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USE OF THE CENTRIFUGAL GOVERNOR MECHANISM AS A TORSIONAL VIBRATION ABSORBER

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Use of the Centrifugal Governor Mechanism as a Torsional Vibration Absorber

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The centrifugal governor mechanism, suitably modified, is shown to be a practical torsional vibration absorber. Equations for predicting its performance are developed, and comparisons are made with other types of absorbers.

Nomenclature

The following nomenclature is used in the paper:

m = mass of absorber, lb sec²/in.
G = center of gravity of absorber mass
R = radius of G from axis of rotation, in.
Ω = angular velocity of disk, radians per sec
θ = angular displacement, radians
θ_o = amplitude of vibration of disk, radians
ω = circular frequency of vibration, radians per sec
t = time, sec
T = torque, in-lb
F = force, lb
a = acceleration, in/sec²
v = velocity, in/sec
r = displacement of mass from equilibrium position, in.
r_o = amplitude of vibration of absorber mass, in.
I = moment of inertia of disk, lb-in. sec²
K_t = torsional spring constant of shaft, in-lb/radian
K = spring constant of absorber system when rotating, lb per in.
k = spring constant of absorber spring, lb per in.
ω_n = circular natural frequency of absorber system, radians per sec
Ω_n = circular natural frequency of main system, radians per sec
N = order number of vibration, cycles per shaft revolution
E_r = theoretical relative effectiveness of two absorbers

Introduction and Description of Absorber Mechanism

If a body is vibrating, one possible way to reduce or eliminate the vibration is by the addition of a dynamic vibration absorber. For torsional vibration, the most common of these devices are the rotating pendulum absorber and the Frahm absorber.[2] It is the purpose of this paper to describe another type of absorber which, although differing in construction from those mentioned, has quite similar characteristics. The mechanism of this absorber is basically that of a centrifugal governor, consisting of a mass elastically mounted on a rotating body and constrained to move in a radial direction when acted upon by centrifugal force.

Fig. 1 shows an idealized diagram of the absorber mounted on a rotating disk. The mass m in the frictionless slot and the spring connecting the mass to the disk make up the absorber. Point G at radius R is the center of gravity of the mass. The disk rotates with a uniform angular velocity Ω, upon which is superimposed an additional velocity $\theta_o\omega \cos \omega t$ due to resonant torsional vibration of the disk. This vibration is excited by a periodic torque $T_o \cos \omega t$ acting on the disk. The radial force acting on the mass is

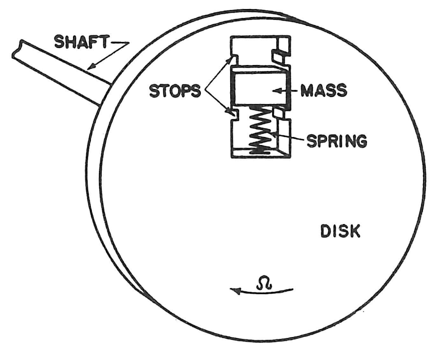

Fig. 1 Vibrating System With Absorber

$$\begin{aligned} F &= mR\,(\Omega + \theta_o\omega \cos \omega t)^2 \\ &= m\,R\,(\Omega^2 + 2\Omega\theta_o\omega \cos \omega t + \theta_o{}^2\omega^2 \cos^2 \omega t) \\ &= (m\,R\,\Omega^2 + {}^1/_2 mR\theta_o{}^2\omega^2) + (2m\,R\,\Omega\,\theta_o\omega \cos \omega t) \\ &\qquad + ({}^1/_2\, mR\theta_o{}^2\,\omega^2 \cos 2\omega t) \qquad [1] \end{aligned}$$

The total radial force is thus the sum of a constant force, a periodic force of frequency ω, and a periodic force of frequency 2ω. The constant force will determine the extension of the spring and hence the radius R. The two periodic forces will excite vibrations of the mass at their respective frequencies. However, the force of frequency 2ω is very small in comparison with the other, a fact which can be verified by substitution of typical values. Furthermore, it will appear later that the natural frequency of the absorber must be equal to ω; therefore the effect of the force of frequency ω will be magnified due to resonance. For these reasons, the exciting force on the mass is taken as

$$\begin{aligned} F' &= 2m\,R\,\Omega\,\theta_o\omega \cos \omega t \\ &= 2m\,R\,\Omega \frac{d\theta}{dt} \qquad [2] \end{aligned}$$

