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ANALYSIS OF SINGLE-PHASE-TO-THREE-PHASE STATIC PHASE CONVERTERS

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Analysis of Single-Phase-to-3-Phase Static Phase Converters

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MEMBER AIEE

A STATIC phase converter is a device which supplies polyphase power to a load from a single-phase power source using only passive circuit elements. Ideally such a device should produce balanced polyphase voltages at the load. Several recent articles[1,2,3] have been published concerning the operation of 3-phase induction motors connected through static phase converters to single-phase lines. These articles discuss the simple form of phase converter using only capacitors. This capacitor-only type of phase converter can be adjusted to give balanced 3-phase power only for loads possessing a lagging power factor of 0.5. For any other load it is impossible to obtain balanced conditions. In contrast, the Add-A-Phase phase converter, consisting of an autotransformer in addition to the capacitors, may be set to supply balanced 3-phase power to any load possessing a lagging power factor regardless of its value. This more general type of phase converter circuit will be referred to in this article as the autotransformer type of phase converter. At the point at which balanced operation occurs with an induction motor load, the speed, torque, horsepower, efficiency, and other motor quantities will be identical to those obtained when the motor is connected to a balanced 3-phase power source of the same voltage.

It is the purpose to present an analysis of the autotransformer phase converter, treating the capacitor-only phase converter as a special case of the more general analysis. The analysis indicates that many of the limitations placed on the application of the capacitor-only phase converter may be relaxed when the autotransformer phase converter is used.

Analysis

The general form of the circuit for phase converters is given in Fig. 1. The voltage at terminal A' may be varied according to the tap setting on the autotransformer. The capacitor X_c in series with phase A may also be varied to obtain balanced conditions. This circuit may be analyzed by the method of symmetrical components by considering the voltages at A', B', and C' as a set of unbalanced 3-phase voltages applied to a circuit with unequal line impedances going to a 3-phase motor. This set of voltages may be expressed in terms of the single-phase voltage applied to the phase converter and the tap setting of the autotransformer. Thus

$$V_{A'B}=NV_1 \tag{1}$$

$$V_{BC}=V_1 \tag{2}$$

$$V_{CA'}=-(1+N)V_1 \tag{3}$$

where

$$N=\frac{|V_{A'B}|}{|V_{BC}|} \tag{4}$$

The positive negative and zero-sequence component voltages are given by

$$V_{A'B_1}=\frac{1}{3}\,(V_{ab}+aV_{bc}+a^2V_{ca}) \tag{5}$$

$$V_{A'B_2}=\frac{1}{3}(V_{ab}+a^2V_{bc}+aV_{ca}) \tag{6}$$

$$V_{A'B_0}=\frac{1}{3}(V_{ab}+V_{bc}+V_{ca}) \tag{7}$$

Substituting equations 1 to 3 into equations 5 to 7 yields

$$V_{A'B_1}=\frac{V_1}{\sqrt{3}}(1\underline{/90^\circ}+N\underline{/30^\circ}) \tag{8}$$

$$V_{A'B_2}=\frac{V}{\sqrt{3}}(1\underline{/-90^\circ}+N\underline{/-30^\circ}) \tag{9}$$

$$V_{A'B_0}=0 \tag{10}$$

Since the sum of the three currents must also equal zero, the zero-sequence component of the current is also zero.

The equivalent circuit of the induction motor is usually expressed in terms of an equivalent wye circuit, with the applied voltage being a line-to-neutral voltage. Expressing the component voltages of equations 8 and 9 in terms of line-to-neutral voltages gives

$$V_{A'n_1}=\frac{V_{A'B_1}}{\sqrt{3}}\underline{/-30^\circ} \tag{11}$$

$$V_{A'n_1}=\frac{V_1}{3}[N\underline{/0^\circ}+1\underline{/60^\circ}] \tag{12}$$

$$V_{A'n_2}=\frac{V_{A'B_2}}{\sqrt{3}}\underline{/30^\circ} \tag{13}$$

$$V_{A'n^2}=\frac{V_1}{3}[N\underline{/0^\circ}+1\underline{/-60^\circ}] \tag{14}$$

