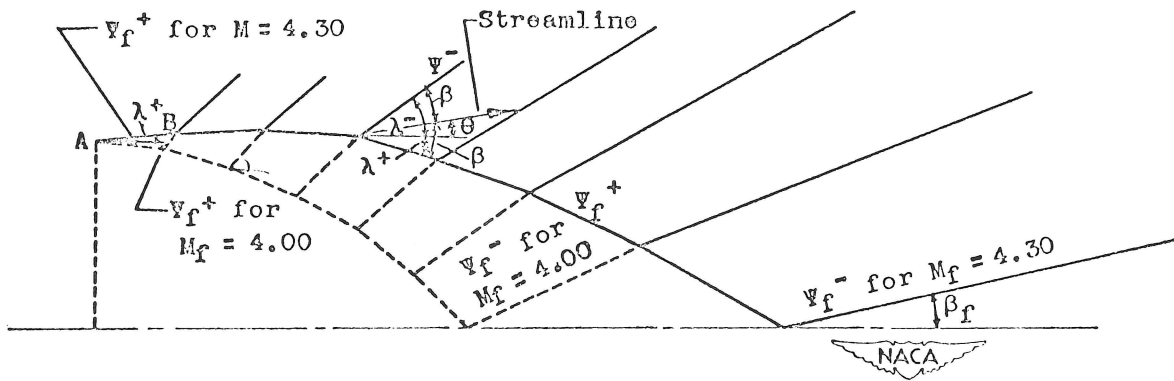


Figure 5. - Method of determining bounding characteristic Ψ_f^+ for a desired Mach number from a known adjacent characteristic.

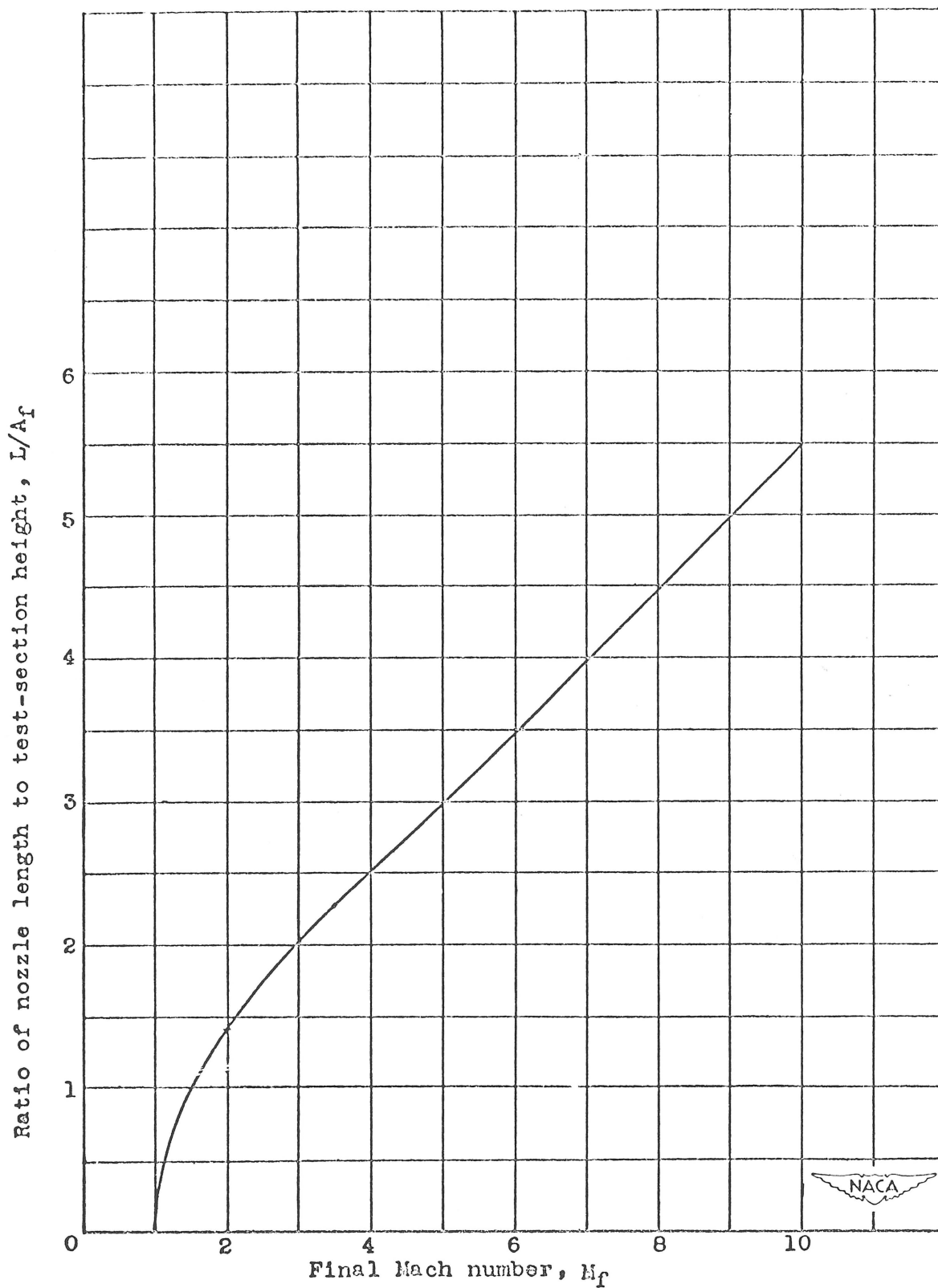


Figure 6. - Length of sharp-edge-throat nozzles.

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Local Identifier PondLove1967

Capture information

Date captured 2018 May

Scanner manufacturer Ricoh
Scanner model MP C4503
Scanning software
Optical resolution 600 dpi
Color settings Grayscale, 8 bit;
File types Tiff

Source information

Format Book
Content type Text
Notes Digitized duplicate copy not retained in collection.

Derivatives - Access copy

Compression LZW
Editing software Adobe Photoshop
Resolution 600 dpi
Color Grayscale, 8 bit; Color, 24 bit
File types Tiffs converted to pdf
Notes Greyscale pages cropped and canvassed. Noise removed from
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