The vibration of the disk thus induces vibration of the absorber mass. The effect of this motion on the disk is shown by writing Coriolis' law between point G and a coincident point G' on the disk

$$a_G = a_{G'} \mathrel{+\!\!\!\rightarrow} a_{GG'} \mathrel{+\!\!\!\rightarrow} 2v_{GG'}\,\Omega \qquad [3]$$

$$= R\,\Omega^2 \mathrel{+\!\!\!\rightarrow} R\frac{d\Omega}{dt} \mathrel{+\!\!\!\rightarrow} \frac{d^2r}{dt^2} \mathrel{+\!\!\!\rightarrow} 2\frac{dr}{dt}\,\Omega \qquad [4]$$

The last two terms of Equation [4] are the result of the motion of the mass. The last term, by virtue of its tangential direction, results in an inertia torque transmitted to the disk by pressure between the slot and mass equal to

$$T_i = 2m\,R\frac{dr}{dt}\,\Omega \qquad [5]$$

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[2] "Mechanical Vibrations," by J. P. Den Hartog, McGraw-Hill Book Company, Inc., New York, N. Y., third edition, 1947, pp. 112–119.

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In Equation [5] the average angular velocity Ω is used instead of the instantaneous velocity $\Omega + \theta_o \omega \cos \omega t$. The inclusion of the latter term results in an expression similar to the last term of Equation [1] and is not important in the present discussion.

Operation of Absorber

The action of the absorber is shown diagrammatically in Fig. 2. In Fig. 2 (a), the instantaneous magnitude of a quantity is the projection of its vector upon the Y-axis. At resonance, T_o will be in phase with the velocity of $\theta_o\omega$, and will lead the amplitude θ_o by 90 deg. Let the natural frequency of the absorber system equal ω. Then the absorber amplitude r_o lags 90 deg behind its exciting force F', which is due to and in phase with the velocity $\theta_o\omega$. The absorber radial velocity $r_o\omega = (dr/dt)$ is then in phase with T_o. The inertia torque T_i is of opposite sign to $r_o\omega$, and therefore joins with the damping torque T_d in opposing T_o. This is shown more clearly in Fig. 2(b), which omits the displacements and velocities.

Equations of Motion

Applying Newton's law to the disk and neglecting damping

$$\Sigma T = I\alpha$$

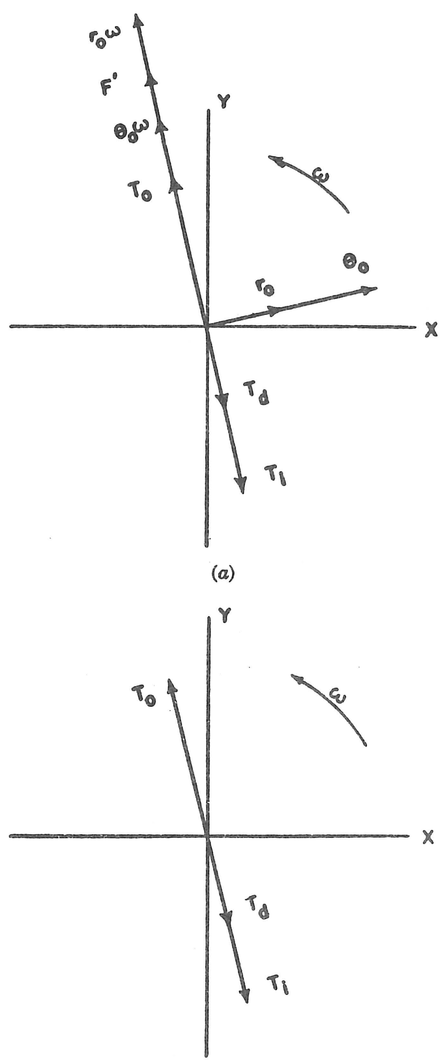

Fig. 2 Vector Diagram of Absorber Action

$$T_o \sin \omega t - 2m R \Omega \frac{dr}{dt} - K_t\theta = I \frac{d^2\theta}{dt^2} \quad \ldots\ldots\ldots [6]$$