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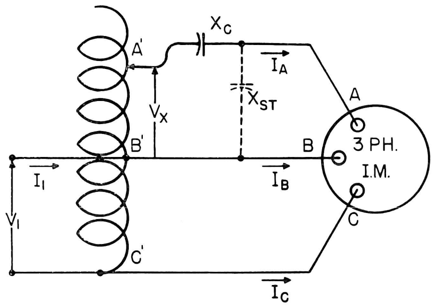

Fig. 1. Circuit of autotransformer type of phase converter

Next the unsymmetrical line impedances must be resolved into a form suitable for application of the sequence rule. The sequence components of these unbalanced line impedances are

$$Z_{A0}=\frac{1}{3}(Z_A+Z_B+Z_C) \tag{15}$$

$$Z_{A1}=\frac{1}{3}(Z_A+aZ_B+a^2Z_C) \tag{16}$$

$$Z_{A2}=\frac{1}{3}(Z_A+a^2Z_B+aZ_C) \tag{17}$$

In the phase-converter circuit

$$Z_B=Z_c=0 \tag{18}$$

Thus equations 15, 16, and 17 become

$$Z_{A0}=Z_{A1}=Z_{A2}=\frac{Z_A}{3} \tag{19}$$

The equations of the circuit may now be written in terms of the sequence components. Thus

$$V_{an1}=I_{a1}(Z_{A0}+Z_{m1})+I_{a2}Z_{A2} \tag{20}$$

$$V_{an2}=I_{a1}Z_{A1}+I_{a2}(Z_{A0}+Z_{m2}) \tag{21}$$

where

I_{a1}, I_{a2} = the positive- and negative-sequence component currents going to the motor respectively.

Z_{m1}, Z_{m2} = the positive- and negative-sequence impedances per phase of the induction machine respectively.

Substituting equations 12, 14, and 19 into equations 20 and 21 yields

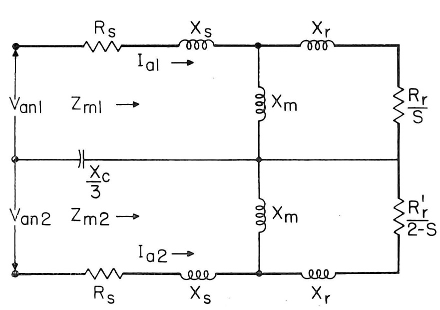

Fig. 2. Equivalent circuit of phase converter and induction motor load

$$\frac{V_1}{3}(N\angle 0^\circ+1\angle 60^\circ)=I_{a1}\left(\frac{Z_A}{3}+Z_{m1}\right)+I_{a2}\frac{Z_A}{3} \tag{22}$$

$$\frac{V_1}{3}(N\angle 0^\circ+1\angle -60^\circ)=I_{a1}\frac{Z_A}{3}+I_{a2}\left(\frac{Z_A}{3}+Z_{m2}\right) \tag{23}$$

Solving for the sequence currents gives

$$I_{a1}=\frac{V_1}{3}\left(\frac{Z_{m2}(N\angle 0^\circ+1\angle 60^\circ)+\frac{Z_A}{\sqrt{3}}\angle 90^\circ}{Z_{m1}Z_{m2}+\frac{Z_A}{3}(Z_{m1}+Z_{m2})}\right) \tag{24}$$

and

$$I_{2a}=\frac{V_1}{3}\left(\frac{Z_{m1}(N\angle 0^\circ+1\angle -60^\circ-\frac{Z_A}{\sqrt{3}}\angle 90^\circ}{Z_{m1}Z_{m2}+\frac{Z_A}{3}(Z_{m1}+Z_{m2})}\right) \tag{25}$$