The corresponding equation for the absorber is

$$\Sigma F = ma_G$$

$$2m R \Omega \frac{d\theta}{dt} - Kr = m \frac{d^2r}{dt^2} \quad \ldots\ldots\ldots [7]$$

Inspection of these equations shows that the following will be a solution

$$\theta = \theta_o \sin \omega t$$

$$r = r_o \cos \omega t$$

Also

$$\sqrt{\frac{K}{m}} = \omega_n$$

$$\sqrt{\frac{K_t}{I}} = \Omega_n$$

Making the foregoing substitutions results in

$$\theta_o = \frac{\dfrac{T_o}{K_t}\left(1 - \dfrac{\omega^2}{\omega_n^2}\right)}{\left(1 - \dfrac{\omega^2}{\Omega_n^2}\right)\left(1 - \dfrac{\omega^2}{\omega_n^2}\right) - \dfrac{(2m R \Omega\omega)^2}{KK_t}} \quad \ldots\ldots [8]$$

$$r_o = \frac{\dfrac{T_o}{K_t}\left(\dfrac{2m R \Omega\omega}{K}\right)}{\left(1 - \dfrac{\omega^2}{\Omega_n^2}\right)\left(1 - \dfrac{\omega^2}{\omega_n^2}\right) - \dfrac{(2m R \Omega\omega)^2}{KK_t}} \quad \ldots\ldots [9]$$

Equation [8] shows that θ_o becomes zero if the natural frequency of the absorber equals the applied frequency.

Owing to the effect of centrifugal force, the absorber-system spring constant and natural frequency vary with the angular velocity as follows

$$K = k - m\Omega^2 \quad \ldots\ldots\ldots [10]$$

$$\omega_n = \sqrt{\frac{k}{m} - \Omega^2} \quad \ldots\ldots\ldots [11]$$

The action of the absorber is due to the fact that it adds another degree of freedom to the original system. An additional natural frequency is added, making a total of two for the case under discussion, neither of which corresponds to the original natural frequency. These new frequencies are found by setting the denominator of Equation [9] equal to zero. A convenient form of the resulting equation, in which the order of vibration N has been introduced and R is the original desired radius, is

$$\left(1 - \frac{\omega^2}{\Omega_n^2}\right)\left[1 - \omega^2\left(\frac{k}{m} - \frac{\omega^2}{N^2}\right)^{-1}\right] - \left(\frac{4m R^2 \omega^4}{IN^2\Omega_n^2}\right) \times \left(1 - \frac{m\Omega_n^2}{k}\right)^2\left(1 - \frac{m\omega^2}{kN^2}\right)^{-2}\left(\frac{k}{m} - \frac{\omega^2}{N^2}\right)^{-1} \equiv 0 \quad \ldots [12]$$

Solving Equation [12] for ω gives the two natural frequencies.

In a practical design, the undesirable effect of the two new natural frequencies is eliminated by means of stops such as shown in Fig. 1. Initial spring force holds the mass against the inner stop until the speed at which the absorber is to operate is approached, at which time it is pulled away from the stop by centrifugal force. At some higher speed the mass is forced against the outer stop. Thus the absorber does not operate at speeds which might allow resonance at the two natural frequencies.

Design of Absorber

An equation useful in proportioning the absorber is obtained from Equation [9] when $\omega = \omega_n$

$$2m\,R\,\Omega r_o \omega_n = T_o \qquad [13]$$

The angular velocity Ω, the natural frequency ω_n, and the torque T_o will be known The mass m and radius R must be chosen so that the amplitude r_o will be reasonable. The stops will be located to eliminate the natural frequencies of Equation [12]. The spring constant is found from Equation [11]. The initial force in the spring must balance the centrifugal force on the mass at the speed at which the absorber is to begin operating.

The details of construction will vary with the application, but a possible design is shown in Fig. 3. The mass is mounted on a cantilever spring, which transmits the absorber torque to the vibrating body. An alternative construction would have the mass pivot on antifriction bearings. Adjustable stops are above and below the mass. The spring is loaded in compression, the initial load being adjustable. The distance between the center of gravity of the mass and the line of action of the spring force is also adjustable, allowing the spring constant to be varied slightly. The type of mounting depends on the shape of the vibrating body. For greater effectiveness, two or more absorbers may be mounted radially about the center of rotation.

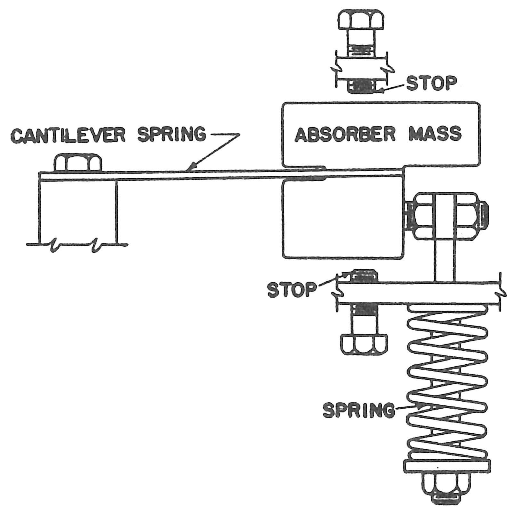

Fig. 3 A Possible Absorber Construction

Comparisons

Fig. 3 emphasizes the fact that, if desired, the absorber may be constructed so that its characteristics are somewhat variable. This would facilitate exact tuning to correct for manufacturing tolerances, approximations in calculations, or wear. Other types of dynamic absorbers, particularly the rotating-pendulum absorber, are often difficult to tune exactly.[3]

An important comparison may be made by writing the equation for either the rotating-pendulum absorber or the Frahm absorber which corresponds to Equation [13]. This equation is

$$m\,R\,r_o\,\omega_n^2 = T_o' \qquad [14]$$

Equation [14] states that a rotating-pendulum absorber of a certain mass, radius, amplitude, and frequency will balance a certain exciting torque T_o'. Equation [13] states that the absorber under discussion of the same mass, radius, amplitude, and frequency will balance a different exciting torque T_o. For the sake of comparison, let the theoretical relative effectiveness E_r be the ratio of T_o to T_o'. Then

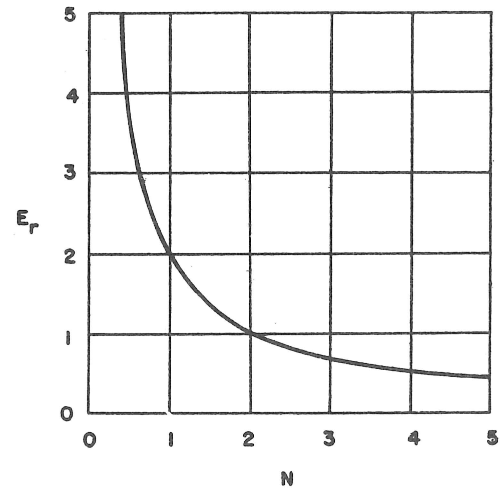

Fig. 4 Absorber Compared With Rotating-Pendulum Absorber

$$E_r = \frac{2m\,R\,\Omega\,r_o\,\omega_n}{m\,R\,r_o\omega_n^2} = \frac{2\Omega}{\omega_n} = \frac{2}{N} \qquad [15]$$

This result is visualized in Fig. 4. For second-order vibration both absorbers are equally effective; in other words, for a given application, the absorbers would be about the same size. For higher orders, the rotating-pendulum absorber would be preferred on the basis of Equation [15]. On the other hand, for orders of vibration lower than the second, the absorber under discussion would be the most effective.