The ideal operating condition for a 3-phase motor is to have balanced 3-phase voltages applied to the terminals of the motor which should result in balanced line currents. Thus, the negative-sequence current must equal zero. From equation 25 it is seen that this condition is satisfied by the relationship

$$Z_{m1}\angle\theta_{m1}(N\angle 0^\circ+1\angle -60^\circ)=\frac{Z_A\angle 90^\circ}{\sqrt{3}} \tag{26}$$

where θ_{m1} is the positive-sequence power-factor angle of the machine. Z_A may be any type of impedance but to minimize losses it is usually a capacitor for inductive loads. Thus, for an induction motor load

$$Z_A=-jX_c \tag{27}$$

Since both the autotransformer tap setting N and the capacitive reactance X_c may be varied, it is possible to satisfy equation (26) for any polyphase induction motor and thus insure balanced operation at that particular load. As the load requirements change, the motor will no longer have exactly balanced currents. It is, therefore, important to adjust initially the values of N and X_c so as to give balanced operation for the particular load condition most usually encountered. Solution of equation 26 gives

$$X_c=\frac{3}{2}\,\frac{|Z_{m1}|}{\text{Sin }\theta_{m1}} \tag{28}$$

and

$$N=\frac{\text{Cos }(\theta_{m1}+30^\circ)}{\text{Sin }\theta_{m1}} \tag{29}$$

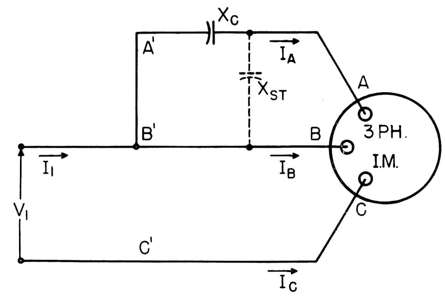

Fig. 3. Circuit of capacitor-only type of phase converter

Equivalent Circuit

Equations 22 and 23 are the equations of the equivelent circuit shown in Fig. 2. It is interesting to note that this circuit is identical in form to the equivalent circuit for a capacitor motor.[4] The circuit for the capacitor motor represents 2-phase symmetrical component currents and voltages while the circuit shown in Fig. 2 gives 3-phase quantities. This similarity in equivalent circuits indicates that the unbalance caused by negative-sequence currents in a phase converter is the same as the unbalance that occurs in capacitor-run motors caused by changing load conditions.

Capacitor-Only Phase Converter

The theory of the capacitor-only phase converters may be treated as a special case of the autotransformer phase converter. Adjusting the tap setting A' to point B' makes voltage $V_{A'B'}$ equal to zero or for this case

$$N=0 \tag{30}$$

The autotransformer has no effect on the operation of the phase converter and the circuit is the same as that used in the capacitor-only phase converter as given in Fig. 3. The negative-sequence current must again be zero to obtain balanced conditions, as specified by equation 26. For the capacitor-only phase converter, this becomes

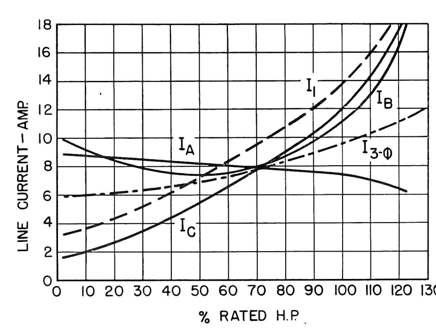

Fig. 4. Current variation caused by load changes for phase converter balanced at 70-per-cent load

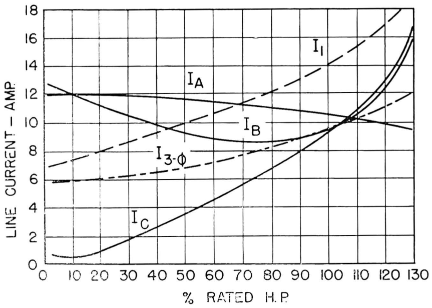

Fig. 5. Current variation caused by load changes for phase converter balanced at 110-per-cent load

$$Z_m \underline{/\theta_{m1} - 60^\circ} = \frac{X_c}{\sqrt{3}} \underline{/0^\circ} \tag{31}$$

This equation can only be satisfied if

$$\theta_m = 60^\circ \tag{32}$$

and

$$X_c = \sqrt{3}|Z_{m1}| \tag{33}$$

From equation 33 it is seen that only the special case of power factor equal to 0.5 can be balanced. This, however, usually occurs only when the machine is very lightly loaded. Thus it may be concluded that the autotransformer is very necessary in phase converters to obtain the desired balanced operation for a motor operating under load conditions.

The same type of analysis discloses that a capacitor-run motor with identical windings in quadrature can only give balanced currents if the motor power factor is 0.707. By making the number of turns on the auxiliary winding different than the number of turns on the main winding, it is possible to obtain conditions for loads with power factors other than 0.707. This change in number of turns for the capacitor-run motor is equivalent to the change in voltage $V_{A'B}$ possible with the autotransformer phase converter and accomplishes the same purpose.

Operating Characteristics

To illustrate the operating characteristics of the autotransformer type of phase converter, experimental test results are given. A 3-phase 220-volt 60-cycle-per-second induction motor was used with a rating of 3 horsepower at 1,165 rpm. Load runs were made with the motor connected directly to a 3-phase line and with the motor connected to a single-phase line through the phase converter. Separate runs were made for the phase converter adjusted to give balanced conditions. Results will be given for the two runs taken when the phase converter was set to give a balance at 70 and 110 per cent of rated output. Since it is desired to have balanced line currents to the motor, an examination of the variation of line current as a function of horsepower output is shown in Figs. 4 and 5. It is seen that the line currents can be balanced at particular values of load, although there is considerable variation in current unbalance as the load increases or decreases away from the point at which balance occurs. Curves given by Habermann[1] show that the currents may never become balanced with a capacitor-only phase converter.

The torque variation as a function of speed is of course the most important factor in matching a motor to a given load. The motor operated through the phase converter has a speed-torque characteristic very similar to the motor operated directly on 3-phase up to loads of about 120 per cent of rated horsepower, as shown in Fig. 6. For loads above this value the speed drops very rapidly and the breakdown torque is much lower than for 3-phase case. For the run connection the phase converter was set to give balanced conditions at 110 per cent of rated load. For certain types of load occasionally requiring excessive torques, it may be necessary to use the next larger size motor.

Voltage variation caused by line drop is a serious problem on rural single-phase lines. Since the voltage drop is proportional to the line current, it is desirable to keep the line current for a given power at a minimum. This is the same as saying that the power factor should be as close to unity as possible. Fig. 7 points out the excellent single-phase power-factor characteristics of the autotransformer phase converter compared to the power factor of the 3-phase motor.

The over-all efficiency on single-phase using the phase converter is almost the same as the motor efficiency on balanced 3-phase; the only losses present in the phase converter being the core and copper losses of the autotransformer. However, additional losses occur in the motor for unbalanced operation because of negative-sequence losses. This will result in over-all efficiencies approximately equal to the efficiency on 3-phase at the points of balance. As the unbalance increases, the over-all efficiency will decrease because of the additional motor losses, as shown in Fig. 8.

To provide the required starting torque, it is necessary to have an additional capacitor X_{ST} in the circuit for starting. This capacitor tends to balance the currents at starting and, after the motor comes up to speed, the capacitor X_{ST} is disconnected by means of a relay circuit

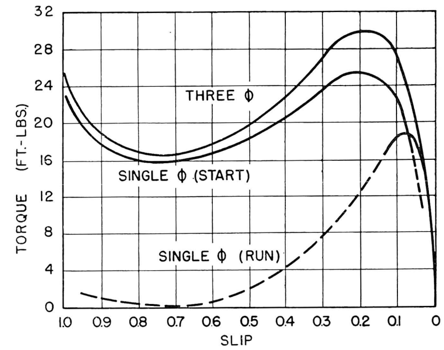

Fig. 6. Speed-torque curves for balanced 3-phase and phase-converter operation

sensitive to voltage V_{AB}. The characteristics during starting and build-up are given in Fig. 6. A tabulation of locked-rotor quantities is given in Table I. Characteristics of 3-horsepower 6-pole single-phase capacitor-start–capacitor-run motors are also given for comparative purposes. It is seen that the autotransformer phase-converter motor combination has the lowest (most desirable) value of kilovolt-amperes per foot-pound at starting. It should be pointed out that the autotransformer is just as essential to the circuit at starting as it is during running conditions. Recent tests[2] on capacitor-only phase converters indicated starting kilovolt-amperes per foot-pound of 0.86, compared to 0.57 for the autotransformer phase converter adjusted to balance at 70 per cent of rated load. If Table I had compared 4-pole motors instead of 6-pole motors, the per-cent torques throughout the whole table would be greater than those given.