Discussion

R. J. Harker.[4] This paper on the use of a radially oscillating mass to suppress torsional vibrations in rotating systems is novel and appears to present some interesting possibilities, particularly with respect to low orders of excitation. The author is to be congratulated for his ingenious proposal and for his contribution to the theory of the tuned dynamic vibration absorber.

Equations [8] and [9] of the paper may be simplified by defining the main system as a single mass at a radius of gyration equal to the radius R, or $I = MR^2$, and by using the following notation

$$f = (\omega_n/\Omega_n) = \text{absorber tuning factor}$$

$$g = (\omega/\Omega_n) = \text{forced-frequency ratio}$$

$$\mu = (m/M) = \text{mass ratio}$$

Then Equations [8] and [9] become

$$\theta_0 = \frac{(T_o/K_t)\,[1-g^2]}{[1-g^2]\left[1-\left(\frac{g}{f}\right)^2\right] - 4\,\mu\left(\frac{g}{f}\right)^2\left(\frac{g^2}{N^2+1}\right)}$$

$$r_0 = \frac{\left(\frac{T_0R}{K_t}\right)\left[\frac{2}{N}\left(\frac{g}{f}\right)^2\right]}{[1-g^2]\left[1-\left(\frac{g}{f}\right)^2\right] - 4\,\mu\left(\frac{g}{f}\right)^2\left(\frac{g^2}{N^2+1}\right)}$$

[3] "Practical Solution of Torsional Vibration Problems," by W. K. Wilson, John Wiley & Sons, Inc., New York, N. Y., second edition, vol. 2, 1941, p. 570.

[4] Associate Professor of Mechanical Engineering, University of Wisconsin, Madison, Wis. Mem. ASME.

If the tuning of the absorber is correct, $f = 1$, and these equations reduce to

$$\theta_o = \frac{\left(\frac{T_0}{K_T}\right)[1 - g^2]}{[1 - g^2]^2 - \frac{4\,\mu g^4}{N^2 + 1}}$$

$$r_o = \frac{\left(\frac{(T_0R)}{K_T}\right)\left[\frac{2g^2}{N}\right]}{[1 - g^2]^2 - \frac{4\,\mu g^4}{N^2 + 1}}$$

The resulting frequency equation for the combined system with exact absorber tuning is then

$$g^4\left[1 - \frac{4\mu}{N^2 + 1}\right] - 2g^2 + 1 = 0$$

with the solution

$$g^2 = \frac{1 \pm \sqrt{\frac{4\,\mu}{N^2 + 1}}}{1 - \frac{4\,\mu}{N^2 + 1}}$$

The exciting frequencies corresponding to infinite amplitudes of the system are seen to be a function of the mass ratio and the order of excitation. It is apparent that for high mass ratios and low orders of excitation the factor $\left(\frac{4\,\mu}{N^2 + 1}\right)$ may be greater than unity. Since g^2 can only be positive, this condition apparently would result in a single resonant frequency. In general, however, there would be two resonant frequencies as indicated.

Author's Closure

The author wishes to thank Professor Harker for his comments. The simplification of Equations [8] and [9] and subsequent derivation of an expression for the resonant frequencies added by the absorber is worth while in view of the fact that Equation [12] given in the paper for this purpose is rather tedious to solve.

To avoid confusion in the use of these equations it is necessary to realize that they apply only if the original main system has but a single degree of freedom. Also, the manner in which the exciting torque acts on the system must be defined. If the excitation is of a definite order, its frequency varies with angular velocity. The natural frequency of the absorber system and radius of the absorber mass, which vary with angular velocity, are then functions of exciting frequency. Equation [12] applies to such a case. On the other hand, the excitation may not be of a definite order, but may vary in frequency at constant angular velocity. Then the natural frequency of the absorber system and radius of the absorber mass remain unchanged. In this instance the absorber tuning factor f will equal unity at all times as assumed in Professor Harker's derivation. Inspection of the equations indicates that there is not too much difference in resonant-frequency values obtained by proceeding upon either assumption, and values obtained in either manner will be approximately correct for the other case.

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