Conclusions

From this analysis of the autotransformer phase converter, the following advantages may be cited:

1. **It is the only static phase converter available whose circuit is capable of providing balanced 3-phase currents to a 3-phase induction motor from a single-phase line.**

2. **The starting kilovolt-amperes required are greatly reduced below that of the 3-**

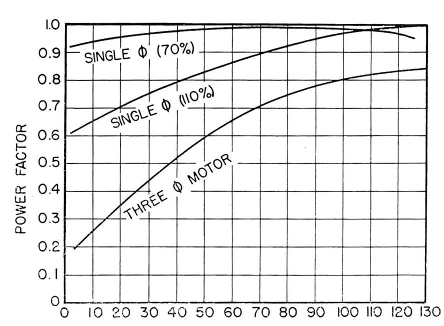

Fig. 7. Power-factor variation caused by load changes

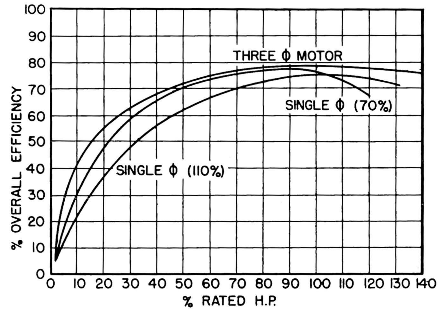

Fig. 8. Efficiency variation caused by load changes

Table I. Comparison of Operating Characteristics of 3-Horsepower Induction Motors

	3-Phase Motor on 3 Phases	3-Phase Motor with Add-A-Phase: 70-Per-Cent Balance	3-Phase Motor with Add-A-Phase: 110-Per-Cent Balance	Single-Phase Motor†
Line volts	220	220	220	230
Breakdown torque, per cent of rated torque*	250	138	145	200
Starting current, amperes	48	53	58	70
Starting torque, per cent of rated torque*	210	155	195	175
Starting kilovolt-amperes	18.3	11.7	12.8	16.1
Starting kilovolt-amperes per foot-pound	0.645	0.57	0.49	0.68

*Rated torque = 13.5 foot-pounds

†Capacitor-start–capacitor-run

phase motor on balanced 3-phase power or that of a single-phase motor.

3. Power factor is improved throughout the complete operating range of the motor.

4. The starting torque is comparable to that of a single-phase motor but only about 85 per cent as much as the 3-phase motor.

5. The starting kilovolt-amperes per foot-pound are smaller than those of the 3-phase or single-phase motor.

6. Cost of autotransformer phase converter plus 3-phase induction motor is competitive with the cost of a single-phase motor.

7. Simplicity of 3-phase motor reduces maintenance required.

8. Where 3-phase power may eventually become available, it permits immediate use of 3-phase motors.

9. Phase-converter units of a given horsepower rating may be used with motors smaller than that rating.

The advantages of the phase converter are as follows:

1. Breakdown torque is less than that of the 3-phase induction motor on balanced 3-phase power or that of a single-phase motor.

2. The starting torque is about 85 per cent of the starting torque of the 3-phase motor and is about the same as the starting torque for a single-phase induction motor.

It should be mentioned that 3-phase motors will increase in usage in rural areas because of the 3-phase power source that will soon be supplied on farm tractors. The 3-phase motor with a phase converter can thus be used on the single-phase lines serving the farm and, in case of emergency, the 3-phase motor can be connected directly to the portable 3-phase power source.

In conclusion, the autotransformer phase converter—3-phase induction motor combination—can be used as a source of power where only single phase is available without the usual excessive voltage drop during starting. For loads where the power required is almost constant, the unit can be adjusted to give balanced operation at a high power factor for that particular load without any derating of the motor. Where danger of explosion exists, the phase converter may be placed in a position remote from the 3-phase motor. Power companies might also permit motors to be used with higher ratings than the usual 5 to 7.5 horsepower when such motors are used in conjunction with a phase converter because of the low kilovolt-amperes taken during starting.

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